Research Thesis

Real Options Valuation for Petroleum Investments

by

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Declaration

Although this thesis is conducted under the supervision of Doctor John van der Hoek, Professor Reidar B Bratvold, Professor Peter Peter Behrenbruch, and Professor Steve Begg, the research underpinning it, is entirely my own work. It has not been submitted in any previous application for a degree. All quotations in the thesis have been distinguished by quotation marks, and the sources of information specifically acknowledged.

Signed:____________________ July28, 2006

NAME: Real Options Valuation for Petroleum Investments PROGRAM: Master Degree of Petroleum Engineering Science

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Whilst this thesis is entirely my own work, many others have contributed to it and shaped it in their own unique way, and I would like to take this opportunity to recognise them.

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Notations

α	Drift parameter
β_r	A replicating trading strategy
F_{t+1}	Commodity future price at time t+1
c_{apex}	Capital expenditure
c_{α}	Pay offs generated from a strategy α
$\tilde{c}$	Variable production costs
$\tilde{c}(t)$	Variable production costs at time t
$c_o(t)$	Operation costs at time t
$c_{fix}(t)$	Fixed costs at time t
$Cf(t)$	Free cash flow at time t
c_{option}	Option costs
C_t	Price of real (delay) option at time t
$c_{waterconing}$	Water coning costs
d	Producing day
D	Dividend
D_{yearly}	Depreciation yearly
d	Down time factor
dz	The increment of a Wiener process
dt	Time interval
$ENPV$	Expected net present value
$E_0\left[\tilde{S}_t\right]$	Expected spot price at time t
$E_0\left[\tilde{S}_{t+1}\right]$	Expected future spot price at time t+1
k_t	Risk-adjusted discount rate
μ	Drift in geometric Brownian motion
L	The economic limit production
$N(\ ):$	The cumulative normal density function
n	Production years
NPV	Net present value
rf	Risk-free interest rate
$R(t)$	Revenue at time t
π	Risk neutral probabilities
σ^2	Variance of project returns
δ_t	Convenience yield at time t
t	Exercise time
$q(t)$	Production rate at time t
S_t	Stock price at time t
S_0	Current stock price
T	Expiration date of an option
$S(t)$	Gas price at time t

p	Probability
Pe	Contract penalty in the fifth year
$PV(t)$	Present value at time t
P_t	The price of a European put option at time t
$q(0)$	Production at time 0
$\lambda(t)$	Taxes at time t
λ	Tax rate
u	Up factor
$\bar{v}$	Upper bound of project value
$\underline{v}$	Lower bound of project value
V_0	Option value at time 0
V_{option}	Project flexibility value
$V_{project}$	Project value
$WACC$	Weighted average capital cost rate
ω_t	Given state of information
X	Exercise price
$x(t,\omega)$	Risky cash flow streams
$\bar{x}$	A long run equilibrium level in Mean Reverting Method

Abstract

In many instances, oil companies struggle with decisions pertaining to petroleum investment. The difficulty partially stems from the uncertainties in many of the inherent variables. Furthermore, conventional investment methods often fail to properly identify available opportunities.

As commonly acknowledged, traditional valuation methods such as Discounted Cash Flow (DCF) and Net Present Value (NPV) analyses are unable to properly portray investment opportunities. Due to large uncertainties and hence risk in Petroleum Exploration and Production (E & P), investors are gradually turning to a more dynamic approach to investment decisions.

Real Options Valuation involves a methodology for evaluating the value of an opportunity, leading to a strategic decision in an uncertain environment. Based on academic research in finance and business management, Real Options Valuation may be extended from option-pricing tools of the finance sector to that of evaluating E & P projects. In other words, although Real Options thinking has been widely accepted and used in some cases, the wider use of the Real Options approach is still a "hot" debate in the petroleum industry.

A permissible definition of "Real Options" may lead to inconsistencies among Real Options approaches. As such, Real Options may be defined as a company having a right, not an obligation, to invest in a future opportunity. The opportunity may involve technical aspects or may be purely commercial in nature. In all cases a quantitative approach is required. In the work by Borison (2003) and Bratvold et al (2005), the authors have listed five Real Option methodologies: the Classic approach, the Subjective approach, the Market Disclaimer approach

(MAD), the Smith approach, and the Luenberger approach. A comparative analysis of these Real Option approaches is presented in this thesis.

In comparing the above-mentioned Real Options approaches, it is apparent that two types of uncertainties may be considered: technical and market. In the study presented, two petroleum cases are considered: a technical uncertainty dominated case and a market uncertainty dominated case. The technical uncertainty dominated case is related to reservoir management. The market uncertainty dominated case involves a Liquid Petroleum Gas (LPG) distribution project. The case studies presented demonstrate the functionality of the five Real Options approaches.

This research is multi-disciplinary in nature, integrating the finance option theory with petroleum engineering projects, as well as project management. As such, it is shown that the petroleum industry could benefit from using Real Options Valuation in their investment strategy, thus improving petroleum business performance.

Chapter 1 Introduction

1.1 Research Background

Business under-performance in the oil and gas industry, and the failure of many projects to return expected results, is due in part to the high risks and uncertainties associated with the industry. In many instances managers make important investment decisions where they tend to trust their experience and intuition instead of relying on a proper project evaluation. Project valuation is the process of determining the actual market worth of an investment. The methodology is based on rigorous mathematics and logic. If this were the case, why would decision makers doubt a valuation and rely on other factors? One of critical reasons is that project valuations are often less rigorous and may be invalid in that uncertainties have not been properly included. Although such valuation "errors" may be numerically slight, there is the potential to incur great economic loss. On the other hand, companies have typically options available to make strategic changes during the project life of the investment, and such strategic actions have value. However, it is often difficult to determine the value of strategic actions due to the uncertainties involved and the use of traditional project valuation tools. As a result, valuation based decisions in the petroleum industry are usually challenged when they involve the commitment of high capital costs.

Compared to traditional methods, the Real Options pricing model (Real Options Valuation) should be more appealing. The attraction of Real Options Valuation lies not in making the estimated project value more accurate, but in taking the strategy (flexibility) value of the project into account. When decision makers face significant uncertainty, strategic and flexible management can dramatically reduce risks, albeit not totally eliminate them. This flexibility, the result of uncertainty, acted upon quickly and decisively, is able to add significant value to a project. Real Option thinking, thus, assigns a value for uncertainty. The more uncertainty, the more flexibility the project has, and thus, the greater the possible upside in project. In other words, flexibility has monetary worth. The Real Options Approach (ROA) thus concentrates on evaluating the monetary worth of flexibility.

Most Real Options approaches have three stages: the identification of uncertainties, the identification of Real Options, and the valuation of flexibilities. The identification of uncertainties involves the recognition of key sources of uncertainty. In principle, there are many sources of uncertainty in petroleum investments, but the two key uncertainties are technical uncertainty and market uncertainty. The second stage of the Real Options approach is to understand the types of flexibility or options. If the flexibility comes by solving technical problems, technical data is applied. In contrast, if the flexibility is related to a market situation, market data would be employed. The third stage is then evaluating, via logic and mathematics, the monetary worth of the established flexibility.

1.2 The Problem

The problem of Real Options Valuation is that researchers and practitioners identify Real Options from different perspectives. A few researchers think Real Options are flexibilities in which the project can be traded in the market. The project option value is thus the project trading value. On the other hand, practitioners believe technical strategies also have value. Real Options then are flexibilities in which investors strategically manage the technical uncertainty. Obviously, there are different types of flexibility and these can, therefore, be evaluated in different ways.

Since the 1980's, a number of approaches have been created for Real Options Valuation. These include, the Classic approach (Paddock, Siegel and Smith, 1988), the Subjective approach (Luehrman, 1998a), the Market Asset Disclaimer (MAD) (Copeland et al, 2000), the Smith approach (Smith and McCardle, 1998a), the Luenberger approach (Luenberger, 1998), and the Elliott et al approach (Elliott et al, 2001). Each approach identifies and values options in its own unique way, and therefore, each approach has its advantages and disadvantages.

To date no single approach has received unanimous support from both academic researchers and project managers. The main reason for the variety of Real Options approaches is a lack of "theoretical precision" in the definition of "Real Options"[1]. Moreover, there is very little written that compares and contrasts all the aspects of the various approaches. Lund (2002) concludes that most Real Options approaches contribute one type of flexibility, "ignoring the interrelations between different flexibility types. They simplify the project description, by including only one stochastic variable". The variety of approaches to value Real Options is thus a consequence of this problem.

The aim of this study then is to review the various Real Options approaches by comparing their valuation frameworks and their models. The study focuses on the two key uncertainties: technical and market related. By analysing these, the functions of modelling uncertainties in each Real Option approach can be seen. Furthermore a technical uncertainty-

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[1] Borison, 2002, pointed out "this situation leaves potential practitioners in troubling circumstances. In principle, the concept seems valuable and appealing. But given the current state of practice, there is a good chance that one could either apply an unsound approach or make inappropriate use of a sound one. The result is not simply a lack of theoretical precision, but mistaken investment decisions and lost value."

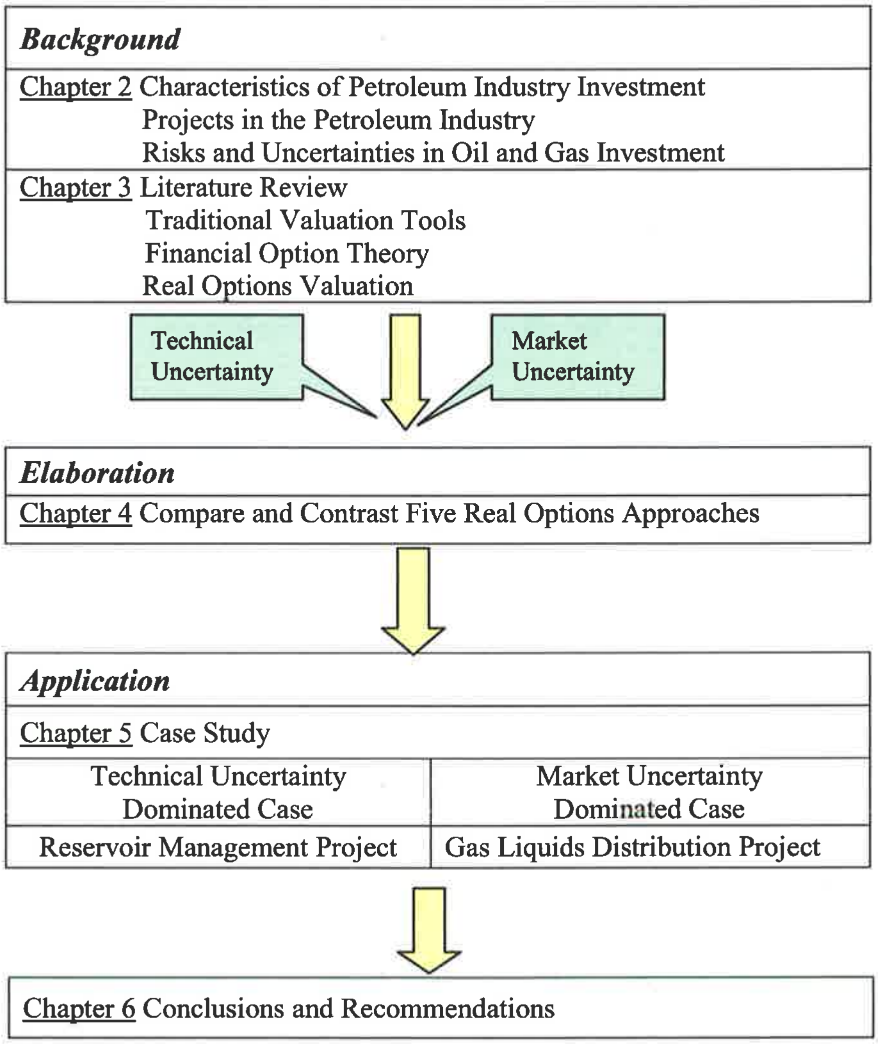
dominated case study together with a market uncertainty-dominated petroleum case is developed to test the conclusions and provide constructive advice for practical use.

1.3 The Structure of the Thesis

The thesis concentrates on comparing Real Options approaches in two aspects: the types of uncertainty (both technical and market) and the types of flexibility (both technical and market).

The thesis is structured in six parts as shown in **Figure 1**. Chapter 2 elaborates on the various risks and uncertainties in the petroleum industry and the types of oil and gas projects, together with the flexibilities or options available to E&P companies. Chapter 3 reviews current literature on the traditional Discounted Cash Flow method, project valuation, Real Options theory, as well as financial option pricing models. Chapter 4 illustrates five Real Options approaches, and compares and contrasts these through two key uncertainties: technical and market. Chapter 5 develops two petroleum cases studies, one involving a reservoir management strategy, the other being related to gas liquids distribution. The former is typical of the technical uncertainty faced by E&P companies, while the latter is a typical market-pricing problem confronting petroleum companies.

Figure 1 Structure of Thesis



Chapter 2 Investments in the Petroleum Industry

Petroleum exploration and development investment is a high-risk business, involving capital-intensive physical assets, as well as long cycle times. There are numerous, complex geological and market uncertainties that have led E&P investments to be at the more risky end of the investment spectrum.

2.1 Uncertainties in the E&P industry

Uncertainty denotes a lack of surety. In the uncertain world, investment decisions largely depend on their assessed value. Project valuation, however, is an estimate of the future value derived from numerous uncertain variables. An inappropriate estimate of petroleum project valuation could cause project managers to make a poor decision, leading, in turn, to potential large financial losses for petroleum companies, or to the failure to capture possible upside value. Hence, it is essential for project managers to understand uncertainties in the petroleum industry. Proper understanding has the potential to bring project valuation closer to real market value.

In addition to such possible downsides, uncertainty can bring unexpected opportunities where greater uncertainty gives rise to greater opportunity. Positive opportunities could create benefits for investors. In investment terms, uncertainty allows investors to not only reduce risks but also have the chance to create more upside opportunities.

Some of the more common uncertainties in the E&P industry are discussed below:

- **Market uncertainty**

Market uncertainty in the petroleum industry mainly results from price volatility. Historically, oil and gas prices have exhibited significant volatility due uncertainty in political situations, and the balance between supply and demand. Viewed from any instant in time, market uncertainty increases with time.

- **Technical uncertainty**

Technical uncertainties may be exploration and reserve related – involving the hydrocarbon pool size, shape, distribution, pressure, and recovery characteristics, etc – as well as field development uncertainties (involving aquifers, field production rate, field decline rate). The first category is related primarily to uncertainty in physical characteristics, which are actually present but not well defined, while the latter relates to man-made implementation which may be well defined a priori but has uncertainty as far as implementation. Although extensive scientific and goetechnical work is indeed essential for successful modern petroleum exploration, it must also be recognized that nearly all of the paremters required to assign an expected montery value to an exploration prospect can only result in an estimate, made under substantial uncertainty (Rose, 2001). The large magnitude of geological uncertainty can initially be reduced by paying for further data or information. Technical uncertainty tends to decrease with time, and as such technical uncertainty may be primarily associated with cost uncertainty.

- **Cost uncertainty**

Costs associated with any petroleum investment consist of two parts, Capital Expenditures (Capex) and Operational Expenditures (Opex). Capex uncertainty arises from uncertainties in future prices of exploration and production equipment or associated project implementation services, such as production facilities and pipelines, and their installation or construction. These costs are estimated with future pricing, and hence, by definition, are uncertain. Opex may include fixed as well as variable costs. In many cases, fixed costs, such as management costs and maintenance costs, are less flexible, and can be predicted with relative certainty. But variable costs, such as material costs, storage costs vary throughout the project life.

- **Timing uncertainty**

Scheduling and timing uncertainty is inherent in all projects and activities, the result of many influencing factors. All are contingent on the various uncertainties previously discussed. Timing uncertainty impacts project valuation in two ways. First, there is the actual time within the valuation cycle that events are predicted to occur, for example, time to first production, whilst, the second is related to the time value of money. Megill (1988) explained that when a corporation invests in petroleum exploration ventures they are anticipating the receipt of a series of future annual cash flow revenues. As Ross (2001) has pointed out, assessing the value of such future cash flows requires understanding of the time value of money, especially the concepts of the future value of money, compounding, present value as well as discounting. The project lifetime is directly related to the time value of money.

Moreover, the large number of uncertainties and their varying magnitudes can complicate the process of project valuation, thereby increasing valuation errors (discussed further in Chapter 3).

Among all possible uncertainties, market and technical uncertainty are fundamental. Cost and timing uncertainties can be correlated to market and technical uncertainties.

2.2 Risks in the E&P industry

2.2.1 High Risks in the E&P industry

Risk is defined as the potential threat of loss in a petroleum investment. The threat of loss is usually measured as monetary investment, i.e. capital. The capital loss results, therefore, directly from numerous uncertainties. Like uncertainties, risk cannot be eliminated, but can be reduced and then managed.

Obviously, risks associated with petroleum investments are very high. The outcome of a project is subject to the valuation process, the implementation and chance. Hence a "bad" outcome can arise from poor estimates as well as "bad" timing about when to deal with uncertainties. Even a "good" decision can bring possible losses for the company because of chance. Bratvold et al (2002) fully elaborated on how a "good" decision can lead to a "bad" outcome in decision-making. Poor estimates derive from an under or over estimate of the uncertainties previously discussed. It is difficult for decision makers to make profitable decisions and precisely predict project returns under uncertainty. Hence, uncertainty and decision quality do not have a linear relationship.

On the other hand, decisions would be often made without having all of the information. Lack of information causes a major threat to investment loss. Firstly, lack information for price trends can result in a project with increased price downside risks. Historically, it has been difficult for managers to forecast price trends. Secondly, as mentioned previously, the estimation of hydrocarbon prospect size involves many estimated parameters, each with its own uncertainty, not to mention various options in evaluation tools. Exploration professionals generally use probability in estimating the chance, as well as potential size, or for a potential discovery define the probability of discovering a certain volume at different confidence levels, typically "proved, or P90", "proved plus probable, or P50", and "proved plus probable plus possible, or P10". The use of such probabilities may bring significant estimation errors into project valuation, and hence potential monetary loss in a particular exploration prospect investment.

2.2.2 Two types of risks

There are two types of risks in the oil and gas industry: market and private.

Market risk is the potential loss that arises from oil (or gas) market uncertainty, where price volatility is a main risk factor. Worldwide oil production directly impacts the oil price. All oil and gas companies are exposed to price volatility. Thus, price volatility is critical for economic returns, and it is crucial for project valuation. Furthermore, potential loss due to market uncertainty also arises from supply and demand situations associated with world economic growth.

Private risk, on the other hand, is associated to potential loss that is related to the unique nature of each project. Different projects have different private risks. In the petroleum industry, private potential loss results from uncertainty of the interplay of geological uncertainty, engineering technology uncertainty, as well as corporate business strategies.

In conclusion, to evaluate any investment, practitioners are required to understand the character of various risks, as well as their sources, so that they could derive at more reasonable and realistic project values.

2.3 Flexibilities and Options in E&P Projects

2.3.1 Types of Oil and Gas Projects

A typical petroleum project may be divided into three “phases”: the exploration phase, the development and production (or exploitation) phase, and concurrently with the latter, the marketing phase.

The exploration phase typically involves seismic surveys and drilling of prospects, etc. Companies conduct these activities to collect geological information of hydrocarbon prospects. High capital costs are inherent in the exploration phase. The drilling of individual offshore prospects costs millions of US dollars or even tenth of millions of US dollars for deepwater projects. With investment of such large capital amounts, companies face a high threat of capital loss, the direct result of large geological uncertainty inherent in this phase. Due to poor exploration results, such as a series of “dry holes”, companies frequently abandon further exploration of a particular region.

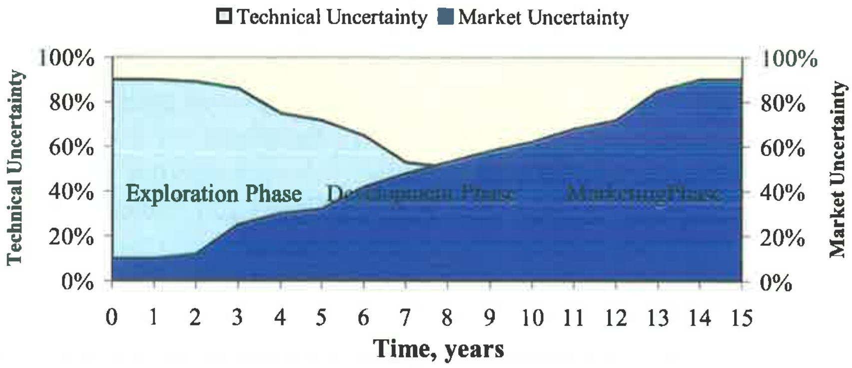
If exploration results, however, are favourable, a company may proceed to the development phase. This phase involves feasibility studies, appraisal and development drilling, and facilities design and construction. In the development phase, the company installs the necessary field production facilities, such as separators, lifting devices and metering equipment for production of hydrocarbons. The aim is to optimise the overall project, resulting in the best financial returns. Marketing involves downstream activities, related to refining, selling, and transporting of hydrocarbons. Such activities are very susceptible to price volatility.

Table 1 summarises typical project aspects, their uncertainties and risks.

Table 1 Uncertainty and Risk Categories of Petroleum Industry Projects

Phase or Business	**Example Activities**	**Example Uncertainties**	**Example Risks**
Exploration	Seismic surveys Drilling (wells)	Trap shape Hydrocarbon migration Integrity of seals Reservoir characterisation Faults and fractures	Seismic costs Drilling costs Logging costs Chance of success
Development	Facility design & implementation Recovery plan Well plan R&D Well testing Well treatment	Size of reservoirs Rock volume Formation thickness Porosity Pemeability Fluid properties Producable area Recovery factor Production rate Production decline rate Aquifer size & strength Reservoir communication	Facility costs Production costs Drilling costs Facilities costs
Marketing	Distribution project Refining	Oil / gas price volality Supply and demand Political issues	Investment costs Distribution costs

Figure 2 Uncertainties in Petroleum Investments



Technical uncertainty decreases with time. Market uncertainty increases with time.

2.3.2 Options in E & P Projects

As mentioned above, uncertainties lead to possible downsides or risk, but the former also create opportunities (or options) related to flexibilities in project investment. As **Table 1** and **Figure 2** show, projects in the petroleum business may have plenty of flexibilities. First, flexibilities derive from technical uncertainties. Technical flexibility is an opportunity that managers create to solve technical problems. Technical flexibility can add to the project value. Technical uncertainty decreases with time. For an oil or gas lease in the exploration phase, the technical uncertainty is dominant. And thus, there are a large number of technical flexibilities. These flexibilities are aimed at increasing the value associated with obtaining new information. The flexibilities are created as the company seeks to reveal more perfect reservoir information by taking advantage of timing and uncertainty. Once the value of information increases, the profit associated with the project will also increase.

Second, when the company proceeds to the marketing phase, market uncertainty is significant. Market uncertainty increases with time. Oil prices will determine the project value, which in turn reflects the value of the underlying asset to the company. The value of oil reserves needs to be converted from a physical asset to a monetary asset. During the conversion, flexibilities derive largely from market uncertainties, and thus the oil price fluctuation. The company can make use of the oil price volatility to obtain the convenience yield and the option value, and hence, obtains more profits.

2.3.3 Evaluating Flexibility

Most petroleum projects are valued using the traditional Discounted Cash Flow and/or decision tree analysis. Decision tree and DCF methods involve the use of a discount rate and deriving expected (risked NPV) values. There is debate over whether a single discount rate can represent all project risks through the whole project life. When the project has a long time-horizon, the single discount rate definitely does not account for all risks. An oil or gas lease usually has 20 to 30 years time horizon, and so O&G projects are extremely susceptible to this problem (this will be discussed in Chapter 3).

Therefore, evaluating flexibilities using Real Options Valuation approaches has received more and more attention in project valuation and decision-making in the petroleum industry.

Chapter 3 Background for Real Options Valuation

3.1 Introduction

In this chapter, firstly, we begin with the basic concept of a real option and its valuation. The key issue in real options valuation (ROV) is to analyse risks and uncertainties, and decide how to model them. We will model uncertainties by two stochastic processes—geometric Brownian motion (GBM) and Mean Reverting Model (MRM) — which are widely used in the real options literature. We discuss how Monte Carlo simulation can be used to estimate the volatility of the project returns, and how risk neutral probabilities are used in real options valuation. Finally, we illustrate real options thinking in decision-making and the value of information (VoI).

3.2 Basic Concepts about Real Options

3.2.1 Definition of Real Options

The real options concept as a branch of financial options was coined by Stewart Myers (1977)[2]. He used concepts from financial option valuation to evaluate real assets.

A real option is a right but not an obligation for investors to make an investment action. Real options are company strategies aiming at maximizing shareholder value. The options can be viewed as strategies and flexibilities to manage an underlying asset such as oil reserves or oil contracts that an oil company owns.

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[2] Also, Stewart Myers (1984) “Finance Theory and Financial Strategy”, p136, wrote:” Standard discounted cash flow techniques will tend to understate the option value attached to growing profitable lines of business. Corporate finance theory requires extension to deal with Real Option”.

In Luenberger (1998)—real options are “a series of operational options dealing with real assets”. Numerous operational options provide investors with managerial flexibilities and investment strategies. Investors exploit positive opportunities in uncertainties to increment positive cash flows by implementing managerial flexibilities and investment strategies.

In Dixit and Pindyck (1994)—Investment opportunities are analogous to call options on a common stock. It gives the right (which we need not be exercised) to make investment expenditures (at the exercise price of the option) and receive the project (a share of stock) the value of which fluctuates stochastically. The model of irreversible investment demonstrates a close analogy between a company’s option to invest and a financial call option.

Real options are options over real assets while financial options are often options over stocks. Both concepts are similar, but investors deal with different assets. In financial options, investors handle financial assets. In real options, investors handle real assets. A financial option contract gives investors the right, but not the obligation, to buy or sell a security at a future stated price. Analogously, with real options, investors (or holders of real assets) have the right, but not the obligation, to exploit positive payoffs from the strategic management in future.

Real options thinking centres attention on two key points: uncertainty and time. Firstly, uncertainty is not necessarily an enemy. Great uncertainty is not always equal to great loss. Great uncertainty can generate positive opportunities. Investors may capture “unexpected” monetary values from the positive opportunities. Secondly, time is on the investors’ side. In other words, investors can take advantage of time to resolve uncertainty and avoid downside risks. Time plays an important role when solving investment uncertainties with real options. “Wait and see”, or “a trial investment” is often used as a strategy with real options. Time is not only associated with chances that generate positive payoffs for the company, but also it is also associated with the time value of money. For example, a petroleum-engineering project (such as a reservoir management project or a drilling project) needs times to operate. Time allows investors to change their minds so as to avoid potential project losses. In fact, real option thinking represents a conservative approach for investment in a long run.

3.2.2 Real Options Categories

As with financial options, there are a number of categories of real options. These categories of real options are based on the types of strategies, such as delaying, abandoning, expanding, and so on.

a) The deferral call option— here the investor has the right to delay capital investment to a deferral date. It is a call option. If the deferral date is stipulated in a contract, this option is analogous to a European call option (ECO). The investors only can start a project at a contracted date. If the deferral date is not stipulated in a contract, it is analogous to an American call option (ACO). The investors can start the project at any time.

b) The abandonment option— here investors have the right to give up a project by selling an underlying asset (or a project) at a given price. It is viewed as an American put option (APO). When the price of the project is not favourable for investors (when the price of an underlying asset decreases), investors can abandon the project at a profitable price. For example, investors may begin with a small trial investment, if the investment results are unsatisfactory, the investors could abandon the project.

c) The expansion option— if an initial investment goes well, investors have the right to expand the project scale by investing additional costs.

d) A compound option—different types of options are being exercised in one project at its different phases. It is a combination of options.

So flexibility is the kernel idea of real options. Due to uncertainty, investors make investments more strategically, and create more opportunities. Investors take advantage of opportunities, and they obtain greater potential benefits from uncertainties. In this way, risks (losses) of investments can be partly reduced.

Real options valuation is a method to appraise an underlying property or a project. One focuses on the value of flexibilities under uncertainty, and the asset present value and its future potential value. The key issue in real options valuation is to assess the value of this flexibility. To do this, investors need to analysis sources of uncertainties. What types of uncertainties bring about risks? How does one model these uncertainties?

Actually, real options valuation is a tool for the project valuation specially to assign value to flexibility. It is difficult to value these flexibilities under uncertainties in other ways.

3.3 Project Valuation and Valuation Errors

Project valuation is a process to appraise the market value of an investment project. With project valuation, decision makers estimate the monetary worth of the investment project. It is a basis by which decision makers make optimal decisions in an uncertain world. However, decision makers may not make decisions based only on the results of the project valuation. The reason is partly due to valuation errors in it. As most researchers and professionals agree, a project valuation always has some valuation errors.

"A valuation error occurs when there is a difference between the market valuation of a contract and the value assigned by a valuation system"[3] (Picoult, 2002). The errors of a valuation system arise from valuation formulas and assumptions of the formulas, the transformation of data, inputs of observed market data, as well as related parameters. The lost project value (positive and negative) in the valuation arises from errors in inputs of the market

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[3] Picoult, E, 2002," Quanfitying the Risks of Trading", edited by Dempster, M.A.H in "Risk management: Value at Risk and Beyond", noted "my experience is that most of the large valuation losses that have been reported by firms in recent years can be attributed to either (a) errors in the values of the market factors used for valuation or (b) errors in the algorithms used to extrapolate from observed market rates to inferred, unobserved rates...That is why it is more precise to speak of a valuation system error rather than "model error".

data and related parameters, and errors in formulas and assumptions of the formulas. A given project might be valued differently with different formulas, even if valuation systems have the same inputs. The errors in inputs of the market data and related parameters are unique, and the errors in formulas and their assumptions are systematic.

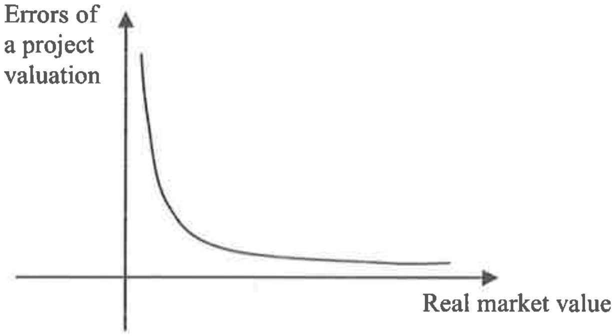
The errors of a project valuation will never be eliminated, but can be decreased (see **Figure 3**). Theoretically, errors are associated with any part of a project valuation system. Uncertain inputs in formulas and uncertain formulas in a valuation system could result in an uncertain value for a project. That is why decision makers are reluctant to take the project valuation as the only criterion in decision-making.

In the finance and corporation management literature, there are two commonly used project valuation methods: the traditional discounted cash flow method (the traditional DCF method) and real options valuation. The errors in the traditional DCF method arise from its formulas and assumptions (it will be discussed again in the Chapter 3.4.2.). As researchers and practitioners agree, the traditional DCF method is very simple and practitioners readily apply it. The "simple" here comes from two aspects: explicit mathematical formulas, and a direct transformation of data. The simple transformation of data would possibly ignore the values of an investment project.

In contrast, real options valuation is a valuation method that focuses on the value of flexibility. The mathematical sophistication of the real options pricing model is often a barrier for practitioners to apply it. Its "sophistication" contains abroad assumptions, such as the assumption of a complete market, and the assumption that markets are arbitrage free. Its "sophistication" also contains the probabilities, such as risk-neutral probabilities. Moreover, its "sophistication" also contains complex formulas for the valuation process. However, the abroad assumptions, complex formulas and the valuation process, which are used for valuing the flexibility, exactly compensate for weaknesses of the traditional DCF method.

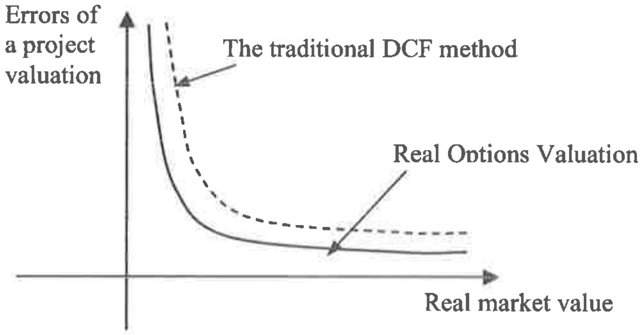
Thereby, real options valuation is intended to revise the valuation errors of the traditional discounted cash flow method. See **Figure 3**.

Figure 3 Project Valuation Errors



The errors of a project valuation will never be eliminated, but can be decreased.

Figure 4 Real Options Valuation Vs Project Valuation Errors



Real Options Valuation can decrease the errors of the traditional DCF method.

3.4 A Review of Traditional Project Valuation Methods

The increasing interest in real options valuation research is triggered by long-term unsatisfactory investment valuations based on the traditional discounted cash flow method. It is widely accepted that the valuation error of the traditional discounted cash flow method can be large, especially when it is used for contingent investment decisions (Laughton, et al 2000, and Copeland et al 2001).

3.4.1 Traditional DCF Method

The traditional discounted cash flow method essentially assumes that a series of future cash flows of a project are certain, and then discounts them according to the project risks, adding them together, using formula 3(1). In the process, project managers need to estimate a discount rate that accounts for the project risks. The total value of net discounted cash flows is Present Value (PV). The Net Present Value (NPV) is then this Present Value minus the initial investment and is assumed to be the project value. By calculating NPV or its derivations, such as the discounted payback (DP), the internal rate of return (IRR), the growth rate of return (GRR), project managers could predict the minimum capital requirement and payoffs of the investment, and the time value of capital. In this approach, the condition NPV>0 indicates that decision makers should invest. Conversely, if NPV<0, then managers should abandon the investment.

$$PV = \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t} \qquad \text{3(1)}$$

Where:

PV = the present value

CF_t = forecast cash flows after corporate taxes at time t

r = the discount rate

T = the project life

3.4.2 Limitations— A valuation system error rather than a model error

The traditional DCF criterion is able to estimate the inherent value of an investment project. It represents the values regarding the time value of money and the minimum capital requirement. When project cash flows are not uncertain, the traditional DCF method provides a project value that is equal to the market value. In another words, the traditional DCF method is suitable for valuing cash flows in an explicit manner.

Despite this, the traditional DCF method is used by many practitioners and has long been used to value oil exploration and development projects. But they have found this approach fails to account for the flexibility of future management.

The main reasons given are the following:

a) The method assumes cash flows are certain.

This assumption in the formulas is too restrictive to reflect the changeable movements of cash flows in real investments. In reality, project cash flows are usually uncertain. The level of uncertainty of cash flows depends largely on investment strategies and managerial flexibilities. When managers respond to unfolding unexpected events, the traditional DCF method does not allow for the changed cash flows. Thus, it does not consider how many project cash flows are impacted by strategic decisions that managers make. Valuation errors of the traditional DCF method occur when uncertain events unexpectedly occur. Thus, the traditional DCF method has a system valuation error rather than a model error.

As Dixit and Pindyck (1994) also argued, many investment realities do not meet the assumptions of the DCF method. When unexpected chances occur, the DCF technique cannot adjust quickly, responding to the new situation, and evaluate the alternatives. The traditional DCF method fails because it neglects to account for the value of strategies and managerial flexibilities.

It is also recognised by Campbell (2001). He argued that, there are three critical pitfalls. The first pitfall is the source of cash flows. All project cash flows come from a single investment case. The investment decision is irreversible. In reality, project cash flows could derive from many sources. An investment decision can be cancelled, changed, as well as reshaped. These decisions bring about changes to cash flows. Project cash flows can be always changed, reshaped, and optimised.

b) The discount rate

Campbell mentioned another critical pitfall results from the discount rate. The risks are liable to occur from any aspect of a project. A single discount rate cannot represent all the risks in different periods of an investment.

As Claeys et al. (1999) and Newendorp et al (2000) also point out, it simply assumes that a risk-adjust discount rate could not reflect future opportunity costs. The opportunity of the capital growth is intangible. Can a risk-adjusted discount rate account for the intangible opportunity costs? When future opportunity values are implicit, the valuation of the opportunity is complex. Simply depending on a single discount rate to value all flexibilities (opportunities), one would not obtain the real market value of an investment.

c) The utility of capital

Different projects have different amounts of invest capital, they involve different risks, but they might have the same NPV. The traditional DCF method does not consider the utility of capital. Newendorp and Schuyler (2000) also elaborated this aspect. For example, the present value of project A is $5000, and the present value of project B is $2000. If their initial investment costs are respectively $3500 and $500, then the net present value of project A is $1500, and the project B is $1500. Their NPVs are same, but investors will more readily choose the project B, since it costs less. The tradition DCF method does not tell investors that the utility of capital in the project B is less.

In summary, the traditional DCF method accounts for the time value of investment capitals. But it ignores management strategies and flexibilities that are taken in response to the uncertain market.

Due to the weaknesses of the traditional DCF method, improvements over these this traditional valuation method is needed. Real options valuation is a new approach to project valuation, and thus, could attract attention from investment managers.

3.5 Previous Studies of Real Options Valuation

Previous research of real options valuation originated from the valuation of natural resources investments, partly because they have higher risks and are more uncertain than other investments, and partly because oil companies use flexibilities for investment managements.

Real options valuation is a valuation method. The research on real options began in the 1980s. Researchers applied financial option theory to value assets in real investments. Financial option pricing models such as Black-Scholes Option Pricing Model (Black and Scholes, 1973) and Binominal Tree Option Pricing Model (Cox, Ross and Rubinsten, 1979) are widely being used to value managerial flexibility of real assets.

Brennan and Schwartz (1985) applied financial options to model the copper mine investment decision problem. According with the fluctuation of the copper price, the miner has three choices: open the mine, defer mining, and abandon mining. These are all managerial options. They developed a one-factor model for this investment decision evaluation.

McDonald and Siegel (1986) also used financial option valuation to appraise a project. They introduced a stochastic geometric Brownian motion to determine the present values and investment costs. They supposed that the life of project is unlimited, and that project cash flows could be zero at times during the life of the investment.

Paddock, Siegel & Smith (1988) wrote a paper using the option theory to study the value of an offshore lease and the development investment timing. The Paddock, Siegel and Smith's model is one of the most popular models for petroleum real options applications. They used the typical Black-Scholes option-pricing model to evaluate real assets. They believed that there is "the market equivalent value" for the real underlying assets. They connected the stock price with the real underlying asset. (We will discuss this in the next chapter.)

Copeland and Antikarov (2001) published a book "Real Options", and systematically illustrated the real options concept and a valuation assumption called—"Market Disclaimer Assets"(MAD). They introduced the binominal tree and the risk-neutral probabilities. The crucial assumption of MAD is: "the present value of cash flows of the project without flexibility (i.e., the traditional PV) is the best unbiased estimate of the market value of a project were it a traded asset." When the flexibility comes in, the real options value is the market value including the flexibility value.

These are advocates for real options valuation. However, there also exist critical voices about real options valuation.

Ross and Lohrenz (1996) used Black–Scholes option pricing model (Black-Scholes Model) to value oil and gas assets. They compared the values from Black-Scholes option pricing model with the one from the traditional DCF valuation. They proved the values from Black-Scholes option pricing model are greater than PV from the traditional DCF method.

Burns, Lewis, and Sick, (1992) argued, though the traditional DCF method "tends to undervalue real investments", they supported "the assertions (from advocates of real options) are correct for assets with values subject only to the effects of the oil and gas price fluctuations, but are not necessary correct and may be horrendously in error for real oil and gas asset values". In other words, they agreed real options valuation that are based on Black-Scholes Model and can account for management flexibilities from the price uncertainty, but it does not correctly account for the project specific uncertainty. Thus, their conclusion is that: there is “no panacea” to apply real options pricing model in every case.

Investors are less happy now to accept real options valuation as a valid method, because real options valuation does not necessarily make the important distinction for decision-making between what is possible and what is do-able. Even if opportunity values look attractive, they add values in project valuations but not in real investments.

The past literature presents pros and cons for real options valuation. As most recent researchers agree, and as Dayer (2002) points out, and as we have discussed above, the “real options approach is intended to supplement, not replace, capital budgeting analyses based on standard DCF methodologies”.

3.6 Real Options Valuation and Decision-making

Decision-making is the cognitive process of selecting a series of actions from among multiple alternatives. Real options valuation is a quantitative analysis of an investment project for decision-makers to judge investment decisions. Thus, real options valuation and decision-making are inter-related. Both are needed for decision makers. Gigerenzer et al (1999) and Kahneman et al (1982) point out: “understanding human decision-making processes has been a central enterprise for the cognitive sciences, as well as the focus of applied research across disciplines like psychology, economics, business, marketing, and the health sciences”.

The mathematical sophistication of real options valuation does not simplify the process and criteria for decision-making. For instance, the real options technique needs to use high-quality probability assumptions. The real options approach does not make decisions easier since making decisions are inherently difficult and complex.

However, an optimal investment decision needs to understand impacts of valuation tools and decision processes. Real options valuation does open up investors' minds in a new way. Real options valuation provides investors and managers with an insight on understanding how to deal with uncertain events.

As a consequence, improving valuation tools with real options valuation is one way that leads to improve decision-making. Despite this, one still needs to prove that with real options valuation there is a decrease in errors in the project valuation.

Therefore, to make an optimal decision, decision makers still need to rely on a combination of real options valuation and improved traditional techniques, as well as other related disciplines.

3.7 Real Options Theory

Real options theory is the analysis for values of a series of strategies and flexibilities in investments. It is related to the finance theory, decision-making, economics, corporation management as well as other related disciplines. Real options theory originates from the financial option theory.

To value flexibilities of an investment project using real options valuation, we necessarily review financial options and their valuations.

3.7.1 Financial Options

3.7.1.1 An Introduction to Financial Options

In finance, an option is a contract where an investor has a right (but not the obligation) to exercise a contract (the option) on or before a future date (the exercise date).

In option markets, trading financial options can have advantages over trading the stocks themselves. Investors trade options to speculate on prices movements of a stock. Generally, it is cheaper to trade options than the stocks themselves. It takes less capital than directly investing into underlying stocks. From this aspect, there are a number of opportunities and flexibilities in the investments.

Basically, there are two kinds of options: American options and European options. An American option permits the owner to exercise at any time before or at its expiration. The owner of a European option can exercise only at its expiration.

There are two popular numerical options pricing models: binomial tree model and Black-Scholes option pricing model.

3.7.1.2 Option Monetary Value

If the option does not have positive net payoffs, it is viewed as “out of money”. If the option yields positive payoffs, it is viewed as “in the money”. When the option is “out of money” (generating negative payoffs) at its expired date, the holders will simply “ abandon the option”. It will expire worthless. As a consequence, based on above definitions for the option, the option itself would never have value less than zero (**Figure 5**).

This aspect is very important for real options valuation. According to the financial option theory, when one evaluates the flexibility using option-pricing models, one always obtains a positive value of the flexibility. However, flexibility creates opportunities, but it is not equal to generate positive payoffs (See **Figure 6**). Only positive opportunities can generate positive payoffs. When one is not able to control uncertainties in investments, even if there are a plenty of flexibilities, investors could not obtain profits.

In other words, when one evaluates the project using real options valuation, one should not create unrealistic strategies. Otherwise, it is same as increasing income in the valuation but not in the real investment.

Figure 5 Option Monetary Values

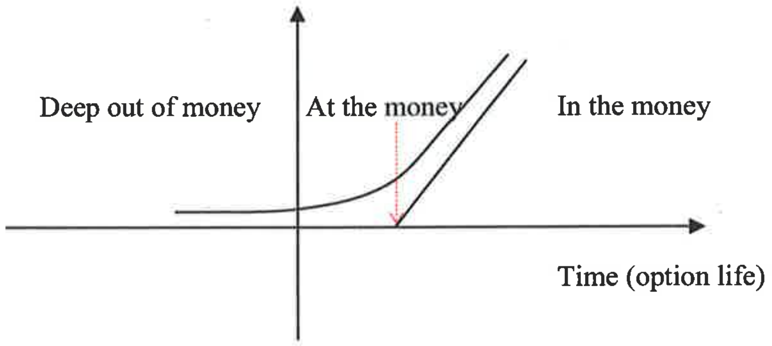
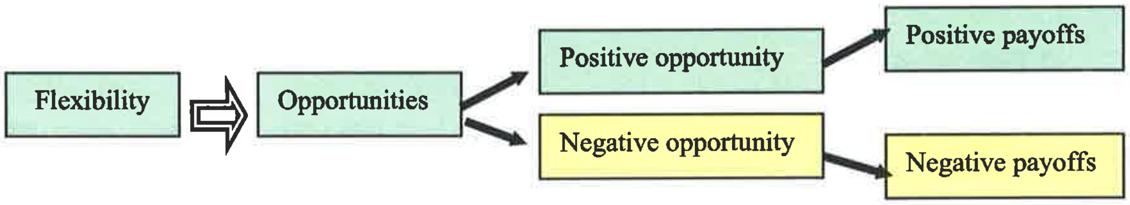


Figure 6 Opportunity ≠ Positive payoffs



3.7.2 Option Pricing Models

Common financial option-pricing models are the Black-Scholes option pricing model (BSOPM) and the binomial tree option model (BTOM), in which the former can be based on the latter. Binomial tree option model is a discrete model for the option valuation. Black-Scholes option pricing model uses a continuous model. Given small time steps in the BTOM, the option value from BSOPM is close to the result from this binomial tree model.

3.7.2.1 The Binomial Tree Options Model

The binomial tree options model (BTOM) provides a numerical method for option valuation. The binomial tree option model was first proposed by Cox, Ross and Rubinstein (1979). The model is a discrete time model to trace the evolution of variable option values, using a binomial lattice. In the model, the prices vary over time. Essentially, the binomial options model is an application of the risk neutral valuation over the life of the option. The valuation process is iterative, starting at the present price, deduce the variable prices from node to node till the final nodes, and then working backwards deducing option costs. The option value is the value at the initial node of the second binomial tree. The binomial tree model process is the following:

- Calculating the magnitude of the movement

a) Time steps (Δt):

In the binomial lattice, the whole option life is divided into given time steps. The given time steps begin at the valuation date, and end at the option expiration date. Each node in the lattice presents a possible price of the option at the given time.

b) Up and down factors (u and d):

It is assumed that the stock price movement is allowed to move in two directions: up and down. An up or down factor is calculated by Equation 3(4) and 3(5) respectively. The up and down factor, and time steps control price changes. Given So as the current price, the price in the next period will either be S_{up} or S_{down}, using equation 3(2) and 3(3). Thus the price tree is produced from the start date to the option expired date.

$$S_{up} = S_0 \times upfactor \text{ ....3(2)}$$

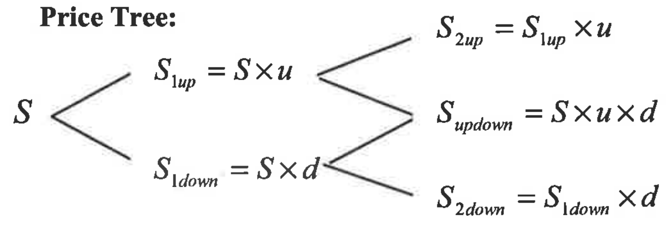
$$S_{down} = S_0 \times downfactor \text{ ....3(3)}$$

The up and down factors are calculated using the stock price volatility σ and the time step t.

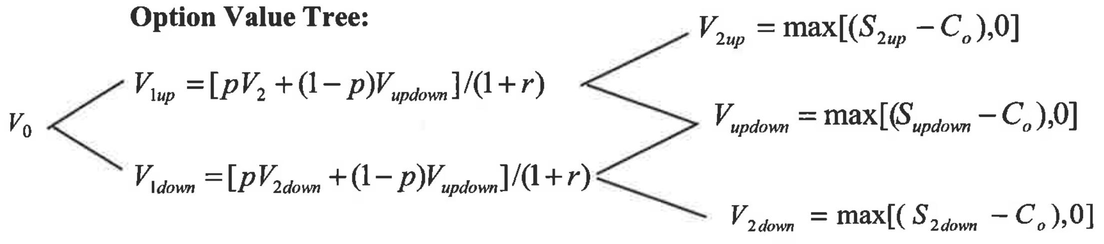
$$u = upfactor = e^{\sigma\sqrt{\Delta t}} \quad \text{.......................................} 3(4)$$

$$d = downfactor = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u} \quad \text{..............................} 3(5)$$

- Generating price trees:



- Calculating of the option value at each node:



Where Co is the exercise cost of the (call) option.

Through the backward calculation, the value V_0 at the first node is the option value.

From the formulas above, the value of the option consists of two components: the intrinsic vaslue and the time value, the first component being the value we obtain if we could exercise the option immediately, and the time value of the option is always positive and declines with time, reaching zero at the expiration date. The up or down factors could also be functions of time and the stock price volatility.

3.7.2.2 The Black-Scholes European Call Option Model

Black-Scholes European call option model (Black-Scholes model) is built upon following assumptions:

- Asset prices adjust to prevent arbitrage;
- Stock prices change continuously;
- Stock returns follow a lognormal distribution;
- The interest rate and the volatility of the stock remain constant over the life of the option;
- The model holds for European call options on stocks with no dividends.

The Black-Scholes European Call Option Pricing Model:

$$C_t = S_t N(d_1) - Xe^{-r(T-t)} N(d_2) \quad \text{.......} 3(6)$$

Where:

C_t = the price of an European call option at time t

S_t = the stock price at the time t

$N(\ )$ = the cumulative normal density function

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \quad \text{.......} 3(7)$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \quad \text{.......} 3(8)$$

X = the exercise price

T = the expiration date of the option

t = the current time

r = the risk-free rate of interest

σ^2 = the variance of the lognormal distribution of the stock return process

Based on Equation 3(6), 3(7), 3(8), one should note that the Black-Scholes option model do not depend on the discount rate that affect investors risk preference. Stock prices, time, the risk free interest rate, and the volatility, do not represent investors attitude for risk tolerances. All these variables are independent on risk preferences. As a consequence, this is risk neutral valuation, which we will discuss again in section 3.11.

With real options, parameters and methods to value the flexibility can be borrowed from financial options. The value of real options depends on six variables (Copeland and Antikarov, 2001):

- The value of the risky assets
- The exercise price
- The expiration time of option
- The standard deviation of the return rate of the underlying risky asset
- The risk-free rate of interest
- The dividends

3.7.3 Differentiating Risk Definitions Between Finance and Decision Analysis

The meanings of the term " risk" that is used in finance and in decision analysis are different. The different definitions of "risk" have consequences for different quantitative analysis.

Picoult (2002) presents an explicit definition on risk. He pointed out: "risk" can be defined as the magnitude of a potential loss, or "risk" can be defined as the standard deviation of the potential revenue (or income) of a trading or investment portfolio over some period of times. In decision analysis, "risk" always is understood as the potential downside monetary loss, where the mean of the project PV is less than zero.

When we discuss "risk" in real options valuation, as usual, we focus on the risk as the potential loss, and the uncertainty as the standard deviation of project returns. These definitions apply throughout the thesis.

3.7.4 Differences Between Real Options and Financial Options

The real options concept replicates the financial option theory to manage risks and uncertainties for physically tangible underlying assets. The real options concept is analogous to financial options concepts, but investors deal with different assets in different markets. Actually, real option is a process of risk management from the perspective of the finance.

Firstly, we will distinguish the markets between finance options and real investments. In finance, the efficient market means that a commodity price is decided by the time and its intrinsic value. The intrinsic value is marginal production costs of a commodity. In other words, the market price reflects marginal production costs and time values of the commodity. The market information is instantaneously available for anybody. There are no risk-less arbitrage chances in the market. Every investor is able to access the commodity that they want to buy at any time. Market liquidity[4] is smooth. The efficient market theory is based on macroeconomics "rational expectation theory"(Muth 1961). The future price change is a function of the price volatility and times. And thus, there is no analysis for human behaviour.

However, referring to real investments, investors are not always rational. Human behaviour effects a decision, its investment process, and of course, its investment results. In real options, investors deal with tangible real assets. Human behaviour plays an important role in the real market and in real options, and impact on the value of real assets.

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[4] Market liquidity is an economics term. It refers to the ability to quickly buy or sell a particular commodity without causing a significant change in the price. The essence of market liquidity is that there are ready and willing buyers and sellers at all times.

Real assets have their own characteristics. There is a comparison between real options and financial options in the table below:

Table 2 Comparison between Real Options assets and financial assets

Items	**Financial options**	**Real Options**
Market	Complete Market	Incomplete Market
Expiration time	Months	Years or months
Arbitrage ability	Limited	High
Carrying option costs	Limited	High
Depreciation of assets	None	Have
Initial value of option	Unchanged (Limited)	Changed according to market volatility and depreciation
Capital liquidity	High	Low
Physical constraints and technology impact	None	Plenty
Sources of uncertainties	Price uncertainty	Price uncertainty Technology uncertainty
The value effected by human behaviours	Market aspect	Market and technology aspects
Flexibility in Management	Low	High

From this comparison, we would conclude that:

- Real Options assets are more complex than financial assets.

Real options deal with two problems: the investment problem and the financial problem (Smith, 1998). Some investment projects do not meet all assumptions of financial option pricing models.

As such, in some cases, a project could not be viewed as a European call option, because they can be exercised at any time. Furthermore, real assets are often not in the complete and efficient market. If one directly replicated the financial option-pricing model to value opportunities, it would lead to valuation errors for real options.

3.7.5 Flexibility in Real Assets

With respect to an option on real assets, the option here pertains to the flexibility (or strategy) in the investment. Despite that, a real option is a right (but not an obligation) for investors to take investment action. They have different definitions of the flexibility with real options. The options can category into two types: flexibilities in the technology management and flexibilities with respect to financial markets. That is, there are options in management and options in pricing and trading.

The flexibility in real options is defined in a ready capability to adapt to new, different, or changing requirements in uncertain markets and in changing the technological management. Based on main types of uncertainties: the market uncertainty and the technical uncertainty, the flexibility consists of two aspects: strategies used for solving market uncertainties, and strategies used for solving technical uncertainties. **Figure 7** lists the origination and valuation of the types of flexibility.

- Strategies for market uncertainty

The strategies for market uncertainty cope with market uncertainty. The market uncertainties are the price uncertainty and the market liquidity uncertainty. As such, they are "trading project strategies" (Smith et al, 1998).

- Strategies for technical uncertainties

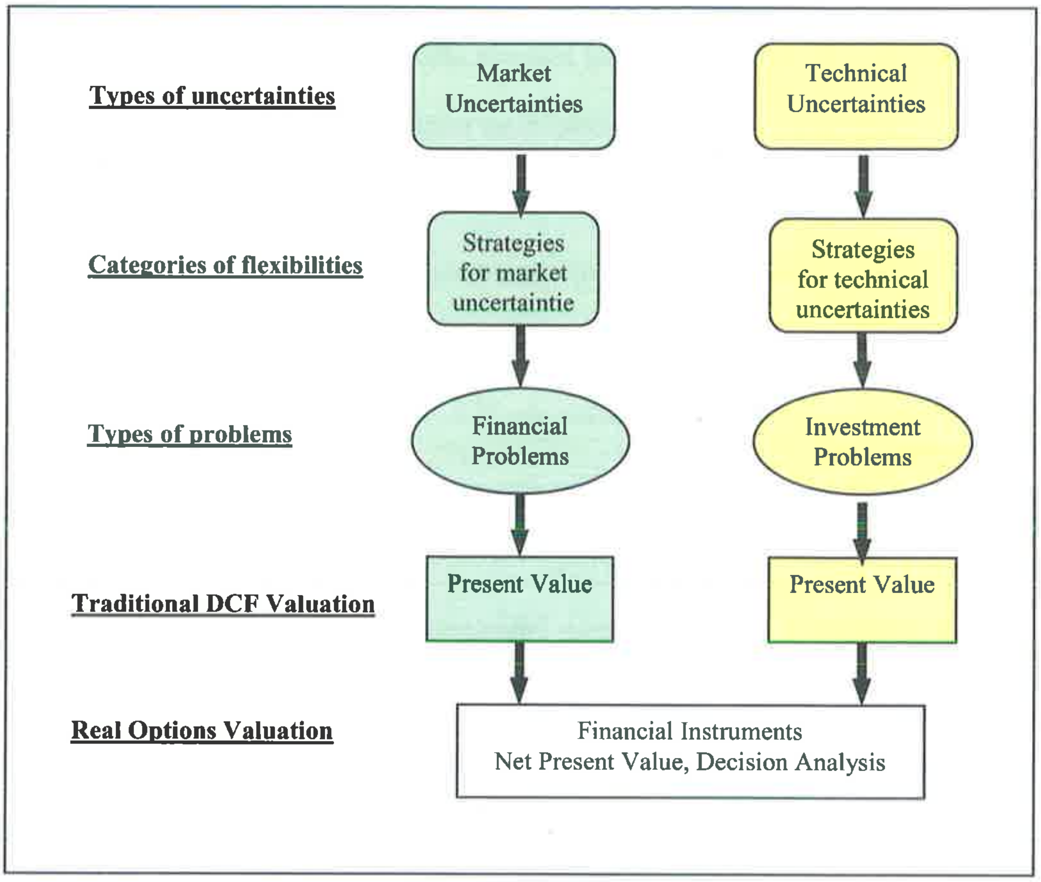
The strategies for technical uncertainties are related with the technology management. Investors use technical strategies, when they cope with technical uncertainties. As such, they could be a well treatment for a production well, or a horizontal drilling strategy for the oil exploration. They are strategies used within the company.

By reviewing a number of real options papers, we led to conclude that there is a belief amongst finance researchers that "the value of a non-traded project is the price the project would have when it is traded in the market" (Mason and Merton, 1985). The fundamental assumptions of real options arise from financial options. The flexibilities of the project are strategies by selling or buying projects after the initial capital has been invested. In other words, the flexibilities could be derived from trading the project. Thus, the flexibility value is mean to be the value of the trading project. These strategies are essentially related with the price volatility. And therefore, based on numerous published papers, most researches focus on the price uncertainty (the market uncertainty). We refer to N. Dickens, Ross and Lohrenz (summer 1996); Brennan, M.J., and Schwartz, E.S., (1985).

Another side comes from management researchers. They believe flexibilities in a project are strategies used for technology management. Managers should actively and quickly response to technically uncertain events. Managers implement strategies in order to optimise technical flexibility values. Strategic management creates opportunities that have positive payoffs. Managers exploit these opportunities to increase the project profits. Thus, the flexibility value is the value of technical strategies. Even if the invested project is not traded in the market, there still exist the flexibility value in technical strategies. As such, Luenberger(1998) discussed the option to expand and how to value it. The project in his example is on a nontraded project. The value of flexibility derives from management strategies. Also, Gallil, Armstrong, and Dias, (2004) and Dias (2002) discuss the valuation of technical strategies as real options. As most researchers agree, there are two parts of the values in investments. One part of the value is the present value, arising from the direct investment. Another part of the value is the managerial flexibility or the opportunity value, arising from flexible investment strategies. Based on the real options literature, the former is evaluated by the traditional DCF method, and the latter is evaluated by real options valuation.

Therefore, based on these two meanings for strategies, numerous real options valuation approaches arise in the real options literature.

Figure 7 Flexibilities and Uncertainties in Real Options Valuation.



3.8 Modelling Uncertainty in Stochastic Processes

Real options thinking (Begg et al 2002) guides people to exploit more uncertainties. They believe that more uncertainties bring investors more opportunities and flexibilities in the investment. If opportunities were correctly dealt with, the more value there would be. The critical issue in real options valuation is how to understand uncertainties, and then how to value flexibilities.

Two mathematical models have been used to describe the price uncertainty in petroleum project valuations: geometric Brownian motion (GBM) and mean reverting oil price model (MROPM). The two processes assume that the oil price varies continuously over time in a random or partly random way. The underlying asset value is stochastic. Therefore, investment decisions are made under uncertainty.

Geometric Brownian motion also can be used to model the cost uncertainty and production uncertainty. (We will discuss these in the Chapter 5. GBM integrates with the petroleum technical production model.)

3.8.1 Geometric Brownian Motion

A geometric Brownian motion (GBM) (Brown, 1828) is a continuous (time) stochastic process. Its mathematical model is appropriate for some financial phenomena, such as the stock price or an option price movement, because absolute numerical changes of the stock price vary stochastically. The equation is:

$$dx = \alpha x dt + \sigma x dz \quad \text{..} 3(10)$$

Where:

dz is the increment of a Wiener process

dt is a time interval

α is the drift parameter. In a price forecast, the drift usually is the risk free rate.

σ is the volatility of the stock price.

Based on the equation 3(10) above, as Dixit and Pindyck elaborated (1994), GBM contains three key assumptions:

- Markov process

This means the future price depends only on its current value not its past value. The future price path is determinate only by the current price.

- Wiener process

The Wiener process describes the changes always exist in the process. Each change in the log price at any time involves an independent increment dz.

- Normal probability distribution

The changes in the log price at any time t are distributed normally. The variance of this distribution could be a function of time.

Geometric Brownian motion is widely used in finance for modelling future stock prices. It will be frequently used in the case study in the Chapter 5.

3.8.2 A Popular Oil Pricing Model – The Mean Reverting Model

Mean reverting model is fundamentally based on Brownian motion.

The general equation of a mean reverting model (Ornstein-Uhlenbeck process) is:

$$dx = \eta(\bar{x} - x)dt + \sigma dz \quad \text{..................................3(11)}$$

A "normal level"— $\bar{x}$ controls the drift in the mean reverting model (Dixit and Pindyck, 1994). $\bar{x}$ is assumed to be a long run equilibrium level. Also, it can be viewed as a long run marginal production cost.

The model represents:

- Despite that the price sensibly oscillates in short term, the price of a commodity follows its marginal production costs in long run.
- The drift can be positive and negative in the process. When the market price goes far from the equilibrium level (becomes "unreasonably" high or low), the equilibrium level

will draw back the price. In other words, the long run marginal production costs constrains the commodity price trend. Like price leverage, the marginal production cost constrains price changes.

- Based on the microeconomics theory, the model reflects the "rational expectations theory" (we discuss this in section 3.7.4). That is, the price of the commodity is decided by time and its intrinsic value. The intrinsic value means the marginal production cost.

As widely agreed by academics and practitioners, the mean reverting process is centred on the long-term marginal production costs. It presents the nature of physically tangible commodities, such as the oil and gas.

Referring to the oil price, although it is volatile due to political issues, geological structures, exploration and production technologies, and labour costs, for example, the long-term oil price is tied with marginal production (exploration and development) costs of OPEC (Organization for Petroleum Exporting Countries).

3.9 Using Monte Carlo Simulation to Determine the Project Volatility

Monte Carlo simulation is used to "simulate repeatedly the random processes by governing random variables (production rates or prices) from statistical or mathematical models" (Dowd, 1998). It is named after the small country famous for its casinos. Each result is a possible value from the statistical or mathematical model. The model repeatedly generates numerous results. The results from Monte Carlo simulation depend on the model that one uses to describe the variables. The mean and variance of the distribution could indicate the model characteristics. Copeland et al (2001) used Monte Carlo simulation to estimate the volatility of the project returns. When the risk factor is non linear with project revenues, Monte Carlo simulation can be used to work out a distribution for project revenues. But the computation is intensive because one needs many repetitions.

In the thesis, in order to determinate the project return volatilities, we will implement @Risk software to run Monte Carlo simulation. We will discuss this in the later chapter (the Chapter 4.4.2).

3.10 Convenience Yield

The existence of the convenience yield is important for the investment management. Financial theory assumes the market is efficient. A price protection (or a price leverage) arises from the efficient market. The asset prices always reflect demands and supplies of the market, and always balance the two. The diversity of investor opinions results in human behaviour that can lead to the price volatile, because investors do not always make reasonable decisions. Thus, the convenience yield actually is arbitrage benefits from the price volatility. Investors could exploit the convenience value by "surfing" the volatility of commodity prices.

3.10.1 Definition of Convenience Yield

In finance, the convenience yield is the dividend on a stock, and it is a yield per unit of a stock price. But for a real commodity, the definition is slightly modified. The convenience yield is the flow of benefits that the marginal stored unit provides (Dixit and Pindyck, 1994), or in other words, a liquidity premium arising from the storage of a commodity (Gibson, 1990). It is a measure of the value of remaining physical control of the commodity.

A firm often holds inventories to be able to smooth production. It is convenient for the firm to access productions. By this method, a firm would avoid future shortfalls. During the high demand market, a firm might be willing to hold the commodity in order to get opportunities to arbitrage. During the high supply market, the firm might be willing to relinquish the commodity to avoid its physical depreciation.

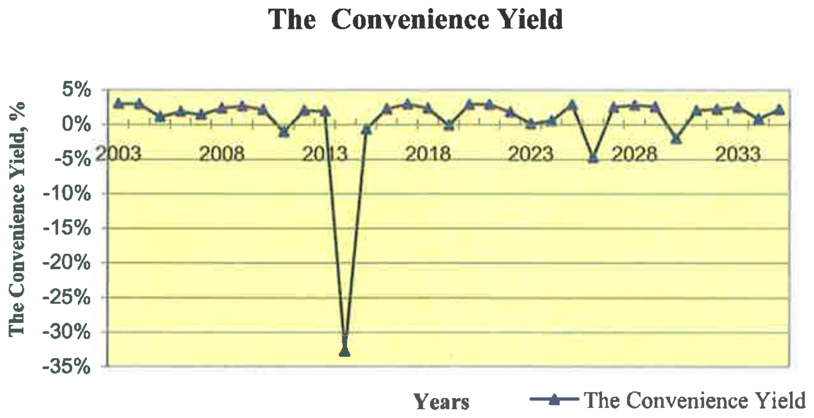
The convenience yield means that the company has benefits when the firm stores a commodity. "Inconvenience" always carries costs. The convenience yield represents the relation between the carrying-commodity-costs and carrying-commodity-benefits. For example, the gold has a zero convenience yield, because its price changes are very small, and generally compliant with changes of the risk-free interest rate. So keeping gold is same as keeping money in the bank.

3.10.2 Convenience Yield and Oil price

Convenience yield is closely related to the oil price. Based on the definition of the convenience yield, it arises from the price changes. If the oil price volatility is zero, the convenience yield is constant and equal to the discounted rate. The convenience yield is an indicator of the oil price risk. **(See Figure 9)**

Although the convenience yield is associated closely with the market risk (public risk) of the project, it makes less sense for private risks.

Figure 8 Convenience Yield



If capital costs is only the time value of money, and the price risk is low, the liquidity prenium(convenience yield) is low.

3.10.3 Convenience Yield and Project Cash Flows

The convenience yield could significantly impact on the firm cash flows. The firm buys or sells commodities (inventories) to obtain or reduce cash flows. By this way, the firm also can optimise cash flow portfolios. From the components of cash flows, the convenience yield could impact on operating costs, annual revenues, and taxes of the firm. The firm-buyer could acquire the tax refunds by arbitraging the convenience value. Different investors can generate different cash flows from a same investment using arbitrage strategies. In essence, this is arbitrage behaviour.

3.10.4 Convenience Yield and Storage Costs

Inconveniently accessing to inventories could cause extra costs, when the firm needs that inventories urgently. The company would pay more, such as long-distant transportation expenses, if there were shortfalls of the commodity in the demanding market. These costs could be avoided by holding suitable inventories for emergency needs. In petroleum investments, investors hold reserves as inventories for the development, and obtain the oil convenience value. Also, investors could hold reserves for trading project purposes.

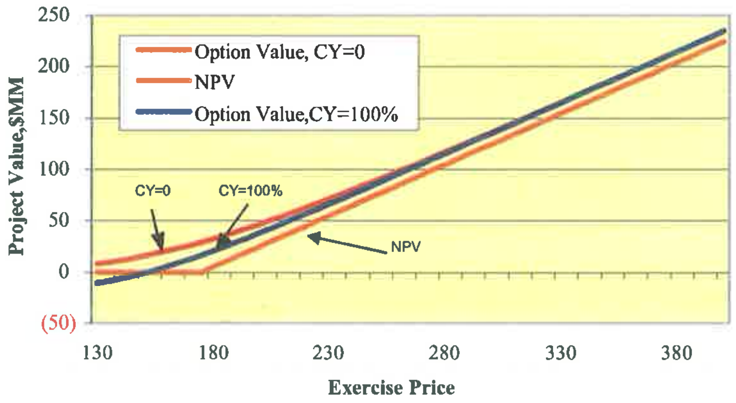
However, it is important to distinguish between gas and oil. In terms of gas, it could have the convenience liquidity premium in theory due to changes of gas price. In reality, its storage is so costly that few investors exploit its convenience value. In this case, the firm could trade the project as a strategy to exploit the real options value, rather than carry the gas as the commodity. Then storage costs can be considered as keeping option costs, such as costs to delay for developing the gas field.

3.10.5 Convenience Yield in Real Options Valuation

In real options, the price volatility pushes investors to delay for developing oil fields. On the contrary, the convenience yield pushes investors to exercise option early. When the weighted average cost of capital (WACC) is larger than the growth rate of the oil price, the convenience yield is positive. The time value of capital that is occupied by invested reserves is higher than the option value of the reserves. Investors keep options with high costs. So, the project should be developed early.

Consider **Figure 9** (Borison, 2003). Assuming the convenience yield is very high at nearly 100%, PV of the project is close to the real options value of it. It represents the situation where there is not too much flexibility in the project. The project should be invested in right now.

Figure 9 Convenience Yield in Real Options Valuation



3.10.6 A Model of Convenience Yield

Based on the concept of the convenience yield, the basic model is defined by:

$$F_{t+1} \times (1+\delta_t) = F_t \times (1+r_f) \quad \text{.............................} 3(12)$$

where:

F_t = Spot price at time t

F_{t+1} = Future prices at time t+1

r_f = Risk free interest rate

δ_t = The convenience yield at time t

According to the formula 3(12), the future value of the commodity with the convenience yield is equivalent to the inherent value of this commodity plus the time value of money. The inherent value of the commodity today is equivalent to the inherent value of the commodity plus its time value tomorrow.

But considering carrying commodity costs and the time value of cash flows occupied by the commodity, Sick (1997) proposed a modified model for the oil convenience value for the petroleum industry. This is:

$$\delta_t = k_t - \ln\left(\frac{E_0[\tilde{S}_{t+1}]}{E_0[\tilde{S}_t]}\right) \qquad 3(13)$$

where

k_t is the risk-adjusted discount rate

$E_0[\tilde{S}_{t+1}]$: the expected future spot price at t+1

$E_0[\tilde{S}_t]$: the expected spot price at t

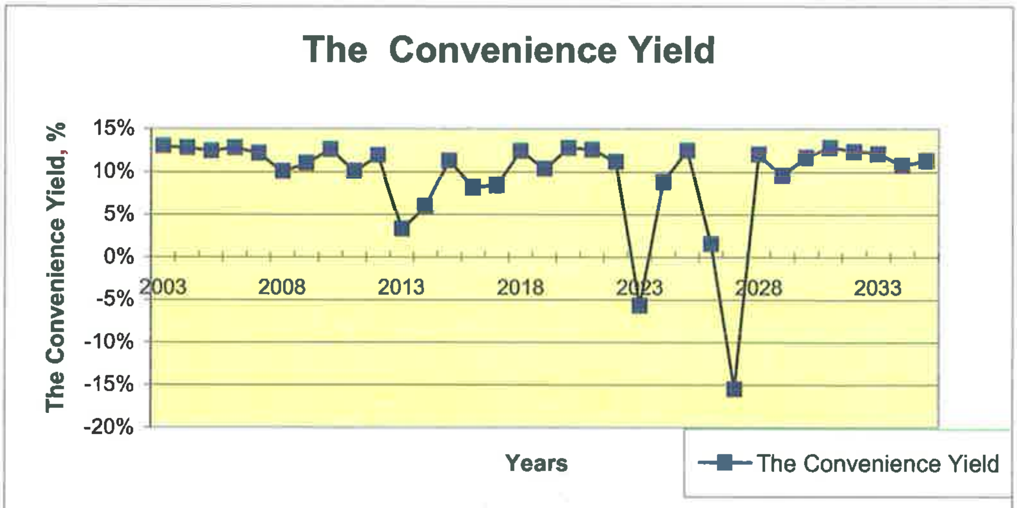
Based on the formula 3(13) above, when the future oil prices decrease, $\ln\left(\frac{E_0[\tilde{S}_{t+1}]}{E_0[\tilde{S}_t]}\right)<0$, and one receives the high convenience value, and a high liquidity premium at present. Investors will not control more physical inventories because the commodity price could be accessed with lower costs in future. When the future oil price increases, $\ln\left(\frac{E_0[\tilde{S}_{t+1}]}{E_0[\tilde{S}_t]}\right)>0$, investors will get the low convenience value, and will control more inventories because of its low liquidity value at present. The equation shows the price volatility is sensitive to the convenience yield.

In equation 3(13), $\ln\left(\frac{E_0[\tilde{S}_{t+1}]}{E_0[\tilde{S}_t]}\right)$ is the growth rate of the expected spot price. The oil convenience value is the discounted value minus the expected growth value of the oil price (that is, WACC minus Growth Rate of the oil price).

Sick considered three aspects into his model, which are the costs of capital, the time value of capital, and investment opportunity costs.

Embedded with real options, if WACC is larger than the growth rate of the oil price, δ_t is high, the time value of capital that are occupied by investment reserves is higher than the growth rate of the oil price. Investors keep options with high costs (**Figure 10**). Investors should develop the project immediately. The option should be exercised early. If WACC is less than the inherent price growth rate, δ_t is low, the time value of capital are lower than the inherent price growth rate, investors keep options with low costs. Investors should delay in exercising the option. If WACC is equal to the growth rate of the oil price, the carrying-option costs is equal to the growth rate of the oil price, there is no benefits to control the physical petroleum assets. Whether one develops the project later, or develops the project now, the investors would receive the same returns.

Figure 10 High Convenience Yield in Real Option



Graph 3 If the cost of capitals is high (using WACC), and the oil price risk is high, the liquidity premium is high.

(All Data above come from Borison paper, " Real Options Analysis: Where are the emperor's clothes?")

3.11 Risk Neutral Probabilities

The term of “risk neutral” in economics is used to describe an investor who only cares about the expected outcome of an investment. He never pays less to avoid risks (the potential loss) or positively take more risks. The investor always sits on the fence for risks. In the risk neutral valuation, the expected return rate of holding an underlying asset is the risk free interest rate. The value of an underlying asset is discounted by the risk free interest rate.

The risk neutral probability is a measurement for risks.

The equations are:

$$u = upfactor = e^{\sigma\sqrt{t}}$$

$$d = downfactor = e^{-\sigma\sqrt{t}} = \frac{1}{u}$$

$$\left[\frac{(1+r_f)-d}{u-d}\right]+\left[\frac{u-(1+r_f)}{u-d}\right]=1 \quad \ldots\ldots\ldots\ldots 3(14)$$

$$up\_probability = \frac{(1+r_f)-d}{u-d} \quad \ldots\ldots\ldots\ldots 3(15)$$

$$down\_probability = 1-up\_probability \quad \ldots\ldots\ldots\ldots 3(16)$$

Where:

σ : the price return volatility

Δt : time step

r_f : risk free rate

Based on Equation 3(14), 3(15), 3(16), one could infer that the risk neutral probability is only related with the time and the price volatility. This proves what “the efficient market theory” holds. The price of a financial underlying asset is a function of time and the price volatility. All (attainable) financial assets have the same expected rate of return, regardless of the price volatility.

However, real assets do not have same expected rate of return, since assets have their own characterized private risks.

3.12 Value of Information

The Value of Information (VoI) is a quantitative measure of the value of knowing the outcome of uncertainty variables prior to making decision. It includes values of "perfect information" and "imperfect information". "Perfect information" describes the fact that is fully revealed. "Imperfect information" means the fact is partly or never being revealed. It is known that information plays an important role in the process of investment decision-making. The value of information can be viewed as a "learning option" (Dias, 2002) in real options, since obtaining information involves monetary costs, so information has monetary values. When high uncertainties arise, perfect information has high value for the investment, and increases the project value.

Value of information is related closely with technical flexibility. This is because the whole process of implementing technical flexibility is essentially a process of revealing information.

Chapter 4 Real Options Approaches

4.1 Introduction

4.1.1 Various Real Options Approaches

Since 1980s, real options have been valued by a variety of approaches. Currently, most oil companies still use the traditional DCF method to evaluate investment projects rather than using real options valuation. The main reason for this is that there are disputes about the correct approach to the valuation of real options.

We elaborate five real option approaches in the thesis. As named by Borison (2003), they are the Classic Approach (Paddock, Siegel and Smith, 1988), the subjective approach (Luehrman, 1998a), the Market Asset Disclaimer (MAD) approach (Copeland et al, 2001), the Smith approach (Smith and McCardle, 1998) and the Luneberger approach (Luenberger, 1998).

All of these focus on valuing the flexibility. They are built upon their own assumptions and mechanisms respectively. The assumptions used in different approaches are aimed to model uncertainty and assign value to flexibility.

The goal of our research is to compare the different real options approaches by applying them to realistic petroleum investment projects, to help petroleum managers comprehend different real option valuation approaches, and to apply real options valuation in the E&P projects. Also, we identify characteristics of the real options approaches, indicate their applicability, and then draw some constructive conclusions.

4.1.2 Introduction to the Borison (2003) Paper

4.1.2.1 Real options approaches described in this paper

Borison published a paper " Real Options Analysis: where are the emperor's clothes?" He discussed five real options approaches in this paper. They are listed as following:

- Classic Approach
- Subjective Approach
- MAD Approach
- Revised Classic Approach (Amram and Kulatilaka, 2000)
- Integrated Approach (Smith Approach)

In addition to these approaches, there is another real options approach – the Luenberger approach. It will be discussed in the section 4.6.

4.1.2.2 Reasons for various real options approaches

Current researchers agree that the real options concept is a valuable innovation for investment management and finance. However, various real options approaches being used are inconsistent. The literature presents different definitions for real options. Real options are "a series of operational options dealing with real assets" (Luenberger, 1998). The "operational options" are composed of two parts: the financing flexibility (trading projects) and the technical flexibility. Financial people would like to think real options as a series of financing operational flexibilities in the market. Technical people would prefer to regard real options as a series of technical operational flexibilities. The options should be evaluated differently because of the different characteristics of the flexibilities. Also Lund (2002) in "Real Options in Offshore Oil Field Development Projects" says that most real options papers simplify the types of flexibility, and focus only on the market uncertain variables. As a result, "it is hard to discern the benefit of flexibility in a realistic oil field development project from contributions reported in the literature."

And as a result, it is difficult for practitioners to choose a sound implementation. Borison mentions, "The difficulty of implementing real options approaches is rarely being discussed in the literature. Moreover, when there are errors in project valuation, wrong investment decisions can be made. These wrong decisions could be costly for shareholders.

Based on their assumptions, applicability, and mechanisms, Borison contrasted and criticized these real options approaches. Also, he studied an example from oil investment projects to interpret these approaches. Through the comparison, Borison pointed out pros and cons for each approach, and suggested how one should apply each real options approach.

4.1.3 Other approaches

In the thesis, we describe another approach—the Luenberger Approach. We will discuss it later in the chapter.

4.2 The Classic Approach

4.2.1 Assumptions (no arbitrage, market data[5])

The Classic Approach is based on Paddock et al. (1988). The assumptions are analogous to those used with financial options.

- It supposes the investment market is complete and efficient.
- There are no arbitrage opportunities in the complete market, so no profitable project exists without paying costs for it.
- There is a replicating portfolio between a financial asset and a real investment. The financial asset and the underlying asset are similar; the underlying asset can thus be traded and priced by financial instruments.

Given a no arbitrage condition, similar assets must necessarily have similar prices. The price of the underlying asset is a stochastic process, and the value follows a geometric Brownian motion.

Paddock et al. demonstrated the market equilibrium for the underlying real asset. By integrating typical Black-Scholes option pricing model to value a delay option of developed petroleum reserves, they derived a value for the real options. They hold that the option valuation methodology can avoid the need to use a risk-adjusted discount rate.

They supported project evaluators need to understand the real asset value (petroleum reserve value) is in equilibrium to the market stock value. They modelled petroleum reserve values with option pricing techniques, valuing the development option and the option to explore.

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[5] In Borison(2002), he defined the Classic Approach is “no arbitrage, market data”. The Luehrman Approach is “ no arbitrage, subjective”. MAD is “two types of uncertainties”. The Luenberger Approach is “two types of risks”.

4.2.2 Mechanism

4.2.2.1 Petroleum Reserves Market Equilibrium

There is equilibrium between the value of petroleum reserves and the market value. The stock price of the oil company is essentially equivalent to the net asset value of the company. For the oil company, the main assets are petroleum reserves. The value of company assets is equivalent to the value of developed oil reserves. Thus, the stock price multiplied by the number of stock shares of the company can be equal to the market value of developed oil reserves. That is the market equilibrium value of developed petroleum reserves.

When the stock price of the oil company goes up, the value of oil reserves goes up accordingly. The present value of holding a proven oil reserve can compensate the owner for the opportunity costs of investing in the reserve.

4.2.2.2 Important Inputs

Table 3 Inputs of the Classic Approach

Current stock price	Value of developed reserve discounted for development lag
Variance of the stock price	Variance of rate of change of the value of a developed reserve (Variance of the oil and gas price)
Exercise price	Per unit development cost
Time to expiration	Relinquishment requirement
Riskless rate of interest	Riskless rate interest
Dividend	Net production revenue less depletion

4.2.2.3 Developed Reserve Value

The market value of developed oil reserves is equal to the value of the oil company stock. It is the market value of the net assets value for the oil company.

4.2.2.4 Variance

The variance of the value of developed reserves is an important input. As Paddock supported, "it is commonly used by industry participants, that the prices of a developed reserve tend to be about one third of crude oil prices". The variance of crude oil prices is used as a proxy for the variance of the values of developed reserve.

4.2.2.5 Expected Exploration and Development Costs

Expected exploration and development cost is one part of the exercise costs.

4.2.2.6 The model to Value Real Options

$$C_t = S_t N(d_1) - X e^{-r(T-t)} N(d_2) \quad \text{.......................................4(1)}$$

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Where:

C_t : Price of Real (Delay) Option at time t

S_t: Value of reserves discounted by delaying lag

$N(\ )$: The cumulative normal density function

X : Exercise price

T : Expiration date of the option

t : Present time

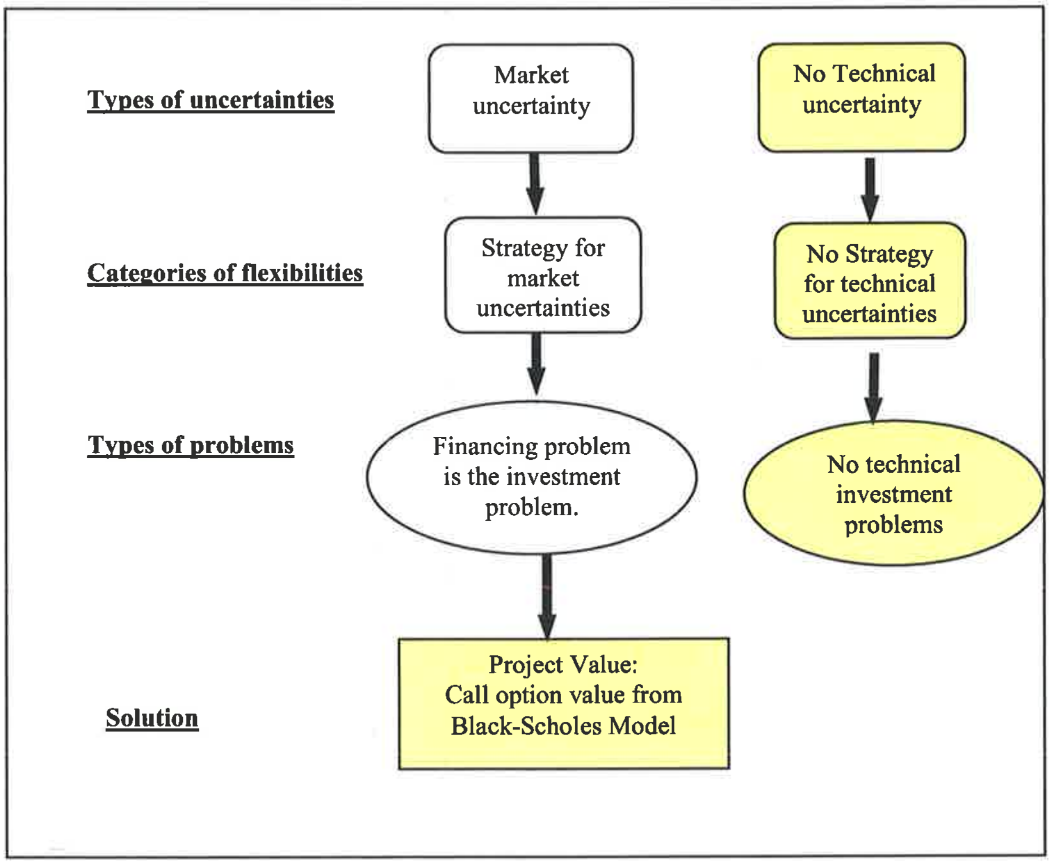
r : Risk-free rate of interest

σ^2: Variance on the return of gas prices

4.2.3 Conclusion

C_t is the real options value for the developed oil reserves. It is a pure financial solution. The real options value is in the value of financial flexibilities. The approach focuses on the price uncertainty. The Classic approach does not have a good solution for valuing technical flexibilities. The value of technical flexibilities is ignored.

Figure 11 Modelling Uncertainties in the Classic Approach



4.3 The Subjective Approach

4.3.1 Assumptions (no arbitrage, subjective data)

The Subject Approach is based on Luehrman, T. A. (1997/1998a/1998b). He agreed that the investment valuation is still based on the capital budget analysis. A corporate investment opportunity is like a call option because the corporate has the right, but not obligation, to acquire payoffs from the opportunity. Thus, the investment value composes of two problems: a capital investment problem and an investment opportunity problem.

The assumptions of Subjective Approach are as follows:

- The real options value is the opportunity value. It can be viewed as the value of a European call option.
- There is a replicating portfolio between a financial asset and a real investment. The project is tradable.
- The project (the underlying asset) is in a partly complete market. There is no arbitrage in the tradable project market.
- It assumes that the dynamic value of a project follows geometric Brownian motion process.

4.3.2 Mechanism

4.3.2.1 Project Value

Luehrman claims "project valuators who want to begin using the option pricing technique need not discard their current DCF-based systems." The net present value is the investment intrinsic value. NPV can be obtained from the traditional DCF method. The conventional NPV of the traditional Discounted Cash Flow valuation is a starting point of a project. The real options value is the value of an opportunity in the project. The real options value can be derived from Black-Scholes option pricing model. The total project value is:

Project Value =NPV + Option Value……………………………….4(2)

NPV= PV-investment costs………………………………………4(3)

4.3.2.2 Inputs

Table 4 Inputs of the Subjective Approach

Financial Option	Symbol	Real Options
Stock price	S	Present value of a project
Exercise price	X	Investment costs
Expiration time	T	Time until the opportunity elapse
Stock price volatility	σ	Standard deviation of project returns
Risk free interest rate	rf	Time value of money

S = Present value of a project, which is from the traditional DCF method.

X= Expenditure required to acquire the project assets.

T= Length of time the decision may be deferred

rf =Risk free interest rate

σ= standard deviation of project returns (see **Chapter 4.3.2.3**)

4.3.2.3 Project Return Volatility

Luehrman suggested “the variance is a summary measure of the likelihood of drawing a value far away from the average value”. It is obviously that high-variance assets are riskier than low-variance assets. Variance is a measure of uncertainty. A time dimension is needed as well: how much prices can change while investors wait depends on how long investors can afford to wait. Time is relevant to option costs. Timing as an opportunity, Investors must pay to keep an opportunity. For business projects, business circumstances could change much more if investors wait two years than if an investors waits only two months. Thus, in the option valuation, Luehrman applied the term “variance per period”. He suggested using “the variance of project returns---percentage gained (or lost) per year” rather than using the variance of project values. The volatility is the square root of variance. He ensured the uncertainty is associated with time and the volatility of project returns.

He proposed that the Monte Carlo simulation technique could be applied to synthesize a probability distribution of project returns, when the project risk is non-linear with project returns.

In the case study, based on his illustration, the solution of the standard deviation is same as in the MAD approach example. The standard deviation of project returns is derived from the Market Asset Disclaimer (MAD) approach (Copeland, 2001). See **Chapter 4.4 2** for details.

4.3.2.4 Evaluating Value of Flexibility

$$C_t = S_t N(d_1) - Xe^{-r(T-t)} N(d_2) \text{ ....} 4(4)$$

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Where:

C_t = Price of real (delay) option at time t

S_t = Project present value

$N(\ )$ = The cumulative normal density function

X = Exercise price (X = costs of delaying for bidding + costs of investment)

T =Expiration date of the option

t = Present time

rf = Risk-free interest rate

σ^2 =Variance of project returns

4.3.3 Conclusion

C_t is the opportunity value of an investment project. From equation 4(4), the opportunity value depends on time, investment costs, and the volatility of project returns. Luehrman suggested that the volatility σ quantifies all elements of uncertainty including technical and market.

- Technical uncertainty

Luehrman assumed that the traditional DCF method and Monte Carlo Simulation could model technical uncertainty. The option-pricing model could value the technical flexibility. This value of the technical flexibility is associated with investment costs. The investors must allocate costs to have this option. Based on equation 4(3), the value of the technical flexibility increases, the investment costs increase, and the net present value of the project decrease as the volatility increases.

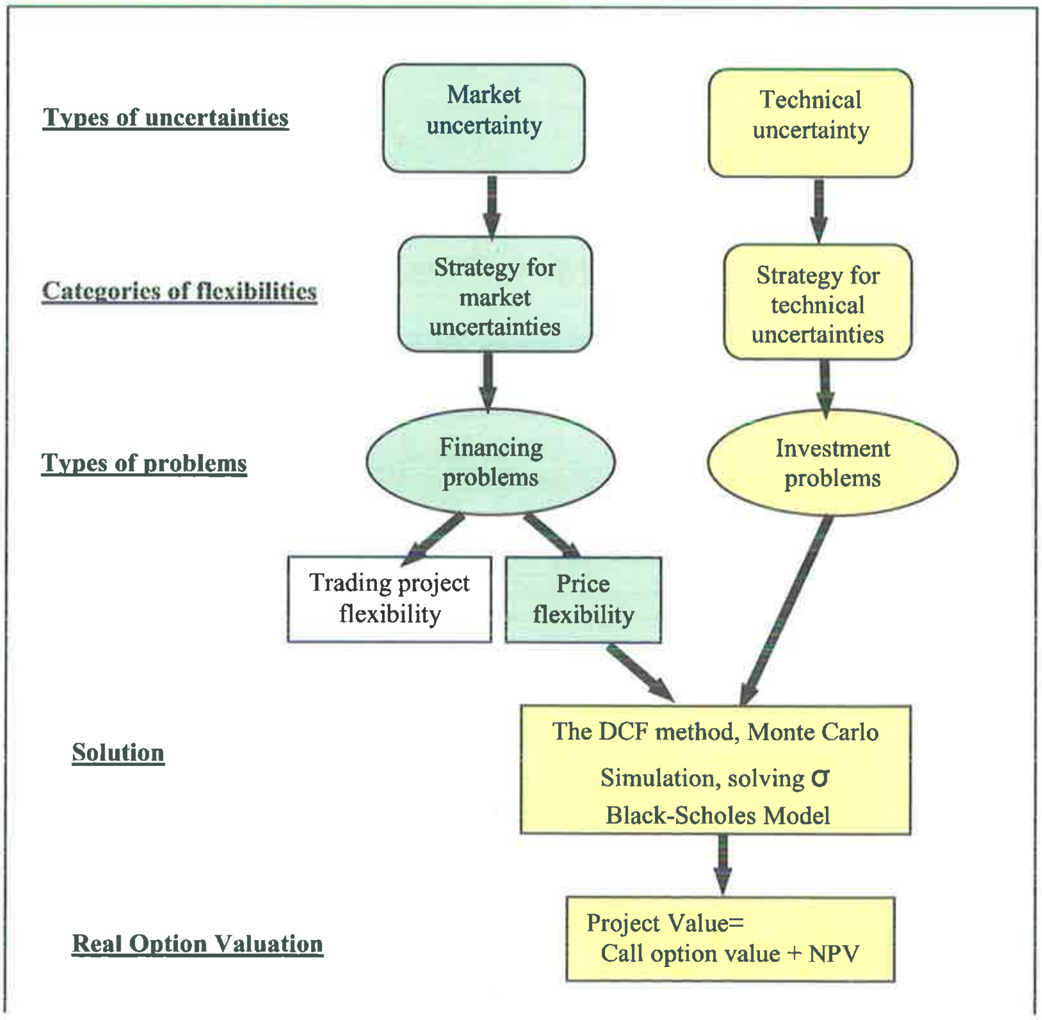
- Market uncertainty

Based on equation 4(3) and 4(4), Luehrman also take the market opportunity value account into the volatility of project returns. The volatility of the project returns also represents the market uncertainty. He claims that the market flexibility derives from the price uncertainty. The price uncertainty is associated with investment costs, which is considered in the net present value.

However, Mason and Merton (1985) pointed out "the value of a nontraded project is the price the project would have when it is traded in the market". The financing flexibility arises from trading projects and managing price fluctuations. When arbitrage opportunities arise in the market, the value of trading projects will be ignored in this approach.

The real options value, as the opportunity value, arises from financing flexibilities and technical flexibilities. Therefore, the subjective approach misses the value of the flexibilities for trading projects.

Figure 12 Modelling Uncertainties in the Subjective Approach



4.4 Market Asset Disclaimer (MAD)

4.4.1 Assumptions

According to Copeland et al (2000), the MAD approach essentially supports "the present value of a traded project is an unbiased estimate of its market value when there are no flexibilities and opportunities in investments". As other approaches, the real options value here arises from technical flexibilities and market opportunities. The magnitude of the value of flexibility depends on the uncertainties. The more uncertainty (or volatility), the higher is the value of the project flexibility.

The assumptions of MAD are as follows:

- The market of real assets is not complete, or it is "partly" and arbitrage free.
- There is no replicating portfolio between a financial asset and a real investment[6].

  The volatility of oil prices in the financial market is not same as the volatility of a commodity (oil reserve) in the real investment.

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[6] Copeland et al (2000), P98. Marketed Assets Disclaimer: it is impossible to find a priced security whose cash payouts in every state of nature over the life of the project are perfect correlated with those of the project.

- The present value is the inherent value of the company investment.

  That is, the project value = present value + real options value.
- The price of the underlying asset moves stochastically. The price follows a geometric Brownian motion.
- Investors' attitude is risk-neutral.

They applied the risk neutral probability approach. They argued that the traditional DCF method could value the intrinsic value for an investment. NPV could not evaluate opportunity. Uncertainties may affect future cash flows throughout the project whole life. And thus, they integrate the traditional DCF method with the option-pricing model to evaluate project opportunities. Instead of the Black-Scholes Option Pricing Model they apply the Binomial Tree Model. Binomial Tree Model makes fewer assumptions than Black-Scholes Model. The Binomial Tree Model permits the underlying asset to be in an arbitrage free and complete market. The model has much wider application than the Black-Scholes Model.

4.4.2 Four Steps in the Market Asset Disclaimer Approach

4.4.2.1 Build Up a Present Value Spreadsheet

The commodity price uncertainty and the production uncertainty are first defined in a present value spreadsheet. The present value is discounted by WACC.

$$PV = \sum_{t=0}^{T} \frac{Cf(t)}{(1+WACC)^t} \quad \text{..} 4(5)$$

PV : Present value at time t

$Cf(t)$: Free cash flow at time t

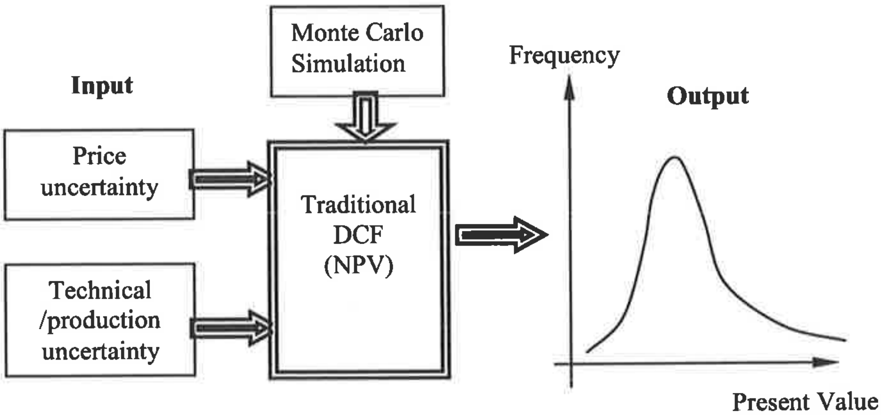
$WACC$: Weighted average capital costs rate

T : Project life

t : Project at time t

4.4.2.2 Define Uncertainty Variables

Because of the non-linear relationship between risks and the project returns, Copeland et al. used Monte Carlo simulation to estimate the volatility of project returns.



$$z = \ln\left(\frac{PV_1 + FCf_1}{E\left(P\widetilde{V}_0\right)}\right) \quad \text{...} 4(6)$$

$$E(P\widetilde{V}_0) = \frac{E(FCf_1)}{(1+WACC)^1} + \frac{E(P\widetilde{V}_1)}{(1+WACC)^1} \quad \text{..................................} 4(7)$$

$$PV_1 = \sum_{t=2}^{T} \frac{\widetilde{FCf}_t}{(1+WACC)^{(t-1)}} \quad \text{.......................................4(8)}$$

When the simulations are run, one will obtain:

$$\sigma = StdDev(z) \quad \text{..4(9)}$$

In the thesis, we replicate the Copeland's example[7] for estimating the volatility (the project returns volatility).

Table 5 Copeland Examples for Project Volatility

Period	0	1	2	3	4	5	6	7
Price/unit		10.00	10.00	10.00	10.00	10.00	10.00	10.00
Quantity		100.00	120.00	139.00	154.00	173.00	189.00	200.00
Variable cost/unit		6.00	6.00	5.70	5.40	4.80	4.20	3.60
Revenue		1000.00	1200.0	1390.0	1540.0	1730.0	1890.0	2000.0
-Variable cash costs		600.00	720.00	792.30	831.60	830.40	793.80	720.00
-Fixed cash costs		20.00	20.00	20.00	20.00	20.00	20.00	20.00
-Depreciation		229.00	229.00	229.00	229.00	229.00	229.00	229.00
EBIT		151.00	231.00	348.70	459.40	650.60	847.20	1031.0
-Cash taxes		61.00	93.00	112.00	122.00	121.00	114.00	96.00
+Depreciation		229.00	229.00	229.00	229.00	229.00	229.00	229.00
-Capex	-1600	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-Increase in working capital		200.00	40.00	24.00	13.00	0.00	13.00	24.00
Free cash flow		119.00	327.00	441.70	553.40	758.60	975.20	1188.0
WACC		0.12	0.12	0.12	0.12	0.12	0.12	0.12
Discount factor1		0.89	0.80	0.71	0.64	0.57	0.51	0.45
PV of free cash flow1		106.25	260.68	314.39	351.70	430.45	494.07	537.39
PV of project		2494.93						
Investment		1600.00						
NPV of project		894.93						
Discount factor2		1.00	0.89	0.80	0.71	0.64	0.57	0.51
PV of free cash flow2		119.00	291.96	352.12	393.90	482.10	553.35	601.88
PV of project2		2794.32						
PV1		2675.32						

$$E(FCf_1) = 118.98$$

---

[7] Copeland (2000), Exhibit 9.3, Chapter 9: "Estimating Volatility: Consolidated Approach", P247.

$$E\left(P\widetilde{V}_1\right)=2675.39$$

$$E\left(P\widetilde{V}_0\right)=2494.97$$

$$PV_1=2675.32$$

$$z=0.11$$

$$\sigma=StdDev(z)=0.25$$

$$Mean(z)=0.07$$

4.4.2.3 Binomial Tree

Binomial tree forecasts the evolution of the project value (including option costs).

4.4.2.3.1 Up and Down Factor

The up and down factor are functions of the time increment and the project return volatility. See equations 4(10) and 4(11).

$$u=e^{\sigma\sqrt{dt}} \qquad \text{4(10)}$$

$$d=\frac{1}{u} \qquad \text{4(11)}$$

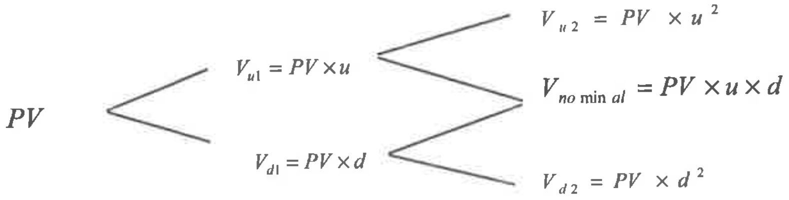
4.4.2.3.2 Risk Neutral Probabilities [p, 1-p]

$$\left[\frac{(1+r_f)-d}{u-d}\right]+\left[\frac{u-(1+r_f)}{u-d}\right]=1 \qquad \text{4(12)}$$

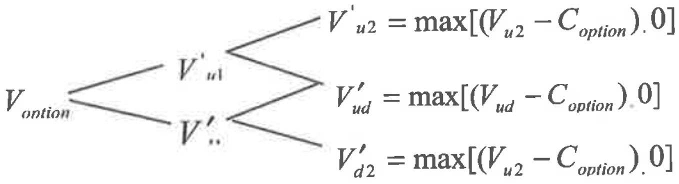
$$p=\frac{1+r_f-d}{u-d} \qquad \text{4(13)}$$

4.4.2.4 Evolution of Project Value and Net Option Value

1) Evolution of the net project value:



2) Net option value:



$$V'_{u1} = [pV'_{u2} + (1-p)V'_{ud}]/(1+r_f) \quad \text{..........} \quad 4(11)$$

$$V'_{d1} = [pV'_{ud} + (1-p)V'_{d2}]/(1+r_f) \quad \text{..........} \quad 4(12)$$

$$V_{option} = [pV'_{u1} + (1-p)V'_{d1}]/(1+r_f) \quad \text{..........} \quad 4(13)$$

$$V_{project} = V_{option} + PV \quad \text{..........} \quad 4(14)$$

Equation 4(13) shows how to compute the project value using risk-neutral probabilities. The value of the project with the (embedded) option is given by 4(14).

4.4.3 Conclusion

As shown in **Figure 13** below, the market asset disclaimer approach is similar as the subjective approach. The difference between the two is that the former uses the Binomial Tree Model while the latter uses the Black-Scholes Model.

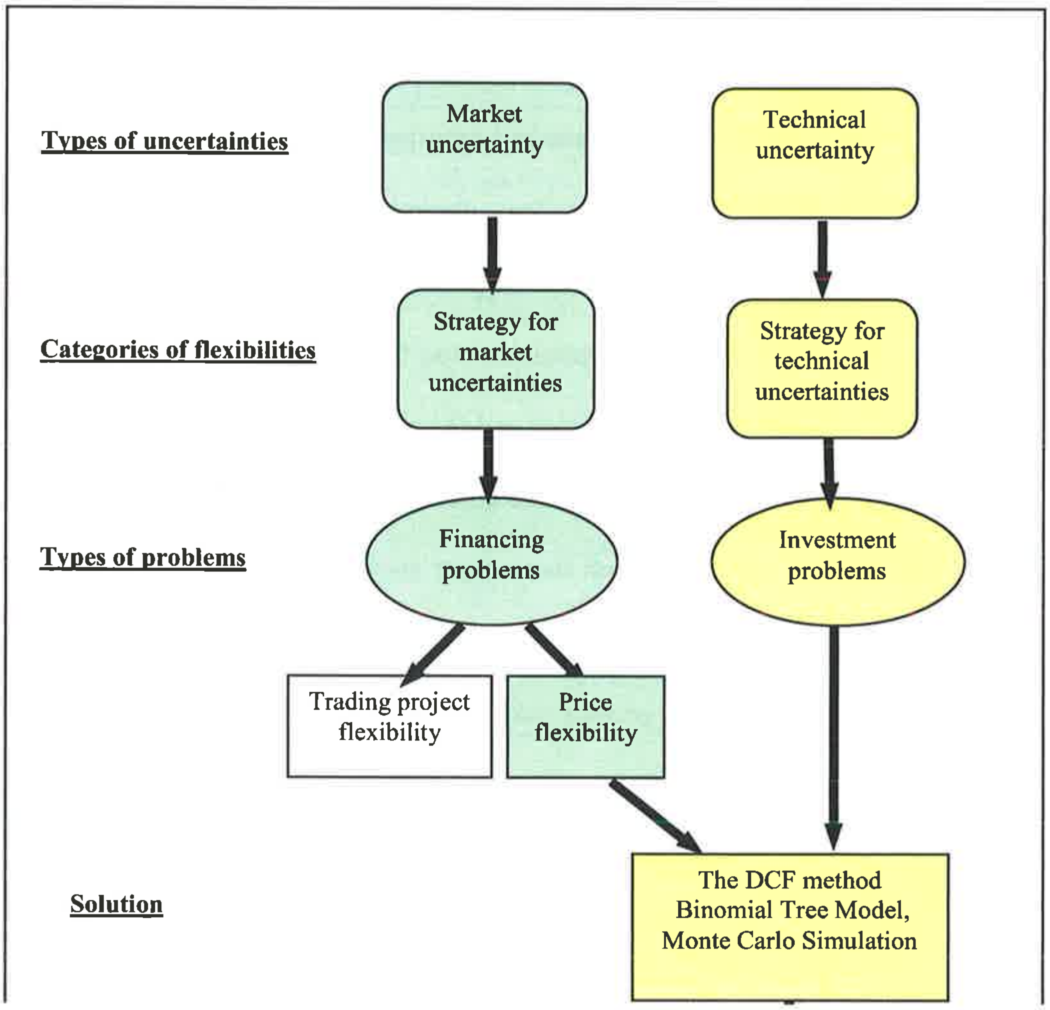
- Technical uncertainty

The MAD approach assumes that the technical uncertainty can be valued by the traditional DCF method. The value of the technical flexibility is the opportunity value in the technical uncertainty and is modelled by the binomial tree model.

- Market uncertainty

The price flexibility is opportunities in the market uncertainty. It is also modelled by a binomial tree model. As the subjective approach, the value of trading projects is also missed in MAD, since the project volatility used is the price volatility.

Figure 13 Modelling Uncertainties in MAD



4.5 The Smith Approach

4.5.1 Assumptions (two types of risks)

Smith et al (1995/1998a/1998b) introduced the decision tree analysis into real options valuation. The assumptions of the Smith approach are presented as follows:

- The market is not complete and arbitraged, which is closer to reality.
- Throughout their papers, they assume the project could be traded as the financial security. The firm can obtain profits by trading the project without taking risks or expending additional capital, because the market is arbitrage free.
- Stochastic Process: the price and the production rate follow random walk.

There are two kinds of risks in the investment: market risk and private risk.

The market risk arises from the market uncertainty. This is a systematic risk. The market risk refers especially to the price risk.

On the other hand, the private risk arises from the individual project. It is unsystematic. The private risk is mainly associated with the technical uncertainty.

These two types of risks should be modelled in different ways.

- An investment decision problem can be separated into two sub-problems.

Smith et al. suggest problem one: the project stage. It is an investment problem, in which project investors focus on the capital budget decisions. Problem two is a financing problem—the security stage, in which investors focus on the opportunity to buy or sell the project.

In the project stage, decision makers face a pure investment problem. Investors invest capital to own the project. There are private risks in this stage. Private risks are associated with uncertainties such as in the commodity quantity (large or small oil reserves) and in the techniques (the success or failure of operated drilling techniques or well treatments). They assume PV (using discounting by the risk-free rate) could represent capital cost; and it is a pure investment issue. Project risks in this stage mainly come from capital investments.

After investors obtain the ownership of the project, the investment enters the second stage—the security stage. Investors now face a financial problem. Uncertainties in this stage come mainly from the market. "If the security market is complete, all project risks can be perfectly hedged by trading the security", Smith suggested. Project managers can trade or borrow the project, just as one can trade a financial security. Downside risks can be hedged in this way.

4.5.2 Integration of Decision Analysis and Option Price Techniques

Smith and Nau[8] (1995) proposed, "Decision analysis and option price techniques can be profitably integrated". The option pricing method—Binomial Tree Model or the risk-neutral pricing technique can value public or market uncertainties. Decision tree analysis can model

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[8] Valuing Risky Projects: Option Pricing theory and decision analysis, P796, 2rd paragraph.

private uncertainties. They also suggested that if the option pricing model and decision analysis methods are correctly applied, two methods must give consistent results.

4.5.3 Three Steps in the Smith Approach

4.5.3.1 The Investment Problem in the Incomplete Market

For private risks, the project is an investment problem in an incomplete market.

- Sketching (constructing) a decision tree for the investment;
- Calculating the present value of each end of the node;

$$s(0)=\sum_{at}\frac{s(t)}{(1+r_f)^t} \quad \text{......................................} 4(15)$$

- Using subjective probabilities and the certainty equivalence.

4.5.3.2 A Financing Problem in the Complete Market

For market risks, the project is a financing problem in a complete market.

- Using risk neutral probabilities;
- Using market information;

$$u=e^{\sigma\sqrt{dt}} \quad \text{..} 4(16)$$

$$d=\frac{1}{u} \quad \text{..} 4(17)$$

$$\left[\frac{(1+r_f)-d}{u-d}\right]+\left[\frac{u-(1+r_f)}{u-d}\right]=1 \quad \text{..................................} 4(18)$$

$$p=\frac{1+r_f-d}{u-d} \quad \text{..} 4(19)$$

- Replacing the expected present value with the present value.

$$s(0)=\sum_{\omega t}\frac{\pi(_t)}{(1+r_f)^t}s(t)=E_\pi\left[\frac{s(t)}{(1+r_f)^t}\right] \qquad \text{4(20)}$$

π is risk neutral probabilities. T is time

4.5.3.3 Choosing Branch with the Maximum Value

$$v^* = \max E_\pi\left[\sum_{t=0}^{T}\frac{c_\alpha(t)}{(1+r_f)^t}\right] \qquad \text{4(21)}$$

- Upper bounds of the project value in the incomplete market;

$$\bar{v} = c(0)+\min_{\beta}\{\beta(0)s(0):[\beta(t-1)-\beta(t)]s(t)\geq c(t)\}, t>0$$

$$\bar{v} = \sup_{\pi\in\Pi} E_\pi\left[\sum_{t=0}^{T}\frac{c(t)}{(1+r_f)^t}\right] \qquad \text{4(22)}$$

- Lower bounds of the project value in the incomplete market

$$\underline{v} = c(0)+\max_{\beta}\{\beta(0)s(0):[\beta(t-1)-\beta(t)]s(t)\leq c(t)\}, t>0 \qquad \text{4(23)}$$

$$\underline{v} = \inf_{\pi\in\Pi} E_\pi\left[\sum_{t=0}^{T}\frac{c(t)}{(1+r_f)^t}\right] \qquad \text{4(24)}$$

Where:

$s(t)$: Project price at time t

ω_t: Given state of information at time t.

$x(t,\omega)$: Risky cash flow streams at time t in state w.

c_α: Pay offs generated from a strategy α

β_r: A replicating trading strategy

4.5.4 Price and Production Models

Project price follows a geometric Brownian motion model:

$$ds(t) = \mu_p s(t)dt + \sigma_p s(t)dz_p(t) \quad \text{.......} 4(25)$$

Production output also follows geometric Brownian motion model:

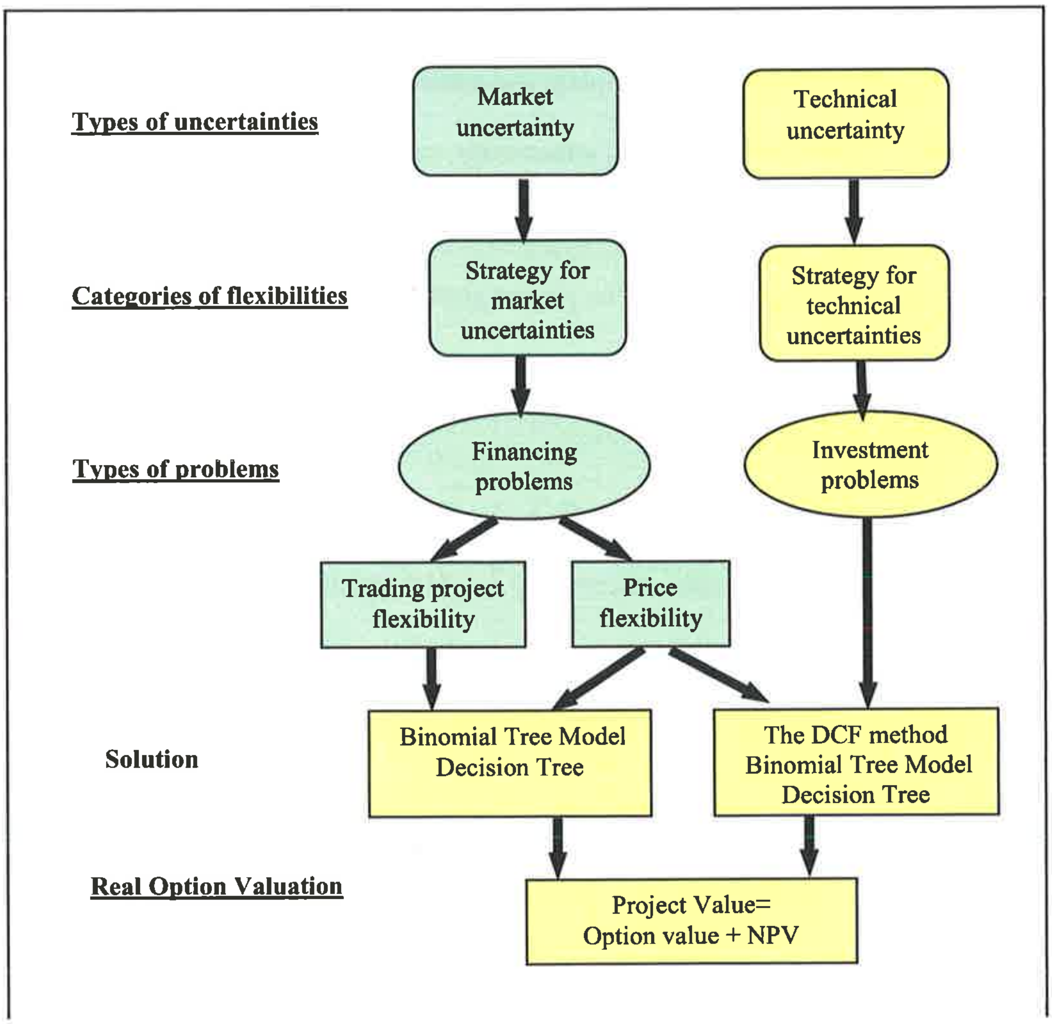
$$dq(t) = \mu_q [q(t) - L]dt + \sigma_q [q(t) - L]dz_q(t) \quad \text{.......} 4(26)$$

4.5.5 Conclusion

- The option-pricing model and decision analysis approaches are able to model the opportunity. The decision tree, or dynamic programming model, and the option-pricing model complement each other. See **Figure 14** below.
- Investing in the project is a priority. The initial investment seems to be a “ticket”. Only after investors buy this “ticket”, and they have rights to implement financing flexibilities. In other words, investors need to invest capital to obtain the project, and then they have opportunities to own flexible options in the trading-project period. During the investment period, investors compute the project NPV by using the risk-free rate.
- During the security period, the project likes a security. Investors could buy or sell the project, just as trading a security in the financial market. There are a variety of trading opportunities in this period.
- Discrete decision tree

  The decision tree in the Smith approach is a "discrete tree", split by time periods of the cash flows. It is different from the traditional decision tree.

Figure 14 Modelling Uncertainties in the Smith Approach



4.6. The Luenberger Approach

4.6.1 Assumptions (Dual decision trees)

Luenberger (1998) also developed a similar approach. He assumed investors should purchase the project and then carry it on as an option.

- The market is not complete and arbitrage free.
- There are two types of uncertainty: market uncertainty and private uncertainty.
- The value of the underlying asset follows the GBM stochastic process.
- Luenberger assumes the project starts at the “zero-level pricing”.

4.6.2 The Mechanism

In his example, he applies subjective probabilities to the increasing and decreasing of oil production with private uncertainty[9]. He implemented the standard binominal lattice approximation to determine the up and down factors, as well as risk neutral probabilities to handle the market uncertainty—the crude oil price uncertainty.

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[9] Luenberger, (1998), “Investment Science”, Chapter 9, p415-p453.

4.6.2.1 Private uncertainty

For the investment problem, the project is in the incomplete market:

- Calculating the present value of each end of the node

$$s(0)=\sum_{\omega t}\frac{s(t,w)}{(1+r_f)^t}\ \cdots\cdots\cdots 4(27)$$

- Using subjective probabilities and the certainty equivalent.

4.6.2.2 Market Uncertainty

- Using market information and the risk neutral probability approach

$$u=e^{\sigma\sqrt{dt}}\ \cdots\cdots\cdots 4(28)$$

$$d=\frac{1}{u}\ \cdots\cdots\cdots 4(29)$$

$$\left[\frac{(1+r_f)-d}{u-d}\right]+\left[\frac{u-(1+r_f)}{u-d}\right]=1\ \cdots\cdots\cdots 4(30)$$

$$p=\frac{1+r_f-d}{u-d}\ \cdots\cdots\cdots 4(31)$$

- Replacing the expected present value with the present value.

$$s(0)=\sum_{\omega t}\frac{\pi(\omega_t)}{(1+r_f)^t}s(t,\omega)=E_{\pi}\left[\frac{s(t)}{(1+r_f)^t}\right]\ \cdots\cdots\cdots 4(31)$$

π is risk neutral probabilities. The solution of the risk neutral probabilities is same as that of the MAD approach.

4.6.2.3 Project Value Model

The project starts at the "zero-level pricing".

V= Revenues – costs +1/R (risk neutral value of next period)…………………4(32)

V=flow*oil price- (fixed costs+ variable cost)+1/1.4*(risk-neutral value of next period)

……………………………………4(33)

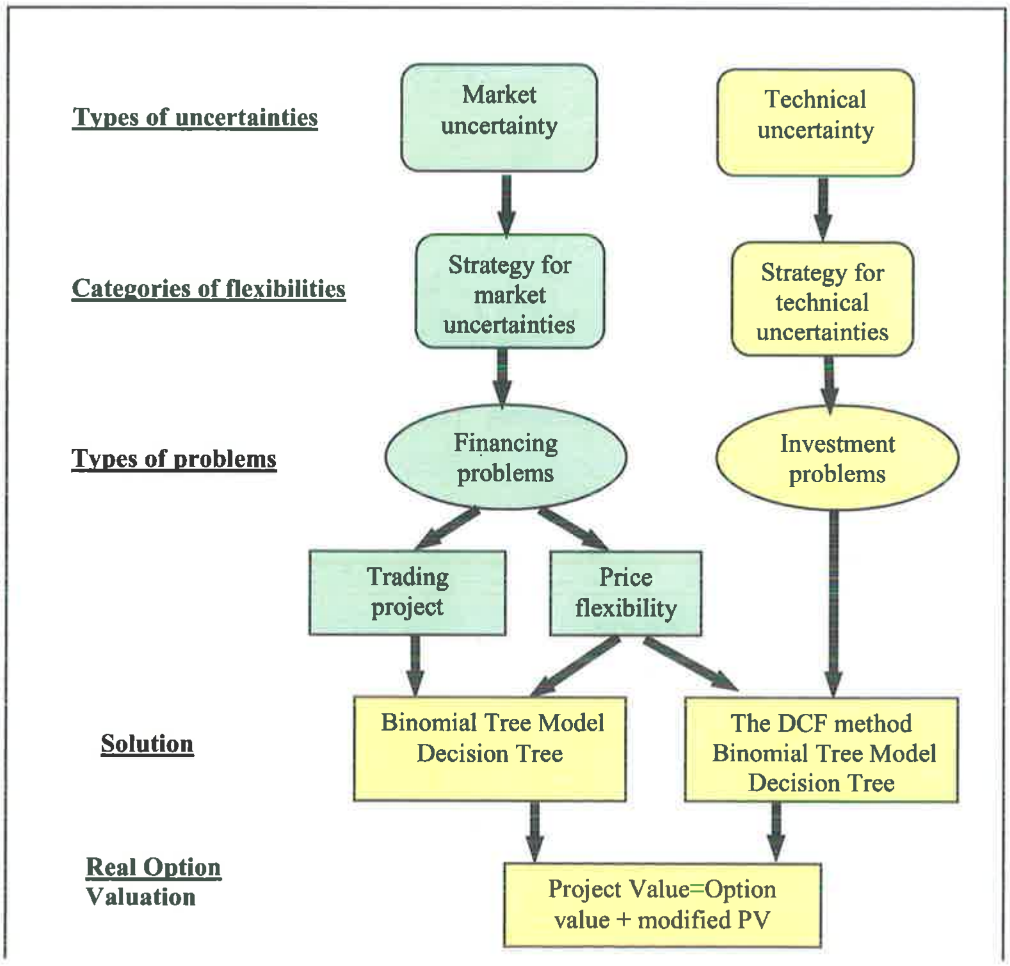
4.6.2.4 Time Discrete Decision Tree

Just as in the Smith Approach, Luenberger also integrates the decision analysis with option price techniques. This also uses a time-discrete decision tree approach. The decision tree is split by time periods of cash flows.

4.6.3 Conclusion

The Luenberger Approach is similar to the Smith Approach. The difference is that the project start at " the zero price" which is the modified present value.

Figure 15 Modelling Uncertainties in the Luenberger Approach



Chapter 5 Case Studies

5.1 Case 1: Reservoir Management Project

5.1.1 Introduction

The first case study is an application of Real Options that is dominated by technical uncertainty. Although there is uncertainty in the market, it is assumed that the commodity price only increases with low volatility and the products have ready markets. Project uncertainty is then related to production rates and reserves. More specifically, production decline rates after drilling of two new lateral bores, and implicitly reserves, are the main sources of uncertainty.

Generally, when technical activities are proposed, they typically involve a number of uncertainties, and while additional information may be obtained to reduce that uncertainty, such information is often very costly. Furthermore, the associated capital expenditure (CAPEX) is often difficult to estimate, and related operational expenditure (OPEX) is equally difficult to define early in a project's life.

5.1.2 Project Background

Ideas for the case study have been taken from the Yang-xing gas field, situated in China. The situation can generally be described as involving a major petroleum company that is struggling to replenish reserves amid increased competition to sell those reserves.

Consider Company A (the Company), which has to make a decision regarding a possible new 5-year gas sales contract. The contract would be to supply 560 million m^3 of specification gas over a 5-year period (with a minimum requirement of 70 million m^3 per year) to a fertilizer plant (the Factory). The Factory would like to sign an agreement with a stable gas supplier. The Company owns the closest gas field (Yang-xing) to the plant and that field appears to have spare capacity from its two primary producing wells.

For the Company this would be a "big" contract, but there are several other gas companies that are also planning to bid for the contract. For the Company to fulfil the envisaged 5-year contract, the two wells would have to deliver a minimum commercial gas quantity of 392 (=560×70%) million m^3. Due to estimated maintenance/workover requirements for the wells, the available amount would actually be reduced to 355 (=200,000×355×5) million m^3, indicating a potential shortfall.

The Company has studied various scenarios to increase production levels. For the case study it is assumed that the Company has identified the following three choices:

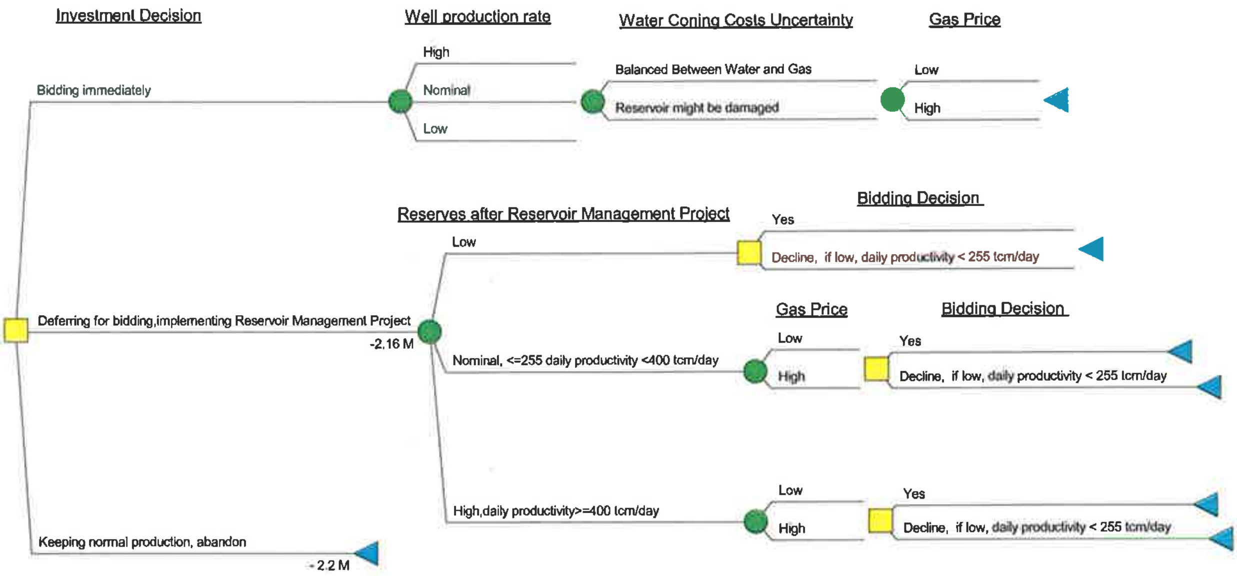
Choice 1: Immediate signing of the contract and subsequent increase in daily production from the two primary producers, to 200,000 m^3/day. As a result, water coning will likely occur, "damaging" the reservoir and leading to reduced ultimate recovery. It is estimated that additional production costs of $0.52 million would be incurred in the third year. In subsequent years, these costs are expected to increase at a rate following a normal distribution with a mean of 3% and a standard deviation of 5%. The increased production is not facility constrained.

Choice 2: Delay 8 months before commencing production for the contract. To preserve the bidding right for the contract, the Company would pay $1.50 million to the owners of the Factory. During the 8 months, the Company would implement a new reservoir management program (discussed in Section 5.1.3) to access new reserves such that the contract volumes can be fulfilled. The company has estimated that it would incur additional costs of $2.16 million for this program.

Choice 3: Immediate signing of the contract, with continued normal production, leading to a likely shortfall in delivery in the fifth year. As a result, it is estimated that a contract penalty of $2.2 million would be payable at the end of the fifth year.

These choices can be represented in the following Decision Tree.

Figure 16 Decision Tree



Relevant Data and Information

Technical details of the Yang-xing gas field, reserve estimates and the gas contract aspects are listed in Tables 6 to 8.

Table 6 General Data for All Reservoirs

Producing wells	17	Wells
Historic production	15	Years
Producing wells: in the Permian-2 reservoir in other thin reservoirs	2 15	Wells
Initial gas in place	2.1	Billion m^3
Gas recovery to date	51%	

Table 7 General Data for Permian 2 Reservoir (principal reservoir)

General information	Heterogeneous formation: water-eroded cave-carbonates, naturally fractured with low permeability matrix, active water drive. Wells -1 and -2 are part of the same pressure system.	
Initial gas in place	1.2	billion m^3
Cumulative production	0.624	billion m^3
Annual production decline rate	mean 3%, SD 5%	lognormal distribution
Producing well numbers (-1and -2)	2	

Current daily production	well-1	150,000	m^3/ day
	well-2	50,000	
	total	200,000	

Table 8 Gas Contract Information for Principal Reservoir

Contract quantity		560	million m^3
Contract period		5	years
Cost for delayed delivery		1.50	million dollars
Period of delay in delivery		8	months
Production coverage of well-1 and -2		70%	
Total production requirement over 5 years		392=560×70%	million m^3
Well downtime		10	days / year
Current gas price Estimated annual increase		55 mean 2%, and SD 4%	\$/ m^3 normal distribution
Drilling costs for mother bore of well-1		0.58	\$ million
Drilling costs for mother bore of well-2		0.31	\$ million
Well life		20	years
Production (variable) costs Yearly increase		42.0 3%	\$/ thousand m^3
Annual fixed costs		0.41	\$ million/well
Costs for brine Estimated annual increase		0.52 mean 3%, SD 5%	\$ million normal distribution
Depreciation		460,000	\$ / year
Tax		12%	
Risk free cash rate		5%	
Weighted average cost of capital (WACC)		12%	
Contract penalty		2.2	\$ million
Costs for reservoir management program	well-1	0.30	\$ million
	well-2	0.26	
	Pre-drilling Reconstruction	0.10	

	In total	0.66	

5.1.3 Reservoir Management Program

The Company has seen evidence from seismic of additional fractured zones in the principal reservoir (Permian 2). It is decided that in order to maintain daily gas production and increase ultimate recovery, two laterals will be drilled to access these fractures. In this case, lateral technology offers a completion configuration where a lateral is drilled from existing main-wellbores. The technology has been recognized as a reliable means of increasing oil/gas production.

The envisaged reservoir management program would encompass the drilling of a single 1200m lateral from well-1, and another 1000m lateral from well-2. After drilling each lateral the wells would be tested. Due to operational constraints, drilling of each lateral would be sequential. It is estimated that the development program will take 8 months. Production loss would be 4 months for each well while drilling and testing is in progress, resulting in reduced production over the 8 months period.

The reservoir management plan is expected to deliver the required amount of gas, 392 million m^3, over the 5-year contract period at a cost of $2.16 million.

5.1.4 Identification of the Option:

Choice 2 creates an Option by deferring the signing of the contract until the company has a greater surety that it can meet the contract obligations. This Option has the following characteristics. The company can delay bidding for the contract for 8 months or else, it can choose to increase production immediately. The 8 month delay is an option for the company, but not an obligation. The company holds the right to bid in 8 months. This option is analogous to a financial option called a deferral call option with characteristics defined in Table 9. The option maturity time **(T)** is 8 months - 0.67 years. The stock price **(S)** is the project's current net present value. The exercise price **(X)** is $2.16 million; inclusive of $1.50 million to hold the bidding right plus $0.66 million for operating and capital costs. The risk free rate **(rf)** is 5%.

Table 9 Analogies between Real Option in Case 1 and Financial Option

Real Option in Case 1	**Value**	**Symbol**	**Financial Option**
Present value		S	Stock price
Exercise option costs	$ 2.16 million	X	Exercise price
Time to expiration	0.67 year	T	Maturity Time
Risk free rate	5%	rf	Risk free rate
Project return volatility		σ	Price volatility

5.1.5 Uncertainty in the Project

Fundamentally, there are two sources of uncertainty in any project: market-based and technology-based. Production rates and gas prices will both vary stochastically over time. The gas price model is based on reality and comes from market observation. It follows a normal distribution with a mean of 2% and a standard deviation of 4%.

5.1.5.1 Gas Price Uncertainty

Although not the primary uncertainty in this case there is market uncertainty based on the volatility in gas price - the gas price is uncertain over the next 5 years. The price uncertainty is correlated with gas supply and demand, increasing with time. It is assumed that the spot price for gas will follow a geometric Brownian motion (GBM), a common model in the Real Options literature, see for example Dixit and Pindyck (1994).

$$dS(t) = 0.02S(t)dt + 0.04S(t)dz(t) \quad \text{..........................5 (1)}$$

If S(0) is given and using formula 5(1), the gas price is assumed to be a lognormal distribution. The distribution of $\ln(S(t)/S(0))$ will have a mean of $\left(\mu - \frac{\sigma^2}{2}\right)t$ and a variance of $\sigma^2 t$. In this case, the gas price follows a positive drift, with μ =2% and volatility of σ = 4%. The forecasted gas price and confidence interval are presented in **Figures 17 and 18**. The gas price starts at \$55.0/thousand m^3. As discussed in the section 3.8.1, based on assumptions of the GBM model, future price depends only on its current value, not its past value. The future price path is therefore only determinate by current information. As can be seen in Figures 17 and 18 the trend in gas price increases with time. Since the volatility of the gas price is low, σ is 4%, the curve has noise, although the model has a positive drift of 2%.

Figure 17 Geometric Brownian Motion for Gas Price Forecast

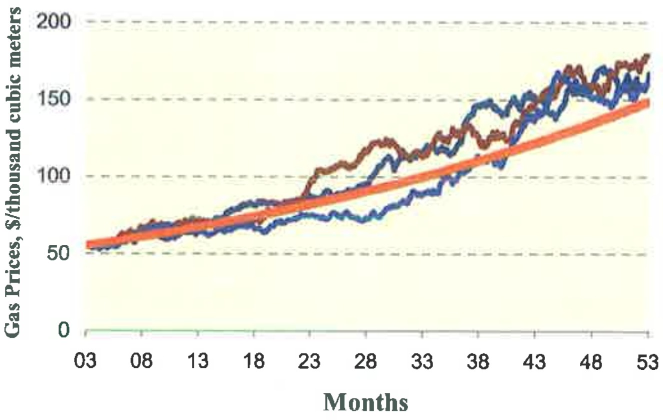
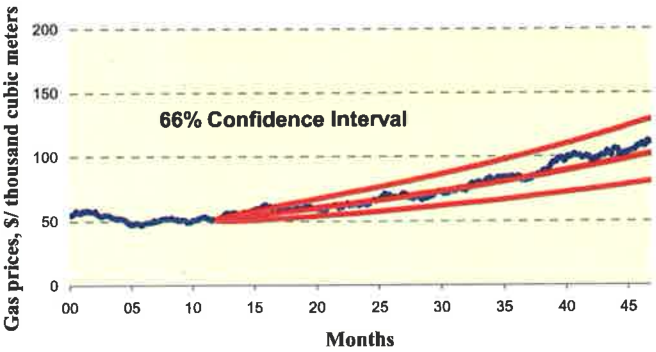


Figure 18 Geometric Brownian Motion - Confidence Interval



5.1.5.2 Technical Uncertainty

The technical uncertainty is a major uncertainty in this case study and is related to the reservoir management program. It comes from two sources: estimated remaining reserves and estimated production rates.

Uncertainty in remaining reserves

Remaining reserves are highly uncertain in this case, due to the heterogeneity of the fractured carbonate reservoir and the active water drive. “Surprises” are common when water drives are active. The use of lateral technology has the potential to increase recovery and then reserves need to be updated.

Commonly in this field, Estimated Ultimate Reserves (EUR) of fractured reservoirs would not be estimated by the volumetric method; because the well bore data does not adequately represent the heterogeneity of the fractured reservoir. It is more common to use the pressure decline curve and pressure build up analysis to forecast the reserve. The parameters for these two methods will be acquired in the programmed well test.

There is a high correlation between the ultimate recovery and connate water production. Once water coning commences, the gas production might become unstable. If the lateral wells intersect the predicted fractures, the reserve may possibly increase. If not, the reserve would remain those estimated prior to the reservoir management program. Hence there is a large range of uncertainty in the size of the remaining reserves. And therefore, there is potential loss due to these uncertain factors.

Initial production rate q(0) after the reservoir management program

Project valuation is an up front estimation. To solve q (0), lateral drilling information, such as the new initial reservoir pressure, the skin factor, etc is needed (See Appendix A: Fetkovich Decline Curve Analysis Features). But the initial production rate q (0) after the reservoir management program is unknown prior to it being implemented. Lack of confirmation of these parameters would result in large uncertainties surrounding the initial production rate. However, an estimate for the range of the production rate q (0) can be based on historical data and experience. Therefore, q (0) is subjectively estimated in the project valuation.

The production decline rate

The production decline rate is also uncertain, since the future production rate and the remaining reserves are unknown. As Li and Horne (2002/2003/2004/2005) analyse, the decline rate is mainly related to the initial reserve pressure (in this case the pressure after the reservoir management program), the water and gas saturation (partially known), the permeability (unknown) and the time to water production (unknown). When the initial production is uncertain, the production decline rate is also uncertain.

In short, the economic valuation of the reservoir management program must be generated from estimates of technical data where uncertainties abound. Unlike the market uncertainty, however, the production uncertainty decreases with the time.

Modelling the production rate uncertainty

Given the enormous uncertainty that exists in the production data, it was decided to model the production rate by integrating the Li-Horne model (Li and Horne, 2002) with geometric Brownian motion (GBM).

GBM is a stochastic process, and is therefore suitable for modelling random uncertainty. Hence its frequent use for modelling price uncertainty in Real Options literatures. According to GBM, the future price is only related to the current spot price. And thus, it fits well in modelling financial stocks.

The Li-Horne model is a deterministic approach, and it is satisfactory for predicting future production rates explicitly. Once the matrix permeability, the skin factor, the cumulative production and the initial production rate are known the future production rate can be determined. The model relates the daily production and the cumulative production as shown in equation 5(2). (See Appendix A and B for details.)

$$q(t) = \frac{a}{R(t)} - b \qquad \text{...5(2)}$$

Where q(t)=daily production rate

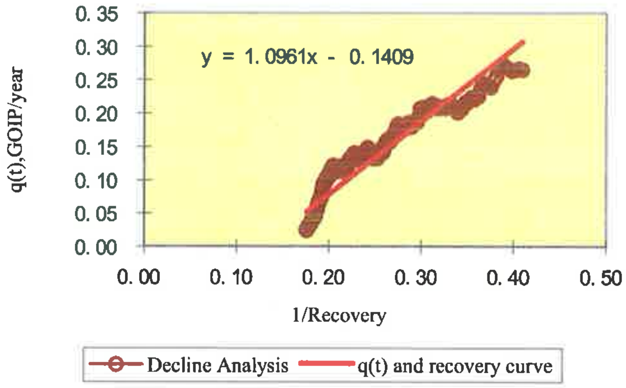
a and b are constants

R(t)=recovery up to time t

“a” is a coefficient associated with capillary forces and “b” is a coefficient associated with gravity. Both these parameters can be derived from core samples (for the model’s details, see Appendix B-Li-Horne Model).

Based on equation 5(2) and Figure 19 below, it is obvious the Li-Horne model does not resolve production forecast at the initial production stage. Thus, the model is insufficient to determine the initial production. The Li- Horne model, however, appears to fit the long-term well production trend.

Figure 19 Production Decline Analysis



$$\mu = \frac{q(t) - q(t-1)}{q(t-1)} \quad \text{...5(3)}$$

Equation 5(3) is used to derive the production decline rate, μ, from the initial and daily production rates, q(t) and q(t-1), which can be estimated using the Li-Horne model.

In order to integrate GBM into the Li-Horne model the production rate q(t) is assumed to follow a lognormal distribution where q(t) has a mean, $q(0)e^{(\mu.t)}$, and a variance, $q(0)^2 e^{2\mu t}(e^{\sigma^2 t} - 1)$. See Equation 5(4):

$$dq(t) = \mu_q [q(t) - L]dt + \sigma_q [q(t) - L]dz_q(t) \quad \text{...............................5(4)}$$

Where L is the economic limit production rate.

Given an initial production rate, q(0) - subjectively estimated from historical production data, the model has a negative drift μ - estimated using Equation 5(3), and a positive volatility, σ. The volatility, σ, of the production rate results from changes in production and is subjectively estimated from historical production data. As previously discussed, different production rates are associated with different decline rates. So in this case study the decline rates and the initial production rates are related to the three choices available to the company. These are shown in Table 10.

Table 10 Decline Rates for Different Gas Production Rates.

Choices	Initial production rate (thousand m^3/day)	Parameters	Yearly	Monthly
1	260	μ_2	-9%	-0.007500
		σ_2	8%	0.023094
2	300	μ_3	-5%	-0.00417
		σ_3	7%	0.020207
3	200	μ_1	-7%	-0.00583
		σ_1	6%	0.017321

1) Increasing production directly, q(0)=260 thousand m^3 /day:

$$dq(t) = -0.09[q(t) - L]dt + 0.08q(t)dz(t) \quad \text{.................5(4a)}$$

2) Implementing the reservoir management program, q(0)=300 thousand m^3 /day:

$$dq(t) = -0.05[q(t) - L]dt + 0.07q(t)dz(t) \quad \text{................5(4b)}$$

3) Keep normal production, q(0)=200 thousand m^3 /day:

$$dq(t) = -0.07[q(t) - L]dt + 0.06q(t)dz(t) \quad \text{.................5(4c)}$$

The forecast production profiles for the three choices are shown in **Figures 20 and 21.**

Figure 20 Production Forecasting using GBM and Li-Horne Model

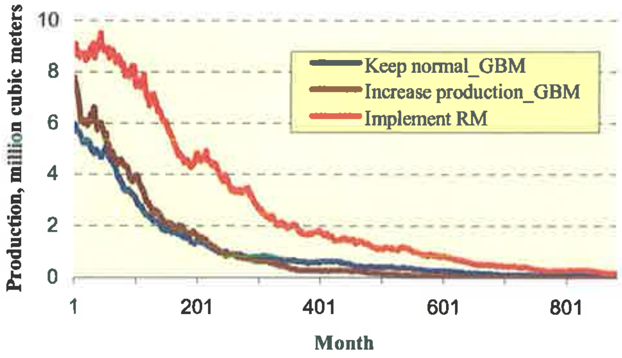
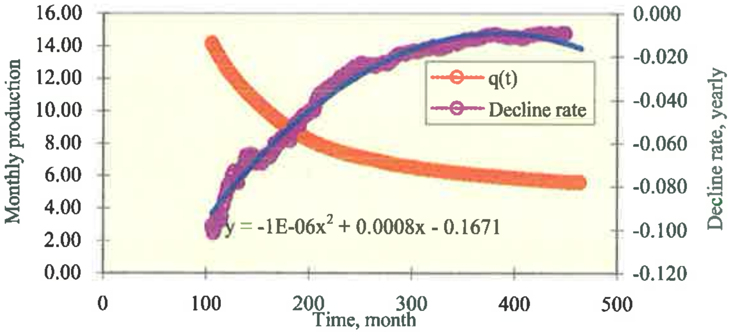
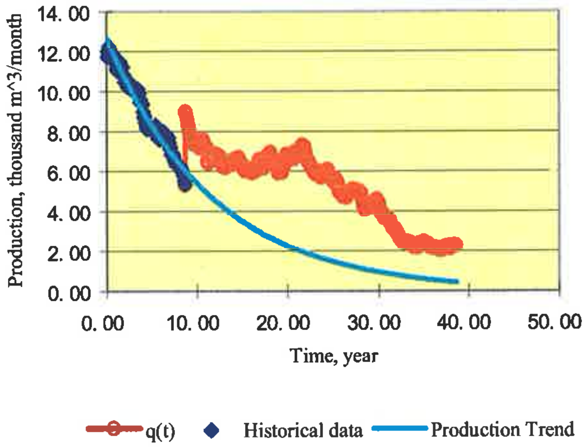


Figure 21 Drift in GBM-Forecasting Production Decline Rate



One applies the Li-Horne model to find out the production decline rate, and embed this decline rate into GBM. The plot shows a decline rate trend (Figure 21), in which the decline rate is a quadratic function with time. The production volatility is assumed subjectively based on the historical production data. The decline rate is negative and it is in a range. Now one could embed the production decline rate and the production volatility to GBM. The integrated production model allows the production rate to change. The production changes are controlled by the decline rate and initial production rate. The different choices have different initial production rates and different decline rates. Using Equation 5(4b), Figure 22 shows the production forecast from the integrated production model.

Figure 22 Production Forecast for Choice 2



5.1.6 Risk and Uncertainty Analysis with the DCF Valuation

The payoffs will be highly dependent on the uncertain production rates and gas prices.

Applying these production and gas price parameters into the traditional DCF method, and running 1000 iterations in Monte Carlo Simulation using @Risk software, risk profiles of the NPV distributions for the three choices were obtained and are presented in **Figure 23**.

Figure 23 Distributions For NPVs of Three Choices

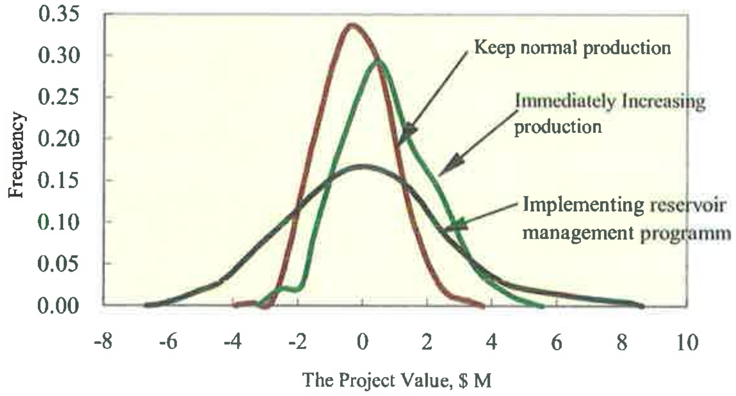


Table 11 Comparisons of Parameters of Project Choices Using the DCF Method

Unit US $, million

Project Choices	Mean of NPV	Standard Deviation
1) Increasing production immediately	0.70	1.45
2) Exercising reservoir management program	0.13	2.46
3) Keeping normal production	-0.21	1.12

Table 11 is a comparison of the mean and the standard deviation of the NPV of the three choices using the DCF method. Based on the traditional DCF method, the risk profile of "increasing production immediately" has the best shape among the three choices. As can be observed, it is narrow, and moves to the right (positive) side. Its standard deviation is $1.45 million - not the largest compared with the other choices. The mean project value, $0.70 million, is the highest. Thus, if the traditional DCF method where used to make the choice, "increasing production immediately" would be chosen. It is apparently informed that the uncertainty of the strategy "increasing production immediately" is lowest, and its project value is highest. Thus, people who support the DCF method would conclude that the strategy "increasing production immediately" is the best.

Figure 24 Cash Flows Streams of the Project

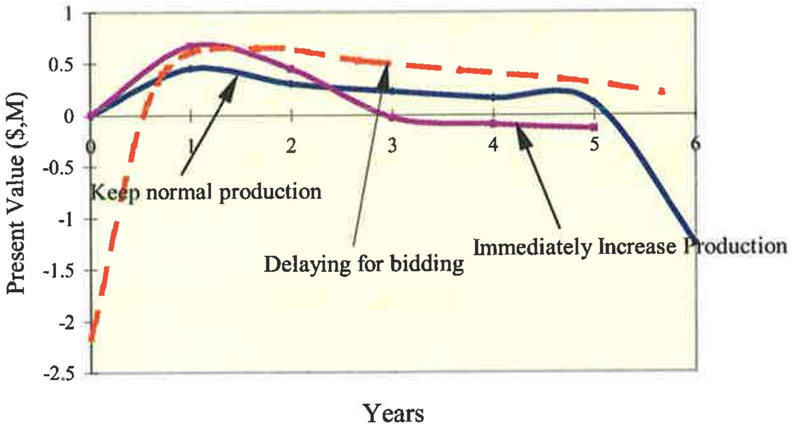


Figure 24, which displays the cashflow associated with the three choices, shows that this choice goes into negative cashflow after 3 years, and Choice 3 is the best option from the cash flow perspective. Which choice is the best strategy for the company?

5.1.7 Real Options Valuation Analysis—Valuing Flexibility

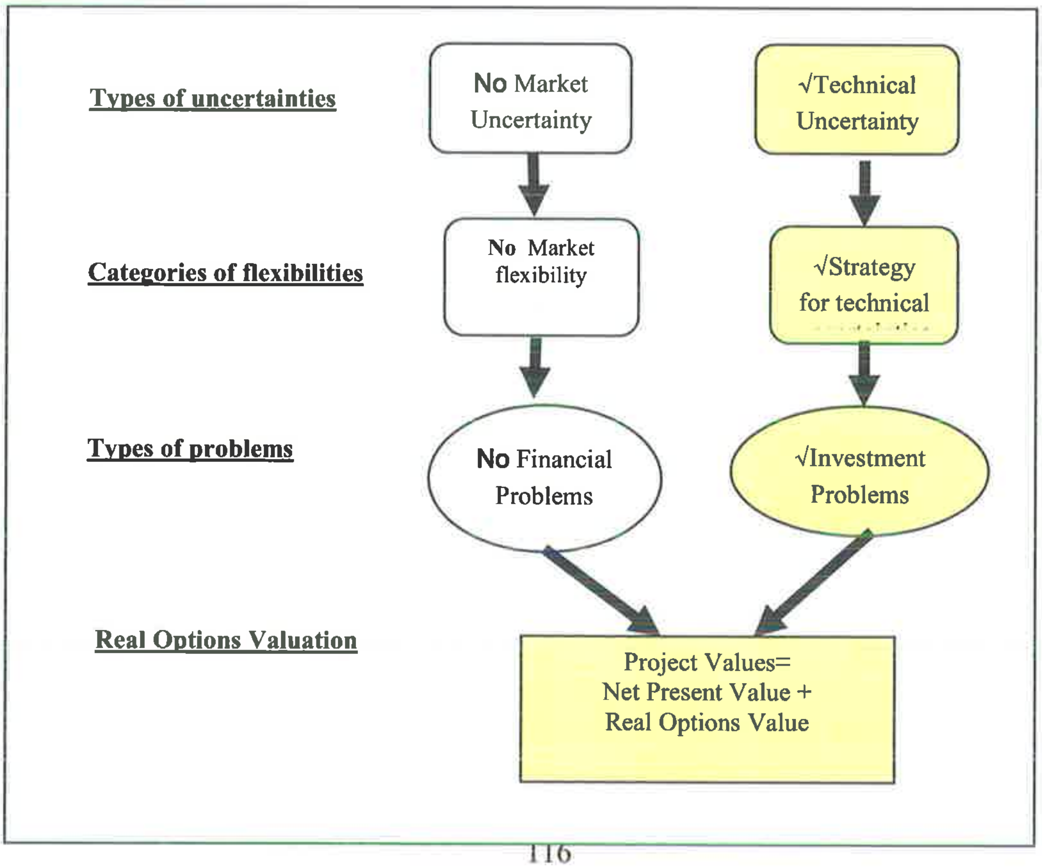
In Real Options Theory, the project value comes from the value of the Real Options (that are: market opportunities and managerial flexibilities), PLUS the inherent project investment payoff—NPV.

The project value = Flexibility value + Inherent value

= Real Option Value + NPV

Real Options Valuation will therefore be used to help determine the optimal project value. Only Choice 2 will be valued because the NPV derived for Choice 1 and 3 is the total project value because there is no option associated with them and hence no extra value. Figure 25 below presents the types of uncertainty, and types of flexibility (strategy) of this case. The project uncertainty is dominated by technical uncertainty. The strategy (in Choice 2) is a technical strategy. The Real Options Value is therefore the value associated with adopting the reservoir management program. In this case there is no market strategy value, since the manager would not consider trading the project in the market, and the price market uncertainty is low. The project value is, hence, the price the project would have if there were the managerial flexibility (strategy) and timing. The time delay offers the value of management flexibility.

Figure 25 Modelling Flexibilities in Case 1



5.1.8 Case Solution and Comparison of the 5 Real Options Approaches

In summary then the company expects to take advantage of the delay of 8 months to solve the reserve uncertainty by implementing the reservoir management program. The project value is expected to go up after implementing the reservoir management program. The option of this choice gives the company the right to bid, but not the obligation to sign the contract. Hence, if the result of the reservoir management program is unsatisfactory; the company will have right to abandon the contract. In order to value the option associated with Choice 2, the five Real Options approaches discussed previously were being used. Table 12 summaries the results of the various approaches

Table 12 Comparison of results of 5 Real Options Approaches

Real Option Approaches	Project value with Real Options V=NPV+ROV ($, MM)
Classic Approach	2.88
Subject Approach	2.73
Market Asset Disclaimer	2.78
Integrated Approach	0.90
Luenberger Approach	1.10
Traditional NPV method	0.10

The project value in each approach is well above the maximum NPV of Choice 1 ($0.70 million). Hence the recommended approach, using the Real Options Valuation method, is to buy the option to delay the contract for 8 months and implement the reservoir management program.

If the program is successful then sign the contract but if in the unlikely case that it is unsuccessful there is no obligation to sign. (See Appendix C elaborates the equations of each approach.)

5.1.9 Real Investment Result

In this case the company did delay bidding for the contract and the results of the reservoir management program were actually successful. Well No 1 intersected a new gas fracture and the daily productionwent up to 350 thousand m^3 per day. The remaining reserve is, however, still ambiguous because of the active water drive and the possibility of other isolated fractures.

The final investment result demonstrates that the Real Options Approach is much more successful in determining the real value of the project and thus assisting in decision making than the traditional DCF method.

5.1.10 Valuation Errors in the Traditional DCF Method

In the DCF method the valuation error arises because the traditional DCF method does not take the timing value into account. This is the critical part of this project valuation. This strategy actually increases the project value. Hence the DCF method ignores the strategy value of the reservoir management program.

As discussed in the Chapter 3, investors could take advantage of timing to solve uncertainties, and thus reduce the downside investment risk. Ignoring this may underestimate the real investment value. Real Options Valuation accounts for this timing value.

5.1.11 Technical Uncertainty and Value of Information

From the case, it can be seen that Value of Technical Information may determine the value of the project. If the company invests before obtaining information, the technical uncertainty of the investment is higher. If the company invested after obtaining information the company could make the better decision. In such a way, the company may avoid downside risks whilst increasing project values.

However, in general, it is important to note that although the information reduces the technical uncertainty, that reduction is not 100% (Dias, M. A. G., 2002). In most cases, the extra information provides a **"partial revelation"** of the true value of the project.

5.2 Liquid Gas Distribution Project

5.2.1 Case 2 Description

Case 2 is a Liquid Petroleum Gas (LPG) distribution project. It is dominated by market uncertainty. It aims to present different real options approaches by modelling market uncertainty in different ways. The project involves uncertainties in oil prices, liquid gas market demands, and sale volumes. These uncertainties stem from the oil market. Thus, the project value is extremely dependent on the volatility of oil prices. Based on this, we will apply different approaches and compare different project values. The investment timing and choices of strategies, as well as crude oil prices determine the project value and the value of the flexibility.

5.2.1.1 Project Background

Company A (the company) is an affiliate of a national oil corporation. Because there has been no big breakthrough in the oil and gas exploration for a decade, its headquarter decides to reform Company A in 2000. The company needs to develop new promising businesses.

Meanwhile, the company is acquainted with the entire range of refinery and application technologies in the liquid gas sector. Liquid gas is one of hydrocarbons, and therefore is composed of propane, butane and mixtures. Under normal atmospheric conditions these are gaseous. These gases are liquefied when they are subjected to a certain pressure. Hence, it is termed as "liquid gas". Liquid gas is characterized by its economical consumption and versatility. Due to its latest technology, a high degree of efficiency is attained. As such it reduces heating costs rapidly. As a result, it is a modern heating fuel, which offers investors a future-oriented business.

Also, the company regards liquid gas a safe energy that meets with the company policy of being environmentally conscious. Liquid gas is indispensable for motors and automobiles.. Consequently, if the liquid gas project is evaluated as being profitable, Company A will concentrate on the liquid gas project.

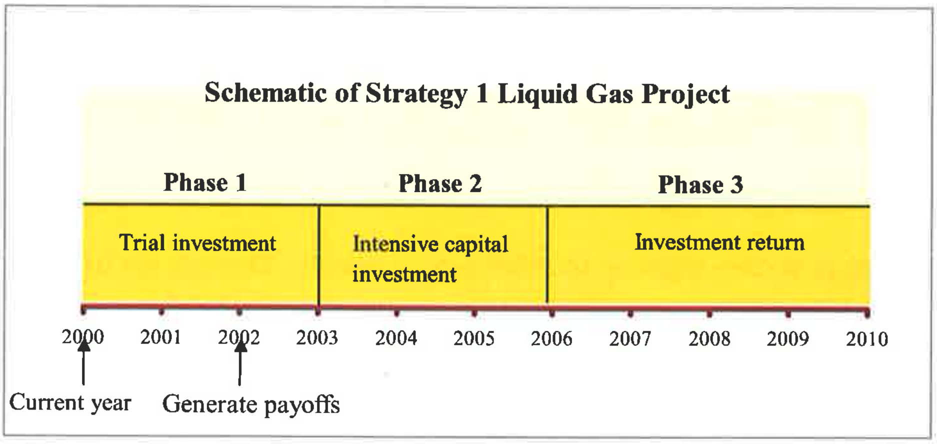
So far (2000), the company can obtain the liquid gas in three ways: using a self–owned extraction factory, using other domestic oil companies, or using international oil companies (importing liquid gas from overseas). The company has a mature oil field and a self–owned extraction factory, where the oil field can deliver crude oil at a rate of 50000 tons per year to the self–owned extraction factory. The extraction factory is able to refine liquid gas at 40000 tons per year. The company is also able to buy the liquid gas from other domestic oil companies. The company is also able to buy liquid gas internationally. The company can develop the liquid gas distribution network based on its proprietary network of oil products.

Before taking action, the company needs to design investment strategies, and fully evaluate these strategies.

5.2.1.2 Strategy 1: Three-phase Liquid Gas Project

The lifetime of the liquid gas distribution project is assumed to be 10 years. The whole investment is designed in three phases: a trial investment period, an intensive capital investment period, and an investment return period. The trial investment—**Phase 1** begins in 2000 and lasts 3 years from 2000 to 2003. The intensive capital investment—**Phase 2** from 2003 to 2006 will be implemented if the trial investment achieves a good result. If the gas price goes down, or the market demands are too small, there is no need to expand investment by then. The investment return period—**Phase 3** is scheduled to start in 2006. In Phase 3, there is no capital investment, and the project is expected to have stable investment returns. After 2-year distribution network construction, the positive payoffs are assumed to come in 2002. **Figure 26** shows the investment plan below:

Figure 26 Strategy 1 Investment Plan



5.2.1.2.1 Phase 1—Trial Investment Period

The trial investment period is planned from 2000 to 2003. By 2003, the company will set up a small supplying and distributing network. The investing items are listed as following:

Table 13 Investing Items in Phase 1-Strategy 1

Investment Items	Number
A gas distribution centre	1
Three gas terminals	3
Several satellite gas stations	6
Bei-he[10] transhipment base	1
LPG transportation track	10
LPG ball tank	9
LGP pump	6
Gas pipeline	2000KM
Sales offices	6
Total investing capitals (Million RMB)	93

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[10] A place near sea

The total investment capital in **Phase 1** is 93 million RMB[11].

During the trial investment, the company focuses on constructing gas terminals, gas sub-stations and a transhipment base (Be-he Transhipment Base). The liquid gas transportations mainly rely on the Be-he Transhipment Base and inland warehouses via railroad tank wagons, highway tank wagons or inland waterway tankers. The first payoffs will be generated in 2002.

The market shares are expected to increase from 3.85×10^4 tons in 2000 to 7.00×10^4 tons in 2003. The distributing network covers 3 cities and 12 towns. By the end of trial investment (2003), the company expects to offer a complete energy service, ranging from the provision of liquid gas in bottles, right up to a full supply facility. Through the trial investment, the storage capacity of the company will enable it to deliver a volume of 50.5×10^4 tons liquid gas each year. Its customers will increase to over 360000; including liquid petroleum gas plants, motor service stations, and retail gas consumers.

5.2.1.2.2 Phase 2—Intensive Capital Investment Period

The intensive capital investment would be conducted if the trial investment performs well. Based on distribution grids of Phase 1, the company will expend the distribution network construction in the intensive capital investment period, which includes as following items:

Table 14 Expend the Distribution Network

Investment Items	Number
A gas distribution centre	2
Gas station terminals	6
Satellite gas service stations	4
Gas pipelines	2000 Km

---

[11] RMB: Chinese Yuan, 1 Us dollar=8.01 Chinese Yuan (RMB) (20/08/2005 http://finance.yahoo.com)

Transportation equipment: Train and track wagons	10
LPG ball tank	6
Sales offices.	10
Liquid Petroleum Gas (LPG) bottling plants	2
Gas bottle testing line	1
Total investing capitals (Million RMB)	66

The investment capital is 66 million (RMB). The phase 3 will be carried on for 2 years from 2004 to 2006. By the end of 2006, the company is expected to be able to distribute liquid gas volumes of 45.6×10^4 tons per year.

During the intensive capital investment period, company payoffs are generated mainly from gas distribution grids built in Phase 1. After Phase 2, the entire distribution network will be composed of items shown below.

Table 15 Entire distribution network

Investment Items	Number
A gas distribution centre	3
Gas station terminals	7
Several satellite gas stations	10
Bei-he transhipment base	1
Transportation equipment (Tracks)	20
Gas tanks	15
Gas pipeline	4000km
Retail service offices.	16
LPG bottling plants	2
Gas bottle testing line	1
Total investing capitals (Million RMB)	159

5.2.1.2.3 Phase 3—Investment Return Period

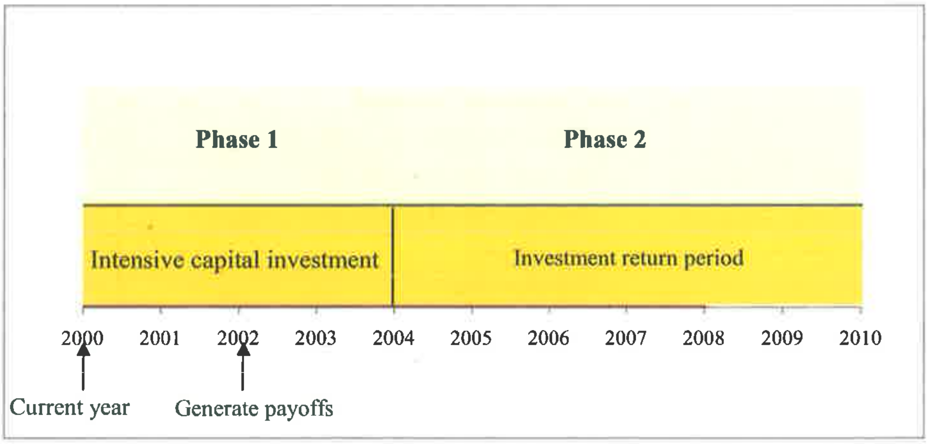
After the intensive capital investment, the project will enter Phase 3—the investment return period. In this phase, the company will not plan to invest capital, but in turn, to receive stable profits from the project. The expected return rate is expected to be 16.1% per annum.

The company's capital has three sources: asset based lending from a bank, joint venture capitals, as well as self-owned capitals.

5.2.1.3 Strategy 2: Two-phase Liquid Gas Project

The company invests a large amount of capital right now, and begins immediately at the intensive capital investment period. The whole investment is designed in two phases: an intensive capital investment period and an investment return period. The intensive capital investment—**Phase 1** begins in 2000, and lasts 4 years from 2000 to 2004. After two year construction, the project positive payoffs are assumes to come in 2002. The investment return period—**Phase 2** starts in 2004. By 2006, the project is expected to have stable investment returns. The liquid gas distribution project life is 10 years. See Figure 27 below:

Figure 27 Schematic of Strategy 2 Investment Plan



5.2.1.3.1 Phase 1—The Intensive Investment Period

As the investment plan, the company shall make investment decision right now. From 2000 to 2004, the company will build up a large distribution network. It will cover 5 cities and 20 towns. The total investment capital is 159 million (RMB). The construction lasts 2 years. The project will generate positive payoffs in 2002. By 2004, the transportation capacity will be 100.5

$\times 10^4$ tons per year, and the company will be able to distribute liquid gas volumes 100×10^4 tons each year.

The investment items are listed as following:

Table 16 Investing items for Phase 1 in Strategy 2

Items	Number
A gas distribution centre	1
Three central gas terminals	3
Several satellite gas stations	6
Bei-he transhipment base	1
LPG transportation equipment (Tracks)	20
LPG ball tank	9
LGP pump	6
Gas pipeline	4000KM
Sales offices	16
Total investing capitals (million RMB)	159

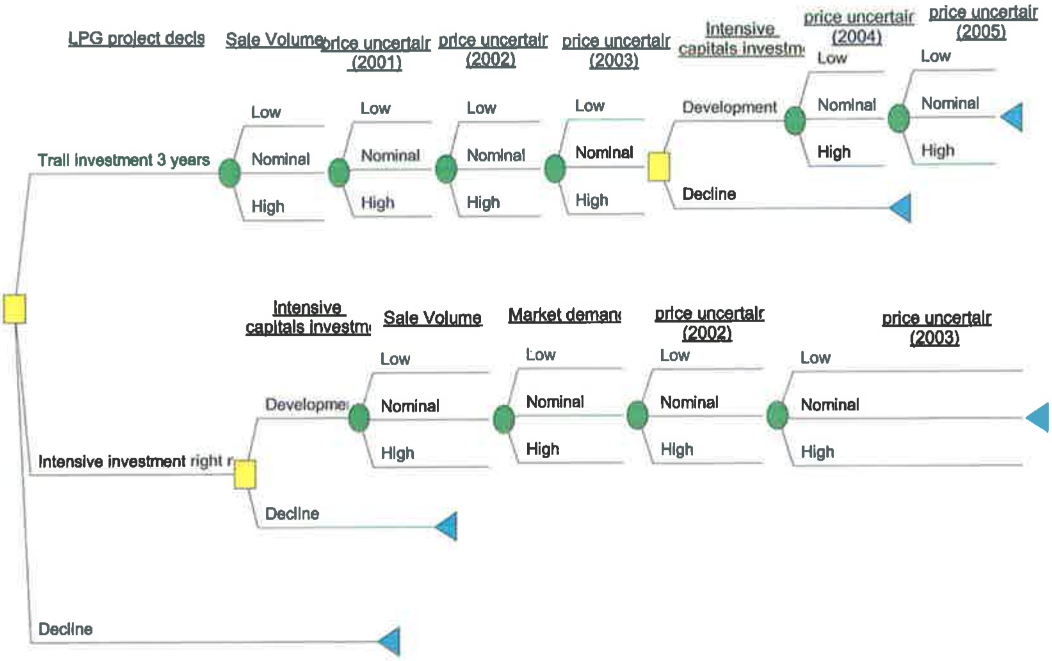
5.2.1.3.2 Phase 2—The Investment Return Period

After the intensive capital investment, the project will enter Phase 2—the investment return period.

5.2.2 Case 2 Decision Tree

The first project analysis is to construct a considered investment strategy map that comprises all uncertain elements in the liquid gas distribution project.

Figure 28 Case 2 Decision Events Tree



5.2.3 Parameters and Variables of the Project

Table 17 Parameters and Variables of the Project Valuation

Parameters	Unit	
Risk free interest rate		5.8%
WACC		15%
Convenience yield		7%
The project life	Years	10
WTI crude oil price (01/02/2000)	$/bbl	45.42
Oil price increase rate		8%
Oil price volatility		24%
Distribution costs	RMB/ton	1776.67
Correlation with WTI oil price		0.70
Increase rate		Lognormal (7%,16%)
Market wholesale price	RMB/ton	2737.61
Increase rate		Lognormal (5%,15%)
Sales volume:		
Strategy 1	Tons	4.3
Strategy 2		6.2
Increase rate		Lognormal (9%,25%)
Correlation between sale volumes and WTI oil price		-0.6
Depreciation	Years	20
Salvage	%	5%
Investment capitals: Strategy 1	Million RMB	159
Strategy 2	Million RMB	159
Fixed costs	RMB/tons	500

Cash tax rate		12%
Option Costs (Discounted)	Million RMB	76.17
Option Costs (Undiscounted)	Million RMB	93

5.2.4 Identification of Real Options

The project price is the value of a contingent claim. The estimated market price of the project is contingent on the investment timing. The timing relates to the decision whether to invest to expand right now, or to postpone expansion through investment to 3 years later. The timing determines the project value. The Strategy 2 contains a real option. The company has a right to delay 3 years to make decision, but not an obligation. To keep the option, the company needs to invest 93 million RMB (undiscounted). The strategic management optimises the timing and investment costs. By postponing the expansion decision, the company can observe the market and the actual price movement. It will invest if the price goes up, but not if it goes down. Thus, it avoids the loss, and then see the price go down. This value of waiting must be traded off against the loss of the trial investment profits.

Identifying real options involves five aspects. It is an expansion call option, which is analogous from a financial call option to a real investment.

Table 18 Identification of Real Options

Option Inputs	Unit	Value
Project present value	Million RMB	
Maturity time	Years	3
Option costs	Million RMB (discounted by WACC)	76.17
Volatility	Different values depending on different ROV approaches	
Risk free interest rate		5.8%

Timing

The opportunity will last 3 years. The company holds the right to expend investment to expand, but not obligation, for 3 years. The option maturity time **T** is 3 years.

Project present value

The stock price **S** is the project present value. Its value is very sensitive to volatility in oil prices.

Exercise price

The exercise price **X** is trial investment capital cost, which is 76.17 million RMB (discounted at 15%).

Risk free rate

The risk free rate **rf** is 5.8%.

Volatility

Referring to the volatility, there are different values depending on different real options approaches. However, the variability in the value of real options in the liquid gas distribution project fundamentally stems from the oil price, as the project uncertainties are dominated by market uncertainty that are dominated by the oil price uncertainty.

5.2.5 Risk and Uncertainty in the Project

The main factors that affect the project value are liquid gas production costs, sale prices as well as sale volumes. These factors are variable and uncertain throughout the life of the project. Thus, the uncertainties in the LPG project are mainly production costs, sale volumes, and sale prices. It is obviously that these uncertain parameters are directly related to the oil price volatility.

We subjectively assume values for several variables shown below.

Table 19 Uncertain Variables

Variables	Standard Deviation	Mean	Correlation with oil price
WTI oil price	8.00%	24%	
Production costs	7.00%	16%	0.7
Market wholesale price	5.00%	15%	0.7

Sales volumes	9.00%	25%	-0.4

5.2.5.1 World Oil Price Uncertainty

Based on the “rational market theory”, the crude oil price always stems from the balance between supplies and demands. If the energy market has no arbitrage opportunities and is complete, the (risk neutral expected) oil price increase rate is always at the risk free interest rate. The oil price is an independent variable, and only related with the current oil price. However, there are many reasons why the oil price may not be determined from a perfect supplies and demands balance. The OPEC partly controls the world oil price, and some other political issues impacts the world oil price. As a result, the oil price is extremely volatile.

On the other side, consumer behaviour impacts significantly on future crude oil price. When new reserves were discovered such as Texas Gulf Coast Field and East Gulf in the 1920s and 1930s, the world oil price dropped. Arlie and Skow et al (1995) had a good analysis on relationship between oil price changes and consumption based on historical analysis. In resent days, the world oil price has hit a new top at $70/bbl. This is also mainly because that the oil consumption in developing countries has increased rapidly. The oil price changing, reserves replenishing, and oil demands volatility, as well as political issues are main causes that lead to the oil price uncertainty higher.

Based on historical WTI oil price data, we plot the actual oil price movement in the past. See Figure 29 below:

Figure 29 Actual Crude Oil Price

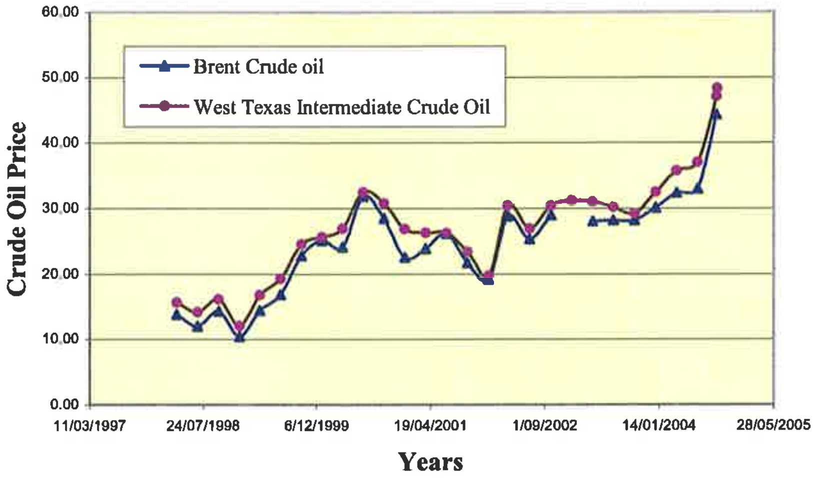
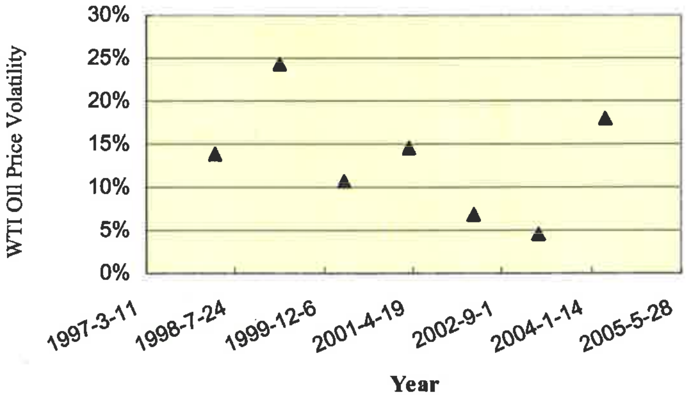


Figure 30 WTI crude oil price volatility



We apply the binomial tree model and the geometric Brownian motion to predict the oil prices respectively. Shown on **Figure 31** and **Figure 32.** The Binomial Tree method can show the oil price trend in a short term, and the red line plotted in **Figure 31** is the actual oil price trend. In contrast, Geometric Brownian Motion can present the oil price trend in a long run.

Figure 31 Price Forecast by Binominal Tree

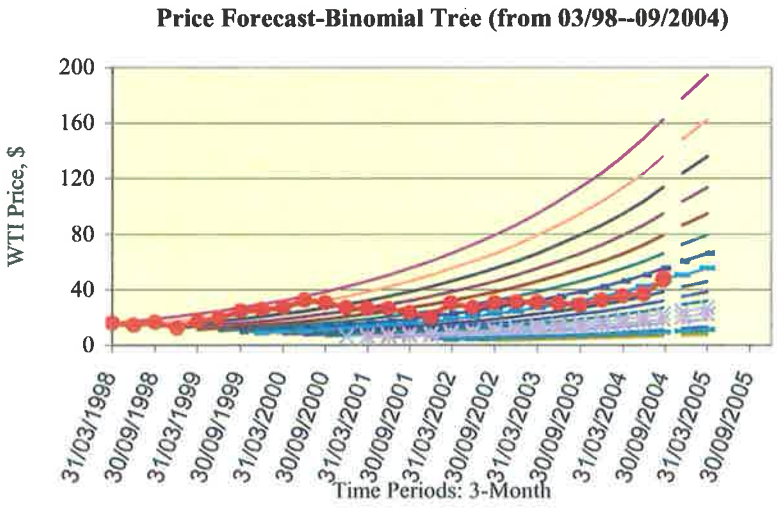
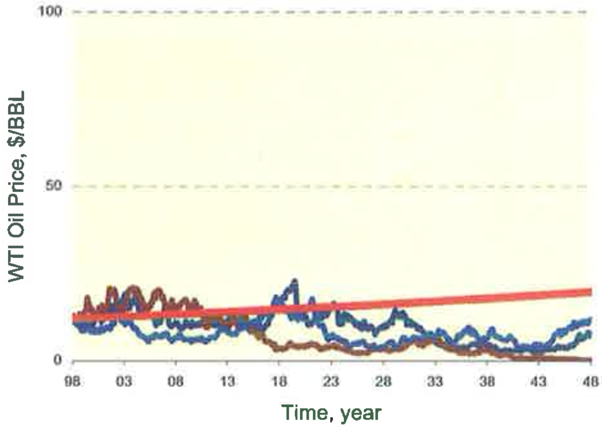


Figure 32 GBM for WTI Oil Price Forecast



The prices modelled here represent spot prices for West Texas Intermediate grade crude oil for delivery in Cushing, Oklahoma. The parameters for the process are based on historical WTI prices using annual data from 1998-2010.

We assume the oil price at any future time is lognormal distributed with the drift 10% and the volatility 24%.

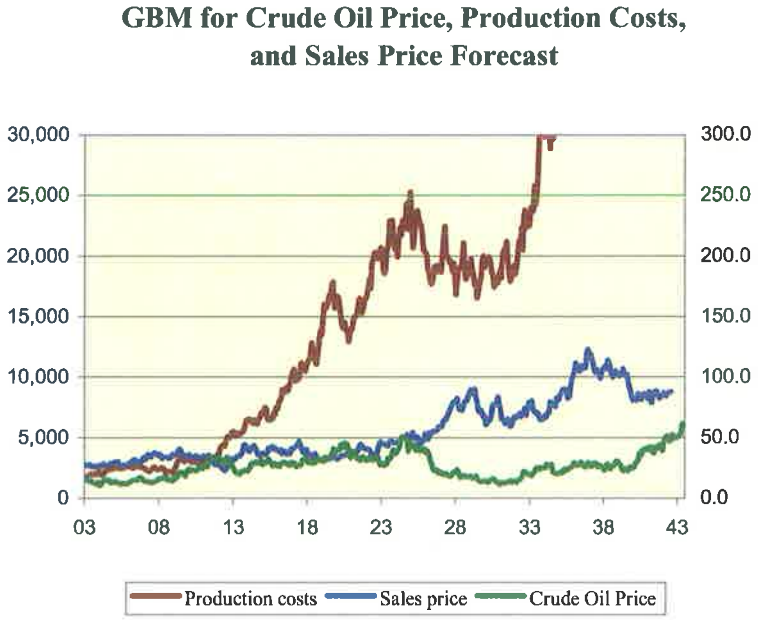
5.2.5.2 Costs Uncertainty and Gas Sale Price Uncertainty

The crude oil price is high uncertain, and this leads to future costs that are uncertain. Future costs affect investment decisions. Liquid gas production costs have high correlation with the crude oil prices. As we know, the raw material of liquid gas is the crude oil. The oil used to produce the liquid gas comes from three sources: self-owned oil fields, domestic oil companies as well as international oil companies. Obviously, the world crude oil price impacts production costs. Though project costs can be partly controlled, production costs are volatile due to oil price uncertainty.

Accordingly, the movement of liquid gas sale price as an output price follows the oil price. When the crude oil price goes up, the liquid gas sale price will go up.

5.2.5.3 Sales Volume Uncertainty

Returning to sales volume of liquid gas, forecasted volumes not only depend on supply conditions of liquid gas products, but also on the oil price fluctuations. First, the supply condition constraints the sales volume, if the distribution capacity is not sufficient for demand. However, the sufficient supply system also supports the sales volume Meanwhile, oil price changes also impacts on consumer behaviour. When oil price goes up, it is extremely unfavourable for oil consumers. Consumer confidence would be frustrated by the continuous increasing oil price. Some consumers would consider changing to alternative heating energy. As a result, the oil price uncertainty also leads to the sales volume uncertainty.



GBM for Crude Oil Price, Production Costs, and Sales Price Forecast

The crude oil price volatility is thus an essential source of the cost uncertainty, sales price uncertainty and the sales volume uncertainty. Even if the crude oil price is high, the project payoffs would go negative. The reason is that production costs would increase with the crude oil price accordingly. When the managers could not control production costs, the project may not generate positive payoffs (See Figure 33). The uncertain variables are forecasted in the table 29, 30, 31。

5.2.6 Risk and Uncertainty Analysis Using the Traditional DCF Method

The task of risk and uncertainty management is to maximize the project's expected return. Based on this issue, investors will be readily to choose Strategy 2.

The reasons to choose the project based the DCF method are:

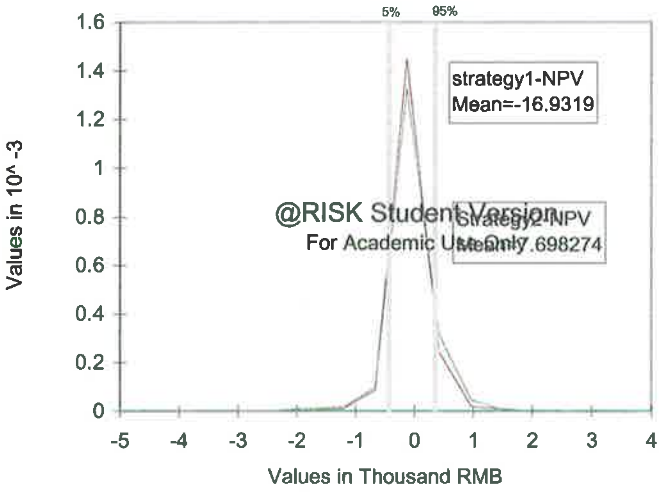
1) The project whose NPV is larger is preferred.

Comparing the two NPVs (see **Table 20**), it is easy to see that Strategy 2 has a higher expected NPV, which is 20.94 million RMB, and Strategy 1 has a lower expected NPV, which is –5.89 million RMB. Although the standard deviation of Strategy 1 (355.34) is lower than Strategy 2 (409.08), the mean value of Strategy 2 (7.70) is much higher than Strategy 1 (-16.9319). Based on the principle of maximized expected project value, investors should take Strategy 2.

Table 20 Comparisons of NPV Distributions of Two Strategies

Name	Strategy1 (Million RMB)	Strategy2 (Million RMB)
Expected NPV	-5.89	20.94
Std Dev	355.338	409.0787
Mean	-16.9319	7.698274
Variance	126265.1	167345.4
Minimum	-4743.909	-2444.812
Maximum	3617.736	3959.894
Skewness	-1.939304	1.334795
Mode	-61.72909	77.57572

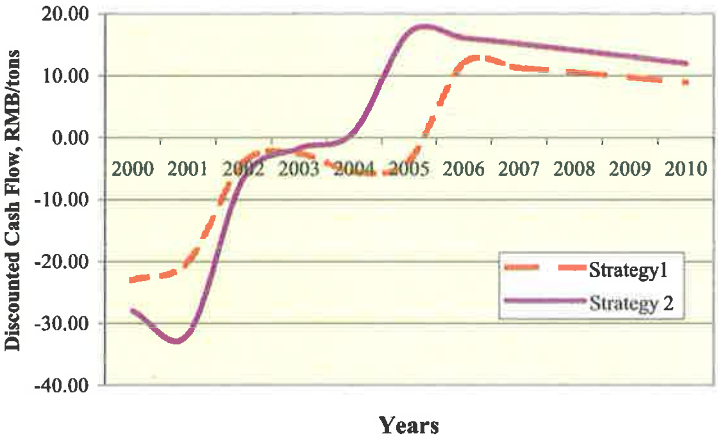
Figure 33 Distributions for NPVs of Two Strategies



2) The project investors could be beneficial from the time-value of money.

The payoffs of Strategy 2 provide profits earlier than for Strategy 1. Based on **Figure 34**, the payback period of Strategy 2 is 4.5 years, and the payback period of Strategy 1 is 5.7 years. Comparing two strategies, one can see Strategy 2 is much better than Strategy 1.

Figure 34 Strategy 1 and Strategy 2 Cash Flows



3) Risk analysis

With regard to risk analysis, the standard deviation of the project present value of Strategy 1 is 355.338 million RMB, and the standard deviation of the project present value Strategy 2 is 409.08 million RMB. Comparing two project present values, Strategy 1 has lower risks than Strategy 2. The difference is 54 million RMB. The present value of Strategy 2 is 20.94, and it is higher 26 million RMB than that of Strategy 1.

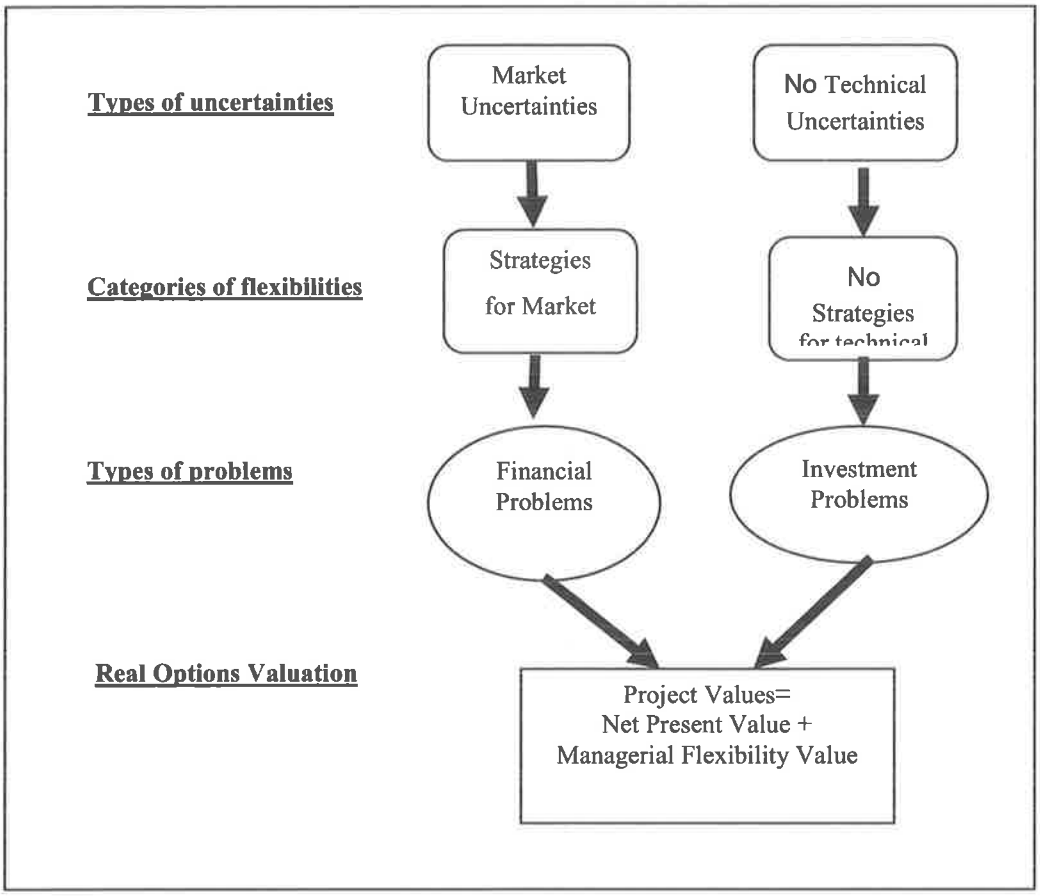
According to the traditional DCF method, the investor should choose Strategy 2 rather than Strategy 1.

However, referring to real options valuation, the project valuation result is quite different.

5.2.7 Real Option Valuation – Valuing Flexibility

Through the above analysis, the company understands the oil price uncertainty, the cost uncertainty, as well as the sales volume uncertainty, are the market uncertainty categories. The project has a timing flexibility. As we elaborated in the previous chapters, the traditional NPV method ignores the value of flexibility. When we apply real option valuation, we may find the value of market flexibility.

Figure 35 Modelling Uncertainties in Case 2



5.2.8 Case Solutions and Comparisons for Five Real Options Approaches

5.2.8.1 The Classical Approach

Table 21 Option-pricing Model Inputs in the Classic Approach

Symbols	Stock Call Option	Interpretation
S	Stock price	Discounted Present Value by delaying lag
X	Exercise price	Delaying cost in the trial investment cost plus trial production costs
σ^2	Variance of rate of return on the stock	Variance of rate of change of the oil price
T	Time to expiration	Relinquishment project
R	Risk-free rate of interest	Risk-free rate of interest

Table 22 Input Parameters in the Classic Approach

Parameters	Value	Unit
Input price (production costs)	1776.67	RMB/tons
Output price(market wholesale price)	2737.61	RMB/tons
Fixed costs	500	RMB/tons
Capex	66	Million RMB
Sales volumes	60	Tons
Option costs	76.17	Million RMB
WACC	15%	
Risk free rate	5.8%	
Convenience yield	7.0%	
Oil price volatility	24%	
WTI oil price	17.53	$/bbl, 2/01/1998
Time to maturity	3	Years

Table 23 Results of the Classical Approach

Current value	$119.82	Million RMB
D1	1.716241332	
D2	1.300549138	
N(D1)	0.956941095	
N(D2)	0.903293522	
C	56.77767428	Million RMB
Project value	56.78	Million RMB

By applying Equation 4(1), we obtain outputs in Table 23.

5.2.8.2 The Subjective Approach

When we assume the oil price and the liquid gas production unit cost has –0.4 correlations with the oil price, the project volatility will be accordingly fluctuating. Based on Equation 4(2), the project volatility has a mean 0.12 and a standard deviation 0.496 (**Figure 36 Distribution for Project Volatility**). Based on Equation 4(3), 4(4), applying Black-Scholes Model, one could obtain a project value of 50.73 million RMB. See **Table 24** below.

Figure 36 Distributions for Project Volatility

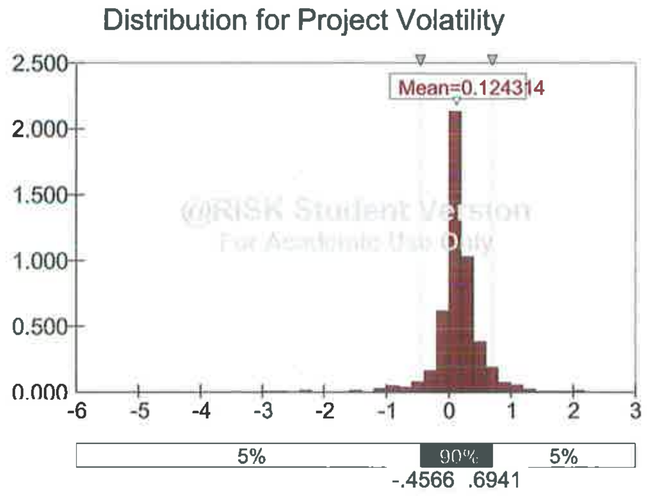


Table 24 Inputs and Outputs of the Subjective Approach

Inputs		
Sale Volumes	55.99	Million tons
Production costs	1776.67	RMB/ton
Market wholesale price	2737.61	RMB/ton
Option Costs	76.17	Million RMB
Capital Costs	82.83	Million RMB
Risk free interest rate	5.8%	

WACC	15%	
Present Value	41.76	Million RMB
Project volatility	49.6%	
Exercise Price	76.17	Million RMB
Time to maturity	3	Years
Outputs		
D1	-0.032074581	
D2	-0.924603613	
N(D1)	0.487206231	
N(D2)	0.177586042	
C	8.968087038	Million RMB
Project value of strategy 1	50.73307547	Million RMB

5.2.8.3 The MAD Approach

We also assume the oil price and the liquid gas production unit cost has a –0.4 correlation, the project volatility will be accordingly fluctuating. The project volatility is same as that of the Subjective approach, which has a mean 0.12 and a standard deviation 0.496 (**Figure 36**). Applying Binominal Model 4(5), 4(6), 4(13), 4(14), we could obtain a project value of 50.98 million RMB. See **Table 25, 26 and 27** below.

Table 25 Inputs of the MAD Approach

Volatility	49.60%	
WACC	15%	
Risk free rate	5.8%	
Option costs	76.17	million RMB
Convenience yield (cy)	7.00%	
Time step	3.00	Year
Binomial step	3.00	
dt (the length of binomial period)	1.00	
Up factor	1.6421	
Down factor	0.6090	
Up factor minus cy	1.5721	
Down factor minus cy	0.5390	
Up probability	0.4976	
1-p	0.5024	
The project value of Strategy 1	50.98	million RMB

Table 26 The evolution of the project (million RMB)

			162.2715
		103.22	
	65.66		55.63386
41.76		35.39	
	22.51		19.07375
		12.13	
			6.539323
Year 1	Year 2	Year3	Year 4

Table 27 The Opportunity Value (million RMB)

			86.10
		40.88	
	19.41		0.00
9.22		0.00	
	0.00		0.00
		0	
			0.00
Year 1	Year 2	Year 3	Year 4

5.2.8.4 The Smith Approach

Based on Equation 4(15), 4(19), 4(20), 4(21), 4(24), we analyse the option value below:

Table 28 Sales Volume Uncertain Parameters

Sales volume	Total (Tons)	Increase rate		
Low	52.54	0.04	0.25	Lognormal Distribution
Nominal	59.70	0.09	0.25	
High	80.14	0.20	0.25	

Figure 37 The Smith Approach Decision Event Tree

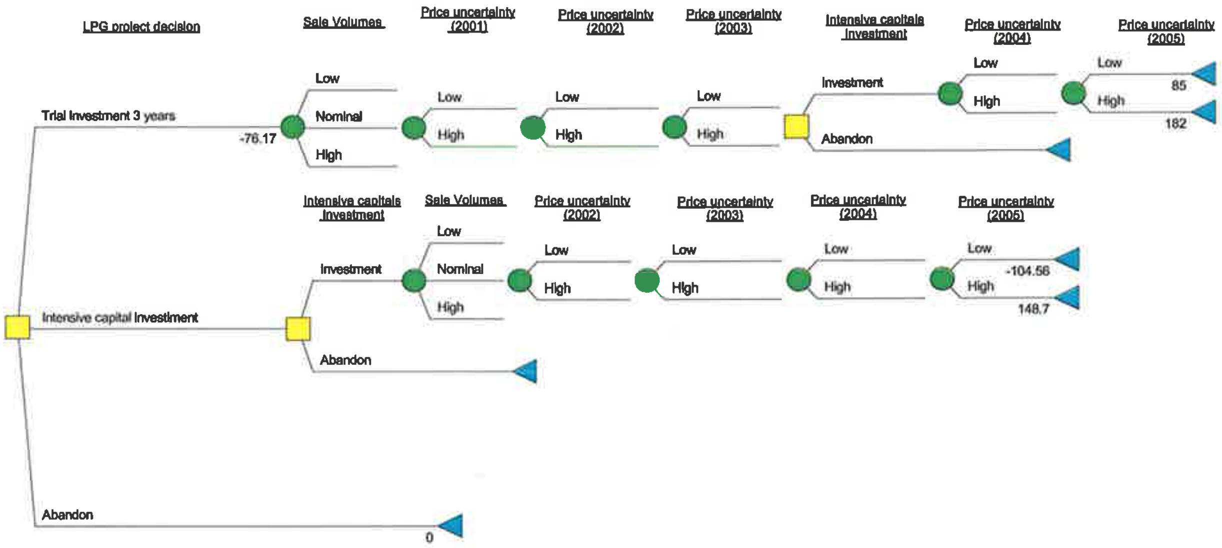
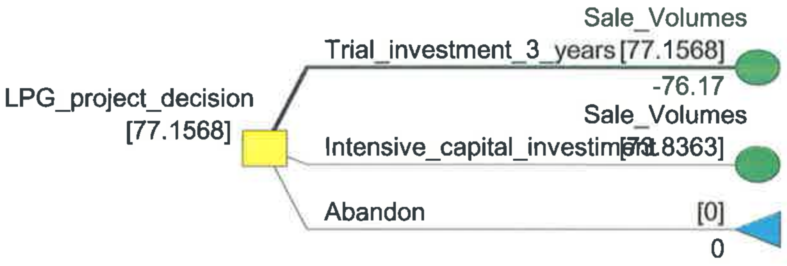


Figure 38 DPL Policy Tree



After running DPL®, we can obtain the project value and the value of each branch. Strategy 1 has a project value of 77.16 million RMB, whilst Strategy 2 has a project value of 73.84 million RMB. Although Strategy 2 has the net present value 73.84 million RMB, its initial capital risk is much higher than Strategy 1. Strategy 2 takes higher capital cost risk.

5.2.8.5 The Luenberger Approach

There are three uncertainty trees in this case, which are the sales price tree, the production cost tree, and the sales volume tree. The sales prices and production costs vary all the time. They follow the oil price fluctuation. Based on Luenberger (1998) and Equation 4(30), 4(31), 4(32), 4(33), we assume the period for one binomial tree step is 2 years for simplicity. Sales volumes and their probabilities are subjectively assumed. By integrating three chance trees, we can obtain the project value of 104.8 million RMB. The project value after tax is 89.08 million RMB.

Table 29 Sales Price Tree

2000	2002	2004	2006	2008	2010
					11570.370
				8672.654	
			6500.652		5569.488
		4872.612		4174.651	
	3652.302		3129.140		2680.917
2737.61		2345.470		2009.501	
	1758.064		1506.236		1290.480
		1129.010		967.289	
			725.038		621.183
				465.612	
					299.011

Table 30 Production Cost Tree

2000	2002	2004	2006	2008	2010
					7508.990
				5628.419	
			4218.823		3614.512
		3162.250		2709.284	
	2370.288		2030.763		1739.874
1776.667		1522.174		1304.135	
	1140.957		977.524		837.502
		732.7104		627.756	
			470.539		403.138
				302.175	
					194.054

Table 31 Sales volume and probability

Sales volume	Probability
95	0.05
70	0.1
50	0.2
40	0.3
20	0.2
10	0.1
5	0.05

Table 32 Other Inputs

Oil price volatility	24.00%	
WACC	15%	
Risk free rate	5.8%	
Option costs	76.17	Million RMB
Fixed costs	500.00	
Convenience yield	7.00%	
Time	10.00	Year
Binomial step	5.00	
dt (the length of binomial period)	2.00	
u	1.4041	
d	0.7122	
u-cy	1.3341	
d-cy	0.6422	
up prob	0.6009	
1-p	0.3991	
WTI oil price	2/01/1998	17.53
Production costs	1776.67	RBM/tons
Sales Price	2737.61	RBM/tons

5.2.9 Summary

Time gives investors the flexibility of investment management. Based on the Table 33, even the lowest value of the real options approach is higher that the traditional DCF. Most projects can accommodate an expansion of its production capacity by additional investments later (Dezen, F et al, 2001). The value of expansion option is the value of expanded project over the NPV. Contrasting five real options approaches, all of them apply the market information and data the oil price volatility. Except for the Classic approach, the MAD and the Subjective approach, the Smith approach and the Luenberger approach start at the traditional DCF method. Among these approaches, the MAD and the subjective approach are similar. The reason of similarity between MAD and the Subjective approach is that both employ the binominal tree model. Also, the Smith approach and Luenberger approach use the decision tree analysis. Thus, their results are closer.

Table 33 Summary of Real Options Approach for Case 2

Real Options Approach	Project Value (Unit Million RMB)
The Classic Approach	56.78
The Subjective Approach	50.73
MAD	50.98
The Smith Approach	77.16
The Luenberger Approach	89.08
Traditional DCF method (NPV)	-5.89

Chapter 6: Conclusion and Recommendation

6.1 Conclusion

Our first conclusion from this study is that the ambiguous definition of "Real Options" is one of reasons that have lead to numerous Real Options approaches. Real Options are operational strategies that companies could have. The flexible strategies are composed of technical strategies and market strategies. The ways to appraise technical strategies and market strategies need not to be same.

The second conclusion from this study is that among the five real options approaches, all real options approaches are still based on the traditional DCF method, except the Classic approach. They still employ the discount rate to value the private risk. Real options valuation is not a replacement for the traditional DCF method. It extends the traditional DCF method. In fact, it is an improvement for the project valuation.

The third conclusion is that there are five techniques applied in real options valuation. These are the traditional DCF method, the Black-Scholes Model, the binominal tree model, decision analysis, and Monte Carlo simulation. We list the techniques applied in the real options approaches below:

Table 34 Techniques Applied in Real Options Approaches

Real Options Approach	Traditional DCF	Option-pricing model		Decision Analysis	Monte Carlo Simulation
		Black-Scholes model	Binominal tree model		
Classic Approach		√			
Subjective Approach	√	√			√
Market Asset Disclaimer	√		√		√
Integrated Approach	√		√	√	√
Luenberger Approach	√		√	√	

Different methods lead to different project values. Similar methods have similar numerical results, because they use similar techniques. The subjective approach and the MAD approach have similar project values, because both of them replicate option-pricing models: Black-Scholes model and the binominal tree model. The Smith approach and the Luenberger approach have similarity due to application of the time-discrete decision tree analysis.

The fourth conclusion is that the real options approaches have their own way to value flexibility. See **Table 35** below:

Table 35 Comparisons of Real Options Approaches in Valuing Flexibilities

Real Options Approaches	Authors	The Value of Flexibility			Real Options Valuation	
		Technical	Market		Value of technical flexibility	Value of market flexibility
			Trading projects	Price		
Classic Approach	Paddock et al 1988	Missing	√	√	--	Black-Scholes Model
Subjective Approach	Luehrman, 1998a	√	Ambiguous	√	Traditional DCF Monte Carlo Simulation	Black-Scholes Model
Market Asset Disclaimer	Copeland et al 2001	√	Ambiguous	√	Traditional DCF Monte Carlo Simulation	Binominal Tree Model Risk Neutral Probabilities
Smith Approach	Smith and McCardle, 1996	√	√	√	Traditional DCF Decision Analysis	Binominal Tree Model, Risk Neutral Probabilities Decision Analysis
Luenberger Approach	Luenberger, 1998	√	√	√	Modified traditional DCF Decision Analysis	Binominal Tree Model, Risk Neutral Probabilities Decision Analysis

√: to be considered.

The fifth conclusion is that most real options approaches are more suitable for valuing the market strategy, because real options valuation is derived from financial option valuation methods. Seldom do the described real options approaches value well the technical strategy. To model technical uncertainty, we need to apply technical information. In addition, the appraisal of the value of technical flexibility is more complicated. As Dixit (1994) pointed out, the decision to invest or to wait depends on the parameters that specify the model, most importantly the extent of uncertainty (which determines the downside risk avoided by waiting) and the discount rate (which measures the relative importance of the future versus the present). And hence, valuing technical flexibility requires high quality parameters. These high quality parameters are derived from the analysis of evaluators who understand the essence of technical problems and real options theory.

6.2 Recommendation

When we focused on oil and gas applications, we need to classify the types of flexibility. In principle, these are technical flexibility and market flexibility. Despite that the valuation of a project starts at PV— the inherent value of the project, technical flexibility and market flexibility in the project should be valued in different ways.

If technical flexibility is dominant in the project, we should concentrate on modelling technical uncertainty. The technical information, such as the reservoir permeability, porosity, reservoir pressures, as well as relative reservoir parameters, etc, will be used for the appraisal of the value of technical flexibility. Based on our analysis and conclusion of the thesis, we suggest applying decision tree techniques, Monte Carlo simulation, and risk neutral probabilities. Since the decision tree analysis and Monte Carlo simulation can value the technical flexibility, when the risk is non-linear with costs. Meanwhile, the risk neutral probability technique can give one the solution of valuing flexibility, because it is able to represent the relationship between uncertainty and time. Therefore, the Smith Approach and the Luenberger Approach are more suitable for valuing the technical flexibility.

On the other hand, if market uncertainty is dominant in the project, to value the market flexibility, we should focus on modelling market uncertainty. It is a financing problem. We need to employ the market information to evaluate the market flexibility. The price volatility, investment timing, oil price forecast, are necessary. In this case, we suggest using Monte Carlo simulation, risk neutral probabilities, financial option-pricing instruments, and decision tree technique as well. These techniques are suitable for the appraisal of market flexibility. Consequently, we recommend the subjective approach, MAD, the Smith Approach, and the Luenberger Approach. These approaches are suitable for market flexibility valuation.

However, we could not agree that one method is the best beyond the other approaches, since each approach has its own strengths and weaknesses for the flexibility valuation. The choice and application of a real options approach could be determined on the case-by-case basis.

Further research needs researchers to understand the flexibility variables that are used for modelling technical uncertainty and market uncertainty, and to value real options in a realistic, and "option-doable" way.

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Appendix A: Fetkovich Decline Curve Analysis Features

The Fetkovich Type Curve represents an integration of analytical[1] and empirical[2] models for the rate decline performance of oil and gas reserves operating at nearly constant bottom-hole flowing pressure.

The assumptions of the Fetkovich Type Curve are (1) single-phase flow (2) no water injection and no water production (3) production rate is proportional to $\bar{p}-p_{wf}$. $\bar{p}$ is the average pressure in the reservoir and p_{wf} is the bottom pressure in the production well. When a well first starts to produce, whether at a constant rate or flowing bottom hole pressure, the pressure disturbance so created propagates radically from the well bore out into the producing zone until a natural or man-made boundary or fluid contact is encountered. Prior to the pressure disturbance reaching a boundary, the well is said to behave as if the producing zone were infinite in size. After reaching a natural or man-made no-flow outer boundary, the well is said to behave in "depletion mode", hereafter referred to as Late Time behaviour. The time at which infinite-acting behaviour ends and depletion behaviour begins is known as the time required to reach "pseudo-stead-state" behaviour. A useful equation relating the time required to reach pseudo-steady-state behaviour to reservoir parameters for a homogeneous producing zone exhibiting classical radial flow follows:

$$t_{pss} = (379\phi\mu_i c_{ti} A)/k \quad \text{.....................................A(1)}$$

Where: t_{pss}:time, hours

ϕ:porosity, fraction

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[1] Mathematical models based on the physics describing fluid flow in porous media.

[2] Statistically derived equations found via observation to adequately model the behaviour of oil and gas wells.

μ_i: viscosity, cp

c_{ti}:total system compressibility, psi^{-1}

A :area, ft^2

Analytical solution can describe infinite-acting and pseudo-steady-state behavior. The solution can be represented in the curves. The X and Y axes are presented in the form of dimensionless variables. Early Time Curves Dimensionless Y axis is shown below:

$$q_{Dd} = \frac{q(t)141.2\mu\beta}{kh\left[P_i - P_{wf}\right]} \times \left[\ln\left(\frac{r_e}{r_{wa}}\right) - \frac{1}{2}\right] \text{.............................A(2)}$$

X axis is present as Equation A(3):

$$t_{Dd} = \frac{0.00634kt}{\phi\mu c_t r^2{}_{wa}} / \frac{1}{2} \times \left[\left(\frac{r_e}{r_{wa}}\right)^2 - 1\right]\left[\ln\left(\frac{r_e}{r_{wa}}\right) - \frac{1}{2}\right] \text{..................A(3)}$$

$$s = -\ln\left[\frac{r_{wa}}{r_w}\right] \text{..................................A(4)}$$

Where r_{wa}: apparent well bore radius.

r_w: actual well bore radius

s well's skin factor.

The empirical Arps[3] decline equation represents the relationship between production rate and time for oil wells during pseudo steady-state period and is shown as follows:

$$\text{Hyperbolic Equation: } q(t) = \frac{q_i}{(1 + bD_i t)^{1/b}} \text{.............................A(5)}$$

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[3] Arps, J.J., "Analysis of Decline Curves," Trans. AIME (1945) 160, 228-247

where q(t) is the oil production rate at production time t and q_i is the initial oil production rate. b and Di are two constants. Equation A(1) can be reduced in two special cases: b=0 and b=1. b=0; Equation A(5) can be converted to be an exponential decline model. See Equation A(6).b=1; Equation A(5) can be converted to an Harmonic model A(7).

$$q(t) = q_i e^{D_i t} \quad \text{..A(6)}$$

$$q(t) = \frac{q_i}{(1 + D_i t)} \quad \text{..A(7)}$$

Fetkovich developed an equation between the exponential decline model and the analytical solutions for a well/reservoir exhibiting pseudo-steady-state behaviour:

Equation A(8) shows the relationship between empirical and analytical solution:

$$q_i = \frac{kh\left[p_i - p_{wf}\right]}{141.2\mu\beta} \times \frac{1}{\left[\ln\left(\frac{r_e}{r_{wa}}\right) - \frac{1}{2}\right]} \quad \text{..................................A(8)}$$

$$D = \frac{2(0.00634)k}{\phi\mu\beta c_t (r_e^2 - r_{wa}^2)\left[\ln\left(\frac{r_e}{r_{wa}}\right) - \frac{1}{2}\right]} \quad \text{..................................A(9)}$$

The Fetkovich Type Curve can be used in two ways: (1) as a diagnostic plot against which to compare a log-log plot of a well or reservoir decline rate; (2) as a source of quantitative information (reservoir parameters from Early Time data curve matching and rate forecast from Late Time data curve matching.)

Appendix B: Li-Horne Model: Decline Analysis Models for Production Forecasting

Li et al (2003/2004/2005) supported the idea that the exponential decline hyperbolic curve tends to undervalue reserves and production rates and that the harmonic decline curve has a tendency to overvalue the reservoir performance. They agreed that in some production cases, the decline data fitted neither the exponential nor the harmonic model but was a cross over of the entire set of curves.

Li-Horne Model is shown in following equations:

$$q(t) = a_0 \frac{1}{R(t)} - b_0 \quad \text{.........} B(1)$$

Where:

$$a_0 = \frac{AM_e^*(S_{wf} - S_{wi})}{L} P_c^* \quad \text{.........} B(2)$$

$$b_0 = AM_e^* \Delta \rho g \quad \text{.........} B(3)$$

A is area of the reservoir.

L is the length of the reservoir or core.

S_{wf} is the water saturation behind the water front.

S_{wi} is the initial water saturation.

$\Delta\rho$ is the density difference between water (the wetting) and oil (the nonwetting) phases.

g is the gravity constant.

P_c^* is the capillary pressure at S_{wf}.

M_e^* is the global mobility in which relative permeability data of oil and water are included.

The Li-Horne Model presents the relationship between production rate and the cumulative production. Based on the Li-Horne Model, a linear trend can be obtained by plotting X axis q versus the reciprocal of cumulative production (or the recovery). The maximum recovery or the recoverable reserve is shown as the follow equation:

$$R_{\max} = \frac{1 - S_{wi} - S_{or}}{1 - S_{wi}} \quad \text{..........} \text{B(4)}$$

Where S_{or} is the residual oil saturation.

$$M_e^* = \frac{M_0^* M_w^*}{M_0^* + M_w^*} \quad \text{..........} \text{B(5)}$$

$$M_0^* = \frac{kk_{ro}^*}{\mu_0} \quad \text{..........} \text{B(6)}$$

$$M_w^* = \frac{kk_{rw}^*}{\mu_w} \quad \text{..........} \text{B(7)}$$

Where k is the permeability of the rock sample.

k_{ro}^* is the oil relative permeability at a specific water saturation.

k_{rw}^* is the water relative permeability at a specific water saturation.

μ_o μ_w are the oil and water viscosities respectively.

Li et al concluded that the model can match the production data at the later period of production. The maximum value of recovery predicted by the Li-Horne model is greater than the harmonic model and the exponential model. Through comparison with the other two models, they conclude that Li-Horne Model may have a better accuracy than the exponential model and the harmonic model.

Appendix C: Real Option Equations for Case 1

$q(t)$: Production rate at time t

$S(t)$: Gas price at time t

S_0: Gas price at time 0

n: Production years

$\widetilde{c}(t)$: Variable production costs rate

$\widetilde{c}$: Variable production costs

$c_o(t)$: Operation costs at time t

c_{option}: Option costs

$c_{fix}(t)$: Fixed costs at time t

D_{yearly}: Depreciation yearly

$c_{waterconing}$: Water coning costs

Pe: Contract penalty at the fifth year

c_{apex}: Capital expenses

$\lambda(t)$: Taxes at time t

λ: Tax rate

$R(t)$: Revenue at time t

NPV : Net present value of the project

$PV(t)$: Present value at time t

$WACC$: Weighted average capital costs rate

$Cf(t)$: Free cash flow at time t

d : Producing day

C-1. Gas prices model

Gas prices and production rates both vary stochastically over time following a random and conditional random walk respectively.

A: Gas prices model

$$dS(t) = 0.02S(t)dt + 0.04S(t)dz(t) \text{ (yearly)} \quad \ldots\ldots 5(3a)$$

In month, $\mu_{month} = \frac{\mu_{yearly}}{12}$, $\sigma_{month} = \sqrt{\frac{1}{12}} \times \sigma_{yearly}$;

$$dS(t) = 0.001667S(t)dt + 0.011547S(t)dz(t) / monthly \quad \ldots\ldots 5(3b)$$

B: Production rates models

a) Keep normal production model...5(4a)

b) Increasing production directly model.................................5(4b)

c) Implementing a reservoir management program, the production model.5(4c)

C-2 Net Present Value

1) The gas production:

$$Q = q(t) \times d \quad \ldots\ldots 5(5)$$

2) Variable cost at time t:

$$\tilde{c} = \tilde{c}(t) \times Q \tag{5(6)}$$

3) Depreciation in a year (straight-line):

$$D_{yearly} = \frac{1}{n} \times (B_0 - Salvage) \tag{5(7)}$$

Here, salvage is assumed to be zero.

4) Cash taxes at time t:

$$\lambda(t) = \lambda \times (R - \tilde{c} - c_{fix} - D_{yearly}) \tag{5(8)}$$

5) Free cash flow $Cf(t)$:

$$EBIT(t) = R(t) - \tilde{c} - c_{fix} - D_{yearly} \tag{5(9a)}$$

$$Cf(t) = EBIT(t) - \lambda(t) + D_{yearly} \tag{5(9b)}$$

6) Present value and net present value of the project:

$$PV = \sum_{t=0}^{T} \frac{Cf(t)}{(1 + WACC)^t} \tag{5(10a)}$$

$$NPV_{choice3} = PV_{choice3} - Investment \tag{5(10b)}$$

If the company could not fulfil the contract, PV at the 5th year that the company receives:

$$PV_5 = \frac{q(5) \times d - (\tilde{c} + c_{fix} + \lambda(5)) - Pe}{(1 + WACC)^5} \tag{5(10c)}$$

7) Option costs:

$$C_{option} = C_{exercising\_reservoir\_management\_program} + C_{delaying\_for\_bidding} \text{ ....5(11)}$$

8) If the company delays for bidding for decision, implements a reservoir management program, the net present value of project is:

$$NPV_{choice2} = PV_{choice2} - investment - c_{option} \text{ ....5(12)}$$

See **Table 36, Table 37 and Table 38**

The expect value of the strategy "keeping normal productions" is -0.21 million dollars. The expect value of the strategy "increasing production immediately" is 0.64 million dollars. The expect value of the strategy "delaying for bidding and implementing the reservoir management program" is 0.10 million dollars.

Table 366 Project Net Present Values for "Keeping Normal Production "

Current normal productivity	200	Thousand Cubic meters /day			Decline rate		
Producing days	355	Days			Mean	SD	
Contract penalty	2.20	Million dollars			7%	6%	
Period	0	1	2	3	4	5	6
Price		55.00	56.10	57.22	58.37	59.53	
Gas production	293969	71000	61724	57551	53661	50033	
Variable (production) cost		42.00	43.26	44.56	45.89	47.27	
Revenue		3.91	3.46	3.29	3.13	2.98	
-Variable cash costs		2.98	2.67	2.56	2.46	2.37	
-Fixed cash costs		0.41	0.41	0.41	0.41	0.41	
-Depreciation		0.46	0.46	0.46	0.46	0.46	
EBIT		0.05	-0.08	-0.14	-0.20	-0.26	
-Cash taxes		0.01	0.00	0.00	0.00	0.00	
+Depreciation		0.46	0.46	0.46	0.46	0.46	
-Capex	0.22						
Free cash flow		0.51	0.38	0.32	0.26	0.20	
WACC		0.12	0.12	0.12	0.12	0.12	
Discount factor		0.89	0.80	0.71	0.64	0.57	
PV of free cash flow	0	0.45	0.30	0.23	0.16	0.12	-1.25
Present value		1.26					
Contract penalty	-2.20	1.25					
-Capex		0.22					
NPV of project		-0.21					

Table 377 Project NPV for "Increasing productions immediately"

Production	260	Thousand cm /day		Decline rate		9%(Mean)	8%(SD)
Producing days	355	Days					
Later production costs for the water-coning	0.52	Million dollars		Increase rate		3%(Mean)	6%(SD)
Period	Total	0	1	2	3	4	5
Price ($/unit)			55.00	56.10	57.22	58.37	59.53
Gas production (tcm)	363104		92300	77095	70460	64396	58853
Production cost ($/unit)			42.00	43.26	44.56	45.89	47.27
Revenue			5.08	4.33	4.03	3.76	3.50
-Variable cash costs			3.88	3.34	3.14	2.96	2.78
-Fixed cash costs			0.41	0.41	0.41	0.41	0.41
-Depreciation			0.46	0.46	0.46	0.46	0.46
-Water coning costs					0.52	0.54	0.55
EBIT			0.33	0.12	-0.50	-0.60	-0.70
-Cash taxes			0.04	0.01	0.00	0.00	0.00
+Depreciation			0.46	0.46	0.46	0.46	0.46
-Capex		0.22					
Free cash flow			0.75	0.57	-0.04	-0.14	-0.24
WACC			0.12	0.12	0.12	0.12	0.12
Discount factor			0.89	0.80	0.71	0.64	0.57
PV of free cash flow		0	0.67	0.45	-0.03	-0.09	-0.14
PV of project			0.87				
-Capex			0.22				
Contract Penalty		-2.20	1.25				
NPV of project			0.64				

Table 38 Project NPV for "Implementing reservoir management program"

Option costs	2.16	$, Million			Decline rate		5%	7%
Production-Permian 2	300	Thousand Cubic meters /day						
Producing days	355	Days						
Time	Total	0	0.67	1.67	2.67	3.67	4.67	5.67
Period		0	1	2	3	4	5	6
Price ($/unit)		55	55.73	56.85	57.98	59.14	60.33	61.53
Gas production (tcm)	453341		36000	96365	91665	87195	82942	59173
Production cost ($/unit)		42.00	42.42	43.69	45.00	46.35	47.74	49.18
Revenue			2.01	5.48	5.32	5.16	5.00	3.64
-Variable cash costs			1.51	4.21	4.13	4.04	3.96	2.91
-Fixed cash costs			0.10	0.41	0.41	0.41	0.41	0.31
-Deprcciation			0.12	0.46	0.46	0.46	0.46	0.35
-Water coning costs								
EBIT			0.28	0.40	0.32	0.25	0.17	0.08
-Cash taxes			0.03	0.05	0.04	0.03	0.02	0.01
+Depreciation			0.12	0.46	0.46	0.46	0.46	0.35
-Capex		0.22						
Free cash flow			0.36	0.81	0.74	0.68	0.61	0.41
WACC			0.12	0.12	0.12	0.12	0.12	0.12
Discount factor			0.89	0.80	0.71	0.64	0.57	0.51
PV of free cash flow		-2.16	0.32	0.65	0.53	0.43	0.35	0.21
PV of project			2.26					
Option cost			2.16					
NPV of project			0.10					

C-3 The Classical Approach

Comparison of financial option and Real Options

Table 389 Comparisons of Variables in Case 1

Symbols	Stock Call Option	Reserves in Case 1
S	Current stock price	Value of developed recoverable reserves discounted by delaying time.
X	Exercise price	Delaying cost plus reservoir management cost plus production costs
σ^2	Variance of rate of return on the stock	Variance of rate of change of the value of recoverable reserves
T	Time to expiration	Relinquishment project
rf	Risk-free rate of interest	Risk-free rate of interest
D	Dividend	Net production revenue less depletion

Case 1 Solution

Real Option Value:

$$C_0 = S_t N(d_1) - Xe^{-r(T-t)} N(d_2) \quad \text{.................................5(13)}$$

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Where:

C_0: Price of Real (Delay) Option at time t

S_t: Value of reserves discounted by delaying lag

$N(\ )$: The cumulative normal density function

X : Exercise price (X = costs of delaying for bidding + costs of reservoir management+ production costs)

T: Expiration date of the option

t : Exercise time

r : Risk-free rate of interest

σ^2: Variance of gas prices

Gas prices model

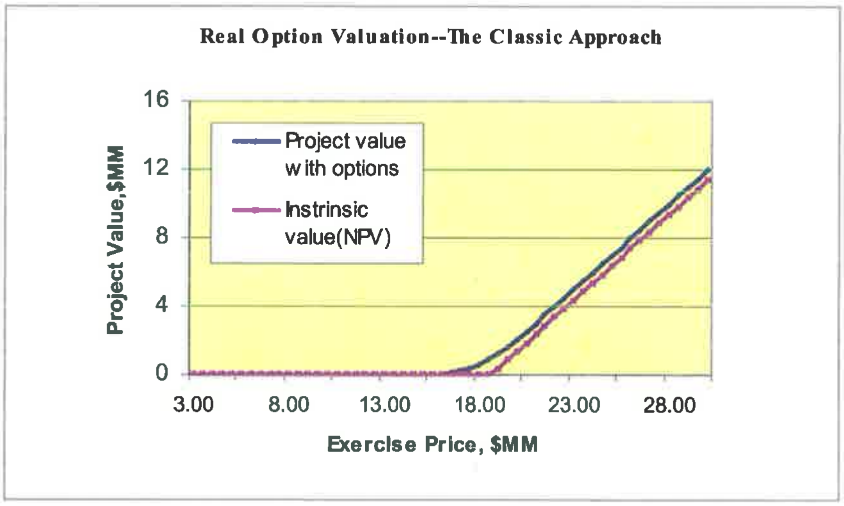
Equation 5(3a) and 5(3b);

In month, $\mu_{month} = \frac{\mu_{yearly}}{12}$, $\sigma_{month} = \sqrt{\frac{1}{12}} \times \sigma_{yearly}$;

$$dS(t) = 0.001667S(t)dt + 0.011547S(t)dz(t) / monthly$$

See Table 41, the project value (the NPV plus Real Option value) is \$2.87 million dollars. Based on the classic approach, while the gas volatility is high, the value of the project is high.

Figure 40 Real Options Valuation-The Classic Approach



C-4 The Subjective Approach

Comparison of variables of stock call options and reserves

The Subjective approach starts at the net present value. The net present value is discounted by WACC. The volatility is the project returns volatility. It applies Monte Carlo Simulation to solve the volatility (we discuss it later). The approach assumes the project value is comprised of the net project value without flexibility, plus the value of flexibility.

Table 40 Parameters in the Subjective Approach

Symbols	Stock Call Option	Reserves in Case 1
Stock	Current stock price	Net present value discounted by WACC
X	Exercise price	Delaying cost plus reservoir management cost
σ^2	Variance of rate of return on the stock	Variance of rate of project returns
T	Time to expiration	Relinquishment project
R	Risk-free rate of interest	Risk-free rate of interest
D	Dividend	Net production revenue less depletion

Solution

$$C_0 = S_t N(d_1) - Xe^{-r(T-t)} N(d_2) \quad \text{.................................5(14)}$$

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \quad \text{..............................5(15)}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \quad \text{...................................5(16)}$$

Where:

C_0: Price of Real (Delay) Option at time t

S_t: Project present value

$N(\ )$: The cumulative normal density function

X : Exercise price (X = costs of delaying for bidding + costs of reservoir management)

T: Expiration date of the option

t : Exercise time

r : Risk-free rate of interest

σ^2 : Variance of project returns

Gas prices model

Equation 5(3a) and 5(3b);

In month, $\mu_{month} = \frac{\mu_{yearly}}{12}$, $\sigma_{month} = \sqrt{\frac{1}{12}} \times \sigma_{yearly}$;

$$dS(t) = 0.001667S(t)dt + 0.011547S(t)dz(t) / monthly \quad \text{......................... 5(17)}$$

Production models

a) Keep normal production model:

Given q(0)=200 thousand m^3/day, using Equation 5(4a);

b) Increasing production directly model:

Given q(0)=260 thousand m^3/day, using Equation 5(4b);

c) Implementing a reservoir management program, the production model:

Given q(0)=300 thousand m^3/day, using Equation 5(4c);

The volatility of project returns

$$PV_{choice2} = \sum_{t=0}^{T} \frac{Cf(t)}{(1+WACC)^t} \quad \text{.......................................} 5(18)$$

$$NPV_{choice2} = PV_{choice2} - Investment \quad \text{.......................................} 5(19)$$

$$E(P\widetilde{V}_0) = \frac{E(FC\widetilde{F}_1)}{(1+WACC)^1} + \frac{E(P\widetilde{V}_1)}{(1+WACC)^1} \quad \text{....................} 5(20)$$

$$PV_1 = \sum_{t=2}^{5} \frac{FCF_t}{(1+WACC)^{t-1}} \quad \text{.......................................} 5(21)$$

$$Z = \ln\left(\frac{PV_1 + FCF_1}{E(PV_0)}\right) \quad \text{.......................................} 5(22)$$

$$\sigma = StdDev(Z) \quad \text{.......................................} 5(23)$$

See Table 16: After running Monte Carlo simulation, one will obtain:

PV_1=1.94; $E(P\widetilde{V}_0)$=2.65; $FC\widetilde{F}_1$=0.95

$\sigma \approx 55\%$

Therefore, the project value (the NPV plus Real Option Value) is $2.73 M.

See **Table 41** the solution of the Subjective Approach.

C-5 The Market Asset Disclaimer

Gas prices model

Equation 5(3a) and 5(3b);

In month, $\mu_{month} = \frac{\mu_{yearly}}{12}$, $\sigma_{month} = \sqrt{\frac{1}{12}} \times \sigma_{yearly}$;

$dS(t) = 0.001667S(t)dt + 0.011547S(t)dz(t) / monthly$

Production models

a) Keep normal production model:

Given q(0)=200 thousand m^3/day,5(4a);

b) Increasing production directly model:

Given q(0)=260 thousand m^3/day,5(4b);

c) Implementing a reservoir management program, the production model:

Given q(0)=300 thousand m^3/day, 5(4c);

Project present value

$$PV = \sum_{t=0}^{T} \frac{Cf(t)}{(1+WACC)^t} \quad \text{...5(24)}$$

$$NPV = PV - Investment \quad \text{...5(25)}$$

The volatility of project returns

$$PV_{choice2} = \sum_{t=0}^{T} \frac{Cf(t)}{(1+WACC)^t} \quad \text{...5(26)}$$

$$NPV_{choice2} = PV_{choice2} - Investment \quad \text{..................................5(27)}$$

$$E(P\widetilde{V}_0)=\frac{E(FC\widetilde{F}_1)}{(1+WACC)^1}+\frac{E(P\widetilde{V}_1)}{(1+WACC)^1} \quad \text{..........................5(28)}$$

$$PV_{1-choice1}=\sum_{t=2}^{5}\frac{FCF_t}{(1+WACC)^{t-1}} \quad \text{..................................5(29)}$$

$$\sigma = StdDev(Z) \quad \text{...5(30)}$$

$$Z=\ln\left(\frac{PV_1+FCF_1}{E(PV_0)}\right) \quad \text{..5(31)}$$

See Table 17: After running Monte Carlo simulation, we obtain:

$PV_{1-choice1}$=1.94; $E(P\widetilde{V}_0)$=2.65; $FC\widetilde{F}_1$=0.95

$\sigma \approx 55\%$

Binominal tree

$$u=e^{\sigma\sqrt{dt}} \quad \text{...5(32)}$$

$$d=\frac{1}{u} \quad \text{...5(33)}$$

$$p=\frac{1+r_f-d}{u-d} \quad \text{...5(34)}$$

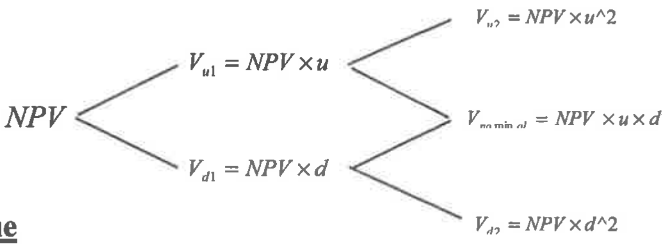
See Table 18

$u=e^{\sigma\sqrt{dt}}$=1.3732

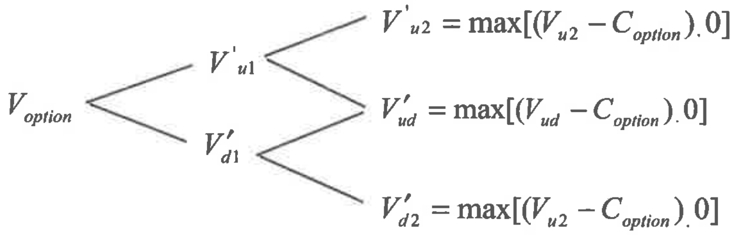
$d=\frac{1}{u}$=0.7282

$p=\frac{1+r_f-d}{u-d}$=0.4989; 1-p=0.5011

Real Option Value with costs



Net Real Option Value



$$V'_{u1} = [pV'_{u2} + (1-p)V'_{ud}]/(1+r_f) \quad \text{.................................5(36)}$$

$$V'_{d1} = [pV'_{ud} + (1-p)V'_{d2}]/(1+r_f) \quad \text{.................................5(37)}$$

$$V_{option} = 0.52MM$$

See Table 43 Evolution of Project Value, and Table 44 Project Option Value.

$$\begin{aligned} V_{project} &= V_{option} + NPV \\ &= 0.52 + 2.26 = 2.78MM \end{aligned} \quad \text{.......................................5(38)}$$

C-6 The Smith Approach

$s(t)$: Price at time t

ω_t : Given state of information

$c(t, \omega)$: Risky cash flow streams

c_α: Pay offs generated from a strategy α

β_r: A replicating trading strategy

π: Risk neutral probability

$$s(0) = \sum_{\omega t} \frac{\pi(\omega_t)}{(1+r_f)^t} s(t,\omega) = E_\pi\left[\frac{s(t)}{(1+r_f)^t}\right] \quad \text{..........................5(39)}$$

Separation theorem (incomplete market) for investment problem:

$$v^* = \max E_\pi\left[\sum_{t=0}^{T} \frac{c_\alpha(t)}{(1+r_f)^t}\right] \quad \text{..5(40)}$$

Separation theorem (complete market) for financing problem:

$$\beta_r(t-1,\omega_{t-1})s(t,\omega_t) = c(t,\omega_t) + \beta_r(t,\omega_t)s(t,\omega_t) \quad \text{........................5(41)}$$

The case is dominated by technical uncertainty. The managers would not trade the project as securities. Based on this assumption, there is no financing problem in the case. We also assume the investors' risk attitude is neutral.

Upper and lower bounds of the project value in the incomplete market:

$$\bar{v} = c(0) + \min_\beta\{\beta(0)s(0) : [\beta(t-1) - \beta(t)]s(t) \geq c(t)\}; t > 0$$

$$\bar{v} = \sup_{\pi\in\Pi} E_\pi\left[\sum_{t=0}^{T} \frac{c(t)}{(1+r_f)^t}\right] \quad \text{...............5(42)}$$

$$\underline{v} = c(0) + \max_\beta\{\beta(0)s(0) : [\beta(t-1) - \beta(t)]s(t) \leq c(t)\}; t > 0 \quad \text{...............5(43)}$$

$$\underline{v} = \inf_{\pi \in \Pi} E_\pi \left[\sum_{t=0}^{T} \frac{c(t)}{(1+r_f)^t} \right] \qquad \text{.....5(44)}$$

Production models

$$dq(t) = \mu_q q(t)dt + \sigma_q q(t)dz_q(t) \qquad \text{.....5(45)}$$

a) If increasing production immediately,

$$q(0) = 300: dq(t) = -0.10q(t)dt + 0.09q(t)dz_q(t) \qquad \text{.....5(45a)}$$

$$q(0) = 260: dq(t) = -0.09q(t)dt + 0.08q(t)dz_q(t) \qquad \text{.....5(45b)}$$

$$q(0) = 200: dq(t) = -0.07q(t)dt + 0.06q(t)dz_q(t) \qquad \text{.....5(45c)}$$

b) If implementing a reservoir management program,

$$q(0) = 400: dq(t) = -0.05q(t)dt + 0.08q(t)dz_q(t) \qquad \text{.....5(46a)}$$

$$q(0) = 300: dq(t) = -0.05q(t)dt + 0.07q(t)dz_q(t) \qquad \text{.....5(46b)}$$

$$q(0) = 200: dq(t) = -0.06q(t)dt + 0.07q(t)dz_q(t) \qquad \text{.....5(46c)}$$

Gas price models

Equation 5(3a) and 5(3b);

In month, $\mu_{month} = \frac{\mu_{yearly}}{12}$, $\sigma_{month} = \sqrt{\frac{1}{12}} \times \sigma_{yearly}$;

$dS(t) = 0.001667S(t)dt + 0.011547S(t)dz(t) / monthly$

The volatility in the project is gas price volatility.

Gas price volatility=4%.

See Table 21 The Smith Approach-Discrete Decision Tree

The company should delay for bidding for the contract, implementing the reservoir management program. The project value with Real Option is 0.92 million dollars, which is higher than the project value of increasing production immediately.

C-7 The Luenberger Approach

Time-discrete decision tree

There is a doubletree approach for production and gas uncertain variables.

Production models

Subjective Production Probabilities in the Luenberger Approach

Gas Production	260	300	350	400	450
Probability	0.1	0.3	0.2	0.3	0.1

Gas price models

Binominal tree model:

$$u = e^{\sigma\sqrt{dt}} \quad \ldots\ldots 5(47)$$

$$d = \frac{1}{u} \quad \ldots\ldots 5(48)$$

$$p = \frac{1 + r_f - d}{u - d} \quad \ldots\ldots 5(49)$$

σ = Gas price volatility……5(50)

See Table 22, the Binominal Tree for the price model.

Project value at zero-level:

V=flow*oil price-fixed cost-variable cost+1/1.05*(risk-neutral value of next period)

Project Value = C0 + 1/R×E(C1) = NPV+ROV

= (Flow*oil price-fixed cost-variable Cost)

+ 1/R*(risk-neutral value of next period)...................................5(51)

The project volatility

Gas price volatility=4%

Table 39 The Classic Approach

Inputs		
Contract production	560	million cubic meter
Original recoverable reserves	1.5	billion Cubic meters
Maximum production	255	thousand cubic meter/day
Permian 2 reservoir production	70%	
Current gas price	55.00	$/thousand cubic meters
Exercise costs	18.62	$, million dollars
Production costs	42.00	$/thousand cubic meters
Option costs	2.16	$, million dollars
Outputs		
Current value	20.87	$, million
Time to maturity	0.67	years
Risk free interest rate	5%	
Gas Price Volatility	10%	
Exercise Price	18.62	$, million
D1	1.839251999	
D2	1.757398471	
N(D1)	0.967060995	
N(D2)	0.960575088	
C	2.875003405	
Project value	2.88	$, Million

Table 40 Project Return Volatility for "Implementing Reservoir Management program" in the Subjective Approach

Operation costs	2.16	$, Million						
Reservoir productions	300	Thousand cubic meters /day				Decline rate	5%	7%
Producing days in first year	355	Days						
Time	Total	0	0.67	1.67	2.67	3.67	4.67	5.67
Period		0	1	2	3	4	5	6
Price/unit		55.00	55.73	56.85	57.98	59.14	60.33	61.53
Gas production	453341		36000	96365	91665	87195	82942	59173
Production cost/unit		42.00	42.42	43.69	45.00	46.35	47.74	49.18
Revenue			2.01	5.48	5.32	5.16	5.00	3.64
-Variable cash costs			1.51	4.21	4.13	4.04	3.96	2.91
-Fixed cash costs			0.10	0.41	0.41	0.41	0.41	0.31
-Depreciation			0.12	0.46	0.46	0.46	0.46	0.35
EBIT			0.28	0.40	0.32	0.25	0.17	0.08
-Cash taxes			0.03	0.05	0.04	0.03	0.02	0.01
+Depreciation			0.12	0.46	0.46	0.46	0.46	0.35
-Capex		0.22						
Free cash flow			0.36	0.81	0.74	0.68	0.61	0.41
WACC			0.12	0.12	0.12	0.12	0.12	0.12
Discount factor2			0.89	0.80	0.71	0.64	0.57	0.51
PV of free cash flow2			0.32	0.65	0.53	0.43	0.35	0.21
PV2 of project			2.26					
PV1			1.94					

Table 41 Project Value (with Option Value) in the Subjective Approach

Parameters	Value	Unit
Current value S	2.26	US$, million
Time to maturity T	0.67	Years
Risk free interest rate Rf	5%	
Volatility	54.94%	
Exercise Price X	2.16	US$, million
PV(x)	2.090872505	US$, million
NPVq	1.080162100	
NPV	0.10	US$, million
D1	0.397984873	
D2	-0.050562829	
N(D1)	0.654679286	
N(D2)	0.479836874	
C	0.475886510	US$, million
Project value with option	2.73	US$, million

Table 42 Market Asset Disclaimer (MAD)-Risk Neutral Probability

Volatility	54.94%	
WACC	12%	
Risk free rate	5%	
Option costs	2.16	Us$, million
T	0.67	year
Binomial step	2.00	
dt (the length of binomial period)	0.33	
u	1.3732	
d	0.7282	
up prob	0.4989	
1-p	0.5011	

Table 43 MAD --Binominal Tree: Evolution of Project Value ($, million Us)

		4.26
	3.10	
2.26		2.26
	1.64	
		1.20

Table 44 MAD- Project Option Value

		2.10
	1.04	
0.52		0.10
	0.05	
		0
The current value of the option	0.33 Years	0.34 Years
The Project Value with Option	$2.78M	

Table 45 the Smith Approach-Discrete Decision Tree

Choice 1		Prob	Gas production (TCM)	EPV	Water Coning Costs Uncertainty		Project Present Value					NPV
							Year1	Year2	Year3	Year4	Year5	
EPV	High	30%	300.00	0.91	High	60%	0.87	0.62	-0.05	-0.18	-0.27	0.76
					Low	40%	0.87	0.62	0.06	-0.05	-0.13	1.14
-0.79	Nominal	40%	260.00	0.44	High	60%	0.71	0.51	-0.14	-0.25	-0.32	0.28
					Low	40%	0.71	0.51	-0.03	-0.12	-0.19	0.67
	Low	30%	200.00	-4.46	High	60%	0.48	0.35	-0.29	-0.36	-0.41	-2.41
					Low	40%	0.48	0.35	-0.17	-0.23	-0.27	-2.03
Choice 2				Year0.67	Year1	Year2	Year3	Year4	Year5	Year6	NPV	
	High	30%	400.00	0.00	0.46	1.14	1.00	0.88	0.76	0.50	2.36	
0.92	Nominal	40%	300.00	0.00	0.33	0.73	0.64	0.56	0.48	0.31	0.67	
	Low	30%	260.00	0.00	0.20	0.42	0.35	0.28	0.23	0.13	0.00	

Table 46 The Luenberger Approach-Inputs

Time periods, years	5
Sigma	4%
Up factor	1.04081
Down factor	0.96079
Up probability	1.11483
Down probability	-0.11483

Table 47 The Luenberger Approach – Gas Price Evolution

55.73	58.01	60.38	62.84	65.40
($/TCM)	53.55	55.73	58.01	60.38
		51.45	53.55	55.73
			49.43	51.45
				47.49
Year1	Year 2	Year 3	Year 4	Year 5