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MATTER

**Hadrons and Quarks in Dense Matter:
From Nuclear Matter to Neutron Stars**

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Signed Statement

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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Dedication

This thesis is dedicated to the memory of my mother Meredith Anne Kite (1960–2009) who impressed upon me from an early age the importance of education.

Abstract

The underlying theme of this thesis is an investigation of the equation of state of strongly interacting matter and the modelling of cold neutron stars. Particular emphasis is placed on the influence of quark degrees of freedom, which we investigate by using relativistic quark level models. More precisely, we study the equation of state for QCD matter in the zero temperature limit, from the confined hadronic phase to the deconfined quark phase.

We begin by exploring the equation of state for nuclear matter in the quark-meson coupling model, including full Fock terms. The comparison with phenomenological constraints can be used to restrict the few additional parameters appearing in the Fock terms which are not present at Hartree level. Because the model is based upon the in-medium modification of the quark structure of the bound hadrons, it can be readily extended to include hyperons and to calculate the equation of state of dense matter in beta-equilibrium. This leads naturally to a study of the properties of neutron stars, including their maximum mass, their radii and density profiles.

Next, we study deconfined quark matter using the three flavour Nambu–Jona-Lasinio model based on one-gluon exchange. The model is implemented by employing Schwinger’s covariant method of proper time regularisation. Comparisons are made with the more commonly used three momentum regularised model with the t’ Hooft determinant term. Hybrid equations of state are constructed using the developed Hartree-Fock quark-meson coupling and Nambu–Jona-Lasinio models. We consider the possibility that deconfinement may be a crossover transition. Using the resulting hybrid equations of state, the properties of hybrid stars are then calculated.

List of Publications

- Whittenbury, D.L., Matevosyan, H. H., Thomas, A.W. *Hybrid Stars using the Quark-Meson Coupling and Proper Time NJL Models* (in preparation)
- Whittenbury, D.L., Carroll, J.D., Thomas, A.W., Tsushima, K., Stone, J.R. *Quark-Meson Coupling Model, Nuclear Matter Constraints and Neutron Star Properties*, Phys. Rev. C **89** 065801 (2014).
- Thomas, A.W., Whittenbury, D.L., Carroll, J.D., Tsushima, K., Stone, J.R. *Equation of State of Dense Matter and Consequences for Neutron Stars*, EPJ Web Conf. **63** (2013) 03004.
- Whittenbury, D.L., Carroll, J. D., Thomas, A. W., Tsushima, K., and Stone, J.R. *Neutron Star Properties with Hyperons*, arXiv:1204.2614 [nucl-th] (unpublished).