

Multi-Particle Baryon Spectroscopy in Lattice Quantum Chromodynamics

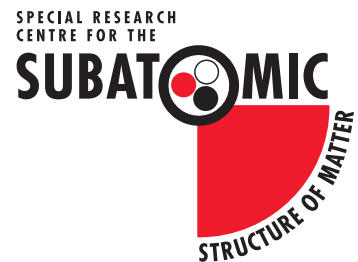
Adrian Leigh Kiratidis

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Abstract

Quantum Chromodynamics (QCD) is widely accepted as the theory that describes the strongest force in Nature (by coupling constant), aptly named the strong nuclear force. The challenge is to understand the phenomena that emerge from this fundamental quantum field theory. Hadronic spectroscopic calculations can be performed utilising the formalism of lattice QCD by discretising space-time onto a hypercube. This is the only known non-perturbative *ab-initio* approach for studying QCD. Equipped with a tractable formalism, we consider some recent work done extracting resonances, in particular the Roper and the $\Lambda(1405)$ resonances studied at the CSSM in Adelaide. These studies are done with three quark interpolators, and as such we expect to be extracting resonances having strong overlap three-quark states. In order to rule out the possibility of contamination from more exotic five-quark states, and to extract multi-particle states in their own right, the use of five-quark interpolators is of considerable interest. We first construct five-quark interpolating fields for the p , Λ and Δ^{++} . The corresponding correlation functions are calculated which can be of considerable size. Relevant elements of the all-to-all propagator (the so-called loop propagator), are calculated using stochastic estimation techniques. Dilution in spin, colour and time are implemented as a means of variance reduction. We conclude by presenting effective mass plots for the five-quark interpolators, the relevant contributions from fully connected and loop containing pieces, and comparing them to the masses extracted from standard three-quark operators.

Declaration

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to Adrian Leigh Kiratidis and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Adrian Leigh Kiratidis

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