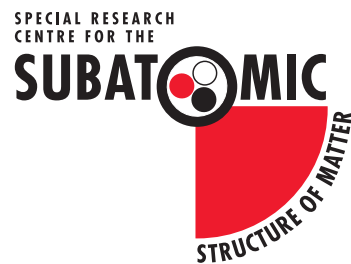


The Influence of Instantons on the Quark Propagator in Lattice Quantum Chromodynamics

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of Adelaide



For someone or something or whatever

Declaration

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Abstract

The non-Abelian nature of quantum chromodynamics means the vacuum cannot be treated as devoid of structure. The non-trivial topology of the vacuum is manifested through instantons, which have important effects on chiral symmetry breaking and dynamical mass generation. We wish to discern the presence and import of instantons on the lattice, first by using the technique of smearing to create lattice configurations which consist solely of instanton-like objects, then calculating the quark propagator on such configurations. Doing so, we hope to understand the role of instantons in the quark propagator.

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