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# **Robust Parameter Estimation in Computer Vision: Geometric Fitting and Deformable Registration**

by

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## *Abstract*

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by Quoc Huy Tran

Parameter estimation plays an important role in computer vision. Many computer vision problems can be reduced to estimating the parameters of a mathematical model of interest from the observed data. Parameter estimation in computer vision is challenging, since vision data unavoidably have small-scale measurement noise and large-scale measurement errors (outliers) due to imperfect data acquisition and preprocessing. Traditional parameter estimation methods developed in the statistics literature mainly deal with noise and are very sensitive to outliers. Robust parameter estimation techniques are thus crucial for effectively removing outliers and accurately estimating the model parameters with vision data. The research conducted in this thesis focuses on single structure parameter estimation and makes a direct contribution to two specific branches under that topic: geometric fitting and deformable registration.

In geometric fitting problems, a geometric model is used to represent the information of interest, such as a homography matrix in image stitching, or a fundamental matrix in three-dimensional reconstruction. Many robust techniques for geometric fitting involve sampling and testing a number of model hypotheses, where each hypothesis consists of a minimal subset of data for yielding a model estimate. It is commonly known that, due to the noise added to the true data (inliers), drawing a single all-inlier minimal subset is not sufficient to guarantee a good model estimate that fits the data well; the inliers therein should also have a large spatial extent. This thesis investigates a theoretical reasoning behind this long-standing principle, and shows a clear correlation between the span of data points used for estimation and the quality of model estimate. Based on this finding, the thesis explains why naive distance-based sampling fails as a strategy to maximise the span of all-inlier minimal subsets produced, and develops a novel sampling algorithm which, unlike previous approaches, consciously targets all-inlier minimal subsets with large span for robust geometric fitting.

The second major contribution of this thesis relates to another computer vision problem which also requires the knowledge of robust parameter estimation: deformable registration. The goal of deformable registration is to align regions in two or more images corresponding to a common object that can deform nonrigidly such as a bending piece of paper or a waving flag. The

information of interest is the nonlinear transformation that maps points from one image to another, and is represented by a deformable model, for example, a thin plate spline warp. Most of the previous approaches to outlier rejection in deformable registration rely on optimising fully deformable models in the presence of outliers due to the assumption of the highly nonlinear correspondence manifold which contains the inliers. This thesis makes an interesting observation that, for many realistic physical deformations, the scale of errors of the outliers usually dwarfs the nonlinear effects of the correspondence manifold on which the inliers lie. The finding suggests that standard robust techniques for geometric fitting are applicable to model the approximately linear correspondence manifold for outlier rejection. Moreover, the thesis develops two novel outlier rejection methods for deformable registration, which are based entirely on fitting simple linear models and shown to be considerably faster but at least as accurate as previous approaches.

## *Declaration*

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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# *Publications*

This thesis is based on the content of the following conference and journal papers:

- Quoc-Huy Tran, Tat-Jun Chin, Gustavo Carneiro, Michael S. Brown, and David Suter. In Defence of RANSAC for Outlier Rejection in Deformable Registration. In *European Conference on Computer Vision (ECCV)*, Florence, Italy, October 2012.  
(DOI: [http://dx.doi.org/10.1007/978-3-642-33765-9\\_20](http://dx.doi.org/10.1007/978-3-642-33765-9_20))
- Quoc-Huy Tran, Tat-Jun Chin, Wojciech Chojnacki, and David Suter. Sampling Minimal Subsets with Large Spans for Robust Estimation. *International Journal of Computer Vision (IJCV)*, July 2013.  
(DOI: <http://dx.doi.org/10.1007/s11263-013-0643-y>)
- Quoc-Huy Tran, Tat-Jun Chin, Julio Zaragoza, Gustavo Carneiro, and David Suter. Outlier Rejection in Deformable Registration with Moving Least Squares. *Submitted to Journal of Computer Vision and Image Understanding (CVIU)*.  
(under review)

In addition, I have co-authored the below paper:

- Julio Zaragoza, Tat-Jun Chin, Quoc-Huy Tran, Michael S. Brown, and David Suter. As-Projective-As-Possible Image Stitching with Moving DLT. *IEEE Transactions on Pattern Analysis and Machine Intelligence (TPAMI)*, November 2013.  
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*Dedicated to my family,  
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