Mass of the *H* Dibaryon

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Recent lattice QCD calculations have reported evidence for the existence of a bound state with strangeness -2 and baryon number 2 at quark masses somewhat higher than the physical values. By developing a description of the dependence of this binding energy on the up, down and strange quark masses that allows a controlled chiral extrapolation, we explore the hypothesis that this state is to be identified with the H dibaryon. Taking as input the recent results of the HAL and NPLQCD Collaborations, we show that the H dibaryon is likely to be unbound by 13 ± 14 MeV at the physical point.

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Our understanding of quantum chromodynamics has been challenged for decades by the apparent absence of multiquark states. The outstanding candidate for such a state has been the H dibaryon, ever since it was suggested that it should be very deeply bound with respect to the $\Lambda - \Lambda$ threshold [1]. Extensive experimental efforts to find this new particle [2–5] have led to the conclusion that it does not appear to be bound. However, the issue has been given new life in the past few months by reports from the HAL and NPLQCD Collaborations, whose lattice simulations find that the H-particle is indeed bound at quark masses somewhat above the physical range [6,7].

Early spectroscopic studies of the baryon and dibaryon spectrum within the bag model [8] showed that the pion cloud contribution was phenomenologically very important in determining whether or not the H was indeed bound. A preliminary analysis of some of the lattice data using a fit linear in m_π^2 suggests that it is bound at the physical quark mass, while an extrapolation linear in m_π, while also consistent with binding, does allow that the H may be slightly unbound [9]. We consider it important that the extrapolation to the physical quark masses should respect the constraints of chiral symmetry, such as ensuring the correct leading nonanalytic behavior. Given the rather large range of quark masses over which current lattice simulations have been made, the technique which offers the best opportunity for a quantitative fit, while preserving the correct nonanalytic behavior, is finite range regularization (FRR) [10–12].

In this Letter we explore the possibility that the H is a compact, multiquark state that may be bound with respect to the $\Lambda - \Lambda$ threshold. We apply the FRR technique to describe the quark-mass dependence of both the octet-baryon masses and the binding energy of the H dibaryon. With few lattice results available for the dibaryon binding energy, it is essential to utilize the fit to the hyperon masses to determine the dependence on the nonsinglet combination of quark masses (i.e., $m_l - m_s$), which plays a critical role in describing the variation of the (nonsinglet) Λ mass. Extrapolated to the physical quark masses, we conclude that the H dibaryon is most likely unbound, with its mass being 13 ± 14 MeV above the $\Lambda - \Lambda$ threshold.

Following the technique described in Ref. [13], we fit the data for octet masses recently published by the PACS-CS Collaboration [14], using an expansion about the SU(3) limit for the light and strange quark masses:

$$M_B = M^{(0)} + \delta M_B^{(1)} + \delta M_B^{(3/2)} + \ldots \tag{1}$$

Here the leading term, $M^{(0)}$, denotes the degenerate mass of the baryon octet in the SU(3) chiral limit, and

$$\delta M_B^{(1)} = -C_{Bl}^{(1)} m_l - C_{Bs}^{(1)} m_s, \tag{2}$$

with the coefficients given in Table I, is the correction linear in the quark masses.

At next order, after the linear mass insertions, one finds quantum corrections associated with the chiral loops involving the pseudo-Goldstone bosons, $\phi \equiv \pi,\ K,\ \eta$. While our formal assessment of the order of a given diagram treats the intermediate octet and decuplet baryons as degenerate, in order to more accurately represent the branch structure near $m_\phi \sim \delta$, we retain the octet-decuplet mass difference (δ) in the numerical evaluations. These loops take the form:

$$\delta M_B^{(3/2)} = -\frac{1}{16\pi f^2}\sum_\phi [\chi_{B\phi} I_R(m_\phi, 0, \Lambda) + \chi_{T\phi} I_R(m_\phi, \delta, \Lambda)], \tag{3}$$

TABLE I. Values for the terms linear in the nonstrange quark mass, $m_l \to m_\pi^2/2$, and the strange quark mass, $m_s \to (m_K^2 - m_\pi^2/2)$, expressed in terms of the leading quark-mass insertion parameters α, β and σ.

B	$C_{Bl}^{(1)}$	$C_{Bs}^{(1)}$
N	$2\alpha + 2\beta + 4\sigma$	2σ
Λ	$\alpha + 2\beta + 4\sigma$	$\alpha + 2\sigma$
Σ	$\frac{5}{3}\alpha + \frac{2}{3}\beta + 4\sigma$	$\frac{1}{3}\alpha + \frac{4}{3}\beta + 2\sigma$
Ξ	$\frac{1}{3}\alpha + \frac{4}{3}\beta + 4\sigma$	$\frac{5}{3}\alpha + \frac{2}{3}\beta + 2\sigma$

where the coefficients $\chi_{B\phi}$, $\chi_{T\phi}$, taken from Ref. [15], are given in Table II. The meson loops involve the integrals:

$$I_R = \frac{2}{\pi}\int dk \frac{k^4}{\sqrt{k^2+m_\phi^2}(\delta+\sqrt{k^2+m_\phi^2})} u^2(k) - b_0 - b_2 m_\phi^2 \tag{4}$$

where the subtraction constants, $b_{0,2}$, are defined so that the parameters $M^{(0)}$, $C_{Bl}^{(1)}$ and $C_{Bs}^{(1)}$ are renormalized (explicit expressions may be found in Ref. [10], or can be readily evaluated numerically by Taylor expanding the integrand in m_ϕ^2). The loop contribution parameters are taken to be $D+F=g_A=1.27$, $F=\frac{2}{3}D$, $C=-2D$, $f=0.0871$ GeV and $\delta=0.292$ GeV. Within the framework of FRR, we introduce a mass scale, Λ, through a dipole regulator, $u(k)=(\frac{\Lambda^2}{\Lambda^2+k^2})^2$. Λ is related to the scale (typically $\sim\Lambda/3$ for a dipole) beyond which a formal expansion in powers of the Goldstone boson masses breaks down [16,17]. However, rather than growing uncontrollably as some power of m_ϕ^2, in this regime the Goldstone loops are actually suppressed, decreasing as powers of Λ/m_ϕ [18–21]. In practice, we choose this mass parameter by fitting the lattice data itself. Extensive studies of the FRR technique have established that the extrapolation is independent of the functional form chosen for the regulator [10]—essentially because of the rapid decrease of the loop contributions which we just explained.

In order to describe the mass of the H, treated as a compact, multiquark state (rather than a loosely bound molecular state), we note that because it is an SU(3) singlet, at the equivalent order of the quark-mass expansion its mass can be expressed as:

$$M_H = M_H^{(0)} - \sigma_H\left(\frac{m_\pi^2}{2}+m_K^2\right) + \delta M_H^{(3/2)}, \tag{5}$$

with

$$\delta M_H^{(3/2)} = -C_H(2D^2+C^2)[I_R(m_\pi,\delta H,\Lambda) + \tfrac{4}{3}I_R(m_K,\delta H,\Lambda) + \tfrac{1}{3}I_R(m_\eta,\delta H,\Lambda)]. \tag{6}$$

As indicated in Eq. (5), because it is a flavour-singlet, at leading order in the quark masses M_H depends only on the sum of the quark masses. We therefore set:

$$\begin{aligned} B_H &= 2M_\Lambda - M_H \\ &= (2M^{(0)} - M_H^{(0)}) - (4\sigma_\Lambda - \sigma_H)\left(\frac{m_\pi^2}{2}+m_K^2\right) \\ &\quad - 2\alpha m_K^2 - 2\beta m_\pi^2 + 2\delta M_\Lambda^{(3/2)} - \delta M_H^{(3/2)} \\ &= B_0 + \sigma_B\left(\frac{m_\pi^2}{2}+m_K^2\right) - 2\alpha m_K^2 - 2\beta m_\pi^2 \\ &\quad + 2\delta M_\Lambda^{(3/2)} - \delta M_H^{(3/2)}, \end{aligned} \tag{7}$$

where B_0 and σ_B are parameters determined by the fit to the lattice data for B_H [6,7]. Of course, α and β are determined by the fit to the baryon octet described above. For simplicity, we keep the regulator mass for the octet and the H the same, while varying the chiral coefficient for the H, C_H, to fit the lattice data.

In order to fully maintain the correlations between the errors associated with all of the fitting parameters, we carried out a simultaneous analysis by minimizing χ^2 for the fit to both the masses of the nucleon octet and to the difference in mass of the H and two Λ hyperons. Of course, the parameters $M^{(0)}$, α, β, σ, as well as the regulator mass Λ, were primarily determined by the fit to the PACS-CS data, which is shown in Fig. 1, with the corresponding parameters given in Table III. We note that, as explained in Ref. [13], the octet data was corrected for small, model independent finite volume effects before fitting. In the figure we show the lattice data after applying both the finite volume correction and a correction (based upon our fit parameters) arising because the PACS-CS simulations used values of the strange quark mass that were somewhat larger than the empirical values. The χ^2 per degree of freedom for the octet data was 0.49 (7.3 divided by $20-5\equiv 15$). This is lower than unity as, without access to the original data, we cannot incorporate the effect of correlations between the lattice data. Nevertheless, the fit is clearly very satisfactory over the entire range of quark masses explored in the simulations and should provide an excellent basis for the study of the possible binding of the H dibaryon. Indeed, the masses of the N, Λ, Σ and Ξ baryons at the physical point are (0.959 ± 0.023, 1.129 ± 0.014, 1.188 ± 0.011, 1.325 ± 0.006) GeV, where all the errors include the correlated uncertainties of all the fit parameters,

TABLE II. Chiral SU(3) coefficients for the octet baryons to octet (B) and decuplet (T) baryons through the pseudoscalar octet meson ϕ.

		$\chi_{B\phi}$			$\chi_{T\phi}$	
	π	K	η	π	K	η
N	$\frac{3}{2}(D+F)^2$	$\frac{1}{3}(5D^2-6DF+9F^2)$	$\frac{1}{6}(D-3F)^2$	$\frac{4}{3}C^2$	$\frac{1}{3}C^2$	0
Λ	$2D^2$	$\frac{2}{3}(D^2+9F^2)$	$\frac{2}{3}D^2$	C^2	$\frac{2}{3}C^2$	0
Σ	$\frac{2}{3}(D^2+6F^2)$	$2(D^2+F^2)$	$\frac{2}{3}D^2$	$\frac{2}{9}C^2$	$\frac{10}{9}C^2$	$\frac{1}{3}C^2$
Ξ	$\frac{3}{2}(D-F)^2$	$\frac{1}{3}(5D^2+6DF+9F^2)$	$\frac{1}{6}(D+3F)^2$	$\frac{1}{3}C^2$	C^2	$\frac{1}{3}C^2$

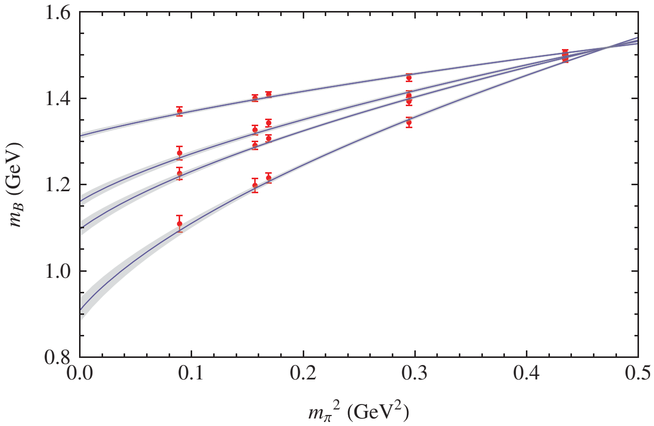

FIG. 1 (color). Fit to the octet data of PACS-CS [14] using Eq. (1). Note that we have fit the data after applying finite volume corrections and we have also used our fit to correct the lattice data for the strange quark mass, which was somewhat larger than the physical value.

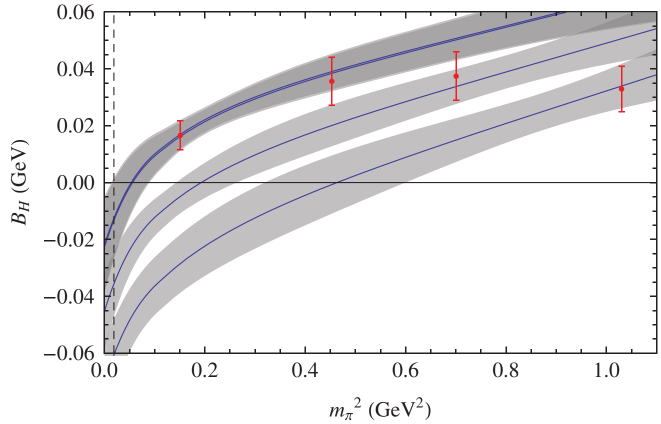

FIG. 2 (color). Binding energy of the H dibaryon versus pion mass squared, resulting from our chiral fit, for several values of the strange quark mass at which the simulations of Refs. [6,7] were carried out.

including the regulator mass, Λ. For comparison we note that the physical octet masses are $(0.939, 1.116, 1.193, 1.318)$ GeV.

With respect to the H dibaryon we have retained only the data from the HAL Collaboration [6] which was generated on the largest lattice volume, namely, 3.87 fm. These data points correspond to large (degenerate) pseudoscalar masses, 1.015, 0.837 and 0.673 GeV, for which the finite volume corrections are expected to be very small. Accordingly we used the reported values without applying any finite volume correction. In the case of the NPLQCD Collaboration [7], where the calculation was performed at $m_\pi = 389$ MeV and $m_K = 553$ MeV, we include in the fit only the value for the binding of the H determined after their extrapolation to infinite volume. Finally, we were unable to include the quenched lattice data of Ref. [22], which also indicated a bound H at large quark mass, because the errors associated with "unquenching" are considerable.

We chose the mass splitting between the H dibaryon and the other dibaryon states appearing in its chiral loop corrections to be the same as the octet-decuplet mass splitting used earlier, namely $\delta H = \delta = 0.292$ GeV. This is compatible with the estimates of Aerts *et al.* [23] calculated within the MIT bag model, as well as with the experimental absence of other nearby states. The sensitivity of our fit to δH is quite small, with an increase (decrease) of δ by 100 MeV increasing (decreasing) the mass difference $2M_\Lambda - M_H$ by only 4 MeV. This small shift is combined in quadrature with the error found from our chiral fit to yield the final, quoted error in the binding of the H.

The best fit parameters describing the binding energy of the H dibaryon are also given in Table III and the actual fit is shown in Fig. 2. As explained above, the data shown in Fig. 2 are from NPLQCD (lowest mass point) and HAL (three largest mass points). In each case the curve nearest the data point illustrates the extrapolation as a function of the light quark mass implied by our fit at the value of the strange quark mass corresponding to that lattice data point. The errors shown are the result of combining in quadrature the statistical and systematic errors quoted by the collaborations. The shaded error bands incorporate the effect of correlations between the fit parameters, including the uncertainty on the regulator mass. We note the remarkable result that the best fit value of the chiral coefficient for the H dibaryon, C_H [which for convenience is normalized with respect to the chiral coefficient for π loops on the Λ hyperon in Eq. (6)], is within 20% of the theoretical value reported by Mulders and Thomas [8], who calculated it using SU(6) symmetry. If instead we retain the Mulders-Thomas coefficient, the H dibaryon is unbound by 30 ± 9 MeV but the quality of the fit is significantly reduced, with a χ^2 per degree of freedom of almost 2.

It is clear from Figs. 1 and 2 that both the octet data and the data on the binding energy of the H dibaryon are very well described. The binding of the H is reduced by a decrease in the masses of the u and d quarks and the s, with significant chiral curvature for m_π below 0.4 GeV. It is important to note that our analysis does not explicitly

TABLE III. Values of the fit parameters for the octet and H dibaryon data corresponding to the fits shown in Figs. 1 and 2.

	Λ (GeV)	M_0 (GeV)	α (GeV^{-1})	β (GeV^{-1})	σ (GeV^{-1})	B_0 (GeV)	σ_B (GeV^{-1})	C_H (GeV^{-2})
best fit value	1.02	0.861	−1.71	−1.20	−0.51	0.019	−2.36	5.65
error	0.06	0.037	0.12	0.10	0.05	0.004	0.20	0.09

include the effect of the change of quark masses on the coupling of the H dibaryon to any of the open baryon-baryon channels. For an SU(3) singlet like the H, the largest couplings are to the $\Xi - N$ and $\Sigma - \Sigma$ channels. As their respective thresholds lie well above the mass of the H at all quark masses, they may be expected to contribute attraction which varies slowly with quark mass. There is somewhat more uncertainty concerning the $\Lambda - \Lambda$ channel, which becomes open as the H becomes unbound. Given where we find the multiquark state, this means that the effect of the $\Lambda - \Lambda$ channel might change from attraction to repulsion, depending on the range of momenta which dominate this continuum channel. It is also true that it would be extremely valuable to have new simulations at lower pion mass and large volume which would further constrain the extrapolation. Nevertheless, the conclusion of our study is quite clear. Even though the H is bound at larger quark masses, we see in Fig. 2 that chiral physics leads to a more rapid decrease of the mass of the Λ as m_π approaches its physical value than we find for the H and as a result one must conclude that at the physical values of the quark masses the H dibaryon is most likely unbound. Our estimate, including the effect of correlations between all the fit parameters, is that the H is unbound by a mere 13 ± 14 MeV at the physical point. That this is so close to the $\Lambda - \Lambda$ threshold will undoubtedly spur investigations into the consequences for doubly strange hypernuclei as well as the equation of state of dense matter.

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[1] R. L. Jaffe, Phys. Rev. Lett. **38**, 195 (1977).
[2] C. J. Yoon *et al.*, Phys. Rev. C **75**, 022201 (2007).
[3] H. Takahashi *et al.*, Phys. Rev. Lett. **87**, 212502 (2001).
[4] R. W. Stotzer *et al.* (BNL E836 Collaboration), Phys. Rev. Lett. **78**, 3646 (1997).
[5] T. Iijima *et al.*, Nucl. Phys. A **546**, 588 (1992).
[6] T. Inoue *et al.* (HAL QCD Collaboration), Phys. Rev. Lett. **106**, 162002 (2011).
[7] S. R. Beane *et al.* (NPLQCD Collaboration), Phys. Rev. Lett. **106**, 162001 (2011).
[8] P. J. Mulders and A. W. Thomas, J. Phys. G **9**, 1159 (1983).
[9] S. R. Beane *et al.*, arXiv:1103.2821.
[10] D. B. Leinweber, A. W. Thomas, and R. D. Young, Phys. Rev. Lett. **92**, 242002 (2004).
[11] R. D. Young, D. B. Leinweber, and A. W. Thomas, Prog. Part. Nucl. Phys. **50**, 399 (2003).
[12] R. D. Young, D. B. Leinweber, A. W. Thomas, and S. V. Wright, Phys. Rev. D **66**, 094507 (2002).
[13] R. D. Young and A. W. Thomas, Phys. Rev. D **81**, 014503 (2010).
[14] S. Aoki *et al.* (PACS-CS Collaboration), Phys. Rev. D **79**, 034503 (2009).
[15] A. Walker-Loud, Nucl. Phys. **A747**, 476 (2005).
[16] G. P. Lepage, arXiv:nucl-th/9706029.
[17] R. D. Young, D. B. Leinweber, and A. W. Thomas, Phys. Rev. D **71**, 014001 (2005).
[18] A. W. Thomas, Nucl. Phys. B, Proc. Suppl. **119**, 50 (2003).
[19] R. E. Stuckey and M. C. Birse, J. Phys. G **23**, 29 (1997).
[20] J. F. Donoghue, B. R. Holstein, and B. Borasoy, Phys. Rev. D **59**, 036002 (1999).
[21] D. B. Leinweber, D.-H. Lu, and A. W. Thomas, Phys. Rev. D **60**, 034014 (1999).
[22] Z.-H. Luo, M. Loan, Q.-G. Lin, and Y. Liu, arXiv:1106.1945.
[23] A. T. M. Aerts, P. J. G. Mulders, and J. J. de Swart, Phys. Rev. D **17**, 260 (1978).