

Hadron Structure in Deep Inelastic Scattering



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Andrew Casey

School of Chemistry & Physics

University of Adelaide

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Contents

Contents	i
Dedication	iv
Abstract	v
Statement of Originality	vii
Acknowledgement	ix
List of Figures	x
1 Contextual Statement	1
1.1 Contextual Statement	1
1.1.1 Calculating Dihadron Fragmentation Function in the NJL-jet model	1
1.1.2 Dihadron Fragmentation Functions from the NJL-jet model and their QCD Evolution	2
1.1.3 Gluon Polarization in the Proton	2
2 QCD and the Parton Model	3
2.1 History	3
2.2 Quantum Chromodynamics	4
2.2.1 QCD Lagrangian	5
2.2.2 Running of α_s and Asymptotic Freedom	8
2.3 Experimental Processes	10

2.3.1	Overview	10
2.3.2	QCD Factorization Theorem	10
2.3.3	Deep Inelastic Scattering	12
2.3.4	Semi-Inclusive Deep-Inelastic Scattering	16
2.4	Parton Model	18
2.4.1	Infinite Momentum Frame	18
2.4.2	Bjorken Scaling	20
2.4.3	DGLAP Evolution Equations	20
2.4.3.1	Parton Distribution Function Evolution Equations	22
2.4.3.2	Single Hadron Fragmentation Function Evolution Equations	25
2.4.3.3	Dihadron Fragmentation Function Evolution Equa- tions	27
2.5	Proton Spin Crisis	30
2.5.1	Polarization of the Gluon in the Proton	31
3	Nambu–Jona-Lasinio model	34
3.1	Concepts and Properties of the Nambu–Jona-Lasinio model	34
3.1.1	Nambu–Jona-Lasinio Model Lagrangian	35
3.1.2	Mass Gap Equation	36
3.2	Bethe-Salpeter Equation and the Bubble Graph	38
3.2.1	Quark-Meson Coupling	41
3.2.2	Meson Decay Constant	42
3.3	Regularization	45
3.3.1	Three-Momentum Cutoff Regularization	45
3.3.2	Light Cone Coordinates	49
3.3.3	Lepage-Brodsky Invariant Mass Cutoff Scheme	51
3.4	Fragmentation Functions from the NJL-jet model	53
4	Portfolio of Publications	57

4.1	Calculating dihadron fragmentation functions in the Nambu–Jona-Lasinio–jet model	58
4.2	Dihadron fragmentation functions from the NJL–jet model and their QCD evolution	71
4.3	Gluon polarization in the proton	85
5	Concluding Remarks	92
5.1	Overview	92
5.2	Summary of Research and its Significance	93
5.3	Issues Encountered	95
5.4	Future Research	97
	Appendix	99
.1	Gell-Mann Matrices	99
.2	Dirac Gamma Matrices	100
.3	Gamma Trace Properties	101
.4	Useful Integral Relations	101
.5	Table of Flavor Factors for Distribution and Fragmentation Functions	102
.6	Errata	102
	References	104

I would like to dedicate this thesis to my loving wife, Andrea, and
my precious daughter, Georgina ...

Abstract

Deep inelastic scattering (DIS) is an experimental process used to probe a wide variety of properties of hadronic matter. It is a process in which leptons collide with hadrons at high energies, resulting in the hadron being broken into a large number of other particles. Information obtained from this process is combined with what is known from studies of the strong force in Quantum Chromodynamics (QCD), to extract details of the hadronic structure. In this thesis, functions that can be extracted from DIS cross sections are discussed including structure functions, parton distribution functions, and fragmentation functions for single hadron and dihadron cases.

This thesis is presented as a portfolio of publications that investigate some of the previously mentioned functions that can be extracted from DIS processes, which includes semi-inclusive deep inelastic scattering (SIDIS). The first paper describes our method for generating the dihadron fragmentation functions (DFFs) within the Nambu–Jona-Lasinio-jet model. These functions describe the probability of detecting two hadrons with particular light-cone momentum fractions. The DFFs for combinations of pions and kaons calculated in the first paper are obtained at the model momentum scale of $Q_0^2 = 0.2 \text{ GeV}^2$. Several properties of these functions are explored, including how they change if strange quarks are included.

In the second paper, the appropriate evolution equations are applied to the NJL-jet model calculated DFFs to determine the DFFs at a typical experimental scale of $Q^2 = 4 \text{ GeV}^2$ for combinations of pions and kaons. A comparison with the results of another model at $Q^2 = 109 \text{ GeV}^2$ are also presented in this paper, with compelling results.

The final paper departs from the DFFs and instead investigates the gluon spin contribution to the spin of the proton, which is extracted from the spin dependent structure function g_1 using renormalization group techniques. An upper bound is suggested at leading order for the value of this contribution, with an estimate of the error calculated as well.

Statement of Originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to Andrew Casey and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Publications for the Portfolio

Calculating Dihadron Fragmentation Functions in the NJL-jet model

Andrew Casey, Hrayr H. Matevosyan, and Anthony W. Thomas,
Physical Review D, 85,114049, 2012.
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Cited as Ref. [1]

**Dihadron Fragmentation Functions from the NJL-jet model
and their QCD Evolution**

Andrew Casey, Ian C. Cloët, Hrayr H. Matevosyan, and Anthony W. Thomas,
Physical Review D, 86, 11401885, 2012.

Copyright 2012 by the American Physical Society.

Cited as Ref. [2]

Gluon Polarization in the Proton

Steven D. Bass, Andrew Casey, and Anthony W. Thomas,
Physical Review C, 83, 038202, 2011.

Copyright 2011 by the American Physical Society.

Cited as Ref. [3]

Signed:

Andrew Casey

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List of Figures

2.1	Diagrammatic representation of QCD factorization theorem	12
2.2	Deep inelastic scattering process	13
2.3	Semi-inclusive deep inelastic scattering process	17
2.4	Parton picture of scattering with a photon probing a hadron (a) before the virtual photon has an affect and (b) after the virtual photon collides, resulting in multiple smaller combinations of partons	19
2.5	Processes of the LO splittings involved in DGLAP evolution equations. Diagram for (a) $P_{qq}(\eta)$, (b) $P_{qg}(\eta)$, (c) $P_{gq}(\eta)$ and (d) $P_{gg}(\eta)$. The incoming parton has a momentum fraction of 1, the parton at the top has a momentum fraction of η and thus the parton at the bottom has momentum fraction of $1 - \eta$	24
3.1	Diagrammatic representation of the mass gap equation in the NJL model. Bold lines represent the propagators for the dynamically generated massive constituent quarks and the thin lines are propagators for the current quarks.	36
3.2	Diagrammatic representation of the Bethe-Salpeter equation shown in Eq. (3.10).	39
3.3	Diagrammatic representation of the rearranged Bethe-Salpeter equation shown in Eq. (3.12).	40
3.4	Feynman diagram for the decay of a meson	42
3.5	Cut diagrams for (a) valence quark distribution functions $f_q^m(x)$ and (b) elementary fragmentation functions $d_q^m(z)$	53
3.6	Cascade of hadrons produced in jet models.	55