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Fisher-information condition for enhanced signal detection via stochastic resonance

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Various situations where a signal is enhanced by noise through stochastic resonance are now known. This paper contributes to determining general conditions under which improvement by noise can be *a priori* decided as feasible or not. We focus on the detection of a known signal in additive white noise. Under the assumptions of a weak signal and a sufficiently large sample size, it is proved, with an inequality based on the Fisher information, that improvement by adding noise is never possible, generically, in these conditions. However, under less restrictive conditions, an example of signal detection is shown with favorable action of adding noise.

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I. INTRODUCTION

Stochastic resonance (SR) is now a well-established cooperative phenomenon wherein the response of a nonlinear system to a weak signal can be optimized at a nonzero noise level [1–11]. Briefly, SR emerged from the field of meteorology [1], and the topic has flourished in physics [2–6] and neuroscience [5–11]. Meanwhile, the promise of applying SR to nonlinear signal processing has been studied over several decades. The improvement of output signal-to-noise ratio of a nonlinear system first attracted much attention [2–5,12–16], and later, noise-enhanced detection was observed in dynamic [17–19] and static nonlinearities [20–29]. An interesting idea explored in Ref. [29] is that, in order to find an optimal processor in the context of SR where injection of more noise into a given signal is an available option, one can continuously update the optimal processor according to the composite noise. Then, as shown by examples in Refs. [27–29], optimal processors acting on the output with added noise can emerge with an improved performance over that of the original optimal processor on the output without added noise.

In this context, it is then useful to seek to identify generic conditions under which it is *a priori* possible to decide whether or not addition of noise can be a favorable option for signal detection.

In this paper we focus on the detection of known weak signals in additive white noise in the context of SR. This detection problem can be viewed as a simple binary hypothesis testing. Under assumptions of a weak signal and a sufficiently large number of observation values, the performance of a locally optimum (LO) detector is demonstrated to be asymptotically optimum that its detection probability is maximized for a desired false alarm probability [30–32]. In order to evaluate the performance of the LO detector with respective to the Neyman-Pearson detector, the asymptotic relative efficiency of two detectors is introduced [30–32]. With regularity conditions [32], the asymptotic relative efficiency can be computed simply as a ratio of their efficacies [see Eq. (7)] of detection procedures based on the sequence of statistics [30–32]. For a given false alarm probability, the detection probability of the LO detector is a monotonically increasing function of its efficacy, which is simply given by the Fisher information (FI) of the noise probability density function (PDF) [31,32]. When independent noise is added to the signal, we update the exact LO detector for each added-noise condition. Then, it is theoretically proven, by using the FI convolution inequality [33,34], that no improvement in detection can be obtained compared to the initial condition with no added noise. However, beyond these restrictive conditions, the SR method can be an appropriate way of improving the detection performance of a detector [22–29]. Here we present a novel instance of detection of a known weak signal in uniform noise with favorable action of the noise through SR. In this case the FI of a uniform noise PDF is infinite, but the LO detector is physically unrealizable, since the output of the LO detector tends to infinity when the input is larger than unity [31]. It is shown that a realizable LO detector can be constructed by adding a type of noise with a continuous PDF. Furthermore, we observe that the detection performance of a fixed dead-zone limiter (DZL) detector can be infinitely enhanced by adding suitable dichotomous noise in order to better detect the known weak signal in uniform noise. This example shows a potential application of SR in signal detection in the case where a LO detector is physically unrealizable.

II. THE OBSERVATION MODEL AND FEASIBILITY OF SR IN SIGNAL DETECTION

Consider the observation vector $X = (X_1, X_2, \ldots, X_N)$ of real-valued components X_n by

$$X_n = \theta s_n + W_n, \quad n = 1, 2, \ldots, N, \tag{1}$$

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where the W_n form a sequence of independent and identically distributed (i.i.d.) random variables with PDF f_w, and the known signal components s_n are with the signal amplitude θ. Here the signal amplitude θ takes values of either $\theta_0 = 0$ (the observations contain no signal) or $\theta_1 > 0$ (the signal is present) [32]. For the known (periodic or aperiodic) signal sequence $\{s_n, n = 1, 2, \ldots, N\}$, it is assumed that there exists a finite (nonzero) bound U_s such that $0 \leqslant |s_n| \leqslant U_s$, and the asymptotic average signal power is finite and nonzero, i.e., $0 < P_s^2 = \lim_{N\to\infty} \sum_{n=1}^N s_n^2/N < \infty$ [32]. Then the detection problem can be formulated as a hypothesis-testing problem of deciding a null hypothesis H_0 ($\theta = \theta_0$) and an alternative hypothesis H_1 ($\theta = \theta_1$) describing the joint density function of X with

$$H_0: f_X(X,\theta_0) = \prod_{n=1}^N f_w(X_n) \text{ for } X_n = W_n,$$
$$H_1: f_X(X,\theta_1) = \prod_{n=1}^N f_w(X_n - \theta s_n) \text{ for } X_n = \theta s_n + W_n. \tag{2}$$

From the generalized Neyman-Pearson lemma and as $\theta_1 \to \theta_0$, the Taylor expansion of the log-likelihood ratio test statistic can be expressed as

$$\ln\left[\frac{f_X(X,\theta_1)}{f_X(X,\theta_0)}\right]\bigg|_{\theta_1\to\theta_0} = \ln\left[\frac{\prod_{n=1}^N f_w(X_n - \theta s_n)}{\prod_{n=1}^N f_w(X_n)}\right]\bigg|_{\theta_1\to\theta_0}$$
$$\approx \sum_{n=1}^N \left[-\frac{f_w'(X_n)}{f_w(X_n)}\right]\theta s_n, \tag{3}$$

with the derivative $df_w(x)/dx = f_w'(x)$ existing for almost all x [30–32]. Then, based on Eq. (3), the LO detector can be written as

$$T_{\mathrm{LO}}(X) = \sum_{n=1}^N \left[-\frac{df_w(X_n)/dX_n}{f_w(X_n)}\right]s_n = \sum_{n=1}^N g_{\mathrm{LO}}(X_n)s_n \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \tag{4}$$

with the decision threshold γ and the nonlinearity $g_{\mathrm{LO}}(x) = -f_w'(x)/f_w(x)$ [30–32].

Under the assumptions of a weak signal and sufficiently large observation data, the detection performance of a LO detector has an optimum. This is because, for a generalized correlation (GC) detector,

$$T_{\mathrm{GC}}(X) = \sum_{n=1}^N g(X_n)s_n \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \tag{5}$$

where the function g is an arbitrary memoryless nonlinearity. We assume the memoryless nonlinearity g has zero mean under f_w, i.e., $\int_{-\infty}^{\infty} g(x)f_w(x)\,dx = \mathrm{E}[g(x)] = 0$, which is not restrictive since any arbitrary g can always include a constant bias to cancel this average [32]. Noting the natural boundary conditions of f_w, the function g_{LO} accords with this assumption of $\mathrm{E}[g_{\mathrm{LO}}(x)] = 0$. In the asymptotic case of $\theta_1 \to \theta_0$ and $N \to \infty$, the test statistic T_{GC}, according to the central limit theorem, converges to a Gaussian distribution with mean $\mathrm{E}[T_{\mathrm{GC}}|H_0] = 0$ and variance $\mathrm{Var}[T_{\mathrm{GC}}|H_0] = NP_s^2\mathrm{E}[g^2(x)]$ under the null hypotheses H_0 [31,32]. Similarly, T_{GC} is asymptotically Gaussian with mean $\mathrm{E}[T_{\mathrm{GC}}|H_1] = \theta NP_s^2\mathrm{E}[g'(x)]$ and variance $\mathrm{Var}[T_{\mathrm{GC}}|H_0] = \mathrm{Var}[T_{\mathrm{GC}}|H_1]$ under the alternative hypothesis H_1 [31,32]. Here we also assume that the derivative $g'(x) = dg(x)/dx$ exists for almost all x.

Given a false alarm probability P_{FA}, the asymptotic detection probability P_D for the GC detector of Eq. (5), for a large sample size N, can be written as [31,32]

$$P_D = Q[Q^{-1}(P_{\mathrm{FA}}) - \sqrt{N}\theta P_s\sqrt{E_{\mathrm{GC}}}\,], \tag{6}$$

with $Q(x) = \int_x^{\infty} \exp[-t^2/2]/\sqrt{2\pi}\,dt$ and its inverse function $Q^{-1}(x)$ [31,32]. Thus, for fixed N and θP_s (since the signal is known), P_D is a monotonically increasing function of the efficacy E_{GC} given by [32]

$$E_{\mathrm{GC}} = \lim_{N\to\infty} \frac{\left\{\frac{d}{d\theta}\mathrm{E}[T_{\mathrm{GC}}(X)]|_{\theta=\theta_0}\right\}^2}{N\mathrm{Var}[T_{\mathrm{GC}}(X)]|_{\theta=\theta_0}} = \frac{\mathrm{E}^2[g'(x)]}{\mathrm{E}[g^2(x)]}$$
$$= \frac{\left\{\int_{-\infty}^{\infty} g(x)[-f_w'(x)/f_w(x)]f_w(x)\,dx\right\}^2}{\int_{-\infty}^{\infty} g^2(x)f_w(x)\,dx}$$
$$\leqslant \int_{-\infty}^{\infty}\left[\frac{f_w'(x)}{f_w(x)}\right]^2 f_w(x)\,dx = I(f_w), \tag{7}$$

with equality being achieved when $g(x) \equiv g_{\mathrm{LO}}(x) = -f_w'(x)/f_w(x)$, as indicated in Eq. (4). Here $I(f_w)$ is the FI of the PDF f_w for location shift [31,32]. This result indicates that the asymptotic optimum detector is the LO detector achieved by the test statistic $T_{\mathrm{LO}}(X) = \sum_{n=1}^N g_{\mathrm{LO}}(X_n)s_n$ [31,32].

Aiming to improve the performance of the LO detector in the context of SR, we add the i.i.d. random variables Y_n with PDF f_y to the given signal X. The updated components $\hat{X}_n = \theta s_n + W_n + Y_n = \theta s_n + Z_n$ and the composite random variable Z_n has a convolved PDF:

$$f_z(x) = \int_{-\infty}^{\infty} f_y(x-u)f_w(u)\,du. \tag{8}$$

Then, based on the deduction of Eq. (7), a new LO detector $\hat{g}_{\mathrm{LO}}$ can be designed according to f_z, and its efficacy $\hat{E}_{\mathrm{GC}} = I(f_z)$ is achieved when the nonlinearity $g(x) = \hat{g}_{\mathrm{LO}}(x) = -f_z'(x)/f_z(x)$.

Since Y_n and W_n are independent, it is known that the FI quantities $I(f_z)$, $I(f_y)$, and $I(f_w)$ satisfy the FI convolution inequality [33,34]

$$\frac{1}{I(f_z)} \geqslant \frac{1}{I(f_y)} + \frac{1}{I(f_w)}, \tag{9}$$

with equality in Eq. (9) occurring when Y_n and W_n are Gaussian distributed [33,34]. Since any $I(f) > 0$, we have

$$\frac{I(f_z)}{I(f_w)} \leqslant 1 - \frac{I(f_z)}{I(f_y)} \Rightarrow \hat{E}_{\mathrm{GC}} = I(f_z) \leqslant E_{\mathrm{GC}} = I(f_w), \tag{10}$$

which implies that the detection performance of the LO detector cannot be improved by adding independent noise to the signal in the sense of asymptotic optimality.

III. NOISE-ENHANCED DETECTION IN GC DETECTORS

From Eq. (3) to Eq. (10), it is seen that, with the asymptotic assumptions of $\theta_1 \to \theta_0$ and $N \to \infty$, the LO detector of Eq. (4) is optimal, since its efficacy E_{GC} in Eq. (7) is maximized as the FI of the noise distribution. It is noted that the memoryless nonlinearity of $g_{\rm LO}$ in Eq. (4) depends on the noise PDF f_w. When we add more noise to the observed data, the nonlinearity of $\hat{g}_{\rm LO}$ should be updated according to the composite noise PDF f_z. In this way the efficacy of the updated LO detector is determined by the FI $I(f_z)$. The FI convolution inequality in Eq. (10) tells us that the detection performance of the updated LO detector is inferior to that of the original LO detector. Therefore, aiming to improve the weak signal detection by a LO detector, the SR method of adding independent noise to a given signal is theoretically proven to be impossible in the considered conditions. Interestingly, under less restrictive conditions, noise-enhanced detection was observed in fixed LO detectors [23], suboptimal detectors [22,24], and the optimal detector with finite sample sizes or nonweak signals [29]. It is noted that these observed noise-enhanced detection phenomena occur outside the asymptotic case of weak signals for sufficiently large data.

We now consider another interesting example of GC detectors, which are not restricted to the conditions of Sec. II, because the LO detector of Eq. (4) is physically unrealizable in this considered example. Consider the generalized Gaussian random variables W_n with PDF

$$f_w(x) = \frac{c_1}{\sigma_w}\exp\left(-c_2\left|\frac{x}{\sigma_w}\right|^{\alpha}\right), \tag{11}$$

where $c_1 = \frac{\alpha}{2}\Gamma^{\frac{1}{2}}(\frac{3}{\alpha})/\Gamma^{\frac{3}{2}}(\frac{1}{\alpha})$, $c_2 = [\Gamma(\frac{3}{\alpha})/\Gamma(\frac{1}{\alpha})]^{\frac{\alpha}{2}}$ for a rate of exponential decay parameter $\alpha > 0$, and the noise root-mean-square (RMS) amplitude is σ_w [31,32]. The normalized LO detector indicated in Eq. (4) [31,32] has the nonlinearity

$$g_{\rm LO}(x) = |x|^{\alpha-1}\mathrm{sgn}(x). \tag{12}$$

The efficacy achieved by this normalized LO detector equals the FI of the PDF [31,32], viz.,

$$I(f_w) = \sigma_w^2\,\alpha^2\Gamma(3\alpha^{-1})\Gamma(2-\alpha^{-1})/\Gamma^2(\alpha^{-1}). \tag{13}$$

When the exponent $\alpha = \infty$, the W_n are i.i.d. uniform random variables, and f_w can be rewritten as

$$f_w(x) = 1/(2b), \tag{14}$$

for $-b \leqslant x \leqslant b$ $(b = \sqrt{3}\sigma_w > 0)$ and zero otherwise. It is noted that the FI of uniform noise PDF in Eq. (13) is $I(f_w) = \infty$ for $\alpha = \infty$ and $0 < \sigma_w < \infty$, but the nonlinearity of Eq. (12) is not realizable as $g_{\rm LO}(x) = \pm\infty$ for $|x| > 1$ in Eq. (12). This is because f_w is not absolutely continuous at $x = \pm b$, so that the regularity assumption is not satisfied [32]. Then we resort to the SR method by adding the i.i.d. random variables Y_n with an absolutely continuous PDF f_y to the signal. Then the PDF f_z of the composite random variables $Z_n = Y_n + W_n$ is

$$f_z(x) = \int_{-b}^{b} f_y(x-u)\frac{1}{2b}\,du = \frac{F_y(x+b) - F_y(x-b)}{2b}, \tag{15}$$

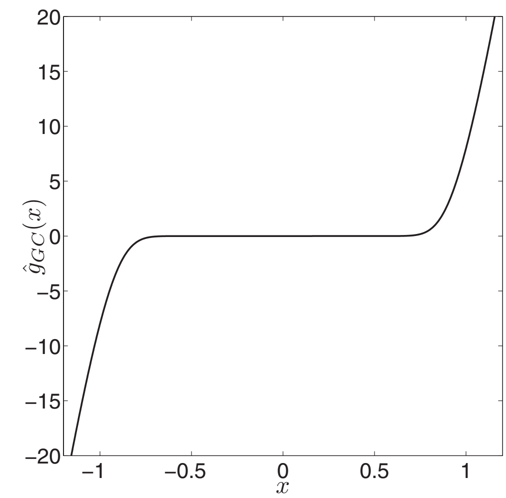

FIG. 1. Nonlinearities of the redesigned nonlinearity $\hat{g}_{\rm LO}(x)$ of Eq. (18) for parameters $\sigma_y = 0.1$ and $b = 1$.

with the cumulative distribution function (CDF) $F_y(x) = \int_{-\infty}^{x} f_y(u)\,du$. Thus, a new realizable LO detector can be constructed with the nonlinearity

$$\hat{g}_{\rm LO}(x) = -\frac{f_z'(x)}{f_z(x)} = -\frac{f_y(x+b) - f_y(x-b)}{F_y(x+b) - F_y(x-b)}. \tag{16}$$

For example, assume Gaussian random variables Y_n with PDF $f_y(x) = \frac{1}{\sqrt{2\pi\sigma_y^2}}\exp[-\frac{x^2}{2\sigma_y^2}]$, and the composite PDF f_z becomes

$$f_z(x) = \frac{Q\left(\frac{x-b}{\sigma_y}\right) - Q\left(\frac{x+b}{\sigma_y}\right)}{2b}. \tag{17}$$

The nonlinearity $\hat{g}_{\rm LO}$ can be expressed as

$$\hat{g}_{\rm LO}(x) = \frac{\exp\left[-\frac{(x-b)^2}{2\sigma_y^2}\right] - \exp\left[-\frac{(x+b)^2}{2\sigma_y^2}\right]}{\sqrt{2\pi}\sigma_y\left[Q\left(\frac{x-b}{\sigma_y}\right) - Q\left(\frac{x+b}{\sigma_y}\right)\right]}, \tag{18}$$

which is illustratively plotted in Fig. 1 for parameters $\sigma_y = 0.1$ and $b = 1$. From Eqs. (7) and (17), the corresponding efficacy $\hat{E}_{\rm GC} = I(f_z) = 3.101$. Thus, it is seen that the addition of extra noise to the given signal can elicit a realizable LO detector but yields a degraded $\hat{E}_{\rm GC} = 3.101$ compared with the original one $E_{\rm GC} = I(f_w) = \infty$ in accordance with Eq. (13).

Can we find an effective and simple detector that has the infinite asymptotic efficacy $E_{\rm GC} = \infty$ by adding further noise to a weak signal in uniform noise? This idea is feasible. Let us consider a fixed GC detector with its nonlinearity $g_{\rm DZL}$ given by [31,32]

$$g_{\rm DZL}(x) = \begin{cases} -1 & \text{for} \quad x < -c, \\ 0 & \text{for} \quad -c \leqslant x \leqslant c, \\ +1 & \text{for} \quad x > c, \end{cases} \tag{19}$$

with response thresholds at $x = \pm c$, which is specifically called the DZL detector. From Eq. (7), the efficacy $E_{\rm GC}$ of the DZL detector is [32]

$$E_{\rm GC} = \frac{\mathrm{E}^2[g'(x)]}{\mathrm{E}[g^2(x)]} = \frac{2f_w^2(c)}{1 - F_w(c)}, \tag{20}$$

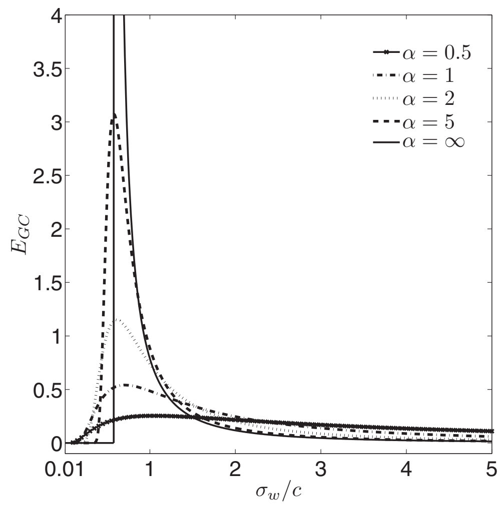

FIG. 2. The efficacy E_{GC} in Eq. (21) of the DZL detector as a function of noise RMS amplitude σ_w/c for different exponents $\alpha = 0.5$, 1, 2, 5, and ∞ in Eq. (11).

where F_w represents the CDF of W_n. Here we focus on the generalized Gaussian noise components W_n with the PDF of Eq. (11).

In order to investigate the role of noise, we rewrite Eq. (20) as

$$E_{\mathrm{GC}} = \frac{1}{c^2}\frac{c^2}{\sigma_w^2}\frac{2\sigma_w^2 f_w^2(c)}{1 - F_w(c)} = \frac{1}{c^2}\left(\frac{c}{\sigma_w}\right)^2 \frac{2 f_{w_0}^2(c/\sigma_w)}{1 - F_{w_0}(c/\sigma_w)}, \quad (21)$$

where F_{w_0} is the CDF of the standardized noise PDF $f_{w_0}(x) = f_w(x/\sigma_w)/\sigma_w$ with unity variance $\sigma_{w_0}^2 = 1$ [21]. Thus, the variation of noise RMS amplitude σ_w, as well as the response threshold c, can improve the efficacy E_{GC} of the DZL detector, as shown in Fig. 2. For a fixed-response threshold c ($c = 1$ without loss of generality), Fig. 2 shows E_{GC} of the DZL detector as a nonmonotonic function of the RMS amplitude σ_w/c for different exponents $\alpha = 0.5$, 1, 2, 5, and ∞ in Eq. (11). It is clearly seen that the SR effect occurs as σ_w/c increases. Of course, E_{GC} is never larger than the FI $I(f_w)$ of Eq. (13), because the DZL detector is not the LO detector of Eq. (12) for generalized Gaussian noise. However, as the exponent $\alpha = \infty$, E_{GC} reaches the FI $I(f_w) = \infty$ of the uniform noise at $\sigma_w = c/\sqrt{3}$ (as c is fixed). This is because, for the standardized uniform noise PDF $f_{w_0}(x) = 1/(2\sqrt{3})$ ($-\sqrt{3} \leqslant x \leqslant \sqrt{3}$) with unity variance $\sigma_{w_0}^2 = 1$, $f_{w_0}(\sqrt{3}) \neq 0$ but $F_{w_0}(\sqrt{3}) = 1$ in Eq. (21). Note that this infinite efficacy E_{GC} can be also achieved by tuning the response threshold into $c = \sqrt{3}\sigma_w$ of the DZL detector (for the fixed σ_w).

When $E_{\mathrm{GC}} = I(f_w) = \infty$ for detecting weak signals in uniform noise, the detection probability P_D in Eq. (6) is approximately unity. This is because, for a fixed false alarm probability P_{FA} and the known weak signal, the decision threshold in Eq. (5) $\gamma = \sqrt{\mathrm{Var}[T_{\mathrm{GC}}|H_0]}\,Q^{-1}(P_{\mathrm{FA}}) \approx \sqrt{N P_s^2 \mathrm{E}[g_{\mathrm{DZL}}^2(x)]}\,Q^{-1}(P_{\mathrm{FA}})$ is of order $\sim\sqrt{N}$ as N increases. Since the uniform noise RMS $\sigma_w = c/\sqrt{3}$, the maximum and the minimum values of W_n are $\pm b = \pm\sqrt{3}\sigma_w = \pm c$. Then, if the weak signal components $s_n > 0$, the probability of the mixture $X_n = \theta s_n + W_n > c$ is $\theta|s_n|/(2c)$, and the same for $s_n < 0$ [with the probability of the mixture $X_n = \theta s_n + W_n < -c$ being $\theta|s_n|/(2c)$]. Computing by the nonlinearity g_{DZL} in Eq. (19), the output of detector is $\sum_{n=1}^N g_{\mathrm{DZL}}(X_n)s_n \approx \sum_{n=1}^N \theta|s_n|^2/(2c) = N\theta P_s^2/(2c)$ of order $\sim N$. Thus, as the observed data number $N \to \infty$, Eq. (5) will certainly take $T_{\mathrm{GC}}(\mathrm{X}) = \sum_{n=1}^N g_{\mathrm{DZL}}(X_n)s_n > \gamma$ for deciding the hypothesis H_1 ($\theta = \theta_1 > 0$), i.e., $P_D \to 1$.

Now we reconsider the mentioned question whether adding independent noise to the given signal can be helpful or not for detection. For the Gaussian noise ($\alpha = 2$) shown in Fig. 2, if the original Gaussian noise RMS $\sigma_w < \sigma_w^* = 0.6098$, which corresponds to the maximum of $E_{\mathrm{GC}}^* = 1.1512$, we can add independent Gaussian random variables Y_n with its RMS amplitude $\sigma_y = \sqrt{\sigma_w^{*2} - \sigma_w^2}$ to increase E_{GC} to $E_{\mathrm{GC}}^* = 1.1512$, because the sum of two Gaussian noise are still Gaussian distributed. This point has been noted in Ref. [25]. However, this approach cannot be used for the uniform noise with the exponent $\alpha = \infty$. If the original uniform noise has the noise RMS amplitude $\sigma_w < c/\sqrt{3}$ ($b = \sqrt{3}\sigma_w < c$), we cannot add more uniform noise to the data to obtain the infinite E_{GC}, since the sum of two uniform random variables is not uniformly distributed. In this case we consider the dichotomous noise components Y_n with the PDF

$$f_y(x) = [\delta(x - \sigma_y) + \delta(x + \sigma_y)]/2, \quad (22)$$

with its RMS amplitude $\sigma_y = c - b$ and $\delta(\cdot)$ is Dirac delta function. In this way $Z_n = W_n + Y_n$, as the sum of uniform and dichotomous random variables has its PDF $f_z(x) = [f_w(x - \sigma_y) + f_w(x + \sigma_y)]/2$ with a maximum bound of $+c$ and a minimum bound $-c$. Since $s(t)$ is corrupted by Z_n, the asymptotic efficacy of Eq. (20) becomes $\hat{E}_{\mathrm{GC}} = 2f_z^2(c)/[1 - F_z(c)] = \infty$, because $f_z(c) = f_w(c - \sigma_y)/2 = f_w(b)/2 \neq 0$ and $F_z(c) = 1$.

IV. CONCLUSION

In this paper we studied the constructive role of noise in detecting known signals in additive white noise. Under the assumptions of weak signals and a sufficiently large number of observation values, the LO detector is asymptotic optimum and its efficacy, i.e., the FI of the noise PDF, is maximal. When the LO detector can be redesigned (optimized) for the new composite noise, the SR method of adding independent noise to a given data set for improving the performance of a LO detector is proved to be impossible using the FI convolution inequality. However, beyond these restrictive conditions, we demonstrated that the SR method can be an appropriate way of improving detection performance. A novel example is shown that, for detecting a weak known signal in uniform noise, the SR method of adding noise can elicit a realizable LO detector. Furthermore, we found that the detection performance of a fixed DZL detector can be infinitely enhanced by adding suitable dichotomous noise to the initial uniform noise in certain cases.

Some interesting open questions arise. Here we consider only the LO detection of known weak signals in additive white noise. It is found that SR becomes an alternative method of improving signal detection in certain cases. Beside the DZL detector, it is also interesting to further explore the possibility of noise-enhanced phenomenon in other noninvertible nonlinearities for signal detection [21]. In practical detection problems, the parameters of signals or background noise are often not known, the signal might be a random variable, or the noise is multiplicative. In these configurations the same question arises regarding the conditions under which the addition of noise to given data is favorable for weak signal detection. We argue that the constructive role of noise in these practical detection problems will be of interest for further studies of signal detection.

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