



Analysis of Rock Performance under Three-Dimensional Stress to Predict Instability in Deep Boreholes

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Abstract

Underground rock formations are always under some stress, mostly due to overburden pressure and tectonic stresses. When a borehole is drilled, the rock material surrounding the hole must carry the load which was initially supported by the excavated rock. Therefore, due to the introduction of a borehole, the pre-existing stress state in the sub-surface rock mass is redistributed and a new stress state is induced in the vicinity of the borehole. This new stress state around the borehole can be determined directly by means of *in situ* measurements, or can be estimated by applying numerical methods or closed form solutions.

In this thesis borehole stability analysis is undertaken by means of the linear elasticity theory. The introduction of a borehole into a block of rock which behaves linearly elastic, leads to stress concentration near the hole. If the rock material around the borehole is strong enough to sustain the induced stress concentration, the borehole will remain stable; otherwise rock failure will occur at the borehole wall. Therefore, a key aspect in stability evaluation of a borehole is the assessment of rock response to mechanical loading.

For borehole stability evaluation in good quality brittle rock formations, which can be considered as isotropic, homogeneous and linearly elastic, stresses around the borehole are usually calculated using a closed form formulation known as the generalised Kirsch equations. These equations are the three-dimensional version of the original form of the well known Kirsch equations for calculating stresses around a circular hole in an isotropic, linearly elastic and homogeneous material. These equations have been widely used in the petroleum and mining industries over the past few decades. However, the boundary conditions on which these equations were based have been poorly explained in the literature and therefore merit further investigation.

In this thesis, in order to eliminate the ambiguity associated with the boundary conditions assumed for deriving the analytical model for stress analysis around the borehole, finite element analysis (FEA) was carried out to create a numerical counterpart of the current analytical solution. It appeared that the assumed boundary conditions for deriving the analytical model, i.e. the generalised Kirsch equations, are incompatible in the physical sense.

A new set of boundary conditions in better compliance with the physics of the problem was introduced in order to modify the analytical model, by reducing the simplifying assumptions made to facilitate the derivation of the closed form solution.

Another key parameter in borehole stability evaluation is the strength of the rock material at the borehole wall. The rock strength is usually evaluated using a failure criterion which is a mathematical formulation that specifies a set of stress components at which failure occurs. A number of different failure criteria have been introduced in the literature to describe brittle rock failure among which the Coulomb and the Hoek-Brown criteria have been widely used in industry; however, they both have limitations. For instance, both the Coulomb and the Hoek-Brown criteria identify the rock strength only in terms of maximum and minimum principal stresses and do not account for the influence of the intermediate principal stress on failure. On the other hand, at the borehole wall where a general stress state ($\sigma_1 > \sigma_2 > \sigma_3$) is encountered, a failure criterion which neglects the influence of the intermediate principal stress on failure seems to be inadequate for rock strength estimation in the borehole proximity.

Although a number of three-dimensional failure criteria have been proposed over the past decades, none of them has been universally accepted. A major limitation in studying the three-dimensional rock failure criteria is the lack of adequate true-triaxial experimental data that can be used for validation of theoretical rock failure models. A number of true-triaxial tests were carried out at the University of Adelaide and the results, along with nine sets of published true-triaxial experimental data, were utilised for comparison and validation of five selected failure criteria. These failure criteria have been developed especially for rock material and include; the Hoek-Brown, the Pan-Hudson, the Zhang-Zhu, the Generalised Priest and the Simplified Priest. A new three-dimensional failure criterion was also developed by modifying the simplified Priest criterion and was identified as a three-dimensional model which best describes the rock failure in three-dimensional stress state, compared to other selected criteria.

In this thesis, a case example is presented where the borehole instability is predicted by comparing the induced major principal stress at the borehole wall to the predicted rock failure stress. The major in situ principal stress around the borehole is calculated by means of the FEA based on the assumption of a new set of boundary conditions. The rock failure stress

under the three-dimensional stress state at the borehole wall is calculated by means of the newly proposed three-dimensional failure criterion.

Statement of Originality

This work contains no material which has been accepted for the award of any other degree or diploma in any University or other tertiary institution and, to the best of my knowledge and belief no material previously published or written by any other person, except where due reference has been made in the text.

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