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A Variational Approach to Hadron Structure in Lattice QCD

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Abstract

In order to understand how hadrons acquire their physical properties from their constituents, we must resort to the underlying theory of the strong interaction, Quantum Chromodynamics (QCD). However, the non-perturbative nature of this theory in the relevant energy scales renders the standard perturbative methods ineffective. The formulation of QCD on a discrete space-time lattice allows for a first principles, non-perturbative approach to studying the strong interaction, in a manner well suited to numerical computation.

Over the past decade, the use of variational techniques has provided an effective framework for spectroscopic studies of the full hadron spectrum. Herein we generalise the use of the variational approach to hadron form factor calculations and examine its use in a number of different hadronic systems. As such an approach allows for the isolation of terms relevant to a single eigenstate or eigenstate transition, we show that this method is both an effective way to remove excited state contamination from the study of ground state systems and an effective framework through which one can study the structure of hadronic excitations.

We begin with an evaluation of the nucleon axial charge, g_A , to investigate the improvement offered through this method and consider the role that excited states play in the discrepancy observed between lattice determinations and experiment. This is followed by a determination of the ρ -meson electromagnetic form factors G_C , G_M and G_Q , and the corresponding radiative transition form factor G_{M1} using near physical masses. We then turn our attention to the electromagnetic form factors of the two lowest-lying negative parity nucleons, where such techniques are required to disentangle the contributions of these two near degenerate states. Here we present the first evaluation of the elastic form factors G_E and G_M for both these low-lying states.

Finally, through careful consideration of the $N\gamma \rightarrow N^*$ vertex, we develop an innovative formalism that allows one to evaluate radiative transition form factors for all spin- $1/2$ nucleon excitations. This novel formalism is implemented to provide the world's first examination of the odd-parity transitions of the nucleon in lattice QCD.

Statement of Originality

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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Benjamin James Owen

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Contents

Abstract	iii
Statement of Originality	v
Acknowledgements	vii
1 Introduction	1
2 Quarks, Hadrons and the Strong Interaction	5
2.1 The Quark Model of Hadrons	6
2.2 Quantum Chromodynamics	10
2.3 Hadron Structure and Form Factors	13
3 Lattice QCD	15
3.1 Path Integrals	16
3.2 Discretising the QCD Action	17
3.3 Improving the Lattice Action	24
3.4 Propagators	27
3.5 The $U + U^*$ trick	31
4 Studying Hadrons on the Lattice	33
4.1 Interpolating Fields	33
4.2 Correlation Functions at the Quark Level	34
4.3 Correlation Functions at the Hadronic Level	38
4.4 The Variational Method	41
5 Nucleon Axial Charge	47
5.1 Calculation Details	49
5.2 Results	52
5.3 Cost–Benefit Discussion	56
5.4 Summary	62

6	Light Meson Form Factors	63
6.1	Extracting Light Meson Form Factors	64
6.2	Calculation Details	68
6.3	Results	70
6.4	Summary	90
7	Meson Transitions on the Lattice	91
7.1	Variational Methods for Transitions Elements	91
7.2	The Pseudoscalar–Vector Transition	92
7.3	Results	94
7.4	Summary	103
8	Negative Parity Electromagnetic Form Factors	105
8.1	Accessing Negative Parity States on the Lattice	106
8.2	Negative Parity Nucleon Spectrum	110
8.3	Electromagnetic Form Factors	114
8.4	Summary	122
9	Negative Parity Transition Form Factors	123
9.1	Generalised Nucleon Transition Vertices	124
9.2	Negative Parity Nucleon Transition Results	133
9.3	Summary	138
10	Conclusion	139
A	Gamma Matrices	143
B	Form Factor Plateaus	147
B.1	Light Meson Form Factors	148
B.2	Light Meson Transition Form Factors	155
B.3	Negative Parity Nucleon Form Factors	157
C	Full Expressions for the Nucleon Transition Matrix Elements	161
C.1	Normal Parity Transition Elements	161
C.2	Abnormal Parity Transition Elements	163
D	Papers by the Author	167