Advanced Analytical Models for Well Injectivity Decline

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To my Mum and Dad,

to my Wife,

to my lovely daughters Tahoora

and Zahra (*I lost her during preparation of this thesis*)

Contents

Abstract

The major fraction of world oil is produced by waterflooding, where the injected water displaces oil and maintains the reservoir pressure. In addition, produced water reinjection (PWRI) is an economic and environmental-friendly option to convert waste to value with waterflooding. However, the major challenge is the drastic decline of well injectivity which has been widely reported in the literature. The main mechanisms of the injectivity decline are capture of particles from injected water in the porous rock and formation of low permeable external filter cake on the well wall followed by its stabilisation. The reliable predictive analytical model for well injectivity behaviour forecast up to the stabilisation stage is not available in the literature.

So, the aim of this thesis is to develop full predictive analytical models for injectivity decline during sea water injection and PWRI.

In order to achieve this aim, a new mathematical model for injectivity stabilisation using mechanical equilibrium of a particle on the cake surface accounting for all colloidal forces is developed in this thesis. It is found that the main empirical parameter of the model, highly affecting the stabilised cake prediction, is the lever arm ratio. The lever arm ratio is calculated from laboratory cross-flow filtration experiments and from well injectivity data. It is also determined from Hertz's theory for the elastic particle deformation. Good agreement between the calculated results for the lever arm ratio validates the developed model.

This thesis presents the derivation of a new analytical model for non-uniform cake thickness profile along injection wells. It is found out that, two regimes of the stabilised cake build-up correspond to low injection rates, where the cake starts from the reservoir top, and for high injection rates, where the cake is formed only on the lower well section. The sensitivity analysis shows that water injection rate, cake porosity, water salinity and Young's modulus are the most influential parameters defining the cake thickness profile.

The thesis presents the development of an analytical model for axi-symmetric two-phase flow with simultaneous deep bed filtration of injected particles, formation of external filter cake and its stabilisation due to particle dislodgement. It also introduces a seven-parameter adjustment method. It is shown that the initial injectivity increase,

induced by varying two-phase mobility, adds three degrees of freedom to one-phase impedance growth model. This additional information is used to tune the models with the Corey relative permeability and the pseudo relative permeability under the viscous-dominant displacement. Good agreement between field data and model prediction validates the developed analytical model for injectivity decline during waterflooding and its adjustment method.

The developed analytical model along with laboratory coreflood test data and probabilistic histograms of injectivity damage parameters are applied to predict the injectivity behaviour during produced water disposal into a thick low preamble sandstone reservoir as a field case study. Unusual convex form of impedance curve is observed in the coreflood test and well behaviour modelling; impedance grows slower during external cake formation if compared with deep bed filtration. Risk analysis method using probabilistic histograms of injectivity damage parameters is also developed and applied to well behaviour prediction under high uncertainty conditions.

The above analytical models, results of laboratory studies and field cases allow recommending the developed models for full prediction of injectivity decline during waterflooding and disposal operations.

Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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Publications

Peer reviewed journal publications

1. **Kalantariasl, A.,** Bedrikovetsky, P. (2013) Stabilization of External Filter Cake by Colloidal Forces in a "Well-Reservoir" System, *Industrial & Engineering Chemistry Research*, **53**(1), 930-945.
2. **Kalantariasl, A.,** Zeinijahromi, A., Bedrikovetsky, P. (2014) Axi-Symmetric Two-Phase Colloidal-Suspension Flow in Porous Media during Water Injection, *Industrial & Engineering Chemistry Research*, **53**(40), 15763-15775.
3. **Kalantariasl, A.,** Zeinijahromi, A., Bedrikovetsky, P. (2014) Modelling of External Filter Cake Profile along the Well during Drilling, *Australian Petroleum Production and Exploration Association (APPEA) Journal*, **54**, 319-328.
4. **Kalantariasl, A.,** Farajzadeh, R., You, Z., Bedrikovetsky, P. (2015) Non-Uniform External Filter Cake in Injection Wells, *Industrial & Engineering Chemistry Research*, **54**(11), 3051-3061.
5. **Kalantariasl, A.,** Schulze, K., Storz, J., Burmester, C., Küenckeler, S., You, Z., Badalyan, A., Bedrikovetsky, P. (2015) Produced Water Re-Injection and Disposal in Low Permeable Reservoirs, *Petroleum Science and Engineering*, (under review, Manuscript: PETROL-S-15-00103).

International conference papers and poster presentations

6. **Kalantariasl**, **A.,** Duhan, S., Bedrikovetsky, P. (2013) Type Curves for Injectivity Decline, presented at *SPE European Formation Damage Conference & Exhibition*, Noordwijk, The Netherlands, 5-6 June, SPE 165112-MS.
7. **Kalantariasl, A.,** Zeinijahromi, A., Bedrikovetsky, P. (2014) External Filter Cake in Dynamic Filtration: Mechanisms and Key Factors, presented at *SPE International Symposium and Exhibition on Formation Damage Control*, Lafayette, Louisiana, USA, 26-28 February, SPE 168144-MS.
8. **Kalantariasl, A.,** Zeinijahromi, A., Bedrikovetsky, P. (2014) External Filter Cake Formation: Experience from Membrane (Micro/Ultra/Nano) Filtration, presented at *SPE workshop "Nano-Technology and Nano-Geoscience in Oil and Gas Industry"*, Kyoto, Japan, 4-7 March.

9. **Kalantariasl, A.,** Zeinijahromi, A., Bedrikovetsky, P. (2014) Modelling of External Filter Cake Profile along the Well during Drilling, presented at *Australian Petroleum Production and Exploration Association (APPEA) Conference and Exhibition*, Perth, Western Australia, Australia, 6-9 April.
10. **Kalantariasl, A.,** Farajzadeh, R., You, Z., Bedrikovetsky, P. (2015) Mathematical Modelling of Non-Uniform External Filter Cake in Long Injection Wells, presented at *SPE European Formation Damage Conference & Exhibition*, Budapest, Hungary, 3-5 June, 2015, SPE-174184-MS.
11. **Kalantariasl, A.,** Schulze, K., Storz, J., Burmester, C., Küenckeler, S., You, Z., Badalyan, A., Bedrikovetsky, P. (2015) PWRI and Disposal in a Thick Tight Formation (Mathematical Modelling, Laboratory Test and Field Case), presented at *SPE European Formation Damage Conference & Exhibition*, Budapest, Hungary, 3-5 June, SPE-174185-MS.
12. **Kalantariasl, A.,** Bedrikovetsky, P. (2015) Formation Damage due to Drilling and Completion: External Cake Formation and Stabilisation, accepted for presentation at *SPE Russian Petroleum Technology Conference*, Moscow, Russia, 26-28 October, SPE-176527-MS.

Publications in preparation

1. **Kalantariasl, A.,** Bedrikovetsky, P. (2015) Type Curves for Injectivity Decline, to be submitted to journal of *Oil and Gas Science and Technology*.
2. **Kalantariasl, A.,** You, Z., Bedrikovetsky, P. (2015) Injectivity Decline in Limited Reservoirs, to be submitted to journal of *Petroleum Science and Engineering*.

Chapter 1

Introduction

1.1. Background and aims

Suspension flow in porous media is encountered in a wide variety of chemical, environmental and petroleum engineering applications. This process is important in solid-liquid separation, water purification, food processing, membrane filtration, fresh water storage in aquifers, contaminant transport in subsurface, drilling operations, water injection into oil and geothermal reservoirs, and waste disposal into subterranean formations (Herzig et al., 1970; Jiao and Sharma, 1994; Bennion et al., 1998; Sharma et al., 2000; Song and Singh, 2005; Abou-Sayed et al., 2007; Torkzaban et al., 2007; Zamani and Maini, 2009; Buret et al., 2010; Civan, 2011).

Attachment and detachment of solid particles in porous media are challenging problems for both scientific research and industrial applications. These challenges are more complex in natural rocks like petroleum bearing formations and aquifers (Bedrikovetsky et al., 2011; Zeinijahromi et al., 2012). Capture of colloidal particles in porous media causes reduction in reservoir permeability and increase in flow resistance.

In petroleum engineering, impairment of the permeability of petroleum-bearing formations is referred as “formation damage” (Schechter, 1992; Civan, 2011). Formation damage can be caused by various factors including hydrodynamic, physio-chemical, biological, chemical, and thermal interactions of fluid, porous formation and particles (Civan, 2011). Formation damage occurs in almost every operation of the oilfields. Drilling, completion, workover, production and injection, and stimulation are potential sources of formation damage that usually affect well productivity in production wells and well injectivity in injection wells (Krueger, 1986; Todd et al., 1990; Khilar and Fogler, 1998; Bennion et al., 2000; Civan, 2011; Tiab and Donaldson, 2011). For injection wells, the reduction in the permeability leads to increase in well injection pressure or reduction in well injection rate and consequently to costly stimulation operations or even to irreversible wellbore damage (Hofsaess and Kleinitz, 2003; Vaz et al., 2006).

Understanding, prevention, assessment, control and remediation of formation damage can be achieved with proper experimental design and development of mathematical models. According to Civan (2011), despite decades of experimental and theoretical studies carried out for understanding the factors and mechanisms involved in formation damage, still a unified theory and approach does not exist.

Water injection plays an essential role in oil and gas recovery (Hill et al., 2012). Almost all forms of improved, enhanced and unconventional extraction involve large volumes of fluid injection. Water is the best choice as a nonhazardous, inflammable and relatively cheap fluid (Rowland and Walsh, 2013). Typical sources of injected water are produced water from oil producing fields and sea water. Drastic decline of well injectivity is widely reported during sea water injection, produced water reinjection and disposal. The main mechanisms of the injectivity decline are capture of suspended particles in the injected water by the reservoir rock (deep bed filtration) which results in reservoir permeability reduction, and formation of low permeable external filter cake on the well wall that causes further decline in well injectivity. On global basis, 20% of waterflood projects have not met the targets due to mismatch between water quality and host reservoir (Costier et al., 2009).

In addition, produced water is the largest-volume by-product stream associated with oil and gas recovery operations, and its management and handling is a major growing challenge for oil and gas industry. As oilfields mature, produced water volumes increase significantly (Paige and Murray, 1994). According to Veil and Clark (2011), 98% of US Exploration and Production (E&P) waste is produced water and 95.2% of produced water was reinjected into subsurface formations for enhanced oil recovery, pressure maintenance and disposal purposes. Water oil ratio is expected to reach an average of 12 bbl/bbl (water cut=92.3%) for onshore crude oil resources by 2025. Current global water cut is around 75% (Dejak, 2013). Therefore, management of produced water is a major challenge for petroleum industry.

Produced water re-injection (PWRI) is an attractive economic and environmentally acceptable solution to convert waste to value via maintaining reservoir pressure and enhancing oil recovery (Hsi et al., 1994; Khatib and Verbeek, 2002; PWRI-JIP, 2003; Abou-Sayed et al., 2007; Buret et al., 2010; Ochi et al., 2014). However, the major challenge of PWRI is high risk of formation damage and drastic decline of well injectivity due to capture of solid particles by porous formations (Buret et al., 2010).

Although, different stages of filtration can reduce the amount of impurities and enhance the quality of injected water, the economic issues and operational considerations limit the total removal of solid particles and hydrocarbon liquid droplets (Costier et al., 2009; Buret et al., 2010; Civan, 2011). Thus, even after massive surface treatment through a complex system, the injected water still contains certain amount of fine particles and oil droplets that are difficult to remove with high potential of pore plugging and injectivity decline (Buret et al., 2010). According to Costier et al. (2009), there is no universal answer for the level of water treatment and thus knowledge of leak-off dynamics into the porous media is essential for establishing minimum required water quality in water injection operations.

An accurate understanding of formation damage mechanisms and subsequent injectivity performance prediction is a key factor in decision making, design and implementation of PWRI projects (Paige and Murray, 1994). Success of many pressure maintenance, enhanced recovery, and disposal operations directly depends on the ability to inject required amount of water into the porous formation of interest at a pressure, in most cases, below the fracturing pressure (to maintain conformance of the injected water) (Bennion et al., 2000).

The solid particles can deposit inside the porous formation (deep bed filtration) or on the injection face (external filter cake formation) followed by stabilisation period with erosion of particles from the cake surface. These processes reduce the overall permeability of the system that usually causes drastic reduction in well injectivity which finally affects the ultimate oil recovery.

Accurate mathematical models are necessary to predict injectivity decline performance, to understand the main affecting factors, and determine strategies to avoid or reduce formation damage effects in water injection wells (Civan, 2011).

Formation damage mechanisms are not fully understood yet. Several methods and models for prediction of injector half-life are still limited with validity and require framing with best practices (PWRI-JIP, 2003; Abou-Sayed et al., 2007). Existing models predict unlimited injectivity decline while several field and experimental data exhibit stabilisation period. Therefore, to the best of our knowledge, a comprehensive model for prediction of well injectivity behaviour does not exist in the literature. The present thesis aims to propose a full predictive tool for prediction of well injectivity performance and enhance interpretation of field observation.

The main objective of this thesis is to improve the understanding of well injectivity performance during water injection into oilfields for improved oil recovery and/or pressure maintenance, and other similar operations such as subsurface water disposal. This understanding is essential for management and design of water treatment facilities, water injection systems, risk assessment, parametric sensitivity analysis, and prediction of future well behaviour. The findings lead to the development of mathematical models that allow performing prediction, sensitivity analysis, and matching and interpretation of experimental data and field observations. Several laboratory and field data are used to evaluate the validity of the proposed mathematical models.

The specific goal of the current thesis is to develop a comprehensive mathematical model for full prediction of well injectivity decline during injection of sea water or reinjection of produced water into oilfields for pressure maintenance or improved oil recovery.

One of the most important features of well injectivity decline is its stabilisation after a sequence of deep bed filtration and external filter cake formation that have been observed in several water injection wells (Sharma et al., 2000; Bedrikovetsky et al., 2005; De Paiva et al., 2006; Zinati et al., 2009; Yuan et al., 2012). However, to the best of our knowledge, a reliable predictive model for stabilised cake is not available in the literature. Basic colloidal forces are used to determine the conditions of stabilised cake on the cake surface. In the current thesis a new modified particle detachment model that describes injectivity stabilisation, is presented and validated with several experimental and field data. Equality of detaching and attaching torques of drag, lifting, permeate, gravitational and electrostatic forces determine the equilibrium cake thickness. Lever arm ratio is defined as an empirical parameter in the torque balance equation and it is found to be the main parameter to determine the stabilised cake thickness. Particle deformation theory is proposed to calculate lever arm ratio and validated with experimental and field data with high accuracy. The proposed theory can be used for calculation of lever arm ratio and prediction of the value of stabilised injectivity and equivalent time of equilibrium cake thickness when experimental/field data are not available. Similar process occurs during migration of fines in petroleum reservoirs where attached fine particles to the rock surface are detached by velocity alteration or perturbation in solution chemistry (Bedrikovetsky et al., 2011; Bradford et al., 2011; Sasidharan et al., 2014). The models then can be applied for prediction of injectivity performance in waterflooding projects and other operations such as geothermal water injection, drilling and water disposal.

In waterflood operations, the cost of achieving minimum water quality through complex water treatment system must be balanced with achievable incremental hydrocarbon recovery. Establishing the desired water quality requires in-depth understanding of fluid leak-off dynamics in reservoir conditions (Costier et al., 2009). The tangential rate in long vertical and horizontal wells declines from the injected value on the reservoir top to zero at the bottom. Therefore, stabilised cake thickness and overall hydraulic resistance significantly increase with depth. A new analytical model developed in this thesis allows prediction of non-uniform external filter cake profile along vertical and horizontal wells. Sensitivity analysis to different physio-chemical and operational parameters is performed. It is shown that, by introducing critical injection rate, depending on the injection rate, the cake can be formed on the overall well surface or only on its lower part.

The combination of three injectivity stages (deep bed filtration, external cake formation and its stabilisation) results in a monotonic impedance growth with further stabilisation. However, in several field cases, it was observed that the injectivity increases from the very beginning of water injection. It was explained by the displacement of higher viscous oil by water causing the timely increase of two-phase fluid mobility around the injection well (Altoe et al., 2004). An analytical model for injectivity decline with the displacement of oil by injected water during deep bed filtration, external cake formation, and cake stabilisation stages is derived in this thesis. It is shown that consideration of two-phase displacement resulting in the initial injectivity increase adds three degrees of freedom to the traditional one-phase impedance growth model. This additional information is used for tuning the Corey relative permeability and the pseudo-relative permeability under the viscous-dominant displacement. The analytical model for colloidal-suspension two-phase flow and a procedure for history matching of observed injectivity data are used for tuning and interpretation of several field data.

In the current thesis, a new insight into formation damage from PWRI and disposal into low preamble formations is presented. Laboratory coreflood test is performed to investigate the impedance behaviour during suspension injection into a low permeable sandstone core sample. Analytical model for well impedance growth, along with probabilistic histograms of injectivity decline parameters is applied to well injectivity decline prediction during produced water disposal in a thick low permeable formation (Völkersen field, Germany). Unusual convex form of impedance curve is observed in both coreflood test and well behaviour modelling; impedance grows slower during external cake formation stage if compared with deep bed filtration stage. This is due to the low ratio between the reservoir and cake permeabilities yielding relatively slower impedance growth during cake formation. Risk analysis method using histograms of injectivity damage parameters is applied to well behaviour prediction under high uncertainty conditions.

Phenomenological injectivity decline prediction models require too many empirical parameters for prediction of well injectivity performance that makes them less attractive for field scale applications (PWRI-JIP, 2003). These parameters must be obtained either from representative laboratory tests or from treatment of field data. In this thesis, analytical models are used to obtain the probabilistic histograms of the main injectivity damage parameters (filtration and formation damage coefficients, external cake permeability and lever arm ratio) from analysis of numerous published field data. The obtained probabilistic histograms enable performing risk analysis and injectivity prediction in the absence of experimental/field data. A method is developed to use these probabilistic histograms of injectivity damage parameters for sensitivity analysis and risk assessment of field case studies.

1.2. Thesis structure

This is a PhD thesis by publications. Eight papers are included in this thesis. In all papers the PhD candidate is the first author. Five papers have been reviewed and published in or submitted to peer reviewed journals and three full conference papers have been presented in Society of Petroleum Engineers (SPE) European Formation Damage Conference and Exhibition.

The thesis body is formed by six chapters and two appendices. The *first chapter* contains the general aims and introduction to the importance of the research in petroleum industry, chemical, and environmental engineering, and other areas.

Paper	Chapter	Paper title	Status
1	3	Stabilization of External Filter Cake by Colloidal Forces in a "Well-Reservoir" System	Published
2	4	Non-Uniform External Filter Cake in Injection Wells	Published
3	4	Modelling of External Filter Cake Profile Along the Well During Drilling	Published
4	5	Axi-Symmetric Two-Phase Colloidal-Suspension Flow in Porous Media during Water Injection	Published
5	6	Type Curves for Injectivity Decline	Published
6	6	Produced Water Re-injection and Disposal in Low Permeable Reservoirs	Submitted
7	Appendix A	Injectivity during PWRI and Disposal in Thick Low Permeable Formations (Field Case, Laboratory and Mathematical Modelling)	Published
8	Appendix B	Mathematical Modelling of Non-Uniform External Filter Cake in Long Injection Wells	Published

Chapter two presents the detailed literature review on mechanisms of formation damage in water injection wells during injection of sea water, produced water or any poor quality water. Water production growth and water management challenges in petroleum industry are highlighted. Laboratory coreflood tests on reservoir core samples for water injection and drilling fluid design purposes and also field data observations are reviewed. In addition, lessons learned from cross-flow filtration experiments of membrane science are briefly summarized. Different mathematical models for prediction of well injectivity decline are reviewed and discussed. It shows the lack of comprehensive mathematical model for prediction of well injectivity decline.

Detailed analysis of different colloidal forces exerted on a single particle at the cake surface is presented in *Chapter three*. A mathematical model based on the equality of attaching and detaching torques is developed to determine the criterion of particle detachment from the cake surface. The Hertz theory of particle deformation is applied to calculate the new defined empirical parameter, lever arm ratio, that mainly controls the particle detachment from the cake surface. Several experimental and water injection well data have been matched to confirm the validity of the proposed model.

In long vertical and horizontal injection wells, tangential flow decreases significantly from top of the formation to the bottom due to fluid leak-off into the adjacent formation.

It results in non-uniform external filter cake along the injection well. A competition between different forces acting on a single particle determines the condition of particle detachment on the cake surface. Coupling torque balance of particles on the cake surface, fluid volume balance and Darcy's law allows development of simple mathematical model to determine the critical injection rate for cake formation and non-uniform distribution of external filter cake in long water injection wells. The mathematical model with detailed derivations is presented in *Chapter four.*

Chapter five presents mathematical modelling and history matching procedure of non-monotonic injectivity performance during two-phase flow of oil and water with colloidal-suspension in porous media during water injection in oil reservoirs. The mathematical models allow coupling of suspension flow with capture of particles that result in injectivity loss and two-phase flow that may cause initial increase in injectivity. A new procedure for history matching of injectivity performance is presented. Both synthetic cases and field data have been well matched.

In *Chapter six* injectivity predictions during PWRI in tight formations are presented as a field case study. Histograms of four injectivity damage parameters are developed using comprehensive field data treatment from the published literature. These histograms are used for risk assessment and sensitivity analysis of a produced water disposal project in a tight sandstone formation. Unusual convex well impedance performance is predicted for low permeable formations and confirmed with laboratory coreflooding test.

Chapter seven summarizes the results and presents conclusions.

Mathematical modelling and laboratory study of impedance growth during disposal of produced water into low permeable sandstone formations is presented in *Appendix A*. An analytical model is derived to obtain injection rate decline *vs* real time from impedance growth curve under constant injection pressure. It is also shown how to use the probabilistic histograms of injectivity damage parameters to perform risk analysis for PWRI and disposal operations. *Appendix B* discusses conditions of particle detachment in long injection wells and presents derivation of analytical model for cake buildup profile along with parametric sensitivity analysis.

1.3. Relation between publications and this thesis

Paper 1 *Stabilization of external filter cake by colloidal forces in a "well-reservoir" system* presents a new mathematical model for analysis of injectivity stabilisation in water injection wells and dynamic filtration of experimental data. Injectivity performance in water injection wells usually exhibits three stages: deep bed filtration of colloidal particles into the porous formation adjacent to the wellbore, buildup of low permeable external filter cake on the well wall and cake stabilisation. In this paper a new torque balance model for particle dislodgment on the cake surface is introduced. Basic colloidal forces of tangential (cross-flow) drag, permeate (normal), gravitational, lifting and electrical van der Walls, electrostatic double layer and Born repulsion forces are applied to investigate the condition of particle detachment on the cake surface. A new empirical parameter, lever arm ratio is defined and found to be the main parameter that controls the equilibrium cake thickness. The lever arm ratio is calculated from laboratory cross-flow filtration experiments and from well injectivity data. It is also determined from the Hertz's theory for the elastic particle deformation on the solid cake surface. The obtained lever arm ratio values show a good agreement between those predicted theoretically and the laboratory- and field-based values. It validates the model proposed and allows using the model for reliable predictions. Applying the Hertz theory eliminates the need for empirical lever ram ratio in the absence of laboratory and field data. The developed model can be used for full reliable prediction of deep bed filtration, external cake formation and its stabilisation during low quality water injection.

In thick formations, tangential rate changes significantly from reservoir top to the bottom of the well due to fluid leak-off into the adjacent formation (according to Darcy's law) and non-uniform cake forms along the injection well. In paper 2 "*Non-uniform cake thickness in long injection wells*", fluid-leak off equation, Darcy's law and mathematical model for particle detachment conditions developed in the *paper 1* are used to develop new analytical models for determination of external filter cake profile in long vertical injection wells. Sensitivity analysis to important physio-chemical and operational parameters of water salinity, cake permeability, injection rate, and particle Young's modulus and Poisson's ratio are investigated. A critical injection rate is defined and formulated that allows determination of two regimes where that external cake can be formed on the whole injection interval or only on the lower section of the long injection well, i.e. cake thickness is zero from top of the formation to the point correspond to critical rate.

Paper 3 *Modelling of external filter cake profile along the well during drilling*, presents sensitivity analysis of different parameters for prediction of non-uniform cake thickness profile during drilling, when the developed torque balance model of attaching and detaching forces in *paper1* and *paper 2* is modified and applied for non-Newtonian drilling fluids. Torque balance of hydrodynamic (lifting, tangential and permeate drag), gravitational and electrostatic (DLVO) forces along with Darcy's law and material balance is used to investigate the conditions of particle detachment on the cake surface. The results of sensitivity analysis show that mud chemistry, particle size, cake permeability, tangential flow velocity, overbalance pressure, and Young's modulus are the most important parameters affecting the steady-state external filter cake thickness and velocity profile.

Injection of colloids and suspensions in natural reservoirs with particle capture results in well injectivity decline. However, some initial improvement in injectivity was observed during waterflooding of oilfields and explained by increasing mobility of two-phase fluid during the displacement of more viscous oil by water. An analytical model for axi-symmetric two-phase flow with simultaneous deep bed filtration of injected particles, formation of external filter cake and its stabilisation due to particle dislodgement is derived in paper 4 *Axi-symmetric two-phase colloidal-suspension flow in porous media during water injection*. This paper introduces a simple method for treatment and interpretation of non-monotonic well injectivity history behaviour with initial improvements in well injectivity during waterflooding of oilfields. The explicit formula for dimensionless pressure drawdown (impedance) yields the type curve for impedance history. It is shown that the initial injectivity increase, induced by varying two-phase mobility, adds three degrees of freedom to one-phase impedance growth model. This additional information is used for tuning the models with the Corey relative permeability and the pseudo relative permeability under the viscous-dominant displacement. A seven-parameter adjustment method is proposed and applied for both synthetic and well data analysis. Treatment of the data from three synthetic cases results in good agreement with the initial data, validating the developed model adjustment method. Data from three field cases have been used. Good agreement between the field and modelling data along with common values of the obtained constants validate the developed analytical model for injectivity decline during waterflooding and its adjustment method.

The mathematical models for injectivity decline that are developed in *paper 1* and used in *paper 4* contain four main empirical injectivity damage parameters (filtration and formation damage coefficients, external filter cake permeability and lever arm ratio) to be determined from experimental investigation or field data analysis. These parameters are necessary for injectivity performance prediction. Analytical models are applied to field data reported in the literature to generate probabilistic histograms of injectivity damage parameters in paper 5 *Type curves for injectivity decline* and used in *papers 6* and *7*. The probabilistic histograms can be used as an input data bank for sensitivity analysis and risk assessment of waterflood projects during injection of sea water or re-injection of produced water in oil and gas reservoirs to investigate optimistic, pessimistic and normal scenarios. It can also be used to predict the injectivity performance in water disposal wells. Moreover, the histograms of four injectivity damage parameters are useful for estimation of further well impairment when limited injectivity data are available.

Injectivity decline prediction during PWRI in tight formations is presented in paper 6 *Produced water reinjection (PWRI) and disposal in a low permeable thick formation* and paper 7 *Injectivity during PWRI and disposal in thick low permeable formations (field case, laboratory and mathematical modelling)* as a field case study. Mathematical models developed in *paper 1* along with histograms of four injectivity parameters generated in *paper 5* are used to study injectivity performance of PWRI in a tight sandstone formation when formation fracturing is not allowed. Laboratory coreflood injectivity test using a low permeable sandstone core sample is performed to investigate the impedance growth shape in low permeable formations. Unusual convex form of impedance curve is observed in both coreflood test and well behaviour modelling; impedance grows slower during external cake formation if compared with deep bed filtration. Good match between mathematical model and experimental data is achieved. In addition *paper 7* derives mathematical model for prediction of injection rate decline *vs* real time from impedance growth curve under constant injection pressure. These two papers provide new insights for management, design and risk analysis of produced water reinjection and disposal into tight formations.

Conditions of particle detachment and cake formation in long injection wells are discussed in paper 8 *Mathematical modelling of non-uniform external filter cake in long injection wells*. Implicate formula for cake thickness profile is derived and parametric sensitivity analysis to different physio-chemical parameters is performed.

Finally, the above mentioned 5 journal papers and three conference papers provide comprehensive and advanced analytical models for prediction of injectivity decline that is essential for decision making, design and implementation of water injection projects and also determining strategies to avoid and/or mitigate formation damage. The proposed models can be applied for prediction of injectivity decline during injection of seawater, reinjection of produced water in oilfields and geothermal reservoirs, waste disposal and, invasion of drilling fluids.

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Chapter 2

Literature Review

2.1. Introduction

Colloidal flow and transport in porous media is encountered in a wide variety of chemical, environmental and petroleum engineering applications. Understanding flow and performance of colloidal particles are vital in different chemical engineering sections such as water treatment, food processing, and membrane and chromatography technologies (Kang et al., 2004; Hwang et al., 2006; Yuan and Shapiro, 2011; You et al., 2014). Transport and capture of virus and other contaminants in underground formations, storage of fresh water in aquifers, and well clogging are important in environmental engineering (Torkzaban et al., 2007; Bradford et al., 2011; Sasidharan et al., 2014). Transport, deposition and detachment of colloidal particles occur during drilling operations of oil, gas and geothermal wells, sea water injection and produced water reinjection (PWRI) into oil reservoirs, reinjection of cold water in geothermal wells, waste disposal in subterranean formations, and fracturing and completion of oil and gas wells (Krueger, 1986; Schechter, 1992; Jiao and Sharma, 1994; Bedrikovetsky et al., 2001; Guan et al., 2006; You et al., 2013; Ochi et al., 2014).

Capture of colloids in porous media results in permeability reduction and increase in flow resistance. According to Schechter (1992) and Civan (2011), impairment of permeability of petroleum-bearing formations is referred as formation damage in petroleum engineering. Formation damage occurs almost in every operation of the oilfields. Drilling, completion, workover, production and stimulation are potential sources of formation damage that usually affect well productivity in production wells and well injectivity in injection wells. Prevention of formation damage is important for both conventional well operations and Enhanced Oil Recovery (EOR). In EOR operations, if injection and production conductivity is damaged, sweep efficiency and recovery factor will be adversely affected. The success or failure of an EOR project may highly depend on the ability to inject planned volume and rate of the fluids (Krueger, 1986; Schechter, 1992; Civan, 2011; Borazjani et al., 2014). Formation damage remediation is usually difficult and costly, and the basic approach should be to prevent damage. Therefore, an accurate and broad knowledge of formation damage mechanisms is essential in prevention of well damage and injectivity/productivity decline (Krueger, 1986; Schechter, 1992; Bedrikovetsky et al., 2001; Civan, 2011; Badalyan et al., 2012).

Water is injected into oil reservoirs for waterflooding (sweeping the oil to the producers) and for pressure maintenance (filling the voidage left by produced fluids) (Palsson et al., 2003). Waterflooding is one of the most economically viable methods for additional oil recovery and an essential part of modern offshore and onshore oilfield operations (Palsson et al., 2003). Water injection also occurs during injection of cold water in geothermal reservoirs, storage of fresh water in aquifers and disposal of waste water in subterranean formation (Hofsaess and Kleinitz, 2003). The injection of water into underground porous layers may lead to several severe problems. The injected water for waterflooding and pressure maintenance frequently contains fine suspended solids which can cause formation damage and well injectivity decline (Vetter et al., 1987; Khatib, 1994; Bennion et al., 1998; Ochi et al., 1999; Sharma et al., 2000; De Paiva et al., 2006; Guan et al., 2006; Yuan et al., 2012; Ochi et al., 2014). Suspended solids in the injected water can penetrate into the porous formation and reduce the near wellbore permeability (deep bed filtration) or deposited on the surface of the porous formation and forming an extremely low permeable layer (external filter cake). Physical, chemical and hydrodynamic mechanisms are responsible for permeability impairment (Eleri and Ursin, 1992). Other mechanisms such as fines mobilisation and capture, scale deposition due to incompatibility between injected water and host reservoir brine can also cause formation damage in water injection wells (Bedrikovetsky et al., 2011). Costier et al. (2009) reported that on global basis, 20% of water injection targets have not been met due to mismatch between water quality and host reservoir.

Injectivity impairment is a major issue in waterflood and disposal operations and has been widely reported in the literature during injection of sea water, reinjection of produced water into oilfields and disposal of produced water. Decline of well injectivity may lead to increase in injection pressure or decrease in water injection rate and consequently increase in costly stimulation operations or even irreducible formation damage (Hofsaess and Kleinitz, 2003).

Two main sources of water injection are sea water and produced water from oil and gas fields. Both sources contain significant level of impurities to be reduced before injection in order to avoid formation damage and injectivity decline. As mentioned by Patton (1990), Zhang et al. (1993), Buret et al. (2010) and Ochi et al. (2014), even after advanced surface treatment, injected fluids still contain solid particle and oil droplets that are difficult to remove and have a high potential risk of plugging. Produced water management is becoming a major challenge for petroleum industry due to the large and increasing stream of produced waters from oil and gas fields and strict environmental regulations. Reinjection of produced water for both reservoir pressure maintenance and EOR methods is among the best options to convert waste to value. According to Al-Abduwani et al. (2005b), Abou-Sayed et al.(2007), Buret et al. (2010) and Veil and Clark (2011), PWRI becomes increasingly the main destination for produced waters. However, the major associated problem is drastic injectivity decline.

Costier et al. (2009) emphasised that in design of PWRI for reservoir management purpose, due to complexity of water treatment system, the cost of minimum water quality must be weighed against achievable incremental reservoir yield. Knowledge of leak-off dynamics into the porous media allows establishing minimum required water quality in water injection operations.

Therefore, accurate understanding of formation damage mechanisms is required to predict water injectivity in terms of number and location of injection wells, spacing between injectors, injection interval and stimulation/ remediation frequency (Costier et al., 2009). Development of reliable mathematical models are of interest to predict, process, and control the formation damage, to find remediation and determine desirable treatment of injected water and waterflood plans, and for drilling fluid sizing.

2.2. An overview of produced water management

Since one of the main sources of water injection in oilfield operations is produced water, this section briefly describes the importance of water management and presents some information about PWRI into oilfields for pressure maintenance, EOR and disposal purposes.

Produced water is a complex mixture of organic and inorganic compounds and the largest-volume by-product stream associated with oil and gas recovery operations (Veil and Clark, 2011). Produced water is brought from underground formations to the surface during different stages of oil and gas production in conventional and unconventional resources. It may include water from the reservoir, injected water, and any chemicals added during production and treatment. Produced water, its treatment and management are growing challenges in almost all oil and gas producing countries. In order to minimize the impacts of the produced water, a multidisciplinary approach integrating subsurface performance, facility design and environmental discharge is required (Fakhru'l-Razi et al., 2009; Veil and Clark, 2011; Hill et al., 2012).

Hill et al. (2012) reported that, in 2011, oil industry has spent more than $50 billion on handling produced water. According to Khatib and Verbeek (2002) estimation, more than 210 million barrels of produced water was generated each day worldwide in 1999. In Shell operations water production increased almost 3 times over 10 years (Khatib and Verbeek, 2002). Based on US Department of Energy's Argonne National Laboratory's estimation, the volume of produced water in the United States is 21 billion barrel a year. Additional water production from the rest of the world is estimated at a volume of more than 50 billion barrels (Dejak, 2013). As an oil field matures, oil production decreases while water production increases. Dejak (2013) reported that the current water/oil ratio (WOR) is estimated at 2:1 to 3:1 worldwide. For a WOR of 3:1 (75% water cut), water production were approximately 250 million bbl/day in 2007 (Dejak, 2013). Detailed produced water by states in the United States provided by Veil and Clark (2011) reveals that water oil ratio (WOR) is 1.06 bbl/bbl for offshore and varies from 2.5 to 42.7 bbl/bbl for onshore operations. Reported water to gas ratio (WGR) range varies from 0.04 to 1200 bbl/MMSCF for different states. US national average onshore and total (onshore and offshore) WOR are presented as 7.6 and 5.3 bbl/bbl respectively. Sharma et al. (2000) mentioned that, most mature oil and gas producing regions of the world have WOR as high as 25. According to Dejak (2013), WOR is expected to reach an average of 12 bbl/bbl (water cut=92%) for onshore crude oil resources by 2025. Figure 1 shows the typical WOR for some oil and gas producers.

Three main reasons for massive increase in produced-water volume are production from mature fields, water based EOR methods and increase in production from unconventional resources (Sharma et al., 2000; Abou-Sayed et al., 2007; Fakhru'l-Razi et al., 2009; Civan, 2011; Shaffer et al., 2013; Keshavarz et al., 2014, 2015).

The production from unconventional gas resources needs massive water production either during dewatering stage for Coal Seam Gas (CSG) reservoirs or fracturing extremely low permeable formations and flow-back water for further gas production. In the rapidly developing shale gas industry, managing the produced water associated with shale gas is a major challenge to maintain the profitability of shale gas extraction and, for environment protection societies that aim to protect public health and the environment (Khanna et al. 2013; Shaffer et al., 2013).

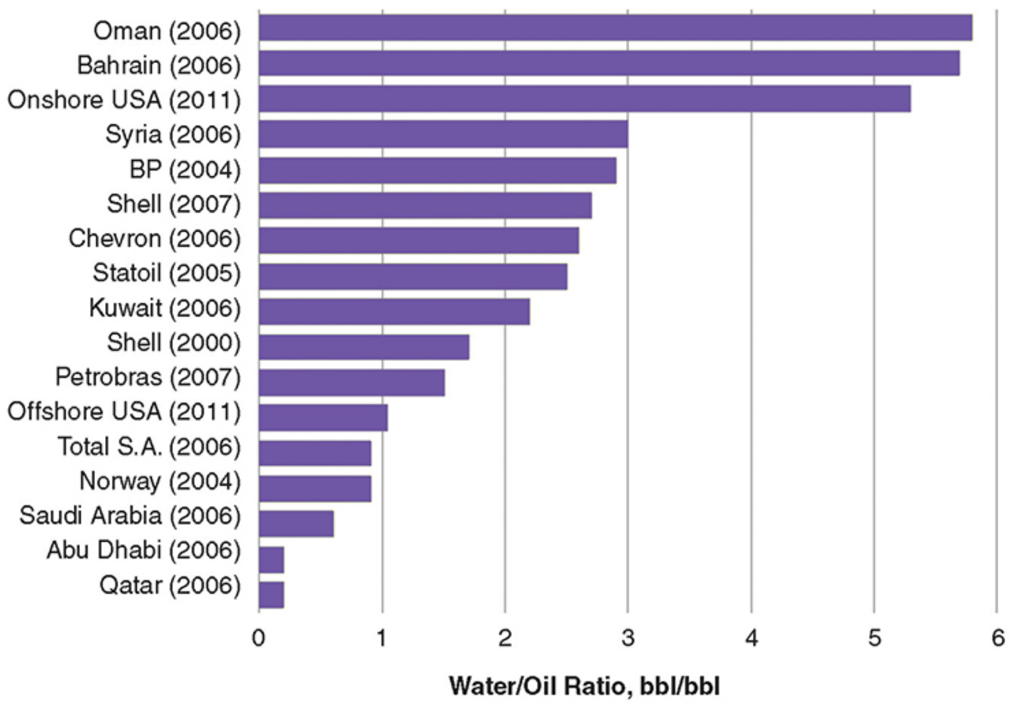

Figure 1. Water Oil Ratio (WOR) for selected producers (Veil and Clark, 2011)

Khatib and Verbeek (2002) reported that across the Shell operating units, 55% of the produced water is re-injected. In United States, 95.2% of the reported volume of produced water in 2007 was injected into underground formations. More than half of the produced water (55.4%, 8.6 billion bbl) was injected for enhanced recovery and more than one-third (38.9%, 6 billion bbl) was injected for disposal. The amount of water production and underground water injection is significant and requires an accurate understanding of injectivity performance.

2.3. Injection water sources

The properties of the injection water are an important factor affecting injectivity decline. Water from different sources can be used for waterflood operations. From injectivity impairment point of view, the important issues that may affect the total quality of the water include: source of the injected water, compatibility between mixed waters, water temperature, pressure path during production and injection operations and possible seasonal variations in the water quality (Bennion et al., 2000).

Common sources of injection/ disposal water include: produced water from conventional oil and gas fields (formation water and/or injected water), sea water, surface water (lake, river, etc.), shallow groundwater (potable), deep aquifer water and produced water from unconventional resources (coal bed methane, shale gas, etc.). A mixture of the above sources (with variety of possible ratios during the life of the injection project) or single source water is usually used for waterflood operations (Bennion et al., 2000). Sea water and produced waters are the most typical source of water injection for pressure maintenance and waterflooding in oil and gas industry. Comparison of advantages and disadvantages of sea water and produced water re-injection have been discussed in a paper by Bader (2007).

2.4. Impurities in the injection water

All water sources contain different organic and inorganic impurities. Injection water quality is affected by several types of contaminates including suspended solids (sand, silts, clay, ect.), scale, oil, bacteria, corrosion products, and marine organisms (Barkman and Davidson, 1972; Mitchell and Finch, 1981). Any of the mentioned impurities may be the predominant source of permeability impairment and injectivity decline for a particular water and formation. The mechanisms of formation damage and subsequent injectivity decline arising from different types of water quality and impurities are different. Among them, suspended solid particles are the major issue in injectivity impairment of water injection wells as stated by Bennion et al. (2000).

Suspended solids originate from naturally occurring fines, clays, sand and silt, etc. in the underground reservoirs; injected solid particles during poor quality waterflooding; solids in drilling, completion or fracturing fluids; sea or shallow waters, organic solids, a wide range of corrosion products from production/injection tubulars, tanks and treating equipment; live/dead bacteria; various precipitates and scales; precipitation of dissolved solids; hydrocarbon solids (waxes, asphaltenes); and biological activities (Barkman and Davidson, 1972; Mitchell and Finch, 1981; Khilar and Fogler, 1998; Bennion et al., 2000; Civan, 2011; Tiab and Donaldson, 2011). The type, concentration, and particle-size distribution of suspended solids in water vary for different source of the injection water.

2.5. Formation damage mechanisms in water injection wells

Formation damage in injection wells can be referred to as a mechanism that alters the permeability, reduce fluid conductivity in the rock and results in injectivity loss. Many factors contribute to formation damage mechanism in water injection wells. The factors involved, effect of each factor and their interaction in formation damage are not fully understood yet (PWRI-JIP, 2003). Experimental and theoretical works suggest that the most general factors affecting formation damage in injection wells are: flow rate and pressure, host formation characteristics, and fluid characteristics (Cavallaro and Baigorria, 2000; PWRI-JIP, 2003).

A literature survey reveals that well injectivity decline may occur due to several reasons including

- incompatibility of injected water and formation brine causing deposition of scale and reduction in rock permeability (Sorbie and Mackay, 2000);
- mobilization of attached fine particles following by particle capture and permeability reduction due to salinity alternation or velocity change (Bedrikovetsky et al., 2011; Hussain et al., 2013, Oliveira et al., 2014);
- invasion of oil droplets and pore blocking (Buret et al., 2010);
- bacterial growth and reduction in rock permeability (Bennion et al., 2000; Civan 2011); and
- solid particle invasion causing near wellbore permeability reduction and/or external cake formation (Bennion et al., 2000; Civan 2011).

Among them, invasion of suspended particles into the porous media by injected water stream is the most common explanation for injectivity reduction in water injection wells as stated by Bennion et al. (2000). Classification of different types of formation damage, corresponding mechanisms and also applications in different petroleum production phases can be found in Bennion et al. (2000), Civan (2011), and Tiab and Donaldson (2011).

2.5.1 Formation damage due to solid particle invasion

Saraf et al. (2010) mentioned that invasion of solid particles into porous formation and subsequent injectivity decline takes place to some degree in most injection wells.

Numerous researchers have studied invasion of fine particles in the injected water into porous media and subsequent formation plugging to understand the potential mechanisms responsible for injectivity decline (Vetter et al., 1987; Eylander, 1988; Pautz et al., 1989; Eleri and Ursin, 1992; Roque et al., 1995; Moghadasi et al., 2004; Al-Abduwani et al., 2005b; Chalk et al., 2012; Yuan et al., 2013). Both experimental investigations and mathematical models have been performed to describe the particle flow, deposition and detachment and permeability alteration in porous rocks. The goal of many laboratory studies was to be able to predict required water quality for water injection wells. Todd et al. (1990), Kumar (1991), Roque et al. (1995), Zamani and Maini (2009) and Henry et al. (2012) reported that the laboratory observations have confirmed that small micron-sized solid particles may result in significant decline in injectivity.

Generally, formation damage due to invasion of solid particles in the injected water can be classified in three stages: deep bed filtration and, external cake formation and it stabilisation.

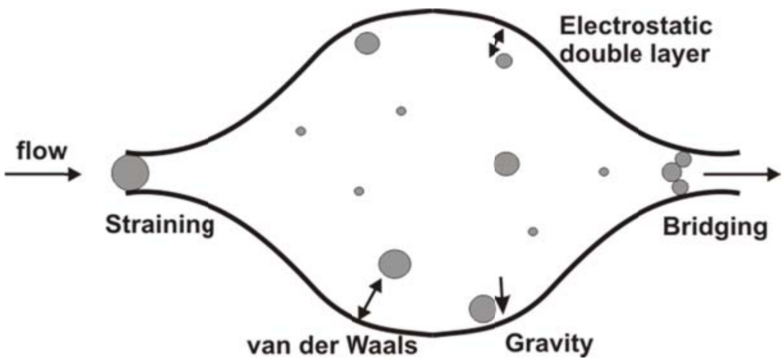

Figure 2. Mechanisms of particle capture at pore scale

2.5.1.1 Deep bed filtration (internal plugging)

The solid particles in the injection water may be in the range of molecules, colloids, micron-sized particles and larger (Kumar, 1991). Deep bed filtration of colloidal and micron-sized particles into porous media was subject of research for decades in chemical, environmental and petroleum engineering (Civan, 2011; Tiab and Donaldson, 2011). Numerous studies by Vetter et al. (1987), Eylander (1988), Pautz et al. (1989), Eleri and Ursin (1992), Roque et al. (1995), Moghadasi et al. (2004), Al-Abduwani et al. (2005b) and Torkzaban et al. (2007) have investigated the effect of different physio-chemical, hydrodynamic and operational conditions on the performance of the particle flow, deposition, and detachment in porous media.

Hydrodynamic and physio-chemical conditions control the particle transport and may cause the particles to be deposited on the rock grain surface (pore walls) or block the pore throat (Fig. 2). A single particle in the porous medium is under influence of various forces acting between particle and pore surface including: gravity, diffusion, van der Walls, electrostatic, Born repulsion, drag and lifting forces. Chemical conditions such as salinity and pH highly affect the particle retention process. Experimental and theoretical studies have proved that a suspension with high salinity and low pH provide a favourable environment for particle deposition (Khilar and Fogler, 1998; Sasidharan et al., 2014).

Three main mechanisms of rock permeability impairment due to foreign solid transport are (Figs. 2 and 3a):

Surface Deposition: Interaction of injected particles with pore surface can cause deposition of particles on the pore wall. In pore scale, different hydrodynamic and chemical forces are exerted on a single particle near the wall of the pore. Deposition is governed by the interaction between suspended particles and the carrier fluid and also by the interactions between particle and the pore surface (Henry et al., 2012). The effect of particle deposition on permeability is significant only if it takes place in pore throats. Thus, permeability reduction is not related to the total amount deposited but only to the fraction deposited in pore throats as discussed in PWRI-JIP (2003) workshop reports.

Pore Bridging: Deposited particles on the surface of the pore throat may cause further capture of the arriving particles, forming a bridge. After formation and consolidation of a bridge, the newly arriving particle accumulate upstream of the bridge.

***Internal Cake Formation*:** Internal cake is formed when the fraction of pore throats with bridging of particles reaches a critical value. So, arriving particles deposit upstream of the bridged pores and also inside the accessible pore bodies forming an internal cake that can cause substantially reduction in near wellbore permeability.

2.5.1.2 External cake formation

According to Pang and Sharma (1997), when deposited particles near the injection face fill the pores up to a critical fraction of initial rock porosity, all injected particles accumulate on the injection face, forming a low permeable external filter cake. External cake can also be formed directly by large particles without penetration into the formation. Based on "1/3-1/7" filtration rule which proposed by Abrams (1977), particles greater than 1/3 of rock pore size can form external cake without invasion into porous formation (Abrams, 1977; van Oort et al., 1993; Sacramento et al., 2015). Cake properties (porosity, permeability, compressibility and thickness) control the injectivity loss during cake formation stage. Formation of low permeable cake on the well wall causes extra reduction in flow conductivity and additional injectivity decline.

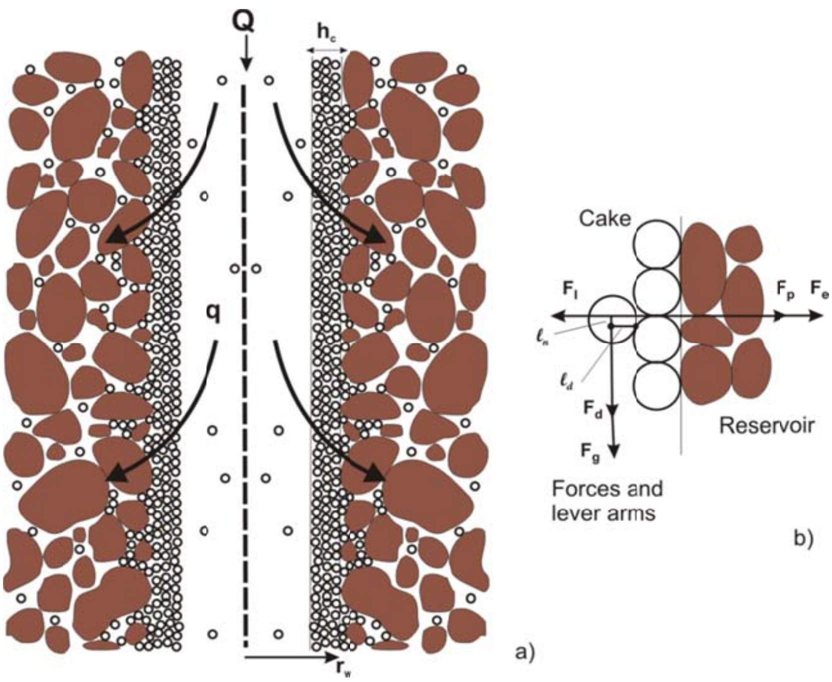

Figure 3. Schema of a) deep bed filtration and external cake formation b) forces exerted on a particle at the cake surface

2.5.1.3 Forces exerting on a particle at the cake surface

Surface interaction forces: Understanding interaction and behaviour of particle-particle and particle-surface is of quantitative significance in design and analysis of any particle-particle-surface system in many industrial projects. In formation damage, both particle-rock grain surface and particle-particle interaction control the particle deposition process and have significant effect on the permeability impairment during deep bed filtration and cake formation.

Deposition and detachment of colloidal particles on both rock surface and deposited particles in the cake layer is usually described by combination of so-called DLVO (Derjaguin-Landua-Verwey-Overbeek) theory and hydrodynamic forces (Jiao and Sharma, 1994; Civan, 1998; Khilar and Fogler, 1998; Bedrikovetsky et al., 2011; Civan, 2011). According to DLVO theory, net interparticle (or particle-surface) force is obtained by summation of attractive van der Walls (VDW), electrostatic (electrical double layer) and Born repulsion forces (Khilar and Fogler, 1998; Elimelech et al., 1998; Torkzaban et al., 2007).

Attractive van der Waals and repulsive electrical double layer interaction energies strongly depend on the separation distance between particle-particle (or particle-surface). Both forces decay rapidly as separation distance between particle-particle (or particle-surface) increases. Attractive van der Waals interaction energy for sphere-plate is described by (Elimelech et al., 1998; Khilar and Fogler, 1998):

$$V_{VDW}(h) = -\frac{A_H}{6}\left[\frac{r_s}{h} + \frac{r_s}{h+2r_s} + \ln\left(\frac{h}{h+2r_s}\right)\right] \tag{1}$$

where, A_H is Hamaker constant between particle-surface separated by an aqueous medium, r_s is the particle radius and h is the separation distance between particle centre and surface. Several other formulae to describe VDW interaction energy for sphere-sphere and sphere-plate geometry can be found in Schechter (1992), Elimelech et al. (1998) and Khilar and Fogler (1998).

Numerous expressions have been used to calculate the electrostatic interaction of particle-particle and particle-plate systems for constant surface potential, constant surface charge and mixed cases of constant surface potential and constant surface charge (Elimelech et al., 1998; Khilar and Fogler, 1998). Electrical double layer interaction energy for constant surface charge case is given by

$$V_{DLR}(h) = \frac{\varepsilon_0 D r_s}{4}\left[2\psi_{01}\psi_{02}\ln\left(\frac{1+\exp(-\kappa h)}{1-\exp(-\kappa h)}\right) - \left(\psi_{01}^2 + \psi_{02}^2\right)\ln\left(1-\exp(-2\kappa h)\right)\right] \tag{2}$$

Here ε_0 is the electric constant (permittivity of free space), D is the dielectric constant, ψ_{01} and ψ_{02} are zeta potentials of particles-particle (or particle-surface), and κ is the inverse Debye length. Debye length is inversely proportional to solution ionic strength. Higher salinity results in lower Debye length and closer packing of particles, while in lower ionic strength minimum separation distance between particle-particle (or particle-surface) increases and less packed system is obtained. Several other formulae are presented in Elimelech et al. (1998) and Khilar and Fogler (1998).

Born repulsion potential accounts for short range structural or hydration forces due to the interaction of solid particles with the absorbed fluid layers. Born repulsion energy for sphere-plate system is given by (Khilar and Fogler, 1998)

$$V_{BR} = \frac{A_H \sigma_{LJ}^6}{7560}\left[\frac{8r_s + h}{\left(2r_s + h\right)^7} + \frac{6r_s - h}{h^7}\right] \tag{3}$$

where σ_{LJ} is atomic collision diameter in Lennard-Jones potential (approximately 5-6 ^{o}A).

The DLVO theory describes the interaction force between charged surfaces in an aqueous medium. The total interaction potential energy (V_T) is obtained by the summation of the attractive and repulsive potentials.

$$V_T = V_{VDW} + V_{EDL} + V_{BR} \tag{4}$$

The net force is then calculated by

$$F_e\left(h\right) = -\frac{dV_T}{dh} \tag{5}$$

Hydrodynamic forces: Figure 3a shows the stream lines for injected water in the wellbore and in the reservoir. The particles carried by water are deposited at the cake surface after the transition time. A particle on the cake surface is submitted to the hydrodynamic drag, permeate, lifting, buoyancy and electrical interaction forces (Fig. 3b). The equality of exerted torques determines the condition of particle erosion from the cake surface.

The expression for tangential drag force exerting the particle on the plane surface which is widely used in modelling of particle attachment and detachment in porous media is

$$F_d = 6\pi\mu r_s u_t C_1 \tag{6}$$

where μ is the water viscosity, u_t is the tangential cross flow velocity of fluid in the centre of the particle, and C_1=1.7 is the correction factor due to wall effect (Goldman et al., 1967).

The lifting force exerted on a single particle is expressed as

$$F_l = \chi\sqrt{\rho_w \mu \left(r_s u_t\right)^3} \tag{7}$$

where χ is the lifting force factor and ρ_w is water density. Jiao and Sharma (1994), Bradford et al. (2011), Kalantariasl and Bedrikovetsky (2013), and Kalantariasl et al. (2014) showed that lifting force exerting on a particle at the cake surface or rock grain can be neglected if compared to other attaching forces. The net gravitational force exerting the particle is expressed as

$$F_g = \frac{4}{3}\pi\Delta\rho g r_s^3 \tag{8}$$

Here $\Delta\rho$ is the density difference between the solid particles and carrier water.

Permeate flow perpendicular to tangential flow cause another hydrodynamic force due to flow in porous media (Fig. 3). The expression for permeate force exerting on the particle placed in front of the plane surface of porous media in the flux perpendicular to the surface accounting for permeate factor is given by

$$F_p = 6\pi\mu r_s u_p \Phi_H \tag{9}$$

Here u_p is the permeate velocity on the well wall (at the injection face entrance). Studies by Goren (1979), Kang et al. (2004), and Kim et al. (2006) have shown the importance of permeate factor Φ_H. As the particle approaches the porous media the permeate force increases by decreasing the separation between particle and porous media. In the present thesis, permeate factor is taken from Sherwood (1988)

$$\Phi_H = 0.36\left(\frac{k_c}{r_s^2}\right)^{-2/5} \tag{10}$$

where k_c is the cake permeability.

2.6. Basic governing equations for transport of particles in porous media

Most particle deposition models are based on the volumetric balance of the suspended and deposited particles in porous media and Iwasaki's particle deposition function (Iwasaki et al., 1937; Herzig et al., 1970; Pang and Sharma, 1997; Saripalli et al., 2000; Bedrikovetsky et al., 2001). When a suspension of particles is injected into a porous medium, some particles are deposited on the pore wall and trapped in the pore throats. Following Saripalli et al. (2000) and Bedrikovetsky et al. (2001) and assuming incompressible flow, negligible change in rock porosity and negligible effect of diffusion, the process of deep bed filtration can be mathematically described as

$$\phi \frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = -\frac{\partial \sigma}{\partial t} \tag{11}$$

Here, ϕ is the porosity, c is the suspended particle concentration, u is the Darcy velocity, and σ is the volume concentration of deposited particles per unit filter volume. Particle capture kinetics is expressed by (Iwasaki et al., 1937)

$$\frac{\partial \sigma}{\partial t} = \lambda c u \tag{12}$$

Filtration coefficient λ, which is the particle capture probability per unit length of its trajectory, is a key factor in particle deposition process and depends on large number of parameters (Pang and Sharma, 1997; Bedrikovetsky et al., 2001). According to PWRI-JIP (2003) workshop reports, constant filtration coefficient was assumed in most cases of reported studies. Pang and Sharma (1997) and Saripalli et al. (2000) mentioned that filtration coefficient can also be obtained from treatment of core (filter) experimental data or can be estimated from empirical correlations. Different functions describing the dependency of filtration coefficient to deposited particles concentration can be found in paper by Zamani and Maini (2009).

Following Pang and Sharma (1997) and Bedrikovetsky et al. (2001) the modified Darcy's law for impaired porous media is

$$u = -\frac{k(\sigma)}{\mu} \frac{\partial p}{\partial x}, \quad k(\sigma) = \frac{k_o}{1 + \beta \sigma} \tag{13}$$

where, k_o is the initial rock permeability and β is formation damage coefficient. Formation damage coefficient represents the increase of reciprocal to permeability per unitary concentration of the retained particles (Bedrikovetsky et al., 2001). Different equations have been used by other researchers for describing permeability reduction due to particle plugging in porous media (Civan, 2011).

Solution of the above system of equations results in calculation of permeability reduction and related decrease in injectivity due to particle plugging inside the porous media. The above equations contain two main empirical coefficients to be determined from experimental analysis or field data treatment: filtration and formation damage coefficients (λ, β). As stated by Bedrikovetsky et al. (2001) knowledge of these two parameters is essential for injectivity decline prediction.

2.7. Mechanical equilibrium of a particle on the cake surface

Stabilisation of injectivity has been observed in several water injection wells and explained by the stabilisation of cake thickness (Sharma et al., 2000; De Paiva et al., 2006; Zinati et al., 2009). Cake stabilisation also has been observed in crossflow (dynamic) colloidal filtration using special coreflood setup for drilling fluids and water injection studies by Jiao and Sharma (1994), Al-Abduwani et al. (2005a), and Costier et al. (2009). As shown in Figure 3b, different attaching (permeate and electrical) and detaching (drag, gravitational and lifting) forces are exerted on a particle at the cake surface. The equilibrium of attaching and detaching torques results in stabilised cake thickness (Kalantariasl et al., 2015).

$$\left(F_p + F_e - F_l\right) l_{\mathrm{n}} = \left(F_d + F_g\right) l_d \tag{14}$$

Equation (14) has been extensively used by Schechter (1992), Khilar and Fogler (1998), Torkzaban et al. (2007), Bedrikovetsky et al. (2011) and Bradford et al. (2011) for modelling of particle detachment in porous media. Moreover numerous experimental investigations of crossflow filtration in membrane science have shown flux declining period followed by stabilization (Elzo et al., 1998; Faibish et al., 1998; Song and Singh, 2005; Singh and Song, 2007). Steady-state flux has been explained by equilibrium cake thickness due to erosion of particles from the cake surface. It should be mentioned that "fouling" is an equivalent term to "formation damage" in membrane science since it represents flux decline due to particle invasion into the membrane pores and accumulation on the membrane surface during constant pressure operation. In cross-flow membrane filtration, fluid containing feed suspension flows parallel to the membrane surface and the permeate flows through the membrane due to differential pressure across the membrane. Permeate flow brings the colloidal particles toward the membrane surface (Fig. 4). The colloidal particles can go through the membrane and deposit on the internal pore surface, block the membrane pores, bridge or simply deposit on the membrane surface (cake formation). That results in flux decline. In contrast to the dead-end filtration, cake thickness is limited to the shear rate resulting from tangential flow. Upon the mechanical equilibrium of the forces exerting the single particle on the cake surface, the cake thickness reaches its equilibrium value and flux stabilizes (Vyas, 1999).

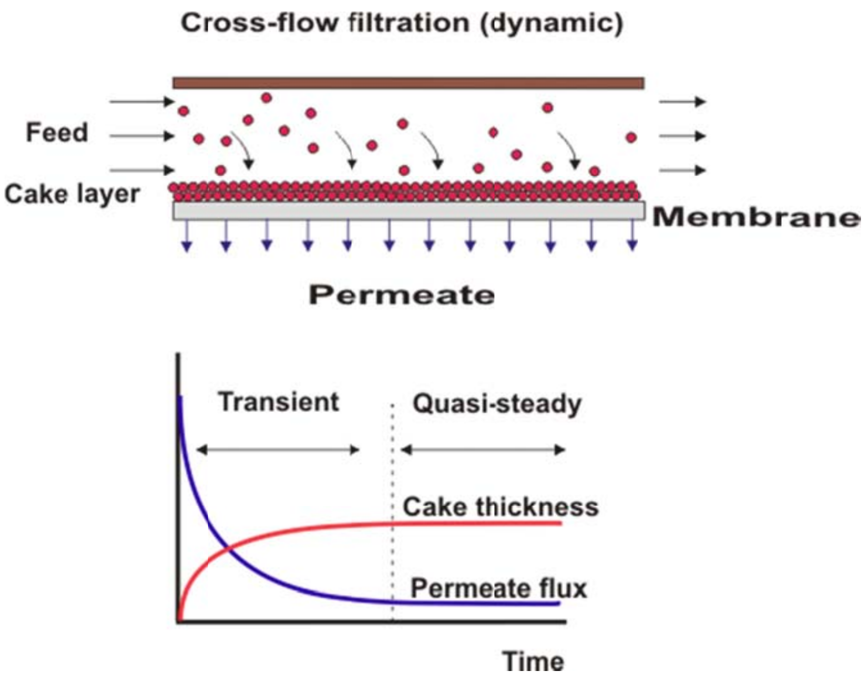

Figure 4. Schema of cross-flow filtration test, permit flux decline and cake growth

The balance between different forces exerted on the solid particle deposited on the cake surface determines the stability of the particle on the cake surface. Gradual deposition of the particles on the cake surface increase the cake thickness and decrease the cross section available for tangential flow and cause increase in shear force. The gradual increase in cake thickness increase the resistance to flow into porous membrane and decease the permeate flux. The shear force exerted by the tangential flow parallel to the membrane surface can cause a particle removal (dislodgment) from the cake surface and keep it at a relatively low level. The thickness of the external filter cake strongly depends on the force/torque balance on the cake surface. Ripperger and Altmann (2002) reported that after a long time, a quasi-steady state flow is reached that means cake thickness is also stabilised at an equilibrium level. Several experimental investigations of crossflow microfiltration in membrane science performed d by Elzo et al. (1998), Faibish et al. (1998), Song and Singh (2005), and Singh and Song (2007) confirmed the strong role of different parameters such as tangential velocity, particle size, particle size distribution, particle shape, salinity and pH of carrier fluid, ion valency of salts, and applied pressure.

2.8. Injectivity decline models

One of the key parameters in designing water injection wells is the prediction of well injectivity with time. Several core flooding tests have provided some insight into different mechanisms of formation dame processes and yielded rules of thumbs and qualitative measures. However, as stated by Todd et al. (1984), Patton (1990) and, Civan (2011), the injectivity decline observed in the core studies is often not observed in practice and application of such results has often been unsatisfactory in the field. For this reason mathematical models are used to predict injectivity performance and determine the economic life of an injector and also optimum intervals for well treatment (Civan, 2011).

According to Pang and Sharma (1997), most of the models have been proposed for prediction of water well injectivity decline; consider either deep bed filtration or cake formation. However in reality both internal deep bed filtration and external filter cake formation occur and those models separating them are oversimplified. Therefore, Liu and Civan (1994), Pang and Sharma (1997), Civan (2011), Tiab and Donaldson (2011) discussed that a proper model for injectivity decline requires coupling of both deep bed filtration and external filter cake.

Injectivity ratio is defined as the ratio of current injectivity index to the initial (non-damage) injectivity index at the beginning of water injection.

$$\theta = \frac{II(t)}{II_0} = \frac{\left(\frac{q}{\Delta p}\right)_t}{\left(\frac{q}{\Delta p}\right)_0} \quad (15)$$

Here q is the injection rate and Δp is the pressure difference between injector and reservoir. The reciprocal of injectivity ratio is referred to as "impedance" which is widely used for modelling of injectivity impairment (Bedrikovetsky et al., 2001; da Silva et al., 2005). In this section a brief overview of injectivity decline models is presented.

Barkman and Davidson (1972) proposed the first model for prediction of well injectivity impairment due to suspended solids by one or more of the four following mechanisms:

Invasion: The suspended solids in the injected water invade the host formation and cause permeability reduction near wellbore by surface deposition, bridging and internal cake formation.

Wellbore narrowing: The suspended solids in the injected water form a filter cake on the well wall without penetration into the formation.

Perforation plugging: The injected solids plug the perforation holes in perforated wells.

Wellbore fill-up: The solids settle to the bottom of the well by gravity and decrease the effective net reservoir thickness.

Pressure drop and particle material balance have been used to develop mathematical models that expresses the time when the current injectivity reduce to a fraction (Θ) of initial injectivity as described in (15). Injector half-life was also defined as the time when the injectivity index has reduced by 50% of its initial value, i.e Θ=1/2.

Injectivity model for particle invasion was based on classical deep bed filtration as described by Herzig et al. (1970). The injector half-life due to particle invasion was expressed as (Barkman and Davidson, 1972)

$$t_{1/2} = \frac{3\pi r_a^2 H \rho_s \phi^2}{\omega \rho_w Q_0} \left(\frac{k_c}{k_o} \right) \ln \left(\frac{r_e}{r_w} \right) \qquad (16)$$

Here, r_a is invasion radius, H is the formation thickness, ρ_s is the solid density, ρ_w is carrier liquid density, ϕ is formation porosity, ω is the weight concentration of solids in water (ppm), r_w is wellbore radius, r_e is drainage radius, Q_0 is the initial injection rate, k_c and k_o are external filter cake and host formation permeabilities respectively.

For a wellbore narrowing mechanism (external cake formation, k_c/k_o>0.05), the injector half-life was given as

$$t_{1/2} = \frac{\pi r_w^2 H \rho_s}{2\omega \rho_w Q_0} \tag{17}$$

The key element in Barkman and Davidson (1972) models is water quality ratio that is the ratio of solid concentration to the filter cake permeability (ω/k_c) which can be obtained by onsite membrane test. Mitchell and Finch (1981) and Hofsaess and Kleinitz (2003) reported that predicted lifetimes of water injection wells are orders of magnitude lower than actually observed values. One of the main reasons of model prediction deviation from field observations was believed to be the determination of model parameters with membrane filtration test as pore geometry of membrane is different from core samples (Vetter et al., 1987; Hofsaess and Kleinitz, 2003). As an alternative approach, core flood data was proposed for injectivity prediction. Hofsaess and Kleinitz (2003) explained that due to lack of documented success with coreflood-based water injectivity prediction, the validity and accuracy of the model is not ascertained.

Davidson (1979), in an attempt to modify the Barkman and Davidson's model, proposed to use sandstone cores to describe the solid particle invasion into the rock and also developed a new model for injector half-life due to particle invasion mechanism assuming a linear decline rate.

Based on experimental results, Eylander (1988) introduced an approach to identify the internal filtration of solid particles by defining a new parameter as filter cake porosity. He also assumed linear resistance of thin external filter cake (due to low concentration of particles in the injected water) and modified the Barkman and Davidson model (PWRI-JIP, 2003). Eylander (1988) finally developed models for both internal and external cake filtration. The modified injectivity decline models for internal solid invasion and external filter cake formation based on experimental investigations are express in (18) and (19) respectively.

$$t_{1/2} \approx \frac{4\pi r_a^2 H \phi (1-\phi_c)}{\omega Q_0} \left(\frac{\phi k_c}{k - \phi k_c} \right) \ln \left(\frac{r_e}{r_w} \right) \tag{18}$$

$$t_{1/2} \approx \frac{4\pi r_w^2 H(1-\phi_c)}{\omega Q_0}\left(\frac{k_c}{k}\right)\ln\left(\frac{r_e}{r_w}\right) \quad (19)$$

In both models (18, 19) the filter cake porosity and permeability should be determined from laboratory tests prior to using the model for injectivity predictions. Eylander (1988) assumed Kozeny-Carman relationship to obtain cake permeability from core-flood data. No specific method was proposed to determine the type of permeability impairment in Eylander's model. Moreover, the inflow equation does not include time-dependent depth of impairment. As reported by PWRI-JIP (2003), the method has not been validated with field data.

van Oortt et al. (1993) developed a semi-empirical radial model for prediction of well injectivity impairment by internal filter cake formation caused by invasion of solid particles in the injected water. Two new parameters, the damage factor and the volume filtration coefficient were introduced that can be obtained from core-flood test. Experimental results were used to obtain model constants for prediction in radial flow. van Oortt et al. (1993) showed that their experimental investigations confirmed "1/3-1/7" filtration rule as initially proposed by Abrams (1977) and suggested this filtration rule as a criterion for identifying internal cake formation when using their model. The model is only applicable for internal filtration. Field data treatment by Hsi et al. (1994) and van der Zwaag and Øyno (1996) showed that the model prediction was not satisfactory compared to real performance in Prudhoe Bay and Norwegian Ula fields. It should be mentioned that it was believed that the model was not fully applicable for prediction of injectivity decline in these fields due to oily water injection and possible thermal fracturing.

Khatib (1994) showed that Eylander's model is sensitive to the type of particles and concluded that type of suspended solids and the compressibility of external filter cake are also important parameter in injectivity prediction. To address this problem, several experiments were performed to determine the relationship between external filter cake porosity and permeability for cake forming compressible matters in water injection wells. Both oil free and oil coated particles have been used. The results of oil coated silt/clay solids showed additional 50% reduction in permeability. Other types of solids also showed similar results. 10 empirical correlations were proposed based on extensive experimental data in exponential and power forms for FeS, $Fe(OH)_3$, $CaCO_3$, $CaSO_4$ and Silt/Clay particles.

All above mentioned models use separately either internal or external cake filtration as the main mechanism of injectivity decline. Pang and Sharma (1997) suggested that both deep bed filtration and external filter cake occur in reality and assumed that both perforation plugging and wellbore fillup, as proposed by Barkman and Davidson (1972), are an extension of external filter cake.

In order to simultaneously model both deep bed filtration and cake formation, Pang and Sharma (1997) introduced the concept of transition time by analysing column experimental data. They showed that particles initially invade the porous media and are captured into the porous spaces (deep bed filtration). Continuous injection of particles into porous media yields more and more deposition of particles mainly very close to the injection face up to the moment when very few particles can penetrate the host formation. From this time on, the injected particles are deposited on the injection face (or well wall in injection wells) and form a low permeable porous layer on the injection face. The time at which penetration of particles into porous media stops and cake formation starts was called "transition time". The transition time occurred when the initial rock porosity near the injection face reached a critical value, i.e. α-th fraction of initial porosity. Different values for critical porosity have been used in the literature. Sharma et al. (2000) used α=0.5 while da Silva et al., (2004) proposed α =0.09 based on laboratory measurements data.

Pang and Sharma (1997) finally, developed a phenomenological model that accounts for both deep bed filtration (internal filtration) and external filter cake formation. The model was developed based on mass balance of suspended and deposited particles with a kinetic equation for particle deposition in deep bed filtration as described in (11-13). The model requires prior knowledge of filtration and formation damage coefficients. They proposed Kozeny-Carman model for cake permeability calculation in the absence of coreflood data. Latter, Sharma et al., (2000) presented injectivity decline equations in terms of flow resistance in series: resistances of undamaged reservoir, internal damaged section and external filter cake. The proposed model was used for matching and interpretation of well injectivity decline data in Gulf of Mexico (Sharma et al., 2000). The model does not account for injectivity stabilisation and predicts unlimited injectivity decline. According to PWRI-JIP workshop reports (2003), too many uncertain input parameters make the model less attractive.

Using system of equations of particle mass balance, capture kinetics and modified Darcy's law, da Silva et al., (2004) developed impedance growth for deep bed filtration and external cake formation in axi-symmetric flow. With the assumption of cake incompressibility, impedance grows linearly with the amount of injected particles. The model contains four empirical constants: filtration and formation damage coefficients, critical porosity fraction and external cake permeability. These parameters can be obtained from matching of field/experimental data. da Silva et al. (2004) also developed an empirical correlation between formation damage coefficient and critical porosity fraction.

$$\beta = 94.949\alpha^{-0.4089} \tag{20}$$

Using extensive laboratory measurements, da Silva et al (2004) also proposed an average value of 0.09 for critical porosity. The model also has been used for theoretical definition of damage zone and treatment of well acidizing performance by Nunes et al. (2010).

As mentioned before, injection of colloids and suspensions in natural reservoirs with particle capture results in well injectivity decline. However, some initial improvement in injectivity was observed during waterflooding of oilfields and explained by increasing mobility of two-phase fluid during the displacement of more viscous oil by water (Altoe et al., 2004).

Recently Hwang and Sharma (2014) studied filtration of solid particles in the injection water in a frac pack and its impact on injectivity decline and injector performance. They also developed new empirical correlations for filtration coefficient at high flow rates in frac packs. Ochi et al (2014) discussed uncertainties in re-injection of produced water using distribution of affecting parameters and proposed a procedure for produced water reinjection projects. Jin and Wojtanowicz (2014) presented a model for oily water injection during linear flow in porous media.

All above mentioned models predict unlimited external cake growth and hence unlimited injectivity decline and impedance growth. Several field and experimental data show stabilisation of injectivity during cross-flow filtration of particles. Different mechanism can be responsible for well injectivity stabilisation. Dynamic fracture propagation due to increase in pressure above formation fracturing pressure, invasion and plugging of small pores in a widely distributed pore size and dynamic equilibrium of particles on the cake surface are common reasons. The latter received more attention in the literature (Al-Abduwani et al., 2005a; de Paiva et al., 2006; Zinati et al., 2009; Yuan et al., 2012). The same process occurs during drilling where upward drilling fluid in vertical well detaches the particles from the mud cake surface. In an attempt to model dynamic filtration of drilling mud, Jiao and Sharma (1994) and Civan (1998) applied force balance equation to determine the conditions of particle erosion from the cake surface. As shown in Figure 3b different forces are acting on a single particle at the cake surface.

Jiao and Sharma (1994) conducted experimental study for filtration of bentonite drilling mud with crossflow filtration apparatus. They developed a simple force balance model to find the maximum particle size that can deposit on the cake surface. Only hydrodynamic permeate and drag forces have been considered. Civan (1998) modelled incompressible external filter cake formation for both linear and radial systems. Deposition and erosion rate was applied to model the net volume of cake forming particles. The erosion rate was assumed to be proportional to the difference between shear stress and critical shear stress on the cake surface, where critical shear stress was the minimum shear stress to dislodge the particle from the cake surface. Shear stress was assumed only based on hydrodynamic force of power-law fluids, i.e. electrostatic forces were neglected (Civan, 1998).

Al-Abduwani et al. (2005a) performed experimental investigation for both deep bed filtration and external filter cake formation using hematite as suspended particles in water. A crossflow filtration set up was used to simultaneously investigate the deep bed filtration and external filter cake formation on the core surface. The experimental results showed that the cake thickness at the inlet of core surface is smaller than at the outlet section. A model was also proposed for calculation of external filter cake profile along a linear and radial system based on force balance approach. Electrostatic forces and factor for permeate drag force were ignored in that study. A constant friction factor in their model is to be obtained from experimental study. Moreover, the presented model has not been validated with experimental and field data.

De Paiva et al. (2006) proposed erosion number to describe the injectivity stabilisation. Average erosion number from treatment of experimental data has been proposed for prediction of well injectivity stabilisation. The proposed erosion number does not fit with well data considering all affecting forces. Zinati et al. (2009) developed a mathematical model for calculation of steady-state cake thickness profile in a radial system using numerical methods. They assumed a constant value for friction factor in force balance equation on the cake surface. The proposed model does not account for electrostatic forces, gravity force and correction factor for permeate drag force. Like Al-Abduwani's model, friction coefficient to be assumed or obtained from laboratory test. Yuan et al. (2012) included gravity force but did not account for electrostatic forces and correction factor for permeate drag force and friction factor was also assumed.

Costier et al. (2009) performed cross-flow experimental test with core samples to investigate the effects of different parameters on permeability impairment during produced water re-injection process. A core-flood rig was designed and built with capability of high injection pressure and much extended test duration. Based on coreflood tests for two oilfields, they concluded that: leak-off dynamics at high pressure difference cannot be extrapolated from low pressure difference; filter cake permeability depended on core sample permeability; steady-state conditions can be achieved in long term exposure, and membrane tests (Barkman and Davidson's method) fail to produce representative external filter cake properties.

With increase of water production from oil and gas fields, and strict environmental regulations, PWRI is the best option. Therefore, clear understanding of formation damage mechanisms and performance in water injection wells is essential for successful control of formation damage and waterflood/disposal design. A critical analysis allows concluding that despite extensive efforts have been done to understand the formation damage, lack of complete understanding persists and prediction of well injectivity performance is still a challenge.

New mathematical models developed in this thesis address the shortcomings of the previous models and provide a full predictive tool for injectivity decline modelling. A mathematical model accounting for all affecting colloidal forces is proposed to investigate the equilibrium conditions of particle on the cake surface. Obtained values of empirical parameters of proposed mechanical equilibrium equation are in good agreement with Hertz theory of particle deformation which can be used to determine the stabilisation time and stabilised impedance value in the absence of experimental or field data. The developed analytical model for prediction of non-uniform cake profile along injection wells in this thesis can be used for sensitivity analysis of different parameters and helps in the design of waterflood and disposal projects.

An analytical model for prediction of non-monotonic injectivity behaviour and a simple procedure is developed and can be used for interpretation of injection well history and reservoir characterization. The performance of water injection in very low permeable formations below fracturing pressure has not been address properly in the literature. Experimental and theoretical studies for re-injection of high salinity produced water are performed in this thesis can provide new insights for injectivity decline performance in low permeable formations. The main phenomenological models required too many input parameters. Probabilistic histograms of empirical injectivity damage parameters are obtained from treatment of several published field data provide a comprehensive tool for full prediction of injectivity decline in sea water injection and PWRI.

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Chapter 3

Mathematical Modelling of Well Injectivity Stabilisation

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Modelling of external filter cake profile along the well during drilling

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ABSTRACT

This paper presents a new mathematical model to predict the steady-state external filter cake thickness distribution and velocity profile along the wellbore during overbalanced drilling. Several models have been suggested for the prediction of external cake thickness using the force balance method. Yet, a comprehensive literature survey reveals that electrostatic forces and the permeate force correction factor have been neglected, while both can significantly change the conditions of particle detachment from the cake surface. Torque balance of hydrodynamic (lifting, tangential and permeate drag), gravity and electrostatic (DLVO) forces along with Darcy's law and material balance is used to investigate the conditions of particle attachment/detachment on the cake surface. The results show strong effects of mud chemistry, particle size, cake permeability, tangential flow velocity, overbalance pressure, and Young's modulus on the external filter cake thickness and velocity profile. The mathematical model can be applied as a predictive tool for the estimation of filter cake thickness. It allows for the calculation of external filter cake distribution using the physiochemical properties of mud and particles.

KEYWORDS

Formation damage, external filter cake, torque balance, modelling, drilling.

INTRODUCTION

Dynamic filtration takes place in drilling, completion, fracturing, and water injection (Fordham et al, 1988; Jiao and Sharma, 1994; Caenn et al, 2011; Elkatatny et al, 2012; Kalantariasl et al, 2013). During overbalanced drilling operations, differential pressure between the well and the adjacent formation causes invasion of drilling fluid with solid particles into the formation. Deposition of invaded particles results in a reduction of permeability in the well vicinity area (Fig. 1). Physical trapping and continuous deposition of particles on the rock grain surfaces yield in pore narrowing and, consequently, significant permeability reduction. Particle penetration into the formation stops when certain amounts of particles are deposited inside the host formation. From this moment, the particles from the mud deposit on the well wall and form an external filter cake while mud filtrate penetrates into the formation (Fig. 1). Formation of external filter cake increases hydraulic resistance to flow and reduces invasion flux into the adjacent porous media.

Cake formation—which limits the invasion of permeable zones by the liquid phase and contributes to several operational issues such as borehole stability, differential sticking and stuck pipes—plays an important role in drilling operations (Vaussard et al, 1986; Ytrehus et al, 2013). Formation of an extremely low permeable cake layer on the well wall plasters the hole, increases the resistance of the formation and holds the wall caving in. Control of fluid loss rate and, consequently, cake thickness is also important to reduce the chance of differential sticking (Leerlooijer et al, 1996).

The properties of the mud cake highly affect the fluid invasion flux. Thus, estimation of filter cake properties (permeability, porosity, thickness profile, etc.) can lead to minimisation of formation damage during drilling, water injection, and production.

A predictive model may be used as a predictive tool to design an optimum drilling fluid to minimise formation damage during drilling operations. A detailed literature survey reveals that a comprehensive theory does not exist for dynamic filtration of drilling mud (Caenn et al, 2011). The existing models do not account for all contributing factors on the formation of external filter cake.

Several experimental studies have shown an initial stage of high fluid loss followed by constant mud invasion rate during dynamic filtration of drilling mud (Jiao and Sharma, 1994; Kerkar et al, 2008). Similar results have been reported during cross-flow filtration in membrane studies (Altman and Ripperger, 1997; Elzo et al, 1998; Hwang et al, 2006).

Deposition of suspended particles and consequent growth of external filter cake during dynamic circulation of drilling fluid is controlled by colloidal forces exerted on the particle at the cake surface (Outmans, 1963; Fordham et al, 1988, Jiao and Sharma, 1994). Tangential drag, gravitational and lifting forces tend to dislodge the particle from the surface while permeate and electrostatic forces attach the particle to the cake surface (Fig. 2). Several studies have shown that the permeate drag force (F_p) on an approaching particle to the impermeable surface becomes infinite at small gaps; that is, very close to the cake surface. Consequently, the permeate force increases as an inverse function of the separation gap and must be modified by a correction factor (Sherwood, 1988; Kang et al, 2004; Hwang et al, 2006). Jiao and Sharma (1994) proposed a mathematical model using torque balance analysis. The proposed model ignores the electrostatic force and assumes the permeate force correction factor is equal to unity. Similar models have been used to predict external filter cake profile in water injection wells (Al-Abduwani et al, 2005; Paiva et al, 2006; Zinati et al, 2009, Yuan et al, 2012). The deposition-erosion model presented by Civan (1998) also ignores electrostatic force in calculation of critical shear stress.

Electrostatic force is a major attaching force in high salinity and low pH solution conditions. Moreover, a correction factor for permeate force is also assumed to be unity. Neglecting the variation of permeate force correction factor results in the permeate force to be underestimated. It significantly changes the condition of particle stability on the cake surface, which can encourage an incorrect prediction of cake thickness.

Despite the significant role of filter cake thickness in drilling operations, it has received very little attention in the literature (Caenn et al, 2011). Prediction of cake thickness is important for cake permeability calculations, fluid loss estimation and formation damage analysis.

A mathematical model based on the torque balance of detaching and attaching forces is developed in this paper. The model includes all affecting forces and accounts for both electrostatic force and permeate force correction factor variations. Steady-state external filter cake profile along a vertical well is calculated and a sensitivity analysis to hydrodynamic and physiochemical parameters are presented.

FORCES AND TORQUES

Figure 2 presents a schematic of all forces and corresponding lever arms exerting on a single particle on the cake surface in a hydrodynamic flow field. Hydrodynamic tangential drag force (F_d) from tangential flow (cross-flow) of suspension fluid (shear stress), permeate (normal) force (F_p) from filtrate flux, hydrodynamic lift force (F_l), net gravitational force (F_g) and electrostatic force (F_e) are acting on a particle. These forces are functions of hydrodynamic and physicochemical properties of the particle-flow system.

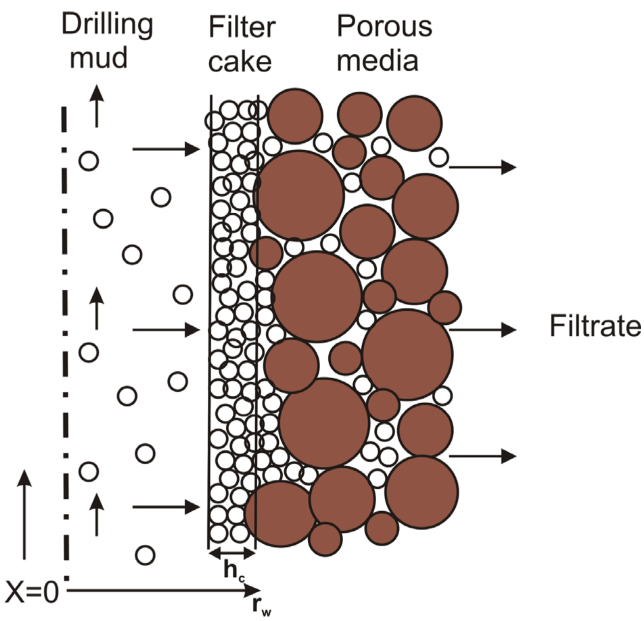

Figure 1. Dynamic filtration of mud.

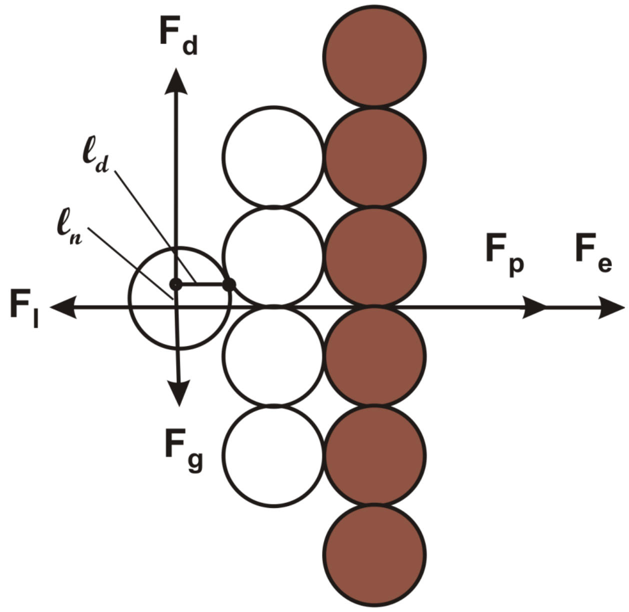

Figure 2. Schematic of forces exerted on a particle at the cake surface.

Tangential drag force

The tangential drag force (F_d) exerted on a spherical particle in contact with a plane wall is determined by the local flow field of fluid from the modified Stokes law (O'Neill, 1968; Schechter, 1992; Altman and Ripperger, 1997; Bradford and Torkzaban, 2008; Bradford et al, 2011) and is given by:

$$F_d = 6(1.9)\pi\mu r_p u_t \tag{1}$$

In Equation 1, μ is the carrier fluid viscosity, r_p is the particle radius and u_t is the fluid velocity at the distance r_p measured from the cake surface. The constant 1.9 in Equation 1 is a correction factor due to the wall effect (O'Neill, 1968). Expressing Equation 1 in terms of wall shear stress, τ, (Sharma et al, 1992) yields Equation 2.

$$F_d = 10.2\pi r_p^2 \tau \tag{2}$$

Wall shear stress for non-Newtonian power-law drilling mud (Jiao and Sharma, 1994; Civan, 1998) is presented by:

$$\tau = k' \gamma^{n'} \tag{3}$$

In Equation 3, k' and n' are the consistency constant and flow index, respectively (Jiao and Sharma, 1994; Civan, 1998), and γ is the shear rate at cake surface in radial system (Civan, 1998) that is determined by:

$$\gamma = \left(\frac{4\bar{u}_t}{r_w - h_c} \right) \tag{4}$$

In Equation 4, r_w is the well radius and h_c is external filter cake thickness.

Permeate drag force

The hydrodynamic permeate drag force can also be presented by the modified Stokes law in terms of the particle Reynolds number. The permeate force is exerted on the particle due to permeate flow velocity, which is normal to the tangential flow. Several studies have shown that the drag force on an approaching particle to the impermeable surface becomes infinite at small gaps. Consequently, the drag force must be modified by a correction factor (Sherwood, 1988; Kang et al, 2004; Hwang et al, 2006; Zinati et al, 2009). The modified form of permeate drag force is expressed by:

$$F_p = \pi r_p^2 u_p^2 \frac{1}{2} \rho_f \frac{24}{(\mathrm{Re})_p} \Phi_H \tag{5}$$

In Equation 5, u_p is the permeate velocity, ρ_f is the carrier fluid density, $(Re)_p$ is the particle Reynolds number, and Φ_H is the correction factor to permeate force. The particle Reynolds number for the power law fluid (Jiao and Sharma, 1994) is described by:

$$(\mathrm{Re})_p = \frac{2^{n'} r_p^{n'} u_p^{2-n'} \rho_f}{3^{n'-1} k'} \tag{6}$$

Substitution of Equation 6 into Equation 5 gives Equation 7, which is the final form of hydrodynamic permeate force for power law fluids.

$$F_p = 12\pi r_p^{2-n'} \frac{3^{n'-1}}{2^{n'}} u_p^{n'} \Phi_H \tag{7}$$

It is worth mentioning that for Newtonian fluids, the rheological parameters k' and n' equal μ and 1, respectively. Equation 7, therefore, is identical to Equation 1, except for correction factors (1.7 and Φ_H) in the case of Newtonian fluids.

The permeate force correction factor—Φ_H—can be obtained by (Sherwood, 1988):

$$\Phi_H = 0.36\left(\frac{k_c}{r_p^{\,2}}\right)^{\left(-\frac{2}{5}\right)} \tag{8}$$

In Equation 8, k_c is the external filter cake (medium) permeability. The value of k_c can be determined in the laboratory. The particle size also can be used to calculate cake permeability by applying different mathematical models such as the generalised Blake-Kozeny equation and Happel cell model (MacDonald et al, 1991; Saripalli et al, 2000; Kim et al, 2006).

It should be mentioned that the permeate force correction factor is highly greater than unity for most operational conditions and increases dramatically with decreasing the separation gap between the particle and surface (Kang et al, 2004). The value of the permeate force correction factor varies with cake permeability and the size of particles and, consequently, changes the permeate force. Equation 8 indicates that the permeate correction factor increases with a decrease in cake permeability and increase in particle size. Figure 3 shows the variation of permeate force correction factor versus a given interval of dimensionless cake permeability ($k_c/r_p^{\,2}$). For a range of particle radii and cake permeabilities investigated in this paper, the permeate force correction factor can vary by up to three orders of magnitude and considerably enhances the permeate force. The sensitivity of the correction factor to the cake permeability and particle size is discussed in detail in the Results and discussions section.

Lift force

The lifting force results from a gradient in the shear flow and acts normal to and away from the cake surface. Following Altman and Reprgaer (1997) and Kang et al (2004), the lifting force is described by:

$$F_l = \chi \gamma r_p^{\,3} \left(\rho_f \tau\right)^{1/2} \tag{9}$$

In Equation 9, χ is the lifting force coefficient. Bergendahl and Grasso (2000) and Kang et al (2004) give a value of 81.2 to χ, whereas Altmann and Ripperger (1997) give a value of 6.1. The lift force is significant for high shear rates and large particle sizes (Hwang et al, 2006). A lifting force coefficient (χ) of 81.2 is used for all calculations in this paper.

Net gravitational force

The net gravitational force exerted on the particle is determined by:

$$F_g = \frac{4}{3}\pi r_p^{\,3}\left(\rho_p - \rho_f\right) g \tag{10}$$

In Equation 10, ρ_p is particle density. The direction of the net gravitational force depends on the well geometry (Fig. 2).

Electrostatic force

The well-known Derjagin-Landau-Verwey-Overbeek (DLVO) theory is commonly used to describe and estimate the net interaction energy and electrostatic force of colloid-surface interaction. The electrostatic force is a derivative of the net total potential energy (Eq. 11).

$$F_e = -\frac{\partial V}{\partial h} \tag{11}$$

In Equation 11, the total energy (V) is the sum of the London-van-der-Waals (Eq. 12, V_{LVA}), double electric layer (Eq. 13, V_{DLR}) and Born (Eq. 14, V_{BR}) potentials, given by DLVO theory (Khilar and Fogler, 1998; Israelachvili, 2006).

$$V_{LVA} = -\frac{A_{132}}{6}\left[\frac{2(1+Z)}{Z(2+Z)} + \ln\left(\frac{Z}{2+Z}\right)\right] \quad ; \quad Z = \frac{h}{r_p} \tag{12}$$

$$V_{DLR} = \frac{\varepsilon_0 D r_p}{4}\left[2\psi_{01}\psi_{02}\ln\left(\frac{1+\exp(-\kappa h)}{1-\exp(-\kappa h)}\right) - \left(\psi_{01}^2 + \psi_{02}^2\right)\ln\left(1-\exp(-2\kappa h)\right)\right] \tag{13}$$

$$V_{BR} = \frac{A_{132}}{7560}\left(\frac{\sigma_{LJ}}{r_p}\right)^6\left[\frac{8+Z}{(2+Z)^7} + \frac{6-Z}{Z^7}\right] \tag{14}$$

$$V = V_{LVA} + V_{DLR} + V_{BR} \tag{15}$$

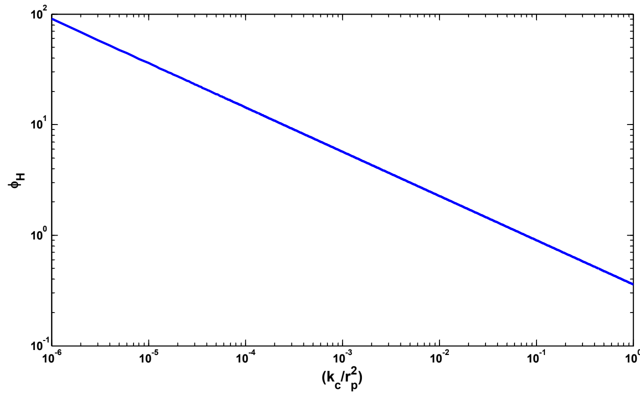

Figure 3. Variation of hydrodynamic permeate correction factor with dimensionless permeability.

In Equations 12–15:

- A_{132} is the Hamaker constant;
- h is the surface-to-surface separation distance;
- ε_0 is the electric constant (permittivity of free space), which equalas 8.854×10^{-12} $C^2J^{-1}m^{-1}$;
- D_e is the dielectric constant, which equals 78.0;
- ψ_{01} and ψ_{02} are the surface potentials of the particles and cake, respectively; and,
- σ_{LJ} is atomic collision diameter in Lennard-Jones potential, which equals 0.5 nm (Khilar and Fogler, 1998).

The inverse Debye length (κ) is:

$$\kappa = \sqrt{\left(\frac{e^2 \sum \nu_i z_i^2}{\varepsilon_0 D k_B T}\right)} \tag{16}$$

In Equation 16, k_B is Boltzmann's constant, n_i is a bulk i-th ion concentration as defined by the number of ions per unit volume, z_i is a valence of the i-th ion, and e is the electron charge ($e = 1.6 \times 10^{-19}$ C). For aqueous solutions under normal temperature (25°C), and using universal constants for k_B, D and ε_0 (Elemelech et al, 1995) Equation 16 simplifies to Equation 17.

$$\kappa = 0.73 \times 10^8 \sqrt{\sum C_{mi} z_i^2} \tag{17}$$

In Equation 17, C_{mi} is the molar i-th ion concentration in moles/m³ (Elimelech et al, 1995).

Since net electrostatic force is a non-monotonic function of separation distance (h), the maximum electrostatic force is used in the calculations; that is, separation distance corresponds to the second derivative of the potential energy. A detailed discussion of this is presented in Bedrikovetsky et al (2011).

DETACHMENT MECHANISMS

The detachment of a particle from the cake surface depends on the equality of detaching and attaching torques. Lifting, sliding and rolling are potential mechanisms of particle detachment from the surface. Force and torque balance analysis have been used to describe the criterion of particle removal from the surface by different mechanisms (Sharma et al, 1992; Jiao and Sharma, 1994; Soltani and Ahmadi, 1994; Bergendahl and Grasso, 2000; Torkzaban et al, 2007; Zoeteweij et al, 2009). If lifting is the prevailing mechanism, the lift force must be greater than the attaching forces (Eq. 18).

$$F_l \geq (F_p + F_e) \tag{18}$$

Figure 4 shows the lifting force is significantly below the attaching electrostatic and permeate forces for a wide range of particle sizes.

Several studies have also neglected the lifting force in cross-flow filtration (Jiao and Sharma, 1994; Hwang et al, 2006). The criterion of particle sliding is presented in Equation 19.

$$\left(F_d - F_g\right) = \mu_f \left(F_p + F_e - F_l\right) \tag{19}$$

In Equation 19, μ_f is the static friction coefficient. A known value of static friction coefficient is needed to evaluate the performance of the sliding mechanism. Several theoretical and experimental investigations have shown that the proportionality coefficient in Equation 19—μ_f—is not constant and depends on flow conditions and particle-surface physical properties (Sharma et al, 1992; Elzo et al, 1996; Altman and Ripperger, 1997; Heim et al, 1999).

Torque balance is used to describe the criterion of rolling mechanisms (Jiao and Sharma, 1994; Soltani and Ahmadi, 1994; Bergendahl and Grasso, 2000, 2003; Torkzaban et al, 2007; Bradford and Torkzaban, 2008; Zoeteweij et al, 2009).

$$\left(F_d - F_g\right) l_d = \left(F_p + F_e - F_l\right) l_n \tag{20}$$

In Equation 20, l_d and l_n are lever arms for tangential and normal forces, respectively, as shown in Figure 2.

The values of lever arms l_n and l_d and the friction coefficient μ_f are related to each other (Eq. 21; Bradford et al., 2011).

$$\mu_f^{-1} = \frac{l_d}{l_n} \tag{21}$$

Sharma et al (1992) and Elzo et al (1996) experimentally showed that rolling is the dominant mechanism of particle detachment from the glass and membrane surfaces, respectively. Their analysis shows that the proportionality coefficient in Equation 19 depends on particle size, and the physical properties of particle-surface and net normal force on the particle attached to the surface. Based on numerous experimental and theoretical studies, it is widely accepted that rolling is the dominant mechanism of micron-size particle detachment from the surface (Sharma et al, 1992; Torkzaban et al, 2007; Zoeteweij et al, 2009; Bradford et al, 2011). Equation 20 is extensively used to describe the particle detachment performance from the grain surface in porous media. According to the different mechanisms, the torque balance in Equation 20 is used to calculate the equilibrium cake thickness profile in a vertical well during drilling.

EQUILIBRIUM CAKE THICKNESS

Darcy's law is applied for the radial geometry of two porous mediums in series (external filter cake and damaged formation; see Fig. 1). The permeate fluid flux per unit length of formation thickness—$q_p(x)$—is described by:

$$q_p(x) = \frac{2\pi k_o}{\mu} \frac{(P_w - P_r)}{\ln\left(\frac{r_e}{r_w}\right)} \left[\frac{k_o}{k_c} \frac{\ln\left(\frac{r_w}{(r_w - h_c(x))}\right)}{\ln\left(\frac{r_e}{r_w}\right)}\right]^{-1} \tag{22}$$

In Equation 22, k_o is the formation permeability, P_w is the well pressure, P_r is the formation pressure, and x is the distance from the bottom of the well (Fig. 1). Permeate velocity u_p is defined in Equation 23.

$$u_p(x) = \frac{q_p(x)}{2\pi\left(r_w - h_c(x)\right)} \tag{23}$$

Tangential flow rate in terms of the average tangential velocity is expressed by Equation 24.

$$Q_t = 2\pi\left(r_w - h_c\right)^2 \bar{u}_t \quad (24)$$

The local tangential volumetric rate along the wellbore changes due to leak-off flux into the formation, $q_p(x)$, and can be determined from volumetric fluid balance (Eq. 25).

$$\frac{\partial Q_t}{\partial x} = -2\pi\left(r_w - h_c\left(x\right)\right)u_p\left(x\right) \quad (25)$$

Substitution of Equations 2, 7, 10 and 11 into Equation 20, and neglecting the lifting force, yields a final mathematical formula (Eq. 26) to calculate the equilibrium thickness of external filter cake (h_{cr}) at each point along the drilled formation.

$$\left(10.2\pi r_p^{\,2} k'\left(\frac{4\bar{u}_t}{\left(r_w - h_{cr}\left(x\right)\right)}\right)^{n'} - \frac{4}{3}\pi r_p^{\,3}\left(\rho_p - \rho_f\right)g\right) l = \left(12\pi r_p^{2-n'}\frac{3^{n'-1}}{2^{n'}}u_p^{n'}\left(x\right)\Phi_H + F_e\right), \quad l = \frac{l_d}{l_n} \quad (26)$$

In Equation 26, u_p is defined from Equation 22 as a function of cake thickness and l is the lever arm ratio.

Determination of the lever arm ratio or static friction coefficient is important for any force/torque balance analysis. The lever arm ratio highly affects the prediction of particle deposition/detachment on the cake surface and, consequently, the cake structure and properties.

Jiao and Sharma (1994) proposed a value of $3^{1/2}$ for the lever arm ratio. Other values of lever arm ratio have been used in literature (Al-Abduwani et al, 2005; Paiva et al, 2006; Zinati et al, 2009; Yuan et al, 2012). As mentioned in the Introduction section, permeate force correction factor and electrostatic force were neglected in the force/torque balance analysis. Several experimental studies have shown the strong effect of salinity and electrostatic force on flux reduction performance during membrane cross-flow filtration (Elzo et al, 1998; Faibish et al, 1998). Previously used lever arm ratios, however, cannot systematically predict the cake thickness values when electrostatic force and permeate force factor are accounted in the torque balance model.

Numerous studies have used the torque balance equation (Eq. 20) to evaluate and predict particle detachment from the rock grain surface in natural and engineered porous media. Johnson-Kendall-Roberts (JKR) theory has been used to calculate the deformation contact radius (l_n, see Fig. 5). According to this theory, l_n varies with particle size, particle-surface physical properties, and net normal force (Sharma et al, 1992; Elzo et al, 1996; Bergendahl et al, 2000; Torkzaban et al, 2007; Bradford et al, 2011).

Recent visualisation studies show the upper layer of the external cake has almost zero porosity; that is, the face of the cake subject to slurry has minimum permeability (Elkatatny et al, 2012). Extremely low permeable external filter cake formation has been reported for both experimental studies (Jiao and Sharma, 1994) and water injection wells (Kalantariasl et al, 2013), which supports the assumption of an almost smooth area of solid filter cake. Moreover, particles in high-salinity conditions also form highly compacted compressible cake due to a small separation distance between deposited particles on the well wall. Similar conclusions have been drawn with theoretical force balance analysis. Increasing cake thickness increases the shear rate and tangential drag force; so, the thicker the cake is, the smaller a particle can deposit on the cake surface and upper layers of the cake become less porous. This explanation supports the assumption of an almost flat surface of the external filter cake (Kalantariasl et al, 2013).

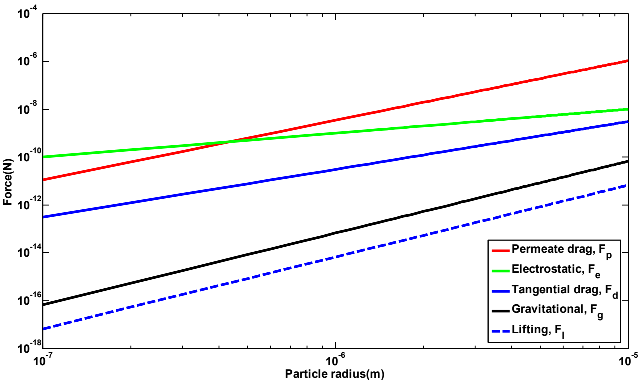

Figure 4. Comparison of forces versus particle radius ($u_t = 0.1$ m/s, $r_w = 0.15$ m, $A_{132} = 3$ zJ, zeta potential = −8 mV, salinity = 3% NaCl, $\Delta\rho = 1{,}600$ kg/m^3, $k_c = 0.01$ mD, $k_o = 500$ mD, $\Delta P = 500$ Psi, $k' = 0.7$ Pas$^{n'}$, and $n' = 0.313$).

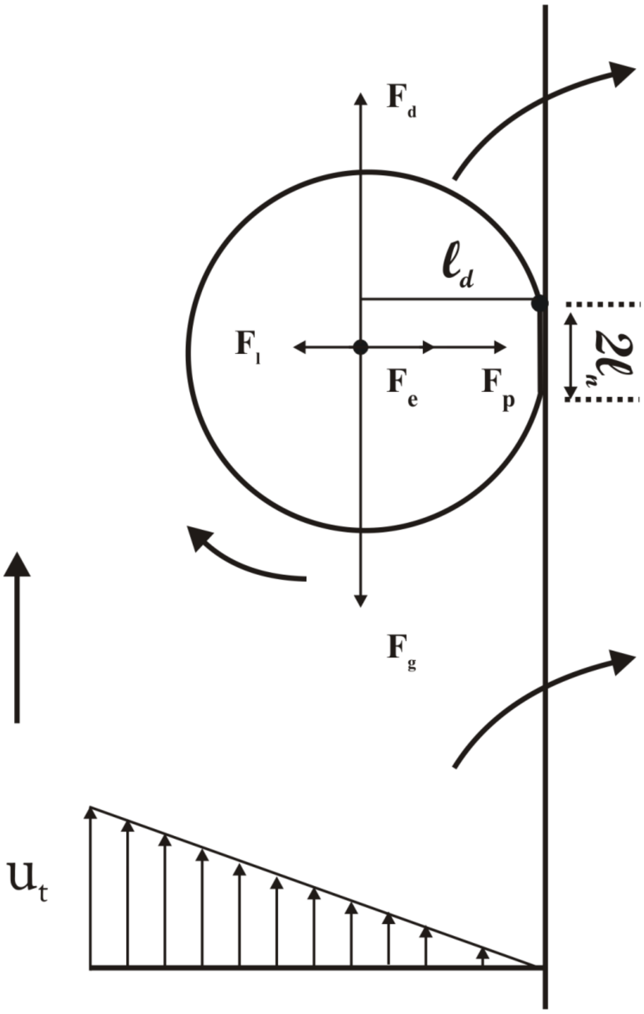

Figure 5. Presentation of lever arms and forces in vertical section upward fluid flow.

In this paper, the Hertz-based theory of deformation (JKR theory) is used to calculate the radius of the contact area that is assumed to be equal to normal lever arm l_n (Fig. 5). It provides a mechanistic basis for the determination of particle deposition-detachment criterion and dynamic cake properties. A similar method was performed to describe the cross-flow microfiltration of lactinum particles (Vyas et al, 2001). Net normal force (permeate, electrostatic), particle radius and physical properties of the particle and cake surface are used to calculate the normal lever arm, l_n, which is expressed as:

$$l_n = \left(\frac{\left(F_e + F_p\right) r_p}{K} \right)^{\left(\frac{1}{3}\right)} \tag{27}$$

In Equation 27, K is the composite Young modulus (Bergendahl and Grasso, 2003) and is described by:

$$K = \frac{4}{3\pi \left(\frac{\left(1-\sigma_p^{\ 2}\right)}{\pi E_p} + \frac{\left(1-\sigma_c^{\ 2}\right)}{\pi E_c} \right)} \tag{28}$$

In Equation 28, σ is Poisson's ratio, E is Young's modulus of elasticity, and subscripts p and c refer to the particle and cake surface, respectively. Both the cake and particle are assumed to have the same Poisson ratios and Young moduli. The contact area size is significantly smaller than the particle radius—$l_n \ll r_s$—(Fig. 5), hence the tangential lever arm (l_d) is assumed to be equal to the particle radius (Vyas et al, 2001; Torkzaban et al, 2007; Bradford et al, 2011; Kalantariasl and Bedrikovetsky, 2014; Kalantariasl et al, 2014).

At the entry point ($x = 0$), tangential velocity is equal to the maximum tangential velocity. Along the tangential flow direction, due to permeate leak-off into the formation, the tangential flow rate, and subsequently the tangential velocity, decreases. Equations 23 and 25–27 are used to calculate the filter cake thickness, local permeate flux and tangential flow rate. The equality of attaching and detaching torques (Eq. 26) determines the equilibrium cake thickness at each point. The system of Equations 23 and 25–27 is solved numerically to find the filter cake thickness profile along the well. The average cake thickness can be calculated from integration of the obtained thickness along the drilled formation column.

RESULTS AND DISCUSSIONS

The mechanical equilibrium of a single particle on the filter cake surface during drilling is described by the equality of detaching and attaching torques. Electrostatic and permeate forces attach the particle to the cake surface while tangential drag, gravitational and lifting forces tend to detach it. Deposition of a particle on the cake surface depends on the equality of detaching and attaching torques from the mentioned forces. If the attaching torque exceeds the detaching torque, particle deposition on the cake surface occurs; otherwise, particles cannot be deposited on the cake surface and are carried away by the flowing mud. The torque balance on the cake surface is used to calculate the equilibrium cake thickness profile and evaluate the effect of physiochemical and hydrodynamic parameters including particle size, cake permeability, tangential flow velocity, differential pressure, and Young's modulus of particles.

Figure 3 shows the plot of permeate force correction factor versus dimensionless permeability (Eq. 8). It can be seen that the permeate force correction factor varies by up to three orders of magnitude when cake permeability changes from 0.1 to 10 mD and particle radius from 0.1 to 10 μm. The large value of correction factor belongs to low cake permeability and large particle sizes implying that the effect of correction factor is more pronounced when large particles are deposited on a low permeable filter cake. Neglecting the permeate correction factor results in underestimation of permeate force and, consequently, an incorrect cake thickness prediction.

The comparison of all effective forces for different particle sizes is shown in Figure 4. It can be seen that all attaching and detaching forces increase with an increase in particle size. For sub-micron particles the electrostatic force highly exceeds the permeate force and is the dominant attaching force. For larger particle sizes the permeate force becomes prominent in comparison with the electrostatic force and can be considered as the major attaching force. Hence, for each flow velocity and solution chemistry conditions, a particular particle size at which the electrostatic force can be ignored in comparison to the permeate force exists; that is, at low salinity and high pH solutions, the permeate force is the dominant attaching force for a wide range of particle sizes. The plot of detaching forces on Figure 4 (blue, black and dashed-blue lines) shows that tangential drag force highly exceeds the gravitational and lifting forces. Hence, the gravitational and lifting forces can be neglected in the calculation of detaching torque; however, the ratio of tangential drag force to gravitational force highly depends on flow velocity and particle size and density. For large particles with a high density at low tangential velocity, the value of gravitational force becomes significant in comparison with tangential drag force and must be considered in the torque balance calculations. It should be mentioned that the rheological parameters of non-Newtonian fluids (flow index and consistency constant) can significantly change the behaviour of hydrodynamic forces.

Sensitivity analysis

The stabilised cake profile along the drilled formation can be calculated using mechanical equilibrium of attaching and detaching torques at each point along the formation. Tangential velocity decreases from the bottom of the well ($x = 0$) to the top of the formation due to the mud permeate leak-off into the formation. A decrease of tangential velocity causes the drag force to decrease along the wellbore length, which can lead to an increase of the cake thickness. Simultaneously, an increase in cake thickness results in a decrease in permeate velocity and, consequently, the permeate force. The increase of cake thickness reduces the cross-section area of the well and results in an increase of the tangential velocity and tangential drag force. The competitive effect of detaching and attaching torques determines the cake thickness at each point along the formation thickness. The equilibrium condition that determines the cake thickness at each point is a function of the particle size and flow conditions including tangential flow velocity, differential pressure, cake permeability, and Young's modulus of particles. Figures 6–11 present the sensitivity study of all mentioned parameters on the cake thickness profile along the well.

EFFECT OF PARTICLE SIZE

Figure 6 shows the effect of the particle radius on the external filter cake profile along the drilled formation. The cake thickness is greater at the top of the formation compared to that at the bottom of the formation for all particle sizes assuming constant filter cake permeability. It should be noted that variation of cake permeability with particle size may result in a different cake profile and has not been considered in the calculations. The cake profile can be explained by the fluid leak-off rate at different depths. The mud leak-off rate into the formation decreases from the bottom ($x = 0$) to the top of formation (Fig. 7); hence, the tangential force is maximum at the bottom of the formation and causes the cake thickness to be minimum. Fluid loss to the formation results in a decrease of tangential mud velocity inside the wellbore from bottom to top, which yields in a decrease of the tangential drag force. The particle deposition on the cake surface is, therefore, higher at the top of formation compared to at the bottom of formation where the flow velocity is higher. The external cake close to the bottom of the formation is thicker for large particle sizes compared to small particles. It can be clearly seen on Figure 4 that the permeate force (red line) grows faster than the tangential drag force (blue line) with an increase in particle size. Close to the bottom of the formation when the cake thickness is small compared to the well radius, the increase of particle size causes the permeate force to increase with a larger slope compared to that for tangential drag force. Consequently, the torque equilibrium on the cake surface occurs at greater values of cake thickness when large particles are being deposited. Thus, large particles can create a thicker cake close to the bottom of the formation. The higher cake thickness at the bottom of the formation for large particles causes the smaller fluid leak-off to the formation and, consequently, the tangential drag forces will be higher at the top of the formation. It can be seen on Figure 6 that the cake thickness close to the top of the formation is greater for small particles due to less tangential flow velocity, which results from higher fluid leak-off at the bottom of the formation (see Fig. 7).

EFFECT OF TANGENTIAL VELOCITY

Figure 8 presents the plot of cake thickness profile along the drilled formation for various tangential flow velocities (mud flowing velocity inside the wellbore). Higher tangential flow velocity yields in higher shear stress on the cake surface and subsequent increase in tangential drag force (Eqs 2–4). The cake thickness, therefore, decreases when the mud flow velocity inside the well bore increases.

EFFECT OF SALINITY

Electrostatic force highly depends on water chemistry. Detachment of particles on the cake surface is more difficult for high salinity conditions (Torkzaban et al, 2007; Bedrikovetsky et al, 2011; Bradford et al, 2011; Kalantariasl et al, 2013). DLVO theory (Eqs 11–15) predicts a higher electrostatic force for higher salinity, which results in an increase of attaching force and, consequently, formation of thicker filter cake for a given tangential flow rate. At the entrance point, the same tangential velocity causes a thicker cake for high salinity water and lower fluid loss into the adjacent formation. On the contrary, lower electrostatic force yields a thinner cake layer and higher fluid loss. Higher tangential velocity at the entrance point causes lower cake thickness for both high and low salinity water. For a given tangential flow rate, along the formation length, tangential flow decreases more for low salinity water, which causes more of a decrease in tangential drag force (F_d). The equilibrium of detaching and attaching torques determines the local stabilised cake thickness.

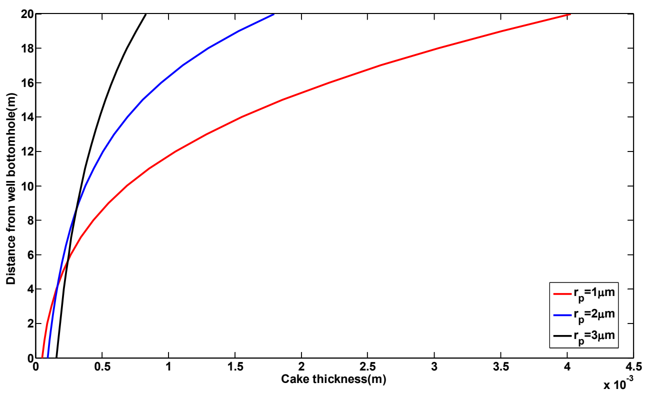

Figure 6. Effect of particle radius on the external filter cake profile ($\sigma = 0.2$, $E = 10$ GPa, $k_c = 0.01$ mD, $\Delta P = 500$ psi, $\Delta\rho = 1{,}600$ kg/m^3, salinity = 3% NaCl, $k_o = 500$ mD, $r_w = 0.1$, $u_t = 0.05$ m/s, $k' = 0.7$ Pas$^{n'}$, and $n' = 0.313$).

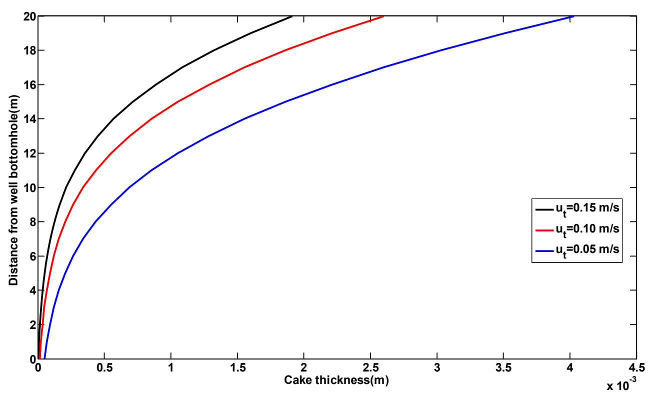

Figure 8. Effect of flow velocity on the external filter cake profile ($\sigma = 0.2$, $E = 10$ GPa, $k_c = 0.01$ mD, $\Delta P = 500$ psi, $\Delta\rho = 1{,}600$ kg/m^3, salinity = 3% NaCl, $k_o = 500$ mD, $r_w = 0.1$, $r_p = 1$ µm, $k' = 0.7$ Pas$^{n'}$, and $n' = 0.313$).

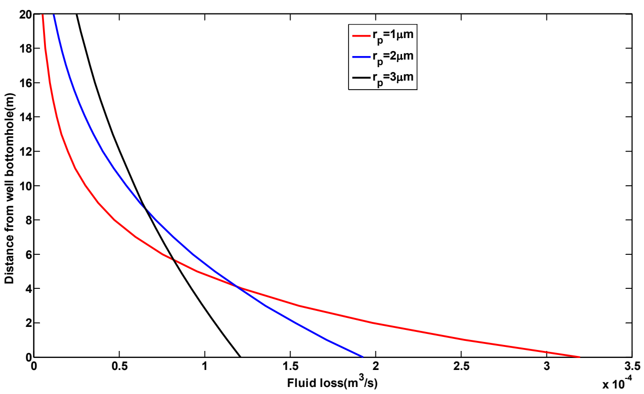

Figure 7. Fluid loss (invasion) distribution during dynamic filtration ($\sigma = 0.2$, $E = 10$ GPa, $k_c = 0.01$ mD, $\Delta P = 500$ psi, $\Delta\rho = 1{,}600$ kg/m^3, salinity = 3% NaCl, $k_o = 500$ mD, $r_w = 0.1$, $u_t = 0.05$ m/s, $k' = 0.7$ Pas$^{n'}$, and $n' = 0.313$).

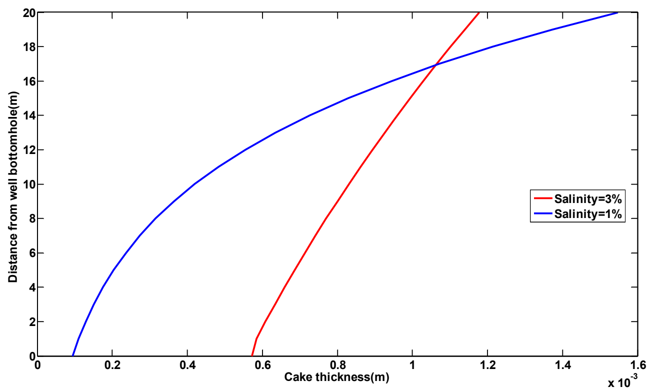

Figure 9. Effect of salinity on the external filter cake profile ($\sigma = 0.2$, E = 5 GPa, $k_c = 0.01$ mD, $\Delta P = 500$ psi, $\Delta\rho = 1{,}600$ kg/m^3, $k_o = 500$ mD, $r_w = 0.1$m, $r_p = 1$ µm, $u_t = 0.05$ m/s, $k' = 0.7$ Pas$^{n'}$, and $n' = 0.313$).

EFFECT OF FILTER CAKE PERMEABILITY

Cake permeability plays an important role in cake thickness profile. It is widely accepted that after cake formation, cake properties control the flow of fluid into the porous formations (Jiao and Sharma, 1994). Figure 10 presents the effect of cake permeability on cake thickness profile. The permeate force is a function of cake permeability by permeate flow (Eq. 22) and permeate force correction factor (Eq. 8). Two competitive effects determine the final effect of cake permeability on cake thickness and its distribution along the well. Substitution of Equations 8 and 22 into permeate force Equation 7 show that the permeate force is inversely proportional to cake permeability (for the flow index used in this study, $n' = 0.313$). It can explain the higher cake thickness for low permeable cake very close to the bottom of the drilled formation (Fig. 10). The low permeable cake, however, yields in small fluid loss to the formation and high flow velocity inside the wellbore at the top of the drilled formation in comparison to that for high permeable cake. Hence, the cake thickness is larger for high permeable cake at some distance from the bottom of the formation.

EFFECT OF THE PARTICLE YOUNG MODULUS

The effect of the particle Young modulus is presented in Figure 11. A larger Young modulus results in a smaller contact deformation (Fig. 5) and, consequently, a smaller normal lever arm (Eq. 27), causing a larger lever arm ratio (Eq. 26). Higher lever arm ratio causes the detaching torque to increase and yields in smaller cake thickness close to the bottom of the formation. Thin external cake at the bottom of the formation results in higher fluid loss into the formation and decreasing tangential fluid rate at some distance from the formation bottom. A decrease of drag force as a result of a decrease in flow inside the wellbore yields formation of a thicker cake towards the top of the well column.

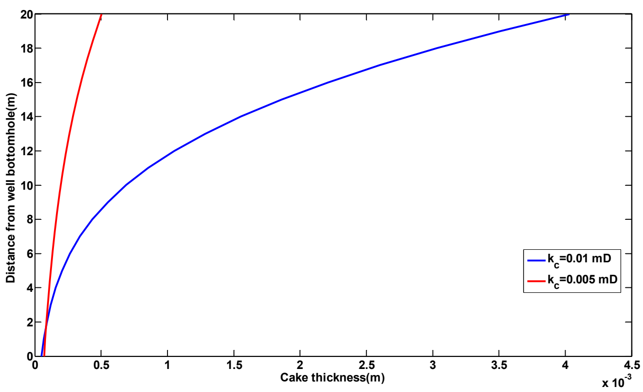

Figure 10. Effect of cake permeability on the external filter cake profile ($\sigma = 0.2$, $E = 10$ GPa, $\Delta P = 500$ psi, $\Delta\rho = 1{,}600$ kg/m^3, salinity = 3% NaCl, $k_o = 500$ mD, $r_w = 0.1$, $u_t = 0.05$ m/s, $r_p = 1$ µm, $k' = 0.7$ Pas$^{n'}$, and $n' = 0.313$).

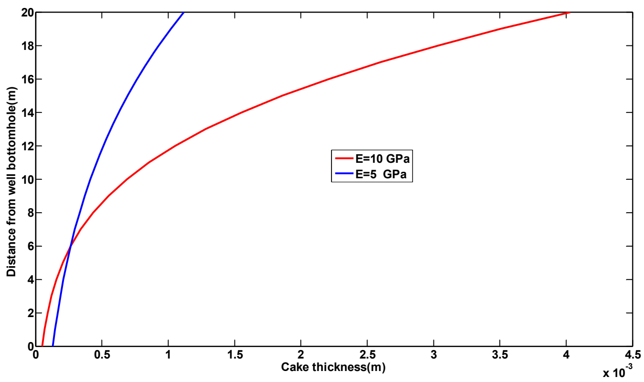

Figure 11. Effect of the particle Young modulus on the external filter cake profile ($\sigma = 0.2$, $k_c = 0.01$ mD, $\Delta P = 500$ psi, $\Delta\rho = 1{,}600$ kg/m^3, salinity = 3% NaCl, $k_o = 500$ mD, $r_w = 0.1$, $u_t = 0.05$ m/s, $r_p = 1$ µm, $k' = 0.7$ Pas$^{n'}$, and $n' = 0.313$).

The numerical Computational Fluid Dynamics (CFD) method can improve calculation of near surface velocity distribution and the permeate force correction factor. Considering non-DLVO forces might change the torque balance condition and cake thickness prediction. Detailed understanding of different colloidal forces helps in drilling particle sizing and design of water injection wells. The proposed model can be extended for more complex geometries (horizontal, slant and fractured wells). A detailed experimental study must be performed to verify the validity of the proposed model.

It must be noted that several restrictive assumptions have been made to model cake thickness due to the complicated nature of drilling mud; hence, the results of the modelling study are indicative only. More realistic estimates of the cake thickness require experimental measurement of the effect of all chemical additives on affecting parameters during the calculation of electrostatic forces.

The modelling results presented in this study are a sensitivity analysis of the effect of different chemical and physical parameters on cake thickness profile. A comparative study between the model prediction and field or laboratory data is required to validate the proposed model.

CONCLUSIONS

- A simple torque balance model is introduced to calculate filter cake profile along the drilled formation.
- Permeate force correction factor and electrostatic force play a significant role in particle deposition/detachment analysis. Ignoring the permeate force correction factor results in underestimating permeate force and, consequently, incorrect cake profile prediction.
- Cake thickness increases from the bottom towards the top of the formation yielding in high fluid loss close to the bottom of the drilled formation.
- Using small particle sizes in the drilling mud results in creation of a thicker external cake, except along a small distance close to the bottom of the formation.
- An increase of mud flow velocity inside the wellbore decreases the external cake thickness and, consequently, increases mud loss in the drilled formation.
- Low permeable cake creates high hydraulic resistance against mud flow into the drilled formation. Hence, the thickness of low permeable cake is smaller in comparison to that for high permeable cake.
- The detaching torque is a function of particle deformation on the cake surface, which depends on the particle Young modulus. Deformation of particles with a small Young modulus yields in a lower lever arm ratio compared to that for particles with a large Young modulus. At the same flow velocity, therefore, particles with a small Young modulus are detached harder than particles with a large Young modulus.

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Chapter 5

Mathematical Modelling of Two-Phase Colloidal-Suspension Flow during Water Injection

Statement of Authorship

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Signature		Date	24/01/15

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Contribution to the Paper	Participating in manuscript writing, help in data analysis		
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Chapter 6

Produced Water Reinjection in Low Permeable Formations: Mathematical Modelling, Experimental Investigation and Field Case Study

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Produced water re-injection and disposal in low permeable reservoirs

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ABSTRACT

Produced water re-injection (PWRI) is an important economic and environmental-friendly option to convert waste to value with waterflooding. However, it often causes rapid injectivity decline. In the present work, laboratory coreflood test using low permeable core sample is performed to investigate the impedance (normalised reciprocal of injectivity) behaviour. Analytical model for well impedance growth, along with probabilistic histograms of injectivity damage parameters, is applied to the well injectivity decline prediction during produced water disposal in a thick low permeable formation (Völkersen field). Unusual convex form of impedance curve is observed in both coreflood test and well behaviour modelling; impedance grows slower during external cake formation if compared with deep bed filtration. This is due to low reservoir permeability and, consequently, high values of filtration and formation damage coefficients causing fast impedance growth during deep bed filtration; while external cake build-up yields relatively slower impedance growth during cake formation. Risk analysis method using probabilistic histograms of injectivity damage parameters is applied to well behaviour prediction under high uncertainty conditions.

Keywords: produced water disposal, PWRI, injectivity decline, coreflood test, mathematical model, risk analysis

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1. Introduction

Well injectivity decline in produced water disposal and re-injection projects has been widely reported in the literature (Nabzar and Chauveteau, 1997; Ochi et al., 1999; Sharma et al., 2000; Civan, 2011; Abou-Sayed et al., 2007; Buret et al., 2010; Bai, 2011; Ding, 2011). The reasons are the permeability decline during deep bed filtration of the injected particles with water, and formation of external filter cake from those particles (Eylander, 1988; Khatib, 1994; Civan,1998; Bedrikovetsky et al., 2001; Rousseau et al., 2008; Zamani and Maini, 2009; Saraf et al., 2010) (Fig. 1a).

Well injectivity decline is described by increase of the so-called well impedance J, which is the normalised reciprocal to well injectivity index

$$J(t_D) = \frac{q_0}{\Delta p_0}\frac{\Delta p(t_D)}{q(t_D)}, \quad t_D = \frac{1}{\pi r_e^2 \phi H}\int_0^t q(\tau)d\,\tau \tag{1}$$

where q is the well injection rate, q_0 is the initial rate, Δp is the pressure drawdown between injection well and reservoir, Δp_0 is the initial pressure drawdown, t_D is the pore volume injected (dimensionless time), r_e is the drainage radius, ϕ is the reservoir rock porosity, and H is the reservoir thickness. The same definition of impedance is applied for linear flow during laboratory coreflood.

Usually, three stages of well injectivity impairment can be distinguished: deep bed filtration, external cake formation and water injection with stabilised skin factor. The analytical models for these three stages have been developed by Barkman and Davidson (1972), Pang and Sharma (1997), Ochi et al., (1999), Sharma et al., (2000) and Kalantariasl and Bedrikovetsky (2013). The analytical model is used for injectivity prediction and well data analysis.

In the present paper, the field case (Völkersen field, Germany) of produced water disposal in aquifer with unusually low permeability and large thickness is investigated. The unusual convex form of the impedance growth curve has been observed in both laboratory coreflood test and well behaviour prediction. It is explained by low rock permeability with consequent high values of filtration and formation damage coefficients, resulting in relatively fast skin build-up during deep bed filtration if compared with that during external cake formation. Under high uncertainty of well behaviour prediction, we develop a risk analysis method using the histograms of injectivity damage parameters. The calculations show that large reservoir thickness can compensate low reservoir permeability, allowing achieving reasonable water injection rate under the conditions of Völkersen field.

The structure of the paper is as follows: Section 2 presents the results of coreflood injectivity test and data analysis on impedance behaviour. A brief description of well impedance growth model accounting for deep bed filtration followed by external cake formation and its stabilisation is presented in Section 3. Selection of injectivity damage parameters for risk analysis is described in Section 4. Model prediction of well injectivity impairment for the field conditions is performed in Section 5. Discussions and conclusions in Section 6 finalise the paper.

2. Laboratory study

In this section, a coreflood injectivity test is carried out on a low permeable sandstone core sample to obtain the impedance growth behaviour during suspension injection.

2.1. Core, particles and water properties

Details of core sample, injected particles and water properties in the experiment are as follows:

Core: Sandstone core sample used in the present study has the following properties: permeability k=0.437 mD, porosity ϕ=0.123, length L=3.75 cm and pore volume 5.40 mL.

Particles: Since the PWRI is applied to sandstone formation (Völkersen field), the sandstone-based particles (quartz, silica, etc.) should be used in the injected suspension. However, due to unavailability of the produced water sample from this field, latex microspheres with radii r_s=0.505±0.005 μm have been used for injection.

Injected water: In order to represent field conditions, the water salinity used in the laboratory test is determined as follows. Accounting for the difference between latex particles injected in the test and particles in field conditions, the equivalent salinity of the injected water is obtained by applying the DLVO theory (Israelachvili, 2011; Yuan et al., 2012), such that the attaching electrostatic force for “latex particle-sandstone” under the calculated salinity is the same as that for “quartz particle-sandstone” under the injected water salinity 1.1 M (83629 mg/L) in the field case. The obtained equivalent salinity for laboratory conditions is approximately 0.18 M NaCl. To be on the safe side, we have applied a higher value of salinity as 0.4 M NaCl in the laboratory test. The value of pH varies from 5.9 to 6.1.

2.2. Experimental procedure

Laboratory setup for the real-time permeability measurement apparatus is shown in Fig. 2. Suspension with volumetric concentration of latex particles of c^0=22.63 ppm was continuously injected at constant volumetric flowrate of 3.8 mL/min (superficial velocity 5.412×10^{-5} m/s). Pressure drop along the core was monitored in real time, which gradually increased from approximately 635 to 672 psi during the experiment. Outlet particle concentrations during the test were measured by portable particle counter PAMAS S4031 GO 25 (PAMAS GmbH, Salzuflen, GERMANY). Insignificantly low particle concentrations at the outlet were registered.

2.3. Experimental results and data analysis

The photographs of the core inlet were taken before and after the experiment. Fig. 3a shows the inlet face of the core before the suspension injection. The image with high resolution magnification is shown in Fig. 3b. The photo of the core inlet after the back flush shown in Fig. 3c presents the image which significantly differs from that before the injection. Yellow spots correspond to accumulated latex particles at the inlet face, and dark background indicates rock surface. High resolution magnification of the inlet face after the back flush is shown in Fig. 3d. The solid and fragile cake is partly destroyed by the back flush. Aggregated latex particles after coagulation under the high-salinity intensive particle-particle attachment are clearly seen at the inlet face.

Comparison between core inlet images before (Figs. 3a and b) and after the test (Figs. 3c and d) shows the formation of external filter cake on the injection face of a low permeable core sample. Particle aggregation at high salinity condition, as observed in Fig. 3d, results in high cake permeability and low formation damage in the case where cake build-up dominates over deep bed filtration in injectivity damage process.

Measured pressure drop versus time along the core is translated into impedance as a function of pore volumes injected (PVI) using the formula (A1). The obtained impedance data from the experiment are shown as red points in Fig. 4. Initial steep rise in the impedance up to approximately 40 PVI accounts for particle deep bed filtration in the core. We attribute the less steep part of the curve to the ripening of an external cake after the transition time, when deep bed filtration stops and all injected particles form the cake. Impedance grows from unity at the beginning of water injection into core sample to above 1.04 up to the transition time, after which the impedance grows with smaller slope. The impedance growth rate in the cake formation stage is lower than that in the deep bed filtration stage during coreflood test. Similar convex form of impedance curve is observed from another coreflood test reported by Pautz et al. (1989).

The experimental data from coreflood test are treated by the impedance model for linear flow (Appendix A). The tuning parameters in the model are: filtration coefficient λ=888 m^{-1}, formation damage coefficient β=381 and cake permeability k_c=0.0086 mD. The modelled impedance result is presented as black curve in Fig. 4. The figure shows good match of impedance growth between the experimental data and modelling results; the coefficient of determination is R^2=0.9937.

Both filtration and formation damage coefficients belong to upper part of the common intervals for those coefficients, which is typical for low permeable rocks (see Pang and Sharma, 1997; da Silva et al., 2004; Rousseau et al., 2008). It determines fast impedance growth during deep bed filtration. “Adding” cake to low permeable core as resistances in series causes slower impedance growth than that during deep bed filtration.

3. Mathematical model for impedance growth during PWRI

The mathematical model for prediction of well impedance growth is briefly presented in this section. It will be applied to the field case study in Section 5.

Well impedance J grows linearly with the amount of injected particles during deep bed filtration stage (Barkman and Davidson, 1972; Pang and Sharma, 1997; Bedrikovetsky et al., 2005)

$$J(t_D) = 1 + mt_D, \quad m = \frac{\beta\phi c^0 (\lambda r_e)^2}{2\ln\frac{r_e}{r_w}}\left(\frac{1}{\lambda r_w} + e^{\lambda r_w} ei(\lambda r_w)\right), \; ei(x) = \int_0^\infty \frac{e^{-y}}{y} d\,y \tag{2}$$

where c^0 and r_w are the injected particle concentration and the well radius, respectively. The impedance slope m is determined by the values of filtration λ and formation damage β coefficients.

Transition time t_{Dtr} is defined as the moment when the retained particle concentration reaches the α-th fraction of reservoir porosity $\alpha\phi$. It is expressed as (Pang and Sharma, 1997; Bedrikovetsky et al., 2005)

$$t_{Dtr} = \frac{2\alpha X_w}{\lambda r_w c^0}, \; X_w = \left(\frac{r_w}{r_e}\right)^2 \tag{3}$$

After the transition time, deep bed filtration stops and injected particles start to form external cake on the well wall.

Following da Silva et al. (2004), the average value for the critical porosity fraction α=0.09 as obtained from analysis of extensive laboratory data is applied to the current field case study (Section 5).

Assuming incompressible cake and using volume balance for the cake-forming particles after the transition time, the cake thickness is proportional to the amount of injected particles (Pang and Sharma, 1997; Ochi et al., 1999; Sharma et al., 2000). Considering impedance at the transition time ($J = 1 + mt_{Dtr}$) and applying Darcy's law to external cake layer, the overall well impedance (including both deep bed filtration and cake formation stages) is obtained as follows

$$J(t_D) = 1 + mt_{Dtr} + m_c(t_D - t_{Dtr}), \; m_c = \frac{k\phi c^0}{2X_w k_c(1-\phi_c)(-lnX_w)} \tag{4}$$

Here m_c is the impedance slope during cake formation stage, k is the initial reservoir permeability, k_c is the cake permeability and ϕ_c is the cake porosity.

A particle on the cake surface is subject to drag (F_d), lifting (F_l), gravitational (F_g), permeate (F_p) and electrostatic (F_e) forces (Fig. 1b). Drag, lifting and gravitational forces detach the particle from the cake surface while permeate and electrostatic forces attach the particle to the cake surface. The equality of attaching and detaching torques determines the stabilised cake thickness and the time when stabilisation occurs (Jiao and Sharma, 1994; Zinati et al., 2009; Kalantariasl and Bedrikovetsky, 2013)

$$(F_p + F_e - F_l)l_n = (F_d + F_g)l_d,\ l = l_d/l_n \tag{5}$$

Here, l_n and l_d are attaching and detaching lever arms respectively, and l is the lever arm ratio. See Kalantariasl and Bedrikovetsky (2013) for expressions of all the above forces. Upon stabilisation of the cake thickness under particle torque balance (5), the impedance will be constant, i.e. impedance growth stops when the cake thickness stabilises.

4. Risk analysis using histograms of injectivity damage parameters

The impedance growth model presented in Section 3 can be applied to the prediction of well impedance with known properties of reservoir and injected water. There are four main injectivity damage parameters in this model: filtration coefficient λ, formation damage coefficient β, external cake permeability k_c and lever arm ratio l. The histograms of these four parameters obtained from data analysis of 35 injection wells (Kalantariasl et al., 2013) are presented in Figs. 5a-d. In this section, we present a method for risk analysis of well injectivity impairment using histograms of these parameters under high uncertainty conditions of injectivity prediction during PWRI.

4.1 Deep bed filtration parameters λ and β

The reservoir permeability on well testing scale varies from 1 to 4 mD. The pore radii r_p obtained from core data using mercury injection vary from 1.5 to 10 μm. Assuming log-normal distribution and applying 10% cut-offs, we obtain the mean pore radius 5.1 μm and standard deviation 4.35 μm.

Since the particle size distribution is not known, for maximum filter cut-offs 1, 2 and 5 μm applied in Völkersen field, the mean particle size is assumed to be equal to half of the maximum size, i.e. the mean particle radii are 0.5, 1.0 and 2.5 μm, respectively.

According to the "1/3-1/7" filtration rule, particles larger than one third of the mean pore radius do not penetrate into the medium. Instead, they start forming cake on the injection face at once (van Oort et al., 1993). So, the particles with mean radius 2.5 μm do not penetrate, since mean particle radius is greater than the one third of mean pore radius, i.e. 2.5>5.1/3=1.7 μm. The jamming ratios (j=particle radius/pore radius) for particles with mean radii 0.5 and 1.0 μm are j=0.5/5.1=0.10 and j=1.0/5.1=0.20, indicating mean values for straining filtration coefficient.

The salt concentration of injected water is 83629 mg/L, which promotes extremely strong attachment of particles from the injected water to the grain surface. Therefore, high filtration coefficient is expected, and the radius of damaged zone is small. The near-well permeability is assumed equal to core permeability. So the filtration and formation damage coefficients are set based on the pore radii obtained from core data. Therefore, we assume high total values of filtration coefficient for particles with mean radii 0.5 and 1.0 μm, i.e. 70 and 100 m^{-1}, respectively (see Pang and Sharma (1997), Sharma et al. (2000) and Bedrikovetsky et al. (2005) for typical values of filtration coefficient).

The formation damage coefficient is determined by both particle attachment and straining. Attachment-dominant process yields low value of formation damage coefficient for fully compacted internal cake. Finally, under weak straining condition, we assume total formation damage coefficients as 50 and 100 for particles with mean radii 0.5 and 1.0 μm, respectively. The chosen values of formation damage coefficient are the most frequent values, according to the histogram of formation damage coefficient (Fig. 5b).

4.2 External cake permeability k_c

Let us show that permeability of external cake formed by filtered particles is proportional to the square of the particle size: $k_{c1}/k_{c2} = (r_{s1}/r_{s2})^2$, where r_{s1} and r_{s2} are the two mean particle radii determined by filter sizes. In the following text, it is referred to as the proportionality formula.

The second column in Table 1 corresponds to the mean particle sizes. Standard deviation is assumed to be equal to the mean particle size (third column), i.e. the coefficient of variation is equal to one (fourth column). Following the breakage algorithm, particle size distribution is assumed to be log-normal. Assuming the same coefficient of variation for distributions of three types of filtered injected particles, the three corresponding particle size distributions are shown in Fig. 6a. The algorithm developed by Chalk et al. (2012) using Descartes' theorem is applied to calculating pore size distribution of the external filter cake from particle size distribution. The mean pore sizes in the cake and their standard deviations are shown in the fifth and sixth columns of Table 1. The three corresponding cake pore radius distributions are shown in Fig. 6b.

The cake permeability is calculated using the hypothesis of parallel tubes intercalated by mixing chambers (Shapiro et al., 2007): $k_c = \frac{\pi}{8}\int_0^\infty f(r_{p_c}) r_{p_c}^4\, dr_{p_c} / \int_0^\infty f(r_{p_c}) r_{p_c}^2\, dr_{p_c}$, where r_{p_c} is cake pore radius and $f(r_{p_c})$ is cake pore radius distribution. The resulting cake permeabilities for three maximum particle sizes are presented in the seventh column of Table 1.

The eighth column in Table 1 shows the obtained external cake permeability from the proportionality formula. It is assumed that cake permeability for mean particle size $\langle r_s \rangle$=0.5 μm is known. Those for $\langle r_s \rangle$=1.0 μm and $\langle r_s \rangle$=2.5 μm are presented in eighth column (Table 1). Comparison between the eighth and seventh columns shows that the proportionality formula is fulfilled with high accuracy.

Sharma et al. (2000) matched several water injection wells data from the Gulf of Mexico fields with k_c=0.6 mD for particle size $\langle r_s \rangle$=1.5 μm, which is taken as a realistic value for cake permeability in the present work. Considering the histogram of cake permeability (Fig. 5c), we define the pessimistic and optimistic values of cake permeability as 0.2 and 3.9 mD, respectively. Cake permeability for different size particles as calculated from the proportionality formula are presented in Table 2.

4.3 Lever arm ratio l

Fig. 5d shows the histogram for lever arm ratio as determined from 9 injection well data. The mean value l=400 is chosen for the realistic case. The values l =200 and l =600 have been chosen for pessimistic and optimistic cases, respectively.

5. Prediction of impedance growth in field case

In this section, mathematical model for impedance growth presented in Section 3 and selected injectivity damage parameters in Section 4 are applied to prediction of injectivity decline and sensitivity analysis during produced water disposal in a low permeable sandstone formation (Völkersen field).

Provided field data are as follows: k=1-4 mD, ϕ=0.12, r_e=1177 m, r_w=0.11 m, μ_w=0.26 cP, H=105 m, reservoir pressure p_{res}=180 bar, formation fracturing pressure p_f=460 bar and injection pressure p_w=0.9p_f. Injected solid concentrations c^0=5, 10 and 100 ppm and the maximum filtered particle radii 1, 2 and 5 µm are used. Constant cake porosity ϕ_c=0.20 is applied.

Let us compare impedance growth using filtered water with different particle sizes versus the amount of injected particles ($c^0 t_D$). It allows comparison between the cases of different injected concentrations using the same plot. Translation from one concentration c^0 to another Nc^0 corresponds to compressing the $c^0 t_D$-axis N times. Fig. 7 shows the impedance dynamics for injection of small, medium and large particles (with mean radii 0.5, 1.0 and 2.5 µm, respectively) in the realistic case of injectivity damage parameters with reservoir permeability 4 mD. The injectivity damage parameters selected in Section 4 are presented in Table 3.

Fig. 7a shows that stabilised values of well impedance reach 16, 4.15 and 1.4 for small, medium and large particles, respectively. The smaller is the particle size, the larger amount of particles must be injected to reach the stabilised impedance.

Fig. 7b shows the zoom of impedance growth at early stage of water injection. Particles with small and medium sizes perform deep bed filtration, followed by the external cake formation. Impedance grows much faster during deep bed filtration stage, compared to that during external cake formation stage. For low permeable reservoirs, high formation damage during deep bed filtration leads to fast growth of impedance; while during cake formation, the low ratio between reservoir and cake permeabilities k/k_c results in slow impedance growth (see Eq. 4). The above two effects of high formation damage during deep bed filtration and low ratio of k/k_c cause the unusual convex shape of impedance growth curve (blue and red curves in Fig. 7b). Large particles do not penetrate into the formation (Section 4). Instead, external cake starts building up from the beginning of large particle injection, leading to impedance growth due to cake formation only (black curve in Fig. 7b). Similar impedance growth behaviour was obtained with reservoir permeability 1 mD.

The pessimistic, realistic and optimistic estimates for impedance growth with injection of medium size particles are shown in Fig. 8a as blue, red and black curves, respectively. The stabilised impedance values for three scenarios are 11.6, 4.15 and 1.7, respectively. Since the contribution of deep bed filtration to the impedance is insignificant (see Fig. 8a), we keep deep bed filtration parameters (filtration and formation damage coefficients) unchanged for the cases of injection of medium and small particles (second and third columns in Table 3). Therefore, impedance during deep bed filtration stage is the same for optimistic, realistic and pessimistic scenarios (Fig. 8b). Similar impedance behaviour during external cake formation occurs for injection of water containing small or large size particles. For a given scenario of pessimistic, realistic and optimistic cases, the stabilised value of impedance increases with particle size reduction, due to decrease in cake permeability formed by smaller particles.

Impedance growth function can be translated to rate decline versus real time during water injection with constant pressure. The effects of particle size and concentration on injection rate decline as calculated for field conditions are shown in Fig. 9. The injection rate q decreases with time due to formation damage (impedance growth) under constant pressure drawdown Δp (see Eq. 1).

Fig. 9a shows injection rate decline for small particle size with various injection concentration. Injection rate decreases from initial value of 2193 to 530, 381 and 161 m^3/d at 50th month of injection for c^0=5, 10 and 100 ppm, respectively. Sharp decline of injection rate at initial period corresponds to deep bed filtration stage where more permeability damage is expected, compared to that in cake formation stage (as explained in Fig. 7). Cake stabilisation is only reached for high concentration during the injection period shown in Fig. 9 (blue curves), due to higher cake thickness formed by higher concentration of injected particles at given time. A similar trend is observed for medium particle size: the higher is the particle concentration, the larger is the rate decline at the same time before cake stabilisation (Fig. 9b). For concentrations c^0=5, 10 and 100 ppm, injection rate declines from 2193 to 921, 701 and 348 m^3/d, respectively.

For large particle injection, deep bed filtration does not occur. Absence of deep bed filtration stage and formation of high permeable cake slows down the injection rate decline. Injection rate decreases from 2193 at the beginning of injection to 1699, 1436 and 912 m^3/d at 50th month of injection for c^0=5, 10 and 100 ppm, respectively (Fig. 9c). Higher injection rate at the same time for large particles is due to larger cake permeability if compared with medium and small particles. Comparison of Figs. 9a,b,c shows that during cake formation stage, larger particle size and lower injection concentration result in higher injection rate at the same time.

The application of impedance growth model to the field case shows that the injectivity decline (impedance growth) is sensitive to both external cake permeability and injected particle concentration. Since the cake permeability depends on the size of particles forming the external cake, injectivity decline is sensitive to the injected particle sizes. Under the field conditions in this study, large filtration coefficient causes early transition from deep bed filtration to external cake formation, resulting in the negligible time interval of deep bed filtration stage compared to that of cake formation stage (Figs. 7 and 8). Therefore, the contribution of deep bed filtration to the total impedance is much smaller than that of cake formation. As a result, the total well impedance is insensitive to the filtration and formation damage coefficients. The lever arm ratio defines the maximum skin value at the stabilised cake thickness and has no effect on the slope of impedance growth.

6. Discussions and conclusions

Corefloods and well history interpretations show that all the injectivity damage parameters (filtration and formation damage coefficients, cake permeability and lever arm ratio) vary in large intervals. Besides, all these parameters strongly affect the injectivity decline. Therefore, prediction of well injectivity decline without either representative coreflood data or injectivity history available from the field has high level of uncertainty. One way around this problem is the risk analysis using probabilistic histograms of the injectivity damage parameters, allowing the comparative study of pessimistic, realistic and optimistic cases.

A concave form of injectivity curves is typical for both laboratory corefloods and well behaviour (Eylander, 1988; Ochi et al., 1999; Sharma et al., 2000; Civan and Rasmussen, 2005; Kalantariasl et al., 2014). However, for the low permeability case discussed in the present study, the unusual convex form was observed during both laboratory coreflood test and well-behaviour modelling. The phenomenon is explained as follows: the impedance grows fast during deep bed filtration due to low reservoir permeability, which leads to high formation damage; it grows slower during external cake formation, due to low ratio between the reservoir and cake permeabilities, since low initial permeability does not exceed significantly the cake permeability. Formation of low permeable external cake on the injection face of a low permeable rock may slow down the impedance growth compared to the deep bed filtration stage. The effect of external cake formation on permeability impairment in low permeable reservoirs is not as significant as that in the conventional high permeable reservoirs.

The above is valid for either disposal of produced water in aquifers or its re-injection into oilfields.

The laboratory coreflood and well impedance modelling allow drawing the following conclusions:

1. Both coreflood test and mathematical modelling on well injectivity performance show that the impedance growth during cake formation is significantly slower than during the deep bed filtration, which leads to the convex form of the impedance curve.
2. Good agreement between the experimental and modelling data has been observed for coreflood test, which validates the model used and allows using laboratory data for well behaviour prediction.
3. Probabilistic histograms of the injectivity damage parameters can be applied to risk analysis of well impairment under high uncertainty conditions in produced water disposal and re-injection projects.
4. Large reservoir thickness can compensate low reservoir permeability, yielding reasonable injection rates.

Nomenclature

c^0	concentration of particles in the injected water, ppm
c_v	coefficient of variation
F_d	drag force, MLT^{-2}, N
F_e	electrostatic force, MLT^{-2}, N
F_g	gravitational force, MLT^{-2}, N
F_l	lifting force, MLT^{-2}, N
F_p	permeate force, MLT^{-2}, N
H	reservoir thickness, L, m
J	impedance
j	jamming ratio
k	reservoir rock permeability, L^2, m^2
k_c	external cake permeability, L^2, m^2
L	core length, L, m
l	lever arm ratio
l_d	lever arm for detaching forces, L, m
l_n	lever arm for normal forces, L, m

m	slope of impedance growth during deep bed filtration
m_c	slope of impedance growth during cake formation
p	pressure, $ML^{-1}T^{-2}$, Nm^{-2}
p_w	wellbore pressure, $ML^{-1}T^{-2}$, Nm^{-2}
q	injection rate, L^3T^{-1}, m^3s^{-1}
r_e	reservoir radius, L, m
r_p	formation (core) pore radius, L, m
r_{pc}	pore radius of external cake layer, L, m
r_s	particle radius, L, m
r_w	wellbore radius, L, m
t	time, T, s
t_D	dimensionless time (PVI)
t_{Dtr}	dimensionless transition time (PVI)
u	injection velocity, LT^{-1}, m s^{-1}
x_w	dimensionless squared well radius

Greek letters

α	critical porosity fraction
β	formation damage coefficient
λ	filtration coefficient, L^{-1}, m^{-1}
μ_w	water viscosity, $ML^{-1}T^{-1}$, $kgm^{-1}s^{-1}$
Δp	pressure drop across the reservoir, $ML^{-1}T^{-2}$, Nm^{-2}
ϕ	reservoir rock porosity
ϕ_c	cake porosity

Abbreviations

PVI	pore volume injected

Subscripts

f	fracturing
res	reservoir
std	standard deviation
0	initial

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Appendix A. Impedance growth during deep bed filtration and external filter cake formation in coreflooding (linear flow)

The linear flow during deep bed filtration and external filter cake formation occurs in laboratory coreflood tests. Following Pang and Sharma (1997) and da Silva et al. (2004), here we present the analytical model of impedance growth in linear flow, which is applied to the treatment of experimental data in Section 2.

Impedance grows linearly with time during deep bed filtration stage as

$$J(t_D) = \frac{q_0}{\Delta p_0}\frac{\Delta p(t_D)}{q(t_D)} = 1 + mt_D, \quad t_D = \frac{1}{\phi L}\int_0^t u(\tau)d\tau \qquad \text{(A1)}$$

in which L is the core length, ϕ is the core sample porosity, u is the injection velocity and the impedance slope m is

$$m = \beta\phi c^0(1 - \exp(-\lambda L)) \qquad \text{(A2)}$$

where λ and β are filtration and formation damage coefficients, respectively.

The dimensionless transition time t_{Dtr} is obtained from particle capture kinetics as

$$t_{Dtr} = \frac{\alpha}{\lambda L c^0} \qquad \text{(A3)}$$

Formation of external filter cake increases the hydraulic resistance to the flow and hence the impedance. The slope of impedance growth during cake formation stage m_c is proportional to the ratio of core and cake permeabilities k/k_c, core porosity ϕ and injected particle concentration c^0. The impedance J during cake formation stage is

$$J(t_D) = 1 + mt_{Dtr} + m_c(t_D - t_{Dtr}), \; m_c = \frac{k\phi c^0}{k_c(1-\phi_c)} \qquad \text{(A4)}$$

where ϕ_c is the cake porosity.

Eqs. (A1-A4) are used for treatment of impedance growth due to particle invasion into porous media and external cake formation in coreflood test.

Table 1

Data used for cake permeability prediction for different particle sizes

Particle type	$\langle r_s \rangle$(μm)	$r_{s_{std}}$(μm)	c_v	$\langle r_{pc} \rangle$(μm)	$r_{pc_{std}}$(μm)	k_c (mD)	$\frac{k_{c1}}{\left(\frac{r_{s1}}{r_{si}}\right)^2}, i = 1,2,3$
1	0.5	0.5	1	0.0531	0.0285	0.0097	0.0097
2	1.0	1.0	1	0.1061	0.0569	0.0397	0.0388
3	2.5	2.5	1	0.2656	0.1422	0.2486	0.2422

Table 2

Pessimistic, realistic and optimistic cake permeabilities for three particle sizes

$\langle r_s \rangle$(μm)	Pessimistic case k_c (mD)	Realistic case k_c (mD)	Optimistic case k_c (mD)
1.5	0.20	0.60	3.90
0.5	0.02	0.07	0.43
1.0	0.09	0.27	1.73
2.5	0.55	1.67	10.83

Table 3

Injectivity damage parameters for the realistic case

Filtered particles with mean radius, $\langle r_s \rangle$ (μm)	Filtration coefficient, λ (m^{-1})	Formation damage coefficient, β	Cake permeability, k_c (mD)	Lever arm ratio, l
0.5	70	50	0.07	400
1.0	100	100	0.27	400
2.5	-	-	1.67	400

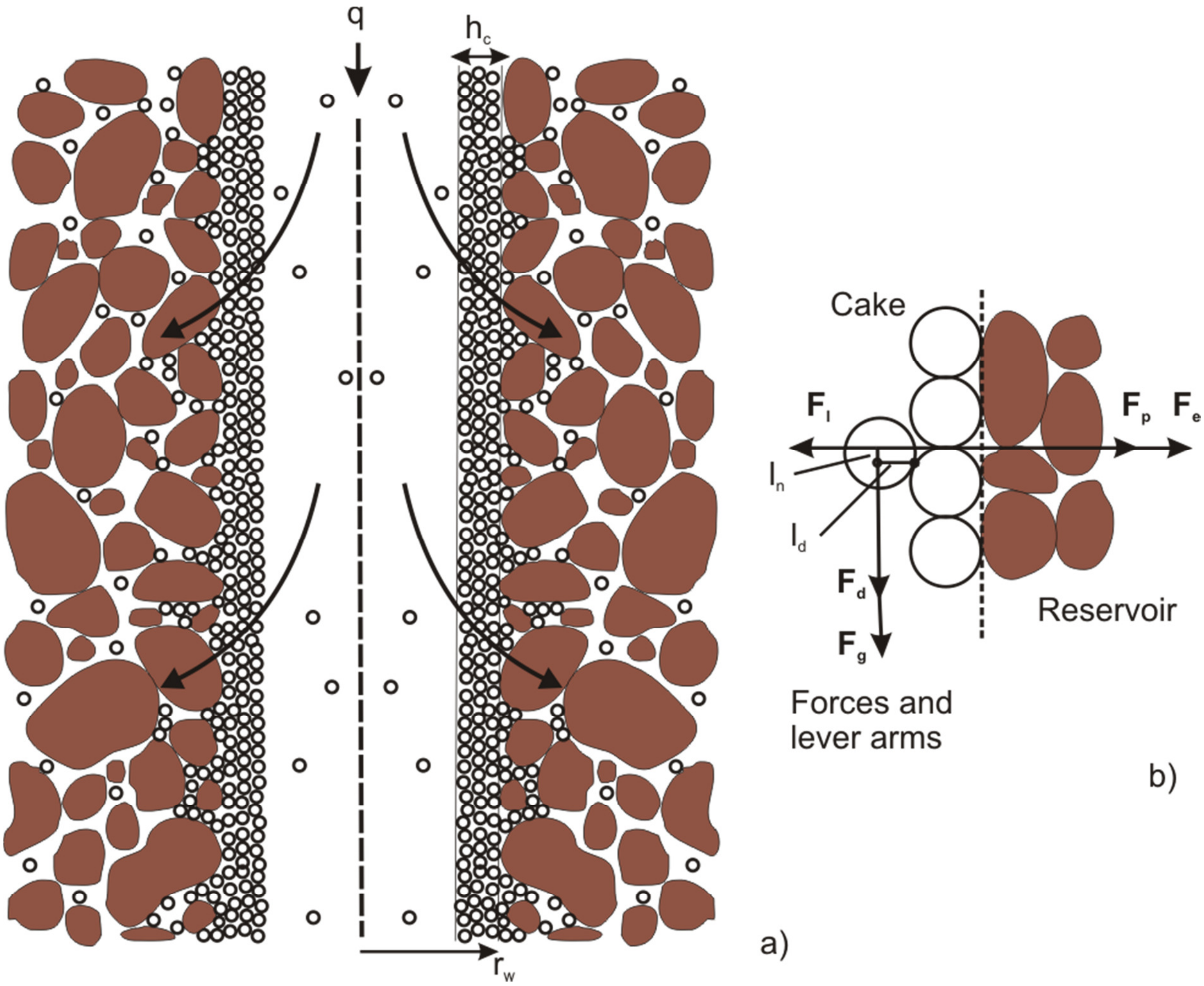


Fig. 1. Three stages of injectivity impairment from the beginning of injection: a) Schema of injectivity decline due to deep bed filtration (Stage 1) and external cake formation (Stage 2); b) Stabilisation of the external filter cake (Stage 3)

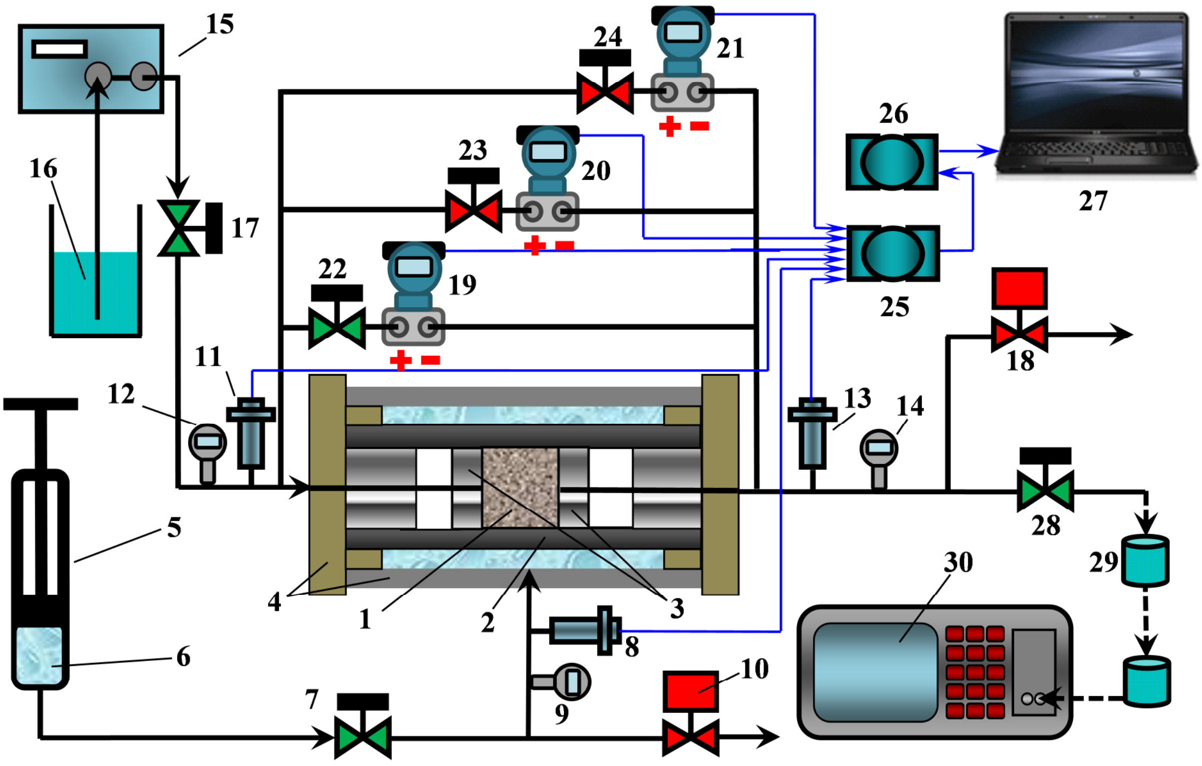


Fig. 2. Schematic of a real-time data acquisition and monitoring system for coreflood test

1 – rock core sample; 2 - Viton sleeve; 3 – stainless steel stoppers; 4 - high-pressure core holder; 5 – manual pressure generator; 6 – distilled water; 7, 17, 22-24, 28 – manual valves; 8, 11, 13 - PA 33X gauge pressure transmitter; 9, 12, 14 – pressure gage with readout; 10, 18 – back-pressure regulators; 15 - HPLC pump; 16 – suspension with latex microspheres; 19-21 – differential pressure transmitters; 25 - ADAM-4019+ data acquisition module; 26 - ADAM-5060 RS-232/RS-485/RS-422 signal converter; 27 - personal computer; 29 - beakers; 30 - PAMAS S4031 GO portable particle counter.

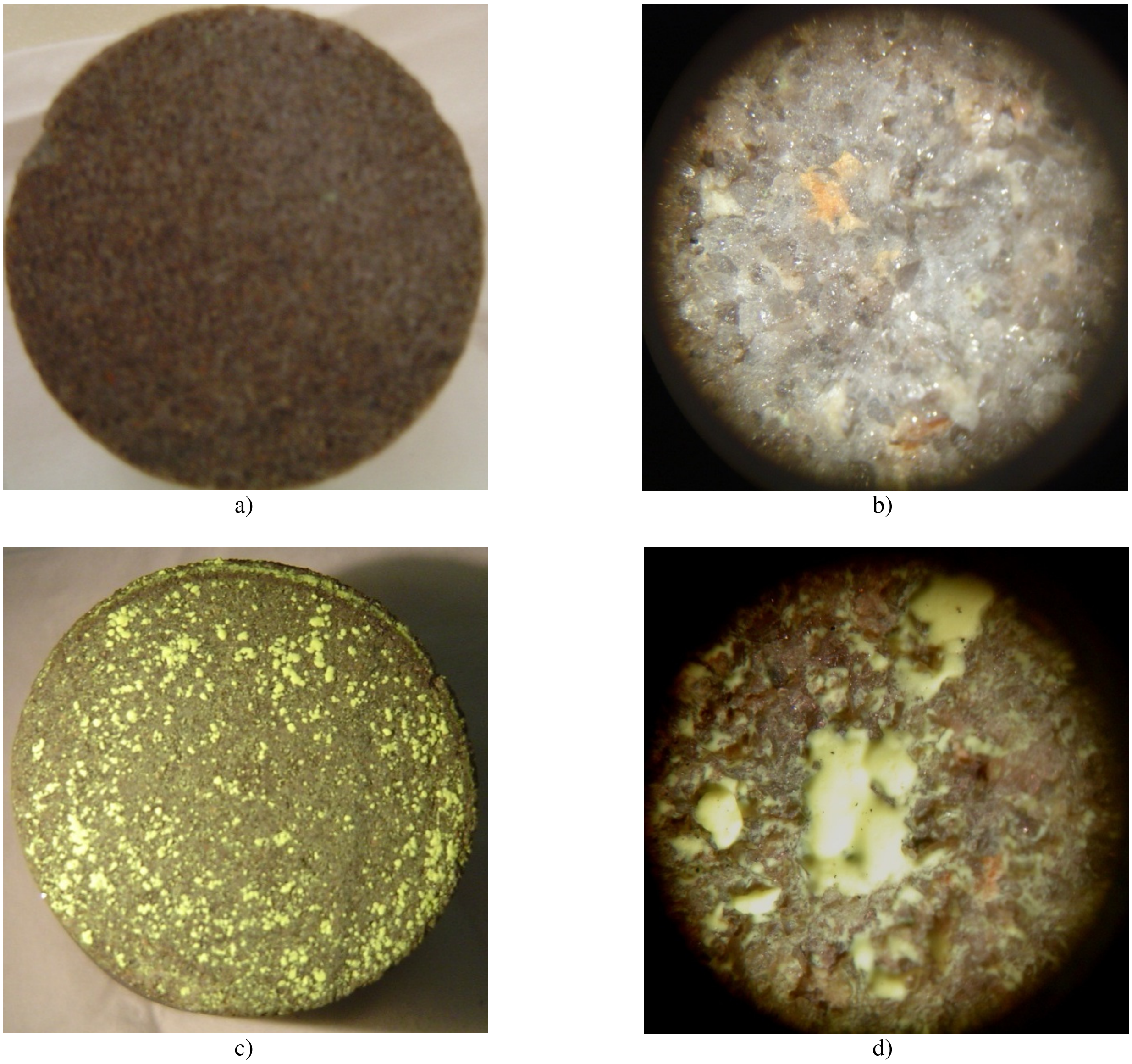


Fig. 3. Inlet face of the core: a) photo image before injection; b) high resolution magnification; c) photo image after the back-flush; d) high resolution magnification

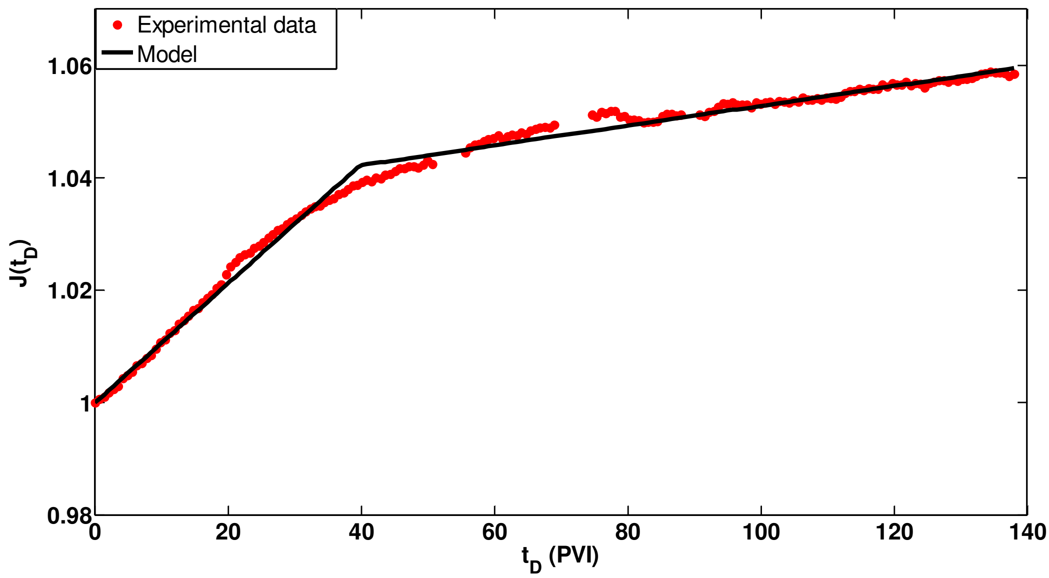
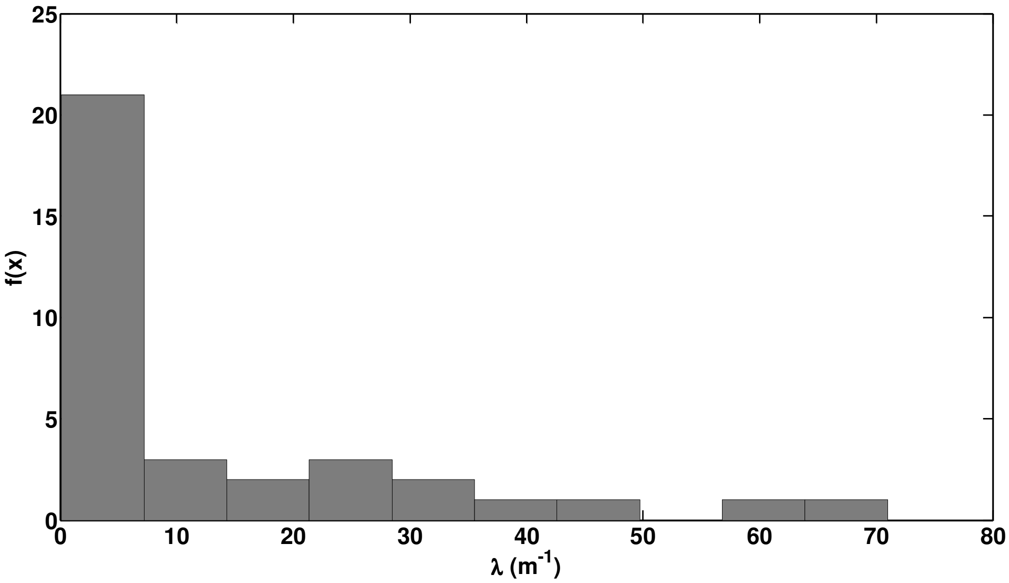


Fig. 4. Comparison of impedance growth between measured data and model prediction



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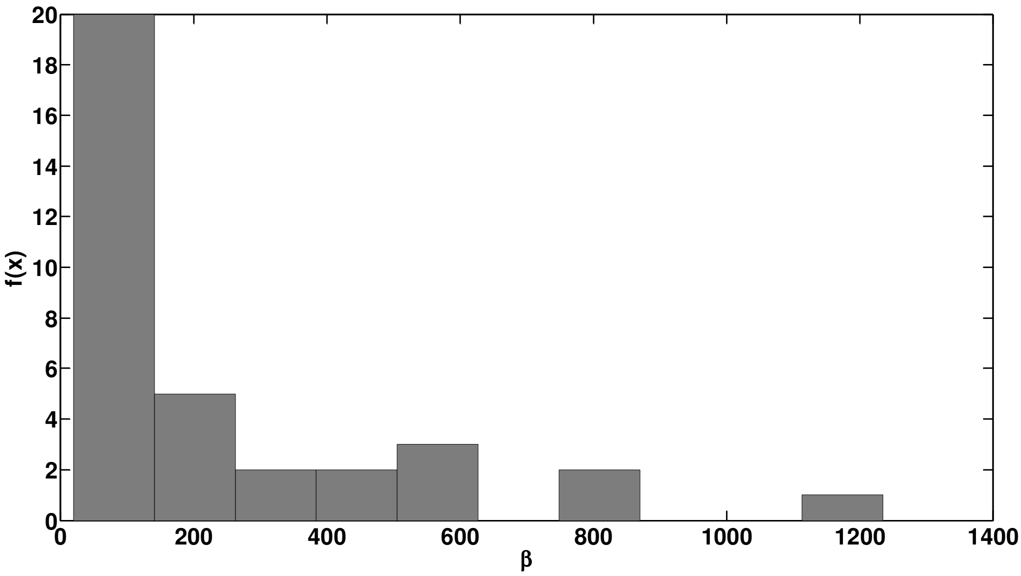
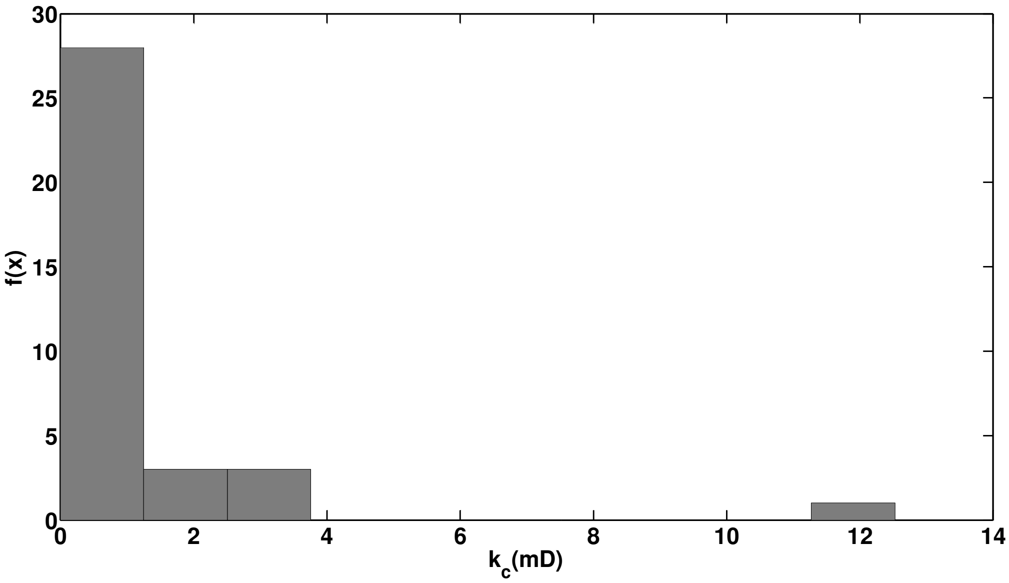
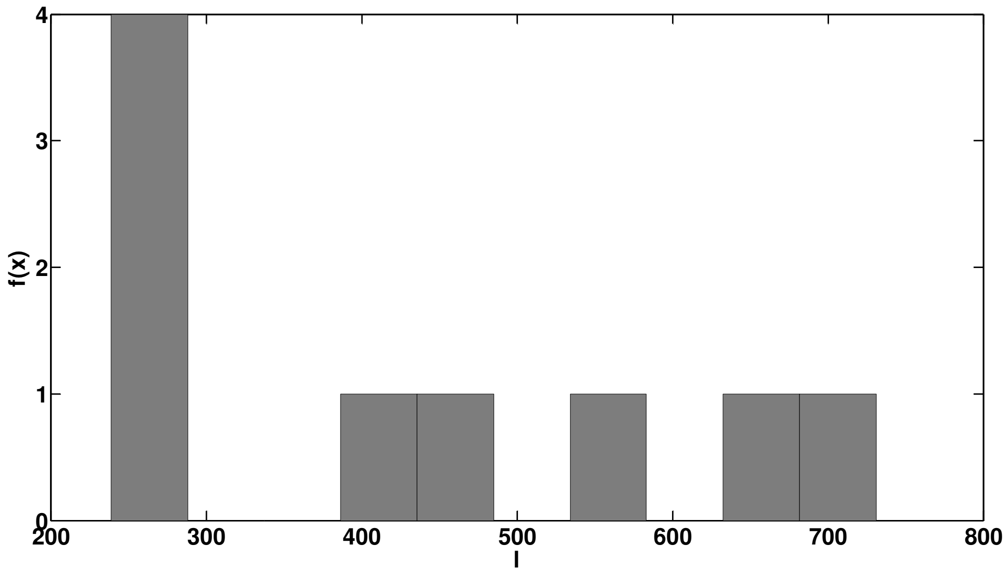


b)

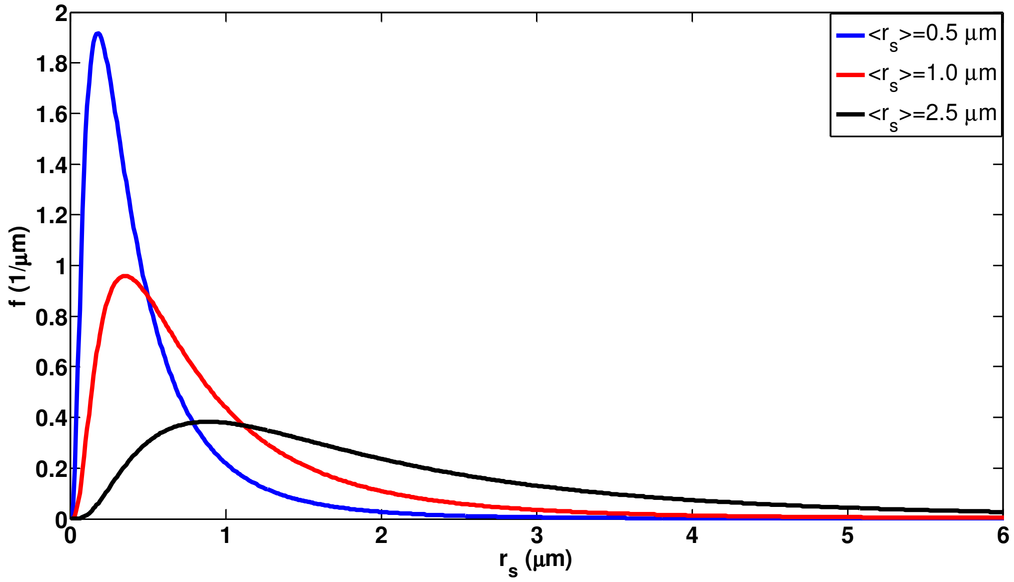


c)



d)

Fig. 5. Probabilistic histograms of injectivity damage parameters as obtained from treatment of 35 field dataset: a) filtration coefficient λ, b) formation damage coefficient β, c) external filter cake permeability k_c and d) lever arm ratio l



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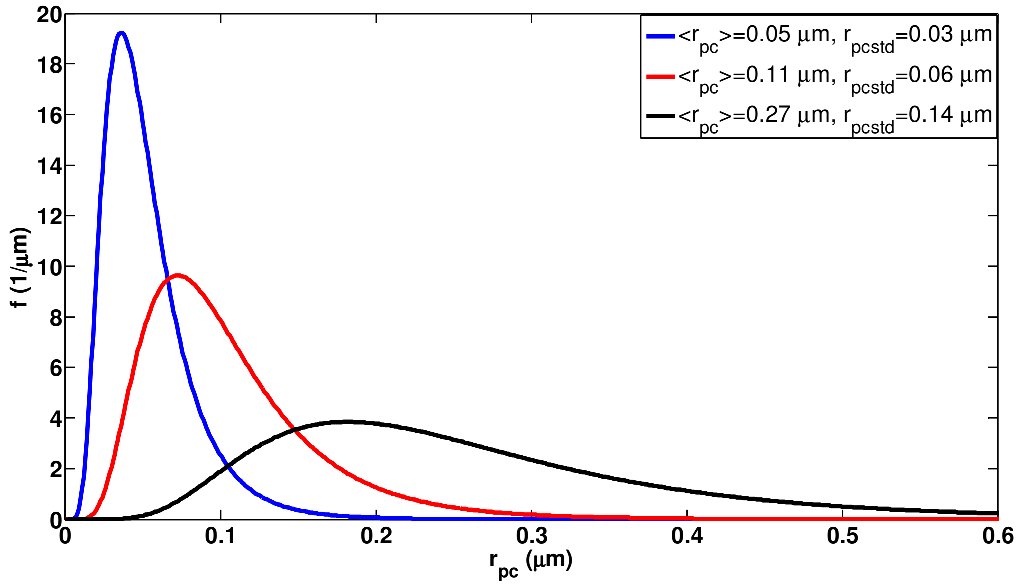
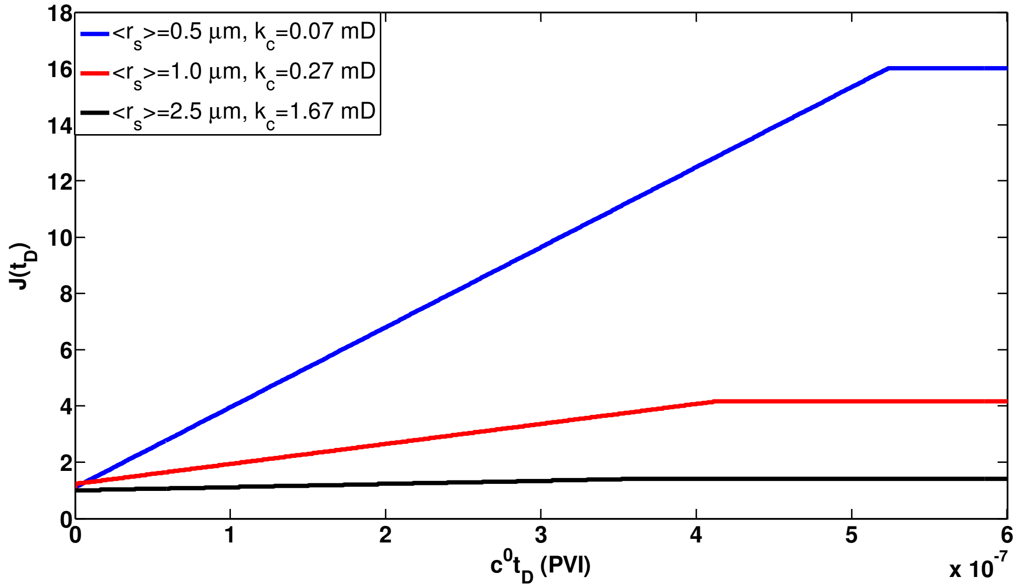


b)

Fig. 6. a) injected particle size distributions for average particle sizes taken as half-maximum-sizes after filtering in order to assess the cake properties; b) pore size distributions in external filter cake as obtained from Descartes' theorem using known particle size distributions



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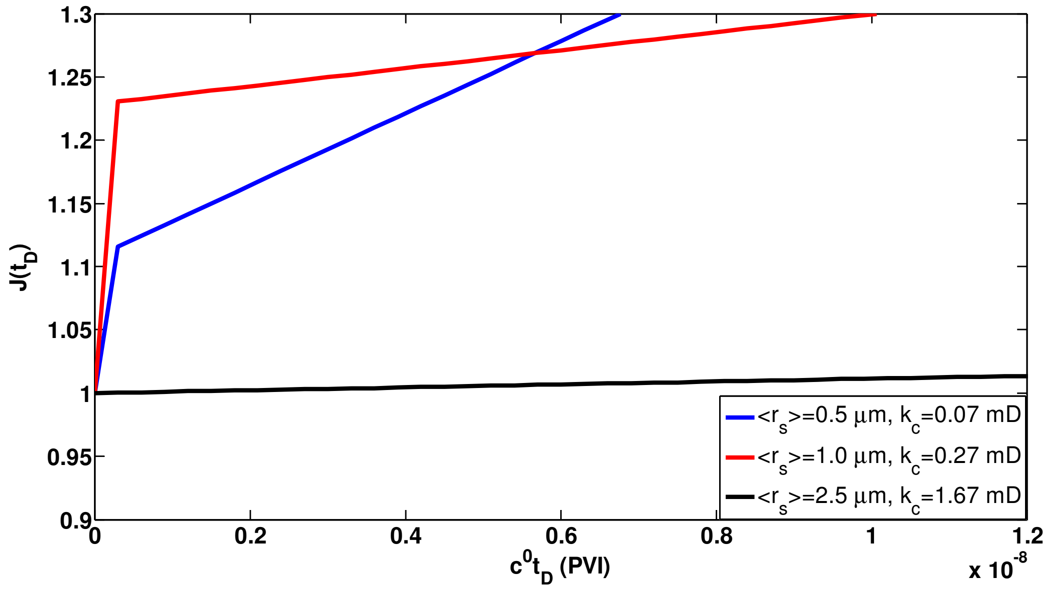
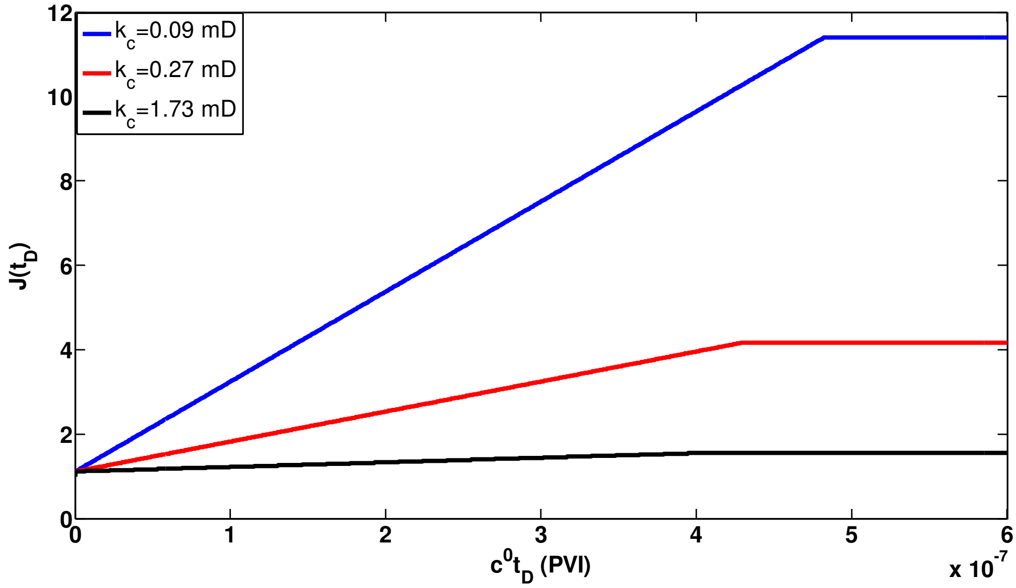
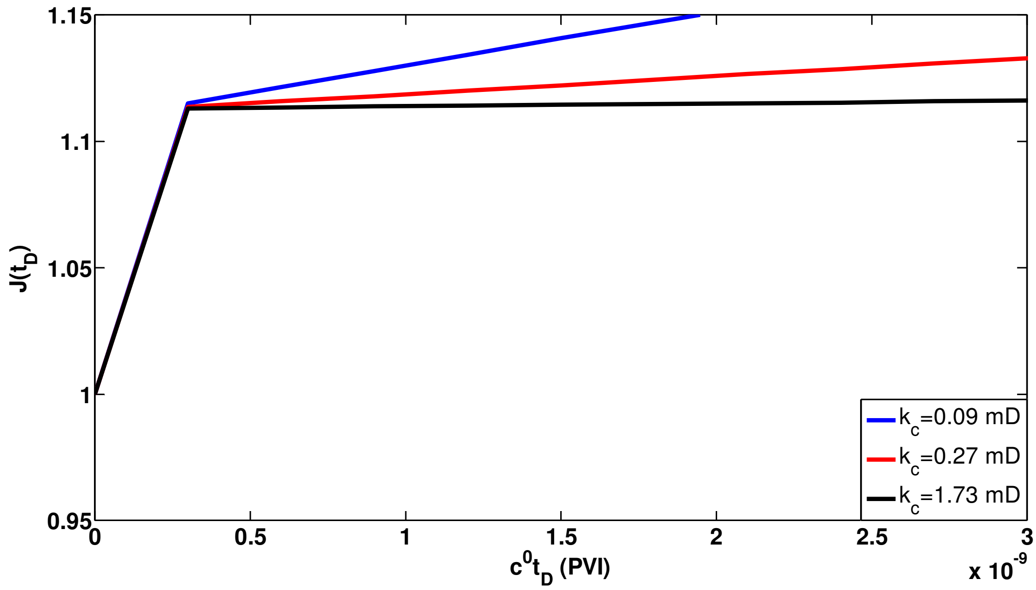


b)

Fig. 7. Well impedance versus amount of injected particles for realistic case of reservoir permeability 4 mD and three particle sizes: a) effect of particle size on impedance growth; b) zoom-in at early stage of water injection

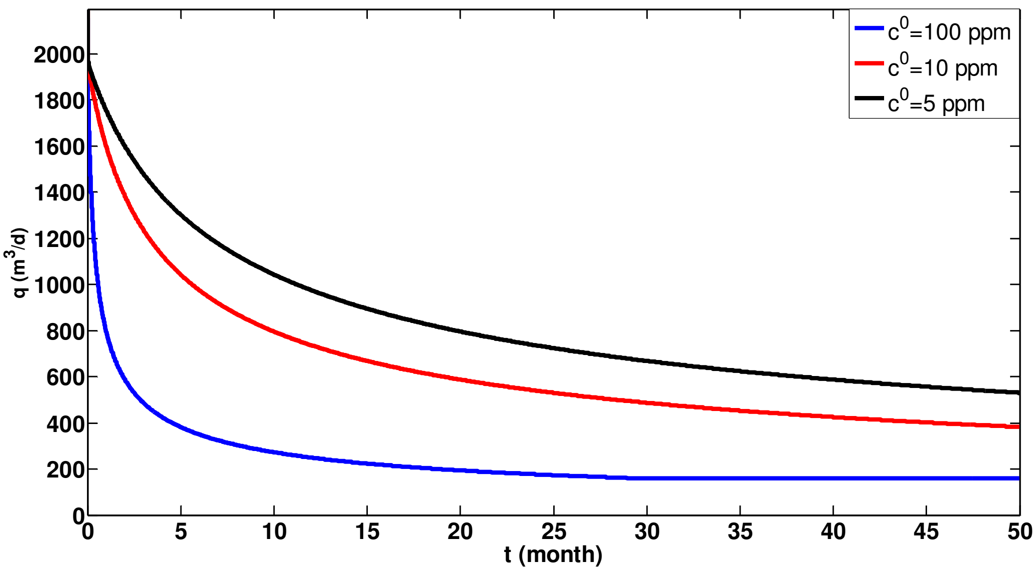


a)

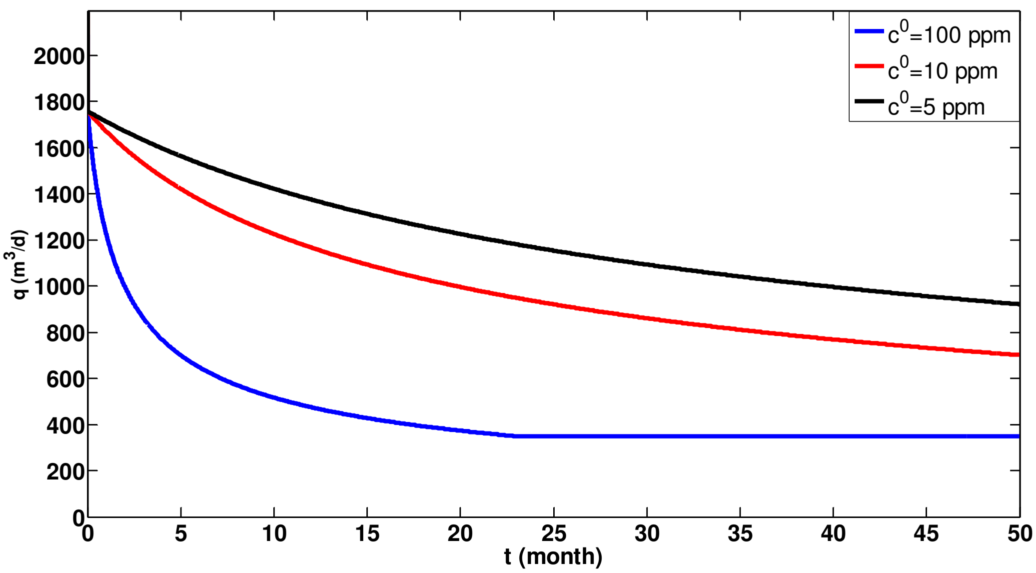


b)

Fig. 8. Pessimistic, realistic and optimistic variants for impedance versus amount of injected particles for medium particle size ($\langle r_s \rangle$=1 μm): a) effect of cake permeability; b) zoom-in at early stage


c0=100 ppm
c0=10 ppm
c0=5 ppm
q (m3/d)
t (month)

a)


c0=100 ppm
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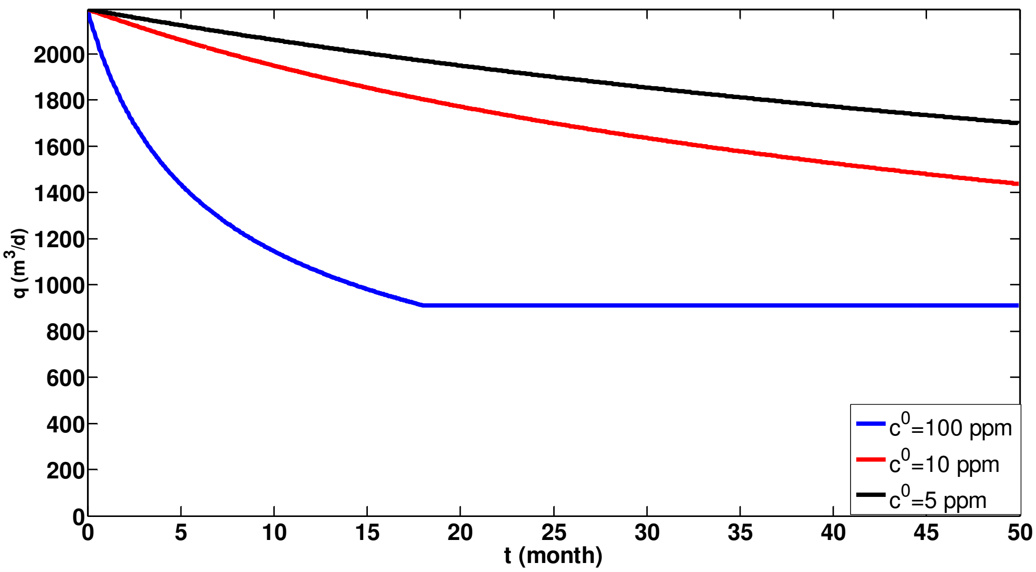
c0=5 ppm
q (m3/d)
t (month)

b)



c)

Fig. 9. Effect of particle size and concentration on the injection rate decline with time: a) $\langle r_s \rangle$=0.5 µm; b) $\langle r_s \rangle$=1.0 µm; c) $\langle r_s \rangle$=2.5 µm

Highlights

- Unusual convex form of impedance curve from PWRI in low permeable reservoirs
- Risk analysis using histograms of injectivity damage parameters for well prediction
- Impedance behaviour obtained from coreflood injectivity test on low permeable core
- Modelling field injectivity decline during PWRI in thick low permeable formation
- Injectivity decline sensitive to injected particle size and concentration

Chapter 7

Summary and Conclusions

Summary and conclusions

This thesis presents new comprehensive mathematical models for full prediction of injectivity decline during injection of sea water, reinjection of produced water, and waste disposal operations. The models for deep bed filtration of particles, external cake formation and its stabilisation presented in this thesis allow prediction of injectivity behaviour and, treatment and interpretation of injection well data during water injection/disposal into aquifers (single phase). The new analytical model, developed for two-phase colloidal-suspension flow in porous media, can be applied to predict well injectivity performance and history matching of field data during waterflooding into oil reservoirs for enhanced recovery (two phase).

In this thesis, a new mathematical model for mechanical equilibrium of a particle on the cake surface at microscale is derived accounting for all colloidal forces. The first paper describes mechanical equilibrium of a particle on the cake surface by torque balance of detaching (drag, lifting and, gravitational) and attaching (permeate, and electrostatic) forces. It is shown that, electrostatic force attaching the particle to the external cake surface can highly exceed other forces and significantly change the value of the stabilised cake thickness. It is also found that permeate force factor can vary up to three orders of magnitude and highly affects the mechanical equilibrium of a particle on the cake surface. Lever arm ratio is defined and found to be the main empirical parameter affecting the stabilised cake prediction in mechanical equilibrium model. The lever arm ratio is obtained by treatment of several experimental cross-flow tests and injection wells data. Close variation intervals for the lever arm ratio, as obtained from the stabilised well impedance and from the laboratory cross-flow experiments, validate the torque balance equilibrium model. The lever arm ratio is also calculated by Hertz theory of particle deformation. Good agreement between the predicted values of the lever arm ratio by the Hertz theory and, from laboratory and field data suggests that the cake mechanical equilibrium is determined by particle deformation rather than by the well surface asperity. The proposed model for lever arm ratio allows reliable prediction of the stabilised injectivity value and corresponding time of cake equilibrium thickness. Finally, the proposed formula for lever arm ratio along with that for deep bed filtration and external cake formation form an analytical predictive model for well index prediction during water injection and disposal operations.

In the current thesis, a new analytical model for prediction of non-uniform cake thickness profile in long injection wells is derived where tangential flow decreases significantly from top to the bottom of the well (papers 2, 3 and 8). Non-uniform profile of external filter cake on a well wall in thick reservoirs during water injection or drilling is described by an implicit formula. Sensitivity analysis for injection flow rate, cake permeability, Young's modulus, Poisson's ratio, water salinity, and applied pressure show that the above parameters are the most influential parameters defining the cake thickness profile. The obtained results show there exists a critical well injection rate corresponding to zero cake thickness. If the injection rate is below the critical value, the external cake is built up over the entire injection interval and if it is above the critical value, there is no cake in the upper part of the wellbore; the cake starts at the depth where the tangential rate reaches the critical value. The developed mathematical model can be used for performing parametric sensitivity analysis to obtain non-uniform stabilised injectivity index in long horizontal and vertical injection well. When the injection rate is above the critical value, then, there is no external cake formation on the upper part of the wellbore; the external cake starts to build up at the well depth where the tangential rate reaches its critical value. The developed mathematical model can be used for performing parametric sensitivity analysis to obtain non-uniform stabilised injectivity index in long horizontal and vertical injection wells.

In this thesis, a new analytical model for displacement of oil by aqueous particle suspension is derived (paper 4). The competition between formation damage effect and displacement of high-viscous oil by water results in non-monotonic impedance behaviour. With introduction of external cake stabilisation, which yields limited impedance value, a type curve for injectivity decline is obtained from the analytical model. Damaged zone radius is introduced to separate the formation damage effect from the two-phase flow effect. This separation allows obtaining analytical model for deep bed filtration, external cake formation and its stabilisation during the displacement of oil by water. It is shown that consideration of two-phase displacement resulting in the initial injectivity increase, adds three degrees of freedom to the traditional four-parametric single-phase injectivity decline model. This additional information is used for tuning the Corey relative permeability and the pseudo-relative permeability under the viscous-dominant displacement. It is also found that the impedance for two-phase damage-free displacement allows for high accuracy approximation by three parametric power-law function. A model adjustment procedure is proposed that consist of a seven-parameter least-square minimization procedure to find four formation damage and three two-phase flow parameters (to match the Buckley-Leverett impedance). Three parameters obtained from Buckley-Leverett impedance curve are used for improved reservoir characterization, determining the relative permeability, or describing the permeability distribution. Good agreement between the data from three field cases and modelling results along with common values of the obtained constants validate the developed analytical model for injectivity decline during waterflooding and the seven-parameter tuning procedure. The developed analytical model can be applied to predict the well injectivity performance and interpretation of field data during waterflooding when water is injected into oil reservoirs for enhanced recovery.

The impedance growth model contains four main injectivity damage parameters (filtration and formation damage coefficients, external cake permeability, and lever arm ratio) that vary in large intervals and strongly affect well injectivity decline. In this thesis, a comprehensive field data analysis –using analytical models- from published literature is carried out to obtain these parameters (paper 5). Four probabilistic histograms of injectivity damage parameters are created. These histograms allow performing sensitivity analysis and risk assessment of water injection and waste disposal projects in the absence of laboratory and/ or field data.

In this thesis, risk analysis and injectivity behaviour in a thick low permeable formation (Völkersen field, Germany) during disposal of produced water is investigated (papers 6 and 7). A new type curve for impedance growth in low permeable formations is presented. A laboratory coreflood injectivity test is designed and carried out on a low permeable sandstone core sample to obtain the impedance growth behaviour during suspension injection. Analytical model for well impedance growth along with probabilistic histograms of injectivity damage parameters are applied to predict the well impedance behaviour using field data. Both coreflood test and mathematical modelling on well injectivity performance showed that the impedance growth during cake formation is significantly slower, and the formation damage is lower, than during deep bed filtration, which yields the unusual convex form of the impedance curve. Good agreement between the experimental and modelling data has been observed for coreflood test, which validates the model used and allows using the laboratory data for well behaviour prediction. Risk analysis method developed in this thesis using the probabilistic histograms of injectivity damage parameters can be applied to predict well impairment under high uncertainty conditions of disposal and re-injection projects.

The presented analytical models in this thesis allow full prediction of injectivity decline during low quality water injection for disposal, pressure maintenance and waterflooding purposes. These models can be also used for similar processes such as fluid invasion, cake formation and its stabilisation during drilling.

Appendix A

Injectivity during PWRI and Disposal in a Thick Low Permeable Reservoir

(Field Case Study)

Statement of Authorship

Title of Paper	Injectivity during PWRI and Disposal in Thick Low Permeable Formations (Field Case, Laboratory and Mathematical Modelling)
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Publication Details	Kalantariasl, A., Schulze, K., Storz, J., Burmester, C., Küenckeler, S., You, Z., Badalyan, A., Bedrikovetsky, P. (2015) Injectivity during PWRI and Disposal in Thick Low Permeable Formations (Field Case, Laboratory and Mathematical Modelling), presented at SPE European Formation Damage, Budapest, Hungary, 3-5 June.

Author Contributions

By signing the Statement of Authorship, each author certifies that their stated contribution to the publication is accurate and that permission is granted for the publication to be included in the candidate's thesis.

Name of Principal Author (Candidate)	Azim Kalantariasl		
Contribution to the Paper	Performed calculations, experimental design, formulation of final conclusions, writing the manuscript		
Signature		Date	28/01/15

Name of Co-Author	Kai Schulze		
Contribution to the Paper	Providing field data, consulting		
Signature		Date	26.01.2015

Name of Co-Author	Joerg Storz		
Contribution to the Paper	Providing field data, consulting		
Signature		Date	27.01.2015

Name of Co-Author	Christian Burmester		
Contribution to the Paper	Providing field data, consulting		
Signature		Date	26.1.15

Name of Co-Author	Soeren Küenckeler
Contribution to the Paper	Providing field data, consulting
Signature	Date 23/01/15

Name of Co-Author	Zhenjiang You
Contribution to the Paper	Participating in calculations, formulation of conclusions, writing the manuscript
Signature	Date 28/1/15

Name of Co-Author	Alexander Badalyan
Contribution to the Paper	Performing coreflood test
Signature	Date 28.01.2015.

Name of Co-Author	Pavel Bedrikovetsky
Contribution to the Paper	Formulation of the problem, supervising, formulation of conclusions, manuscript review and assessment
Signature	Date 28/01/15

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NOTE:
This publication is included on pages 182 - 203 in the print copy of the thesis held in the University of Adelaide Library.

It is also available online to authorised users at:

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Appendix B

Mathematical Modelling of External Filter Cake Profile in Long Injection Wells

Statement of Authorship

Title of Paper	Mathematical Modelling of Non-Uniform External Cake Profile in Long Injection Wells
Publication Status	○ Published, ⊙ Accepted for Publication, ○ Submitted for Publication, ○ Publication style
Publication Details	Kalantariasl, A., Farajzadeh, R., You, Z., Bedrikovetsky, P. (2014) Mathematical Modelling of Non-Uniform External Cake Profile in Long Injection Wells, to be presented at SPE European Formation Damage, Budapest, Hungary, 3-5 June.

Author Contributions

By signing the Statement of Authorship, each author certifies that their stated contribution to the publication is accurate and that permission is granted for the publication to be included in the candidate's thesis.

Name of Principal Author	Azim Kalantariasl		
Contribution to the Paper	Derivation of model, performed calculations, writing programs code, formulation of final conclusions, writing the manuscript		
Signature		Date	28/01/15

Name of Co-Author (Candidate)	Rouhi Farajzadeh		
Contribution to the Paper	Formulation of the problem, manuscript review and assessment		
Signature		Date	23-1-2015

Name of Co-Author	Zhenjiang You		
Contribution to the Paper	Supervising model derivation and calculations, formulation of the conclusions		
Signature		Date	28/1/15

Name of Co-Author	Pavel Bedrikovetsky		
Contribution to the Paper	Formulation of the problem, supervising model development, formulation of the final conclusion, writing the manuscript		
Signature		Date	28/01/15

Kalantariasl, A., Farajzadeh, R., You, Z. & Bedrikovetsky, P. (2015) Mathematical Modelling of Non-Uniform External Cake Profile in Long Injection Wells, presented at *SPE European Formation Damage Conference & Exhibition*, Budapest, Hungary, 3–5 June, SPE 174184-MS

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