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NEW TYPES OF KNOWLEDGE ABOUT  
SYSTEM DYNAMICS FOR INTELLIGENT  
CONTROL SYSTEM DESIGN

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**BY**

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# Abstract

This is a thesis by publication. This thesis comprises ten published/submitted journal articles including eight research articles and two review articles. Five of these journal papers have been already published or finally accepted for publication. This thesis, based on research undertaken in the area of intelligent and non-model-based control, aims at broadening knowledge about system dynamics applicable for control system design. Currently, mathematical models of systems, experimental input-output data and experts' knowledge in the form of fuzzy rules (linguistic expressions) are three types of knowledge about systems dynamics which are employed in control system design. These types of knowledge are used to define the number and the positions of controllers in the control system (architecture of control systems) and/or the mathematical form of controllers (controllers' structure) and/or controllers' parameters. Defining control systems at these three levels (architecture, controllers' structure and parameters) forms the process of control design.

In the area of non-model-based control, some cases of unexpected poor control performance were observed by the author. For instance, neuro-predictive method controls process plants very well. This method is based on input-output data. Thus, this control technique seems to promise a good control performance in general. However, attempts to use this technique in yaw angle control of a model helicopter were unsuccessful regardless of the effort spent on tuning the controller parameters (see Chapter 3); similarly, unsuccessful outcomes resulted for feedback fuzzy control of model helicopter pitch angle (see Chapter 4). This thesis has two main contributions: firstly, this thesis provides explanation for the aforementioned unexpected poor performances through introduction of two new concepts:

Control Inertia (see Chapter 3) and Generalized-Type-Zero (GTZ) Systems (see Chapter 7). It is shown here that high control inertia systems witness a considerable repeating overshoot, and GTZ systems need consistent control input to maintain their desirable control output. Secondly, based on these two new concepts, this thesis offers new control methods with developing new control ideas: fuzzy brakes and steady state control laws which can improve the performance, energy consumption and suitability for implementation of control systems. The proposed methods are shown to be usable for a wide range of systems. The merits of the proposed control approaches were indicated theoretically and practically. As a result, being/not being high control inertia and being/not being GTZ were used as new types of knowledge about system dynamics applicable in control system design.

# Statement of originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution. To the best of my knowledge and belief it contains no material previously published or written by another person, except where due reference has been made in the text.

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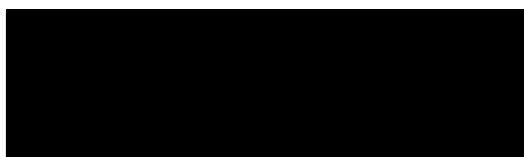


# **Declaration regarding receipt of editorial advice**

I, Morteza Mohammadzaheri, declare that I have received editorial advice in the writing of this thesis. This advice was restricted to the suitability for purpose of the language and illustrations in the publication and to completeness and consistency of English expression in this publication. This nature of this advice was in accordance with Sections D and E of the Australian Standards for Editing Practice. The editor's current or former area of academic specialisation is unrelated to my own.

Morteza Mohammadzaheri

2/09/2010



# Statement of authorship

A Critical Review on Fuzzy Control  
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Intelligent Predictive Control of Model Helicopter's Yaw Angle  
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Design and Stability Discussion of a Hybrid Intelligent Controller for an  
Unordinary System  
Asian Journal of Control, Volume 11, No. 5, Pages 476-488, September 2009.

Double Command Fuzzy Control of a Nonlinear CSTR  
Korean Journal of Chemical Engineering, Volume 27, Number 1, Pages 19-31,  
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Text in manuscript.

Double Command Feedforward-Feedback Control of a Nonlinear Plant  
Korean Journal of Chemical Engineering, Volume 27, No. 5, Pages 1-13,  
September 2010.

Model-Free Double Command Hybrid Control of a Non-thermic CSTR  
Text in manuscript.

Hybrid Intelligent Control of an Infrared Dryer  
Submitted to Control Engineering Practice, 2 September 2010.

**Mr. Morteza Mohammadzaheri (Candidate)**

*Statement of contribution (in terms of the conceptualization of the work, its realization and its documentation)*

Proposed ideas, performed analysis, interpreted data, wrote manuscript and acted as corresponding author

*Certification that the statement of contribution is accurate*

Signed : .....Date **21 FEB 2011**

**Dr. Lei Chen (Supervisor)**

*Statement of contribution (in terms of the conceptualization of the work, its realization and its documentation)*

Supervised development of work, helped in manuscript evaluation

*Certification that the statement of contribution is accurate and permission is given for the inclusion of the paper in the thesis*

Signed ... .....Date **2/02/2011**



# Thesis by Publication

This thesis comprises a portfolio of publications that have been published and/or submitted for publication and/or text in manuscripts in accordance with the ‘Academic Program Rules 2008’ approved by the Research, Education and Development Committee of the University of Adelaide. Five of the research journal articles, presented here, have already been published or finally accepted for publication. All journals to which the articles have been submitted are indexed in the lists of internationally and nationally most reputable research databases: ISI, JSR and ERA.

This thesis is based on the following publications (in the order of appearance in the thesis):

1. Morteza Mohammadzahari and Lei Chen, “A Critical Review on Fuzzy Control”, text in manuscript.
2. Morteza Mohammadzahari and Lei Chen, “A Critical Review on Neuro Control”, submitted.
3. Morteza Mohammadzahari and Lei Chen, “Intelligent Predictive Control of Model Helicopter’s Yaw Angle” Asian Journal of Control, Volume 12, No. 6, Pages 1-13, November 2010.
4. Morteza Mohammadzahari and Lei Chen, “Design and Stability Discussion of a Hybrid Intelligent Controller for an Unordinary System”, Asian Journal of Control, Volume 11, No. 5, Pages 476-488, September 2009.
5. Morteza Mohammadzahari and Lei Chen, “Double Command Fuzzy Control of a Nonlinear CSTR”, Korean Journal of Chemical Engineering, Volume 27, Number 1, Pages 19-31, January 2010.
6. Morteza Mohammadzahari and Lei Chen, “Intelligent Control of a Nonlinear Tank Reactor”, Asian Journal of Control (in press).
7. Morteza Mohammadzahari and Lei Chen, “A Design Approach for Feedback-feedforward Control Systems”, text in manuscript.
8. Morteza Mohammadzahari and Lei Chen, “Double Command Feedforward-Feedback Control of a Nonlinear Plant” Korean Journal of Chemical Engineering, Volume 27, No. 5, Pages 1-13, September 2010.
9. Morteza Mohammadzahari and Lei Chen, “Model-Free Double Command Hybrid Control of a Non-thermic CSTR”, text in manuscript.
10. Morteza Mohammadzahari and Lei Chen, “Hybrid Intelligent Control of an Infrared Dryer”, submitted.

In addition, several conference papers have been presented from the research reported in this thesis. Two of them are included in this thesis as appendices:

1. Morteza Mohammadzahari and Lei Chen, “Intelligent Modeling of MIMO Nonlinear Dynamic Process Plants for Predictive Control Purposes”, 17th IFAC World Conference, 6-11 July 2008.
2. Morteza Mohammadzahari and Lei Chen, “Anti-overshoot Control of Model Helicopter’s Yaw Angle with Combination of Fuzzy Controller and Fuzzy Brake”,

International Conference on Intelligent and Advanced Systems (IEEE sponsored),  
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# Introduction

## I. SYSTEM DYNAMICS AND CONTROL

A control system consists of architecture, structure of controller(s) and parameters of controller(s)<sup>1</sup>. The architecture defines the number and the position of controllers in the control system, for example feedforward or cascade. Controller(s) structure is the mathematical form of controllers, for example fuzzy or proportional-integral. The controller(s) structure includes parameters. These three levels of a control system (architecture, structure and parameters) may be defined through a design process. The design process of a control system starts from system dynamics. For design purposes, in practice, system dynamics includes all knowledge about a system available for the designer and applicable for control system design. Since the early era of automatic control, input-output data and mathematical models are two types of knowledge about a system which partially represent system dynamics in control system design [1, 2]; for example, linear transfer functions (as models) and step response of systems (as input-output data) were used to design PID controllers [3, 4].

The use of mathematical models was widened with emerging new forms of analysable mathematical models such as state space models [5, 6]. Based on linear forms of mathematical models, internal model control[7], state vector feedback control (including linear quadratic regulators [8]) and model predictive control [9] were developed. Companion nonlinear mathematical models led to the feedback linearization method [10]. Even for robust controllers designed based on  $H_\infty$ [11] and sliding mode [10] techniques, imprecise or

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<sup>1</sup> In linear control, the role and the mathematical structure/parameters of controllers may change without changing the control system; for example, a feedforward controller may be replaced by a feedback controller with changing the structure and/or parameters of controllers.

regionally valid mathematical models are needed as the main type of knowledge about system dynamics. Mathematical models are still the dominant type of knowledge about systems usable in control.

Similar to mathematical models, the application of input-output data in control system design increased with the progress of automatic control theory. In the frequency domain, input-output data were used in control system design with the critical assumption that the system is linear [12]. Design of adaptive controllers was another application of input-output data [13]. In adaptive control, input-output data went further than the response of the system to impact, unit step or sinusoidal inputs. However, the prominent progress in using input-output data happened after the application of artificial neural networks in control [14] (see Chapter 2). In neuro controllers, the main type of knowledge used in control system design is input-output data.

In the 1970's, fuzzy control appeared [15], and it was established in the 1980's and the 1990's [16]. In many types of fuzzy control, knowledge of an expert in the form of linguistic expressions is used in control system design [17]. The aforementioned linguistic expressions are stated as fuzzy rules. After being used in control, fuzzy rules became the third type of knowledge about system dynamics after mathematical models and input-output data.

Now control designers choose and design control systems based on their available knowledge of systems dynamics in the form of mathematical models, input-output data and fuzzy rules. The research presented in this thesis was aimed to broaden this knowledge by introducing new types of knowledge about system dynamics and addressing their applicability in control system design. This project was undertaken in the area of intelligent and non-model-based control. Any model based design reported in this thesis (e.g. Chapters 6 and 8) was eventually evolved to a non-model-based design in the course of this research.

## II. INTRODUCTION TO THESIS STRUCTURE

It is a thesis by publication; in total, two review and eight research articles are included in this thesis. The thesis is formed by an introduction, the body and three appendices. The body of the thesis is comprised of four parts:

- 1) Literature Review (Chapters 1 and 2)
- 2) Control Inertia and Fuzzy Brakes (Chapter 3)
- 3) Hybrid ‘Fuzzy-Steady State’ Control Systems (Chapters 4,5 and 6)
- 4) A General Feedforward-feedback Approach for Process Control (Chapters 7,8,9 and 10)

Each part commences with a synopsis, explaining the main idea of that part, followed by some (one to four) chapters. Except for Chapters 1 and 2, each chapter is a journal article; Chapters 1 and 2 include a review journal article and a complementary manuscript.

**Table 1. Contents of the body of the thesis (ten chapters, C1~C10)**

	Title Ch	apters		Publication Status
<b>Part1</b>	Literature Review	C1	A Critical Review on Fuzzy Control	Text in Manuscript
		C2	A Critical Review on Neuro Control	Submitted
<b>Part2</b>	Control Inertia and Fuzzy Brakes	C3	Intelligent Predictive Control of Model Helicopter’s Yaw Angle	Published
<b>Part3</b>	Hybrid ‘Fuzzy-Steady State’ Control Systems	C4	Design and Stability Discussion of a Hybrid Intelligent Controller for an Unordinary System	Published
		C5	Double Command Fuzzy Control of a Nonlinear CSTR	Published
		C6	Intelligent Control of a Nonlinear Tank Reactor	In press
<b>Part4</b>	A General Feedforward-feedback Approach for Process Control	C7	A Design Approach for Feedback-feedforward Control Systems	Text in Manuscript
		C8	Double Command Feedforward-Feedback Control of a Nonlinear Plant	Published
		C9	Model-Free Double Command Hybrid Control of a Non-thermic CSTR	Text in Manuscript
		C10	Hybrid Intelligent Control of an Infrared Dryer	Submitted

Two conference papers cited in body chapters are used as appendices. The last appendix is an explanation of the experimental setup used in the research reported in Chapter 10 followed by a brief conclusion to the thesis and appendices.

In this research, three different case studies have been used to test the proposed control ideas: two in simulation and one in experiment.

Part 1 of the thesis contains two chapters, including two review articles on fuzzy control and neuro control, which are two principal techniques used in this research. Each chapter also has a complementary manuscript which indicates how the critical review (review article) is linked to the research undertaken in this project.

Part 2 has one chapter which includes a published journal article. In this article, initially, the results of neuro-predictive control of the yaw angle of a model helicopter were mentioned which were unexpectedly inadequate. These results were observed despite many attempts in tuning of controller parameters. Neuro-predictive (NP) is a non-model-based control method which has controlled many systems appropriately and works merely based on input-output data of the system. However, the same methodology leads to poor control performance in other systems. It was concluded that a characteristic of systems must exist that differentiates the model helicopter and systems which are controlled well by the NP method. A characteristic was sought that can be recognized/estimated in the absence of mathematical models. This characteristic can be used as a new type of knowledge about the system. In the end, Control Inertia was introduced in this article which is a characteristic of systems and can be diagnosed through overshoot (in closed loop). According to this article, the model helicopter (with a NP controller) was a high control inertia system, and its inadequate performance was explained based on this newly introduced concept. Afterwards, a special algorithm, namely 'fuzzy brake', was suggested to compensate for the undesirable effect of

having high control inertia. The application of this algorithm drastically improved the performance of yaw angle control of the model helicopter with the NP controller, and thus the merit of the proposed algorithm was indicated. Furthermore, it was proved that if a closed loop system is stable, adding a fuzzy brake does not make it unstable. As a result of this part, being/not being high control inertia can be used as a type of knowledge about system dynamics, and an appropriate technique to use this knowledge is also available.

The ideas of ‘control equilibrium point’ and ‘steady state control’ are presented in Part 3 of the thesis. These ideas lead to a feedforward control law; artificial neural networks were employed to play the role of this control law in the absence of precise mathematical models. In this part, a feedback-feedforward control methodology is developed including the aforementioned feedforward control law and an error based fuzzy controller. The case studies are pitch angle control of a model helicopter and single command and double command concentration control of a Catalytic Stirred Tank Reactor (CSTR). In the three chapters of Part3, the merit of the proposed control methodology is indicated. Each chapter includes a published or in press journal paper. This part also involves detailed discussions on fuzzy control design, and the stability analysis of the proposed control systems. In the first chapter, the stability is proved by a relatively ad-hoc approach. However, in next the two chapters, a number of practical assumptions were used in stability discussions; this approach suggests new ways in conducting stability analysis in fuzzy control. In this part, in Chapters 4 and 6, BIBO stability of closed loop systems is proved, and in Chapter 5, asymptotic Lyapunov stability of the control system is proved.

In Part4, generalized zero type (GZT) systems are introduced. These are systems which need a persistent control input to maintain their desired situation. This definition is closely related to the definition of steady state control command offered in Part 3. If a system is GTZ, it is concluded that a steady state control command is applicable; therefore, being or



not being GTZ is a form of knowledge about the system usable in control system design.

This part contains four chapters, each in the form an article, one of which is accepted for journal publication. In terms of control methodology, in this part, for a class of systems, a feedforward-feedback control system is introduced which includes the steady state control law defined in Part 3 as the feedforward control law, and a proportional controller with an arbitrary high gain as the feedback control law. The stability of the proposed control system is proved. Chapters 7 and 8 in this part introduce this control system design approach and offer control systems with guaranteed stability and very quick convergence towards the reference. For a class of systems, trade-off between the performance and the stability seems to be solved using this design method. However, two common undesirable situations may interrupt the stability proof offered in the Chapters 7 and 8: variables coupled with the control output(s) and time-delay. For instance, in the concentration control of the CSTR, if the concentration is aimed to be controlled only, the height of liquid level (a variable coupled with the control output) may converge to zero if it is not considered; that is, the reactor may be evacuated. This problem is addressed in Chapter 9: variables coupled with control output(s) are taken into account in the design of feedforward neural network control law. In the proposed general approach in Chapters 7 and 8, the stability proof is based on the availability of the immediate error or the error which is affected by the most recent control input. In the presence of time-delay or dead-time, such an error is not available. Chapter 10 addresses this issue through a predictive approach. The two last chapters generalize the control methodology proposed in Chapters 7 and 8 and make it ready for real-time application as this control idea is successfully tested in experiment in Chapter 10.

This thesis contains three appendices. Appendices 1 and 2 are conference papers produced during this PhD research and cited in journal articles that form this thesis. These conference papers are not a part of the thesis story; however, their content has been mentioned in thesis

chapters, so they have been presented as Appendices for probable reference. Appendix 3 is a brief explanation of the experimental set up used in the research reported in Chapter 10.

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## **Part 1**

# **Literature Review**

### **SYNOPSIS**

Part 1 includes two review articles on fuzzy control and neuro control, two principal areas of intelligent control used in the research reported in this thesis. These papers offer an inclusive explanation of major techniques in fuzzy/neural network control in a chronological order.

In Chapter 1, a review article on research into fuzzy control shows that the mathematical design and stability analysis have been drastically advanced recently, in return, of human expert knowledge in fuzzy control system design has been diminished. Following this, a complementary manuscript explains how the aforementioned trend of fuzzy control has been addressed in this thesis with mathematical analysis of fuzzy inference systems using practical assumptions, particularly in Part 3. In the proposed method, both mathematical analysis and heuristic design based on experts' knowledge are effectively employed.

Chapter 2 presents a chronological review of artificial neural network (neuro) control in the format of a review article. It shows that initial neural network controllers could be easily used together with other controllers easily; and then gradually, in the 1990's, neuro controllers started to play the role of the sole controller of control systems. However, the wide usage of neuro controllers in conjunction with conventional controllers has now been returned. Also, it shows that some neuro controllers may suit some applications and may not suit others; there is no established way to explain this phenomenon. A complementary

manuscript of this chapter notes that the use of neuro controllers in this project follows the recent trend of these controllers (being used together with other controllers) . It also notes that the concept of control inertia, introduced in Part 2, can explain the unexpected discrepancy between the performances of a neuro control method in different systems.

## **Chapter 1**

### **A Critical Review on Fuzzy Control**

Text in manuscript.

# A Critical Review on Fuzzy Control

Morteza Mohammadzaheri, Lei Chen

**Abstract**-In this paper, main types of fuzzy control systems and their key features are reviewed chronologically. The addressed fuzzy control systems include conventional or heuristic fuzzy control (introduced in 1973), self-tuning adaptive fuzzy control (introduced in 1989), fuzzy PID control (introduced in 1990), neuro-fuzzy control (introduced in 1991), sliding mode fuzzy control (introduced in 1993), model reference adaptive fuzzy control (introduced in 1994) and Sugeno model based fuzzy control (introduced in 1996). All these types of fuzzy control are still used and investigated. In the literature on fuzzy control, a general trend was observed: the improvement of the capacity of systematic design and stability analysis and the decrease in the role of heuristic rules based on human expertise.

## I. FUZZY LOGIC

Fuzzy logic, established by Zadeh in 1965[1], is a bridge between human knowledge in the form of linguistic terms, from one side, and mathematics and computer science, from another side. In other words, we can analyse our knowledge in the form of linguistic expressions using mathematical/computer tools in order to make decisions in a wide variety of applications. In fuzzy logic, fuzzy or linguistic values (i.e. cold and fast) are designated to variables (i.e. temperature and velocity); these fuzzy values are defined using membership functions. A ‘membership function’ is a function which receives the crisp (numeric) value of a variable (i.e. 25°C) and returns another number in the range of [0,1] namely ‘membership grade’. This membership grade determines that how much the numeric value (i.e. 25°C) accords to the membership function (i.e. cold). For instance, the membership grade of 25°C for fuzzy value of cold is 0.1 (according to an arbitrary definition of cold). Membership functions are defined by the designer or derived through data processing.

Conditional sentences formed by fuzzy values are called ‘fuzzy rules’, like

**if *fuel flow is high* then *temperature is high* [can be used in a fuzzy model]**

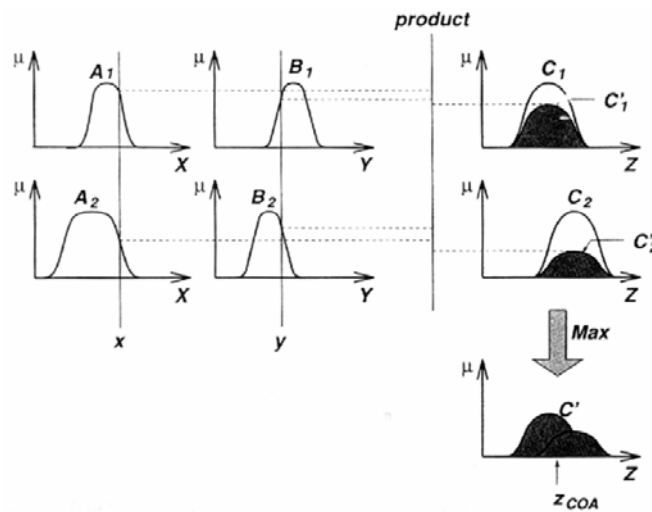
Or **if *temperature is high* then *fuel flow is low* [can be used in a fuzzy controller]**

A set of fuzzy rules is named a ‘fuzzy inference system’(FIS) which receives numeric input values and outputs numeric values too.

Fuzzy inference systems have two main types: Mamdani-type where both antecedents and consequents of rules have fuzzy values ,and Sugeno-type where the consequent part of rules are crisp functions [2].

***I-A. Mamdani-type fuzzy inference system***

In a Mamdani-type FIS, both antecedents and consequents are of linguistic (fuzzy) values; therefore, defuzzification is done as the final stage of inference to generate a numeric output for the fuzzy inference system. Figure 1 shows a scheme of a typical Mamdani-type fuzzy inference system.  $A_i, B_i$  and  $C_i$  are fuzzy values for the first and the second input and the output, respectively. This type of fuzzy inference system has been found different industrial applications [3-7] .



**Figure 1. A scheme of a Mamdani-type fuzzy model [2]**



### I-B. Sugeno-type fuzzy inference systems

A scheme of a Sugeno-type system is shown in Fig.2. Similar to Mamdani-type fuzzy inference systems, in Sugeno-type FISs, the antecedents of rules are formed by linguistic (fuzzy) values. Sugeno-type systems differ from Mamdani-type fuzzy inference systems in that the consequents of rules are crisp (numeric) functions of input numeric values and independent of antecedent membership grades; so, defuzzification is not needed. Sugeno-type fuzzy controllers are applied in a wide range of applications [8-12].

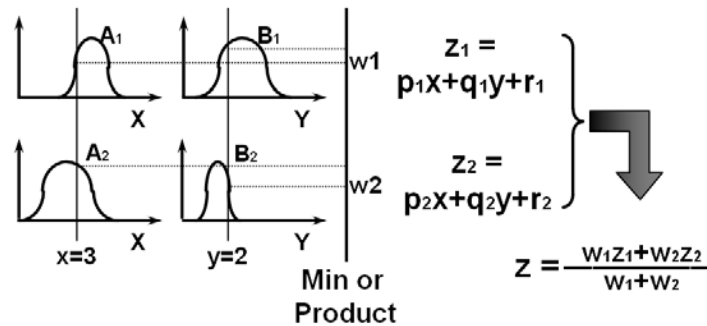


Figure 2. A scheme of a Sugeno-type FIS [2]

## II. FUZZY LOGIC CONTROL

Control system design, in general, is the process of making a path from system dynamics to a control system. A control system consists of an architecture (i.e. feedforward or cascade), controller(s) structure (mathematical form) and controller(s) parameters. In other words, control system design process starts from system dynamics. System dynamics includes all the knowledge about the system available for the designer and applicable for control system design. In model-based control techniques like LQR, feed-back linearization and model-predictive control, a mathematical model usually represents the system dynamics. It can be very difficult to find a suitable model for a system and in this case model-free controllers are employed. With these controllers, the system dynamics applicable to control system design includes other types of knowledge about the system rather than the mathematical model.

Fuzzy logic control was originally introduced and developed as a model-free control system design approach [13]. In 1973, Professor Mamdani, at the University of London, used fuzzy logic to control and stabilize the speed of a small steam engine [14, 15]. Since then, fuzzy logic controllers have found many successful applications in different areas of industry.

Since the era of the pioneers of fuzzy control, the majority of fuzzy logic controllers are still designed heuristically, based on the knowledge achieved by human observation/expertise. As a consequence, membership functions and rule structures are more or less subjective [16]. From the mid 1990's, in academic communities, the lack a systematic stability analysis and control design approach for fuzzy control systems was felt [13]. In order to deal with this issue, attempt were made to benefit stability analysis and design techniques which are popular in classical and model-based controller design in fuzzy control. Then, new generations of fuzzy controllers gradually emerged with a variety of advantages and disadvantages. In this paper, this path is briefly explained. Throughout this paper the following crucial issues are addressed for any fuzzy control technique:

- 1) What type of knowledge about the system is used in fuzzy control system design? Is a mathematical model of the system needed? Is human expert/observer knowledge used effectively in the design?
- 2) Is there any reliable approach for stability analysis?
- 3) Is there any systematic approach in control system design?

### ***II-A. Conventional Fuzzy Control***

As introduced by Mamdani and Assilian [14, 15] in 1974~75, conventional fuzzy controllers are a set of heuristic fuzzy rules based on human knowledge. In 1976 Kickert and Limke [17] and in 1977, Ostergaard [18] published their works in fuzzy control of a warm water

plant and a heat exchanger. This type of fuzzy controller found many applications and is still extensively used and investigated [19].

Conventional fuzzy controllers are essentially heuristic and model-free; even though their performance may be satisfactory, in the absence of mathematical models, the study of closed loop system stability is known to be a difficult problem, and a reliable general approach of stability analysis, without using a mathematical model was not found in the literature. Furthermore; the design approach of conventional fuzzy controllers is heuristic to a large extent, and a general systematic design approach is not available. Now answers to the crucial questions:

- 1) What type of knowledge about the system is used in fuzzy control system design? Is mathematical model of the system needed? Is human expert/observer knowledge used effectively in the design?

Conventional fuzzy controllers benefit human observation/expertise knowledge effectively and are independent of the mathematical model of the system.

- 2) Is there any reliable approach for stability analysis?

No, there is not.

- 3) Is there any systematic approach in control system design?

No, there is not. Some general advice, such as using look-up tables, is available.

### ***II-B. Self-tuning Adaptive Fuzzy Control***

The key idea to the adaptive fuzzy control, as the first non-conventional fuzzy control method, is to adjust parameters of a fuzzy control system in order to have satisfactory control behaviour while the system dynamics is subject to change. A wide variety of control systems are referred to generally as “adaptive fuzzy control”. Initial signs of such controllers appeared in 1985 [20], but the first really adaptive fuzzy controller was introduced by Graham and

Newell from University of Queensland, Australia, in 1989 [21]. It was a self-tuning fuzzy adaptive controller. In these controllers, first, fuzzy identification is performed, and then, based on the achieved fuzzy model, a model-based fuzzy controller (i.e. a restricted inverse model of the system) is designed and tuned by an adaptive algorithm. Universal function approximation capability of fuzzy systems [22] justifies their application in system identification. Tuning operation is repeated for each control cycle. In self-tuning fuzzy control, the controller is not necessarily a model-based fuzzy control; it can be a conventional fuzzy controller [23].

- 1) What type of knowledge about the system is used in fuzzy control system design? Is mathematical model of the system needed? Is human expert/observer knowledge used effectively in the design?

If fuzzy controller is model-based, an initial mathematical model (not necessarily an accurate one) is needed. Also, input-output data of the system are used for both tuning and initial system identification. If the fuzzy controller is not model-based, human expert knowledge is also used.

- 2) Is there any reliable approach for stability analysis?

No, there is not.

- 3) Is there any systematic approach in control system design?

Yes. There are systematic approaches to design model-based self-tuning adaptive fuzzy controllers (i.e. identification and finding inverse model).

### ***II-C. Fuzzy PID controllers***

PID controllers are still the most widely used controllers in industry; fuzzy PID controllers are a combination of these controllers with fuzzy control technique (having nonlinear characteristics). Fuzzy PID controllers emerged in the early 1990's with "gain-scheduling"

fuzzy PID controllers [24]. In these control systems, a fuzzy inference system (i.e. a conventional fuzzy controller) adjusts the gains of PID controllers [24, 25]. Analytic design and stability analysis [16] was one of the main motivations behind moving from conventional fuzzy control to fuzzy PID control. This aim was not satisfied completely in spite of some efforts on stability analysis of these controllers [26, 27].

In the late 1990's, another type of fuzzy PID was introduced, namely “direct-action” fuzzy PID controller [28]. In these controllers, fuzzy inference system is used/installed in the structure of a PID controller in discrete domain generating control input increments, so the structure of PID is maintained to some extent. These controllers can be written in the form of classical controllers, and conventional PID controllers may be replaced by them directly [16]. As a result, design and stability study techniques usable in PID control are applicable for these controllers, especially if a linear model of the system is available.

Direct-action fuzzy PID controllers are briefly explained here. Classical PD controllers, in discrete domain, generate control input increments as shown in (1~3)[16]:

$$\begin{cases} d(nT) = \frac{e(nT) + e(nT - T)}{T} \\ v(nT) = \frac{e(nT) - e(nT - T)}{T} \end{cases} \quad (1)$$

$$\Delta u(nT) = K_p d(nT) + K_D v(nT) \quad (2)$$

$$u(nT) = -u(nT - T) + T \Delta u(nT) \quad (3)$$

where  $e$  is error,  $T$  is sampling time,  $u$  is control input or control command and  $K_p$  and  $K_D$  are controller gains.

In direct-action fuzzy PD controllers, control input increment ( $\Delta u$ ), is the outcome of a fuzzy inference system (FIS), or

$$\Delta u(nT) = F_{FIS}(d(nT), v(nT)) \quad (4)$$

Also, in (5),  $T$  is replaced by another parameter ( $K_u$ ) may be different from sampling time (not necessarily):

$$u(nT) = -u(nT - T) + K_u \Delta u(nT) \quad (5)$$

The fuzzy inference system is formed by triangular membership functions, so in the end, for different operation areas, (4) will be a linear function of  $d(nT)$  and  $v(nT)$ .

Other controllers from the direct-action fuzzy PID family are designed in a similar manner.

Optimisation techniques [29-31] and robust design approaches [32] have been employed to tune/optimize the parameters. In general, it has been indicated that fuzzy PID controllers are usually more successful than classical PID controllers in control of nonlinear systems [24, 25, 30, 31, 33-39]. These controllers continue to be designed and investigated [32, 40]. Despite all the similarities of fuzzy PID s to classical PID controllers, and the establishment of PID design and analysis techniques, there is no systematic design method for fuzzy PID controllers that guarantees a satisfactory performance [13]. Recently, a method has been proposed to stabilize the closed loop system if a linear model of the system is available [40].

The following provides the answers to the critical questions:

- 1) What type of knowledge about the system is used in fuzzy control system design? Is mathematical model of the system needed? Is human expert/observer knowledge used effectively in the design?

Human knowledge is employed in both gain-scheduling and direct-action types of fuzzy PID controllers. In gain-scheduling method, the designer can use his/her knowledge more freely. Mathematical model is not necessarily needed for design.

- 2) Is there any reliable approach for stability analysis?

For gain-scheduling fuzzy PIDs, almost there is no reliable method of stability study. For direct-action type, stability analysis is possible in case of having a mathematical model of the system. Robust techniques may be employed, although they are very conservative.

3) Is there any systematic approach in control system design?

In fuzzy PID controllers, the mathematical structure of the controller is known to some extent, but there is no general systematic approach for design.

#### ***II-D. Neuro-fuzzy controllers***

This class of fuzzy controllers can be considered as a sub-class of adaptive fuzzy controllers; however, due to their importance, a separate section is dedicated to them. Furthermore, this section solely covers neuro-fuzzy controllers which are designed through a training process and excludes Sugeno-Model-Based Fuzzy Control and Model Reference Adaptive Fuzzy Control whose mathematical structures may be identical to neuro-fuzzy networks.

Neural network controllers or neuro-controllers are a group of controllers with some considerable similarities to fuzzy ones. Both are originally model-free. That is; their design process does not start from a mathematical model necessarily, and they use other sources of knowledge about the system (human expertise/observation in fuzzy control and input-output data in neuro control). Neuro and fuzzy control form the main body of intelligent control.

In 1981, from a neurological point of view, Darocha found connections between fuzzy logic and natural neural networks, and tried to explain the behaviour of natural networks using fuzzy logic, and put those together [41]. However, that attempt was not enough to motivate mathematics and engineering experts to make a connection between fuzzy inference systems and artificial neural networks. Neuro-fuzzy systems appeared in the early 1990's [42] and immediately found applications in control [42, 43]. Artificial neural networks provide learning capabilities for fuzzy systems which can present expert knowledge. As a result, back

propagation was employed to tune membership functions of fuzzy controllers to improve their control behaviour [44-49] Genetic algorithm was also used for training neuro-fuzzy networks [50-53]. Moreover, as a control application, neuro-fuzzy networks have been used for tuning PID controllers [54]. In another study, the stability of neuro-fuzzy controllers was investigated using small gain theorem [55].

On-line or real-time training capability puts neuro-fuzzy network in the category of adaptive controllers [56]. Adaptive neuro-fuzzy controllers are usually Sugeno-type fuzzy inference systems. Sugeno-type FISs have linear consequent parts, so the method of the least square of errors can be used for finding the best parameters for consequent. Besides, adaptive neuro-fuzzy inference systems (ANFISs) have remarkable capabilities in modelling [57]. In summary, by combining fuzzy logic and neural networks, neuro-fuzzy systems use both input-output data and human expert/observer knowledge; this is as an important advantage for a method of an intelligent or model-free control. However, there is no general way to analytically prove the stability of closed loop systems and the convergence of learning algorithms [13].

- 1) What type of knowledge about the system is used in fuzzy control system design? Is mathematical model of the system needed? Is human expert/observer knowledge used effectively in the design?

Both human knowledge and input-output data are used in neuro-fuzzy control, and there is no need to mathematical model.

- 2) Is there any reliable approach for stability analysis?

No, also the convergence of learning algorithms is not guaranteed.

- 3) Is there any systematic approach in control system design?

Yes, they are. Well established neuro controller design techniques can be used by neuro-fuzzy control.



### II-E. Fuzzy Sliding Mode Control

After the combination of PID (1990) and neuro control (1991) with fuzzy control, sliding mode control was the other well-known control technique which was combined with fuzzy control in 1993 [58]. As a reminder of classical sliding mode control, let us consider a system with a mathematical model in the form of (6):

$$x^{(n)} = f(\mathbf{x}) + b(\mathbf{x}) u \quad (6)$$

where  $\mathbf{x} = [x \ \dot{x} \ \dots \ x^{(n-1)}]$  and  $n$  is the system's order. In (6),  $f$  and  $b$  are not necessarily known functions but they are upper bounded by a known function. For a general tracking problem with the desired  $\mathbf{x}$  of  $\mathbf{x}_d$ ,  $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{x} \ \dot{\tilde{x}} \ \dots \ \tilde{x}^{(n-1)}]$  is defined as tracking error vector.

Now a scalar equation is defined as  $s(\mathbf{x}, t) = \left( \frac{d}{dt} + \lambda \right)^{n-1} \tilde{x}$ . (7)

If  $\mathbf{x}(0) = \mathbf{x}_d$  and  $s \equiv 0$ , the system keeps the desired value forever.  $s \equiv 0$  will lead to a surface in  $R^n$  space, namely "sliding surface" [59]. If a control law can maintain (8):

$$\frac{d}{dt} s^2 \leq 0 \quad (8)$$

the distance of the state vector ( $\mathbf{x}$ ) to the sliding surface decreases continuously. In case of availability of a mode like (6), control law can be generated based on approximate model of (6) and its bounds. Due to model imprecision and disturbances, the model cannot predict system's behaviour accurately and, system response cannot maintain the sliding surface perfectly. As a result, the reference (desired states of the system)  $\mathbf{x}_d$  may be passed. Therefore, in order to avoid passing the sliding surface,  $sign(s)$  should be used in control law. Consequently, as the sliding surface is passed, control command changes to push the system back towards the sliding surface. As a result of this change, the reference may be passed

again by the system, then  $sign(s)$  changes again, and this situation may repeat several times. This situation is called the “chattering” phenomenon [59].

As Chen noted in 1998 “The sliding mode control approach guarantees the robustness and stability of the resulting control system, which can systematically be achieved at the cost of the chattering side effect” [50].

Fuzzy logic control can be employed to lessen this side effect. To do so, a fuzzy control law, in the form of a conventional fuzzy controller, is employed. The input to this FIS is  $s(\mathbf{x}, t)$  and its output is control command/input. Fuzzy values are designated to  $s$ ; therefore, using the capabilities of fuzzy logic, the changes to  $s$  will affect the control input/command much more smoothly compared to conventional sliding mode control laws. By eliminating the troubling term of  $sign(s)$ , the problem of chattering diminishes drastically [50, 60, 61]. In this paper, this category of controller is called the type-I fuzzy sliding mode control system. In this category, genetic algorithm [50, 62] and adaptive techniques [45, 63-70] have been used to find membership functions parameters of the fuzzy control law.

In some control systems, fuzzy controller is designed based on other methods (conventional, fuzzy PID or ...) then a “compensating” control command, generated by a sliding mode controller, is added to guarantee the stability [71-73]. This category is called type-II fuzzy sliding mode control systems. Some information about the mathematical model of the system is usually needed to design these controllers.

In general, as an important advantage, both types of fuzzy sliding mode control systems benefit systematic stability analysis of sliding mode control to make a framework for design and stability analysis [16, 50, 61, 74, 75]. However, it cannot be claimed that there is a general and systematic design approach for fuzzy sliding mode controllers. The following answers the critical questions about fuzzy control systems.

- 1) What type of knowledge about the system is used in fuzzy control system design? Is mathematical model of the system needed? Is human expert/observer knowledge is used effectively in the design?

In both methods of fuzzy sliding mode, human expert/observer knowledge is used. An approximate mathematical model is needed in type II. In type I, there is no need to a mathematical model. In type I, somehow, sliding mode is a framework to design a conventional fuzzy controller.

- 2) Is there any reliable approach for stability analysis?

Yes, there is (in both types I and II).

- 3) Is there any systematic approach in control system design?

There is no certain systematic approach for design, and heuristic fuzzy system design is a part of design procedure; although, there are methods to find control parameters (i.e. genetic algorithm).

### ***II-F. Model Reference Adaptive Fuzzy Control***

Besides self-tuning type and neuro-fuzzy control (introduced in sections II-B and II-D), another prominent type of adaptive fuzzy control is Model Reference Adaptive Fuzzy (MRAF) control. In model reference adaptive control, there exists a reference model to be tracked by the system. Also, there is a controller and an adaptive algorithm. Based on the error, which is the discrepancy between reference model's and system's response, adaptive algorithm adjusts the parameters of the controller. If any of these components or some of them (reference model, adaptive algorithm or controller) are fuzzy, the system is a MRAF controller. In these control systems, if the reference model is stable and the internal loop (including controller, system and adaptive algorithm) is stable either, then the whole system is stable. Kovacic and Bogdan, in 1994, used MRAF to control the angular speed of a

permanent magnet synchronous motor drive for the first time. This technique is still widely investigated [76-78].

Some other interesting topics in this area are robust adaptive fuzzy control with respect to external disturbances [79-81], the application of genetic algorithm in fuzzy adaptive control [82] and the comparison of fuzzy and conventional adaptive control [83]. Now, answers to critical questions:

- 1) What type of knowledge about the system is used in fuzzy control system design? Is mathematical model of the system needed? Is human expert/observer knowledge used effectively in the design?

For MRAF, both reference model and mathematical model of the system should be available and accurate. Also input-output data are used. Human knowledge (in the form of fuzzy terms) is not needed in the design of MRAF controllers.

- 2) Is there any reliable approach for stability analysis?

Yes. Providing that, both the reference model and the internal loop are stable, the system is stable.

- 3) Is there any systematic approach in control system design?

Yes. There are some systematic approaches to design MRAF control systems.

### ***II-G. Sugeno-Model-Based Fuzzy Control***

Sugeno type fuzzy inference systems can be considered as a set of linear systems smoothly connected by fuzzy membership functions. These fuzzy models can be used to describe complicated nonlinear systems [84-89]. If Sugeno-type fuzzy models are employed as nonlinear mathematical models and model-based nonlinear controllers are designed for them, this approach is called ‘Sugeno-Model-Based Fuzzy Control’. In this approach, human expert/observer knowledge, as the main motivation of the emergence of fuzzy control is not

used any more. Based on this method, optimal and stability-guaranteed controllers have recently been designed [90]. Now, answers to critical questions:

- 1) What type of knowledge about the system is used in fuzzy control system design? Is mathematical model of the system needed? Is human expert/observer knowledge used effectively in the design?

Mathematical model is essentially needed for this design method in the form of a Sugeno-type fuzzy inference system.

- 2) Is there any reliable approach for stability analysis?

Yes, there is.

- 3) Is there any systematic approach in control system design?

Yes, optimal design approach is available now for these fuzzy control systems.

### **III. SUMMARY**

In this paper, the development of fuzzy logic control is tracked. Fuzzy logic was introduced by Zadeh [1] in 1965. The main advantage of fuzzy logic was its capacity to allow expression of knowledge by linguistic terms in the form of fuzzy rules; decisions could be made based on this knowledge. Membership functions let the fuzzy logic evade cutting edges of 0 and 1. Fuzzy logic bridges mathematics, which is the most important tool in decisions analysis, and linguistic terminology, which is the most popular form of human knowledge.

Fuzzy control was introduced in 1973 by Mamadani and Asilian, to benefit the capabilities of fuzzy logic in control. In this paper, control system design is assumed as a pathway between system dynamics (the available knowledge about the system) from one side and control system architecture, structure and parameters from the other side. As a prominent progress, fuzzy logic introduced a new type of knowledge about systems (fuzzy rules based on human knowledge/observation) which can be used in control system design alongside

other well-known types of knowledge: recorded input-output data and the mathematical model which may be unavailable. In practice, fuzzy control permitted the designers to use their own and others' expertise and understanding about the systems which is mainly in the form of linguistic terms in control system design. Besides such a great advantage which made fuzzy one of the most popular control systems in industry, two drawbacks were proposed: Since human knowledge is subjective, unlike model-based control, conventional fuzzy control does not have any certain systematic design approach. In addition, the stability of initial fuzzy control systems, as model-free control systems, seemed impossible to be checked by well-known analytical approaches.

In order to address these drawbacks, research community started to conduct design approaches for fuzzy control based on well established model-based controllers to benefit their design/stability analysis techniques in fuzzy control. In 1985, self-tuning fuzzy control appeared which had a systematic design approach to a large extent and was much less subjective and heuristic than conventional fuzzy control. In 1990, discrete PID controllers were combined with fuzzy control to make them analysable. In 1991, neuro-fuzzy control emerged, with a more systematic design approach. A neuro-fuzzy controller can be designed almost without use of human expert knowledge; that is, it is barely subjective. In 1993, fuzzy sliding mode, not only offers rather systematic design methods; but also, offers a reliable framework for stability analysis. In fuzzy sliding mode, there was still enough room for human expert to apply its knowledge. In 1994, model-reference adaptive fuzzy control appeared, followed by Sugeno-model-based fuzzy control in 1995. In these two recent methods, a systematic approach for design and stability analysis was available; however, there was almost no need to human knowledge in design. Input-output data and mathematical model are the main types of knowledge used in these methods. In Sugeno-model-based fuzzy control, a fuzzy inference system is merely used as a nonlinear mathematical model, and as a

model-based method, an optimal Sugeno-type controller with guaranteed stability is designed for given models. From one viewpoint, two proposed drawbacks against fuzzy control had completely vanished; however, from the other side, the most recent fuzzy control methods have lost the main advantages of fuzzy control: model-free control design and the usage of human expert knowledge in control as a type of knowledge about system dynamics.

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## **How the article “Critical Review on Fuzzy Control” is linked to the research undertaken in this project?**

As mentioned in the article, lack of a rigorous design approach and lack of a stability analysis method were the main drawbacks of conventional or heuristic fuzzy control. Although, these shortages were fixed using recent model-based fuzzy control methods, in these methods human expert knowledge is not used in design anymore and this source of knowledge is eliminated from the design process. In this thesis, two main shortages of conventional or heuristic fuzzy control systems are addressed.

In Chapters 4, 5 and 6 (of Part3) and Appendix 3, attempts have been made to offer a reasonable design methodology that links the main design ideas to the rules of fuzzy controller. In Chapter 6, “damper rule” is introduced which is active around the reference to damp the chattering-like bahviour of the control system. This concept has also been partially used in Chapters 4 and 5.

In addition, a stability analysis method is offered in Chapters 4,5 and 6 for conventional (non-model-based) fuzzy control systems. The basis of the proposed stability analysis is the usage of a number of practical assumptions about the system expressed in the form of fuzzy rules. These assumptions become gradually more plain and evident within Part 3 of the thesis.

In summary, in Part 3 of this thesis, conventional fuzzy control system are designed and successfully used and two main drawbacks of these non-model-based fuzzy control systems including lack of a systematic design approach and lack of stability analysis are tackled.

## **Chapter 2**

### **A Critical Review on Neuro Control**

Submitted Journal Article

# A Critical Review on Neuro Control

Morteza Mohammadzaheri, Lei Chen

**Abstract**-In this review article, the most popular types of neural network control systems are briefly introduced and their main features are reviewed. Neuro control systems are defined as control systems in which at least one artificial neural network (ANN) is directly involved in generating the control command. Initially, neural networks were used to model systems' dynamics inversely to produce a control command which pushes the system towards a desired or reference value of the output (1989). At the next stage, neural networks were trained to track a reference model, and ANN model reference control appeared (1990). In this recent method, ANNs were used to extend the application of adaptive reference model control, which was a well-known control technique. This attitude towards the extension of the application of well-known control methods using ANNs was followed by the development of ANN model-predictive (1991), ANN sliding mode (1994) and ANN feedback linearization (1995) techniques. As the first category of neuro controllers, inverse dynamics ANN controllers were frequently used to form a control system together with other controllers, but this attitude faded as other types of ANN control systems were developed. However, recently, this approach has been revived. In the recent decade, control system designers started to use ANNs to compensate/cancel undesired or uncertain parts of systems' dynamics to facilitate the usage of well-known conventional control systems. The resultant control system usually includes two or three controllers. In this paper, the application of different ANN control systems is also addressed.

## I. ARTIFICIAL NEURAL NETWORKS USABLE IN CONTROL

Artificial neural networks (ANNs) are special mathematical models inspired by human neural networks. An ANN usually includes neurons, connections and biases. Neurons are arranged in 'layers'. There are a variety of neural networks, suitable for different purposes. In neuro control, it is a difficult and unsolved problem to find the best ANN structure for each specific application; thus, a fairly large ANN is usually employed to deal with relatively complex approximation problems [1]. In this paper, the most common types of artificial neural networks in the area of control are introduced: Multi Layer Perceptrons (MLPs) and Radial Basis Function Networks (RBFNs). Both of them are known as universal approximators of systems [2, 3]. Regardless of the main structure, if the output of an ANN depends not only on the current input to the network but also on the current or previous inputs, outputs, or states of the network, it is a recurrent ANN [4]. In other words, in recurrent networks, the (delayed)



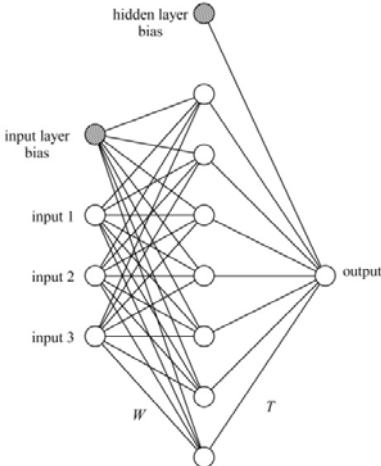
output of each neuron may be returned to the neurons of the same or previous layer. This does not happen in feedforward or static ANNs.

***I-A. Multi Layer Perceptrons***

Multi Layer Perceptrons (MLPs) are the most popular neural networks used in control [5]. In perceptrons, a neuron includes a sum operator and an activation function. All the inputs to a neuron are added and then the result (sum of all the inputs) passes through a function, namely ‘activation function’. The neurons are connected together by ‘connections’. Connections are of ‘weights’. The output of each neuron passes a connection and is multiplied by the connection’s weight, and then the product enters the next neuron. Biases are constant numbers (usually one) installed in the structure of the ANN. They do not have inputs, and their output is their constant value; their outputs pass connections and enter neurons. Figures 1 and 2 show a neuron and a (feedforward) MLP neural network respectively.

NOTE:  
This figure is included on page 38 of the print copy of  
the thesis held in the University of Adelaide Library.

**Figure 1. A typical neuron of an ANN [4]**



**Figure 2. A perceptron with three layers of neurons and two layers of connections**

In MLPs, during the training process, activation functions are fixed and weights are subject to change. Multi layer perceptrons have been investigated for control purposes [6-8] comprehensively and a wide variety of control applications such as the control of printing devices [9], heat exchangers [10] and spacecraft manoeuvre [11] have been found for perceptrons.

### ***I-B. Radial Basis Function Networks***

Radial basis function networks (RBFN's) are the second most popular neural networks for control purposes with a variety of applications [12-15]. These ANNs usually have two layers of neurons: the first layer is completely different from and the second layer is similar to perceptrons' layers. Figure 3 shows the first layer of a typical RBFN.

NOTE:  
This figure is included on page 39 of the print copy of  
the thesis held in the University of Adelaide Library.

**Figure 3. A scheme of first layer of a RBFNN [4]**

There is a radial basis neuron (shown in Fig.3) in the first layer. The  $\| \text{dist} \|$  box in this figure receives the input vector  $\mathbf{p}$  and the weight vector  $\mathbf{w}$ , and produces the dot product of these two vectors; the outcome is multiplied by the bias  $b$  which is the input to the radial basis transfer function. A typical RBFN is shown in Fig.4.

NOTE:  
This figure is included on page 39 of the print copy of  
the thesis held in the University of Adelaide Library.

**Figure 4. A typical RBFNN [4]**

## II. AN INTRODUCTION TO NEURO CONTROL

Control system design is the process of making a path from the system dynamics to a control system that consists of architecture (i.e. feedforward or cascade), a controller(s) structure (mathematical form) and controller(s) parameters. In other words, control system design process starts from the system dynamics. System dynamics includes all the knowledge about the system available for the designer and applicable for control system design. Input-output data of the system have been used in control design from the 1940's [16] :for instance, systems' response to step or sinusoidal inputs have been used for linear control systems design [16, 17]. However, artificial neural networks magnified the value of systems input-output data as a type of knowledge about the system drastically. Input-out data were used to design a variety of nonlinear controllers using ANNs, which was unprecedented in control. Different classifications are found in the literature for neuro controllers [5, 18, 19]. In this paper, the five most popular neuro control methods are briefly introduced in a chronological order, so as to cover both well-established and newly emerged methods. In this article, it is assumed that in neuro control systems, neural networks are directly involved in generating the control command; thus, control systems with ANN observers and without neuro controllers [20-22] are not addressed in this paper.

In the late 1980's, artificial neural networks were employed to map between the measured output and the control command. Then the ANNs, trained using such data, could produce a control command which would lead to a desired output. This approach is called inverse dynamics or inverse modeling method and used to be the dominant approach in the era of the pioneers of neuro control [23, 24] without directly mentioning the word 'inverse' as the name of the method. The phrases including 'inverse' gradually appeared to introduce this approach in the early 1990's [25, 26]. Narendra and Parthasarathy used two ANNs for modeling and control at the same time and devised model reference neuro control [27]. In

the 1990's, ANNs were employed to generalize well-known nonlinear control methods [28]. In 1991, ANN model predictive (neuro-predictive) control emerged as the generalized version of nonlinear model predictive control. A neuro-predictive controller includes a predictive model (ANN) and an optimisation/control approach which may be an ANN [29]. Neuro-predictive method was followed by ANN sliding mode control [30] and ANN feedback linearization control [31]. Later, ANNs were employed to compensate uncertain or complicated parts of the system dynamics not operating through the routine approach of feedback linearization[32]. From the early 2000's, in some research works, after ANN compensation, the resultant dynamics was still nonlinear which was controlled using well-known nonlinear control methods [33]. In this paper, these control systems are named 'Control Systems with ANN Compensation'. In this paper, following classes of ANN control systems are addressed in more detail:

- 1) ANN Inverse Dynamics Control Systems
- 2) ANN Model Reference Control Systems
- 3) Neuro-predictive Control Systems
- 4) ANN Feedback Linearization Control Systems
- 5) Control Systems with ANN Compensation

### ***II-A. ANN Inverse Dynamics Control Systems***

Consider the mathematical model of a first order single-input-single-output (SISO) system, without time delay:

$$y(k+1) = F_D(y(k), u(k)) \quad (1)$$

where  $y$ ,  $u$  and  $k$  are output, input and numerator respectively.  $F_D$  is the direct model of the system.  $F_I$ , as described below, will be the inverse model of the same system:

$$u(k) = F_I(y(k+1), y(k)) \quad (2)$$

If  $F_I$  is known, the input which leads to a desired value of output at the next stage can be found:

$$u(k) = F_I(y_d, y(k)) \quad (3)$$

Then the inverse model can be used as a control law. If an ANN is employed as the inverse model, the system will be an ANN inverse dynamics control systems. These control systems are either used as sole feedforward controllers [26, 34-36] or along with a feedback controller such as proportional[23], proportional-derivative[37] and proportional-integrator-derivative [24]controllers or another ANN [38]. Since 1993, the stability of ANN inverse dynamics control systems has been addressed [37-40].

The reference (setpoint) is often employed as the input to ANN inverse dynamics controllers [23, 24, 26, 34, 38, 39]; disturbance has also been used as the input to these controllers [41] since the late 1990's. Both reference and measurable disturbances have either been used as the input to these neuro controllers [42].

If only off-line (recorded) data are used in training ANN inverse models, the control system is not adaptive [24, 26, 35, 39]; however, since 1991, many ANN inverse controllers benefit from on-line training capacity of neural networks and are adaptive [34, 36-38].

Inasmuch as internal model control (IMC) is based on inverse modelling [43] in many cases, ANN inverse dynamics control systems are called ANN IMC systems [25, 39, 44-46].

## **II-B. . ANN Model Reference Control Systems**

In model reference control, an ANN is often used as a feedforward controller. Usually, this ANN receives the reference and outputs the control command. It is expected that the generated control command makes the plant track a stable mathematical model called the 'reference model' [27]. The weights of the ANN controller are adjusted to minimise a cost function involving the discrepancy of output of the plant and the reference model. The reference model can be a gain of one; in this case, the ANN controller is simply trained to

force the system to follow the reference, and the approach becomes closely similar to the ANN inverse dynamics control method and may be called ANN direct adaptive control [47, 48]. ANN model reference control method is an adaptive approach by its nature [47-50]. Stability of these control systems have also been well addressed [48, 50]. Sometimes, an ANN model is first made to be used instead of the plant in training ANN model reference controllers prior to implementation. In these control systems, there will be two different neural networks: a model and a controller [5, 27]. Figure 5 shows a schematic of an ANN model reference control system.

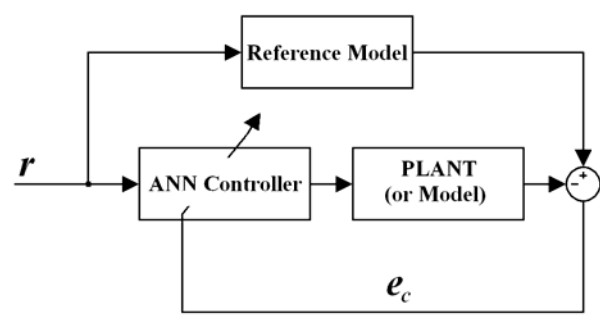


Figure 5. A schematic of an ANN model reference control system.

where  $e_c$  is the control error, and  $r$  is the reference. Occasionally, ANN model reference controllers are used together with feedback controllers [50] or as feedback controllers (with the input of control error) [51].

### II-C. Neuro Predictive Control Systems

The neuro predictive controllers often use a neural network model of a nonlinear system to predict the response of the system for a period of time in the future. This period of time or the number of instants (prediction time divided by sampling time) is called the ‘horizon’ (see Fig.6).

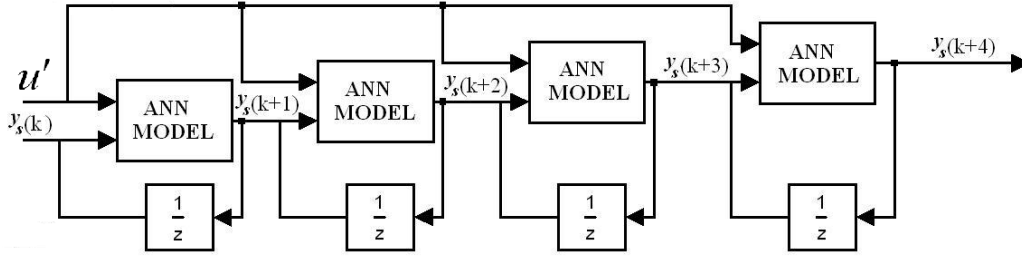


Figure 6. Prediction of the output value with the horizon of 4[52]

where  $y_s$  is the estimated output of the system,  $u'$  is the tentative control input and  $z^{-1}$  is unit delay. After estimation stage, a performance function is defined which usually includes the predicted errors and the change in control input value. Then, a controller or optimiser calculates the control input that will minimise the defined performance function.

In terms of predictive modeling, the most popular structures are multi layer perceptrons [52-61], radial basis function networks [62-64] and neuro-fuzzy networks [65], all in recurrent form. In some cases, on-line (real-time) training of ANNs was effectively used and adaptive neuro predictive controllers were designed [58, 62, 64, 66, 67].

A wide variety of optimisation/control algorithms have been employed in neuro-predictive control, such as sequential quadratic programming [56-58, 60-64], linear quadratic Gaussian [68] or self-tuning PI [58] control, Levenberg-Marquardt [52], fuzzy gradient-descent [54] or genetic [69] optimisation algorithms. An additional ANN can also play the role of controller/optimiser [53, 66, 70]. In some research works, the model and the control law have been so designed that the whole control system is stable [58, 66]. Nonlinear chemical and thermal process are the main area of the application of neuro predictive controllers [54, 56, 58, 61-64, 66]; however, the neuro-predictive approach has also been utilized in the control of model helicopters [52, 71], robots [53], insulin injection (for diabetic patients) [55] and manufacturing processes [57].

### II-D. ANN Feedback Linearization Control Systems

Feedback linearization is aimed at cancelling the nonlinearities. This technique can be easily applied on a class of nonlinear systems which can be described by ‘companion form’ models.

For a SISO system, a companion form model is shown in (4):

$$\mathbf{y}^{(n)} = f(\mathbf{y}) + b(\mathbf{y})u, \quad (4)$$

where  $u$  is the scalar control input,  $y$  is the scalar output of interest and  $\mathbf{y}$  is the state vector.

$$\mathbf{y} = [y, \dot{y}, \dots, y^{(n-1)}]^T, \quad (5)$$

$f(\mathbf{y})$  and  $b(\mathbf{y})$  are nonlinear functions of the states. In classical feedback linearization, this sort of control problems can be transformed to linear control problems (e.g. solvable by pole-placement) using an auxiliary control input [72]. That auxiliary control input pass a function then  $u$  is generated. The combination the aforementioned function and the system is a linear system.

Considering realizability issues, for SISO systems, a companion form model in discrete domain is defined as following[4]

$$y(k+d) = f(\mathbf{y}, \mathbf{u}) + b(\mathbf{y}, \mathbf{u}).u(k+1) \quad (6)$$

where  $\mathbf{y} = [y(k), y(k-1), \dots, y(k-n+1)]^T$ ,  $\mathbf{u} = [u(k), u(k-1), \dots, u(k-m+1)]^T$ , and  $n$  and  $m$  are the order of  $y$  and  $u$  in the system respectively. If such a model can be fitted to a system, at the instant of  $k$ , the control input at the next instant  $u(k+1)$  can be so defined that  $y(k+d)$  converges towards the reference ( $y_d(k+d)$ ) with the following control law:

$$u(k+1) = \frac{y_d(k+d) - f(\mathbf{y}, \mathbf{u})}{b(\mathbf{y}, \mathbf{u})} \quad (7)$$

Neural networks are employed to approximate the functions  $f$  and  $b$ . It is obvious that all systems cannot be fitted to such a model and it is a restriction for this method [4, 5]. In practice, NARMA-L2 is another name for this method [4, 5, 73-75]. Sometimes, control



increment ( $\Delta u$ ) appears in companion form of (6) instead of control input itself ( $u$ ), so the control algorithm calculates  $\Delta u$  [1]. In some cases, classical feedback linearization is enhanced by neural networks, e.g. for uncertainty compensation [22, 76]; these instances are not considered as ANN Feedback Linearization systems in this paper. MLPs [73-75, 77, 78] and RBFNs [1, 79], both in the recurrent form, are the most popular neural networks in ANN feedback linearization. In many cases, using the capability of ANNs in on-line learning, the designed ANN feedback linearization control system is adaptive [1, 77, 79-81]. In these control systems, ANN(s) structure and learning laws can be so defined that the control system is stable [1, 77-79]. Despite neuro-predictive method, ANN Feedback linearization has been widely used to control second order mechanical systems [74, 75, 77, 79]. First order processes/mechanical systems [73, 78, 81] have also been controlled by this method.

### ***II-E. Control Systems with ANN Compensation***

Well-known conventional control methods offer significant advantages in terms of, for example, stability and robustness. However, these methods usually can be applied on systems with a particular dynamics. These particular types of system dynamics usually do not match real systems completely. Recently, neural networks have been employed to fix this problem, mainly by cancelling/compensating undesirable or uncertain parts of system dynamics. In this category of control systems, the neural network is not the sole control law and there usually exist two [76, 82-86] or three [87-90] controllers working jointly. ANN control laws has already been used together with proportional [91], proportional-integral-derivative [86, 90], sliding mode (conventional [88] and intelligent [84, 85]), back stepping [82, 87, 89],  $H_\infty$ -based robust [76], feedback linearization [76] and model reference adaptive [92] controllers. These hybrid control systems, in almost all the cases referred in this paper, are adaptive and stable. Different types of ANNs have been used in these control systems, such as MLPs [87,

90, 93], RBFNs [76, 84], neuro-fuzzy networks [88, 89], wavelet-based ANNs [85] and Sigma-pi neural networks [86].

Inasmuch as ANN design in this control approach is highly dependent on the complementary conventional controller(s) and the dynamics of the system, there is no routine for control system design, unlike previously introduced neuro control systems. Roughly speaking, two different general models are introduced in this paper which match most of control problems solved through control systems with ANN compensators. The first model [76, 90] is

$$\text{Model I: } x^{(n)} = f(\mathbf{x}) + b(\mathbf{x})u + f_B(\mathbf{x}, u) \quad (8)$$

where  $\mathbf{x}$ ,  $u$  and  $n$  are the states vector of the system, system's order and control input respectively, and index  $B$  stands for bad. That is,  $f_B(\mathbf{x}, u)$  contains undesirable or uncertain part of the system dynamics which will be compensated by the compensator control

$$\text{command of } u_c = -\frac{\hat{f}_B(\mathbf{x}, (u_c + u_o))}{b(\mathbf{x})}, \quad (9)$$

where  $u_o$  is the resultant of all other control commands except for the compensator command and  $\hat{f}_B(\cdot)$  is a neural network which approximates  $f_B(\cdot)$ . If  $f_B(\mathbf{x}, u)$  is compensated/cancelled, (8) will be transformed to a companion form equation. However, there is no guarantee that (9) always results in a solution. If  $f_B(\cdot)$  is a function of  $\mathbf{x}$ , this problem will be easier to solve [76]. With the aforementioned difficulties in solving (9), it is not surprising that ANN compensation approach is not popular and applicable for systems with models like (8).

$$\text{The second model (Model II) is } u = f_G(\mathbf{x}) + f_B(\mathbf{x}) \quad (10)$$

where index  $G$  stands for good. That is, providing that  $f_G(\mathbf{x})$  was the sole function of the right hand side of (10), the equation would suit one of well-known conventional control

methods. If a neural network is employed to approximate  $f_B(\cdot)$ , then the compensator control command will be  $u_c = \hat{f}_B(\mathbf{x})$ , and if  $u = u_c + u_o$ , (10) can be written as

$$\begin{cases} u_o = f_G(\mathbf{x}) \\ u_c = \hat{f}_B(\mathbf{x}) \end{cases} \quad (11)$$

As shown in (11), the neural network is used as the compensator control law straightforward. The systems with dynamics that match Model II are very good cases for ANN compensation control [82-89, 92, 93]. These systems are often mechanical systems whose behaviour is explained through Newton's or Euler's laws. The control input is force or torque which appears alone (not multiplied by system parameters or states or as a function of them) in the corresponding model equations. In these problems, if linear or angular displacement is addressed, the control problem will be second order [76, 85, 87-91], and if the linear or angular velocity is controlled, the problem will be first order [82, 84, 92].

### III. SUMMARY

In this paper, the progress of neuro controllers was reviewed from their appearance more than 20 years ago. The main types of neuro controllers were introduced in brief, and it was indicated that the outstanding capability of neural networks in function/system approximation was the motivation behind their application in control. Two main points can be mentioned to summarize the chronological review of neuro controllers.

First, as a general trend, the initial neuro controllers (ANN inverse dynamics controllers) were used in conjunction with other controllers (mainly conventional ones). The next generations of neuro controllers gradually lost this feature and started to play the role of the sole control law. These controllers were the extended version of well-known nonlinear controllers. However, as a return, the most recent type of neuro controllers always accompany other controllers. ANNs are not the extended version of any other controllers in

this recent type. It seems, neural network controllers had a journey from auxiliary controllers to sole controllers and again to auxiliary controllers. However, in their return, newly designed hybrid ANN controllers frequently offer stability and addictiveness to the control system which was rare in initial ANN control systems.

Second, in terms of application, some of neuro controllers are more suitable for some specific applications. Neural networks are trained using input-output data only, so it may seem that ANN control approaches can work for all the systems, provided that enough training data are available. However, this is not correct. For instance, neuro predictive controllers suit process plants exceptionally well and ANN compensation control systems are particularly appropriate for mechanical systems; although, these methods have not specially designed for the aforementioned purposes. This point has not been mentioned explicitly in the literature. The research community may need to define some characteristics of systems to guide designers in choosing appropriate neuro controllers; these characteristics may be an area of novelty in future investigations.

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## **How the article “Critical Review on Neuro Control” is linked to the research undertaken in this project?**

To summarize the article, two points were mentioned. The first point states that initial ANN controllers could be/were used together with other controllers, usually suffering from lack of an appropriate stability analysis; then ANN controllers started to play the role of sole controllers, usually, as the generalized version of a well-known controller (adaptive model reference, model-predictive, sliding mode or feedback linearization). Lately, ANN control has witnessed a trend of application together with other controllers again, this time with proven stability. ANN controllers proposed in Parts 3 and 4 of this thesis all are belonging to this very recent group. They are used together with error-based feedback controllers and the stability of the control system is proven.

As the second point about neuro control, noted in the article, some of neuro controllers are more suitable for some specific applications. The question is how to choose an appropriate neuro control system for a particular application. Some characteristics of systems may be defined to guide designers in selecting the type of neuro controllers. These characteristics should be independent of mathematical model to keep the method non-model-based. Control inertia, introduced in Part 2, can be one of these characteristics, as it is indicated in Part 2, low control inertia systems suit neuro-predictive control.

## **Part 2**

# **Control Inertia and Fuzzy Brakes**

### **SYNOPSIS**

Part 2 includes a published journal article. The concept of ‘Control Inertia’ is introduced in this article. Repeating overshoot shows that a (closed loop) system is high control inertia. Fuzzy brakes are designed to improve the control performance in this case; it is proved that if a closed loop system is stable, adding a fuzzy brake does not make it unstable.

## **Chapter 3**

### **Intelligent Predictive Control of a Model Helicopter's Yaw Angle**

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# Intelligent Predictive Control of a Model Helicopter's Yaw Angle

Morteza Mohammadzaheri, Lei Chen

*Abstract*-In this paper, the concept of “Control Inertia” is introduced, and then based on this concept, unexpectedly inadequate control behaviour of “High Control Inertia” systems is explained, then fuzzy compensators are suggested to improve the control behaviour. This work is in the area of non-model-based control. In order to indicate the merit of the proposed concept/technique, a neuro-predictive (NP) control is designed and implemented on a highly non-linear system: a model/lab helicopter, in a constrained situation. It is observed that the behaviour of the closed loop system under NP controller is either very fluctuating (with a low value of a particular design parameter) or very slow (with high values of the same design parameter). In total, the control behaviour is very poor in comparison to the existing fuzzy controller, whereas NP is used effectively in control of some other systems. Considering the concept of “Control Inertia”; a Sugeno-type fuzzy compensators is added to the control loop to modify the control command. Newly designed neuro-predictive control with fuzzy compensator (NPFC) improves the performance of the closed loop system significantly by the reduction of both overshoot and settling time. Furthermore, it is shown that the disturbance rejection of NPFC controlled system is satisfactory as well as its parameter robustness.

**Key words:** Neuro-predictive Control, Fuzzy, Yaw Angle, Model Helicopter, Control Inertia

## I. INTRODUCTION

Predictive control, as a method of using predicted outputs to generate control inputs, was initially introduced by classical model predictive controllers (MPCs) [1]. It is obvious that a “model” is needed for “prediction”; in classical MPCs, linear state space models are used [2]. In some cases, artificial neural networks (ANN) can be constructed and trained as linear models with limited validity area for non-linear systems; such models can also be used in classical predictive control [3]. But nonlinear models are usually needed in order to predict the behaviour of nonlinear systems. Soloway and Haley used nonlinear artificial neural networks as a model for predictive control purposes [4]. In order to generate the control command in the presence of nonlinear ANN models,

nonlinear optimisation methods are often used [5-7], although an additional ANN can also perform this task [8]. A variety of applications have been found for neuro-predictive controllers such as control of food or chemical processes , air/fuel ratio of engines, hybrid water and power supplies, robots and insulin pump of diabetic patients [9-14]. This control algorithm is entirely non-model based and almost same for different case studies (both in terms of ANN training and control algorithm).

In this research, as a work in the area of non-model based control, neuro-predictive approach is used to control the yaw angle of a model helicopter. Despite several successful applications of NP method in control, specially process control, in case of model helicopter yaw angle control, the control behaviour is unexpectedly unsatisfactory; although the control algorithm is same and model-independent. This research is focused on explaining why the control behaviour of NP is so poor in this case study and how to improve the control behaviour. The concept of “control inertia” explained the discrepancy and, in the end, after adding a special compensator, a very good control performance is achieved.

## II. NEURO-PREDICTIVE CONTROL

In neuro-predictive control, an optimisation method generates the control command (represented by  $u'$ ) based on minimising a performance function involving the predicted errors. (1) is a typical performance function (represented by  $J$ ) in neuro-predictive

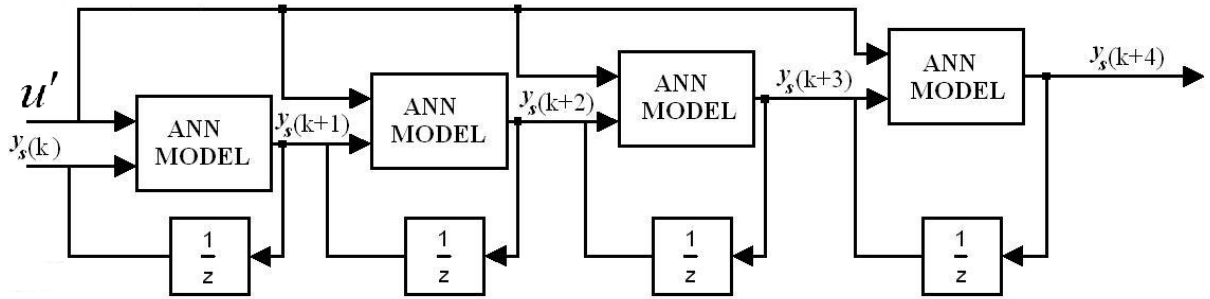
control [15].

$$J(k) = \sum_{i=1}^H [y_s(k+i) - y_d]^2 + \rho [u'(k) - u(k-1)]^2. \quad (1)$$



$y_s$  and  $y_d$  are the estimated and desired outputs of the system respectively, and  $u'$  and  $u$  are tentative and actual control inputs;  $\rho$  represents the importance of the constancy of control input.

In order to generate tentative control command, the performance function ( $J$ ) should be calculated. To do so, the output values of system should be predicted for  $H$  future instants (see (1)), so the nonlinear model (neural network) should be used  $H$  times.  $H$  is called the horizon of prediction.



**Figure 1. Prediction of the output value with the horizon of 4**

If current output and previous output/input of the system (the measured data) and  $u'$  are known, all other arguments of  $J$  will be definitely known (see Fig. 1). So these arguments can not be subject to modification by optimisation algorithms. However  $u'$  can be changed arbitrarily, freely from the measured input/output signals, and this change affects other arguments of  $J$ , then the performance function itself. Therefore, in the optimisation for control purposes, it can be assumed:

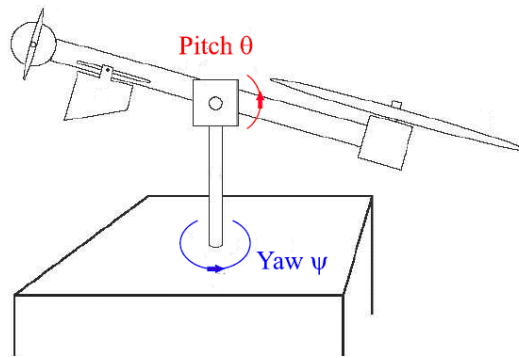
$$J = J(u'). \quad (2)$$

In summary, in the design of neuro-predictive controllers, two main tasks should be performed, neural network modeling and optimisation.

### III. MECHANICAL MODELING

The model helicopter used in this research has two degrees of freedom, the first possible motion is the rotation of the helicopter body around the horizontal axis (which changes pitch angle) and the second is rotation around the vertical axis (which change yaw angle). The helicopter can rotate from  $-170^\circ$  to  $170^\circ$  in yaw, and from  $-60^\circ$  to  $60^\circ$  in pitch. System inputs are voltages to main and rear rotors, and yaw and pitch angles are considered as its outputs.

A mechanical modeling is done using Newton and Euler laws [16]. After modeling, equations (3-7) are setup for the system [16].



**Figure 2. A scheme of the model helicopter**

In this research, a special situation is studied. In this situation, the motion is so constrained that the rotation around horizontal axis (change in pitch angle) is impossible; moreover, the input voltage to main rotor is set to “zero”. As a result, the only input to the system is the input voltage to rear rotor ( $U_s$ ). Also, the yaw angle (the angle in horizontal plane) is considered as the unique output. In real helicopters, while the helicopter is rotating in the horizontal plane (around its vertical axis), the angular velocity of main rotor is often likely to be constant, so according to Euler’s law, a very little

torque (ideally zero) is exerted on the body by the main rotor. This situation is very similar to the discussed situation in this research. In this situation, (3) can represent the behaviour of system. Since there is no change in pitch angle, gyroscopic torque does not exist; furthermore, main rotor does not generate any torque.

$$\frac{d\omega_\psi}{dt} = \frac{1}{I_V} (T_{RearRotor,V} - T_{friction,V}), \quad (3)$$

where:

$$T_{RearRotor,V} = r_S k_{FS} \text{sign}(\omega_S) \omega_S^2, \quad (4)$$

$$T_{friction,V} = c_{\mu V} \omega_\psi. \quad (5)$$

The equations defining the behaviour of this first order system can be set up altogether as:

$$\dot{\psi} = \omega_\psi, \quad (6)$$

$$\dot{\omega}_\psi = \frac{1}{I_V} (r_S k_{FS} \text{sign}(\omega_S) \omega_S^2 - c_{\mu V} \omega_\psi). \quad (7)$$

where  $\omega_\psi$  is the angular velocity of helicopter body in yaw direction and  $\omega_S$  is the angular velocity of rear rotor blades which is a nonlinear function of input voltage to rear rotor.

The parameters can be found in nomenclature.

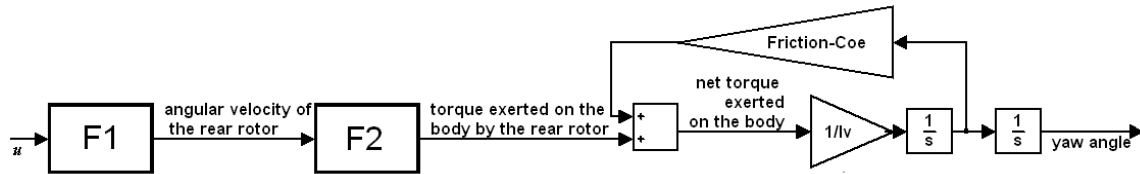


Figure 3. A block mechanical model of the system

## IV. DESIGN OF HYBRID CONTROLLER

### IV-A. Neural Network Model

In order to train an artificial neural network, first of all, training data should be obtained and arranged appropriately. A set of 550 input-output recorded data of system are used

for training. In order to obtain such a data set, random signals ,with sampling time of 0.4 second, are sent to the system in 110 seconds and the output value is recorded accordingly (with the sampling time of 0.2 second). During the test, a fixed value may be added to random signal to simulate a wide variety of system's behaviour. The data are normalized before training. Figure 4 shows input-output data in training stage.

A three layer recurrent perceptron is used to model the system. The number of neurons in input and output layers are 6 and 1(see Fig.4), both having linear activation functions with slope of one. The order (the number of delay functions) and sampling time are defined by the designer based on its familiarity with system's dynamics and controllers designed for similar systems such as the controller designed in reference [17]. After several trials, 20 sigmoid neurons were selected for hidden layer. In this research, Levenberg-Marquardt algorithm is applied both for off-line and on-line training. In off-line training batch back-propagation is employed and in on-line training, single-pattern back-propagation is used. A scheme of neural network is shown in Fig. 4.

In off-line training, ANN is trained in 100 iterations. The performance function of training is the sum of squared errors. After off-line training stage, the neural network should be checked.

Checking data are entirely different from training data, obtained through sending two different sinusoidal signals to the system as the input (the input is the voltage to the rear rotor of helicopter). During checking, after very first instants, the estimated output returns to the model as the input to the neural network.

That is, after initial estimations, all the delayed outputs which are used as the inputs to the model are estimated ones. As a result, any error comes back to the model again and error accumulation phenomenon occurs in case of lack of accuracy of the model.

Figures 5 and 6 show the checking result for the trained ANN; it should be noted that fewer than seven estimated outputs are usually used in predictive control. A criterion is defined for prediction accuracy for  $N$  future instants, namely PAN:

$$\text{PAN} = \frac{\sum_{i=1}^N |y_s - y|}{N} \quad (8)$$

where  $y$  and  $y_s$  are real and predicted outputs.

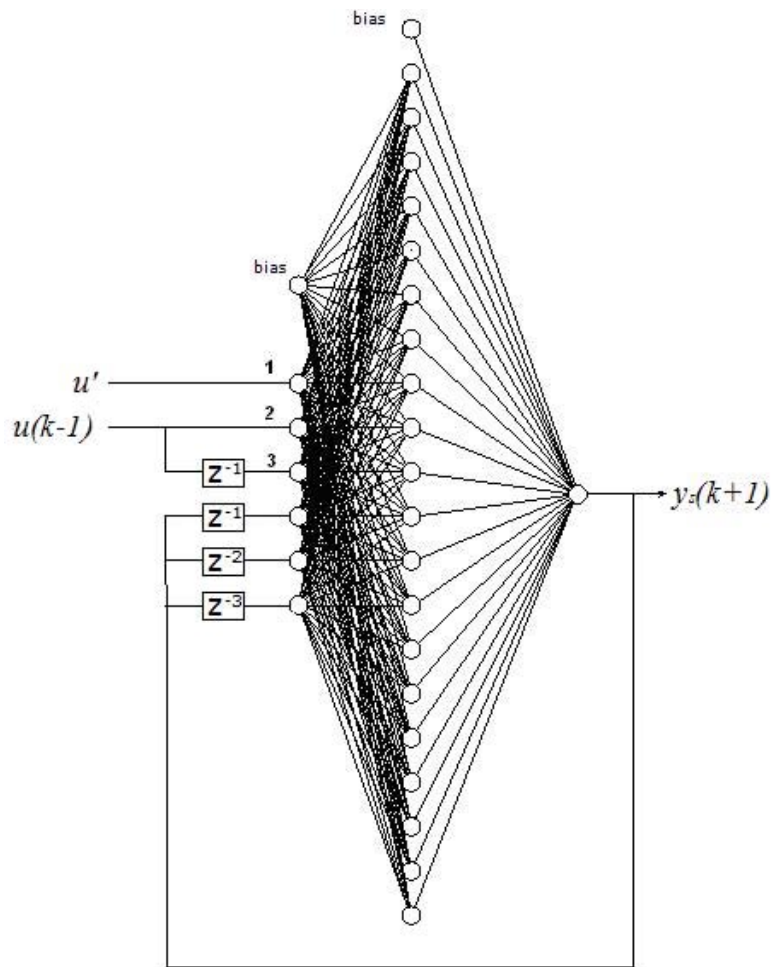
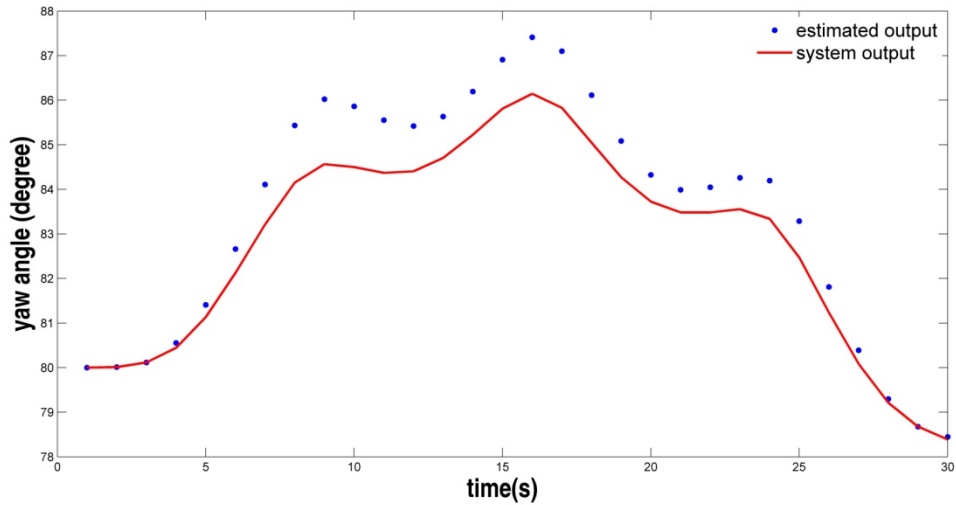
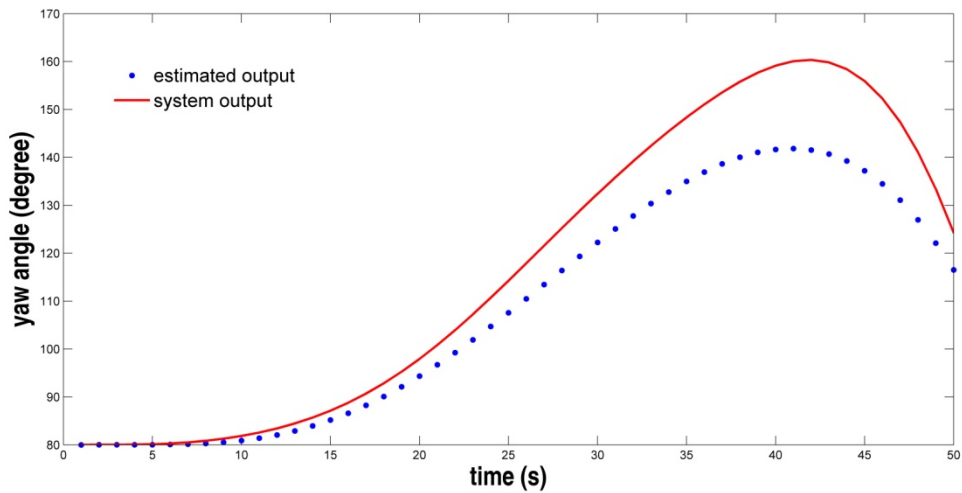


Figure 4. Neural network structure and input-output data in the training stage



**Figure 5. Outputs of ANN model and the real system**



**Figure 6. Outputs of ANN model and the real system**

For Fig.5, PA10 is  $0.2989^\circ$  and PA30 is  $3.1930^\circ$ , for Fig.6, PA10 is  $0.0954^\circ$  and PA30 is  $0.7280^\circ$ .

The trained neural network is used to predict future outputs of the system. For the first prediction step, neural network inputs and outputs are same as Fig.4. For the second step of prediction, connection 2, in Fig.4, is connected to tentative control  $u'$  (instead of

$u(k-1)$ ) and connection 3 is connected to  $u(k-1)$ . In other steps of prediction, all connections 1, 2 and 3 are connected to  $u'$ .

#### IV-B. Predictive Control

According to (1), the purpose of predictive control is to define tentative control input  $u'$  so that  $J$  is minimized. In order to obtain predicted output values, at each instant, ANN should be used  $N$  times (see (1)). In this research  $N = 7$ . Let's consider seven sequential identical neural networks that the output of any of them (except the last one) provides one of the inputs to the next ANN. Such a neural model, namely "Neural Predictive Model" obtains the estimated (predicted) output values of system ( $y(k+i)$ ,  $i = 1 \sim 7$ ) using previous and current values of the output of system ( $y$ ), previous values of control input ( $u$ ) and tentative control input ( $u'$ ). As a result of this stage, having  $\rho$  (see (1)), the value of performance function ( $J$ ) will be available, as shown in Fig.7.

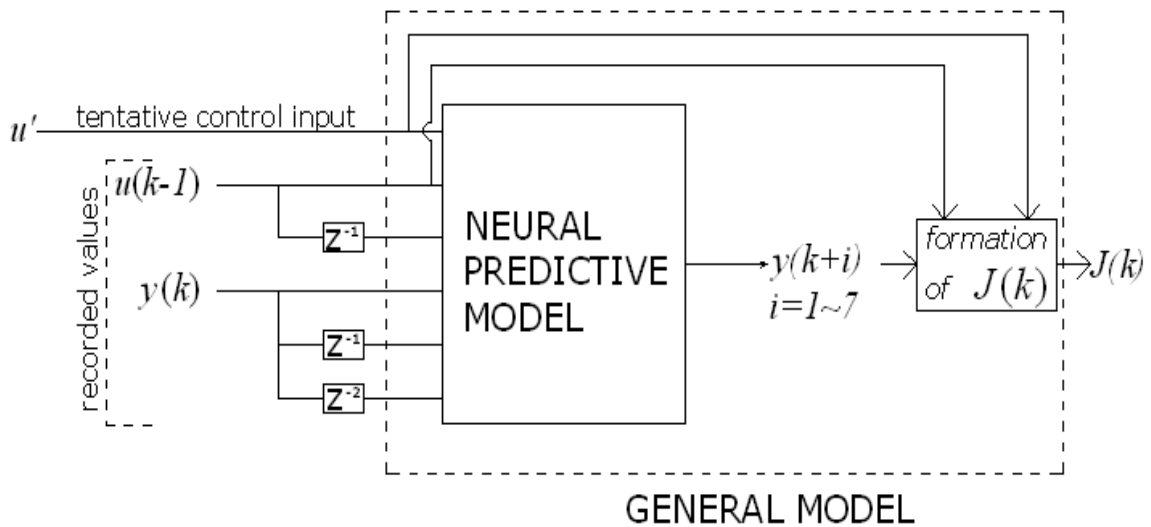
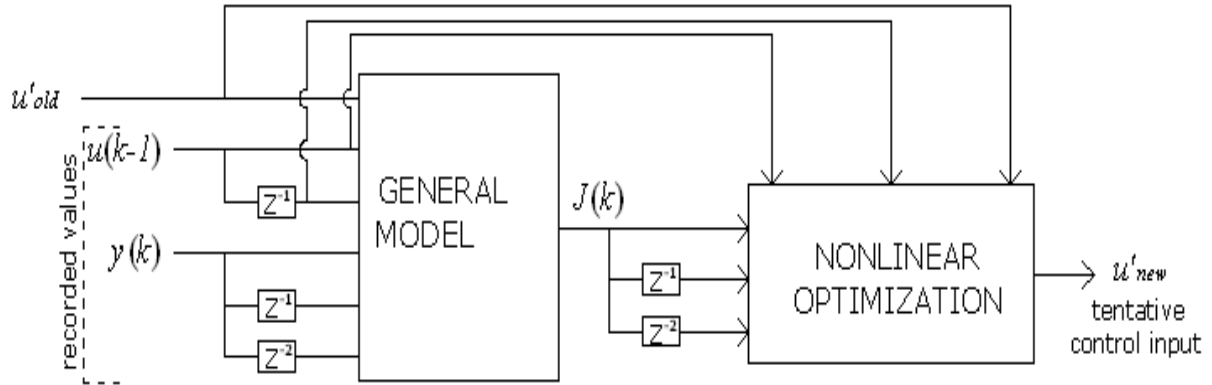


Figure 7. The process of calculation of the performance function



**Figure 8. Neuro-predictive controller**

#### ***IV-C. Control behavior of the system under neuro-predictive control***

Considering (1), nonlinear predictive control is a trade-off between quick convergence to the setpoint and the constancy of control input. Low values of  $\rho$  ( $<2$ ) are usually used in neuro-predictive control. For instance, in the control of stirred tank reactors [15, 18], neuro-predictive control with popular values of  $\rho$  leads to a very good control response. In contrast, in the control of helicopter yaw angle, the same method leads to considerable repeating overshoot and very long settling time (only an extremely high value of  $\rho$  may avoid change of control input and repeating overshoots at the price of very slow convergence to the setpoint). In other words, no matter how big/small  $\rho$  is, the control behaviour is not adequate (Fig.9). In this research, a characteristic of systems, namely “Control Inertia” is proposed, and the aforementioned discrepancy in control behaviour is explained based on this new concept.



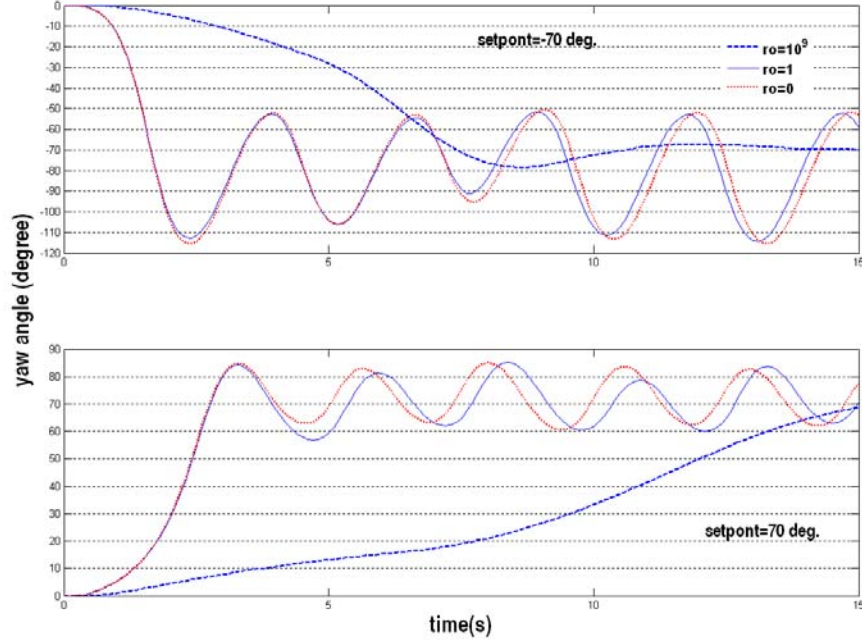


Figure 9. Closed loop response under neuro-predictive control for different values of  $\rho$

#### IV-D. Control Inertia

Definition: The control inertia is defined as the ratio of control input to second temporal derivative of the controlled system's output:

$$\text{control inertia} = \frac{u}{\ddot{y}} \quad (9)$$

where;  $u$  is the control command (control input) and  $y$  is the system's output. Based on this definition, in high control inertia systems,  $\ddot{y}$  is rather small; that is, the rate of  $\dot{y}$  is small. As a result, when the system approaches the desired condition (setpoint),  $\dot{y}$  needs a significant time or energy to decrease; therefore, when the system reaches the setpoint,  $\dot{y}$  is still of a considerable value; and the system passes the setpoint and faces a big overshoot. As the system re-approaching the setpoint, this situation is repeated and leads to a long settling time and a great deal of energy consumption. Considerable output rate around the setpoint is the main feature of high control inertia systems.

Physical models of a wide variety of systems can be approximated by following equation:

$$u = \dots + a_2(\mathbf{x})\ddot{y} + a_1(\mathbf{x})\dot{y} + a_0(\mathbf{x})y + a_{-1}(\mathbf{x}) \quad (10)$$

$\mathbf{x}$  is the states' vector, and  $u$  and  $y$ , are the output and input of the system. Force, pressure, energy or flow-rate can be a form of the input, and position, temperature, level or concentration can be a form of the output. According to the offered definition of control inertia, the value of  $a_2(\mathbf{x})$  is control inertia.

In many chemical reactors,  $a_2(\mathbf{x})$  is theoretically zero (practically, very low); therefore, these systems are low control inertia, whereas mechanical systems (explained by Newton/Euler laws) have significant values of  $a_2(\mathbf{x})$  which is mechanical inertia. So, the difference in the value of control inertia is the reason behind different control behaviour of model helicopter (as a mechanical system) and chemical reactors controlled similarly by NP algorithm [18, 19].

#### ***IV-E. Fuzzy Compensators***

As previously stated, for systems with high control inertia, based on considerable output rate around the setpoint, while the system is approaching the desired condition, it is unlikely to stop at the setpoint or its vicinity and these systems usually pass the setpoint and become far from the desired conditions. In order to prevent this situation, a fuzzy compensator is added to the control system. The input to this fuzzy inference system is the absolute value of error and its output (modification coefficient) is multiplied by  $u'$  (tentative control input, the output of neuro-predictive algorithm) to achieve a modified control input. This fuzzy compensator is a Sugeno-type FIS with two rules:

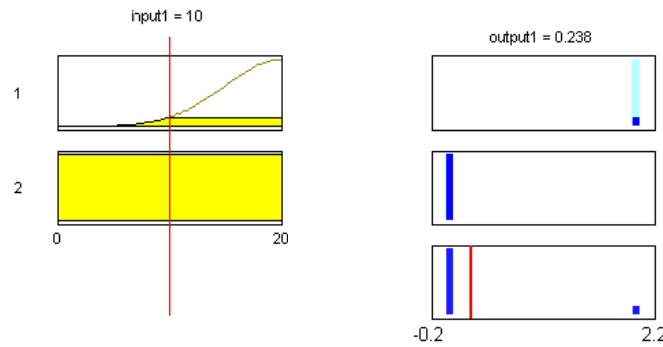
if *absolute error* is *HIGH* then *modification coefficient*=2  
 if *absolute error* is any value then *modification coefficient*=0

*HIGH* is a Gaussian membership function whose membership grade is defined in (11):

$$\mu_{HIGH} |e| = \begin{cases} \exp[-\frac{1}{2}(\frac{|e|-20}{5})^2], & \text{for } |e| < 20 \\ 1 & , \text{for } |e| \geq 20 \end{cases} \quad (11)$$

where  $e$  is the error (of yaw angle).

A scheme of this fuzzy inference system is shown in Fig.10.



**Figure 10. A schematic of fuzzy compensator**

The second fuzzy rule is always fully active (its fire strength is 1). When the absolute error is higher than  $20^\circ$ , the first rule is also fully active, and the output is 1, otherwise the output has a value between 0 and 1. The role of fuzzy compensator is to reduce control input (rear rotor voltage) when the absolute error is low. The main idea of the design on compensator is based on fuzzy logic, but obviously, it can be written in the form of an ordinary mathematical relation:

$$\text{modification coefficient } t = \frac{2\mu_A}{1 + \mu_A} = \frac{2}{1 + \mu_A^{-1}} = \begin{cases} \frac{2}{1 + \exp[0.05(|e|-20)^2]}, & \text{for } |e| < 20 \\ 1 & , \text{for } |e| \geq 20 \end{cases} \quad (12)$$

## V. SIMULATION RESULTS

The response of the closed loop system (Fig.11) is shown in Fig.12 for four various setpoints and three various model-free controllers. The helicopter is rotated from stationary situation to some different desired values of yaw angle by rear rotor, while rear rotor is controlled by existing “fuzzy controller”, “NP controller with  $\rho = 1$ ” or “NP controller with  $\rho = 1$  and a fuzzy compensator (NPFC)”.

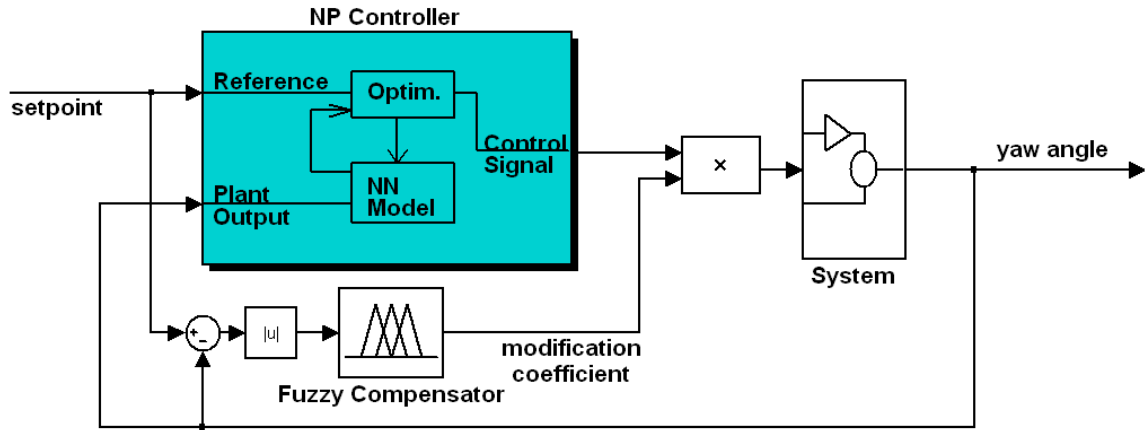


Figure.11: Control System

Mean of absolute error ( $MAE$ ) and energy consumption criterion ( $ECC$ ) are defined as below:

$$ECC = \int_0^T |u(t)| dt, \quad (13)$$

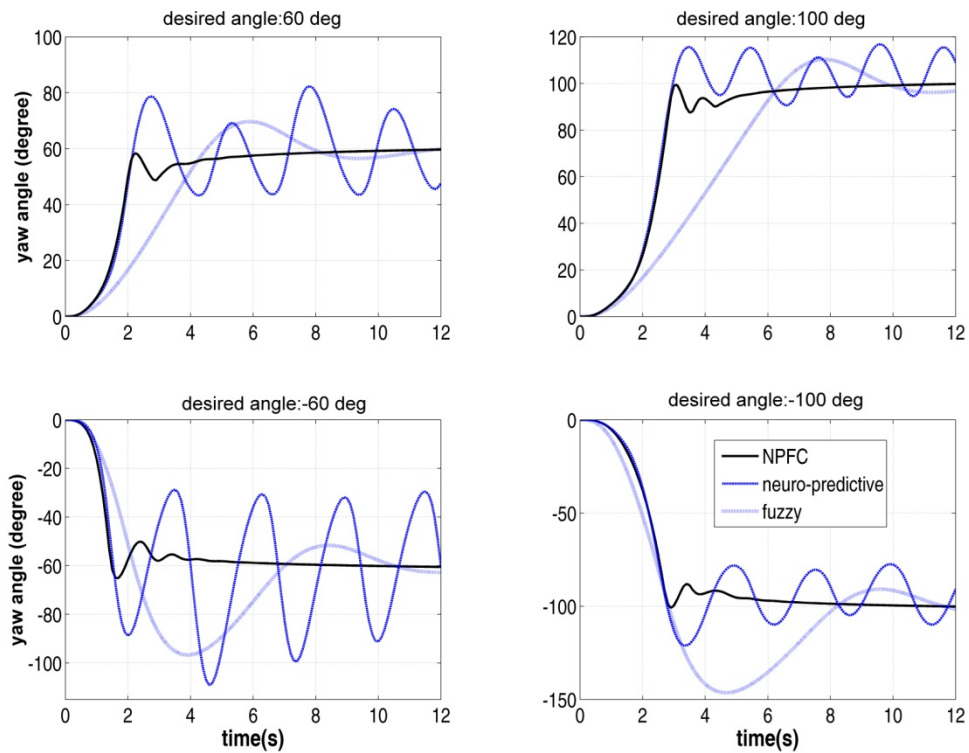
$$MAE = \frac{\int_0^T |\psi(t) - r(t)| dt}{T}, \quad (14)$$

where  $T$  is the final time for simulation and  $u(t)$  is the input voltage to the rear rotor of helicopter or control input and  $r(t)$  is reference or setpoint. Control input can be in the range of  $[0 \ 10]v$ . The control input during simulation is much smaller than  $10v$ , so the

saturation function regarding the actuator limit was ignored in Fig.11. Since neuro-predictive controllers are essentially designed to reduce the deviation of control input rather than the absolute value of control input, another criterion namely, input deviation criterion *IDC* is defined as well, calculated as below:

$$IDC = \int_{\tau}^T |u(t) - u(t - \tau)| dt, \quad (15)$$

where  $\tau$  is the sampling time of system. Table.1 includes these two criteria for all the plots shown in Fig.12 as well as maximum overshoot of various simulations. Furthermore, the time needed for yaw angle to get closer than 5 degrees to its desired value so that its error remains less than 5 degree, settling time, is also noted in Table 1.



**Figure 12. The behaviour of different controllers for different setpoints**

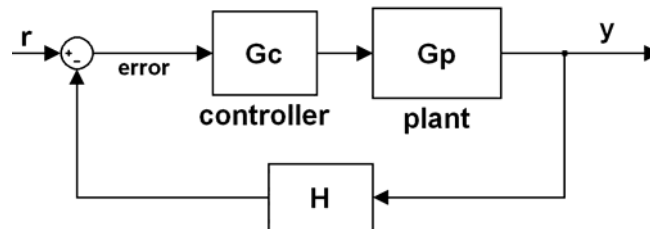
It is clearly observed that newly designed NPFC controller improves the performance of system by reduction of both overshoot and settling time in comparison to existing fuzzy controller which is considered as a satisfactory controller for this high control inertia system. Control input generated by NPFC controller does not exceed permitted range for input voltage although NPFC consumes more energy.

**Table 1. Operational information of different controllers**

Set point	Controller Type	<i>ECC</i> (V.s)	<i>IDC</i> (V.s)	<i>MAE</i> (deg.)	Maximum Overshoot (deg)	Settling time (s)
-60 deg	Fuzzy	5.434	0.326	18.58	36.367	7.4
	NP	11.268	2.839	19.13	49.238	-
	NPFC	9.784	0.798	7.35	9.124	2.7
-100 deg	Fuzzy	6.758	0.3324	30.68	45.222	10.8
	NP	11.806	0.975	26.28	24.781	-
	NPFC	12.510	0.699	19.78	10.041	5
60 deg	Fuzzy	4.1016	0.163	16.23	10.239	6.9
	NP	21.141	2.752	17.15	22.023	-
	NPFC	22.647	1.327	10.40	7.931	4.1
100 deg	Fuzzy	5.9511	0.163	34.06	16.734	9.0
	NP	27.929	1.958	24.88	18.913	-
	NPFC	28.383	1.414	21.51	6.247	5.3

## VI. THE EFFECT OF A FUZZY BRAKE ON STABILITY

According to small gain theorem [20], in the control system shown in Fig.13:



**Figure 13. A typical feedback control system**

The control system is stable, if:  $|G_c \times G_p \times H| < 1$  (16)

In this section, neuro-predictive controllers are adapted to typical feedback controllers, and then the theorems which are valid for feedback control systems (i.e. small gain theorem) can be used for them. Considering:  $e(k) = y(k) - y_d$  (17)

and  $e_s(k+i) = z^i e(k)$  (18)

where  $e_s$  : estimated error. A neuro-predictive controller can be shown as in Fig.14 (see Fig.8).

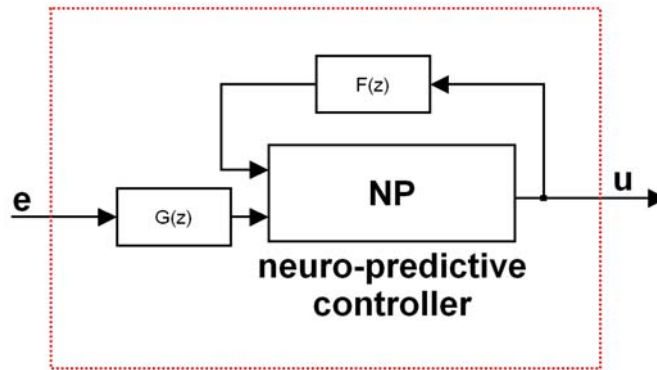


Figure14. Neuro-predictive controller as a feedback controller

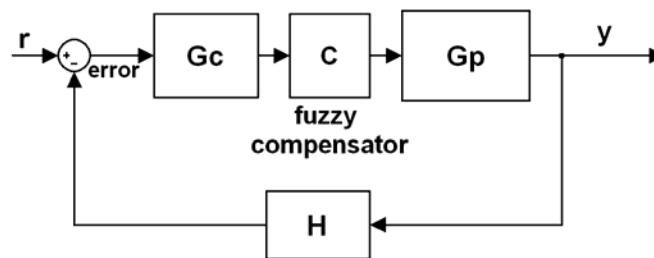


Figure 15. A feedback control system with a (fuzzy) compensator

The components confined in the dashed rectangle in Fig.14 form a general neuro-predictive controller, including neuro-predictive controller and delay functions. Since the value of reference (setpoint) is also needed in neuro-predictive controller block (to derive

$y(k)$  from error). If reference is subject to change, the block shown in Fig.14 is considered time-varying. It is a feedback controller, and at any value of error and initial conditions has an equivalent control gain and returns a control input.

Fuzzy compensator is shown as a classic block (Fig.15), its equivalent gain is definitely in the range of  $[0 \ 1]$  according to Fig.10 and (12), or:

$$0 < |C| < 1 \quad (19)$$

$$\text{As a result, if (16) is satisfied, then: } |G_c \times G_p \times H \times C| < 1 \quad (20)$$

That is, (fuzzy) compensator does not weaken stability conditions. This statement is correct for any type of controller which can be put as  $G_c$  in the circuit shown in Fig.15.

## VII. ROBUSTNESS ANALYSIS

### *VII-A. Disturbance rejection*

At the first step, a NPFC controller with  $\rho = 2$  is designed. In order to elaborate disturbance rejection, the helicopter is exposed to sudden impacts causing  $30^\circ$  rotations in the direction or against the direction of rotation. The disturbances (impacts) are exerted around the sixth second of system's operation, at the same time that the error is about  $2^\circ$  and converging to zero. Desired yaw angles are  $80^\circ$  and  $-80^\circ$ . The assumed impacts are considerably severer than impacts may be encountered in reality. Control system damps these disturbances successfully. Fig.16 shows the response of NPFC control systems under the mentioned disturbances.



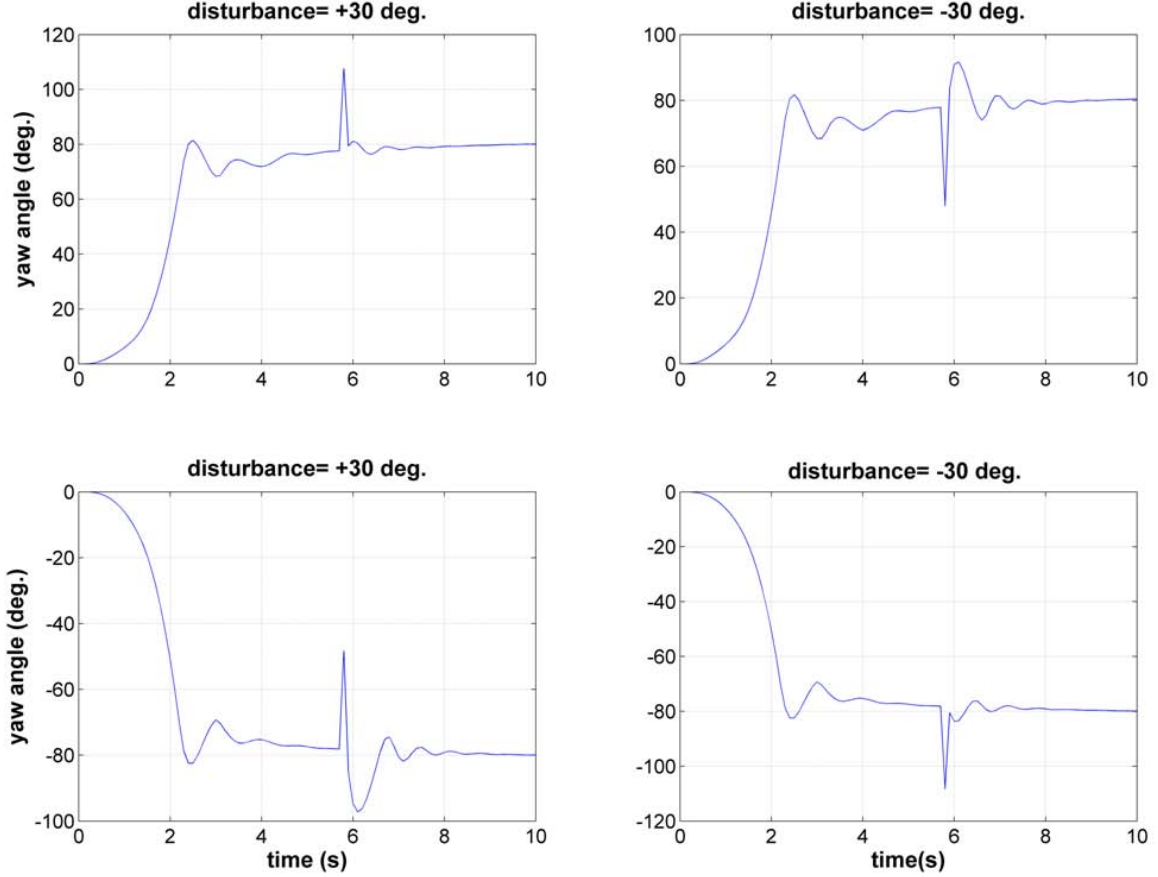


Figure 16. The behaviour of NPFC controlled system with an instantaneous disturbance

### VII-B. Parameter robustness

According to section III, (6 and 7) define systems' dynamic:

$$\dot{\psi} = \omega_{\psi}, \quad (6)$$

$$\dot{\omega}_{\psi} = \frac{1}{I_V} (r_S k_{FS} \text{sign}(\omega_S) \omega_S^2 - c_{\mu V} \omega_{\psi}). \quad (7)$$

There are four parameters in these equations; Moment of Inertia for the helicopter body around its vertical axis or  $I_V$ , the distance of rear rotor from the joint of helicopter body with its basis or  $r_S$ , rear rotor blade constant or  $k_{FS}$  and friction coefficient for rotation around vertical axis or  $c_{\mu V}$ . Among these parameters,  $I_V$  and  $r_S$  are constant geometrical

parameters. As to the operation environment of model helicopter,  $k_{FS}$  is also assumed as a constant. Therefore, the only variant parameter of system is  $c_{\mu V}$  whose original value for a properly lubricated joint is 0.0095. Dust or lack of fabrication may make this parameter increase.

Fig.17 shows the effect of a sudden increase of this parameter by 300% during operation. The increase of  $c_{\mu V}$  occurs at the fourth and sixth second of operation. NPFC control system can pass this situation satisfactorily.

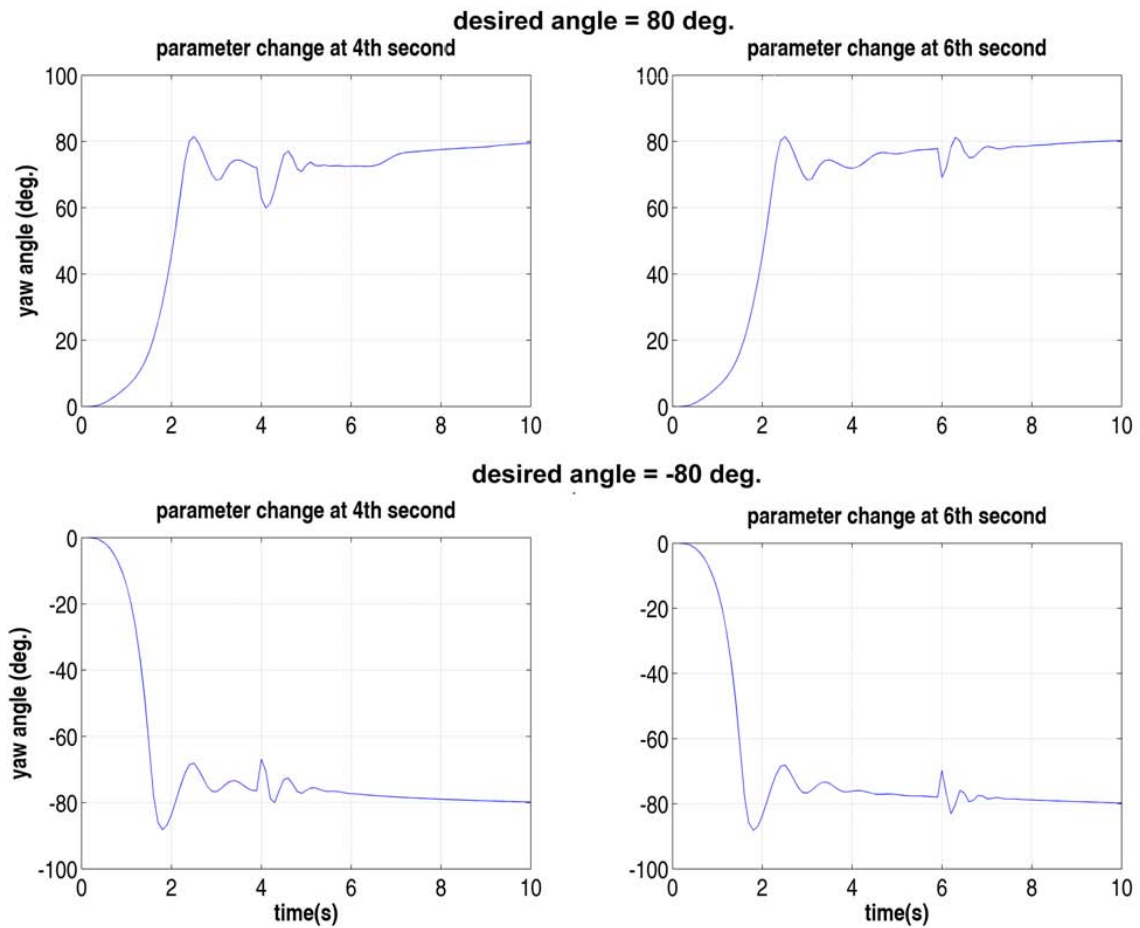


Figure 17. The behaviour of NPFC control system with a sudden parameter change

## VIII. CONCLUSION

In this research, control inertia was proposed, as a type of knowledge about the system usable in the design of intelligent (non-model-based) controllers in conjunction with input-output data and fuzzy expressions. Yaw angle control of a model/lab helicopter was addressed in this article. First, neuro-predictive technique was employed which is very effective in control of process plants. Neuro-predictive technique is merely based on input-output data of systems. Using NP technique, the control behaviour of model helicopter was not satisfactory. This discrepancy in control behaviour (in processes plants and model helicopter) with NP was clearly interpreted using the proposed concept of control inertia: model helicopter is high control inertia; whereas, process plants are generally low control inertia. Then, a fuzzy compensator was designed and added to control system. In neuro-predictive controller with fuzzy compensator (NPFC), during operation, the control input is adjusted through multiplying by a below one positive number generated by fuzzy compensator. It was indicated that the application of the designed hybrid controller improves the control behaviour of closed loop system drastically. Moreover, the disturbance rejection and parameter robustness of system were adequate as well.

## IX. APPENDIX: THE OPTIMISATION METHOD FOR NEURO-PREDICTIVE CONTROL

It has already been found that for nonlinear predictive control purposes:

$$J = J(u'). \quad (2)$$

Now,  $u'$  should be so determined that  $J$  has its minimal value. To do so, Taylor's series is written for performance function up to the second order:

$$J(u' + \Delta u') \cong J(u') + \frac{\partial J(u')}{\partial u'} (\Delta u') + \frac{1}{2} \frac{\partial^2 J(u')}{\partial u'^2} (\Delta u')^2, \quad (21)$$

the derivation of (21), leads to (22):

$$\frac{\partial J(u' + \Delta u')}{\partial (\Delta u')} \cong \frac{\partial J(u')}{\partial u'} + \frac{\partial^2 J(u')}{\partial u'^2} (\Delta u'). \quad (22)$$

In order to minimise  $J(u' + \Delta u')$ , its derivative is set equal to zero. Consequently:

$$\Delta u' \cong -\left[\frac{\partial^2 J(u')}{\partial u'^2}\right]^{-1} \frac{\partial J(u')}{\partial u'}. \quad (23)$$

Right-hand side of (23) is called Newton's direction [21].

Since (23) is an approximate relation; in order to guarantee that the performance function decreases at any stage. (23) is written in the form of (24):

$$\Delta u' \cong -\left[\frac{\partial^2 J(u')}{\partial u'^2} + \lambda\right]^{-1} \frac{\partial J(u')}{\partial u'}. \quad (24)$$

In practice (24) is used:

$$\Delta u' = u'_{new} - u'_{old} = -\eta \left[\frac{\partial^2 J(u')}{\partial u'^2} + \lambda\right]^{-1} \frac{\partial J(u')}{\partial u'}, \quad (25)$$

$$\text{where; } \lambda = d \times \frac{\partial^2 J(u')}{\partial u'^2}. \quad (26)$$

An initial value is assigned to  $d$  (i.e. 0.001), then  $\delta$  is generated:

$$\delta = \frac{\frac{\partial J(u')}{\partial u'}}{(d+1) \frac{\partial^2 J(u')}{\partial u'^2}}. \quad (27)$$

Then: if  $E(u' + \delta) < E(u')$  then  $d=d/10$ ; otherwise  $d=d \times 10$ ;

(10 is a modification factor, it can be of another value)

As a value of  $d$  is repeated, it is the result.

(25) represents Levenberg-Marquardt method for the optimisation of a single variable function [21].

In this method,  $g_k$  is numerically considered as performance function gradient:

$$\frac{\partial J(u')}{\partial u'} = g_k = \frac{J(k) - J(k-1)}{u'(k) - u(k-1)}, \quad (28)$$

Moreover,  $G_k$  is defined as:

$$\left[ \frac{\partial^2 J(u')}{\partial u'^2} \right]^{-1} = G_k = \frac{u'(k) - u(k-1)}{(g_k - g_{k-1})}. \quad (29)$$

So (25) can be rewritten in this form:

$$\Delta u' = u'_{new} - u'_{old} = -\eta(1+d)G_k g_k. \quad (30)$$

Using (30):

$$J(u'_{new}) = J(u'_{old} - \eta(1+d)G_k g_k), \quad (31)$$

$$\text{or: Argument of } J = u'_{old} - \eta(1+d)G_k g_k. \quad (32)$$

Both  $u'_{old}$  and  $(1+d)G_k g_k$  are known in this stag; with changing  $\eta$ , *Argument of J* moves along a line. There is an optimum point on this line that minimizes  $J$ . Such an optimisation problem is classified as a linear search. Backtracking method, introduced by Dennis and Schnabel [22] is selected for linear search. The modified  $u'(u'_{new})$  is used as new control input.

## X. NOMENCLATURE

ANN	Artificial Neural Network
ECC	Energy Consumption Criterion
IDC	Input Deviation Criterion
MPC	Model Predictive Control
NP	Neuro-predictive
NPFC	Neuro-predictive Controller with Fuzzy Compensation
$e$	Error of yaw angle, degree
$g$	Error gradient
H	The horizon of prediction
$I$	Moment of Inertia for the rotor (rear or main), $m^4$
$J$	Performance function
PAN	Prediction Accuracy
t	Time (s)
$T$	Torque (N.m)
$U, u$	Input voltage to the rear rotor, V, Control input
$u'$	Tentative control input
$r$	Distance (m)
$w$	Weight
$c, k$	Constant and numerator

### **Greek Letters**

$\theta$	Pitch angle (degree)
$\rho$	Factor of the constancy of control input
$\psi$	Yaw angle (degree)
$\tau$	Sampling time (s)
$\omega$	Angular velocity, rad/s
$\mu$	Membership grade

### **Subscripts**

$d$	Desired
S	Regarding rear motor
$s$	Estimated
H	Regarding horizontal axis
V	Regarding vertical axis
FS	Regarding friction of blades and air
$\theta$	Regarding pitch angle
$\psi$	Regarding yaw angle
$\mu$	Regarding mechanical friction

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## Part 3

# 'Fuzzy-Steady State' Control

### SYNOPSIS

This part includes three chapters and each chapter is a published or finally accepted journal article. In this part, a feedback-feedforward control methodology is developed. The feedforward control law is derived based on the concepts of 'control equilibrium point' and 'steady state control'. These concepts are first introduced in an Appendix of Chapter 4 and become clearer in Chapters 5 and 6. Introducing these concepts and indicating their merit in control is the main feature of this part. Artificial neural networks are employed to play the role of feedforward control law in the absence of precise mathematical models in Chapters 4 and 5.

Fuzzy inference systems are employed as feedback controllers in this part. Fuzzy inference system generates either control command (in Chapters 4 and 5) or the increments of control command (in Chapter 6). The effect of fuzzy rules associated with membership functions around the reference is checked in this part, leading to the development of damper rules in the last chapter of the part (Chapter 6).

Chapter 4 addresses pitch angle control of a model helicopter, and Chapters 5 and 6 address single command and double command concentration control of a Catalytic Stirred Tank Reactor (CSTR). The stability proof offered in Chapter 4 is much related to controller parameters; however, in Chapters 5 and 6, a number of practical assumptions were used in

stability discussion. As an innovation, the stability of heuristic fuzzy control systems is analysed in these chapters. Lack of mathematical stability analysis is a major shortage of heuristic fuzzy control systems.

## **Chapter 4**

### **Design and Stability Discussion of a Hybrid Intelligent Controller for an Unordinary System**

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# Design and Stability Discussion of a Hybrid Intelligent Controller for an Unordinary System

Morteza Mohammadzaheri, Lei Chen

**Abstract-** In this paper, the pitch angle control of a lab model helicopter is discussed. This problem has some specific features. As a major unusual feature, it is observed that the steady state control command is completely dependent on the setpoint, and for different setpoints, different steady state control commands are needed to keep the error around zero. Moreover, the system is one with highly oscillating dynamics. In order to solve this control problem, two controllers are designed: an artificial neural network, whose input is the setpoint, is used to provide steady state control command, and a fuzzy inference system, whose input is error, is used to provide transient control command. The total control command is the sum of the two aforementioned control commands. It is proven that both ANN and FIS are boundary-input boundary-output (BIBO) systems. Using this fact and considering two experimental assumptions, the closed-loop stability is also proven.

**Key words:** Neural Networks, Fuzzy, Steady State Control, Model Helicopter, Pitch Angle.

## I. INTRODUCTION

Both artificial neural networks and fuzzy controllers have been extensively investigated for use in the control of helicopters since the 1990s [1, 2]. In the field of neural networks, Enns and Si used the technique of approximate dynamic programming to control a model helicopter using ANN [3]. A few research works have also been performed on the control of a YAMAHA model helicopter using double ANN controllers and a combination of ANN with nonlinear controllers [4, 5]. ANN has also been employed as a part of a robust nonlinear feedback control of a model helicopter by researchers at the Georgia Institute of Technology [6, 7]. Neural networks have been applied in combination with linear controllers for helicopter control purposes as well [8]. As the bridge of fuzzy logic and neural networks, neuron-fuzzy networks have also been involved in helicopter control [8-11]. Among the two main types of fuzzy inference systems, the Mamdani type has attracted more attention for helicopter control purposes [12-16] and Sugeno models have

been rarely used [17]. In many cases, fuzzy controllers have been applied in the presence of some other types of controllers such as PID, LQR, and sliding mode controllers or in the presence of other artificial intelligence components, such as neural networks and genetic algorithms [1, 2, 12, 13, 16]. Although model or real helicopters usually have 2 to 6 degrees of freedom, the research projects are usually concerned with constrained situations to reduce the variables and degrees of freedom to allow complicated theories to be tested [10, 14-16, 18] yet complicated MIMO models of helicopters have also been controlled by fuzzy and ANN controllers [3, 12, 13].

## II. BACKGROUND THEORY

This section includes a brief introduction and the main features of the utilized artificial neural network and fuzzy inference system.

### *II-A. Main Features of the Utilized Artificial Neural Network*

In this research, a fully connected perceptron with three layers of neurons and two layers of connections is used. The neurons of the input and output layers have linear activation functions with a slope of 1, and hyperbolic tangent function is employed as the activation function of hidden layer neurons. The mean of squared errors is used as the performance function and a Nguyen-widrow method is utilized to designate initial values of the connection weights. The training algorithm is a Levenberg-Marquardt batch error back propagation.

### *II-B. Main Features of the Utilized Fuzzy Inference System*

In this research, a non-weighted zero-order Sugeno-type fuzzy inference system with AND connectors is used as fuzzy controller. The scheme of such a system is shown in Fig. 1. Similar to other types of fuzzy inference systems, in Sugeno-type FIS, each rule includes two main parts: antecedent and consequent.

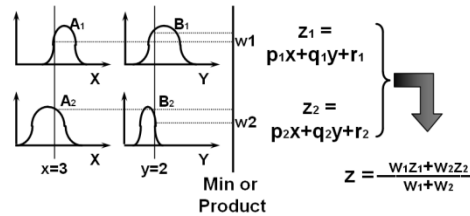
Antecedents are linguistic (fuzzy) values with membership functions. Accordance of current condition (current values of FIS inputs) with the membership function of each fuzzy value determines the membership grade. As a result, in each rule, the number of membership grades equals the number of fuzzy values in the antecedent. All these membership grades (in the range of 0 and 1) pass through a function, namely T-norm. The output of the T-norm is the weight of the rule.

$$\text{weight of rule}(w_i) = \text{Tnorm}(\text{all membership grades}), \quad (1)$$

In this research, the T-norm function is “minimum”. For instance, if the membership grade of the  $j$ th membership function of the  $i$ th rule (having  $M$  membership functions) of FIS is shown as  $\mu_j^i$ , the weight of

$$\text{the } i \text{th rule is: } w_i = \text{Min}_{j=1}^M \mu_j^i. \quad (2)$$

A weight associated with each rule ( $w_i$ ) emerges from this step. In a zero-order Sugeno-type FIS, the consequents of rules are constant numbers ( $r_i$ ), independent from current conditions or antecedents.



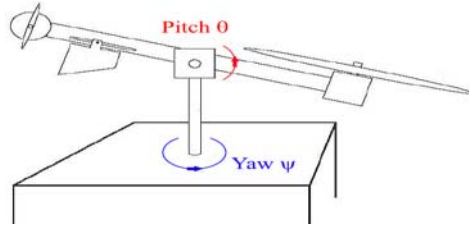
**Figure 1. A scheme of a Sugeno-type FIS**

The total output of FIS, having  $N$  rules, is calculated using following equation:

$$\text{output of FIS} = \frac{\sum_{i=1}^N r_i w_i}{\sum_{i=1}^N w_i}. \quad (3)$$

### III. MECHANICAL MODELING

The model helicopter used in this research is a two input-two output system. The model helicopter has two degrees of freedom, with the first possible motion being the rotation of the helicopter body around the horizontal axis (change in pitch angle) and the second being rotation around the vertical axis (change in yaw angle). The helicopter is produced to rotate from  $-170^\circ$  to  $170^\circ$  in yaw and from  $-60^\circ$  to  $60^\circ$  in pitch. System inputs are voltages of the main and rear rotors, and yaw and pitch angles are considered as the outputs.



**Figure 2. A scheme of model helicopter**

Mechanical modeling is done using Newton and Euler laws, and the following differential equations are derived for the system [14]:

$$\frac{d\omega_R}{dt} = \frac{1}{I_R} (T_{Main\ Rotor\ /\ Electrical} - T_{Main\ Rotor's\ Air\ Friction} - T_{Main\ Rotor's\ Mechanical\ Friction}),$$

(4)

$$\frac{d\omega_S}{dt} = \frac{1}{I_S} (T_{Rear\ Rotor\ /\ Electrical} - T_{Rear\ Rotor's\ Air\ Friction} - T_{Rear\ Rotor's\ Mechanical\ Friction}), \quad (5)$$

$$\frac{d\theta}{dt} = \omega_\theta, \quad (6)$$

$$\frac{d\omega_\theta}{dt} = \frac{1}{I_H} (T_{Main\ Rotor,R} - T_{Rear\ Rotor,R} - T_{friction,R} - T_{weight,R} + T_{Change\ in\ Rotational\ Plane,R}), \quad (7)$$



$$\frac{d\psi}{dt} = \omega_\psi, \quad (8)$$

$$\frac{d\omega_\psi}{dt} = \frac{1}{I_V} (T_{\text{RearRotor},S} - T_{\text{MainRotor},S} - T_{\text{friction},S} + T_{\text{Change in Rotational Plane},S}). \quad (9)$$

where  $\omega_R$  : Main rotor angular velocity

$\omega_S$  : rear rotor angular velocity

$\theta$  : pitch angle

$\psi$  : yaw angle

$I_R$  : main rotor moment of inertia

$I_S$  : rear rotor moment of inertia

$I_H$  : body moment of inertia around horizontal axis

$I_V$  : body moment of inertia around vertical axis

In this research, a special situation is studied. In this situation, the input voltage of the rear rotor ( $U_S$ ) is set to “zero”, so the rear rotor angular velocity ( $\omega_S$ ) equals zero as well. As a result, the only input to the system is the input voltage to the main rotor ( $U_R$ ). Moreover, the pitch angle is considered as the unique output. Inasmuch as the effect of the rear rotor on pitch angle is negligible, the studied situation can be useful for pitch angle control of the MIMO system. In the studied situation, the rear rotor does not generate any torque; consequently, in the case of a change in the rotational plane, the pitch angle is not influenced by the rear rotor. Therefore, (7) is simplified (10). This equation, along with (4) and (6), can represent the system behavior:

$$\frac{d\omega_\theta}{dt} = \frac{1}{I_H} (T_{\text{MainRotor},R} - T_{\text{friction},R} - T_{\text{weight},R}), \quad (10)$$

$$\frac{d\theta}{dt} = \omega_\theta, \quad (6)$$

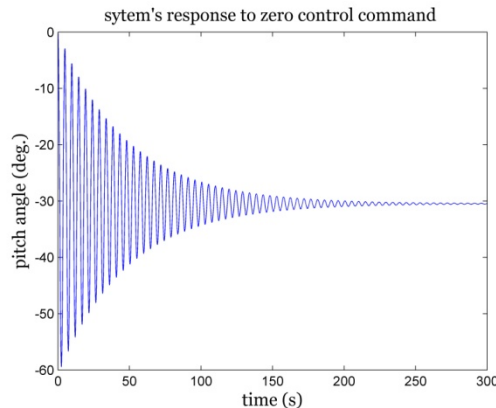
$$\frac{d\omega_R}{dt} = \frac{1}{I_R} (T_{Main\ Rotor\ /\ Electrical} - T_{Main\ Rotor's\ Air\ Friction} - T_{Main\ Rotor's\ Mechanical\ Friction}), \quad (4)$$

In these equations, the torques can be defined as a function of system's parameters, and the pitch angle can be available after integration.

#### IV. PROBLEM STATEMENT

As previously stated, the controlled system is a highly nonlinear model helicopter whose input is the voltage of the main rotor and the pitch angle is considered as the output. The aim is the control of the pitch angle so that the pitch angle approaches the desired value quickly enough. The desired pitch is in the range of  $[-50^\circ, 35^\circ]$ . This system's dynamic has two main uncommon features, which cause this problem to be considered an unusual and difficult control problem.

1) In this system (open loop), it usually takes a very long time for the system to settle into the steady state situation (even for zero input). The steady state situation is obtained only after tens of severe fluctuations (Fig. 3). If quick convergence is aimed for, this problem should be overcome.



**Figure 3. Open loop response to zero input and initial condition**

2) In the discussed model helicopter, in order to obtain a specific pitch angle (setpoint), a particular steady state control command is needed. For instance, after reaching the setpoints of  $-30^\circ$  and  $30^\circ$ , two entirely different steady state control commands are needed so that the error remains around zero. These steady state control commands are independent of the initial conditions but highly dependent on the setpoint.

## V. CONTROL LAW

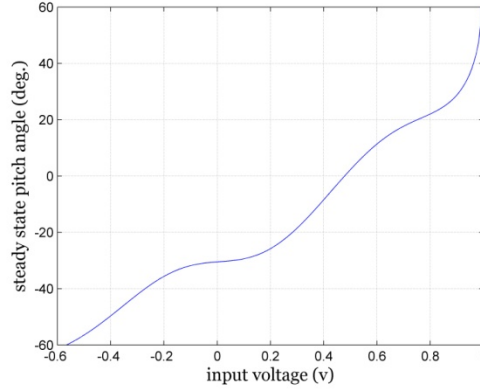
In this problem, steady state control commands are significantly dependent on the setpoint. In order to solve this control problem, the setpoint is directly considered in the control algorithm (see Appendix A), the control command is defined as the sum of two different commands, namely “steady state” ( $u_{ss}$ ) and “transient” ( $u_{tr}$ ) control commands:  $u = u_{ss} + u_{tr}$ . (11)

A hybrid intelligent control is designed, including an artificial neural network and a fuzzy inference system. ANN is responsible for generating steady state control command and transient control command is created by FIS. The command generated by the ANN controller is not influenced by the error and is generated based only on the setpoint, whereas FIS is an error-based controller.

### V-A. Design of Utilized Artificial Neural Network

Through tests, it is known that, under laboratory conditions, any tested input voltage in the range of  $[-0.55v \ +0.99v]$  leads a specific steady state pitch angle in the permitted range of  $[-60^\circ, 60^\circ]$ , independent of initial conditions. In the aforementioned range of input voltages, all voltages at intervals of  $0.01v$  are exerted on the system in the simulation environment, and their

relevant steady state pitch angle is recorded (Fig.4). As a result, a series of data is obtained. 90% of these data are selected as training data.



**Figure 4. Input voltage versus steady state pitch angle**

These data are trained to an ANN inversely; that is, the steady state pitch angle is considered as the inputs and input voltage is assumed as the output. A single input single output perceptron with a 10-neuron hidden layer having hyperbolic tangent activation functions is used for training; in this research, training is finished after 300 epochs. After training, it is expected that the ANN receives the desired pitch angle ( $\theta_d$ ) and returns the voltage approaching the system to that angle in the long term (10~15 minutes). The achieved ANN is checked by the data not used in the training. The average checking error is  $0.1134 \times 10^{-3}$  v. This error is about 10 times less than the minimum interval between voltages of training data. This checking accuracy is acceptable and unlikely to be beaten by other methods. For instance, adaptive neuron-fuzzy inference system (ANFIS) has also been tried, and the obtained checking error by ANFIS is higher than the utilized ANN by 500%. This ANN is used to calculate steady state control command ( $u_{ss}$ ). (12) shows the relation of  $u_{ss}$  and  $\theta_d$ :

$$u_{ss} = \sum_{i=1}^{10} [T_i \tanh(W_i \theta_d + {}_1b_i)] + {}_2b \quad (12)$$

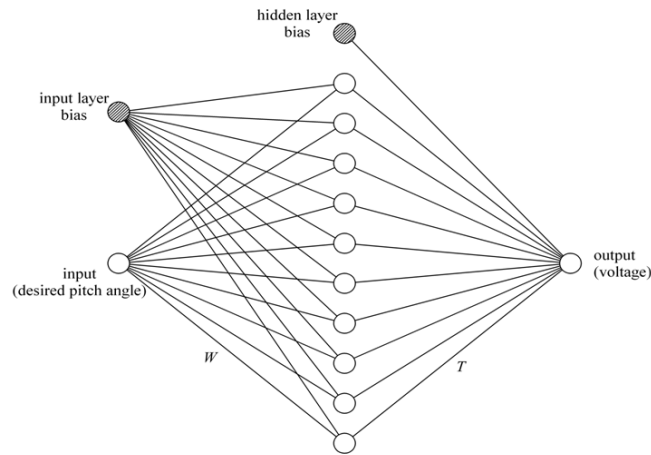
where  $w_i$  : the weight of  $i$ th connection between the input and hidden layers

$T_i$  : the weight of  $i$ th connection between the hidden and output layers

${}_1b_i$  : the weight of connection between the bias of the input layer and the  $i$ th neuron of the hidden

layer

${}_2b$  : the weight of connection between the bias of the hidden layer and output neuron



**Figure 5. A scheme of ANN controller**

#### V-B. Design of Fuzzy Inference System

As previously stated, the fuzzy controller is error-based. Control error “ $e$ ” and its differential “ $de$ ” are input signals of the fuzzy controller, in which “ $de$ ” is the difference between the current and previous error:  $de(k) = e(k) - e(k-1)$ , (13)

where;  $e(k) = s - \theta(k)$ , (14)

and the output signal is the transient control command ( $u_r$ ). In (14),  $s$  stands for setpoint.

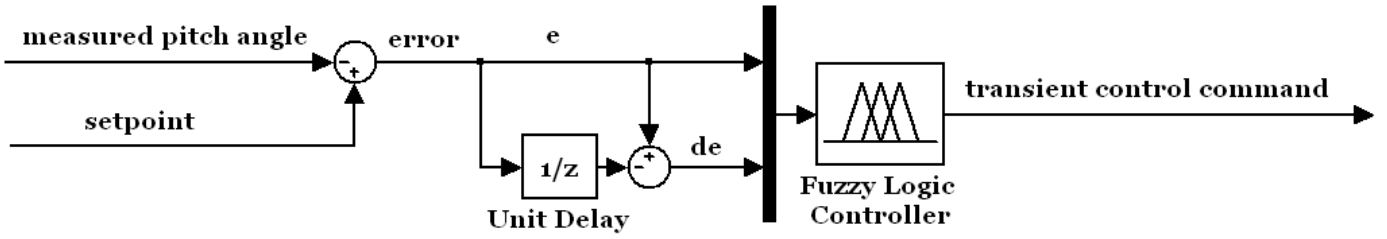


Figure 6. Fuzzy controller input signals

“*e*” has two membership functions, namely, “negative” and “positive”; Furthermore, three membership functions are associated with “*de*” namely “negative,” “positive,” and “good”.

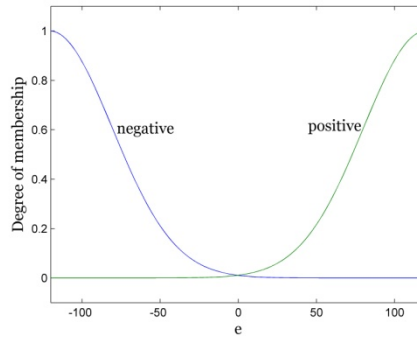


Figure 7. Membership functions of “*e*”

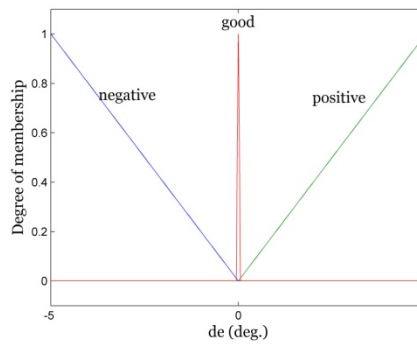


Figure 8. Membership functions of “*de*”

The membership functions of “*de*” are triangular, but the membership functions of “*e*” are Gaussian. Each Gaussian membership function has two variables (*c* and  $\sigma$ ). (15) shows a typical Gaussian membership function:

$$\mu_A^x = \exp\left[-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2\right] \quad (15)$$

For the positive membership function of “*e*”,  $c = 120$  and  $\sigma = 30$ , and for the negative membership function  $c = -120$  and  $\sigma = -30$ .

An approximate relation (shown in Table 1) can be distinguished by a field experiment which may be helpful in the design of fuzzy controllers.

**Table 1. Relation between input voltage of main rotor and pitch angle**

Input voltage ( $U_R$ or $u$ )	Impact on pitch angle ( $\theta$ )
positive	increasing (counter clock wise rotation)
negative	decreasing(clock wise rotation)

The fuzzy logic controller involves three main general design ideas. These general ideas are derived from experiments and the observation of the system’s behavior:

1) When the error absolute value is high and getting higher, force the system to rotate in the direction of error to make it vanish (see (14) and Table 1). For instance, if the setpoint is  $20^\circ$  and the current pitch angle is  $30^\circ$ , the error is  $-10^\circ$ . In this case, providing that *de* is negative (absolute value of error is increasing), according to the first general idea, a negative voltage is exerted on the system to rotate it in negative (clockwise) direction. This idea leads to the two following fuzzy rules:

R1: If *e* is *negative* and *de* is *negative* then  $u'_{rr} = -0.1(\text{volt})$

R2: If *e* is *positive* and *de* is *positive* then  $u'_{rr} = 0.1(\text{volt})$

2) When the error differential is very small, set the transient input equal to zero. This general idea of fuzzy control design is designated for the steady state situation. After approaching the setpoint, because of the fluctuating nature of the model helicopter, a trivial error may appear at any time. This error causes some control input, and chattering appears. The rule generated based on the second general idea avoids chattering effectively.

R5: If  $e$  is *good* then  $u'_{tr}=0$

3) When the error absolute value is high and getting lower, strongly force the system to rotate in the direction opposite the error direction. This odd design idea is the key point of successful control of the system. This control idea practically commands the system not to get close the setpoint when the system is approaching it. In this unusual system, quicker convergence to the setpoint is not the main concern. In reality, the biggest problem is that the system easily passes the setpoint after reaching it. In this system, severe and repeating overshoots are observed, which should be overcome for the control of system. Steady state control command ( $u_{ss}$ ) is enough for the system to reach the setpoint quickly, and halting the overshoot is the role of the transient control command ( $u_{tr}$ ). If  $u_{tr}$  accelerates the system towards the setpoint, the overshoot is magnified. The alternative is that  $u_{tr}$  decelerate the system when approaching the setpoint to avoid the overshoot. This idea leads to the 4<sup>th</sup> and 5<sup>th</sup> rules of the fuzzy controller.

R3: If  $e$  is *negative* and  $de$  is *positive* then  $u'_{tr}=0.3$  (volt)

R4: If  $e$  is *positive* and  $de$  is *negative* then  $u'_{tr}=-0.3$ (volt)

According to (2) and (3) in the Background Theory section, the transient command control will be calculated below:



$$u_{tr} = \frac{\sum_{i=1}^4 u'_{tri} \times \prod_{j=1}^2 \mu_j^i}{\sum_{i=1}^4 \prod_{j=1}^2 \mu_j^i + \mu_{good}(de)} \quad (16)$$

$i$ : the numerator of the fuzzy rules.

Figure 9 shows the total control system.

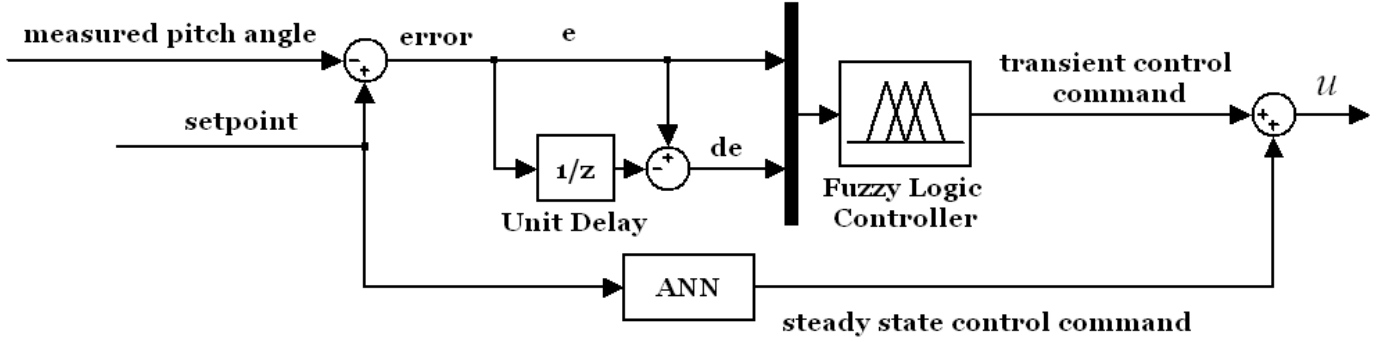


Figure 9. total control system

## VI. STABILITY REMARKS

The stability is studied based on these three assumptions: the first two are experimental assumptions as the result of numerous experiments and the last is mathematically proven:

- A. It is assumed that the contents of Table 1 are correct, for  $u \in [-1.5, 1.5]$ . The rightness of this assumption has been observed in all the experiments.
- B. It is assumed that, if the control command is in the range of  $[-0.50, 0.95]$ , the pitch angle remains in the range of  $[-50^\circ, 35^\circ]$ . This is observed through the experiments.

$$C. \text{ if } -50^\circ \leq \theta_d \leq 35^\circ \text{ then: } \begin{cases} -0.5 \leq u_{ss} \leq 0.95 \\ -0.3 \leq u_{tr} \leq 0.3 \end{cases}$$

The control system is designed to make the model helicopter's pitch angle approach the setpoints in the range of  $[-50^\circ, 35^\circ]$ . As a result, for pitch angles higher than  $35^\circ$ , the error is always negative (see (14)) and for pitch angles lower than  $-50^\circ$ , the error is always positive:

$$\theta < -50^\circ \Rightarrow e > 0 \quad (17)$$

$$\theta > 35^\circ \Rightarrow e < 0 \quad (18)$$

In the case of instability, the absolute value of the error increases; that is,  $de$  and  $e$  have the same sign. Moreover, when the system becomes unstable, the system passes out of the range of  $[-50^\circ, 35^\circ]$ . In order to check stability, the system is studied in such critical situations (pitch angle out of  $[-50^\circ, 35^\circ]$  and  $e \cdot de \geq 0$ ).

$$\text{critical situation} : \begin{cases} \theta \notin [-50^\circ, 35^\circ] \\ e \cdot de \geq 0 \end{cases} \quad (19)$$

From assumption C, it is concluded that:

$$\text{for } -50^\circ \leq \theta_t \leq 35^\circ : -0.80 < u < 1.25 \quad (20)$$

Now, considering (20) and the above assumptions, the stability is checked. Based on the control command value (see (20)), three situations are considered for the system:

I.  $-0.5 < u < 0.95$

II.  $u \geq 0.95$

III.  $u \leq -0.5$

Situation I: According to Assumption B, the system in Situation I is stable.

Situation II: if  $u \geq 0.95$  and  $e > 0$ , considering Table 1 and (13) and (14), the error will decrease towards zero and instability does not occur in this situation. If, however,  $u \geq 0.95$  and  $e < 0$ , the

system's error decreases ( $de < 0$ ) and moves to an unbounded negative value and the system becomes unstable (having an unbounded negative output with a bounded setpoint). This specific situation is named as the first critical situation:

$$\text{The first critical situation: } \begin{cases} u \geq 0.95 \\ e < 0 \\ de < 0 \end{cases} \quad (21)$$

Situation III: if  $u \leq -0.5$  and  $e < 0$ , considering Table 1, the error decreases and the pitch angle, which is currently positive (13), decreases towards zero. In this situation, the system is not subject to unbounded output and instability. In contrast, when  $u \leq -0.5$  and  $e > 0$ , the currently negative pitch angle starts to decrease to an unbounded negative value. That is, the error increases consistently ( $de > 0$ ). This is the second critical situation:

$$\text{The second critical situation: } \begin{cases} u \leq -0.5 \\ e > 0 \\ de > 0 \end{cases} \quad (22)$$

The aforementioned critical situations can lead the system to an unstable situation. Now, it is proven that these situations are impossible.

**Proof:**

The first critical situation: if  $e < 0$  and  $de < 0$ . As a result, only the 1<sup>st</sup> and the 5<sup>th</sup> rules of the fuzzy controller may be active in this situation whose outputs are -0.1 and 0. Therefore, according to Equation (3), for the first critical situation:  $-0.1 \leq u_r \leq 0$ . Considering Assumption C,  $-0.5 < u_{ss} < 0.95$ . Consequently,  $-0.6 \leq u < 0.95$ , and the first critical situation is impossible.

The second critical situation: This situation ( $e > 0$  and  $de > 0$ ) may activate the 2<sup>nd</sup> and the 5<sup>th</sup> rules of fuzzy controller. Considering (3),  $-0.1 \leq u_{rr} \leq 0$ , therefore (considering assumption C),  $-0.5 < u \leq 1.05$ . Consequently, the second critical situation never happens.

It can be concluded that the system doesn't approach an unstable situation.

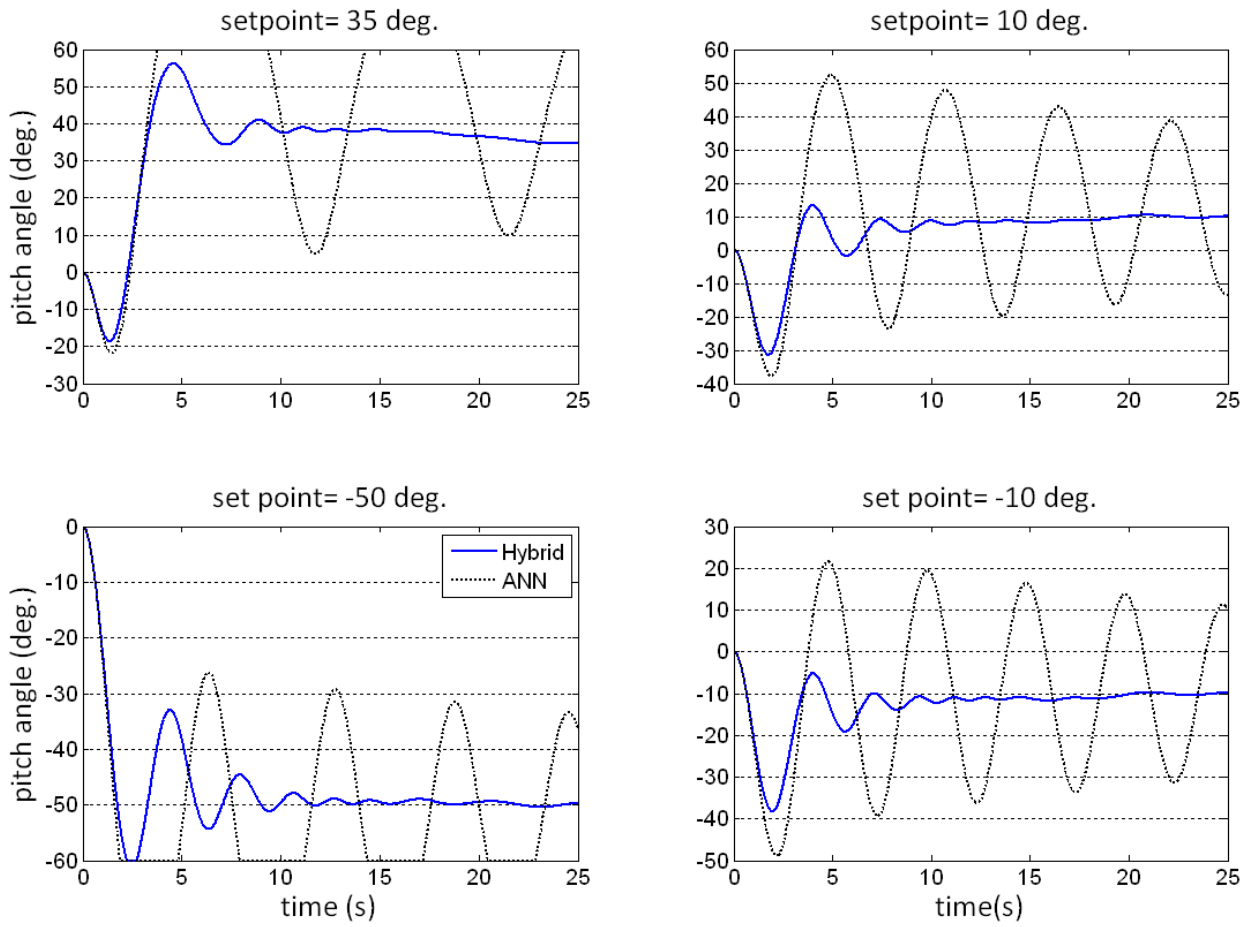
It should be noted that the basis of this proof is the existence of mathematically proven BIBO controllers (Assumption C, see Appendix B and C) and experimental assumptions A and B (without mathematical proof). In reality, since the mathematical model of the system is not involved in the stability discussion (despite model-based control), two experimental assumptions are involved instead of the mathematical model, and a totally mathematical proof is ignored. This viewpoint can be helpful in stability study of intelligent controllers as non-model based controllers. In this research, the controller is a hybrid intelligent one, not a single fuzzy or ANN, so the study of robustness as another alternative would not be easy.

## VII. SIMULATION RESULTS

Figure 10 shows the controlled system response with an initial value of zero for setpoints of  $-50^\circ$ ,  $35^\circ$  (maximum and minimum setpoints),  $10^\circ$  and  $-10^\circ$ , for both ANN and hybrid controllers.

Adding the FIS improves the performance; that is, system response converges towards and settles in the setpoint's vicinity much quicker. Also, using the hybrid controller causes the consumed energy to decline and leads to a lower maximum overshoot. In order to represent energy consumption, an Energy Consumption Criterion is defined as

$$ECC = \int_0^T |u(t)| dt . \quad (23)$$



**Figure 10. closed loop response with ANN and hybrid controller**

**Table 2. Settling time and ECC for plots of Fig. 10**

Setpoint	Controller	Type	ECC (V.s)	Settling time (s)
-50°	ANN		10.12	82.62
	Hybrid		9.49	8.18
-10°	ANN		9.63	91.52
	Hybrid		9.40	6.29
10°	ANN		14.85	99.52
	Hybrid		14.07	6.68
35°	ANN		23.56	96.39
	Hybrid		23.55	9.33

Table 2 shows *ECC* and settling time for ANN and the hybrid controller for the setpoints shown in Fig. 10. Settling time is considered to be the time needed for the controller to reduce the absolute value of the error to lower than  $5^\circ$  so that the system response is maintained in the neighborhood of the setpoint and the error no longer exceeds  $5^\circ$  (unless under the exertion of a disturbance or a change in the system's parameters or setpoint ).

Inasmuch as error-based controllers are unlikely to be able to control the pitch angle of this model helicopter as stated in the Problem Statement section (unless controllers are designed for only one setpoint, not a wide range of setpoints), it is impossible to compare the newly-designed hybrid intelligent control with more common ones as a part of simulation results. Nevertheless, considering the highly oscillating nature of the system, the achieved results seem acceptable.

## VIII. CONCLUSION

In this research, a hybrid intelligent control system is designed for pitch angle control of a lab model helicopter. It is observed that, to obtain and maintain the desired situation of the system, a steady state control command (regarding error = zero) is needed that is completely dependent on the setpoint. That is, for different setpoints, there are different steady state control commands. Therefore, an artificial neural network is designed and trained so as to provide the steady state control command using setpoint. To do so, the different values of control input are applied to the system, then the final value of the pitch angle after a long time (steady state output) is recorded. A series of these control input-output data are used to train the neural network. After training, the neural network can predict control input needed to move the system towards a special pitch angle (setpoint) and maintain the desired situation. In addition to this neural controller, a fuzzy controller is designed to generate the transient control command to be added to the steady state

control command. Fuzzy control pushes the system towards the setpoint and shortens the settling time significantly. The sum of the two control commands (the outputs of ANN and FIS) causes the system to approach its desirable pitch angle quickly and efficiently. In addition, based on two experimental assumptions, BIBO stability is proven.

## IX. APPENDIX

### *IX-A. Discussion over control problem*

Controllers that generate the control command based on the “error” are called error-based controllers, which are widely used for control purposes. In error-based controllers, the control algorithm is not practically sensitive to the setpoint. In some error-based controllers, like state vector feedback and sliding mode controllers:

$$\text{control input} = u = F_c(e, e', \dots, e^{(r)}), \text{ (in continuous form)} \quad (24)$$

$$\text{or } u = F_c(e(k), \dots, e(k-r)). \text{ (in discrete form)} \quad (25)$$

where  $e$ : error,  $r$ : system’s order,  $e^{(r)}$  is the  $r^{\text{th}}$  order temporal derivative of error, and  $F_c$  is the control law or control function(linear or nonlinear).

“Steady state control command” ( $u_{ss}$ ) is defined as the control command at the control equilibrium point where error and its derivatives are zero.

$$\text{At the equilibrium point: } e^{(n)} = 0, n = 0, \dots, \text{system's order}. \quad (26)$$

$$\text{Therefore, for controllers, follow (24, 25), } u_{ss} = F_c(0, 0, \dots, 0). \quad (27)$$

So, for this category of controllers,  $u_{ss}$  is a definite value (independent of the setpoint) if controller parameters are invariant, whereas in our control problem,  $u_{ss}$  is variant and highly dependent on the setpoint. Although integrator terms may make a way for the setpoint to

influence the control command through the history of error, past errors are also affected by initial conditions and disturbances; furthermore, the effect of previous errors through integrator terms tends to zero in the steady state situation (especially in the discrete domain). It seems the consideration of the setpoint is unlikely to be possible if only error-based controllers are employed. Therefore, a neural controller is also designed to work just based on the setpoint.

A. The proof of:  $-50^\circ \leq \theta_d \leq 35^\circ \Rightarrow -0.5 \leq u_{ss} \leq 0.95$

The output of ANN is calculated, as below:

$$u_{ss} = \sum_{i=1}^{10} [T_i \tanh(W_i \theta_d + {}_1b_i)] + {}_2b$$

(12)

**Table 3. Weights of designed ANN**

$i$	$W_i$	$T_i$	${}_1b_i$	$\beta_i$	${}_2b$
1	0.1103	0.2155	-13.9495	0.0238	0.0256
2	0.3462	0.0017	-10.4470	0.0006	
3	-0.0978	0.0140	5.3300	-0.0014	
4	0.2163	0.0591	-4.4890	0.0128	
5	0.0746	0.0946	-1.2272	0.0071	
6	0.1565	0.0690	4.8111	0.0108	
7	-1.9003	-0.0434	-57.7793	0.0824	
8	-0.0152	-0.8253	-0.7601	0.0125	
9	-0.5048	-0.0559	-15.3745	0.0282	
10	-0.1393	-0.0968	-9.0363	0.0135	

All values of  $T$  and  $W$  are known from training and presented in Table 3. The output values of ANN, subject to input values of  $-50^\circ$  and  $35^\circ$ , are  $-0.4046$  and  $0.9418$ , which are limited and in the permitted range of  $[-0.5, 0.95]$ . Providing that  $u$  is an increasing (non-decay) function of  $\theta_d$  in the range of  $[-50^\circ, 35^\circ]$ , the outputs values of ANN subject to inputs values of  $-50^\circ$  and  $35^\circ$



are the minimum and maximum output values of ANN if  $-50^\circ \leq \theta_d \leq 35^\circ$ . That is,  $-50^\circ \leq \theta_d \leq 35^\circ \Rightarrow -0.5 \leq u_{ss} \leq 0.95$ . So, one just needs to prove that the derivative of  $\theta$  to the input ( $u$ ) is positive throughout this range.

Main proof:

$$\frac{d[\tanh(\theta_d)]}{d\theta_d} = \text{sech}(\theta_d), \quad (28)$$

$$\text{the } \frac{du}{d\theta_d} = \sum_{i=1}^{10} T_i W_i [\text{sech}(W_i \theta_d + b_i)]. \quad (29)$$

The derivative is re-written in this form:

$$\frac{du}{d\theta_d} = \sum_{i=1}^{10} \beta_i [\text{sech}(\alpha_i)], \quad (30)$$

where:  $\beta_i = T_i W_i$ ,  $\alpha_i = W_i \theta_d + b_i$ .

$$\forall x \in R : \text{sech}(x) = \frac{2}{e^x + e^{-x}} > 0 \quad (31)$$

$$\text{then } \forall i | 1 \leq i \leq 10 : \text{sech}(\alpha_i) > 0 \quad (32)$$

$$\text{Considering Table.3: } \forall i | 1 \leq i \leq 10, i \neq 3 : \beta_i > 0. \quad (33)$$

Surprisingly, there is only one negative value of  $\beta$ . It should be proven that this negative value

never leads to a negative value of  $\frac{du}{d\theta_d}$ .

$$\text{Since: } \forall i | 1 \leq i \leq 10, i \neq 3 : \beta_i [\text{sech}(\alpha_i)] > 0. \quad (34)$$

(both  $\text{sech}(\alpha_i)$  and  $\beta_i$  are positive)

If it could be proven that:

$$\exists i | 1 \leq i \leq 10, i \neq 3 : \beta_i [\text{sech}(\alpha_i)] > |\beta_3 [\text{sech}(\alpha_3)]|, \quad (35)$$

it would be concluded that the derivative value is positive in the range.

The proof of (35):

$$\text{Using (31), } \text{sech}(x) = \text{sech}(-x) = \text{sech}|x|, \quad (36)$$

$$\text{then: } \text{sech}(\alpha_3) = \text{sech}|\alpha_3| = \text{sech}|-0.0978\theta_d + 5.3300|, \quad (37)$$

$$\text{sech}(\alpha_5) = \text{sech}|\alpha_5| = \text{sech}|0.0746\theta_d - 1.2272|. \quad (38)$$

In the defined range of  $[-50^\circ, 35^\circ]$  both  $|\alpha_3|$  and  $|\alpha_5|$  are linear functions of  $\theta_d$  and, consequently, are partially monotonic. Furthermore, in the aforementioned range,

$$\text{Min}|\alpha_3| > \text{Max}|\alpha_5|;$$

It can be concluded that:

$$\forall \theta_d \mid -50^\circ \leq \theta_d \leq 35^\circ : |\alpha_3| > |\alpha_5|.$$

(39)

$$\text{Lemma: } \forall (x_1, x_2 \in R^+) : x_1 > x_2 \Rightarrow \text{sech}(x_1) < \text{sech}(x_2)$$

Considering Lemma, the above relation leads to

$$\text{sech}|\alpha_5| > \text{sech}|\alpha_3| > 0 . \quad (40)$$

$$\text{Besides, if } i = 5, \text{ it is observed that } \beta_i = \beta_5 = 0.0071 \text{ then } |\beta_5| > |\beta_3| \quad (41)$$

$$((40), (41)) \Rightarrow |\beta_5 \text{sech}|\alpha_5|| > |\beta_3 \text{sech}|\alpha_3||. \quad (42)$$

$$\text{Considering (36): } |\beta_5 \text{sech}(\alpha_5)| > |\beta_3 \text{sech}(\alpha_3)| . \quad (43)$$

$$\text{Since both } \beta_5 > 0 \text{ and } \text{sech}(\alpha_5) > 0, \text{ then } \beta_5 \text{sech}(\alpha_5) > |\beta_3 \text{sech}(\alpha_3)|. \quad (44)$$

Which satisfies the condition offered in (35).

End of proof

The proof of Lemma:

$$\frac{d}{dx} \operatorname{sech}(x) = -\operatorname{sech}(x) \tanh(x) . \quad (45)$$

$$\forall x \in R : \operatorname{sech}(x) = \frac{2}{e^x + e^{-x}} > 0 . \quad (31)$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \Rightarrow \tanh(0) = 0 . \quad (46)$$

After derivation:  $\frac{d}{dx} \tanh(x) = \operatorname{sech}(x) > 0$

Therefore, “tanh” is a monotonically increasing (non-decay) function. Since  $\tanh(0) = 0$  (46), it is concluded that:  $\forall x \in R^+ : \tanh(x) > 0$ . (47)

Considering (31) and (45),  $\forall x \in R^+ : \operatorname{sech}(x) \tanh(x) > 0$ . (48)

As a result of (45) and (48),  $\forall x \in R^+ : \frac{d}{dx} \operatorname{sech}(x) < 0$ .

So, it is proven that, for positive real numbers, “sech” is a monotonically decreasing (decay) function.

End of proof of Lemma

**B. The proof of:**  $-50^\circ \leq \theta_d \leq 35^\circ \Rightarrow -0.3 \leq u_{tr} \leq 0.3$

$u'_{tr}$ , as the output of fuzzy controller, is calculated using (16).

$$u_{tr} = \frac{\sum_{i=1}^4 u'_{tri} \times \operatorname{Min}_{j=1}^2 \mu_j^i}{\sum_{i=1}^4 \operatorname{Min}_{j=1}^2 \mu_j^i + \mu_{good}(de)} . \quad (16)$$

Since  $0 \leq \mu_{good}(de) \leq 1$ , the higher  $\mu_{good}(de)$ , the lower the absolute value of  $u'_{tr}$ . Inasmuch as  $u'_{tr}$  is of both negative and positive values, its minimum and maximum values should have the

greatest absolute values. Therefore, for maximizing or minimizing values of  $u_{tr}$ , the value of  $\mu_{good}(de)$  should be zero. As a result, the minimum and maximum values are only influenced by the first four rules and  $\mu_{good}(de)$  can be ignored. With this assumption, (16) can be re-written as:

$$\bar{u}_{tr} = \frac{\sum_{i=1}^4 u'_{tri} \times \text{Min}_{j=1}^2 \mu_j^i}{\sum_{i=1}^4 \text{Min}_{j=1}^2 \mu_j^i}. \quad (49)$$

where;  $\bar{u}_{tr}$  is  $u_{tr}$  without  $\mu_{good}(de)$ . In fact,

$$\max(\bar{u}_{tr}) = \max(u_{tr}),$$

(50)

$$\text{and, } \min(\bar{u}_{tr}) = \min(u_{tr}). \quad (51)$$

According to (2), (49) can be re-written:

$$\bar{u}_{tr} = \frac{\sum_{i=1}^4 u'_{tri} \times w_i}{\sum_{i=1}^4 w_i}. \quad (52)$$

where;  $w_i$  is the weight of rules and,

$$\text{since } \forall i, j \in R : 0 < \mu_j^i < 1, \quad (53)$$

$$\text{then; } \forall i \in \{1,2,3,4\} : 0 \leq w_i \leq 1. \quad (54)$$

$$\text{therefore, } 2w_1 + 4w_2 + 6w_3 \geq 0. \quad (55)$$

$$(55) \text{ can be re-written as } w_2 - w_1 + 3w_3 \geq -3w_1 - 3w_2 - 3w_3. \quad (56)$$

$$\text{After adding } -3w_4 \text{ to both sides, } w_2 - w_1 + 3w_3 - 3w_4 \geq -3w_1 - 3w_2 - 3w_3 - 3w_4 \quad (57)$$

$$\text{or } w_2 - w_1 + 3(w_3 - w_4) \geq -3(w_1 + w_2 + w_3 + w_4). \quad (58)$$

$$\text{According to (54), } w_1 + w_2 + w_3 + w_4 \geq 0. \quad (59)$$

$$\text{then } \frac{w_2 - w_1 + 3(w_3 - w_4)}{(w_1 + w_2 + w_3 + w_4)} \geq -3, \quad (60)$$

$$\text{by dividing both sides by 10, } \frac{0.1(w_2 - w_1) + 0.3(w_3 - w_4)}{(w_1 + w_2 + w_3 + w_4)} \geq -0.3. \quad (61)$$

Since  $u'_{rr1} = -0.1$ ,  $u'_{rr2} = 0.1$ ,  $u'_{rr3} = 0.3$  and  $u'_{rr4} = -0.3$ , the left-hand side of (61) is the right-hand side of (52).

$$\bar{u}_{rr} = \frac{\sum_{i=1}^4 u'_{rri} \times w_i}{\sum_{i=1}^4 w_i} \geq -0.3. \quad (62)$$

$$\text{Additionally, it can be written as } 2w_1 + 4w_2 + 6w_4 \geq 0 \quad (63)$$

$$\text{Consequently, } 3w_1 + 3w_2 + 3w_4 \geq w_2 - w_1 - 3w_4. \quad (64)$$

$$\text{After adding } 3w_3 \text{ to both sides, } 3(w_1 + w_2 + w_3 + w_4) \geq w_2 - w_1 + 3(w_3 - w_4). \quad (65)$$

$$\text{According to (58), } \frac{w_2 - w_1 + 3(w_3 - w_4)}{(w_1 + w_2 + w_3 + w_4)} \leq 3. \quad (66)$$

$$\text{By dividing both sides by 10, } \frac{0.1(w_2 - w_1) + 0.3(w_3 - w_4)}{(w_1 + w_2 + w_3 + w_4)} \leq 0.3. \quad (67)$$

Since  $u'_{rr1} = -0.1$ ,  $u'_{rr2} = 0.1$ ,  $u'_{rr3} = 0.3$  and  $u'_{rr4} = -0.3$ , the left-hand side of (67) is the right-hand side of (52).

$$\bar{u}_{rr} = \frac{\sum_{i=1}^4 u'_{rri} \times \text{Min}_{j=1}^2 \mu_j^i}{\sum_{i=1}^4 \text{Min}_{j=1}^2 \mu_j^i} \leq 0.3, \quad (68)$$

(50), (51), (62), (68) prove that:  $-0.3 \leq u_{rr} \leq 0.3$

End of proof

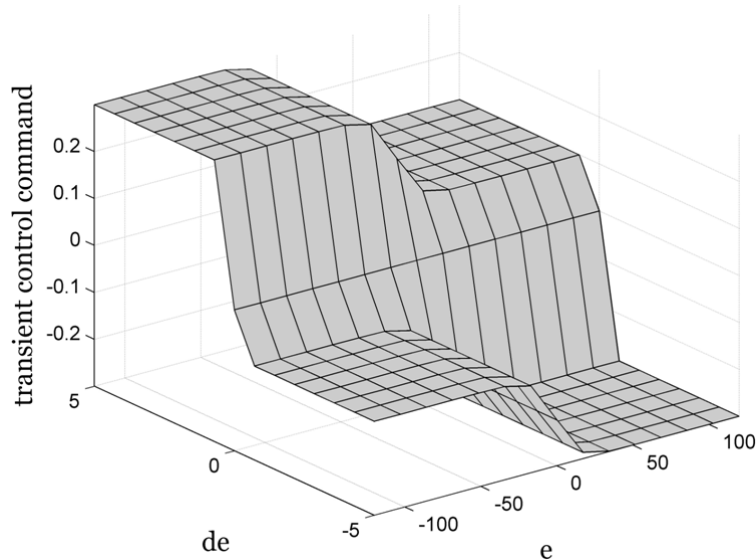


Figure 11. the response of fuzzy logic controller

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## **Chapter 5**

### **Intelligent Control of a Nonlinear Tank**

A brief version of this article will be published in Asian Journal of Control



# Intelligent Control of a Nonlinear Tank Reactor

Morteza Mohammadzaheri, Lei Chen

*Abstract-* In this paper, intelligent control of a particular Catalytic Continuous Stirred Tank Reactor (CSTR) is addressed. Control command is the sum of two components: steady state and transient commands. A fuzzy controller generates transient control command pushing the system towards the setpoint. Steady state control command is generated to maintain the steady state situation at the setpoint (based on the concept of 'control equilibrium point'). This research was performed in the simulation environment; however, the mathematical model of the system was not used during stability analysis; this retained the usefulness of the methodology in the face of considerable uncertainties; instead, certain facts about the system, in the form of fuzzy rules were used for stability analysis, referred to here as the 'fuzzy rough model'. Using this technique, Lyapunov asymptotic stability of the control system was proved. For comparison, the case study was also controlled by neuro-predictive algorithm. The studied CSTR is known as a very good example of neuro-predictive control application; however, the newly proposed hybrid intelligent method leads to much better setpoint tracking as well as less change in control command which is a very important consideration in implementation of the control system.

**Key words:** Fuzzy, Neural Network, Steady State Control, CSTR, Lyapunov Stability.

## I. INTRODUCTION

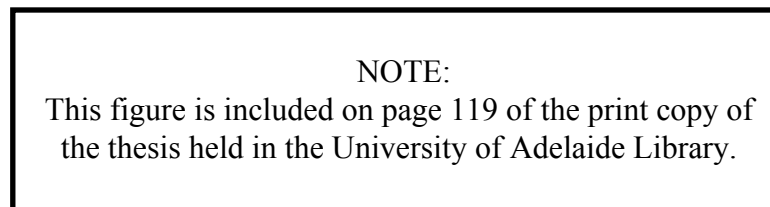
As a well-known benchmark, Catalytic Continuous Stirred Tank Reactors (CSTR)s have been extensively studied in control. These systems are multi-input and multi-output and sometimes highly nonlinear. Self-tuning PID [1, 2], robust controllers [3] adaptive-like control systems [4] and different kinds of nonlinear predictive controllers [5, 6] have been successfully tested on this class of chemical systems. The CSTR is known as an outstanding example of the application of neuro-predictive controllers [7] as a subset of nonlinear predictive controllers. Fuzzy logic controllers are the other types of controllers used in the control of CSTR, and are used to generate either the control command directly [8, 9] or control command increment [10, 11]. In addition to quick convergence to the reference (setpoint), stability guarantee [8] and disturbance rejection [12, 13] are the other motivations for the application of fuzzy control systems in CSTRs.

In this research, a fuzzy controller is so designed that the system is asymptotically stable

according to the Lyapunov theorem. There are many theoretically outstanding works in the literature regarding the stability analysis of fuzzy systems using Lyapunov theorem [14-18] and in some of them fuzzy models contain some uncertainties [16-18]. These research works, generally, use a Sugeno-type fuzzy model, as a weighted summation of linear functions, and based on this model, a controller is designed and the stability of the closed loop system is addressed. Stability analysis usually leads to some inequalities to be satisfied [15, 16, 18]. However, in this research, instead of rather precise Sugeno-type fuzzy models, a fuzzy rough model, which is unable to predict the system's response, has the main role in stability analysis. This fuzzy rough model is a Mamdani-type fuzzy inference system based on the certain facts about the system's behaviour.

## II. S YSTEM DYNAMICS

The case study is a Catalytic Continuous Stirred Tank Reactor (CSTR). A diagram of the process is shown in the following Figure:



**Figure1. A schematic of the studied CSTR [7]**

Two flows of liquid enter the reactor with the concentration of  $C_{b1} = 24.9 \text{ (kmol/m}^3\text{)}$  and  $C_{b2} = 0.1 \text{ (kmol/m}^3\text{)}$ . The flow rates of input flows are named  $u_1$  and  $u_2$ . The reactor outlets another flow of liquid with the concentration of  $C_b$  and the flow rate of  $w$ . The height of liquid in the reactor ( $h$ ) is another variable of the system.

A simplified mathematical model of the system, achieved by mass equilibrium equations

[7, 19-21] is

$$\frac{dh(t)}{dt} = u_1(t) + u_2(t) - 0.2\sqrt{h(t)}, \quad (1)$$

$$\frac{dC_b(t)}{dt} = [C_{b1} - C_b(t)]\frac{u_1(t)}{h(t)} + [C_{b2} - C_b(t)]\frac{u_2(t)}{h(t)} - \frac{k_1 C_b(t)}{1 + k_2 C_b(t)^2}, \quad (2)$$

where  $k_1$  and  $k_2$  are the coefficients related to outlet valve resistance; both of these coefficients are considered equal to 1 in this paper. The effect of output flow mass rate on the output concentration is the main source of nonlinearity. If the concentration of the output flow and the height of liquid level are considered as the outputs of the system ( $w = 0.2\sqrt{h}$ ), the total system can be shown in Fig.2:

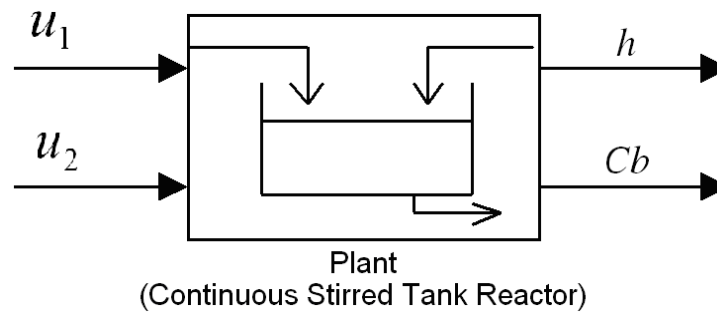


Figure 2. The studied CSTR as a MIMO system

### III. CONTROL SYSTEM

In this problem,  $u_2 = 0.1$  litres/min and the output concentration ( $C_b$ ) is regulated by adjusting  $u_1$ . That is, the control system is single-input single-output. A schematic of the control system is shown in Fig.3. As previously stated, total control command ( $u_1$ ) is a combination of two control commands, transient and steady state (3).

$$u_1 = u_t + u_{ss} \quad (3)$$

The saturation function bounds control command in the range of [0 4].

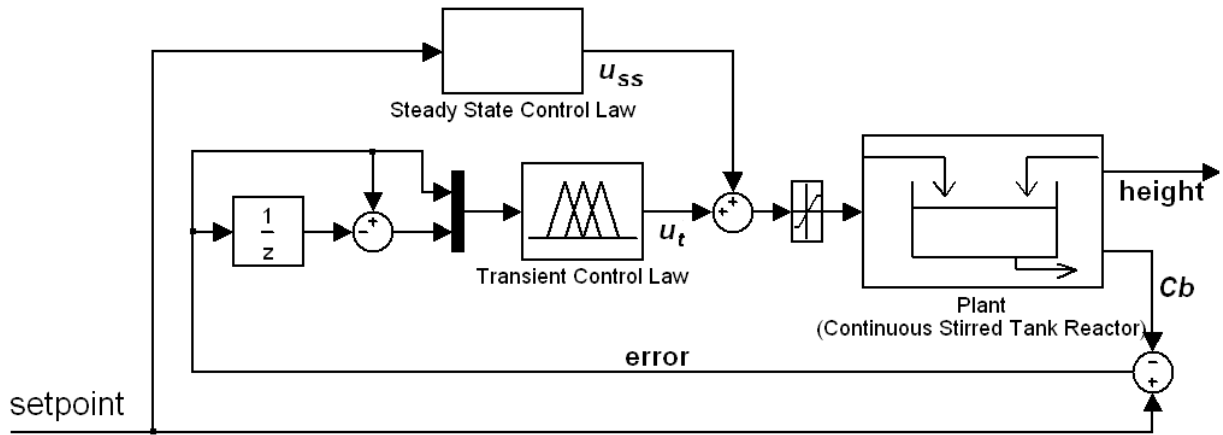


Figure 3. A Schematic of Control System

#### IV. STEADY STATE CONTROL COMMAND

The studied system has an interesting feature: if no control command is applied to the system, the temporal derivative of the concentration never equals zero before thorough evacuation of the reactor. Therefore, a continuous steady state control command is needed to maintain the desired situation. In this research, we employ the concept of ‘control equilibrium point’ (defined in (4)) to find out this steady state control command.

$$\frac{d^{(i)}e}{dt^{(i)}} = 0, i = 0, \dots, \text{system's order} \quad (4)$$

If  $C_d$  stands for the setpoint, the error is  $e = C_d - C_b$  (5)

$$\frac{de(t)}{dt} = [-24.9 + C_b(t)] \frac{u_1(t)}{h(t)} + [-0.1 + C_b(t)] \frac{0.1}{h(t)} + \frac{C_b(t)}{1 + C_b(t)^2}. \quad (6)$$

As previously stated, the height of liquid level ( $h$ ) is not very important and can be ignored in this problem;  $h$  is not considered as a state from control viewpoint. At the control equilibrium point

$$\begin{cases} e = 0 \\ \dot{e} = 0. \end{cases} \quad (7)$$

The solution of (7) generates a “steady state control command” maintaining the steady state

situation for the system at the setpoint.

From system dynamics, it is observed that  $u_{ss}$  must be dependent on the setpoint and the height. Steady state control law is derived as below:

$$u_{ss} = h \left[ (-0.1 + C_d) \frac{0.1}{h} + \frac{C_d}{1 + C_d^2} \right] (24.9 - C_d)^{-1}. \quad (8)$$

However, in order to offer a general methodology working in the absence of a mathematical model, an artificial neural network is also designed to find out the control command which maintains the setpoint. This neural network plays the role of steady state control law (8).

In order to design and train a neural network, a set of training/checking data is needed. Since initial conditions have negligible effect on the steady state situation, it can be considered that both level height and concentration are the functions of the control input, in this situation. So different control inputs in a special range of [0.2 1.5] (*litres / min*), with an increment rate of 0.001 (*litres / min*) are applied to the system and the steady state concentration after 500 seconds is recorded. As a result, in the steady state situation, several sets of data are collected in the following form:

$$[C_b(k) \ u_1(k)], k=1, \dots, n \quad (9)$$

A perceptron having a 5-neuron hidden layer is trained. The activation function of hidden layer is

$$f(x) = \frac{1}{1 + e^{-x}} \quad (10)$$

where  $C_b$  is considered as the input to the neural network, and  $u_1$  ( $u_{ss}$ ) is the output. 90% of the collected data are used for training; the rest of data are used for checking.

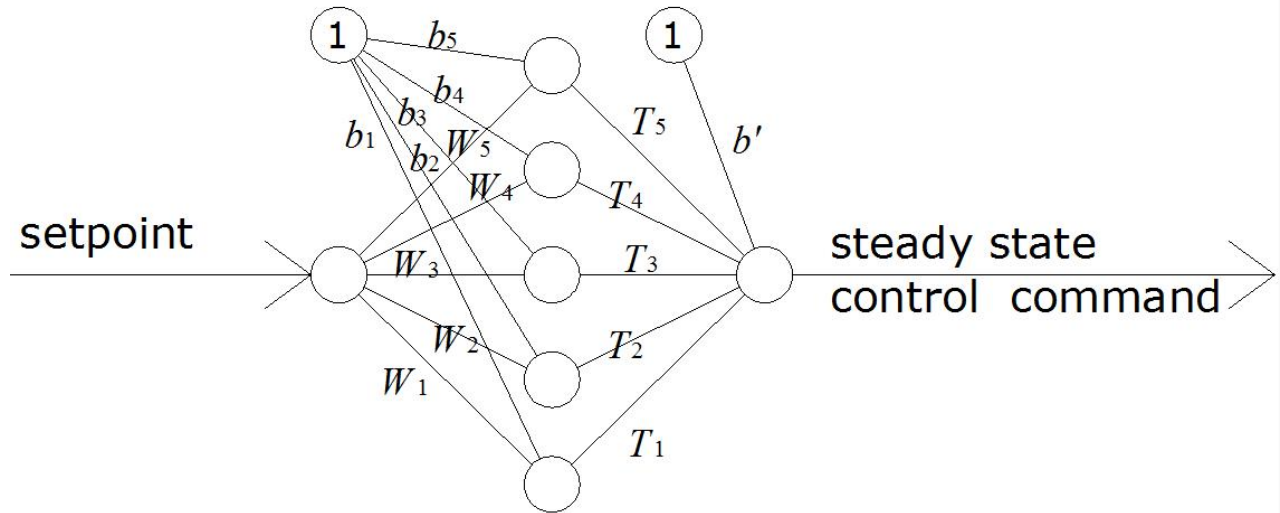


Figure 4. The artificial neural network used in this research

The accuracy of the trained ANN is very good:

$$\text{Checking(Verification)Error} = \frac{\sum_{i=1}^N |e_i|}{N} = 3 \times 10^{-5} (\text{litres/min}) \quad (11)$$

The values of weights and biases are listed in Table.1.

Table 1. Weights of the neural network

$W_1$	0.429	$T_1$	1.7564	$b_1$	-3.129	$b'$	0.121
$W_2$	18.16	$T_2$	0.0004	$b_2$	-22.69		
$W_3$	-19.38	$T_3$	-0.0007	$b_3$	17.623		
$W_4$	19.73	$T_4$	0.0090	$b_4$	-11.98		
$W_5$	-20.43	$T_5$	-0.0010	$b_5$	6.898		

With such good accuracy, the designed neural network can be employed to generate the steady state control command.

## V. TRANSIENT CONTROL COMMAND

A fuzzy controller is used to generate transient control command to be added to the steady state control command. The inputs to fuzzy controller are the error ( $e$ ) and its differential

(de):

$$de = e(1 - z^{-1}) \quad (12)$$

where  $z^{-1}$  is unit delay.

Fuzzy controller outputs transient control command ( $u_t$ ), and is formed by seven rules. The main idea of rules 1 to 4, address two situations: when the error is increasing (rules 1 and 4) and when the error is decreasing (rules 2 and 3); in this recent situation the transient control input is less in order to reduce the overshoot similar to rule 7 which works in the neighborhood of the reference. Rules 4 and 5 address extreme situations. Fuzzy controller is so designed that the stability of the closed loop system can be proved by Lyapunov method.

R1: if  $e$  is *positive* and  $de$  is *positive* then  $u_t = 10e$

R2: if  $e$  is *positive* and  $de$  is *negative* then  $u_t = 4e$

R3: if  $e$  is *negative* and  $de$  is *positive* then  $u_t = 4e$

R4: if  $e$  is *negative* and  $de$  is *negative* then  $u_t = 10e$

R5: if  $e$  is *very positive* then  $u_t = 4$

R6: if  $e$  is *very negative* then  $u_t = -2$

R7: if  $e$  is *zero* then  $u_t = 0$

Membership functions are shown in Figs.5 and 6.

Zero is a very narrow Gaussian membership function:

$$\mu_{zero}(x) = \exp[-0.5(\frac{x}{0.001})^2]. \quad (13)$$

*positive* and *negative* are also Gaussian membership functions.

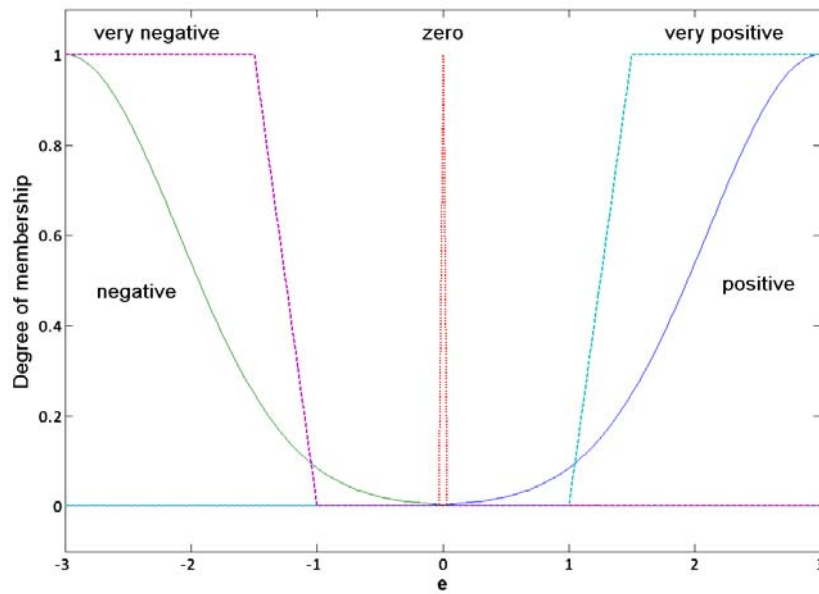
$$\mu_{positive}(x) = \exp[-0.5(\frac{x-3}{0.9})^2], \quad (14)$$

$$\mu_{negative}(x) = \exp[-0.5(\frac{x+3}{-0.9})^2]. \quad (15)$$

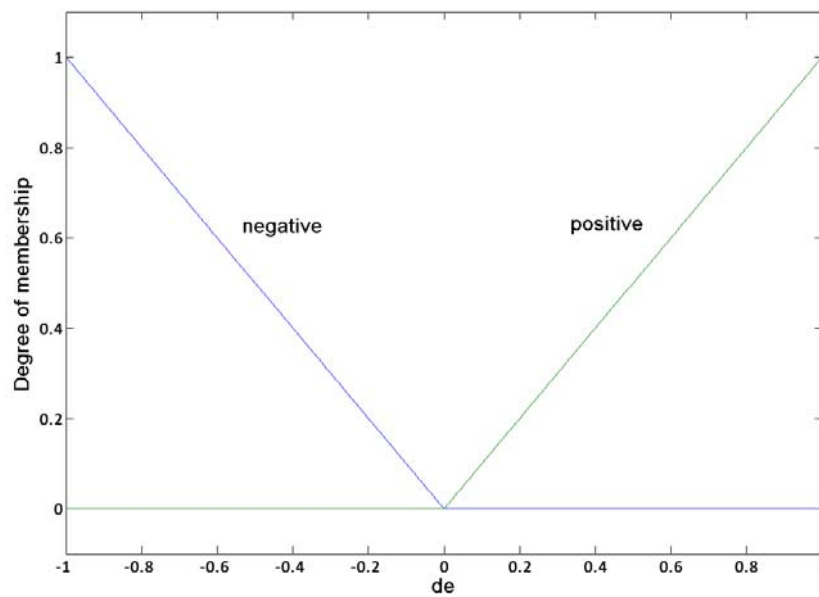
*very negative* and *very positive* are trapezoidal:

$$\mu_{\text{very positive}}(x) = \begin{cases} 0, & x \leq 1 \\ 2(x-1), & 1 \leq x \leq 1.5 \\ 1, & x \geq 1.5 \end{cases} \quad (16)$$

$$\mu_{\text{very negative}}(x) = \begin{cases} 0, & x \geq -1 \\ -2(x-1), & -1.5 \leq x \leq -1 \\ 1, & x \leq -1.5 \end{cases} \quad (17)$$



**Figure 5. Membership functions of error**



**Figure 6. Membership functions of error differential**



## VI. SIMULATION RESULTS

In order to indicate the merit of the designed Lyapunov-based Intelligent (LI) controller, simulation results are offered. Simulation was done both for the LI controller and the Neuro-predictive (NP) controller with  $\rho = 0.1$  and  $\rho = 0.01$  (For further information about NP technique, see the Appendix 3). This problem is known as a successful application of NP control [7, 19]. Moreover, the mathematical model is not used directly in the design of neuro-predictive controllers or the proposed LI method.

In the presented simulation, the initial height of the tank was 30 cm and the initial value of the outlet concentration of CSTR was  $18 \text{ kmol/m}^3$ . In this example, setpoint (reference) changed very quickly to show the capabilities of the controllers.

Two design criteria were defined; the error integral (EI) and the input change (IC):

$$EI = \frac{\int_0^{\tau} |e| dt}{\tau}, \quad (18)$$

$$IC = \frac{\int_0^{\tau} |u(t + \Delta t) - u(t)| dt}{\tau}. \quad (19)$$

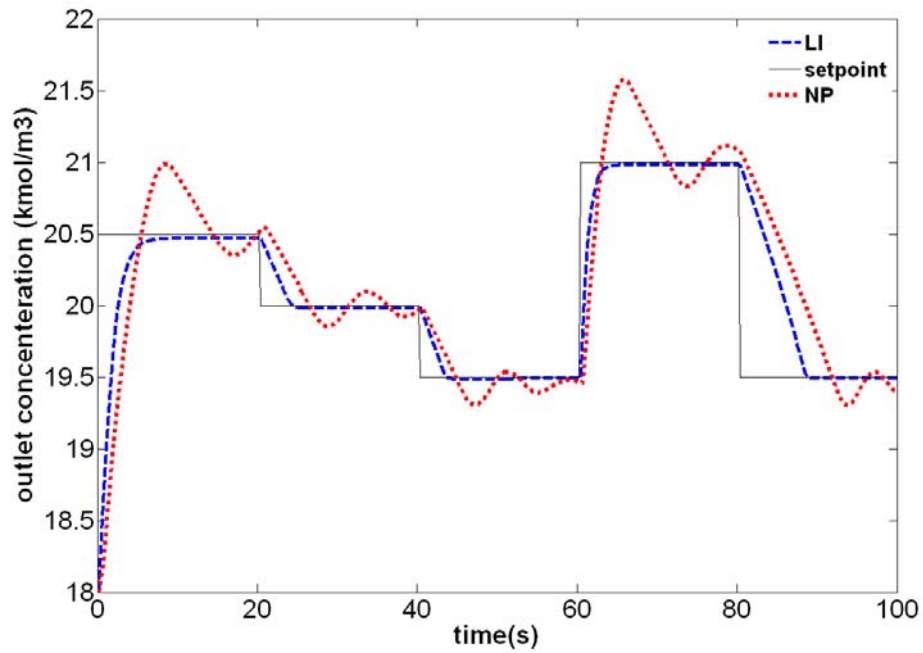
The results are shown in Table II and Fig.7. ‘Time’ is the time needed to simulate 100 seconds of the operation of the closed loop system, with the sample time of 0.2 s, using a dual core processor (4200 MHz) and MATLAB/Simulink software. Figure 8 shows the control input generated by LI control system during the operation period shown in Fig.7.

Lower values of  $\rho$  may lead to a better performance of the Neuro-predictive controller in simulation, but this is at the price of increasing change in control input which makes the controller practically impossible to implement. For instance, with  $\rho = 0.01$ , a very low error integral of  $0.2278 \text{ kmol/m}^3$  is obtained with an excessively high input change of  $0.1938 \text{ litres/min}$ . Even with such low values of  $\rho$ , the performance of the proposed Lyapunov-based intelligent controller is not beatable by NP. Moreover, the proposed algorithm is much

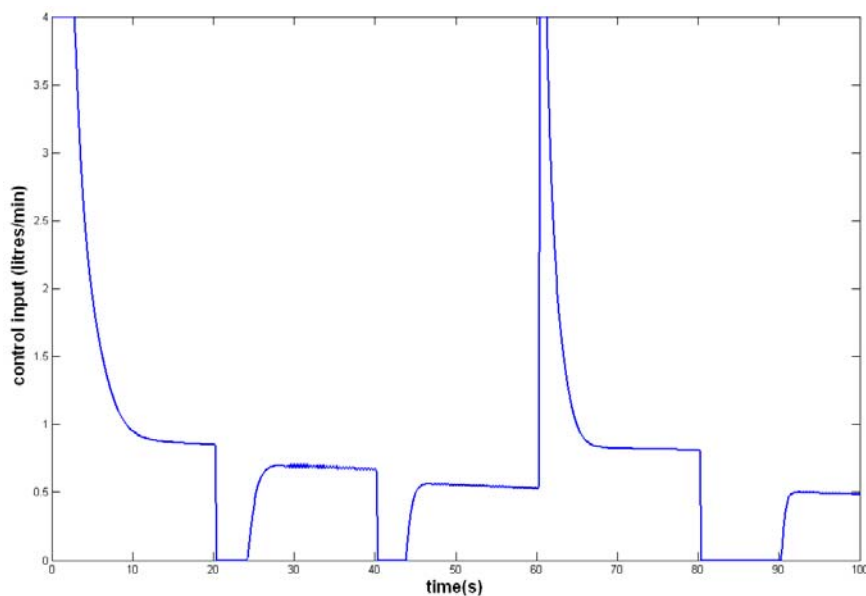
more efficient than the NP algorithm because it needs much less computation time.

**Table 2. Simulation results for different optimization methods**

Optimisation Method	$EI$ ( $kmol/m^3$ )	$IC$ (litres/min)	Time (s)
LI	0.1529	0.0293	8
NP( $\rho=0.1$ )	0.3084	0.0606	225
NP( $\rho=0.01$ )	0.2278	0.1938	225



**Figure 7. Simulation results for Lyapunov-based Intelligent control and Neuro-predictive control with  $\rho=0.1$**



**Figure 8. Control input during the operation period shown in Fig.7**

## VII. DISTURBANCE REJECTION

In this section, a very severe disturbance is exerted on the system to check the disturbance rejection capabilities of the proposed Lyapunov-based intelligent (LI) controller. In the controller, rules 1 and 4 are peculiar to disturbance rejection. It was assumed that 4 litres of liquid with a concentration of  $0.1 \text{ kmol/m}^3$  or  $24.9 \text{ kmol/m}^3$  was suddenly poured into the reactor within 0.2 second, when the system was already in steady state situation at the setpoint of  $17 \text{ kmol/m}^3$ . A successful disturbance rejection was observed. This disturbance was simulated by adding an input of 1200 litres/min to the system for 0.2 second.

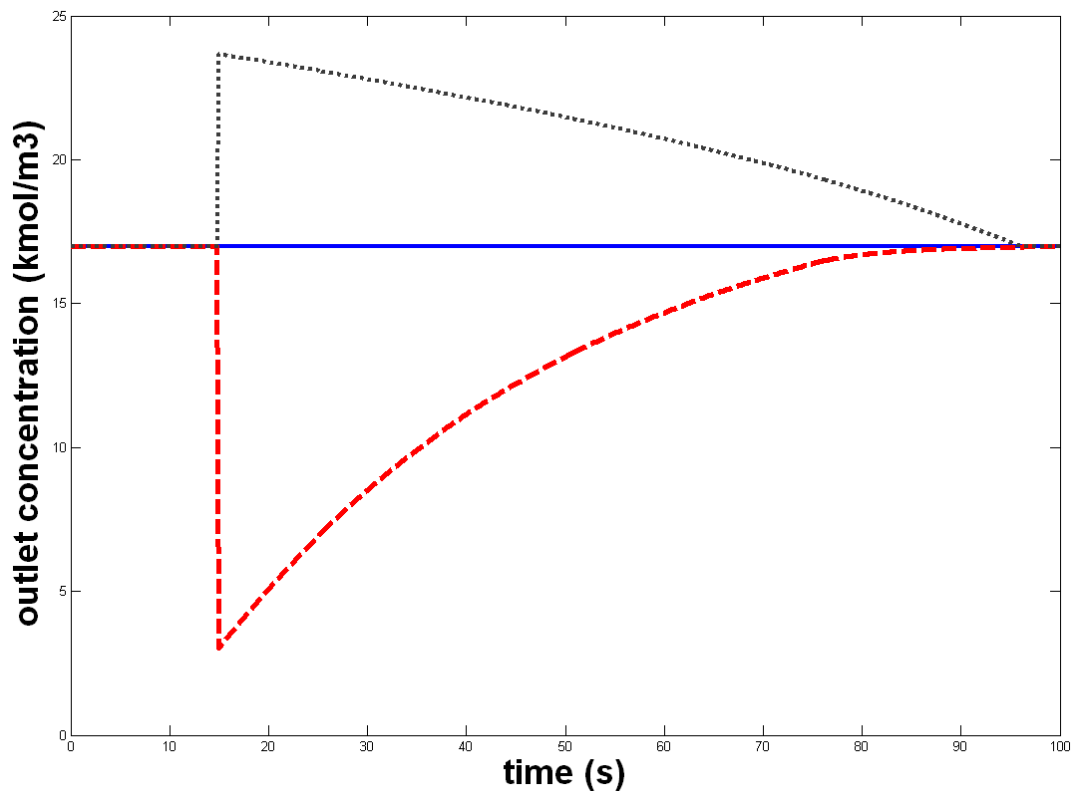


Figure 9. System response against the disturbance

## VIII. CONCLUSION

In this research, the case study for control is a reactor losing its desired situation in the absence of a continuous control command. There are many other control problems with a

similar situation (mainly flow and temperature control problems). Without consistent exertion of a control input, such systems cannot have (natural) equilibrium points at their desired situations (setpoints). In this paper, the concept of control equilibrium point is presented to derive a setpoint-based feedforward control law maintaining the desired situation; in other words, the combination of the aforementioned control law and the system will have an equilibrium point at the setpoint (reference). Feedforward steady state control command removes the steady state error, and this theoretically alleviates the need for integrator or integrator-like terms in feedback control. In order to avoid the effect of model uncertainties, in the absence of an accurate mathematical model of the system, an artificial neural network generates the steady state control command.

Transient control command which pushes the system towards setpoint is an error-based fuzzy controller with a design based on Lyapunov direct method. This research was performed in the simulation environment; however, the use of the mathematical model in design/analysis was avoided to make the results more reliable in practice. Therefore, in the study of satisfaction of Lyapunov conditions, a fuzzy rough model was used instead of the mathematical model. A fuzzy rough model includes fuzzy rules expressing some certain facts about the system that do not the other facts. Similar fuzzy rough models can be made for many other process plants. Using this attitude, asymptotic stability of the control system was (approximately) indicated. Also, the proposed control system was shown to offer very good performance compared to the neuro-predictive method, one of the well known nonlinear methods in stirred reactors' control. In addition to lower error integral, the change of input flow is much lower in the proposed method which makes it more suitable for real applications. Furthermore, compared to the neuro-predictive method, the proposed method is much more computationally efficient. If the control output changes when there is no control input and if the assumptions, similar assumptions proposed in Appendix 1, are true about the

system; this method can be used in other systems. For instance, temperature control problems can be very good cases for the proposed control method.

## IX. APPENDIX 1: STABILITY REMARKS

Lyapunov function used in this research is error square and the states vector is  $\mathbf{x} = [e \ \dot{e}]$ :

$$\begin{cases} V(\mathbf{x}) = e^2 \\ \dot{V}(\mathbf{x}) = 2e\dot{e}. \end{cases} \quad (20)$$

According to the Lyapunov theorem,  $V(\mathbf{x})$  should have a continuous first temporal derivative and meet the following criteria:

$$\begin{cases} V(\mathbf{x}) > 0 \\ \dot{V}(\mathbf{x}) \leq 0 \end{cases} \quad (21)$$

$$\text{If } \dot{V}(\mathbf{x}) = 0 \Rightarrow \mathbf{x} = \mathbf{0} \text{ ,} \quad (22)$$

the equilibrium point and the system is globally asymptotically stable.

According to (20), (22) means that  $e\dot{e} = 0 \Rightarrow \begin{cases} e = 0 \\ \dot{e} = 0 \end{cases}$ . Consequently, the system is

asymptotically stable ((21) is met), if:

Condition 1:  $e\dot{e} \leq 0$ , and

Condition 2:  $\dot{e} = 0 \Leftrightarrow e = 0$ .

Not using a mathematical model, in order to indicate the stability of the system, three practical assumptions were used which are clearly observed during simulation.

Assumption 1) There are negative and positive values for control command (for negative or positive errors) which are able to push the system towards the setpoint (reference) immediately after exertion at any initial conditions.

Assumption 2) Provided that just steady state control command ( $u_{ss}$ ) is exerted on the

system, in the vicinity of setpoint,  $\dot{C}_b|_{ss} = \left. \frac{dC_b}{dt} \right|_{u=u_{ss}} \approx 0$  (23)

Assumption 3) The higher (lower)  $u_1$ , the higher (lower)  $\dot{C}_b$ .

These assumptions can be expressed in the form of fuzzy rules. Such fuzzy rules, which are usually easy to obtain through observation, form fuzzy rough models.

Fuzzy rough model: A fuzzy rough model is a set of fuzzy rules which are always correct (in the operation area) and do not contradict other facts about the system.

These approximate models cannot predict the systems' response; they can, however, explain systems' behaviour correctly and are especially suitable for systems with significant uncertainties.

A combination of a fuzzy controller and a fuzzy rough model explains the behaviour of the closed loop system.

According to assumptions 1~3 and the information obtained by observation, following fuzzy rules can explain the behavior of the system (when  $u_2 = 0.1 \text{ litres/min}$ ):

M1: if  $u_1 = 4$  then  $\dot{C}_b$  is *positive* .

M2: if  $u_1 = 0$  then  $\dot{C}_b$  is *negative* .

M3: if  $u_1 = u_{ss}$  and  $e$  is *zero* then  $\dot{C}_b$  is *zero* .

Both *zero* membership functions are convex membership functions:

$$\mu_{zero}(x) = \begin{cases} 1, & x = 0 \\ 0, & x \gg 0. \end{cases} \quad (24)$$

M4: if  $u_1$  is *high* then  $\dot{C}_b$  is *high*

M5: if  $u_1$  is *low* then  $\dot{C}_b$  is *low*

*low* and *high* (for both  $u$  and  $\dot{C}_b$ ) are monotonic membership functions with the following

characteristics:

$$\mu_{low}(x) = \begin{cases} 1, & x \rightarrow -\infty \\ 0, & x \rightarrow +\infty \end{cases} \quad (25)$$

and

$$\mu_{high}(x) = \begin{cases} 0, & x \rightarrow -\infty \\ 1, & x \rightarrow +\infty. \end{cases} \quad (26)$$

More detailed information was not available through observation.

Three operation areas were considered, based on the closeness to the reference. At each of them, a fuzzy rough model was presented and the engaged rules of fuzzy controller were specified. Then the aforementioned stability conditions were checked.

**A. The first area:  $e > 1.5$ .**

In this operation area, a useful fuzzy rough model is:

M1: if  $u = 4$  then  $\dot{C}_b$  is *positive*.

This fuzzy rule is correct in the whole operation area, and does not contradict any of the assumptions. Therefore, it can play the role of a fuzzy rough model solely.

Rules of the controller, engaged with this operation area, are

R1: if  $e$  is *positive* and  $de$  is *positive* then  $u_t = 10e$ ;

R2: if  $e$  is *positive* and  $de$  is *negative* then  $u_t = 4e$ ;

R5: if  $e$  is *very positive* then  $u_t = 4$ .

$u_{ss} \geq 0$  and the value of control command generated by the engaged rules is obviously higher than 4 when  $e > 1.5$ , so  $u_t = 4$  *litres/min* (after passing the saturation function). Considering the fuzzy rough model,  $\dot{C}_b > 0$ .

Considering (10),  $\dot{e} = -\dot{C}_b < 0$ .

So  $e\dot{e} \leq 0$ ; that is, the first stability condition is satisfied.  $e = 0$  cannot occur in this operation

area, so  $e = 0 \Rightarrow \dot{e} = 0$  is always true. Also, based on M3 and M4,  $\dot{e} = 0$  cannot occur ( $u_1 = 4 > u_{ss} \Rightarrow \dot{C}_b > \dot{C}_b|_{ss}$ ); therefore,  $\dot{e} = 0 \Rightarrow e = 0$  is always true as well, and the second condition is also satisfied.

### **B. The second area: $e < -1.5$**

In this operation area, a useful and correct fuzzy rough model is

M2: if  $u_1 = 0$  then  $\dot{C}_b$  is *negative*.

Rules of the controller, engaged with this operation area, are

R3: if  $e$  is *negative* and  $de$  is *positive* then  $u_t = 4e$ ;

R4: if  $e$  is *negative* and  $de$  is *negative* then  $u_t = 10e$ ;

R6: if  $e$  is *very negative* then  $u_t = -2$ .

In this area, the value of the transient control command is obviously lower than -2; since  $0.2 \leq u_{ss} \leq 1.5$  then  $u_t < -u_{ss}$ , after passing the saturation function  $u_1 = 0$  (see (3)).

Considering the fuzzy rough model,  $\dot{C}_b < 0$  then  $\dot{e} > 0$  and  $e\dot{e} \leq 0$ . Also, based on M3 and M5,  $e \neq 0$  and  $\dot{e} \neq 0$ . As a result, similar to the first operation area, stability conditions are satisfied.

### **C. The third area: $-1.5 \leq e \leq 1.5$**

The area of  $-1.5 \leq e \leq 1.5$  is assumed to be the setpoint's vicinity or the support of *zero* membership function of the error. The engaged rules of the fuzzy controller with this operation area are

R1: if  $e$  is *positive* and  $de$  is *positive* then  $u_t = 10e$ ;



R2: if  $e$  is *positive* and  $de$  is *negative* then  $u_t = 4e$  ;

R3: if  $e$  is *negative* and  $de$  is *positive* then  $u_t = 4e$  ;

R4: if  $e$  is *negative* and  $de$  is *negative* then  $u_t = 10e$  ;

R7: if  $e$  is *zero* then  $u_t = 0$  .

In this area, these three fuzzy rules form the fuzzy rough model.

M3: if  $u_1 = u_{ss}$  and  $e$  is *zero* then  $\dot{C}_b$  ( $\dot{C}_b|_{ss}$ ) is *zero*;

M4: if  $u_1$  is *high* then  $\dot{C}_b$  is *high* ;

M5: if  $u_1$  is *low* then  $\dot{C}_b$  is *low* .

According to the engaged rules of the fuzzy controller, if  $e > 0$  then  $u_t > 0$  , so  $u_1$  will be higher than  $u_{ss}$  (8). Consequently, according to the fuzzy rough model (M4), « $\dot{C}_b$  will be higher<sup>1</sup> than  $\dot{C}_b|_{ss}$ ». Moreover, in this operation area, « $\dot{C}_b|_{ss}$  is *zero*» (close to 0) according to fuzzy rough model (M3). Therefore,  $\dot{C}_b > 0 \Rightarrow \dot{e} < 0 \Rightarrow e\dot{e} < 0$  .

Similarly, in the case  $e < 0$

$$u_t < 0 \stackrel{(3)}{\Rightarrow} u_1 < u_{ss} \stackrel{M5}{\Rightarrow} \dot{C}_b < \dot{C}_b|_{ss} \stackrel{M3}{\Rightarrow} \dot{C}_b < 0 \stackrel{(5)}{\Rightarrow} \dot{e} > 0 \Rightarrow e\dot{e} < 0 .$$

In order to check the second condition of stability ( $\dot{e} = 0 \Leftrightarrow e = 0$ ), the engaged fuzzy rules (specially rule 7) were considered: at  $e=0$ ,  $u_t = 0$  , then  $u_1 = u_{ss}$  so  $\dot{C}_b = \dot{C}_b|_{ss}$  . According to M3 at  $e=0$ ,  $\dot{C}_b|_{ss} \cong 0$  , therefore  $\dot{C}_b \cong 0 \Rightarrow \dot{e} \cong 0$  (for a fixed setpoint). In summary,  $e = 0 \Rightarrow \dot{e} \cong 0$  .

In order to check the correctness of  $\dot{e} = 0 \Rightarrow e = 0$  , proof by contradiction was used. In other words, it was indicated that  $e \neq 0 \Rightarrow \dot{e} \neq 0$  :

$$e \neq 0 \stackrel{R1-4}{\Rightarrow} u_t \neq 0 \stackrel{(3)}{\Rightarrow} u_1 \neq u_{ss} \stackrel{M4-5}{\Rightarrow} \dot{e} \neq 0$$

---

<sup>1</sup> higher: having a higher membership degree in *high* membership function, see fuzzy rough model

In total, both stability conditions are (approximately) satisfied in the area of  $-1.5 \leq e \leq 1.5$ .

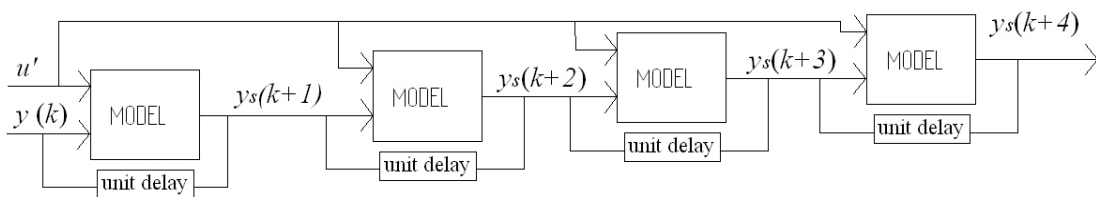
## XI. APPENDIX 2: NEURO-PREDICTIVE CONTROL

In neuro-predictive control, an artificial neural network (ANN) is used to predict the behaviour of nonlinear systems, and an optimisation method generates the control command (represented by  $u'$ ) based on minimising a performance function involving the predicted errors. (27) is a typical performance function (represented by  $J$ ) in neuro-predictive control:

$$J(k) = \sum_{i=1}^N [y_s(k+i) - y_d]^2 + \rho [u'(k) - u(k-1)]^2, \quad (27)$$

where  $y_s$  and  $y_d$  are the estimated and the desired outputs of the system respectively, and  $u'$  and  $u$  are tentative and actual control inputs;  $\rho$  represents the importance of the constancy of the control input.

At the first stage of generating the tentative control command, the performance function ( $J$ ) should be calculated. To do so, the output values of the system should be predicted for  $N$  future instants (see (27)), so the nonlinear model (neural network) should be used  $N$  times.  $N$  is called the horizon of prediction.



**Figure 10. Prediction of output values with the horizon of 4**

If the current output and the previous output/input of the system (the measured data) and  $u'$  are known, all other arguments of  $J$  will be definitely known (see Fig.10). So these arguments can not be subject to modification by optimisation algorithms. However,  $u'$  can be changed arbitrarily freely from the measured input/output signals and this change affects the

other arguments of  $J$ , and consequently, the performance function itself. Therefore, in the optimisation for control purposes, it can be assumed that  $J = J(u')$ . (28)

In summary, in the design of neuro-predictive controllers, two main tasks should be performed, neural network modeling and optimisation.

#### ***A. Neural Modeling of CSTR (with sole control command of $u_1$ )***

The modeling was performed particularly for the purpose of predictive control. The studied CSTR has two control inputs,  $u_1$  and  $u_2$ ; the second input flow (with the concentration of  $0.1 \text{ kmol/m}^3$ ) is set to the constant value of  $0.1 \text{ litres/min}$ . As a result, this value is not considered in modeling as an input signal anymore. Moreover, the order of two is assumed for the model.

NOTE:  
This figure is included on page 136 of the print copy of  
the thesis held in the University of Adelaide Library.

**Figure 11. Dynamic model of CSTR, when the flow rate of an input flows is fixed [20]**

This model (presented in Fig.11 and (29)) is used to return the first estimated value of  $C_b$  :

$$[\hat{C}_b(k+1), \hat{h}(k+1)] = F[u_1(k-1), u_1(k), C_b(k-1), C_b(k), h(k-1), h(k)] \quad (29)$$

or

$$\hat{C}_b(k+1) = F_1[u_1(k-1), u_1(k), C_b(k-1), C_b(k), h(k-1), h(k)] \quad (30)$$

$$\hat{h}(k+1) = F_2[u_1(k-1), u_1(k), C_b(k-1), C_b(k), h(k-1), h(k)]. \quad (31)$$

After very first instants of the prediction,

$$[\hat{C}_b(k+1), \hat{h}(k+1)] = F[u_1(k-1), u_1(k), \hat{C}_b(k-1), \hat{C}_b(k), \hat{h}(k-1), \hat{h}(k)] \quad (32)$$

or

$$\hat{C}_b(k+1) = F_1[u_1(k-1), u_1(k), \hat{C}_b(k-1), \hat{C}_b(k), \hat{h}(k-1), \hat{h}(k)] \quad (33)$$

$$\hat{h}(k+1) = F_2[u_1(k-1), u_1(k), \hat{C}_b(k-1), \hat{C}_b(k), \hat{h}(k-1), \hat{h}(k)] \quad (34)$$

where variables with hat are the estimated ones.

Although only one predicted value is often used in predictive control (i.e.  $C_b$ ), all the outputs should be estimated, because most of the systems are dynamic and the outputs are usually coupled (in this problem,  $h$ , as the representative of liquid volume, affects the value of  $C_b$ ).

Therefore, the estimated values of all the outputs are needed to predict any of them for a period of time in the future.

After the definition of the model's order, the training data should be normalized and arranged. 8000 sets of data (including  $u_1$ ,  $h$  and  $C_b$ ) with the sampling time of 0.2 second are utilized in training. The normalized data were arranged as below:

$$\left[ \begin{array}{cc|cc|cc|cc} \overbrace{u_1(1)} & \overbrace{u_1(2)} & \overbrace{C_b(1)} & \overbrace{C_b(2)} & \overbrace{h(1)} & \overbrace{h(2)} & \overbrace{C_b(3)} & \overbrace{h(3)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \underbrace{u_1(7998)} & \underbrace{u_1(7999)} & \underbrace{C_b(7998)} & \underbrace{C_b(7999)} & \underbrace{h(7998)} & \underbrace{h(7999)} & \underbrace{C_b(8000)} & \underbrace{h(8000)} \end{array} \right] \quad (35)$$

A four-layer recurrent perceptron was trained using the prepared data (35) [20]. The input layer of the utilized perceptron has six neurons (equal to input signals). This ANN has one nonlinear (with sigmoid activation functions) and one linear (with linear activation functions with slope of 1) hidden layers. Both hidden layers have 13 neurons. The output layer also has

two neurons with linear activation functions with the slope of 1. The linear hidden layer may seem useless at first glance, because even without this linear hidden layer, a linear combination of the outputs of the neurons of the nonlinear hidden layer is generated at the output layer. In other words, the structure of the mathematical relation between the input and the output of ANN is the same with or without the linear hidden layer; however, adding an extra linear hidden layer improved the accuracy in practice. It seems a wider variety of adjusting parameters allowed the model to be trained more successfully. The training method was Levenberg-Marquardt error back propagation. The (batch) training was performed in 100 epochs and the performance function was the sum of squared errors (MSE).

In this research, two different series of checking data were used. Both series were entirely different from the training data.

A criterion is defined for the predictive accuracy of the models, namely PAN:

$$PAN = \sum_{i=1}^N |\hat{C}_b(i) - C_b(i)| \quad (36)$$

Table III shows PA10 and PA30 (the sum of absolute error of prediction for 10 and 30 future instants or next 2 or 6 seconds), for two different series of checking data.

**Table 3. Prediction accuracy of the neural network**

Criterion	PA10 ( $kmol/m^3$ )		PA30( $kmol/m^3$ )	
	1 <sup>st</sup> series	2 <sup>nd</sup> series	1 <sup>st</sup> series	2 <sup>nd</sup> series
Perceptron Model	0.018	0.022	0.051	0.033

### **B. Optimisation**

It has already been found that for nonlinear predictive control purposes:

$$J = J(u'). \quad (28)$$

Now,  $u'$  should be so determined that  $J$  has its minimal value. To do so, Taylor's series is written for performance function up to the second order:

$$J(u' + \Delta u') \cong J(u') + \frac{\partial J(u')}{\partial u'}(\Delta u') + \frac{1}{2} \frac{\partial^2 J(u')}{\partial u'^2}(\Delta u')^2. \quad (37)$$

The derivation of (37), leads to (38):

$$\frac{\partial J(u' + \Delta u')}{\partial (\Delta u')} \cong \frac{\partial J(u')}{\partial u'} + \frac{\partial^2 J(u')}{\partial u'^2}(\Delta u'). \quad (38)$$

In order to minimise  $J(u' + \Delta u')$ , its derivative is set equal to zero. Consequently:

$$\Delta u' \cong -\left[\frac{\partial^2 J(u')}{\partial u'^2}\right]^{-1} \frac{\partial J(u')}{\partial u'}. \quad (39)$$

The right-hand side of (39) is called Newton's direction [22].

Since (39) is an approximate relation. In order to guarantee that the performance function decreases at any stage, (39) is written in the form of (40):

$$\Delta u' \cong -\left[\frac{\partial^2 J(u')}{\partial u'^2} + \lambda\right]^{-1} \frac{\partial J(u')}{\partial u'}. \quad (40)$$

In practice (41) is used:

$$\Delta u' = u'_{new} - u'_{old} = -\eta \left[\frac{\partial^2 J(u')}{\partial u'^2} + \lambda\right]^{-1} \frac{\partial J(u')}{\partial u'}, \quad (41)$$

$$\text{where } \lambda = d \times \frac{\partial^2 J(u')}{\partial u'^2}. \quad (42)$$

An initial value is assigned to  $d$  (i.e. 0.001), then  $\delta$  is generated:

$$\delta = \frac{\frac{\partial J(u')}{\partial u'}}{(d+1) \frac{\partial^2 J(u')}{\partial u'^2}}. \quad (43)$$

Then, if  $E(u' + \delta) < E(u')$  then  $d=d/10$ ; otherwise  $d=d \times 10$ ;

(10 is a modification factor; it can have another value)

As a value of  $d$  is repeated, this value is the result ( $d$  in (42)).

(41) represents the Levenberg-Marquardt method for the optimisation of a single variable function [22].

In this method,  $g_k$  is numerically considered as the performance function gradient:

$$\frac{\partial J(u')}{\partial u'} = g_k = \frac{J(k) - J(k-1)}{u'(k) - u(k-1)}. \quad (44)$$

Moreover,  $G_k$  is defined as

$$\left[ \frac{\partial^2 J(u')}{\partial u'^2} \right]^{-1} = G_k = \frac{u'(k) - u(k-1)}{(g_k - g_{k-1})}. \quad (45)$$

So (41) can be rewritten in this form:

$$\Delta u' = u'_{new} - u'_{old} = -\eta(1+d)G_k g_k. \quad (46)$$

Using (46), we will have:

$$J(u'_{new}) = J(u'_{old} - \eta(1+d)G_k g_k), \quad (47)$$

$$\text{or Argument of } J = u'_{old} - \eta(1+d)G_k g_k. \quad (48)$$

Both  $u'_{old}$  and  $(1+d)G_k g_k$  are known at this stage; then with changing  $\eta$ , *Argument of J* moves along a line. There is an optimum point on this line that minimizes  $J$ . Such an optimization problem is classified as a linear search. Backtracking method, introduced by Dennis and Schnabel [23], is selected for linear search. The modified  $u' (u'_{new})$  is used as the new control input.

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## **Chapter 6**

### **Double Command Fuzzy Control of a Nonlinear CSTR**

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# Double-command Fuzzy Control of a Nonlinear CSTR

Morteza Mohammadzaheri, Lei Chen

**Abstract-** In this research, double-command control of a nonlinear chemical system is addressed. The system is a stirred tank reactor; two flows of liquid with different concentrations enter the system through two valves and another flow exits the tank with a concentration between the two input concentrations. Fuzzy logic was employed to design a model-free double-command controller for this system in the simulation environment. In order to avoid output chattering and frequent change of control command (leading to frequent closing-opening of control valves, in practice) a damper rule is added to the fuzzy control system. A feedforward (steady state) control law is also derived from the nonlinear mathematical model of the system to be added to feedback (fuzzy) controller generating transient control command. The hybrid control system leads to a very smooth change of control input which suits real applications. The proposed control system offers much lower error integral, control command change and processing time in comparison with neuro-predictive controllers.

**Keywords:** Fuzzy Control, Hybrid Control, CSTR, Nonlinear System, Steady State Control.

## I. INTRODUCTION

Catalytic Continuous Stirred Tank Reactors (CSTR)s have been extensively used as a benchmark for testing different control systems. These systems are multi-input and multi-output and may be highly nonlinear. Self-tuning PIDs [1, 2]robust controllers [3], adaptive-like control systems [4] and different kinds of nonlinear predictive controllers [5, 6] have been successfully tested on this class of chemical systems. The CSTR is also known as an outstanding example for the application of neuro-predictive controllers [7] which are a subclass of nonlinear predictive controllers. Moreover, fuzzy logic controllers are used in the control of CSTRs to generate either the control command directly [8, 9] or control command increments [10, 11]. As well as improving the performance, other aims achieved by the application of fuzzy control systems on CSTRs are stability guarantee [8, 12]and disturbance rejection [13, 14].

In this paper, at first, successful control of a non-thermic CSTR by neuro-predictive

technique is reported. In this test, the flow mass rate of one of two entering flows is subject to adjustment in order to control outlet concentration of the tank. Although, this problem is known as a good example for neuro-predictive control [7, 15]. This technique worked both ineffectively (in terms of offering improper performance) and inefficiently (in terms of needing heavy computation) when it was tried for double-command control (to adjust the mass rate of both inlet flows). This paper then presents a hybrid control system designed to adjust the flow rates of both entering flows simultaneously. In the presented control system, the control commands are the sum of a transient control command whose increments are generated by a non-model-based fuzzy logic controller and a steady state control command generated by a set-point dependent control law. Finally, the control system was tested in simulation environment. In order that the results are applicable in practice, the “input constancy” is particularly addressed; that is, the proposed control system is designed to reduce the change of control inputs as well as the error.

## **II. THE UTILIZED FUZZY INFERENCE SYSTEM**

In this research, a non-weighted first-order Sugeno type fuzzy inference system, with AND connectors, is used as the fuzzy controller. A schematic of such a system is shown in Fig.1. Each fuzzy rule includes two main parts: antecedent and consequent. Antecedents contain linguistic (fuzzy) values with membership functions. A ‘membership function’ is a function which receives the crisp (numeric) value of a variable (e.g. 25°C) and returns another number in the range of [0,1], namely ‘membership grade’. As a result, in each rule, the number of membership grades equals the number of fuzzy values in the antecedent. All these membership grades (in the range of 0 and 1) pass through a function namely T-norm. The output of the T-norm is the fire strength of the rule:

$$\text{fire strength of rule } (w_i) = \text{Tnorm}(\text{all membership grades}). \quad (1)$$

The fire strengths of rules ( $w_i$ ) are the outcome of this step. In a first-order Sugeno-type FIS, the consequents of rules are linear crisp functions of the inputs, independent of antecedent fuzzy values. For a FIS with  $M$  inputs:

$$z_i = \sum_{j=1}^M a_j x_j + a_{M+1}. \quad (2)$$

$z_i = \text{output of } i^{\text{th}} \text{ rule}$ ,  $x_j = j^{\text{th}} \text{ input}$ .

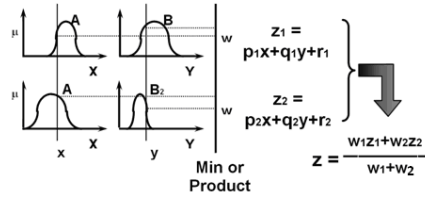


Figure 1. A scheme of a Sugeno-type FIS

The total output of a Sugeno-type FIS, having  $N$  rules, is calculated using following equation:

$$\text{output of FIS} = \frac{\sum_{i=1}^N z_i w_i}{\sum_{i=1}^N w_i}. \quad (3)$$

### III. PROBLEM STATEMENT

The case study is a Catalytic Continuous Stirred Tank Reactor (CSTR). A diagram of the process is shown in the following figure:

NOTE:  
This figure is included on page 146 of the print copy of the thesis held in the University of Adelaide Library.

Figure 2. A schematic of the studied CSTR [7]

Two flows of liquid enter the reactor with the concentration of  $C_{b1} = 24.9 \text{ (kmol/m}^3\text{)}$  and  $C_{b2} = 0.1 \text{ (kmol/m}^3\text{)}$ . The flow rates of high and low concentration input flows are named  $u_1$  and  $u_2$ , respectively. The reactor outputs another flow of liquid with the concentration of  $C_b$  and the flow rate of  $w$ . Another important variable is the height. A simplified mathematical model of the system, achieved by mass balance equations, is:

$$\frac{dh(t)}{dt} = u_1(t) + u_2(t) - 0.2\sqrt{h(t)}, \quad (4)$$

$$\frac{dC_b(t)}{dt} = [C_{b1} - C_b(t)] \frac{u_1(t)}{h(t)} + [C_{b2} - C_b(t)] \frac{u_2(t)}{h(t)} - \frac{k_1 C_b(t)}{1 + k_2 C_b(t)^2}. \quad (5)$$

where the concentration of outlet flow and the height of liquid are considered as the outputs ( $w = 0.2\sqrt{h}$ ).

The control of outlet concentration of the aforementioned system is addressed in this paper. This control problem is a successful case study for neural or neuro-fuzzy predictive control [7]. In neuro-predictive control, usually the low concentration flow ( $u_2$ ) is set to a fixed value (e.g. 0.1 liters/s) and the control algorithm adjusts the other flow ( $u_1$ ).

In spite of all the advantages of neuro-predictive control, formed by neural network models and derivative based control algorithms; it was observed, this method cannot be effectively used for double-command control of this system. In neuro-predictive control, using both flows does not lead to any improvement in the performance of the control system compared to a single-command one. Furthermore, in neuro-predictive control, an optimisation problem should be solved to generate the control command. This process is highly time-consuming for double-command control, especially if the second order optimisation algorithms (which are more reliable) are employed. In the next section, a brief explanation and the results for single command control of the studied CSTR system is presented.

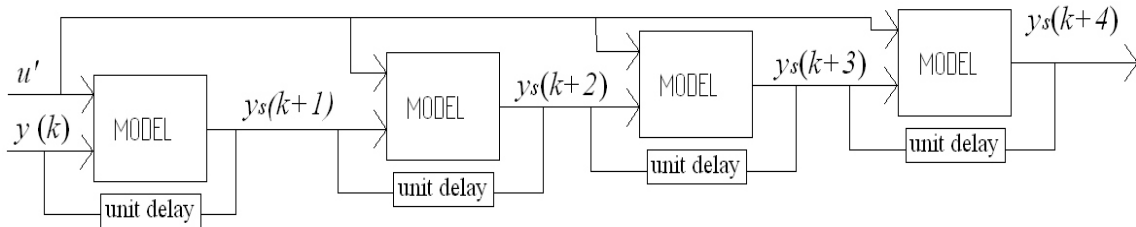
#### IV. SINGLE COMMAND NEURO-PREDICTIVE CONTROL OF CSTR (A BRIEF REVIEW)

In nonlinear predictive control, nonlinear models are used to predict the behaviour of nonlinear systems, and an optimisation method generates the control command based on minimising a performance function involving predicted errors (such as following function):

$$J(k) = \sum_{i=1}^N [y_s(k+i) - y_d]^2 + \rho [u'(k) - u(k-1)]^2. \quad (6)$$

where  $y_s$  and  $y_d$  are the predicted and desired outputs of the system, respectively, and  $u'$  and  $u$  are tentative and actual control inputs. Additionally,  $\rho$  is a factor defining the importance of the constancy of control input.

(6) is a typical performance function (represented by  $J$ ), which is usually used in neuro-predictive control. In discrete domain, at the instant of  $k$ , the output of the system is known ( $y(k)$ ), and the tentative control command ( $u'(k)$ ) is subject to optimisation.



**Figure 3. Prediction of output values with the horizon of 4**

If current output and previous output/input of the system and  $u'$  are known, all other arguments of  $J$  will be definitely known. Figure 3 shows how an estimated output is generated using tentative input and the current output of the system. However  $u'$  can be changed arbitrarily and freely from the recorded input/output data and this change affects other arguments of  $J$ , and consequently, the performance function itself. Therefore, in the optimisation for control purposes, it can be assumed that

$$J = J(u'). \quad (7)$$

Finding  $u'$  so as to minimise the performance function is the final stage at nonlinear predictive control.

Two design criteria are defined, being the error integral and the constancy of input (usually in the form of a flow rate):

$$EI = \frac{\int_0^{\tau} |e| dt}{\tau}, \quad (8)$$

$$IC = \frac{\int_0^{\tau} |u(t + \Delta t) - u(t)| dt}{\tau}. \quad (9)$$

Simulation results of NP control of the system with different optimisation methods are shown in Table 1 and Fig.4 . The initial height of tank is 30 cm and the initial value of outlet concentration of tank is  $20 \text{ kmol/m}^3$ . In Table 1, ‘Simulation Time’ is the time needed to simulate 100 seconds of operation of the closed loop system, with sampling time of 0.2 s, using a dual core processor (4200 MHz) and MATLAB software. In this example, the reference changes very quickly to test the capabilities of the controllers. In optimisation methods, LM stands for Levenberg Marquardt, SD stands for Steepest Descent and FSD stands for Fuzzy Steepest Descent. More details about neuro-predictive control of CSTR are available in the appendices. Appendix 1 is about neural network modeling of the system (with one control command), and Appendix 2 is about the utilized optimisation algorithms.

**Table 1. Simulation results for different optimisation methods**

Optimisation Method	EI ( $\text{kmol/m}^3$ )	IC ( <i>litres</i> )	Simulation Time (s)
LM	0.3651	0.0939	225
FSD	0.3729	0.0944	25
SD	0.7738	0.2553	24



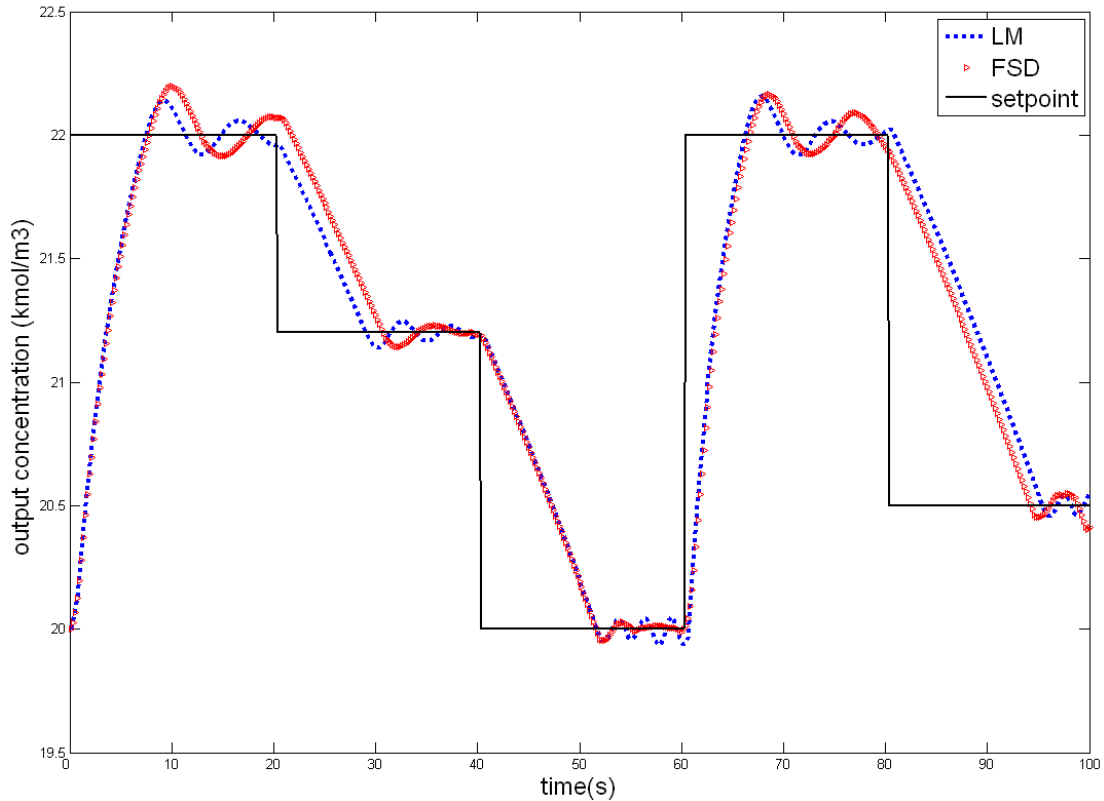


Figure 4. Simulation results for different optimisation methods

## V. DOUBLE COMMAND CONTROL OF CSTR

In this section the design of a control system to command both inlet flows (valves) is addressed. The control command is the sum of a transient control command (whose increments are adjusted by a fuzzy inference system) and a steady state control command. Transient control command pushes the system towards the desired situation, and steady state control command maintains the desired situation. At first, the fuzzy controller (generating the increments of transient control command) is designed, then steady state control law will be introduced.

### A. Fuzzy Controller/Transient control command

In this sub-section, initially the architecture of the transient control system is proposed, then the fuzzy rules forming the fuzzy logic controller are addressed. The absolute value of error

(E) and its differential (dE) are the inputs to the fuzzy controller.

$$dE = E(1 - z^{-1}) \quad (10)$$

where  $z^{-1}$  is unit delay.

The maximum output of the fuzzy controller is set to 0.1 due to practical limitations of the valves. If the error is positive (that is, the concentration is lower than its desirable value) the output of fuzzy controller is added to  $u_1$ , otherwise it is added to  $u_2$ . This decision is made by **f1** function shown in Fig. 5. After adding fuzzy controller output to the delayed control commands, both control commands pass through **f2** function. In **f2**, if the error is positive,  $u_2$  is set to zero; otherwise  $u_1$  is set to zero. Also, if  $E > 2$ , non-zero control command is set to four. Saturation functions guarantee that the generated control command (transient control command) is in the range of [0 4], the acceptable range for the control valves.

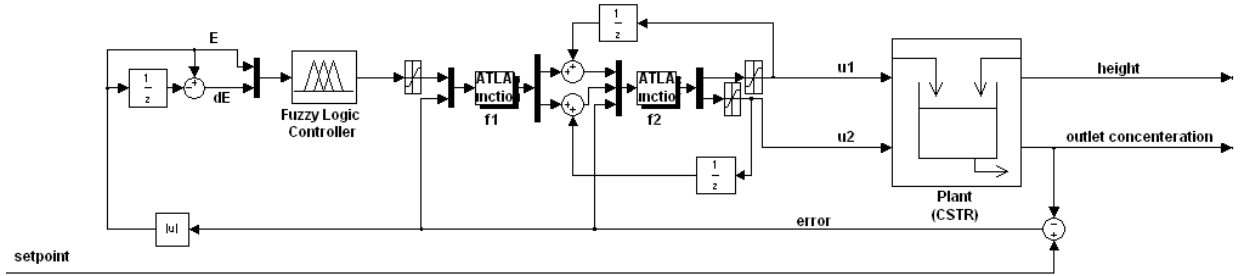


Figure 5. Control circuit for transient mode of control

Before addressing fuzzy rules, the membership functions of E and dE are introduced. In general, membership function should have following characteristics:

$$\begin{cases} \lim_{E \rightarrow be} \mu_{medium}(E) = 0 \\ \lim_{E \rightarrow 0} \mu_{medium}(E) = 0 \end{cases} \quad (11)$$

(12)

$$\begin{cases} \lim_{E \rightarrow 0} \mu_{high}(E) = 0 \\ \lim_{E \rightarrow be} \mu_{high}(E) = 1 \end{cases}$$

$$\left\{ \begin{array}{l} \lim_{dE \rightarrow 0} \mu_{good/bad}(dE) = 0 \\ \lim_{dE \rightarrow -bde} \mu_{good}(dE) = 1 \\ \lim_{dE \rightarrow bde} \mu_{bad}(dE) = 1 \end{array} \right. \quad (13)$$

$$\left\{ \begin{array}{l} \lim_{E \rightarrow ce} \mu_{zero}(E) = 0 \\ \lim_{E \rightarrow 0} \mu_{zero}(E) = 1 \end{array} \right. \quad (14)$$

where, *be* stands for big absolute error (E), *ce* stands for considerable absolute error (E), and *bde* stands for big error differential (big dE)

These parameters are chosen by the designer based on his/her knowledge about the system. The selected membership functions, in this article, are shown in Figs. 6 and 7.

Among E membership functions, “medium” is triangular; and high and zero are Gaussian:

$$\mu_{high}(E) = \exp\left(-\frac{1}{2}\left(\frac{E-3}{0.9}\right)^2\right) \quad (15)$$

$$\mu_{zero}(E) = \exp\left(-\frac{1}{2}\left(\frac{E}{0.01}\right)^2\right) \quad (16)$$

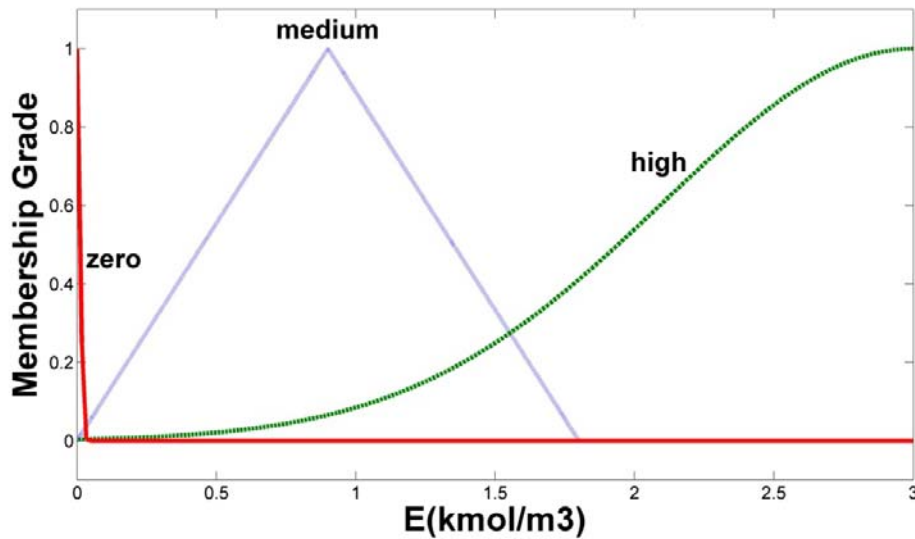


Figure 6. Membership functions of E (absolute error)

dE membership functions are shown in Fig. 7 and (17):

$$\left\{ \begin{array}{l} dE > 1 \Rightarrow \mu_{bad}(dE) = 1 \\ dE < -1 \Rightarrow \mu_{good}(dE) = 1 \end{array} \right. \quad (17)$$

Membership function “zero” is only used in the damper rule which will be introduced later in this section. Other membership functions (two for E and two for dE) are used in the design at this stage. With two fuzzy values (membership functions) for any inputs to the fuzzy controller, four different compositions can be made for the antecedent part of fuzzy rules. These quadruple compositions need to be arranged in terms of the criticalness of the system situation (the value and the trend of error). Table 2 presents such an order. For instance, the worst situation occurs when E is high and dE is bad (which means E is increasing).

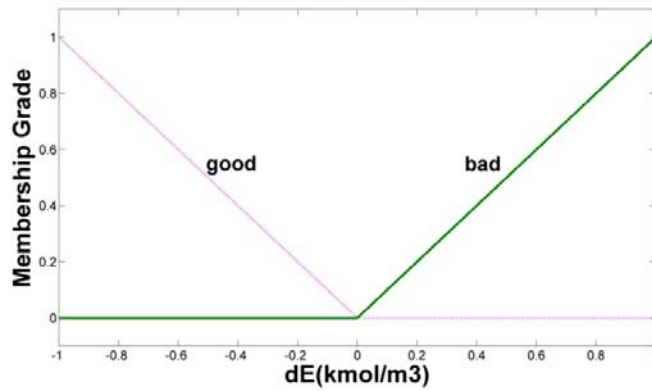


Figure 7. Membership functions of dE

Table 2. The order of possible situations of the system in terms of criticalness

	E	dE
1	medium	good
2	medium	bad
3	high	good
4	high	bad

Four principal rules of the controller are defined based on the aforementioned situations of the system.

If the absolute error is high, the two following rules make the system decrease the error very quickly.

Rule1: IF E is high AND dE is bad THEN  $du_I = 0.1$

Rule2: IF E is high AND dE is good THEN  $du_{II} = 0.3E$

If the error is not very high, the following rules will be activated to moderate the

approaching speed towards the setpoint/reference and to avoid overshoot.

Rule3: IF E is medium and dE is bad THEN  $du_{III} = 0.2E$

Rule4: IF E is medium and dE is good THEN  $du_{IV} = 0.1E$

Figure 8 shows the control behaviour of the system when a fuzzy controller containing the aforementioned fuzzy rules is applied.

With the same initial values as the plot shown in Fig. 4, the error integral and input constancy of the system (represented by (8) and (9)) are  $IC=0.0605$  litres and  $EI=0.1772$   $kmol/m^3$ . The results are acceptable compared to single-command neuro-predictive control in terms of having reasonable values of  $IC$  and  $EI$ , which are important from the viewpoint of implementation and accuracy respectively.

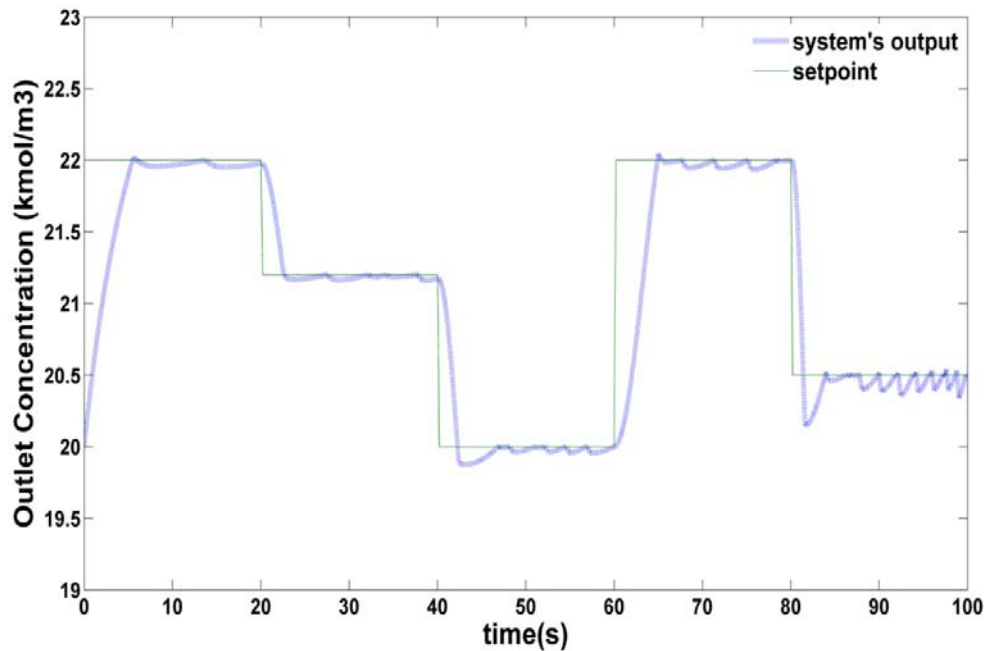


Figure 8. The response of system with fuzzy control (without damper rule)

However, a chattering is observed when the system's output is close to the setpoint/reference. Figure 9 shows control commands (inputs) versus time for the operation period, for which the response is shown in Fig. 8. It is observed that in time periods such as

50s~60s or 90s~100s, changes in control input occur which lead to chattering. Frequent opening and closing of the valve is problematic in practice.

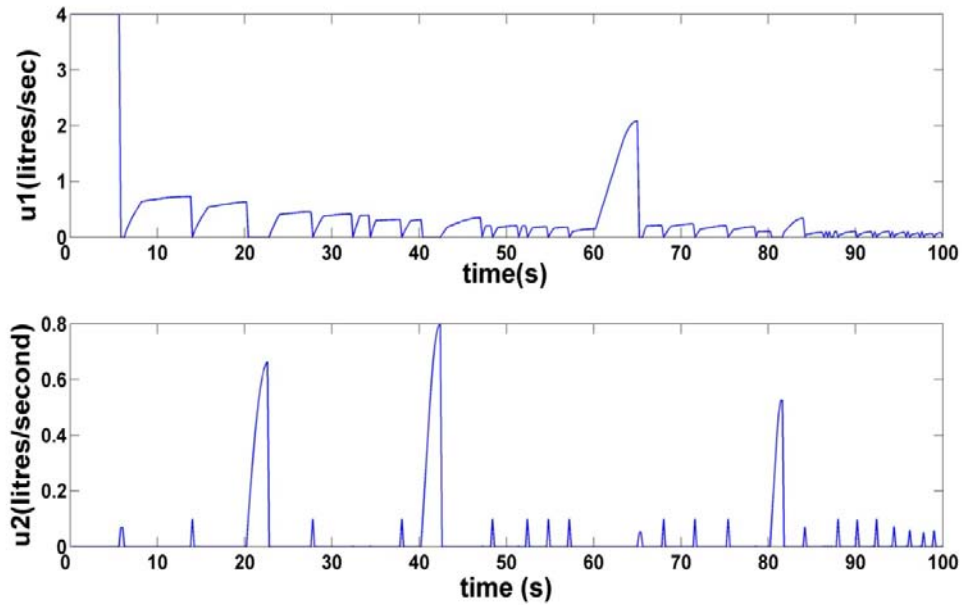


Figure 9. Control commands generated by fuzzy control system of Fig. 5 (without damper rule)

In order to avoid chattering in the vicinity of the setpoint, a damper rule is added to the fuzzy controller:

Rule5: IF E is zero THEN  $du_v = 0$  (damper rule)

This rule improves both design criteria,  $IC = 0.0383$  litres, and  $EI = 0.1700$  kmol/m<sup>3</sup>. Zero membership function is intentionally selected very narrow; a wider zero causes a considerable loss in error integral. The main role of the fifth rule is the diminution of frequent changes of control inputs (and its consequent chattering) when the system's output is in the vicinity of the setpoint.

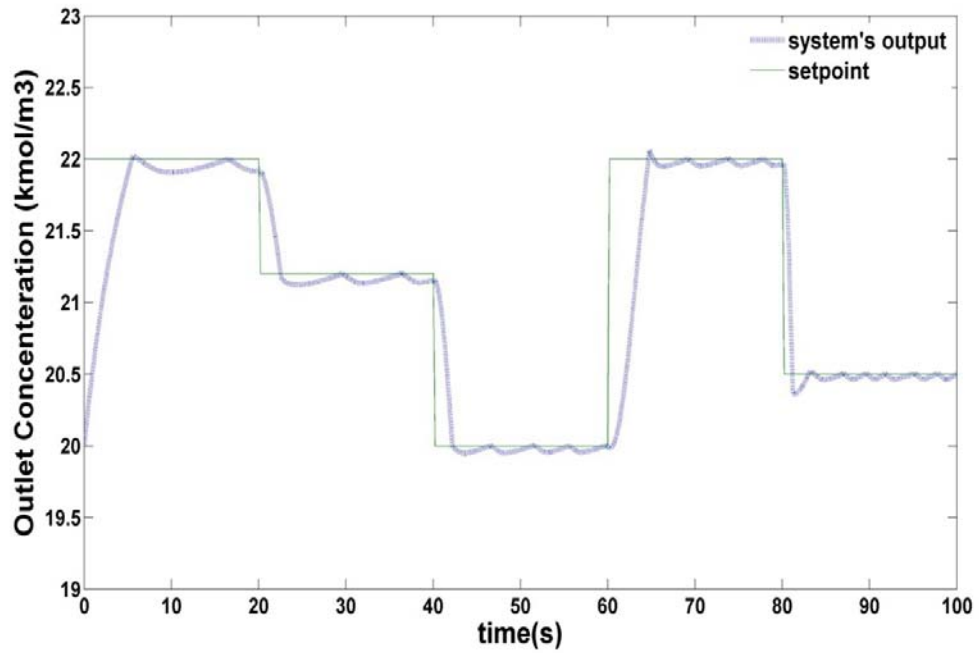


Figure 10. The response of system with fuzzy control (with damper rule)

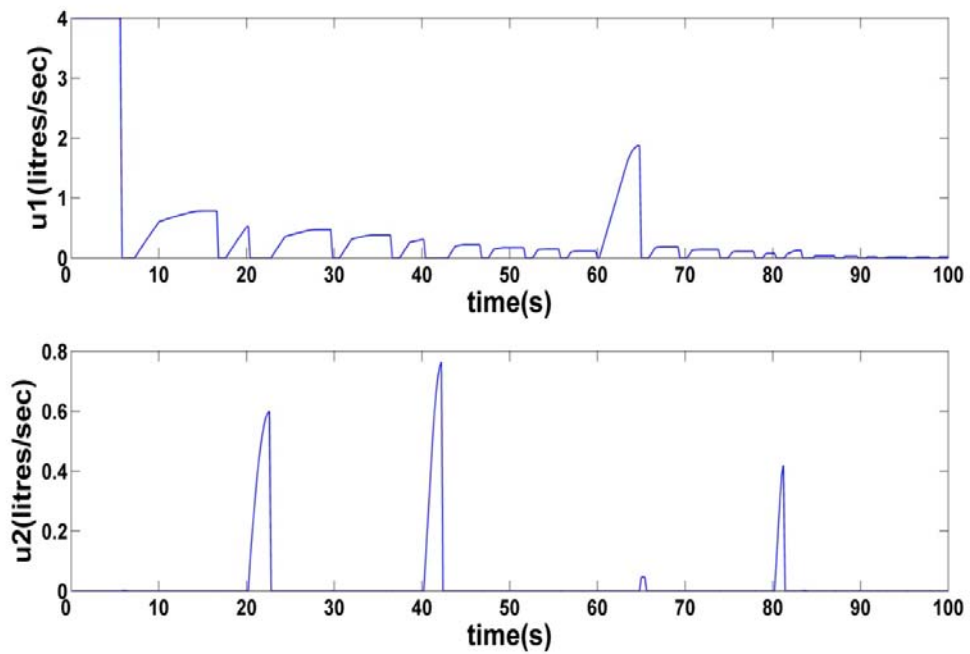


Figure 11. Control commands generated by fuzzy control system of Fig. 5 (with damper rule)

## B. Steady State Control

The designed fuzzy controller is an error-based controller; that is, the only entering signal to the controller is the error. Considering Figs.8~11, it is observed that even when the error is zero (or very close to zero), the system's output and consequently the control input is subject to change (consider, for example time periods of 25s~40s, 45s~60s or 85s~100s). In the designed fuzzy controller, if the error is zero, the transient control input is zero. The value of zero for the control input cannot maintain the desired situation (Figs. 8 and 10). As a result, a steady state control command is added to transient control command (generated by fuzzy controller); when the error is around zero, transient control command approaches zero and steady state control command maintains the desirable situation (keeps the system's response fixed).

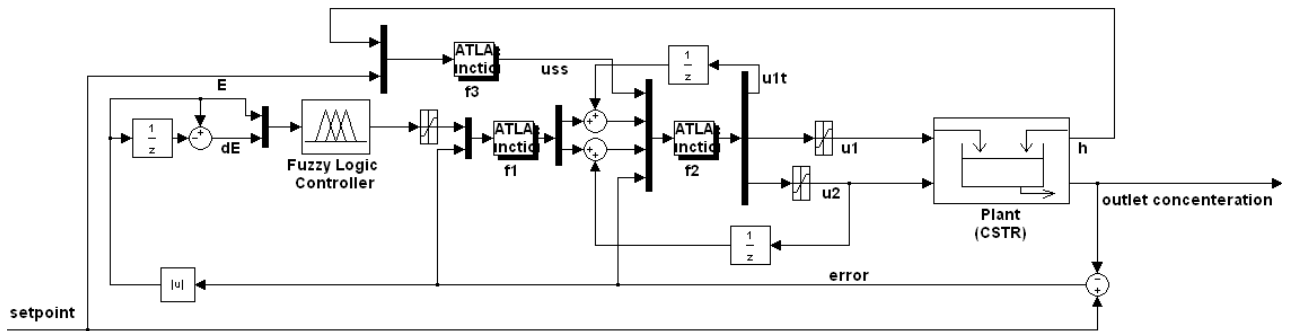


Figure12. Control circuit

In total, at steady control situation:

$$\begin{cases} u_1 = u_{ss} \\ u_2 = 0 \\ \frac{dC_b(t)}{dt} = 0 \\ C_b(t) = r(t) \end{cases} \quad (18)$$

where  $r(t)$  = reference.

In order to find steady state control command, the physics-based model offered in (5) is used:



$$\frac{dC_b(t)}{dt} = [C_{b1} - C_b(t)] \frac{u_1(t)}{h(t)} + [C_{b2} - C_b(t)] \frac{u_2(t)}{h(t)} - \frac{k_1 C_b(t)}{1 + k_2 C_b(t)^2}. \quad (5)$$

Considering the aforementioned conditions, (5) changes to:

$$[24.9 - r(t)] \frac{u_{ss}(t)}{h(t)} - \frac{k_1 C_b(t)}{1 + k_2 C_b(t)^2} = 0, \quad \text{or}$$

$$u_{ss}(t) = \frac{k_1 C_b(t) h(t)}{(1 + k_2 C_b(t)^2)(24.9 - r(t))}. \quad (19)$$

The obtained steady state control command, dependent on both level height and the reference (setpoint), is calculated in **f3** function (Fig. 12). Having steady state control command, the first valve is always open ( $u_1 > 0$ ), so when error is negative (the outlet concentration is higher than its desirable value), the performance decreases and it takes longer for the system to overcome negative error (since a positive  $u_1$  increases the concentration). In order to compensate this drawback, **f2** function sets  $u_{ss}$  (the steady state control command) to zero as error  $< -0.5 \text{ kmol/m}^3$ . Moreover, the final control command is generated in **f2** (by adding two control commands).

Adding steady state control command improves input constancy significantly to  $IC = 0.0305 \text{ litres}$ , but the error integral increases to  $EI = 0.1788 \text{ kmol/m}^3$ . As the main advantage, the control input changes very smoothly, so this combined control command is more suitable for real application.

## VI. STABILITY REMARK

Stability discussion is done based on two practical assumptions (rather than mathematical model) which are evident for the case study and can be paraphrased for a wide class of systems. These assumptions are the basis of stability discussion.

$$\forall C_b \mid C_{b1} > C_b > C_{b2},$$

A1: if ( $u_1 = 4 \text{ litres/min}$  &  $u_2 = 0$ )  $\Rightarrow \dot{C}_b > 0 (\dot{e} < 0)$

A2: if ( $u_2 = 4 \text{ litres/min}$  &  $u_1 = 0$ )  $\Rightarrow \dot{C}_b < 0 (\dot{e} > 0)$

In the first order systems which control input affect the first derivative of the output (e.g. heat-related systems); there usually exists a value of control input that changes the sign of the first temporal derivative of the output immediately after application. As a result, two proposed assumptions can be re-stated for other similar systems.

In order to address bounded-input bounded-output (BIBO) stability, it is proved that if error square (or the absolute value of the error) is higher than a bounded value, this value definitely decreases. So, the error (and the output) always remains bounded if it is bounded in the beginning. The decrease of error square means the derivative of error square is negative.

Since  $\frac{d}{dt}e^2 = 2e\dot{e}$ , as the stability criterion, it is needed to prove if the absolute value of error

(or error square) is higher than a bounded value then :

$$e.\dot{e} \leq 0 \quad (20)$$

In this section, the value of  $3 \text{ kmol/m}^3$  is considered as the boundary of absolute value of error, and it is proved that as the absolute value of error is higher than  $3 \text{ kmol/m}^3$ , error square decreases, and error will not be un-bounded.

$$e > 3 \text{ kmol/m}^3 \xrightarrow{f2} (u_1 = 4 \text{ \& } u_2 = 0) \xrightarrow{A1} \dot{e} < 0 \Rightarrow e\dot{e} < 0,$$

$$e < -3 \text{ kmol/m}^3 \xrightarrow{f2} (u_1 = 0 \text{ \& } u_2 = 4) \xrightarrow{A2} \dot{e} > 0 \Rightarrow e\dot{e} < 0.$$

## VII. S UMMARY OF RESULTS

In this section, with a new set of setpoints/references and initial height (60cm), all three control systems are checked again, including fuzzy controller system (FC), fuzzy control system with damper rule (FCDR) and the proposed hybrid control system (Fig. 13, Table 3).

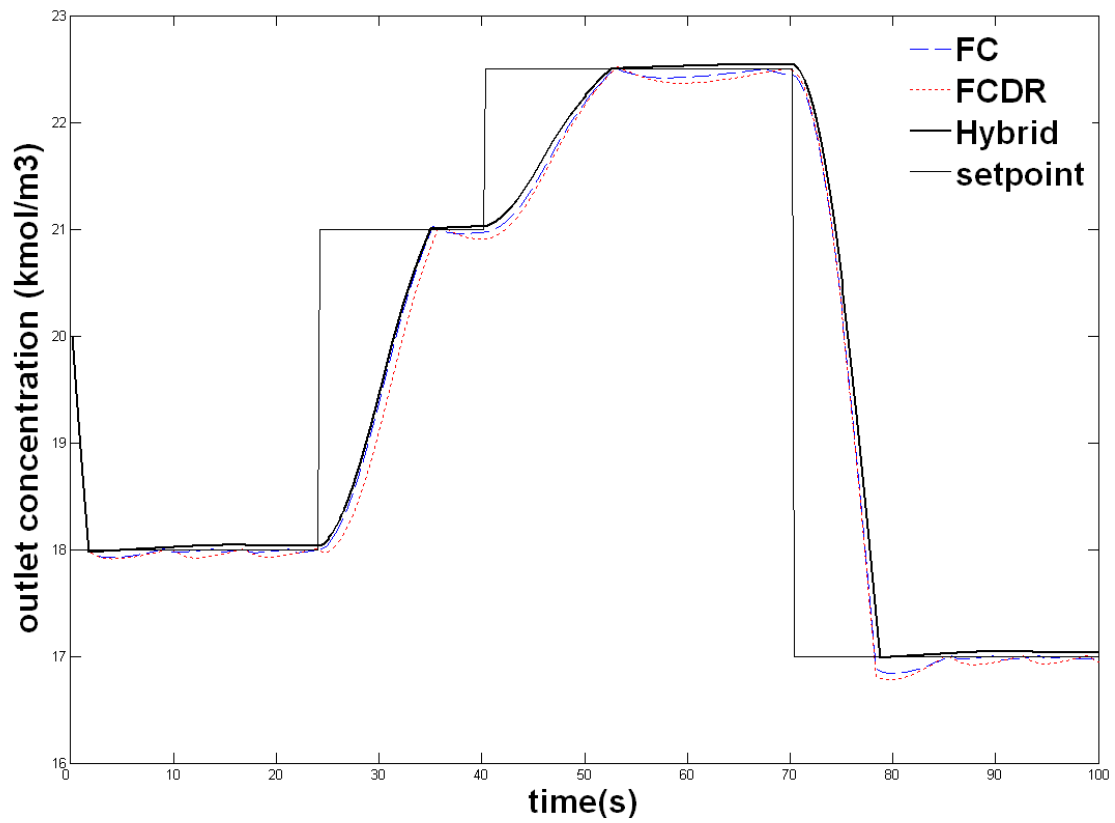


Figure 13. The response of system

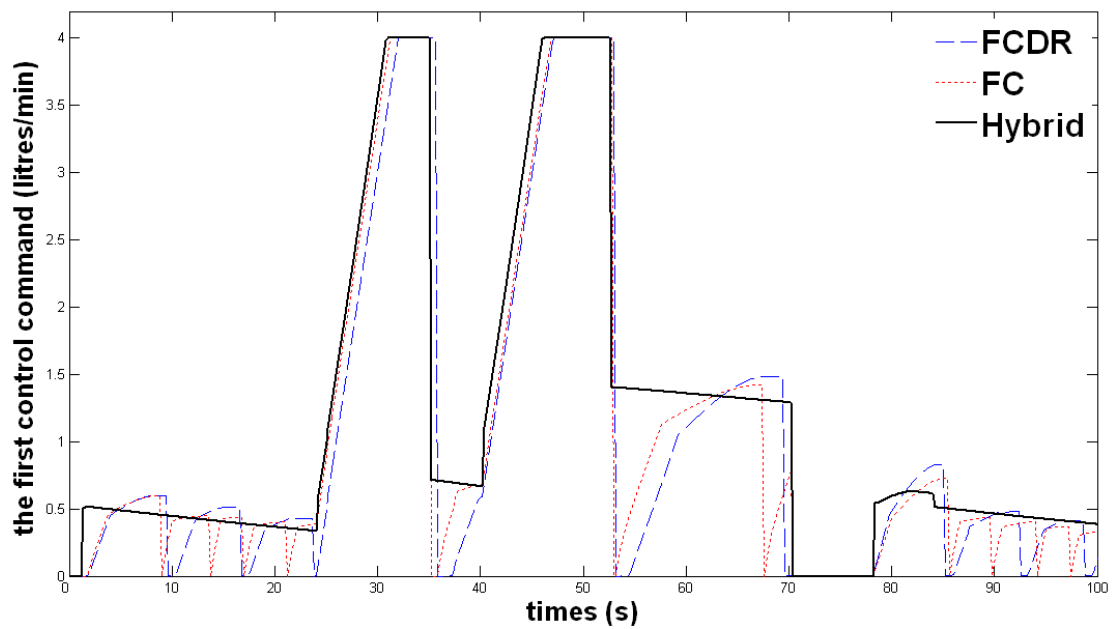


Figure 14. The first control command during operation shown in Fig.13

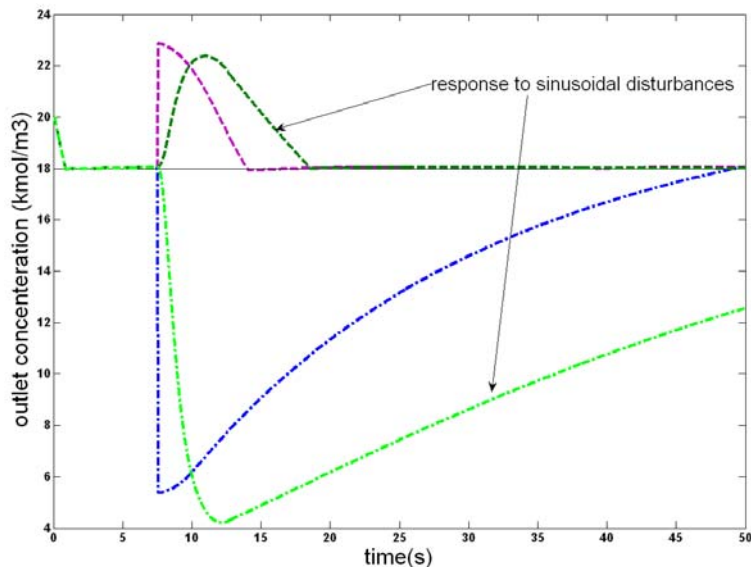
As it is indicated in Fig. 14, the change of control input in the proposed hybrid control system is very smooth. Also, in all the simulations shown in Fig. 13, the time needed for simulation is around 5 seconds (compare with Table 1). There are two more relevant plots in Appendix 3.

**Table 3. Simulation results for different control systems**

	EI ( $kmol/m^3$ )	IC (litres)
FC	0.6088	0.0819
FCDR	0.6528	0.0743
Hybrid	0.6043	0.0556

### VIII. DISTURBANCE REJECTION

In order to check the capability of the hybrid control system in rejecting disturbances, four very severe disturbances were applied to the system. First, 4 litres of low or high concentration liquid were poured suddenly into the reactor (in 0.2 second). Both disturbances (high and low disturbances) were successfully rejected as shown in Fig. 15 (also, Fig. 20 in Appendix 3 offers some complementary information). Then 4 litres of low or high concentration liquid were poured to the system in 10 seconds as poured liquid was a sinusoidal disturbance function of time.



**Figure15. The response of system to disturbances**

## IX. ROBUSTNESS AGAINST NOISES AND UNCERTAINTIES

The feedforward controller is the only model-based part of the proposed hybrid control system. Feedforward controllers are usually more sensitive to noise and parameter uncertainties. High sensitivity to uncertainties or noises (due to model-based feedforward controller) could be a serious defect of the proposed control system. In order to address the aforementioned issue, feedforward control command was multiplied by a ‘Test Coefficient’ before being added to feedback (transient) control command. This means that feedforward control command would no longer remain as accurate (this inaccuracy can be caused by noise or uncertainty, in reality). The effect of this situation on control system was then checked. Table 4 shows the values of design criteria for different values of ‘Test Coefficient’ (see (8) (9) and Table 3). Even a considerable inaccuracy (50%) does not deteriorate control behaviour significantly. Error integral remains reasonably small. Furthermore, the value of input change remains low, as a crucially important advantage in terms of implementation, at all values of test coefficient shown in Table 4.

**Table 4. Simulation results for hybrid system at different test coefficients**

Test Coefficient	EI ( $kmol/m^3$ )	IC ( <i>litres</i> )
0.5	0.6081	0.0608
0.7	0.6090	0.0569
1	0.6043	0.0556
1.3	0.6523	0.0573
1.5	0.6761	0.0609

## X. C ONCLUSION

After disappointing results of using neuro-predictive technique in double command control of the outlet concentration of a nonlinear CSTR, a hybrid control system was designed in this research to adjust simultaneously both entering flows of a nonlinear CSTR. The proposed

hybrid includes a model-free error-based fuzzy controller and a model-based (but very robust against model uncertainties) non-error-based steady state control law, The fuzzy controller pushes the system towards the reference. A fuzzy rule (damper rule) is deliberately added to the fuzzy controller to reduce the control input's frequent change and consequent output's chattering. Steady state control maintains a desirable situation. The input to steady state control law is the reference (setpoint) and measurable states of the system which are not subject to control (height, in this case study). The performance, efficiency and input constancy, achieved by the proposed hybrid control system, are significantly higher than those of single-command neuro-predictive controllers which have already been designed by the authors and known as a successful method in this case study. The proposed method particularly suits real application in terms of trifle computations and control input change. Furthermore, it is proved that the proposed control system is BIBO stable.

## **XI. APPENDIX 1: NEURAL NETWORK MODELING OF CSTR (with sole control command of $u_1$ )**

The modeling was performed particularly for the purpose of predictive control. The studied CSTR has two control inputs,  $u_1$  and  $u_2$ . One control command is used in predictive control, so the flow rate of the second input flow (with the concentration of  $0.1 \text{ kmol}/\text{m}^3$ ) is set to the constant value of  $0.1 \frac{\text{liters}}{\text{s}}$ . As a result, this value is not considered in modeling as an input signal anymore. Moreover, the order of two is assumed for the model. This model (presented in (21)) is used to find the first estimated value of  $C_b$ :

$$[\hat{C}_b(k+1), \hat{h}(k+1)] = F[u_1(k-1), u_1(k), C_b(k-1), C_b(k), h(k-1), h(k)] \quad (21)$$

or

$$\hat{C}_b(k+1) = F_1[u_1(k-1), u_1(k), C_b(k-1), C_b(k), h(k-1), h(k)] \quad (22)$$

$$\hat{h}(k+1) = F_2[u_1(k-1), u_1(k), C_b(k-1), C_b(k), h(k-1), h(k)] \quad (23)$$

After very first instants of the prediction:

$$[\hat{C}_b(k+1), \hat{h}(k+1)] = F[u_1(k-1), u_1(k), \hat{C}_b(k-1), \hat{C}_b(k), \hat{h}(k-1), \hat{h}(k)] \quad (24)$$

or

$$\hat{C}_b(k+1) = F_1[u_1(k-1), u_1(k), \hat{C}_b(k-1), \hat{C}_b(k), \hat{h}(k-1), \hat{h}(k)] \quad (25)$$

$$\hat{h}(k+1) = F_2[u_1(k-1), u_1(k), \hat{C}_b(k-1), \hat{C}_b(k), \hat{h}(k-1), \hat{h}(k)] \quad (26)$$

where variables with hat are the estimated ones.

Although often only one predicted value is used in predictive control (i.e.  $C_b$ ), all the outputs should be estimated, because most of systems are dynamic and the outputs are usually coupled (in this problem,  $h$ , as the representative of liquid volume, affects the value of  $C_b$ ). Therefore, the estimated values of all the outputs are needed to predict any of them, for a period of time in the future.

After the definition of the model's order, the training data should be normalized and arranged. 8000 set of data (including  $u_1$ ,  $h$  and  $C_b$ ) with the sampling time of 0.2 second are utilized in training. The normalized data are arranged as below:

$$\left[ \begin{array}{cc|cc} \overbrace{u_1(1) \quad u_1(2)}^{\text{input}} & & \overbrace{C_b(1) \quad C_b(2)}^{\text{input}} & & \overbrace{h(1) \quad h(2)}^{\text{input}} & & \overbrace{C_b(3) \quad h(3)}^{\text{output}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_1(7998) \quad u_1(7999) & & C_b(7998) \quad C_b(7999) & & h(7998) \quad h(7999) & & C_b(8000) \quad h(8000) \end{array} \right] \quad (27)$$

A four-layer recurrent perceptron is trained using the prepared data (27). The input layer of the utilized perceptron has six neurons (equal to input signals). This ANN has one nonlinear (with sigmoid activation functions) and one linear (with linear activation functions with slope of 1) hidden layers. Both hidden layers have 13 neurons. The output layer also has two

neurons with linear activation functions with a slope of 1. Linear hidden layer may seem useless at first glance, because a linear combination of the outputs of nonlinear hidden layer neurons is generated at the output layer, even without this linear hidden layer; in other words, the structure of mathematical relation between input and output of ANN is the same with or without the linear hidden layer; however, adding linear hidden layer improves the accuracy, in practice. It seems a wider variety of adjusting parameters let the model be trained more successfully. The training method is Levenberg-Marquardt error back propagation. The (batch) training has been performed in 100 epochs and the performance function is sum of squared errors (MSE).

In this research, two different series of checking data were used. Both series are entirely different from training data.

A criterion is defined for the predictive accuracy of models, namely PAN:

$$PAN = \sum_{i=1}^N |\hat{C}_b(i) - C_b(i)|. \quad (28)$$

Table 5 shows PA10 and PA30 (the sum of absolute error of prediction for 10 and 30 future instants or next 2 or 6 seconds), for two different series of checking data.

**Table5. Prediction accuracy for different trained models**

Criterion	PA10 ( $kmol/m^3$ )		PA30( $kmol/m^3$ )	
	1 <sup>st</sup> series	2 <sup>nd</sup> series	1 <sup>st</sup> series	2 <sup>nd</sup> series
Complete Perceptron (double output)	0.018	0.022	0.051	0.033



## XII. APPENDIX 2: OPTIMISATION

From section IV, it is found that for nonlinear predictive control purposes:

$$J = J(u'). \quad (7)$$

Now,  $u'$  should be so determined that  $J$  has its minimal value. To do so, Taylor's series is written for performance function up to the first and second order:

$$J(u' + \Delta u') \cong J(u') + \frac{\partial J(u')}{\partial u'} (\Delta u'), \quad (29)$$

$$J(u' + \Delta u') \cong J(u') + \frac{\partial J(u')}{\partial u'} (\Delta u') + \frac{1}{2} \frac{\partial^2 J(u')}{\partial u'^2} (\Delta u')^2. \quad (30)$$

Based on these expansions, two different methods are used for optimisation in this paper: Levenberg-Marquardt method, based on (30), which is a very good derivative based optimisation method [16, 17]. This method is currently used for neuro-predictive control [7]. The second method, based on (29), is a combination of steepest descent and fuzzy logic.

### ***XII-A. Levenberg-Marquardt (LM)***

In this method, after derivation, (31) will be obtained:

$$\frac{\partial J(u' + \Delta u')}{\partial (\Delta u')} \cong \frac{\partial J(u')}{\partial u'} + \frac{\partial^2 J(u')}{\partial u'^2} (\Delta u').$$

(31)

In order to minimise  $J(u' + \Delta u')$ , its derivative is set equal to zero. Consequently,

$$\Delta u' \cong -\left[\frac{\partial^2 J(u')}{\partial u'^2}\right]^{-1} \frac{\partial J(u')}{\partial u'}. \quad (32)$$

The right-hand side of (32) is called Newton's direction [16].

Since (32) is an approximate relation, in order to guarantee that the performance function decreases at any stage, (32) is written in the form of (33):

$$\Delta u' \cong -\left[\frac{\partial^2 J(u')}{\partial u'^2} + \lambda\right]^{-1} \frac{\partial J(u')}{\partial u'}. \quad (33)$$

In practice, the following relation is used:

$$\Delta u' = u'_{new} - u'_{old} = -\eta \left[ \frac{\partial^2 J(u')}{\partial u'^2} + \lambda \right]^{-1} \frac{\partial J(u')}{\partial u'}. \quad (34)$$

$$\text{where; } \lambda = d \times \frac{\partial^2 J(u')}{\partial u'^2}. \quad (35)$$

An initial value is assigned to  $d$  (e.g. 0.001), then  $\delta$  is generated:

$$\delta = \frac{\frac{\partial J(u')}{\partial u'}}{(d+1) \frac{\partial^2 J(u')}{\partial u'^2}}, \quad (36)$$

$$\text{Then : if } E(u' + \delta) < E(u') \text{ then } d=d/10; \text{ otherwise } d=d \times 10; \quad (37)$$

(10 is a modification factor, it can be of another value)

As the value of  $d$  is repeated, the algorithm stops.

(34) represents Levenberg-Marquardt method for the optimisation of a single variable function [16]. In this method,  $g_k$  is numerically considered as performance function gradient:

$$\frac{\partial J(u')}{\partial u'} = g_k = \frac{J(k) - J(k-1)}{u'(k) - u(k-1)}, \quad (38)$$

moreover,  $G_k$  is defined as:

$$\left[ \frac{\partial^2 J(u')}{\partial u'^2} \right]^{-1} = G_k = \frac{u'(k) - u(k-1)}{(g_k - g_{k-1})}. \quad (39)$$

So (34) can be rewritten in this form:

$$\Delta u' = u'_{new} - u'_{old} = -\eta (1+d) G_k g_k. \quad (40)$$

Using (40), we will have

$$J(u'_{new}) = J(u'_{old} - \eta (1+d) G_k g_k), \quad (41)$$

$$\text{or} \quad \text{Argument of } J = u'_{old} - \eta (1+d) G_k g_k. \quad (42)$$

Both  $u'_{old}$  and  $(1+d)G_k g_k$  are known in this stage; then, with changing  $\eta$ , *argument of J*

moves along a line. There is an optimum point on this line that minimizes  $J$ . Such an optimization problem is classified as a linear search. Backtracking method, introduced by Dennis and Schnabel [18], is selected for linear search. The modified  $u'$  ( $u'_{new}$ ) is used as a new control input.

### ***XII-B. Fuzzy steepest descent (FSD)***

A fuzzy-derivative optimisation method is specially designed to suit optimisation tasks for predictive control purposes.

$$\text{In (29), if } \Delta u' \text{ is replaced by } -\eta \frac{\partial J(u')}{\partial u'} \text{ then } J(u' + \Delta u') \cong J(u') - \eta \left( \frac{\partial J(u')}{\partial u'} \right)^2. \quad (43)$$

$$\text{If } \eta > 0, \text{ it can be concluded (approximately) } J(u' + \Delta u') \leq J(u'). \quad (44)$$

Any positive value can be used for  $\eta$ . For nonlinear predictive purposes,  $\eta$  can be generated by a fuzzy inference system (FIS) to reduce the alteration of the control system's response when it is approaching the setpoint/reference. This FIS has two rules:

Rule 1: if  $|e|$  is low then  $\eta = 0$

Rule 2: if  $|e|$  is high then  $\eta = 1$

Figure 16 shows the membership function of high and low.

The designed FIS can be simplified to a simple function of  $|e|$ .

$$\mu_{HIGH} = |e| \quad (45)$$

$$\text{For } |e| < 0.5 : \quad \mu_{LOW} = -2|e| + 1, \quad (46)$$

$$\text{According to (3), } \eta = \frac{[0 \times (-2|e| + 1)] + [1 \times |e|]}{(-2|e| + 1) + |e|} = \frac{|e|}{1 - |e|}. \quad (47)$$

$$\text{For } |e| > 0.5 : \quad \mu_{LOW} = 0 \quad (48)$$

$$\text{According to (3), } \eta = \frac{[0 \times 0] + [1 \times |e|]}{(0) + |e|} = 1. \quad (49)$$

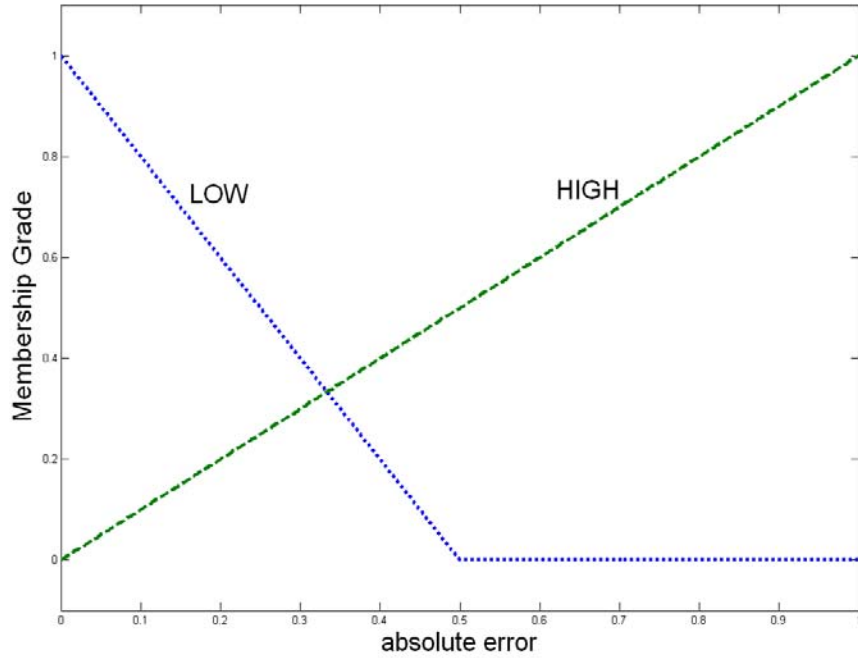


Figure 16: Membership functions of high and low fuzzy values in fuzzy steepest descent optimisation

In summary:

$$\begin{cases} |e| < 0.5 \Rightarrow \eta = \frac{|e|}{1-|e|} \\ |e| \geq 0.5 \Rightarrow \eta = 1 \end{cases} \quad (50)$$

For nonlinear predictive control purposes, the proposed fuzzy steepest descent method leads to much better control performance in comparison with classical steepest descent methods. In terms of control performance, fuzzy steepest descent is comparable with Levenberg-Marquardt method with around ten times less computation (Mohammadzaheri and Chen, 2008).

### XIII. APPENDIX 3: COMPLEMENTARY PLOTS

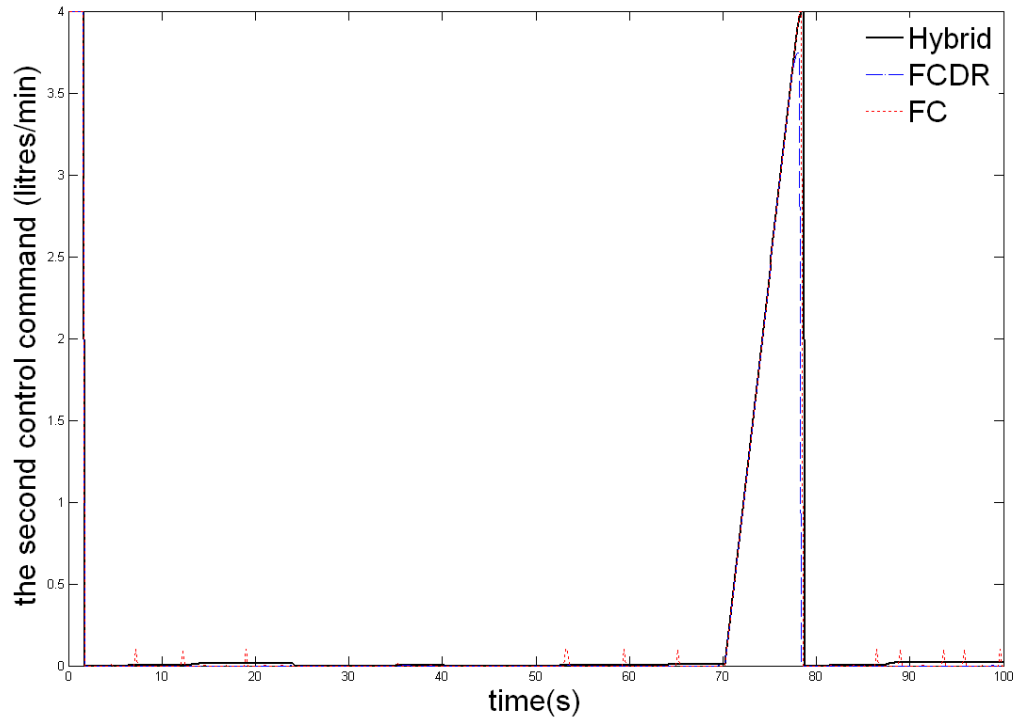


Figure 17. The second output versus time, for the operation shown in Fig. 13

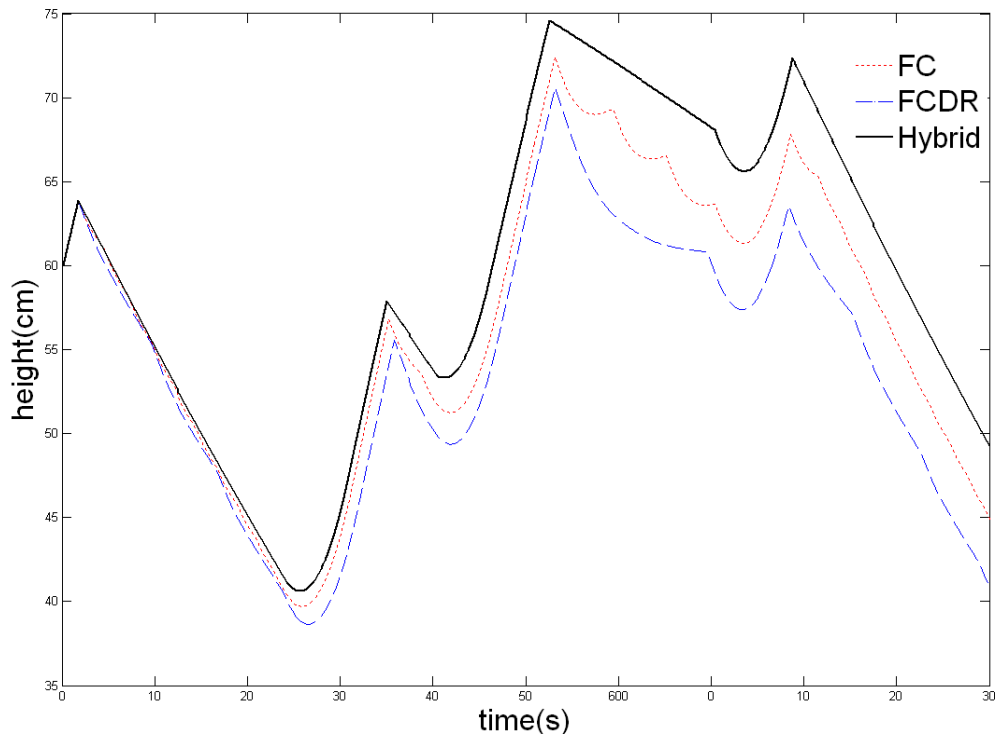


Figure 18. The level height versus time, for the operation shown in Fig. 13

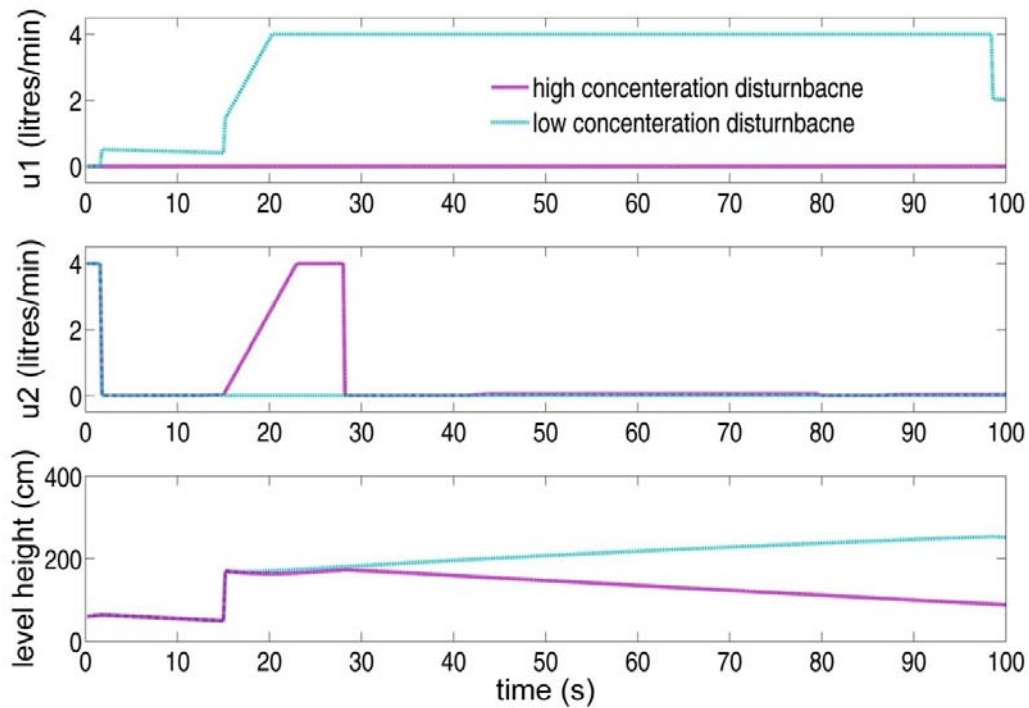


Figure 19. Level height and control inputs during operation shown in Fig. 15 (instantaneous disturbances)

#### XIV. NOMENCLATURE

$a$	:Parameters of consequent part of fuzzy rules	<b>superscript</b>
A	:Assumption	$\hat{\quad}$ :Estimated
BIBO	:Bounded input- bounded output	<b>subscripts</b>
$C_b$	: Outlet concentration of CSTR( $kmol/m^3$ )	$d$ :Desired
$C_{b1}$	: The concentration of the first inlet flow to CSTR( $kmol/m^3$ )	$k$ : Numerator
$C_{b2}$	: The concentration of the second inlet flow to CSTR( $kmol/m^3$ )	$s$ :Predicted
CSTR	:Catalytic Stirred Tank Reactor	ss :Steady state
$du_{1-v}$	:Consequents of fuzzy controller rules (litres/min)	tr :Transient
$e$	:error( $kmol/m^3$ )	
E	:Absolute value of error( $kmol/m^3$ )	
EI	:Error Integral ( $kmol/m^3$ )	
F, $f$	:function	
FC:	:With fuzzy controller	
FCDR	:With fuzzy controller having damper rule	
FSD	:(with) Fuzzy steepest descent method	
$g$	:Defined in (34)	
$G$	:Defined in (35)	

$h$	:Level height ( $cm$ )
IC	:Input constancy criterion ( $litres$ )
$J$	:Performance function
$K_{1,2}$	:CSTR parameters relevant to resistance of valves
LM	:(with) Levenberg-Marquardt method
PAN	:Prediction accuracy for N next instants ( $kmol/m^3$ )
$r$	:Reference ( $kmol/m^3$ )
SD	:(with) Steepest Descent
$t$	: Time (s)
Time	: Time needed for the simulation of control system for 100 seconds(s)
$u$	:Control input ( $litres/min$ )
$u'$	: Tentative control input ( $litres/min$ )
$u_1$	: Volume rate of high concentration input to CSTR ( $litres/min$ )
$u_2$	: Volume rate of low concentration input to CSTR ( $litres/min$ )
$w$	: Weight of a fuzzy rule
$x$	: Input to a fuzzy rule
$y$	: Output of the system
$z$	: Output of a fuzzy rule
$z^{-1}$	: Unit delay function ( $1/s$ )
$\rho$	:a factor defining the importance of the constancy of control input
$\mu$	: membership grade
$\tau$	: time period of operation (s)

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## Part 4

# A General Feedforward-feedback Approach for Process Control

### SYNOPSIS

Part 4 contains four chapters and each chapter is in the form a journal article. One of these journal articles is now accepted and will be published in September 2010 (Chapter 8) and others are submitted. In this part, generalized zero type (GZT) systems are introduced based on the concept of steady state control command defined in Part 3. Also, the applicability of feedforward controllers for GTZ systems is indicated, and in this case, steady state controllers, defined in Part 3, are employed to form the feedforward control laws. As a result, being or not being GTZ, as a new form of knowledge about the system dynamics, is usable in control system design.

A feedback-forward control system, including the aforementioned feedforward control law and a proportional feedback controller, is proposed in this part. For a class of process plants, the stability of the proposed control system is proved with no limit on the value of the gain. It means that the gain can be extremely high and the system can converge to the reference as quickly as possible with guaranteed stability, considering practical limits of the actuator. This methodology is well explained in Chapters 7 and 8 for single command and double command control applications.

However, the stability conditions of the proposed control system may be invalidated in the case of existence of at least one variable coupled with the control output or system dead-time.

For example, in concentration control of the catalytic stirred tank reactor, the height of liquid level is coupled with the concentration (control output). The control system proposed in Chapters 7 and 8 may lead to evacuation of the reactor (height of liquid level is zero) if this variable is not considered. This issue is addressed in Chapter 9 and is resolved satisfactorily by employing a special feedforward neural network control law. In the proposed general approach in Chapters 7 and 8, the availability of the immediate error or the error which is affected by the most recent control input is essential; thus, time-delay is a serious issue. This issue is addressed through a predictive approach in Chapter 10. After the final modification in Chapter 10, the proposed method seems prepared for real application. The experimental results presented in Chapter 10 support this idea.

## **Chapter 7**

### **A Design Approach for Feedback-feedforward Control Systems**

Text in manuscript.

# A Design Approach for Feedback-feedforward Control Systems

Morteza Mohammadzaheri, Lei Chen

**Abstract**— In this paper, a general design approach is proposed to derive the feedforward control law in feedback-feedforward control systems based on the concept of ‘Control Equilibrium Point’. This methodology determines whether a feedforward controller is needed, and if so, it provides a procedure to develop a feedforward control law based on the system mathematical model. In the event of significant uncertainties, an artificial neural network is designed and trained as feedforward control law. Outlet concentration of a non-thermic CSTR is controlled using the proposed methodology. While the feedback controller is a gain (proportional), the stability of the control system is proven with no limit on the value of the gain. Stability proof is based on some evident practical facts rather than the mathematical model, so the proof remains valid in case of change in parameters and it can be extended to a wide class of process plants.

**Key words:** Feedforward Control, CSTR, Artificial Neural Networks, Lyapunov Stability, GTZ systems.

## I. INTRODUCTION

FEED-FORWARD control laws are extensively used in control system, sometimes together with feedback control laws. The most popular input to feedforward controllers is the reference or setpoint signal [1-6], although measurable disturbance signals may also play this role [7, 8]. Special algorithms based on the dynamics of the system [4, 7], linear transfer functions/filters [2] or fixed gains [1] have been applied as feedforward control laws; another popular feedforward control laws is the inverse model of the system [2, 4, 6, 9]. Artificial neural networks have also been used to generate the feedforward control command [5, 10, 11]

Despite all advantages afforded by feedforward control commands, when the disturbance rejection is not the main purpose, there is no general methodology to determine whether a feedforward controller is useful, and if so, to find the feedforward control law. This paper addresses these two issues using the concepts control equilibrium point and steady state control command which have already been introduced by authors [5, 10].

## II. CONTROL EQUILIBRIUM POINT AND STEADY STATE CONTROL COMMAND

Let us define the control equilibrium point of a (under control) system as below:

$$\frac{d^{(i)}e(t)}{dt^{(i)}} = 0, i = 0, \dots, \text{system's order}, \quad (1)$$

where  $e(t)$  is the error. In other words, at the control equilibrium point (CEP) the error is zero and remains zero. If (1) is satisfied; that is, the system is at the desired situation (reference) and stays at this situation. In this paper, the control input obtained by the solution of (1) is called *steady state control input* which is the control input at the control equilibrium point.

From the view point of the value of steady state control input, systems can be divided into two categories. In many systems, at the control equilibrium point, the control input is not zero, meaning, the control equilibrium point is maintained only by the continuous exertion of a control input. In linear systems, type-0 systems have such a feature. In this paper, these systems (systems with non-zero steady state control command) are called generalized type zero (GTZ), whether they are linear or nonlinear. Some instances of so called GTZ systems are level control of water tank with a hole at the bottom, all temperature control problems with the reference different from the environment temperature, and position control of mechanical systems influenced by gravity.

Other systems, with zero steady state control command which maintains the desired situation not needing a control input after reaching and settling at the setpoint (reference), are called generalized- non- type-zero (GNTZ). Level control of a water tank and position control of mechanical systems not influenced by gravity are good examples of such systems.

In summary,

in GNTZ control systems, steady state control input is zero ( $u_{ss} = 0$ ).

in GTZ control systems, steady state control input is not zero ( $u_{ss} \neq 0$ ).

A GTZ control system cannot be controlled just by a gain (P-action) feedback controller. On the other side, a GNTZ control system may be controlled appropriately using a gain error feedback controller.

### III. CASE STUDY

A diagram of the studied non-thermic CSTR is shown in Fig.1:

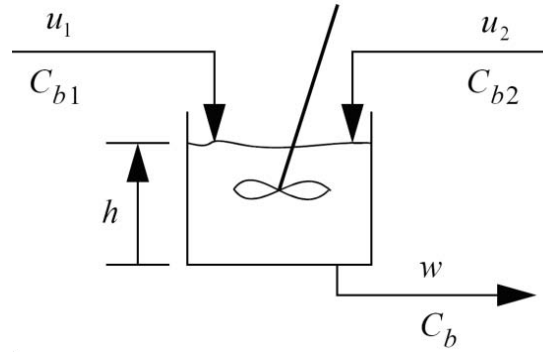


Figure 1. A schematic of the studied CSTR

Two flows of liquid enter the reactor with the concentration of  $C_{b1} = 24.9 \text{ kmol/m}^3$  and  $C_{b2} = 0.1 \text{ kmol/m}^3$ . The flow rates of input flows are named  $u_1$  and  $u_2$ . The reactor outlets another flow of liquid with the concentration of  $C_b$  and the flow rate of  $w$ . The height of liquid in the reactor ( $h$ ) represents  $w$  ( $w(t) = 0.2\sqrt{h(t)}$ ).

A simplified mathematical model of this MIMO and nonlinear system, achieved by mass equilibrium equations, is:

$$\dot{h}(t) = u_1(t) + u_2(t) - 0.2\sqrt{h(t)}, \quad (2)$$

$$\dot{C}_b(t) = [C_{b1} - C_b(t)] \frac{u_1(t)}{h(t)} + [C_{b2} - C_b(t)] \frac{u_2(t)}{h(t)} - \frac{k_1 C_b(t)}{(1 + k_2 C_b(t))^2}, \quad (3)$$

where  $k_1$  and  $k_2$  are parameters relevant to the resistance of valves located in the path of liquid flow.

#### IV. CONTROL SYSTEM

The control of the outlet *concentration* is addressed in this paper. The low concentration flow ( $u_2$ ) is set to a fixed value (0.1 *litres/min*) and the control algorithm commands the other flow ( $u_1$ ). In order to consider the actuator limit, the maximum of 4 *litres/min* and the minimum of zero are considered for  $u_1$  in simulation.

In order to find steady state control command, following equations were used, representing the error and its derivatives. The control output is the concentration.

$$e(t) = C_d(t) - C_b(t), \quad (4)$$

$$\dot{e}(t) = [C_b(t) - C_{b1}] \frac{u_1(t)}{h(t)} + [C_b(t) - C_{b2}] \frac{u_2(t)}{h(t)} + \frac{k_1 C_b(t)}{(1 + k_2 C_b(t))^2}. \quad (5)$$

At the control equilibrium point (where  $\dot{e} = e = 0$ ) :

$$C_b(t) = C_d, \text{ and } u_{1,ss}(t) = \frac{h(t)}{C_{b1} - C_d(t)} \left[ \frac{k_1 C_d(t)}{(1 + k_2 C_d(t))^2} + (C_d(t) - C_{b2}) \frac{u_2}{h} \right]. \quad (6)$$

$u_{1,ss}$ , steady state control command, has a non-zero value, so the system is GNTZ. (6) was used as the feedforward control law. A positive gain ( $k_c$ ) was selected as the feedback (transient) controller. It is proven that the value of this gain could be arbitrarily high without harming the stability (see the appendix), so the stability and the highest convergence speed towards the setpoint, considering actuator limits, were obtained simultaneously and there was no need to employ more complicated feedback controllers.

$$u_1 = u_{1,ss} + k_c e, \quad (7)$$

where  $u_{1,ss}$  is shown in (6).

The level height was not subject to control in the proposed algorithm (there is only one control input).

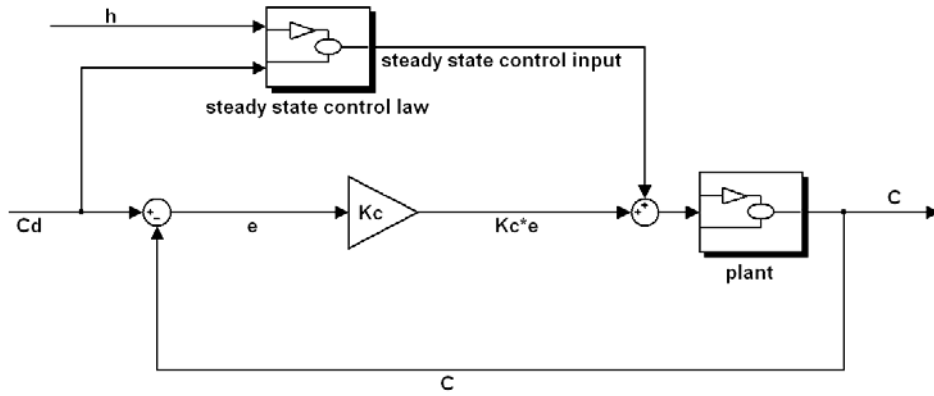


Figure 2. Control system, with a model-based feedforward controller

In the case of substantial uncertainty in the mathematical model, an artificial neural network (ANN) can be designed, trained and employed as the feedforward (steady state) control law. The steady state response, the concentration, is the input to this neural network, and the control input is the output of ANN. Applying any value of the control input to the system leads to a unique pair of the concentration and the level height in the steady state situation, so each value of the steady state concentration has its corresponding unique value of the steady state level height. Thus, when the steady state concentration is used in training neural network, there is no need to use the level height too. Therefore, the training data include a collection of control input and steady state concentration pairs.

For this case study, a perceptron with five sigmoid neurons in the hidden layer was designed and trained as the steady state control law (see Fig.3 and Table 1). The designed ANN offers a good checking accuracy in the area of  $[0.2 \ 1.5]$  *litres/min* for control input.

$$Checking\ Error = \frac{\sum_{i=1}^N |e_i|}{N} \cong 3 \times 10^{-5} \text{ litres/min} \quad (8)$$

where  $e$  stands for error and  $N$  stands for the number of checking input-output data (130).



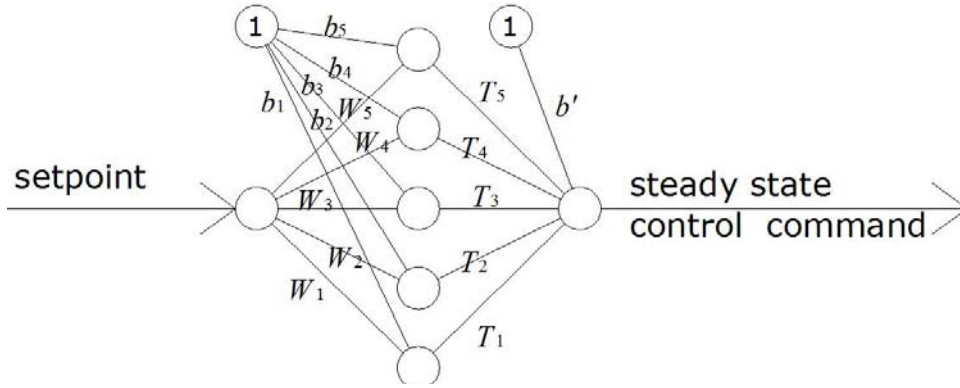


Figure 3. Neural network as the steady state controller

Table 1. Weights of neural network connections

$W_1$	0.429	$T_1$	1.7564	$b_1$	-3.129	$b'$	0.121
$W_2$	18.16	$T_2$	0.0004	$b_2$	-22.69		
$W_3$	-19.38	$T_3$	-0.0007	$b_3$	17.623		
$W_4$	19.73	$T_4$	0.0090	$b_4$	-11.98		
$W_5$	-20.43	$T_5$	-0.0010	$b_5$	6.898		

## V. SIMULATION RESULTS

With  $k_1 = k_2 = 1$  and the sampling time of 0.2 s, the response of the aforementioned control system is shown in Fig.4 (from  $t=20s$ ) with  $k_c = 50$ . This response was compared to the response of the system controlled by neuro-predictive method.

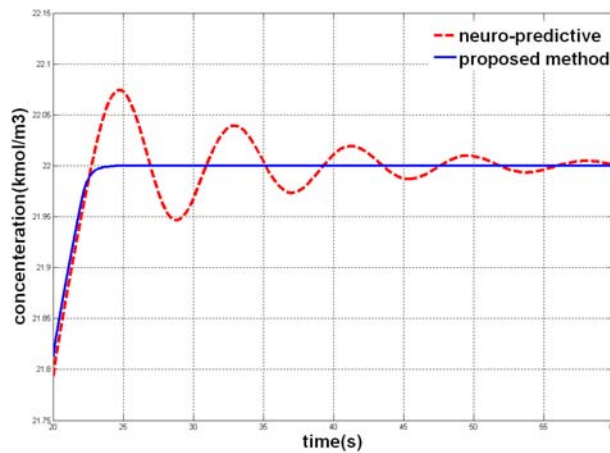
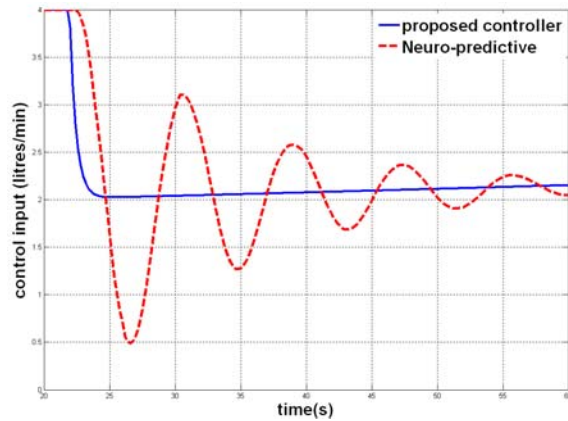


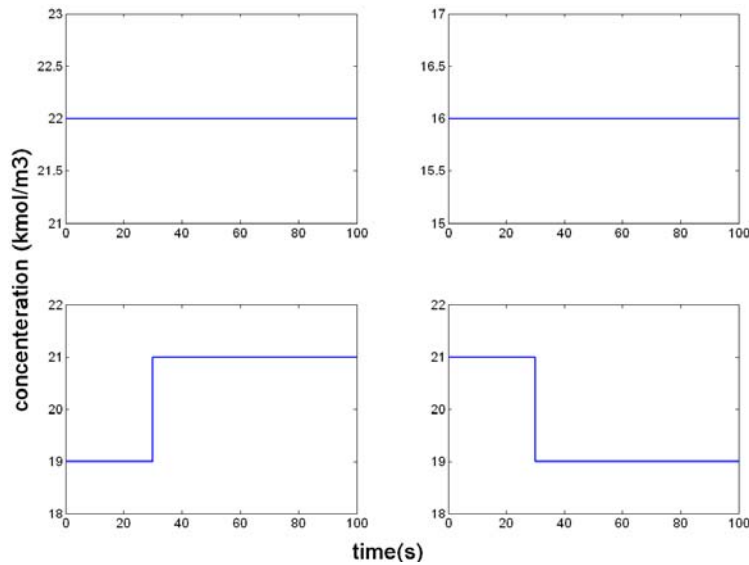
Figure 4. the response of system with different control systems

Figure 5 shows the control input during operation leads to the response shown in Fig.4. The initial concentration was  $15 \text{ kmol/m}^3$  and the initial height was 30 cm. The response of the proposed control system with model-based feedforward and ANN control laws are very similar together.



**Figure 5. Control command of different control systems**

In another simulation, for a set of fixed setpoints (shown in Fig. 6), following results were achieved.



**Figure 6. Setpoints used for simulation**

**Table 2. Results of simulation**

	MAE ( $kmol/m^3$ )
NP	0.4862
Proposed Controller (model-based)	0.4604
Proposed Controller (with ANN)	0.4650

## VI. CONCLUSION

In this paper, based on the concept of ‘control equilibrium point’ and the steady state control command, generalized zero type (GZT) systems were introduced. According to the proposed control approach, a model-based steady state control law (which generates the steady state control command) was employed as a feedforward controller for GZT systems. If a precise model was lacked, an artificial neural network could be designed and trained to play the role of the steady state controller. Such a feedforward control law together with a P-action feedback controller was used to control the outlet concentration of a non-thermic CSTR. Lyapunov asymptotic stability of the control system was proven, regardless of the value of the gain. That is, feedback gain can have an arbitrarily high value and the system can converge towards the reference as quickly as possible with no instability or overshoot. The only constraint on the performance is the actuator limits. The offered stability proof is not dependent on the mathematical model of the system; however, it is based on some generic practical assumptions which can be easily extended to many other process plants. The proposed control system resulted in an excellent control performance as well as a satisfactory disturbance rejection and robustness in this research work.

## VII. NOMENCLATURE

A	:Assumption	<b>subscripts</b>
ANN	:Artificial Neural Network	$d$ :Desired
$b_i$	:Bias of the $i^{th}$ neuron of the first layer of the ANN connections	$i$ :Numerator

$b'$	:Bias of the second layer of the ANN connections	ss	:Steady state
$C_b$	:Outlet concentration of CSTR( $kmol/m^3$ )		
$C_{b1}$	:The concentration of the first inlet flow to CSTR( $kmol/m^3$ )		
$C_{b2}$	:The concentration of the second inlet flow to CSTR( $kmol/m^3$ )		
$C_d$	:The desired outlet concentration of CSTR( $kmol/m^3$ )		
CSTR	:Catalytic Stirred Tank Reactor		
$e$	:Error( $kmol/m^3$ )		
$\dot{e}_{ss-C_d}$	: Temporal derivative of the error when $u_{1,ss}$ is applied on the system solely ( $kmol/m^3$ )		
GNTZ	:Generalized non-zero-type		
GTZ	:Generalized zero-type		
$h$	:Level height ( $cm$ )		
$k_{1,2}$	:CSTR parameters relevant to resistance of valves		
$k_c$	:Feedback controller gain		
MAE	:Mean of absolute error		
MIMO	:Multi-input multi-output		
N	:The number of checking data		
NP	:Neuro-predictive		
$t$	:Time (s)		
$T$	:The weights of the second layer of the ANN connections		
$u$	:Control input ( $litres/min$ )		
$u_1$	:Volume rate of high concentration input to CSTR ( $litres/min$ )		
$u_2$	:Volume rate of low concentration input to CSTR ( $litres/min$ )		
V	:Lyapunov Function		
$w$	:Volume rate of CSTR output ( $litres/min$ )		
$W$	:The weights of the first layer of the ANN connections		
$\mathbf{x}$	:States' vector		

## VIII. ACKNOWLEDGEMENT

The authors thank Ms. Karen Adams for assistance in the English editing of this manuscript.

## IX. APPENDIX: STABILITY PROOF

According to Lyapunov theorem, an equilibrium point is globally stable if a scalar function

(V) with continuous first temporal derivative can be found so that [12]

Condition 1:  $V(\mathbf{x})$  is positive definite;

Condition 2:  $\dot{V}(\mathbf{x})$  is negative definite;

Condition 3:  $\|\mathbf{x}\| \rightarrow \infty \Rightarrow V(\mathbf{x}) \rightarrow \infty$ .

For the first order system shown in (3) and (5),  $\mathbf{x}=[e \ h]$ ,  $h$  is not subject to control, so it is not considered in stability analysis. The Lyapunov function is the error square or

$$V(\mathbf{x}) = e^2. \quad (9)$$

With this Lyapunov function conditions 1 and 3 are evidently satisfied.

In order to prove condition 2, it should be proven that

$$\begin{cases} \mathbf{x} \neq 0 \Rightarrow \dot{V}(\mathbf{x}) < 0 \\ \mathbf{x} = 0 \Rightarrow \dot{V}(\mathbf{x}) = 0 \end{cases} \quad (10)$$

$\dot{V}(\mathbf{x}) = 2e\dot{e}$ , so in this problem, (10) is written in the form of (11):

$$\begin{cases} e \neq 0 \Rightarrow e\dot{e} < 0. \\ e = 0 \Rightarrow e\dot{e} = 0. \end{cases} \quad (11)$$

Below equation of (11) is evident. As a result, if  $e \neq 0 \Rightarrow e\dot{e} < 0$  is proven, the system is asymptotically stable and error square decreases continuously with no overshoot.

The stability study in this paper is based on two evident assumptions rather than the mathematical model. So the stability proof remains valid in case of change in parameters and it can be extended to systems which similar assumptions are valid for them.

Following statement is the first assumption:  $u_{1,ss}(C_d, h)$  maintains the output concentration at  $C_d$  or

$$\mathbf{A1}: u_1 = u_{1,ss}(C_d, h) \ \& \ C_b < C_d [C_b > C_d] \Rightarrow \dot{C}_b > 0 [\dot{C}_b < 0].$$

or (see (4))

$$\mathbf{A1}: u_1 = u_{1,ss}(C_d, h) \ \text{AND} \ e > 0 [e < 0] \Rightarrow \dot{e} < 0 [\dot{e} > 0].$$

If  $\dot{e}_{ss-C_d}$  is the temporal derivative of the error when  $u_{1,ss}(C_d, h)$  is applied on the system solely. A1 can be written as:

$$\mathbf{A1}: e > 0 [e < 0] \Rightarrow \dot{e}_{ss-C_d} < 0 [\dot{e}_{ss-C_d} > 0].$$

Here is the second assumption, if the control input (the flow of high-concentration liquid) increases [decreases], the temporal derivative of the concentration increases [decreases] and the error rate decreases [increases] (see (4)). In summary,

**A2:** The higher [lower]  $u_1$ , the lower [higher]  $\dot{e}$ .

Using these assumptions (A1 and A2), the stability (or (11)) is proven (at any level height).

*Proof:*

At the concentration of  $C_b$  (see (4)):

$$e > 0 \left\{ \begin{array}{l} \stackrel{(7)}{\Rightarrow} u_1 = u_{1,ss}(C_d, h) + k_c e > u_{1,ss}(C_d, h) \stackrel{A2}{\Rightarrow} \dot{e} < \dot{e}_{ss-C_d} \\ \stackrel{(A1)}{\Rightarrow} \dot{e}_{ss-C_d} < 0 \end{array} \right\} \Rightarrow \dot{e} < 0 \Rightarrow e\dot{e} < 0 \quad (12)$$

$$e < 0 \left\{ \begin{array}{l} \stackrel{(7)}{\Rightarrow} u_1 = u_{1,ss}(C_d, h) + k_c e < u_{1,ss}(C_d, h) \stackrel{A2}{\Rightarrow} \dot{e} > \dot{e}_{ss-C_d} \\ \stackrel{(A1)}{\Rightarrow} \dot{e}_{ss-C_d} > 0 \end{array} \right\} \Rightarrow \dot{e} > 0 \Rightarrow e\dot{e} < 0 \quad (13)$$

*End of the proof*

This proof is valid just in the case that  $u_{1,ss}(C_d, h)$  is applicable by the actuator. If the steady state control command is outside actuator limits, [0 4] *litres/min* in this problem, the proof will be invalid. For the current problem, steady state control does not exceed 2.5 *litres/min* in practice for any value of the steady state concentration. The abovementioned invalidity condition (the actuator can not apply the steady state control command) can make the system impossible to control in general, so it is not a major restriction for the proposed control system.

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## **Chapter 8**

### **Double Command Feedforward-Feedback Control of a Nonlinear Plant**

Korean Journal of Chemical Engineering, Volume 27, No. 5, Pages 1372-1376, September 2010.



# Double-Command Feedforward-Feedback Control of a Nonlinear Plant

Morteza Mohammadzaheri, Lei Chen

**Abstract**— In this paper, a design approach is proposed for feedforward-feedback control systems. The basis of the proposed approach is a steady state control law which maintains the desired control output of the system and is employed as the feedforward controller. With this feedforward controller, for a wide class of systems, the stability of the system is proved if the feedback controller is a gain with an arbitrarily high value; that is, the only limit for the feedback (transient) control command is the actuator's practical limit. Moreover, in continuous domain, there will be no overshoot. In this article, the proposed method has been applied to a Catalytic Stirred Tank Reactor (CSTR) to control the flow of two valves simultaneously and resulted in an excellent control response.

**Keywords:** Process Control, Feedforward, Lyapunov, CSTR, Control Equilibrium Point

## I. INTRODUCTION

FEEDFORWARD control commands are used extensively in control, sometimes together with feedback control commands. Some recent applications for feedforward control include power systems [1], medical engineering [2], aircraft/helicopter control [3, 4], vibration and noise control [5], manufacturing [6] and robotics [7].

The most popular input to feedforward controllers is the reference or setpoint signal [1-4, 6], although measurable disturbance signals may also play this role [5, 8, 9]. However, especially when the disturbance rejection is not the main issue, there is no general methodology to determine whether a feedforward controller is useful or not, and if so, to find the feedforward control law. In this paper, these questions are addressed and a general methodology is offered to answer them. Based on the introduced methodology, in simulation environment, a double-

command feedforward-feedback control system is designed for the outlet concentration of a non-thermic Catalytic Stirred Tank Reactor (CSTR).

## II. DESIGN METHODOLOGY

Let us define the control equilibrium point of an under control system as below:

$$\frac{d^{(i)}e(t)}{dt^{(i)}} = 0, i = 0, \dots, \text{system's order} , \quad (1)$$

where  $e(t)$  is the control error. In other words, at the control equilibrium point (CEP) the error is zero and remains zero.

*The control input obtained by the solution of (1) is called steady state control input.*

In some systems, the control equilibrium point is maintained only by continuous exertion of a control input. In other words, a (steady state) control input should be consistently applied on the system to maintain a desirable control output similar to type zero linear transfer functions. In this paper, such systems are called 'generalized type zero' (GTZ) systems. Some instances of so called GTZ systems are level control of water tank with an outlet at the bottom, all temperature control problems with the reference different from environment temperature, and position control of mechanical systems influenced by the gravity. Other systems which can retain their desired output with no control input are called 'generalized non-type zero' (GNTZ) systems.

In summary:

In GNTZ control systems: steady state control input is zero ( $u_{ss} = 0$ ).

In GTZ control systems: steady state control input is not zero ( $u_{ss} \neq 0$ ).

In the proposed method, steady state control command which satisfies (1) is employed as the feedforward control command which is used together with a feedback (transient) control command ( $u_{tr}$ ).

$$u = u_{tr} + u_{ss} \quad (2)$$

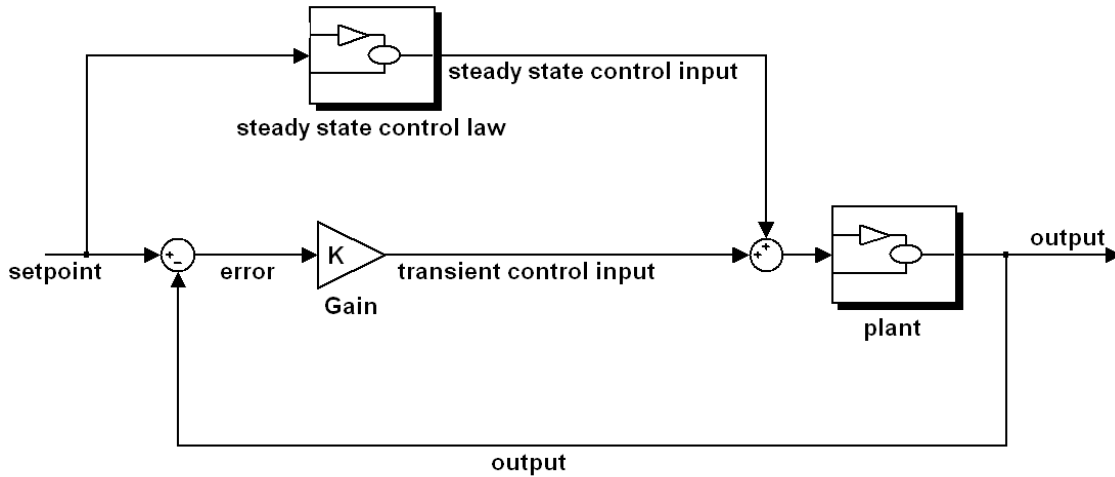


Figure 1. Feedforward-feedback control system for a SISO system

### III. CASE STUDY : CONCENTRATION CONTROL OF A NON-THERMIC CATALYTIC STIRRED TANK REACTOR

A diagram of the studied CSTR is shown in Fig.2:

NOTE:  
This figure is included on page 192 of the print copy of the thesis held in the University of Adelaide Library.

Figure 2 A schematic of the studied CSTR [10]

Two flows of liquid enter the reactor with the concentration of  $C_{b1} = 24.9 \text{ (kmol/m}^3\text{)}$  and  $C_{b2} = 0.1 \text{ (kmol/m}^3\text{)}$ . The flow rates of input flows are named  $u_1$  and  $u_2$ . The reactor outlets another flow of liquid with the concentration of  $C_b$  and the flow rate of  $w$ . The level height of liquid in the reactor ( $h$ ) represents  $w$  ( $w = 0.2\sqrt{h}$ ).

A simplified mathematical model of the system, derived by mass equilibrium equations, is:

$$\dot{h}(t) = u_1(t) + u_2(t) - 0.2\sqrt{h(t)}, \quad (3)$$

$$\dot{C}_b(t) = [C_{b1} - C_b(t)] \frac{u_1(t)}{h(t)} + [C_{b2} - C_b(t)] \frac{u_2(t)}{h(t)} - \frac{k_1 C_b(t)}{1 + k_2 C_b(t)^2}, \quad (4)$$

where  $k_1$  and  $k_2$  are parameters regarding the resistance valves located in the path of input and output flows respectively, and  $h$  is the height of liquid level. The concentration of the outlet flow and the height of liquid level are considered as the outputs.

#### IV. CONTROL SYSTEM

The control of the outlet concentration is addressed in this paper. There are two control inputs available ( $u_1$  and  $u_2$ ) with the maximum volumetric flow rate of 4 *litres/min*.

First the system is checked to determine if it is GTZ. To do so, the following equations are used, representing the error and its first derivative.

$$e = C_d - C_b(t), \quad (5)$$

$$\dot{e}(t) = [C_b(t) - C_{b1}] \frac{u_1(t)}{h(t)} + [C_b(t) - C_{b2}] \frac{u_2(t)}{h(t)} + \frac{k_1 C_b(t)}{1 + k_2 C_b(t)^2}. \quad (6)$$

where  $C_d$  is the reference (desired outlet concentration). According to (4), if both control inputs equal zero, the outlet concentration still changes; as a result, a control input is needed to maintain the desired output, and the system is GTZ.

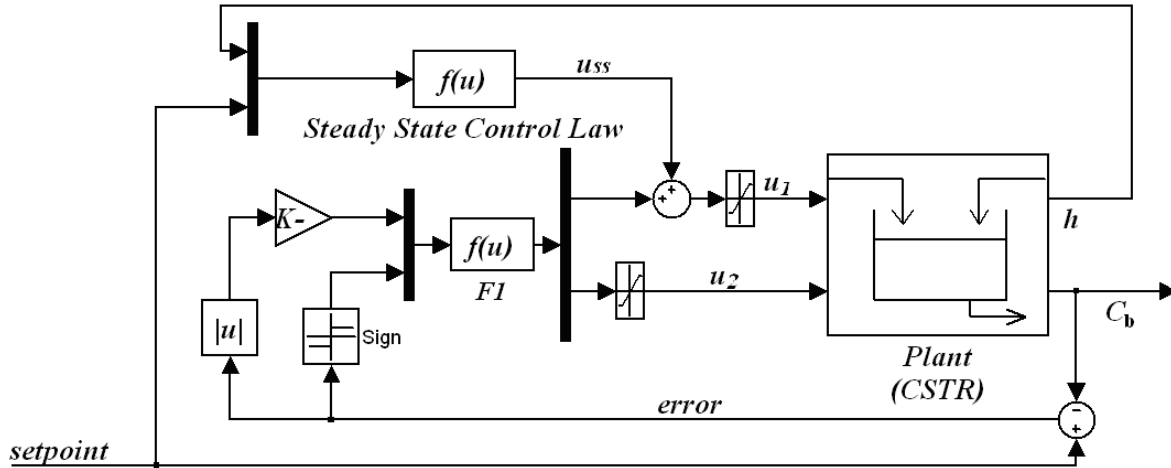


Figure 3. Control system, with a model-based feedforward controller

There are two variables ( $u_1$  and  $u_2$ ) in (6), so the steady state control law derived from solving (5) and (6) would not output a unique couple of control commands.

If both control inputs are zero, the error increases (according to (6)) and concentration decreases. Therefore, the role of the steady state control command will be to increase outlet concentration to some appropriate extent. Consequently,  $u_1$  (with the concentration of  $24.9 \text{ kmol}/\text{m}^3$ ) can play this role solely. In other words, since the second flow decreases the concentration, it is cut and  $u_1$  is the only control input in the steady state situation. In such a situation, the solution of (1), (5) and (6) results in

$$C_b = C_d, \quad \text{and} \quad \begin{cases} u_{ss)1} = u_{ss} = \frac{h(t)}{C_{b1} - C_d} \left[ \frac{k_1 C_d}{1 + k_2 C_d^2} \right] \\ u_{ss)2} = 0 \end{cases} \quad (7)$$

It is observed that the steady state control command is a function of the setpoint/ reference ( $C_d$ ) and the height of liquid level.

Feedback controller is the modified form of an arbitrarily high gain ( $K$ ) (transient control command is generated in F1, in Fig.3). In total, (8) is the general control law:

$$\begin{cases} u_1 = Ke(\text{if } e > 0) + u_{ss} \\ u_2 = -Ke(\text{if } e < 0) + 0 \end{cases} \quad (8)$$

where  $u_{ss}$  was defined in (7).

## V. STABILITY

According to Lyapunov theorem, for a system with the states vector of  $\mathbf{x}$ , an equilibrium point and the system is globally stable if a scalar function ( $V(\mathbf{x})$ ) with continuous first temporal derivative can be found so that [11]

Condition 1:  $V(\mathbf{x})$  is positive definite.

Condition 2:  $\dot{V}(\mathbf{x})$  is negative definite.

Condition 3:  $\|\mathbf{x}\| \rightarrow \infty \Rightarrow V(\mathbf{x}) \rightarrow \infty$  [11].

For the first order system shown in (3) and (6),  $\mathbf{x}=[e \ h]$ .  $h$  is not subject to control, so it is not considered in stability analysis. Lyapunov function is the error square or

$$V(\mathbf{x}) = e^2. \quad (9)$$

With this Lyapunov function conditions 1 and 3 are evidently satisfied.

In order to prove condition 2, it should be proved that

$$\begin{cases} \mathbf{x} \neq 0 \Rightarrow \dot{V}(\mathbf{x}) < 0 \\ \mathbf{x} = 0 \Rightarrow \dot{V}(\mathbf{x}) = 0 \end{cases} \quad (10)$$

$\dot{V}(\mathbf{x}) = 2e\dot{e}$ , so in this problem, (10) is written in the form of (11):

$$\begin{cases} e \neq 0 \Rightarrow e\dot{e} < 0 \\ e = 0 \Rightarrow e\dot{e} = 0 \end{cases} \quad (11)$$

Below equation of (11) is evident. As a result, if  $e \neq 0 \Rightarrow e\dot{e} < 0$  is proved, the system is asymptotically stable (error square decreases continuously with no overshoot).

Two assumptions are used in this stability study:

$$\mathbf{A1:} \begin{cases} u_2 \geq 0[\leq 0] \\ u_1 \leq u_{ss}(C_b, h)[\geq u_{ss}(C_b, h)] \end{cases} \Rightarrow \dot{C}_b < 0[> 0] \text{ or } \dot{e} > 0[< 0] \quad \text{for the height of } h \text{ and the}$$

concentration of  $C_b$ .

( $C_b$  is used instead of  $C_d$  in (7), it means that  $u_{ss}(C_b, h)$  is the control input which maintains the system at  $C_b$ )

**A2:** The higher (lower) *setpoint*, the higher (lower)  $u_{ss}$  (at the same level height)

Using these evident assumptions, the stability is proved (at any height).

*Proof:*

For current concentration of  $C_b$  and setpoint of  $C_d$  and the height of  $h$ :

$$e < 0 \Rightarrow \begin{cases} C_b > C_d[\text{see(5)}] \\ u_2 > 0[\text{see(8)}] \end{cases} \Rightarrow \begin{cases} u_{ss}(C_b, h) \stackrel{A_2}{>} u_{ss}(C_d, h) \stackrel{(8)}{=} u_1 \\ u_2 > 0 \end{cases} \Rightarrow \begin{cases} u_1 < u_{ss}(C_d, h) \stackrel{A_1}{>} \dot{e} > 0 \\ u_2 > 0 \end{cases} \quad (12)$$

$$e > 0 \Rightarrow \begin{cases} C_b < C_d [\text{see}(5)] \\ u_2 = 0 [\text{see}(8)] \end{cases} \Rightarrow \begin{cases} u_{ss}^{(8)}(C_b, h) < u_{ss}^{(8)}(C_d, h) = u_1 \\ u_2 = 0 \end{cases} \Rightarrow \begin{cases} u_1 > u_{ss}^{(8)}(C_d, h) \\ u_2 = 0 \end{cases} \stackrel{A_1}{\Rightarrow} \dot{e} < 0 \quad (13)$$

Therefore  $e \neq 0 \Rightarrow e\dot{e} < 0$ .

*End of proof*

It was indicated that the system is stable and the squared error decreases continuously at any value of control gain ( $K$ ). So, theoretically, control gain can be arbitrarily high to improve the performance. The performance is bounded only by the actuators' practical limitations (saturation functions in Fig.3). That is, the control behaviour can be described as *the quickest possible convergence to the reference (setpoint) without overshoot* (provided that assumption A1 and A2 are valid). Evident assumptions of A1 and A2 are the basis of the stability proof rather than a mathematical model and the mentioned assumptions are valid for a wide variety of process plants, so this methodology can be extended to other process plants easily.

## VI. SIMULATION RESULTS

Having  $k_1 = k_2 = 1$ , in (6), and a sampling time of 0.2 s, with the initial concentration of 20  $kmol/m^3$  and the initial height of 40 cm, the outlet concentration of CSTR with the proposed control system is shown in Figs.4 and 5, with  $K = 10$  (see (8)) with two different sets of references. This response is compared to the response of a well-designed double command fuzzy controller whose merit has been well indicated [10]. Lyapunov stability was proved in continuous domain; however, in discrete domain it takes 0.2 seconds (sampling time) for the



control system to change the control command, and within this time setpoint might be passed.

This is the reason for the tiny overshoot seen in Fig.5.

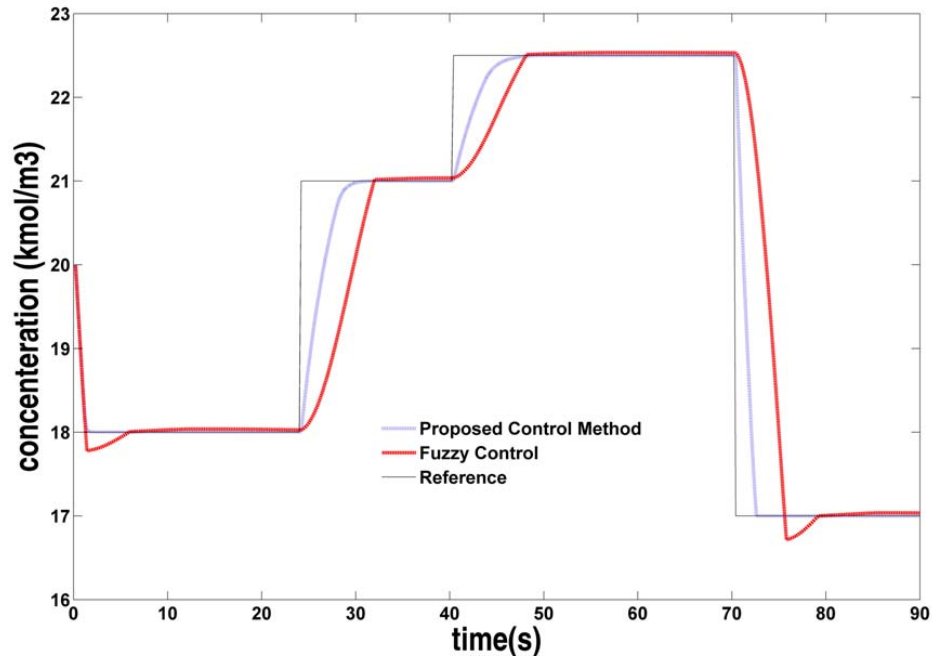


Figure 4. The response of system with different control systems

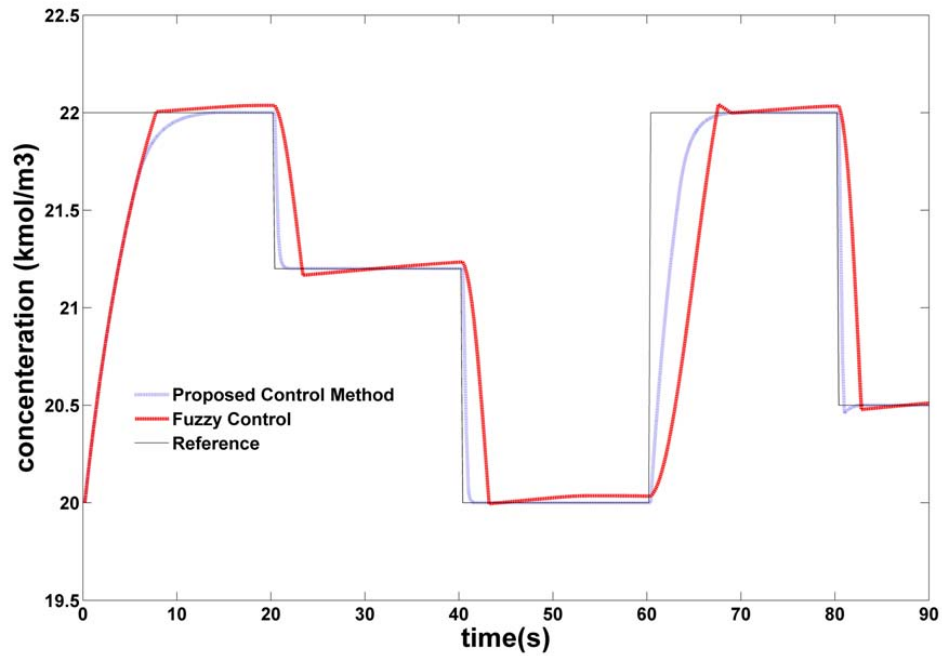
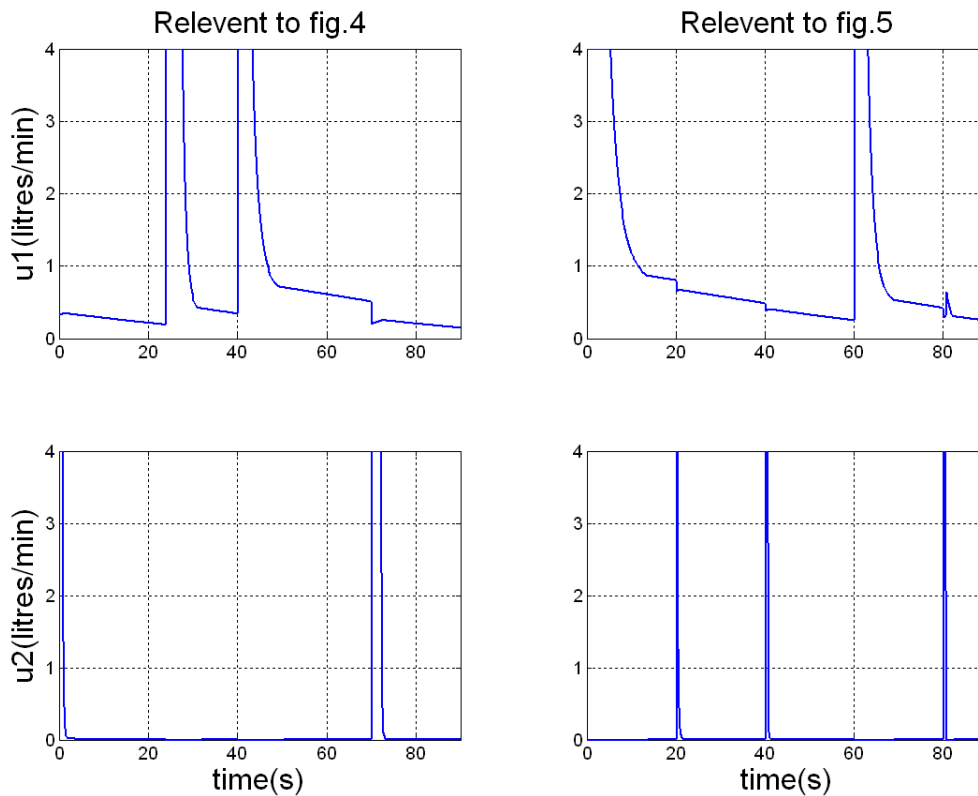


Figure 5. Control command of different control systems

Table 1 shows the mean of absolute error (MAE) for both offered figures.

MAE ( $kmol/m^3$ )	Figure4	Figure5
Proposed Control	0.1357	0.2468
Fuzzy Control	0.1849	0.4796

Figure 6 shows the control input during operation leads to the response shown in Figs.4 and 5. Apart from occasions of significant setpoint change, which rarely happen in reality, the change of control input is smooth and non-oscillating.



**Figure 6. Control inputs during simulation shown in Figs. 4 and 5**

## VII. CONCLUSION

In this article, based on the concept of ‘control equilibrium point’ a steady state control law is derived from system’s mathematical model. The aforementioned control law is used as a feedforward controller while the feedback controller is a proportional (P-action) controller. The proposed control system is applied on a CSTR to command both control valves simultaneously. Lyapunov asymptotic stability is proved based on two evident assumptions (not mathematical model), so the proof can be easily extended to a wide range of plants. In stability proof, there is no limit on the amplitude of control gain and it can be arbitrarily high to accelerate control response in the area of actuator’s capabilities. The simulation results are outstanding, very quick convergence to the reference, with almost no overshoot.

## VIII. ACKNOWLEDGEMENT

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## IX. NOMENCLATURE

A	:Assumption	
$C_b$	:Outlet concentration of CSTR( $kmol/m^3$ )	
$C_{b1}$	:The concentration of the first inlet flow to CSTR( $kmol/m^3$ )	<b>subscripts</b>
$C_{b2}$	:The concentration of the second inlet flow to CSTR( $kmol/m^3$ )	$d$ :Desired
$C_d$	:The desired outlet concentration of CSTR( $kmol/m^3$ )	$i$ :Numerator
CEP	: Control Equilibrium Point	ss :Steady state
CSTR	:Catalytic Stirred Tank Reactor	tr :Transient
$e$	:Error( $kmol/m^3$ )	
F, <b>f</b>	:Function	
GNTZ	:Generalized non-type-zero	
GTZ	:Generalized type-zero	
$h$	:Level height ( $cm$ )	
$k_{1,2}$	:CSTR parameters relevant to resistance of valves	
$K$	:Feedback controller gain	
MAE	:Mean of absolute error	

$t$	:Time (s)
$u$	:Control input ( <i>litres/min</i> )
$u_1$	:Volume rate of high concentration input to CSTR ( <i>litres/min</i> )
$u_2$	:Volume rate of low concentration input to CSTR ( <i>litres/min</i> )
$V$	:Lyapunov Function
$w$	:Volume rate of CSTR output ( <i>litres/min</i> )
$\mathbf{x}$	:States' vector

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## **Chapter 9**

### **Model-Free Double Command Hybrid Control of a Non-thermic CSTR**

Text in manuscript.

# Model-Free Double Command Hybrid Control of a Non-thermic CSTR

Morteza Mohammadzaheri, Lei Chen

*Abstract*— In this paper, a new methodology for feedforward-feedback control system design is proposed. Initially, the concept of control equilibrium point is introduced. Using this concept, steady state control command is defined and a non-model-based feedforward control law is conducted in the form of an artificial neural network. Feedback controller is a gain pushing the system towards the reference point. In this article, the case study is the concentration control of a non-thermic Catalytic Stirred Tank Reactor (CSTR). Using the proposed control system, the value of feedback controller gain can be arbitrarily high with a guaranteed BIBO stability. The mathematical model of the system is used neither in design nor in stability analysis, and the stability of control system is addressed using some evident practical assumptions which can be extended to many other systems. So the proposed design method can be employed in many other process plants. In this case study, the level height of the reactor is not particularly subject to control but the control system is so designed that this variable never goes lower than a specified limit. The proposed method returns surprisingly good results in comparison with the results with an iteratively-tuned IMC-based PI control system.

**Keywords:** Feedforward Control, Nonlinear Control, Artificial Neural Networks, CSTR.

## I. INTRODUCTION

FEEDFORWARD control commands are extensively used in control, sometimes together with feedback control commands. Some of recent applications for feedforward control include power systems [1-3], medical engineering [4], aircraft/helicopter control [5-7], vibration and noise control [8, 9], manufacturing [10] and robotics [11]. Different feedback controllers have been utilized together with feedforward controllers such as PID [4, 12], PD [5, 13], state vector feedback [10], adaptive [14] and iterative [1, 13] controllers as well as nonlinear controllers designed based upon feedback linearization [2], optimal tracking control [15] and fuzzy logic [7].

The most popular input to feedforward controllers is the reference or setpoint signal [1-7], although measurable disturbance signals may also play this role [9, 15, 16]. The input signal

passes a function/algorithm to generate the feedforward control command. Special algorithms based on the dynamics of the system [4, 15], linear transfer functions/filters [2, 16], or fixed gains [3] have been applied in feedforward control; moreover, one of the most popular feedforward control laws has been the inverse model of the system [2, 6, 11, 13]. Artificial neural networks have also been used to generate the feedforward control command [5, 7, 11, 16].

In general, feedforward control is added to feedback control systems to improve the performance (approaching more quickly towards reference), but overshoot reduction [8, 12] or uncertainty compensation [11] have also been reported as the other purposes for using feedforward control commands.

In some control problems, feedforward control is employed to reject a disturbance [16, 17]. Apart from these problems, there is no general methodology to find out whether a feedforward controller is useful or not? If so, how to find the feedforward control law? In this paper, these questions are addressed and a general non-model-based methodology is offered to answer them, in particular in the area of process control.

In a previous paper of the authors, in the simulation environment, the concentration of a non-thermic Catalytic Stirred Tank Reactor (CSTR) was controlled by simultaneous adjusting two inlet control flows based on the concepts of control equilibrium point and steady state control (see section II) [18]. However, the feedforward controller was purely model-based and the level height was not subject to any kind of control. In this paper, the control approach is model-free and guarantees that the reactor level height does not become lower than a specified value. Also, a nonlinear PI control system was designed based on internal model control technique, and then it was iteratively tuned for comparison with the proposed control system.



## II. DESIGN CONCEPT

Let us define the control equilibrium point of an under control system as below:

$$\frac{d^{(i)}e(t)}{dt^{(i)}} = 0, i = 0, \dots, \text{system's order} \quad (1)$$

where  $e(t)$  is the control error. In other words, at the control equilibrium point (CEP) the error is zero and remains zero.

*The control input obtained by the solution of (1) is called steady state control input.*

In some systems, the control equilibrium point is maintained only by continuous exertion of a control input. In other words, a steady state control input should be consistently applied on the system to maintain the desirable situation; as a result, a P-action feedback controller, leading to no control input at the reference point, cannot control such systems. From this aspect, this class of systems is similar to linear systems with type zero linear transfer functions. In this paper, such systems are called ‘generalized type zero’ (GTZ) systems. Level control of a water tank with an outlet at the bottom, all temperature control problems with the reference different from environment temperature, and position control of mechanical systems influenced by the gravity are some instances of so called GTZ systems. Other systems, which can retain their desired output with no control input, are called ‘generalized non-type zero’ (GNTZ) systems.

In summary,

in GNTZ control systems, steady state control input is zero ( $u_{ss} = 0$ );

in GTZ control systems, steady state control input is not zero ( $u_{ss} \neq 0$ ).

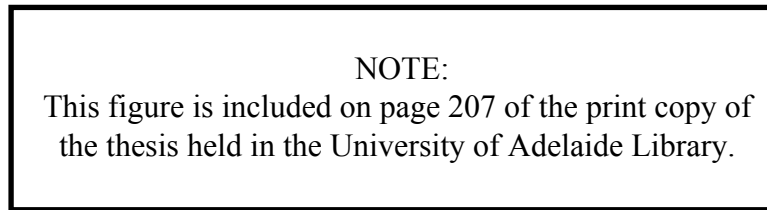
In the proposed method, steady state control command (which satisfies (1)) is employed as the feedforward control command ( $u_{ff}$ ) which is used together with a feedback (transient) control command ( $u_t$ ).

$$u = u_t + u_{ff} \quad (2)$$

In some exceptional occasions where the control output is far from the reference point, in order to improve control performance  $u_{ff}$  and  $u_{ss}$  may be different (see section IV).

### III. CASE STUDY: CONCENTRATION CONTROL OF A NON-THERMIC CATALYTIC STIRRED TANK REACTOR (CSTR)

A diagram of the studied non-thermal CSTR is shown in Fig.1:



**Figure 1. A schematics of the studied CSTR [19]**

Two flows of liquid enter the reactor with the concentration of  $C_{b1} = 24.9 (kmol/m^3)$  and  $C_{b2} = 0.1 (kmol/m^3)$ . The flow volume rates of input flows are named  $u_1$  and  $u_2$ . The reactor outlets another flow of liquid with the concentration of  $C_b$  and the flow volume rate of  $w$ .  $h$  is the height of liquid level in the reactor which is related to  $w$  ( $w = 0.2\sqrt{h}$ ).

A simplified mathematical model of the system, achieved by mass equilibrium equations, is:

$$\dot{h}(t) = u_1(t) + u_2(t) - 0.2\sqrt{h(t)}, \quad (3)$$

$$\dot{C}_b(t) = [C_{b1} - C_b(t)] \frac{u_1(t)}{h(t)} + [C_{b2} - C_b(t)] \frac{u_2(t)}{h(t)} - \frac{k_1 C_b(t)}{1 + k_2 C_b(t)^2} \quad (4)$$

where  $k_1$  and  $k_2$  are the parameters regarding valves resistance.

#### IV. STEADY STATE CONTROL COMMAND

The control of output concentration is addressed in this paper. There are two control inputs available,  $u_1$  and  $u_2$ , with the maximum flow of 4 litres/min.

In the studied CSTR, if both inlet flows are cut, the concentration decreases (see (4)). Therefore, the system is GTZ ; that is, a control input is needed to maintain the desired situation, and steady state control law is applicable.

In order to offer a general model-free methodology, the mathematical model is not used to derive steady state control law. Instead, an artificial neural network (ANN) was employed for this purpose. The input to this ANN is the reference (setpoint), and the outputs are a pair of steady state control commands for two input flows. The level height of liquid was also considered in this design.

A pair of control inputs ( $u_1$  and  $u_2$ ) lead to a special value of concentration ( $C_b$ ) and level height after a long period of operation/simulation, which is the steady state output of the system with those control inputs. If a series of such data is available, in the form of [ $C_{ss}$   $u_{1,ss}$   $u_{2,ss}$ ], an artificial neural network can be trained to output steady state control commands for a special value of concentration . In other words, this ANN control law generates  $u_{1,ss}$  and  $u_{2,ss}$  which lead to  $C_{ss}$ . There is a subtle point here: although  $h$  is not controlled, this value should not approach zero; moreover, this value should not be very high. The higher  $h$ , the more liquid in the reactor and the slower change of the concentration. As a result, when the only under-control output is the concentration, a low value of  $h$  is desirable. In this research, the low value of 5cm is considered desirable for the level height ( $h$ ) in the steady state situation. In order to collect the training data for ANN (steady state control law), a pair of  $u_{1,ss}$  and  $u_{2,ss}$  (steady state control commands) should be found so that if they apply to the

CSTR, the system approaches the steady state concentration of  $C_{ss}$  and the level height of 5cm.

The circuit shown in Fig.2 was designed to collect training data. An integrator pushes  $h$  towards 5cm by adjusting  $u_1$ . Different constant values of  $u_2$  (or  $u_{2,ss}$ ) in the range of [0.01 0.25] litres/min were applied to the system. For any value of  $u_2$  (or  $u_{2,ss}$ ), after 500 seconds of simulation, steady state values of concentration ( $C_{ss}$ ) and  $u_1(u_{1,ss})$  were recorded. This pair of  $u_{1,ss}$  and  $u_{2,ss}$  push the system towards  $C_{ss}$  and the level height of 5cm.

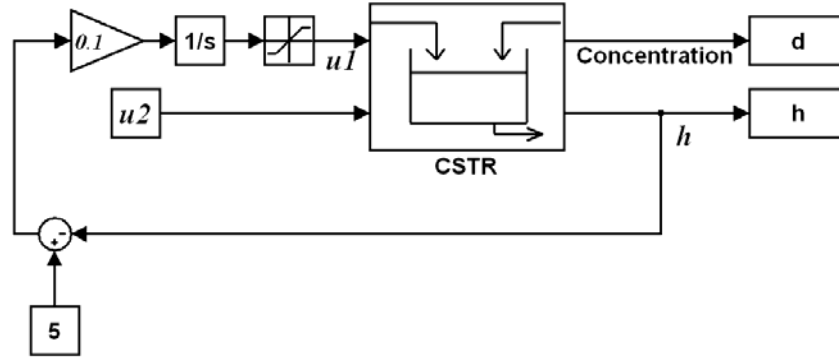


Figure 2. Setup for collecting training data

In the end, 243 series of  $[C_{ss} u_{1,ss} u_{2,ss}]$  were available, 227 series were used in training, and the rest were used for checking. A perceptron with four layers of neurons was designed. Hidden layers have 7 and 20 neurons with sigmoid activation function ( $f$ ) shown below:

$$f(x) = \frac{1}{1 + \exp(-x)} \quad (5)$$

The activation function of input (having one neuron) and output (having two neurons) layers are linear with the slope of one. Levenberg-Marquardt back-propagation method [20] was used for training, and the performance function was the mean of squared errors (MSE). Training was accomplished in 2000 iterations. Checking error is defined in (6),

$$ce_i = \frac{\sum_{i=1}^N |u_{i,s} - u_{i,r}|}{N} \quad (6)$$

where  $u_{i,s}$  is the estimated value of  $i^{th}$  control command by ANN (ANN  $i^{th}$  output), and  $u_{i,r}$  is the real value of  $i^{th}$  control command (a recorded value).

$ce_1 = 1.2448 \times 10^{-5}$  litres/min and  $ce_2 = 1.3345 \times 10^{-5}$  litres/min . The accuracy is acceptable; thus, if  $C_{ss}$  is given to the designed ANN, the pair  $u_{1,ss}$  and  $u_{2,ss}$  is generated which eventually pushes the system towards the concentration of  $C_{ss}$  and the level height of 5cm. Consequently, if in the steady state situation  $u_{1,ss}$  and  $u_{2,ss}$  are exerted on the CSTR, and the system is already at the desirable concentration of  $C_{ss}$  and the level height of 5cm, the concentration and the height level do not change. For  $h=5$ cm,

$$\begin{cases} u_1 = u_{1,ss} \\ u_2 = u_{2,ss} \\ e = 0 \end{cases} \Rightarrow \begin{cases} \dot{e} = 0 \\ \dot{h} = 0 \end{cases} \quad (7)$$

where  $e = C_d - C_b$ , (8),

and  $C_d$  is the desired concentration (reference point).

## V. CONTROL SYSTEM

The proposed control system has a feedforward-feedback structure, shown in Fig.3. For this double-command control problem, (2) is written in the form of (9):

$$\begin{cases} u_1 = u_{1,t} + u_{1,ff}(C_d, h_{ss}) \\ u_2 = u_{2,t} + u_{2,ff}(C_d, h_{ss}) \end{cases} \quad (9)$$

where

$$\begin{cases} u_{1,ff} = \begin{cases} 0 & (\text{if } e < -\varepsilon) \\ u_{1,ss} & (\text{if } e \geq -\varepsilon) \end{cases} \\ u_{2,ff} = \begin{cases} 0 & (\text{if } e > \varepsilon) \\ u_{2,ss} & (\text{if } e \leq \varepsilon) \end{cases} \end{cases} \quad (10)$$

As an example, if the error has a big positive value meaning concentration is too low, applying  $u_{2,ss}$  decreases the concentration and decelerates the control system. Feedforward control law (10) was designed to avoid such situations by setting feedforward control command equal to zero.

Feedback control (transient control) law, defined in (11), always generates a positive control command (in F1 block, in Fig.3),

$$\begin{cases} u_{1,t} = \begin{cases} K|e| & (\text{if } e > 0) \\ 0 & (\text{if } e \leq 0) \end{cases} \\ u_{2,t} = \begin{cases} 0 & (\text{if } e \geq 0) \\ K|e| & (\text{if } e < 0) \end{cases} \end{cases} \quad (11)$$

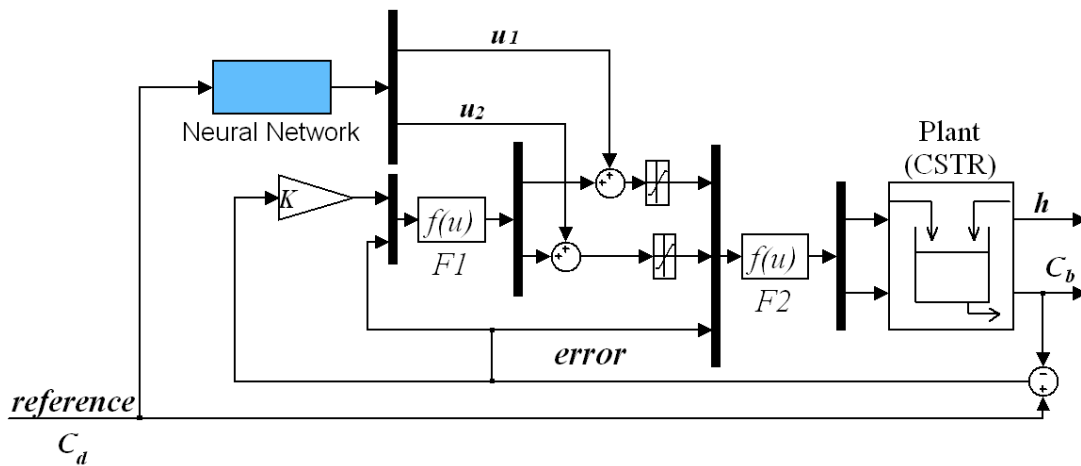


Figure 3. The proposed model-free feedforward-feedback control system

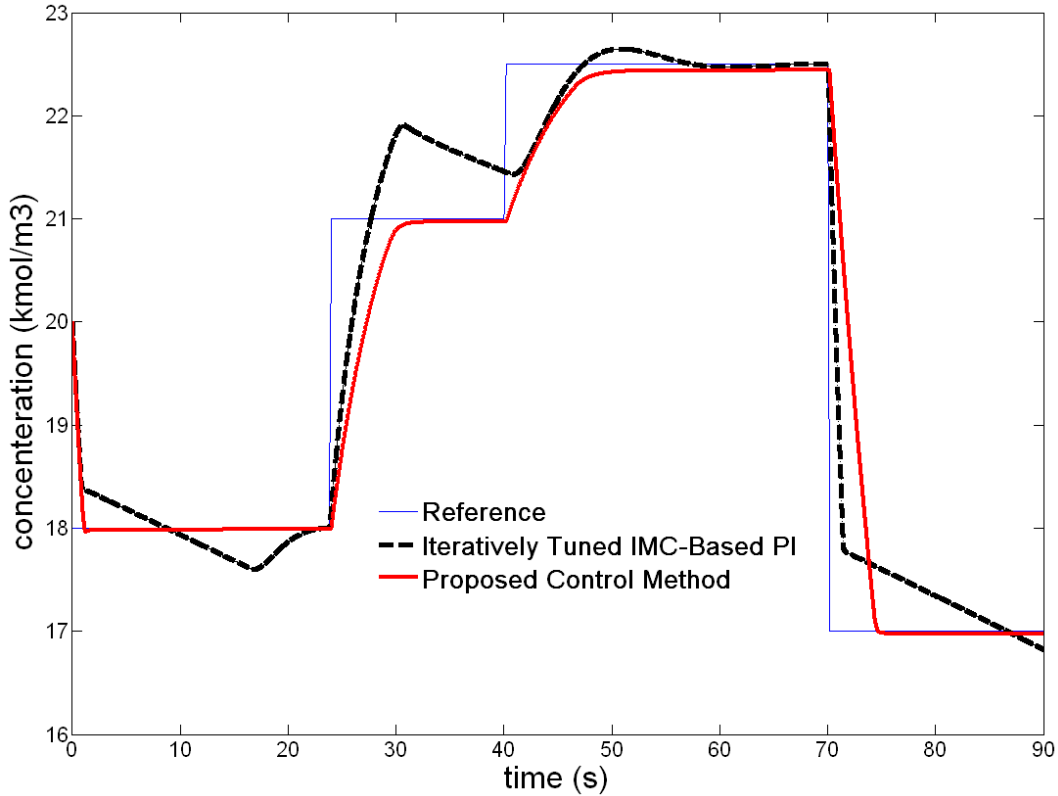
Based on (9),(10) and (11), for analysis purposes, the whole operation area can be divided into four different regions shown in table.1 (see (8)).

**Table 1. Control Commands in Different Operation Areas**

concentration	error	$u_{1,r}$	$u_{2,r}$	$u_{1,ff}$	$u_{2,ff}$	$u_1$	$u_2$
$C_b > C_d + \varepsilon$	$e < -\varepsilon$	0	$K e $	0	$u_{2,ss}$	0	$K e  + u_{2,ss}$
$C_d + \varepsilon \geq C_b \geq C_d$	$-\varepsilon \leq e \leq 0$	0	$K e $	$u_{1,ss}$	$u_{2,ss}$	$u_{1,ss}$	$K e  + u_{2,ss}$
$C_d \geq C_b \geq C_d - \varepsilon$	$0 \leq e \leq \varepsilon$	$K e $	0	$u_{1,ss}$	$u_{2,ss}$	$K e  + u_{1,ss}$	$u_{2,ss}$
$C_b < C_d - \varepsilon$	$e > \varepsilon$	$K e $	0	$u_{1,ss}$	0	$K e  + u_{1,ss}$	0

## VI. SIMULATION RESULTS

In Fig.4, the response of the system with the proposed control system is compared to the system's response with an iteratively-tuned nonlinear IMC-based PI control system (see Appendix 3). For this simulation,  $k_1 = k_2 = 1$  (in (4)) and sampling time is 0.01 seconds. Initial concentration is  $20 \text{ kmol/m}^3$  and the initial level height is 40cm. For the proposed control system,  $\varepsilon = 0.5$  and  $K = 15$  (see (10 and 11)).



**Figure 4. The response of system with different control systems**

Table 2 shows the mean of absolute error (MAE) for Fig.4.

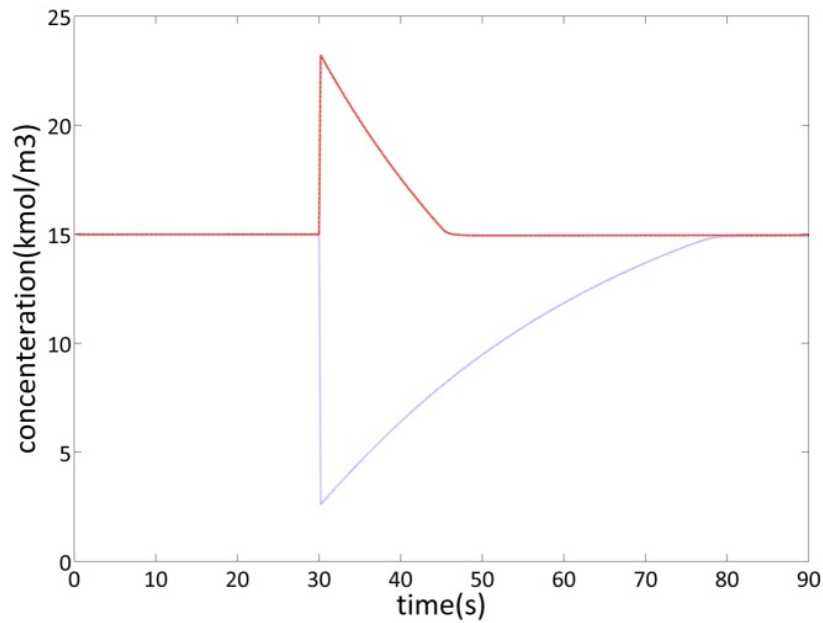
**Table 2.The Results of Simulation**

	MAE ( $kmol/m^3$ )
Proposed Control	0.3134
Iteratively Tuned IMC-based PI	0.3683

In Appendix 1, it is proved that the control system is BIBO stable and the height will never become lower than 5cm ( $h_{ss}$ ). Complementary figures and information on simulation results are offered in Appendix 2.

## VII. DISTURBANCE REJECTION

In order to check the capability of the proposed control system (shown in Fig.3) in disturbance rejection, a very severe disturbance was applied on the system.



**Figure 5. Control system's response against very severe disturbances**

In this test, when the system was already in the steady state situation at the concentration of  $15 kmol/m^3$ , one litre of liquid with the concentration of  $0.1 kmol/m^3$  or  $24.9 kmol/m^3$



was abruptly poured into the reactor (within 0.2 second, in simulation). The effect of disturbances was dampened down shortly without any overshoot (Fig.5).

## VIII. CONCLUSION

In this article, a hybrid feedforward-feedback control design approach was introduced. First, Control Equilibrium Point (CEP) was defined as the point at which the error and its derivatives are zero. Control command at CEP is called steady state control command. Steady state control command preserves the system at the reference (leading to zero error derivative). In order to have a model-free control approach, an artificial neural network was designed, trained and employed as the steady state control law, which generates steady state control commands. The steady state control law was employed as feedforward controller, in the vicinity of the reference (with the radius of  $\varepsilon$ ) in the proposed control system. Outside this vicinity, the feedforward control command is zero. A P-action feedback controller was used along with the aforementioned feedforward controller. The concentration of the output flow of a chemical reactor was controlled in this research with two input flows which both of them were adjusted simultaneously by the proposed control system. It was proved that the gain of the feedback controller can be arbitrarily high (leading to the quickest possible convergence to the reference) without harming the BIBO stability. That is, the absolute value of the error definitely decreases if it is higher than the low value of  $\varepsilon$ , and if the absolute value of the error becomes less than  $\varepsilon$  it never exceeds  $\varepsilon$  (see Appendix 1). As a result, an excellent reference tracking was offered by the proposed control system, with a very high control gain, compared to a well-tuned IMC-based PI control system. The only limit was actuators' constraints. Furthermore, it was proved that the level height of the reactor never becomes lower than a pre-specified value ( $h_{ss}$ ), so the reactor is never evacuated using the proposed

control system. The capability of the offered control system in damping disturbances was also indicated.

## IX. APPENDIX 1: STABILITY DISCUSSION

In the first order systems that the control input affects the first derivative of the output ; there usually exist high values of control input, which can change the sign of the first temporal derivative of the output immediately after application and make the output increase or decrease at once. As an example, in CSTR studied in this paper, when  $u_2$  (flow volume rate of low concentration input liquid) is equal to or less than its steady state value ( $u_{2,ss}$ ), and if  $u_1$  (flow volume rate of high concentration input liquid) has a very high value of  $\beta = 3$  *litres/min* or more, evidently the output concentration increases ( $\dot{C}_b > 0$ ) regardless of the current value of the concentration and level height. Such generic assumptions can be extended to other first order systems to facilitate non-model-based stability analysis. In this problem, the following evident assumptions are used for stability analysis:

A1: if  $u_1 \geq \beta$  and  $u_2 \leq u_{2,ss}$  (*litres/min*) then  $\dot{C}_b > 0$  ( $\dot{e} < 0$ )

A2: if  $u_1 \leq u_{1,ss}$  and  $u_2 \geq \beta$  (*litres/min*) then  $\dot{C}_b < 0$  ( $\dot{e} > 0$ )

Based on these assumptions, it is proved that with the proposed control system for a wide range of  $K$  (see (11)), if the absolute value of the error is higher than the small value of  $\varepsilon$ , the absolute value of the error (or the squared error) certainly reduces. In other words, with a bounded reference point and initial conditions, system's output and the error will never be unbounded or the system is BIBO stable. Mathematically, this stability is equivalent to

$$|e| > \varepsilon \Rightarrow \frac{d}{dt}(e^2) < 0. \quad (12)$$

$$\text{Or } |e| > \varepsilon \Rightarrow e\dot{e} < 0. \quad (13)$$

*Proof*

Considering that  $u_{1)ss}$  and  $u_{2)ss}$  are both positive, for a positive real number of  $\varepsilon$ , if  $K \geq \frac{\beta}{\varepsilon}$  (in

(11)),

$$e > \varepsilon \stackrel{Table 1}{\Rightarrow} \begin{cases} u_1 = K|e| + u_{1)ss} > K\varepsilon \geq \beta \\ u_2 = 0 \end{cases} \Rightarrow \begin{cases} u_1 > \beta \\ u_2 = 0 \end{cases} \stackrel{A1}{\Rightarrow} \dot{e} < 0,$$

and

$$e < -\varepsilon \stackrel{Table 1}{\Rightarrow} \begin{cases} u_1 = 0 \\ u_2 = K|e| + u_{2)ss} > K\varepsilon \geq \beta \end{cases} \Rightarrow \begin{cases} u_1 = 0 \\ u_2 > \beta \end{cases} \stackrel{A2}{\Rightarrow} \dot{e} > 0.$$

As a result, the error approaches the range of  $[-\varepsilon \ \varepsilon]$ , and does not leave this vicinity after entering it.

*End of the proof*

Furthermore during the operation, the tank should not be evacuated thoroughly or  $h$  should not approach zero. The following lemma guarantees that the level height will not be less than  $h_{ss}(5cm)$ , providing that the initial level height ( $h_0$ ) is higher than  $h_{ss}(5cm)$ .

Lemma1:  $h_0 \geq h_{ss} \Rightarrow h \geq h_{ss}$ .

where  $h_0$  is the initial height, and  $h_{ss} = 5cm$  in this problem.

*Proof*

When the level height decreases from its initial value, at  $h = h_{ss}$ , for  $K \geq \frac{\beta}{\varepsilon}$ ,

$$|e| \leq \varepsilon \stackrel{Table 1}{\Rightarrow} u_1 + u_2 = u_{1)ss} + u_{2)ss} + K|e|;$$

and

$$|e| > \varepsilon \stackrel{Table 1}{\Rightarrow} u_1 + u_2 \geq K|e| > \beta.$$

Consequently  $u_{1)ss}(C_d, h_{ss}) + u_{2)ss}(C_d, h_{ss})$  is the minimum input flow, providing that  $\beta > [u_{1)ss}(C_d, h_{ss}) + u_{2)ss}(C_d, h_{ss})]$  which is always correct in this case study (with  $\beta = 3 \text{ litres/min}$ ).

The output flow is a function of  $h$ , so for  $h = h_{ss}$ , the output flow is fixed and  $\dot{h}$  at  $h = h_{ss}$  is solely dependent on the input flow.

The minimum input flow ( $u_{1)ss}(C_d, h_{ss}) + u_{2)ss}(C_d, h_{ss})$ ) leads to  $\dot{h} = 0$  (see (7)). That is, at  $h = h_{ss}$ ,  $\dot{h} \geq 0$  or  $h$  always increases or remains fixed. As a result,  $h$  does not go lower than  $h_{ss}$  providing that the initial level height is higher than  $h_{ss}$  (5cm).

*End of proof*

As an interesting feature, in this stability discussion, there is no upper limit for control gain of  $K$ . This means control gain and approaching speed towards the reference can have the highest possible values without harming the stability, considering actuators limits and the level height.

## X. APPENDIX 2: COMPLEMENTARY SIMULATION RESULTS

Figure 6 shows the level height during the operation period shown in Fig.4.

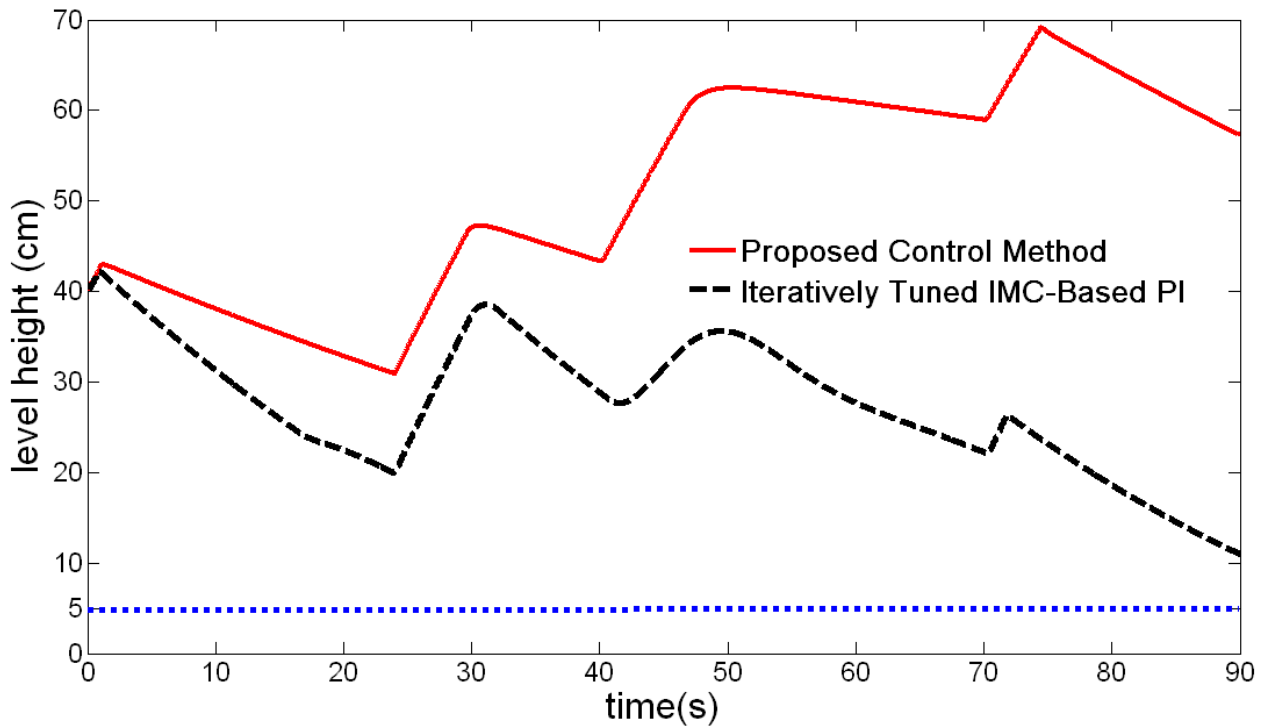


Figure 6. Level height of the reactor during the simulation shown in Fig.4

As the setpoint (reference) jumps, there is a jump in the level height (due to large amount of the applied control input) and outside these moments the level height has a decreasing trend towards  $h_{ss}$  (5cm).

Figure 7 shows the control input during the operation which leads to the response shown in Fig.4. Apart from occasions of significant change in the reference, which rarely happen in reality, the change of control input is smooth and non-oscillating, which is a prominent advantage from practical viewpoint.

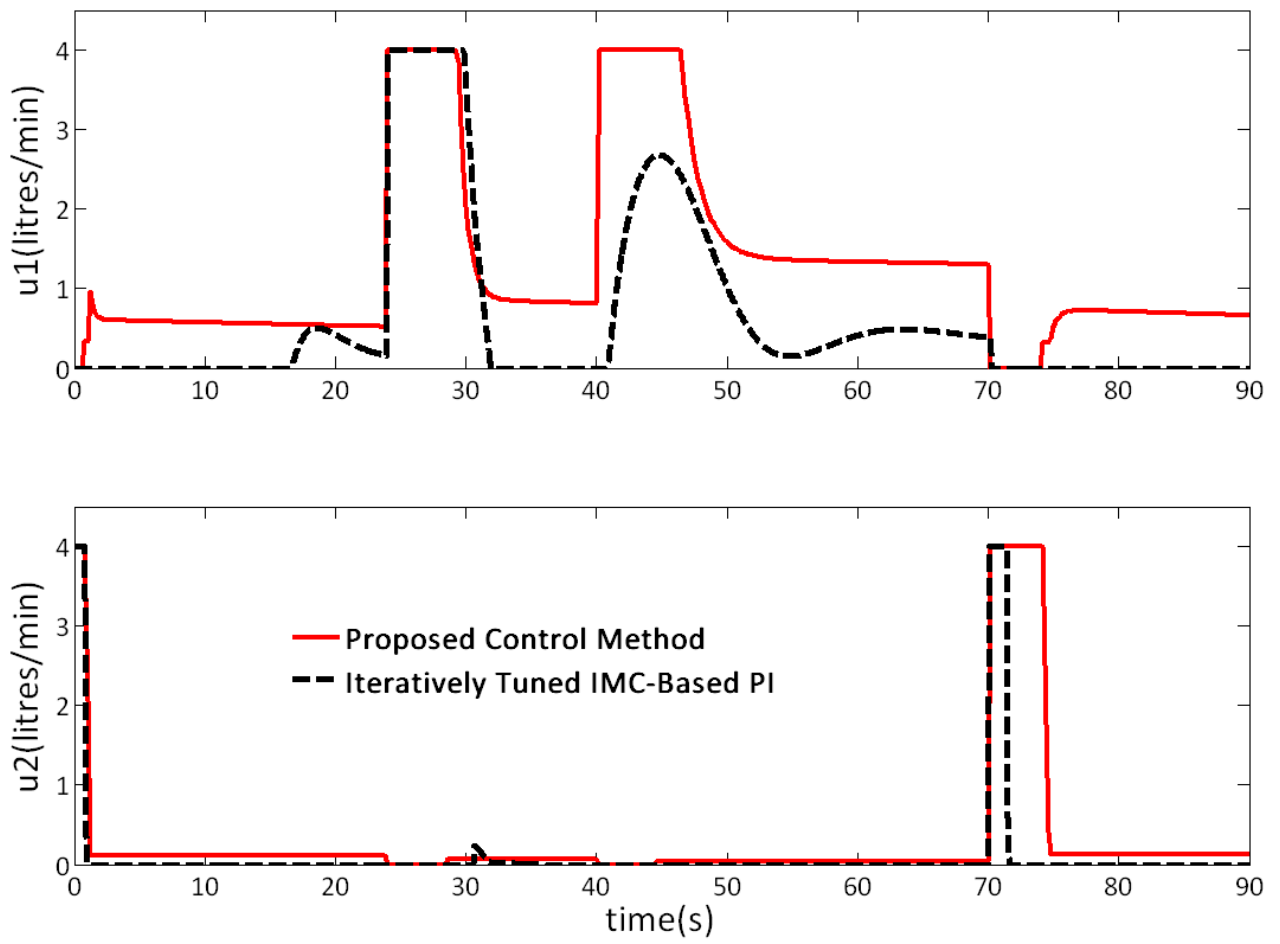


Figure 7. Control commands generated by the proposed control system during the simulation shown in Fig.4

## XI. APPENDIX 3: PI CONTROL SYSTEM DESIGN

### *XI-A. Nonlinear PI Control System for Concentration*

For the purpose of classical control design, attempt was made to eliminate the sources of system nonlinearity. Considering (4):

$$\dot{C}_b(t) = [C_{b1} - C_b(t)] \frac{u_1(t)}{h(t)} + [C_{b2} - C_b(t)] \frac{u_2(t)}{h(t)} - \frac{k_1 C_b(t)}{1 + k_2 C_b(t)^2}. \quad (4)$$

$h$  as a denominator is a main source of nonlinearity. Two new auxiliary variables are employed to eliminate it:

$$\bar{u}_1(t) = \frac{u_1(t)}{h(t)}, \quad (14)$$

and

$$\bar{u}_2(t) = \frac{u_2(t)}{h(t)}. \quad (15)$$

As a result

$$\dot{C}_b(t) = [C_{b1} - C_b(t)] \bar{u}_1(t) + [C_{b2} - C_b(t)] \bar{u}_2(t) - \frac{k_1 C_b(t)}{1 + k_2 C_b(t)^2}. \quad (16)$$

Then a first order linear model, including  $G_1(s)$  and  $G_2(s)$  in the Laplace field, is approximately fit to this first order nonlinear system:

$$C_b(s) = G_1(s) \bar{u}_1(s) + G_2(s) \bar{u}_2(s) \quad (17)$$

For system identification purposes, a wide variety of input signals are applied to the CSTR for 500 seconds and its response is recorded. These data were fitted to a first order transfer function using least square of error method which is the best method for linear modeling [21]. As there is no time delay (dead time) in the original nonlinear model, no time delay (dead time) was considered for the final linear models. The model shown in (18) and (19) is achieved:

$$G_1(s) = \frac{828.17}{1+137.52s}, \quad (18)$$

$$G_2(s) = \frac{-422.46}{1+24.432s}. \quad (19)$$

Both transfer functions are stable.

After modeling, a sinusoidal input signal was applied to the system in 50 seconds for checking; a mean of absolute error of 1.4769 *kmol/litres* was achieved in a range of [13 18] *kmol/litres*. A higher accuracy was not available because of non-negligible nonlinearity of the system. Without employing the proposed auxiliary variables, the system was so nonlinear that even such an approximate linear model was not achievable.

A PI controller is designed for each model based on internal model control (IMC) method, which is a well-known model-based algorithm for process control [22].

For a plant with the transfer function of  $G_P(s) = \frac{k_p}{1+\tau_p s}$  (20), IMC-based PI controller will be

$$G_C(s) = \left[ \frac{\tau_p}{k_p \lambda} \right] \frac{\tau_p s + 1}{\tau_p s}. \quad (21)$$

where  $\lambda$  is the time constant of the filter of internal model controller (see Appendix 4). The smaller  $\lambda$ , the higher convergence speed. However, very small values of  $\lambda$  decrease the robustness of the control system. The tentative time constant of  $\lambda=1$  s was chosen for the control of both transfer functions which have the time constant of 137.52 s and 24.432 s. According to the IMC algorithm (see Appendices 3 and 4), the following PI controllers are designed for  $G_1(s)$  and  $G_2(s)$ , respectively:

$$G_{C1}(s) = 0.166 \left( \frac{137.52s + 1}{137.52s} \right) = \frac{0.166s + 0.0010271}{s}, \quad (22)$$

$$G_{C2}(s) = -0.58 \left( \frac{24.432s + 1}{24.432s} \right) = \frac{-0.58s - 0.024}{s}. \quad (23)$$

### ***XI-B. A Controller to Address the Issue of Level Height***



The designed IMC-based control system could control the system satisfactorily; however,  $h$  was ignored from the control design process and  $h$  is likely to fall towards zero with this control system. In order to avoid this problem, another complementary controller is designed and employed to keep the level height from approaching zero or becoming too high.

(3) describes  $h$  :

$$\dot{h}(t) = u_1(t) + u_2(t) - 0.2\sqrt{h(t)}. \quad (3)$$

$\sqrt{h(t)}$  was expanded around  $h=30\text{cm}$ , which is a prevalent level height in operation (see Fig.7) , so:

$$\sqrt{h(t)} \cong \sqrt{10} + (h-10)\left(0.5h^{-0.5} \Big|_{h=10}\right) = 3.1623 + 0.1581(h-10) = 0.1581h + 1.5811. \quad (24)$$

$$\text{Therefore } \dot{h}(t) \cong u_1(t) + u_2(t) - 0.0316h + 0.3162. \quad (25)$$

$$\text{Another auxiliary variable was defined as } u_h(t) = u_1(t) + u_2(t) + 0.3162. \quad (26)$$

$$\text{Then } \dot{h}(t) + 0.0316h \cong u_h(t), \quad (27)$$

$$\text{or } \frac{h(s)}{u_h(s)} \cong \frac{1}{s + 0.0316} = \frac{31.6228}{1 + 31.6228s}. \quad (28)$$

Comparing (20) and (28),  $k_p = \tau_p = 31.6228$ , with a  $\lambda = 1$ . So the controller is designed (according to (21)):

$$G_{Ch} = \frac{31.6228s + 1}{31.6228s}. \quad (29)$$

The input to this controller is the error of  $h$  (the discrepancy between current and desired level height) and the output is  $u_h$ .  $u_h$  can be changed through  $u_1$  or  $u_2$ . In order to minimize the undesirable effect of the level height controller on concentration control,  $u_2$  is utilized to control the level height. In the control of the concentration,  $u_2$  repeatedly equals zero. When  $u_1$  is known, the value of  $u_2$ , which guarantees that the control input is equal to or higher than  $u_h$  (to prevent reactor evacuation) is

$$u_{2h}(t) = u_h - u_1(t) - 0.3162. \quad (30)$$

This value is added to  $u_2$  for level height control purposes to avoid  $h$  become lower than the desired level height.

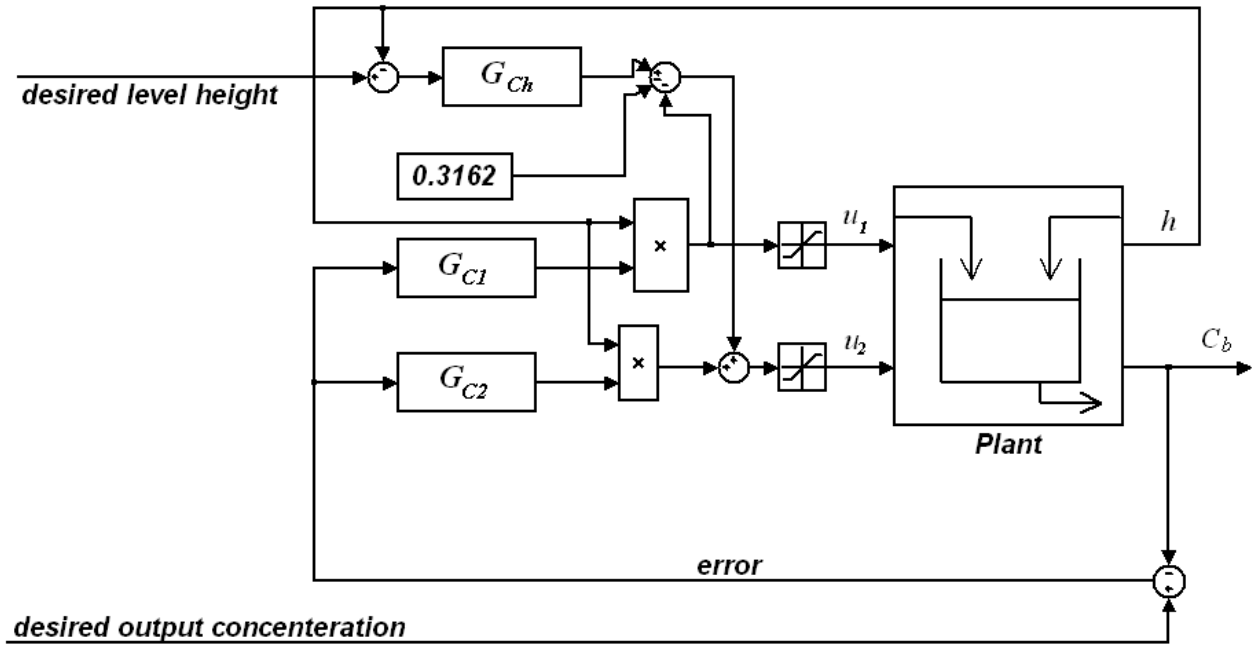


Figure 8. PI Control System

### XI-C. Minimising the Interruption of Height Controller to Concentration Control System

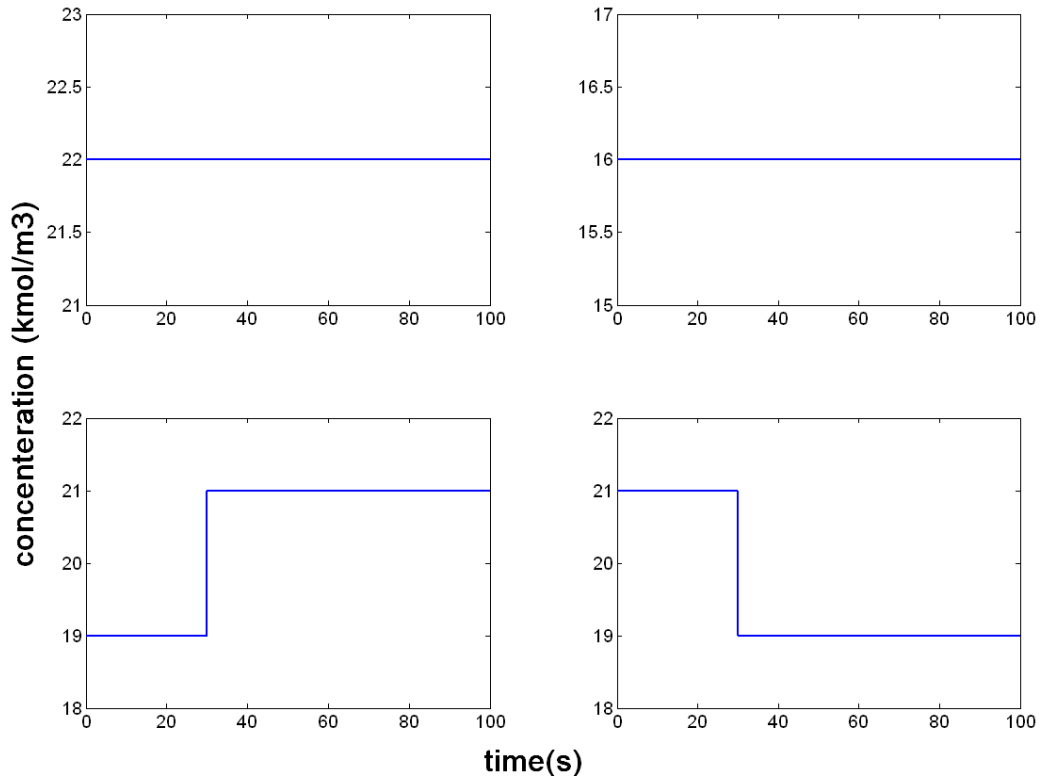
Undoubtedly, height control interrupts concentration control, so the quadruple parameters of PI concentration control system, defined in (22) and (23), were tuned by an iterative algorithm to minimise a performance function defined in (31).

$$PF = \sum |C_b(t) - C_d(t)| + \rho \sum (|u_1(t) - u_1(t - T_s)| + |u_2(t) - u_2(t - T_s)|) \quad (31)$$

where  $T_s$  is simulation sampling time (0.01s) and  $C_d(t)$  is the reference;  $\rho$  represents the importance of input change reduction in performance function.

During optimization of concentration control system, height controller which is not very important was kept fixed. The performance function was evaluated for four different sets of setpoints (see Fig.9) in the operation area rather than one set to end up with a more realistic design/optimization. The initial

conditions are  $h=30\text{cm}$  and  $C_b = 20\text{kmol/litres}$ . The sum of the values of performance function (SPF) for four different sets of setpoints (references) was calculated (with  $\rho = 0.001$ , considering much higher values of control input compared to the error).



**Figure 9. Different sets of reference (setpoints) for optimization purposes**

After finding SPF for the initial set of control parameters, a new set of parameters are considered as below:

$$p_n(i) = p_o(i)[1 + 0.5(2rand - 1)] \quad i=1,2,3,4 \quad (32)$$

where  $p_n$  and  $p_o$  represent new and original controller parameters respectively and  $rand$  represents a random number between 0 and 1. The simulation was run with new control parameters and SPF (the sum of the values of performance function for four different series of setpoints) was calculated. Then the following sub-algorithm was employed:

$$\begin{cases} p_{m}(i) = p_n(i) - \eta \frac{SPF_n - SPF_o}{p_n(i) - p_o(i)} \\ p_o(i) = p_n(i) \\ p_n(i) = p_m(i) \end{cases} \quad (33)$$

Started at SPF=1.3665, after one hundred iterations with  $\eta = 10^{-8}$ , the following tuned controllers were found (SPF=0.8657):

$$G_{C1r}(s) = \frac{0.1344s + 0.5333}{s} \quad (34)$$

and

$$G_{C2r}(s) = \frac{-0.1441s - 0.0512}{s} \quad (35)$$

## XII. APPENDIX 4: IMC-BASED PI CONTROLLER DESIGN

Internal Model Control (IMC) is based on the concept of inverse dynamics control. In this method, the inverse dynamics of a system is employed as a feedforward controller, so the response will be the same as the reference (setpoint). This feedforward controller is called  $q(s)$  in IMC (in Laplace field). The inverse dynamics of a system may be physically unrealizable (the order of the numerator is higher than the denominator). In these cases, a filter is added to the controller to make it realizable. For instance, for

$$\text{a system with the first order transfer function of } G_p(s) = \frac{k_p}{1 + \tau_p s}, \quad (20)$$

$$\text{a first order filter is employed, } f(s) = \frac{1}{1 + \lambda s}, \quad (36)$$

where  $\lambda$  is the time constant of the filter. The whole IMC is:

$$q(s) = G_p(s)^{-1} f(s) = \left( \frac{1 + \tau_p s}{k_p} \right) \frac{1}{1 + \lambda s} = \frac{1 + \tau_p s}{k_p (1 + \lambda s)}. \quad (37)$$

In order to use IMC technique for feedback control system design, a feedback control circuits (Fig.11) is considered as an equivalent to classical IMC control system (Fig.10). Output to reference ratio is determined for both circuits:

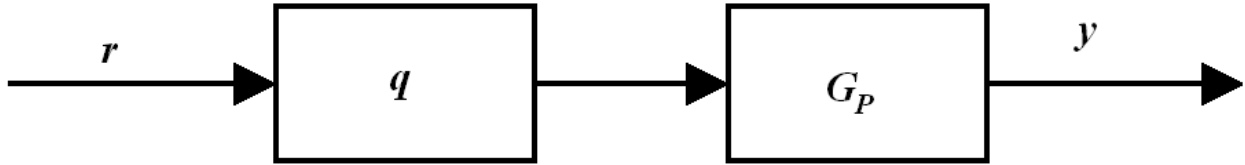


Figure 10. Original Internal Model Control circuit

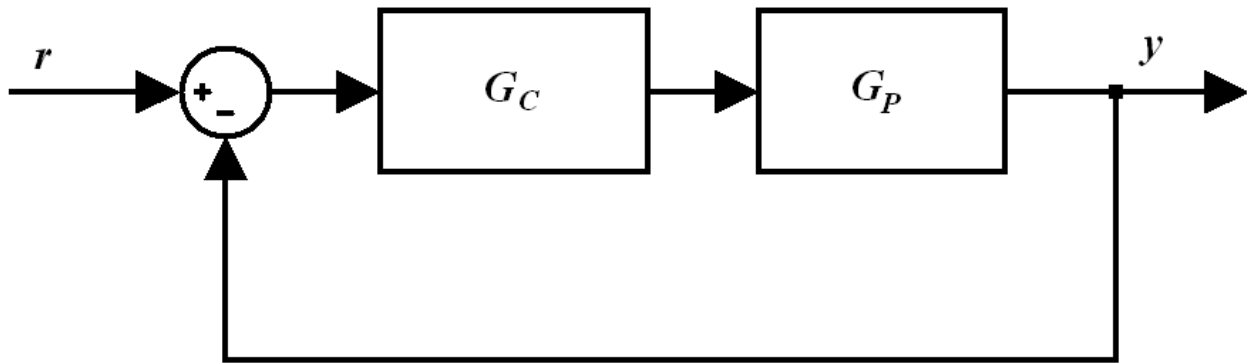


Figure 11. Feedback control circuit equivalent to Internal Model Control circuit

$$\frac{y(s)}{r(s)} = q(s)G_P(s) = \frac{G_P(s)G_C(s)}{1 + G_P(s)G_C(s)}, \text{ and as a result } G_C(s) = \frac{q(s)}{1 - G_P(s)q(s)}. \quad (38)$$

Considering (37) and (38), for a first order system shown in (20):

$$G_C(s) = \left[ \frac{\tau_p}{k_p \lambda} \right] \frac{\tau_p s + 1}{\tau_p s}. \quad (21)$$

### XIII. ACKNOWLEDGEMENT

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## XIV. NOMENCLATURE

<p>A : Assumption</p> <p>ANN : Artificial Neural Network</p> <p><math>C_b</math> : Outlet concentration of CSTR(<math>kmol/m^3</math>)</p> <p><math>C_{b1}</math> : The concentration of the first inlet flow to CSTR(<math>kmol/m^3</math>)</p> <p><math>C_{b2}</math> : The concentration of the second inlet flow to CSTR(<math>kmol/m^3</math>)</p> <p><math>C_d</math> : The desired outlet concentration of CSTR(<math>kmol/m^3</math>)</p> <p><math>C_{ss}</math> : The steady state outlet concentration of CSTR(<math>kmol/m^3</math>)</p> <p><math>ce</math> : Checking Error</p> <p>CEP : Control Equilibrium Point</p> <p>CSTR : Catalytic Stirred Tank Reactor</p> <p><math>e</math> : Error(<math>kmol/m^3</math>)</p> <p>F, <b>f</b> : Function</p> <p><math>f(s)</math> : Linear filter in Laplace field</p> <p><math>G(s)</math> : Linear model in Laplace field</p> <p>GNTZ : Generalized Non-Type-Zero</p> <p>GTZ : Generalized Type-Zero</p> <p><math>h</math> : Level height (<math>cm</math>)</p> <p>IMC : Internal Model Control</p> <p><math>k_{1,2}</math> : CSTR parameters relevant to resistance of valves</p> <p><math>K</math> : Feedback controller gain</p> <p>MAE : Mean of absolute errors</p> <p>MSE : Mean of squared errors</p> <p><math>p</math> : Parameter</p> <p><math>q(s)</math> : Inverse model in Laplace field</p> <p>SPF : Sum of performance function values</p> <p><math>t</math> : Time (s)</p> <p><math>T_s</math> : Simulation sampling time (s)</p> <p><math>u</math> : Control input (<math>litres/min</math>)</p> <p><math>u_1</math> : Volume rate of high concentration input to CSTR (<math>litres/min</math>)</p> <p><math>u_2</math> : Volume rate of low concentration input to CSTR (<math>litres/min</math>)</p> <p><math>w</math> : Volume rate of CSTR output (<math>litres/min</math>)</p> <p><b>Greek Letters</b></p> <p><math>\beta</math> : A big value of control input leading to an instant change on the concentration sign</p> <p><math>\varepsilon</math> : the range of the acceptable error</p> <p><math>\lambda</math> : the time constant of filter in IMC method</p>	<p><b>Subscripts</b></p> <p><math>d</math> : desired</p> <p><math>ff</math> : feedforward</p> <p><math>h</math> : level height</p> <p><math>i</math> : numerator</p> <p><math>n</math> : new</p> <p><math>o</math> : old</p> <p><math>r</math> : real</p> <p><math>s</math> : estimated</p> <p><math>ss</math> : steady state</p> <p><math>tr</math> : transient</p>
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## **Chapter 10**

### **Hybrid Intelligent Control of an Infrared Dryer**

Submitted Journal Article

# Hybrid Intelligent Control of an Infrared Dryer

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**Abstract**-In this research, a hybrid control system was proposed to address temperature control of an infrared dryer. The control system includes a feedback-predictive controller and a neural network steady state control law. Feedback-predictive controller outputs the amplified value of the predicted error as the transient control command. The predictive model was employed to suppress the undesirable effect of the dead-time of the system. A multilayer perceptron was designed and trained based on the idea of control equilibrium point and steady state control to be used as feedforward controller. The stability of control system in continuous domain was proved with no limit on the amplification gain of the predictive-feedback controller. In other words, there is no concern about losing the stability with accelerating convergence towards the reference. The entire control system was constructed in Simulink and compiled to a C code and applied on the experimental setup. Experimental results are outstanding in comparison with the results of an interactively tuned IMC-based PID controller.

## I. INTRODUCTION

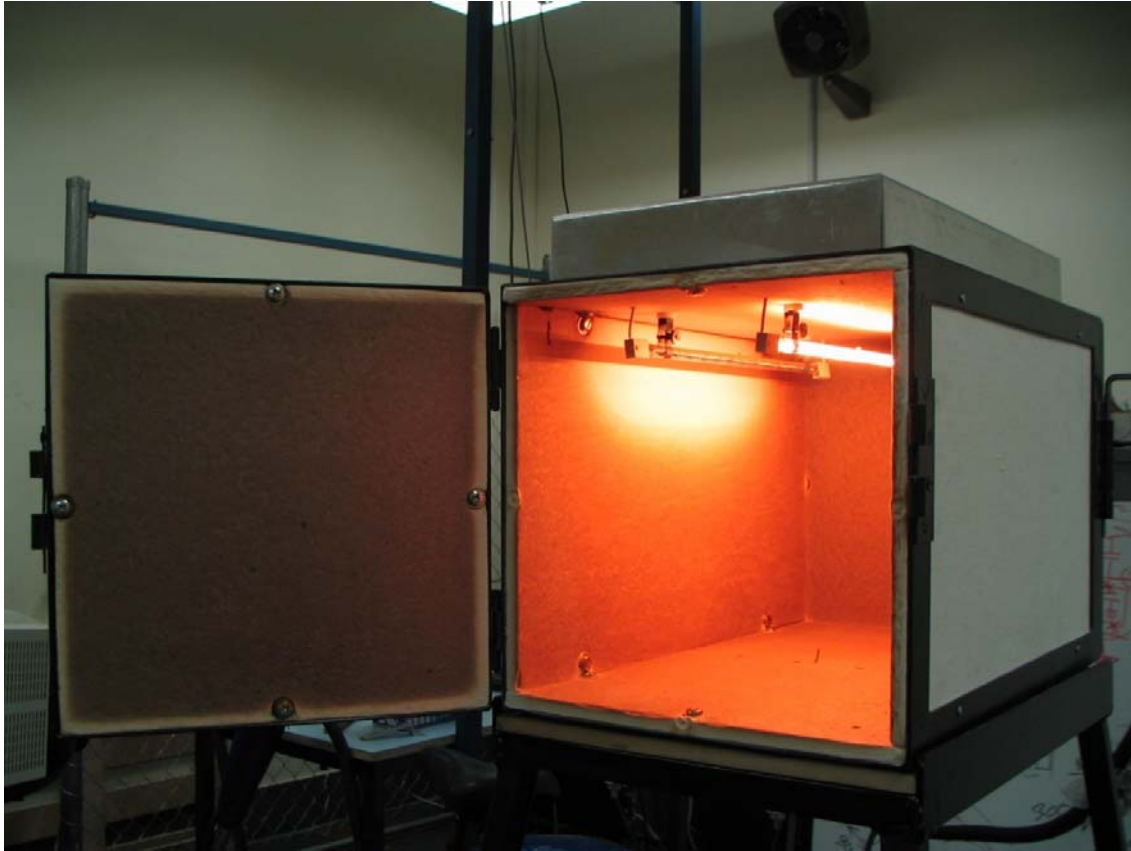
FEEDBACK-FEEDFORWARD control systems have many applications include power systems [1-3], medical engineering [4], aircraft/helicopter control [5-7], vibration and noise control [8], manufacturing [9]and temperature control of different processes [10-12]. A variety of feedback controllers have been used in these systems such as PID [6], PD [5, 10, 13, 14], state vector feedback [9], adaptive [15], iterative learning [1, 13]and predictive controllers [11, 12], as well as nonlinear controllers designed by feedback linearization[2], optimal tracking control [15]and fuzzy logic [7].

As a category, in some cases, feedforward control is utilized to compensate the disturbances [8, 12, 16], and its input is the measured or observed disturbance. In the other category, the reference or setpoint is the input signal to the feedforward controller; this area of feedforward-feedback control system design lack a general well-established methodology and still witness emerging ideas. In this category, the most popular feedforward control laws are the inverse models of the systems [2, 6, 9-11, 14]which are not always available, and

artificial neural networks have also been used to generate the feedforward control command [5, 7, 16] in this category. In 2009, authors of this paper introduced the concepts of control equilibrium point and steady state control briefly in an appendix of an article concerning pitch angle control of a model helicopter [7]. In that research, a neural network was designed and trained to play the role of steady state control law and used as the feedforward controller. Later, the concepts of control equilibrium point and steady state control were defined and explained more clearly and comprehensively, and some feedforward-feedback control systems with proven stability were designed based on these concepts and applied successfully in new case studies in simulation [17-19]. However, none of previous case studies of the proposed methodology encountered the issue of dead-time or time-delay which challenges design and stability analysis; furthermore, none of the aforementioned control systems was tested experimentally in real time environment. In this paper, first the dynamic of the case study, the infrared dryer, is explained, then the concepts of control equilibrium point, steady state control and generalized zero-type systems are introduced in section III. Control system and its stability proof are offered in sections IV and V. However, none of feedback and feedforward control laws, as introduced in these sections, are easily available. In sections VI and VII, the difficulties of achieving feedforward and feedback control laws are addressed and handled. Section VIII and IX consist of experimental results and conclusion followed by references and three appendices.

## II. CASE STUDY

The case study is a radiating dryer with the dimensions of 50cm×30cm×30cm. In this research, one halogen lamp, hung from the top surface, is used as the heat source, and a thermocouple at the bottom surface of the dryer is used as the sensor, both located asymmetrically (Fig.1).



**Figure 1. The infrared dryer**

A mathematical model of the system can be achieved by the first law of thermodynamics. The furnace can be assumed as an enclosure with Diffuse-Gray Surfaces. The following assumptions are considered for thermo-dynamics-based modeling [20]:

1. The surface properties are non-uniform.
2.  $\varepsilon_k$ , the emissivity factor of the surfaces, is independent of the wavelength and the direction of radiation.
3. All the energy is emitted and reflected diffusely.

Incident and reflected energy flux is non-uniform; as a result, the enclosure (dryer) boundary must be subdivided to infinitesimal areas.

In order to find the mathematical model of the dryer, an element on the bottom surface was considered (Fig.2):

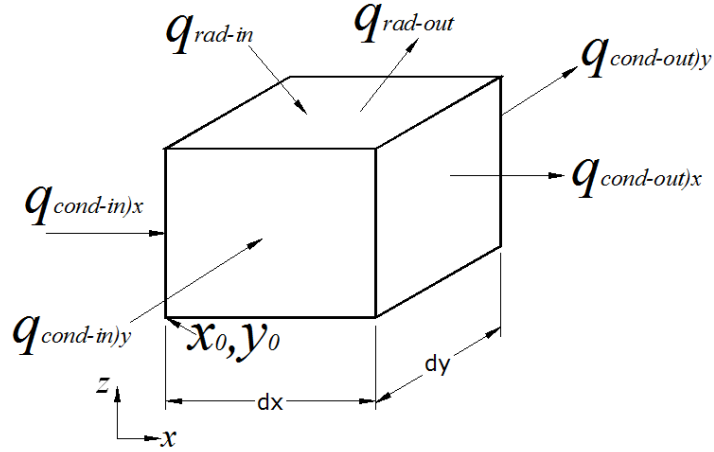


Figure 2. An element at the bottom surface of the furnace

In general, in terms of energy:

$$E_{\text{Input}} - E_{\text{Output}} + E_{\text{Generation}} - E_{\text{Consumption}} = E_{\text{Accumulation}} \quad (1)$$

The input energy to this element comes from the conduction in  $x$  and  $y$  directions, the radiation from the halogen lamp and the radiation from other surfaces:

$$E_{\text{Input}} = -k_{\text{cond}} \left. \frac{\partial T(x, y, t)}{\partial x} \right|_{x=x_0} l dy - k_{\text{cond}} \left. \frac{\partial T(x, y, t)}{\partial y} \right|_{y=y_0} l dx + q_{\text{lamp}}(x, y, t) + \left( \sum_{j=1}^N q_{o,j} F_j(x, y) \right)_{(x,y)=(x_0,y_0)} dx \cdot dy \quad (2)$$

where  $k_{\text{cond}}$  is the heat conduction coefficient ( $W/mK$ ),  $t$  is time (s),  $l$  is the thickness of the dryer body ( $m$ ),  $T$  is temperature ( $K$ ),  $q$  is heat flux ( $W$ ),  $j$  is the index of elements on the surfaces where the studied element is not located on them, and  $N$  is the number of these elements.  $F_j(x, y)$  is the exchange factor of the inner surface of  $j^{\text{th}}$  element and the element located at  $(x, y)$ .  $q_{o,j}$  is the output radiation heat of the  $j^{\text{th}}$  element on the surfaces where the studied element is not located on them.

The output energy can be divided in three categories: output energy through conduction, radiation due to high temperature of the element and the reflecting radiation:

$$E_{Output} = -k_{cond} \frac{\partial T(x, y, t)}{\partial x} \Big|_{x=x_0+dx} ldy - k_{cond} \frac{\partial T(x, y, t)}{\partial y} \Big|_{y=y_0+dy} ldx + \varepsilon_k \sigma \bar{T}^4 . dx.dy$$

$$+ \left( (1 - \varepsilon_k) \sum_{j=1}^N q_{o,j} F_j(x, y) \Big|_{(x,y)=(x_0,y_0)} \right) dx.dy \quad (3)$$

where  $\sigma$  is the Stefan-Boltzmann Constant= $5.6703 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$  and  $\bar{T}$  is the average temperature of the inner side of the element.

According to Taylor's series:

$$\frac{\partial T(x, y, t)}{\partial x} \Big|_{x=x_0+dx} \cong \frac{\partial T(x, y, t)}{\partial x} \Big|_{x=x_0} + \frac{\partial^2 T(x, y, t)}{\partial x^2} \Big|_{x=x_0} . dx , \quad (4)$$

and

$$\frac{\partial T(x, y, t)}{\partial y} \Big|_{y=y_0+dy} \cong \frac{\partial T(x, y, t)}{\partial y} \Big|_{y=y_0} + \frac{\partial^2 T(x, y, t)}{\partial y^2} \Big|_{y=y_0} . dy . \quad (5)$$

$$E_{Generation} = E_{Consumption} = 0 \quad (6)$$

$$E_{Accumulation} = \rho V C_p \frac{\partial T(x, y, t)}{\partial t} = \rho (l . dx . dy) C_p \frac{\partial T(x, y, t)}{\partial t} \quad (7)$$

where  $V$  is the volume of the element and  $C_p$  is the specific heat capacity at a constant pressure. After considering (2-8) in (1) and dividing both all terms by  $V$ .

$$k_{cond} \frac{\partial^2 T(x, y, t)}{\partial x^2} \Big|_{x=x_0} + k_{cond} \frac{\partial^2 T(x, y, t)}{\partial y^2} \Big|_{y=y_0} - \frac{\varepsilon_k}{l} \sigma \bar{T}^4 + \frac{\varepsilon_k}{l} \sum_{j=1}^N q_{o,j} F_j(x, y) \Big|_{(x,y)=(x_0,y_0)}$$

$$+ \frac{q_{lamp}(x, y, t)}{V} = \rho C_p \frac{\partial T(x, y, t)}{\partial t} \quad (8)$$

Due to insulation, the following boundary conditions can be considered for the dryer (see Fig.3):

$$T(x, y, 0) = T_0, \quad (9)$$

$$\frac{\partial T(l_1, y, t)}{\partial x} = 0, \quad (10)$$

$$\frac{\partial T(x, l_2, t)}{\partial y} = 0. \quad (11)$$

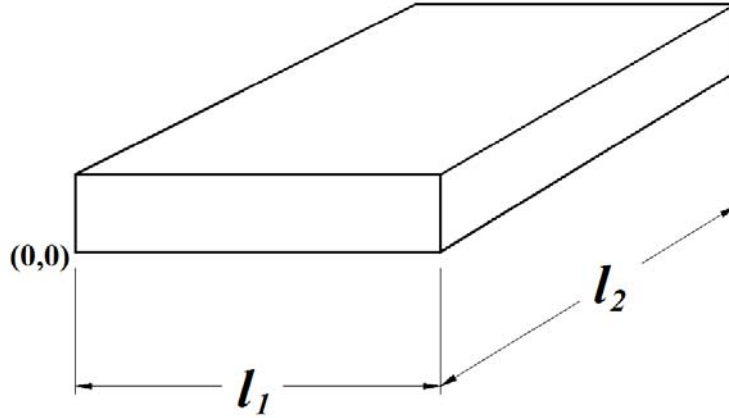


Figure 3. A wall of the furnace body

### III. CONTROL EQUILIBRIUM POINT AND STEADY STATE CONTROL

Let us define the control equilibrium point of a system as

$$\frac{d^{(i)}e(t)}{dt^{(i)}} = 0, i = 0, \dots, \text{system's order} \quad (12)$$

where  $e(t)$  is the control error. That is, at the control equilibrium point (CEP), the error is zero and remains zero.

The control input obtained by the solution of (12) is called *steady state control input*.

In some systems, the control equilibrium point is maintained only by continuous exertion of a control input. In other words, a steady state control input should be consistently applied on the system to maintain the desirable control output; as a result, a P-action feedback controller, leading to no control input at the reference point, cannot control such systems appropriately. From this aspect, this class of systems is similar to linear systems with type zero linear transfer functions (transfer functions without built-in integrators). In this paper, such systems are called ‘generalized type zero’ (GTZ) systems. Some instances of so called

GTZ systems are level control of a water tank with an outlet at the bottom, all temperature control problems with the reference different from environment temperature, and position control of mechanical systems influenced by gravity. Other systems, which can retain their desired output with no control input, are called ‘generalized non-type-zero’ (GNTZ) systems.

In summary,

in GNTZ control systems, steady state control input is zero ( $u_{ss} = 0$ );

in GTZ control systems, steady state control input is not zero ( $u_{ss} \neq 0$ ).

#### IV. CONTROL SYSTEM

In this paper, the steady state control law of  $F_{ss}(\cdot)$ , which generates the steady state control command ( $u_{ss}$ ), is employed as the feedforward controller. Feedback controller is a P-action with the positive gain of  $k_c$  :

$$u = u_{ss} + k_c e \quad (13)$$

where  $e$  is the control error. In this control problem,  $e = T_d - T$ . (14)

$T$  is the existing temperature and  $T_d$  is the desired temperature or the reference temperature, and  $u$  is the input voltage to the lamp.

In this temperature control problem, if the effect of environment temperature is ignored, the only states of the system are the temperature and its derivatives. In the steady state situation (see (12)), all of them equal zero except for  $T$  which equals  $T_d$ . So the steady state control input is solely dependent on the desired or reference temperature:

$$u_{ss} = F_{ss}(T_d) \quad (15)$$

In the next section, the stability of the proposed control system is addressed in case that all the terms of right hand side of (13) are available. In sections VI and VII, practical difficulties



in finding feedforward and feedback control commands are dealt with using artificial neural networks.

## V. STABILITY ANALYSIS

According to the Lyapunov theorem [21], for a system with the states vector of  $\mathbf{x}$ , an equilibrium point and the system are globally stable if a scalar function of  $V(\mathbf{x})$  with continuous first temporal derivative can be found, so that

Condition 1:  $V(\mathbf{x})$  is positive definite;

Condition 2:  $\dot{V}(\mathbf{x})$  is negative definite;

Condition 3:  $\|\mathbf{x}\| \rightarrow \infty \Rightarrow V(\mathbf{x}) \rightarrow \infty$ .

For this first order case study,  $x=[e]$  (see(8 and 14)). The Lyapunov function is the error square:

$$V(\mathbf{x}) = e^2. \quad (16)$$

With this Lyapunov function, conditions 1 and 3 are evidently satisfied.

In order to prove condition 2, it should be proved that

$$\begin{cases} \mathbf{x} \neq 0 \Rightarrow \dot{V}(\mathbf{x}) < 0 \\ \mathbf{x} = 0 \Rightarrow \dot{V}(\mathbf{x}) = 0 \end{cases} \quad (17)$$

$\dot{V}(\mathbf{x}) = 2e\dot{e}$ , and thus in this problem, (17) is written in the form of (18)

$$\begin{cases} e \neq 0 \Rightarrow e\dot{e} < 0 \\ e = 0 \Rightarrow e\dot{e} = 0. \end{cases} \quad (18)$$

Below equation of (18) is evident. As a result, the system is asymptotically stable, providing that  $e \neq 0 \Rightarrow e\dot{e} < 0$ .

The stability study in this paper is based on two evident assumptions rather than the mathematical model. So the stability proof remains valid in case of change in parameters and it can be extended to many similar process control problems.

According to the definition of control equilibrium point and steady state control command , in this temperature control problem,  $u_{ss}$  maintains the desired temperature, and if the temperature is higher [lower] than the desired temperature ( $T_d$ ),  $u_{ss}$  decreases [increases] the temperature towards  $T_d$  :  $T > [<]T_d$  and  $u = u_{ss} \Rightarrow \dot{T} < [>]0$

Considering (14),  $e < [>]0$  and  $u = F_{ss}(T_d) \Rightarrow \dot{e} > [<]0$ .

When  $u = F_{ss}(T_d)$ ,  $\dot{e}$  is named  $\dot{e}_{ss-T_d}$ . So, at any temperature, the previous phrase can be written as

A1:  $e < [>]0 \Rightarrow \dot{e}_{ss-T_d} > [<]0$ .

This evident practical assumption is used in stability analysis.

The higher the input heat flux, the higher the temperature rate or the greater the increase in temperature. It is the second practical assumption: the higher  $u$ , the higher  $\dot{T}$ .

Considering (14),

A2: the higher  $u$ , the lower  $\dot{e}$  (at any temperature).

*Proof:*

At the temperature of  $T$  :

$$e > 0 \left\{ \begin{array}{l} \stackrel{(13)}{\Rightarrow} u = F_{ss}(T_d) + k_c e > F_{ss}(T_d) \stackrel{A2}{\Rightarrow} \dot{e} < \dot{e}_{ss-T_d} \\ \stackrel{(A1)}{\Rightarrow} \dot{e}_{ss-T_d} < 0 \end{array} \right\} \Rightarrow \dot{e} < 0 \Rightarrow e\dot{e} < 0$$

$$e < 0 \left\{ \begin{array}{l} \stackrel{(13)}{\Rightarrow} u = F_{ss}(T_d) + k_c e < F_{ss}(T_d) \stackrel{A2}{\Rightarrow} \dot{e} > \dot{e}_{ss-T_d} \\ \stackrel{(A1)}{\Rightarrow} \dot{e}_{ss-T_d} > 0 \end{array} \right\} \Rightarrow \dot{e} > 0 \Rightarrow e\dot{e} < 0$$

*End of the proof*

As a result, the system is stable and the squared error decreases continuously at any positive value of the control gain  $k_c$ . Theoretically, control gain can be arbitrarily high without

harming the stability. The control performance (approaching speed towards the reference) is bounded only by actuators' practical limits. That is, the control behaviour can be described as 'the quickest possible convergence to the reference (setpoint) without overshoot, provided that assumptions A1 and A2 are valid and both feedback and feedforward terms of the proposed control law in (13) are available.

## VI. NEURAL NETWORK FEEDFORWARD CONTROL LAW

In order to apply the control law proposed in (13),  $F_{ss}(\cdot)$  introduced in (15) is needed. In this research, an artificial neural network (ANN) is employed as feedforward control law or  $F_{ss}(\cdot)$ . For this purpose, a set of control inputs (input voltages to the lamp) were applied to the system, which led to temperatures in the range of [140°C 180°C] after the sufficiently long period of time of five minutes. In total, 15 pairs of heat flux and steady state temperature are collected to be trained to an ANN. A multilayer perceptron was designed (Fig.4), with one neuron in input and output layers and ten neurons in the hidden layer. Activation functions of input and output layers are linear with the slope of one, and the activation function of the hidden layer is a hyperbolic tangent function. After training, the input to this ANN is the desired temperature ( $T_d$ ), then this ANN determines what heat flux ( $u_{ss}$ ) leads to the specified desired temperature in the steady state situation.

The following equation presents the relation of  $u_{ss}$  and  $T_d$  in ANN feedforward control law:

$$u_{ss} = F_{ss}(T_d) = \sum_{i=1}^{10} [T_i \tanh(W_i T_d + b_i)] + b, \quad (19)$$

where  $W_i$  is the weight of  $i^{\text{th}}$  connection between input and hidden layers,

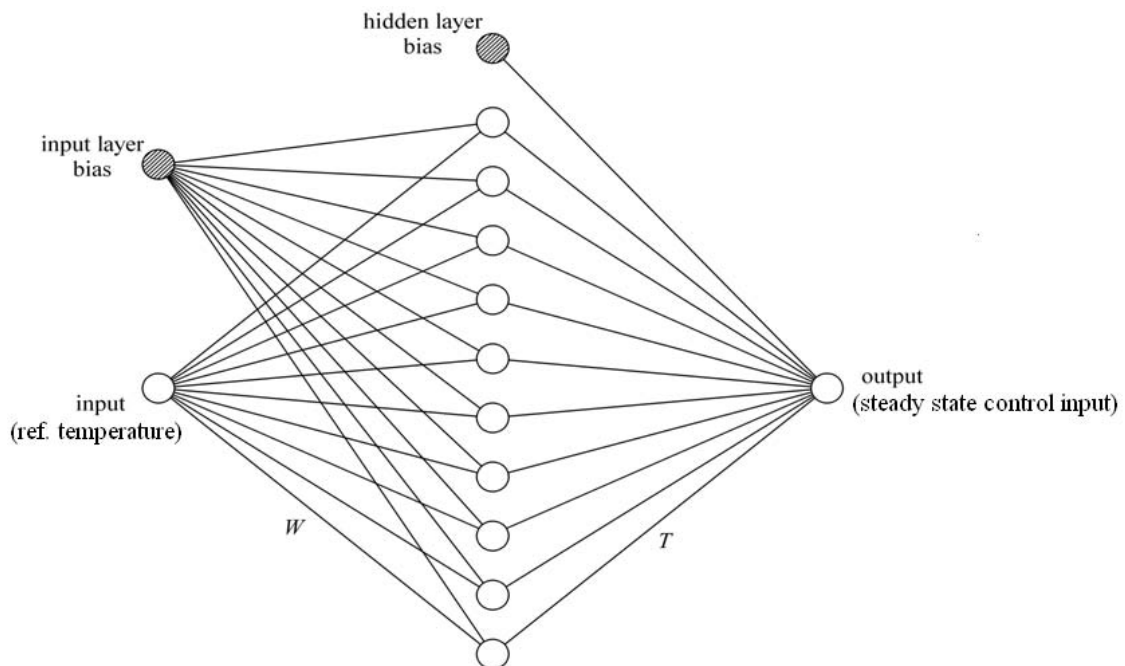
$T_i$  is the weight of  $i^{\text{th}}$  connection between hidden and output layers,

${}_1b_i$  is the weight of the connection between the bias of input layer and the  $i^{\text{th}}$  neuron of the hidden layer, and

${}_2b$  is the weight of the connection between the bias of the hidden layer and the output neuron.

**Table 1. Weights of ANN Feedforward Control Law**

$i$	$W_i$	$T_i$	${}_1b_i$	${}_2b$
1	-0.3629	-1.2404	-126.0059	0.0117
2	-0.5974	1.1946	122.8894	
3	0.8669	0.2522	-119.7773	
4	-0.8640	-0.0485	116.6657	
5	-0.5227	0.5455	113.5567	
6	-0.7281	-0.1626	110.1284	
7	-0.7288	-0.0165	107.3639	
8	0.0388	0.2595	-7.1092	
9	0.7386	-0.6470	-101.3311	
10	0.6486	-0.1691	-97.9999	



**Figure 4. A scheme of ANN feedforward controller**

The weights of the proposed ANN can be tuned continuously to be adopted with the probable changes of system characteristics. In other word, the proposed feedforward control law can be adaptive.

## VII. PREDICTIVE APPROACH FOR FEEDBACK CONTROL AND FINAL CONTROL SYSTEM

It usually takes time (say  $\theta$  seconds) for a control input to affect the control output of the system; therefore, at time  $t$ , the instantaneous error feedback signal is influenced by the control input applied on the time  $t - \theta$ . However, the design and analysis offered in sections IV and V are valid only providing that the temperature or the error made by a control input is immediately available (i.e. A2 in section V). A predictive approach was applied to simulate such an environment. In this approach, the control law (13) was written in the following form:

$$u(t) = F_{ss}(T_d(t+\theta)) + k_c e_p(t+\theta), \quad (20)$$

where  $e_p$  is the predicted error :

$$e_p(t+\theta) = T_d(t+\theta) - T_p(t+\theta), \quad (21)$$

where  $T_p$  is the predicted temperature.

With the sampling time of  $t_s = 0.1s$ , the order of one is considered for both temperature and control input:

$$T_p(t+t_s) = F_p(T(t), u(t-\theta)) \quad (22)$$

where  $F_p(\cdot)$  is a dynamic model of the system. This function is estimated by an artificial neural network (see Appendix 1).

Considering (18 ~22) and the concept of dead-time,

$$T_p(t+t_s) = \begin{cases} F_p(T(t), u(t-\theta)) = F_p(T(t), [k_c(T_d(t)-T(t)) + F_{ss}(T_d(t))]) & \text{for } t > \theta \\ F_p(T(t), 0) & \text{for } t \leq \theta \end{cases} \quad (23)$$

Providing that the values of the reference temperature ( $T_d$ ) and  $F_p(\cdot)$  are available in advance, the temperature of the next instant can be predicted using (23). For a known  $k_c$ ,  $F_{pp}(\cdot)$  is defined based on (23) so that (24) is equivalent to (23):

$$T_p(t+t_s) = F_{pp}(T(t), T_d(t), \theta). \quad (24)$$

and,

$$T_p(t+nt_s) = F_{pp}(T_p(t+(n-1)t_s), T_d(t+(n-1)t_s), \theta). \quad (25)$$

In practice, in order to make the future values of  $T_d$  available, the values of temperature reference are arranged in another data row:

$$r(t) = T_d(t+\theta). \quad (26)$$

$[T_d(0), \dots, T_d(\theta-t_s)]$  are used as initial values of  $r(t)$  in delay functions (see Fig.5).

(25) can be written as

$$T_p(t+nt_s) = F_{pp}(T_p(t+(n-1)t_s), r(t+(n-1)t_s - \theta), \theta); \quad (27)$$

so no future value of function inputs ( $T_p$  and  $r$ ) is needed for prediction purposes and this function ( $F_{pp}$ ) is applicable in real time operation. Figure 5 shows the system which was used to predict  $T_d(t+\theta)$  with  $\theta = 1.3s$  and  $t_s = 0.1s$ .

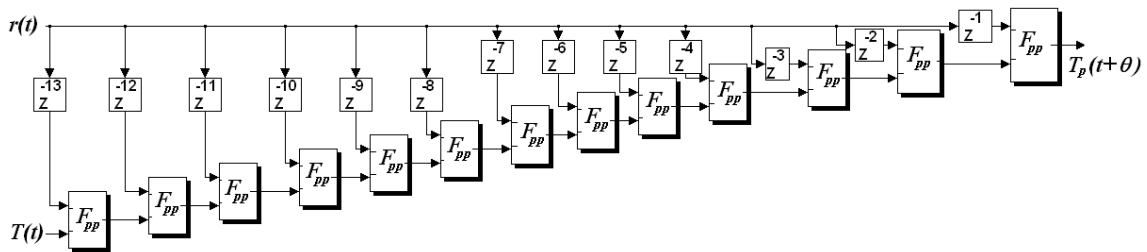


Figure 5. General Predictive Model

The whole system shown in Fig.5 is called the general predictive model and is shown as

$F_{GP}(\cdot)$ :

$$T_p(t+\theta) = F_{GP}(T(t), r(t), \theta). \quad (28)$$

Considering (21), (26) and (28), the control law introduced in (20) is applicable, providing that  $F_{GP}(\cdot)$  and  $F_{ss}(\cdot)$  are known:

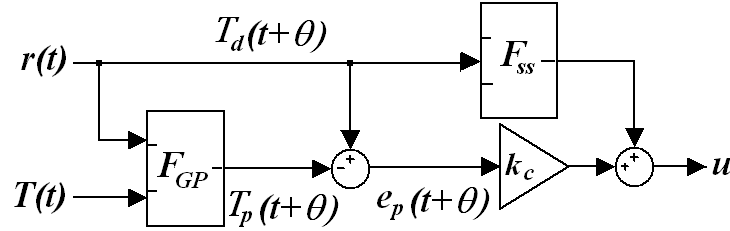


Figure 6. Control System

## VIII. EXPERIMENTAL RESULTS

The control system, explained in sections IV, VI and VII, was implemented on the dryer explained in section II and Appendix 2. Real-Time Windows Target (RTWT) toolbox of MATLAB/Simulink [22] was employed for implementation together with an I/O card (see Appendix 2). For two different setpoints, the results shown in Fig.7 are comparable with those in Figs. 14 and 15.

According to section V, theoretically in continuous domain, there should be no jump in control input after reaching the setpoint and no overshoot. However, the experiment was done in discrete domain with a sampling time of 0.1 s. Also the predictive model has inevitable inaccuracies, and thus a tiny overshoot (around 1°C) along with a few jumps in control input were observed. Compared to an iteratively tuned IMC-based PID (see Appendix 3), the proposed controller resulted in a very small overshoot and a very short reaching time as well as a remarkably little change in control input as a significant advantage from the

implementation viewpoint. This advantage makes this technique suitable for other application, particularly if the essence of control input is material rather than energy (i.e. a liquid flow).

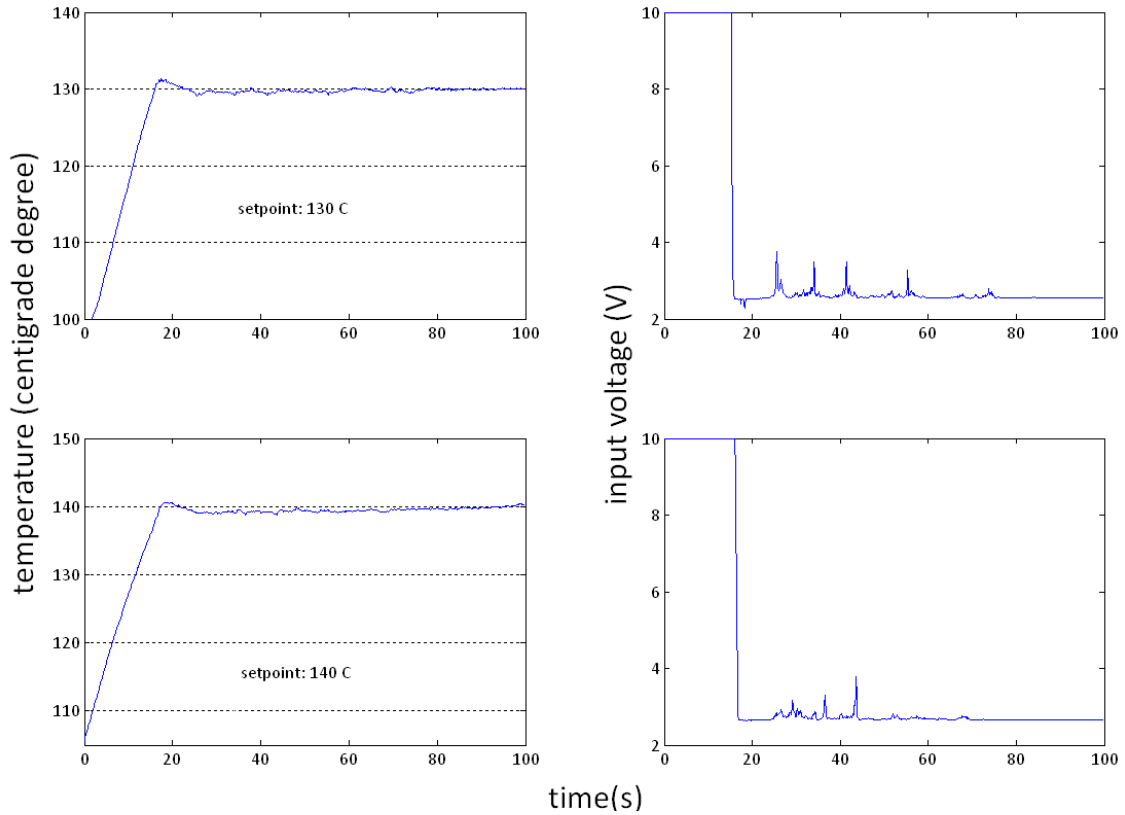


Figure 7. Experimental results for the proposed control system

## IX. CONCLUSION

In this paper, a method for feedforward-feedback control system design was introduced and experimentally applied for temperature control of an infrared batch dryer. The feedforward control law is an artificial neural network designed based on the idea of steady state control. The feedback controller consists of a general predictive model which generates the predicted



error, to overcome the effect of the dead-time, and a proportional gain on the path of the predicted error signal.

It was proved that in continuous domain, the system is stable at any value of the feedback controller gain. This means the system can approach the reference (setpoint) as fast as possible, considering the limits of the actuator (lamp), without harming the stability. The proof is based on a number of evident practical assumptions which can be extended to many other processes. The aforementioned assumptions are independent of the exact values of system parameters, and hence the stability proof remains valid in the case of change in system parameters if feedforward ANN control law is available for the altered system. In other words, the control system is robust in the occasion of parameter change. Furthermore, feedforward ANN control law can be trained/adjusted consistently so the control system can be adaptive.

As another advantage, the proposed method can be employed to control both linear and nonlinear systems, and nonlinearity of the system does not arise as a problem in this method. Experiments show a very fast convergence towards the reference as well as very little overshoot. Of particular advantage, the change in control input is diminutive compared to an iteratively tuned IMC-based PID controller.

## X. APPENDIX 1: ANN IDENTIFICATION OF THE DYNAMIC SYSTEM

Two different dynamic ANN models were designed and trained in this research to approximate  $F_p(.)$  introduced in (22), one of them (namely DANN1) was designed to be used in predictive control (section VII) and the second one (namely DANN2) is a transfer function which was used in IMC-based PID design (Appendix 3). In both cases, for training/checking purposes, at the initial temperature of approximately 100°C/90°C, a sinusoidal/triangular input signal was applied on the furnace for 300 s/100 s (see Fig.8), and the temperature was recorded. A time delay of  $\theta=1.3$  s was also considered, based on comparing the peaks of input and output signals.

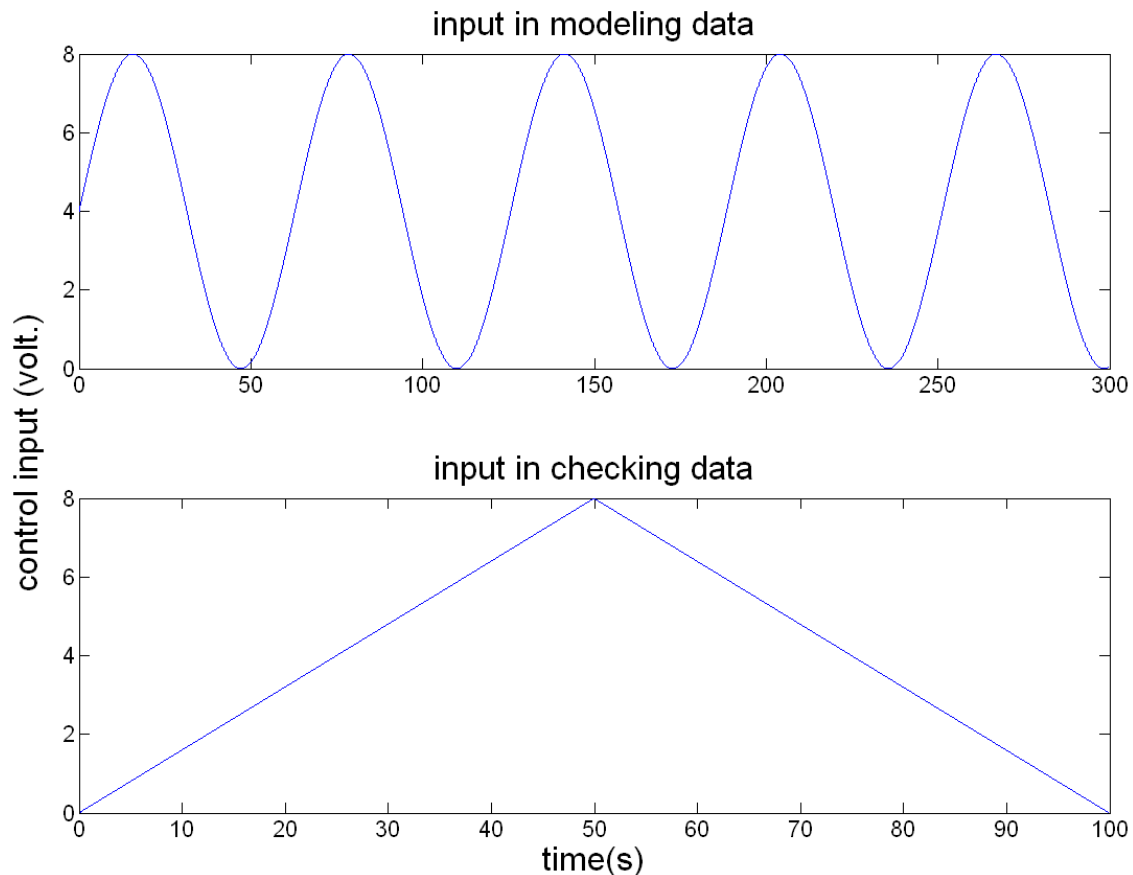


Figure 8. Input signals for ANN training and checking purposes

All activation functions are linear with the slope of one in both neural networks. A schematic of DANN1 and DANN2 are shown in Figs. 9 and 10, and corresponding weights are in Tables 2 and 3.

Before modeling, the temperature and voltage data columns were normalized by dividing the temperature/voltage values by the absolute values of their corresponding data column (approx. 36010 and 865, respectively). The training algorithm is Levenberg-Marquardt batch back propagation.

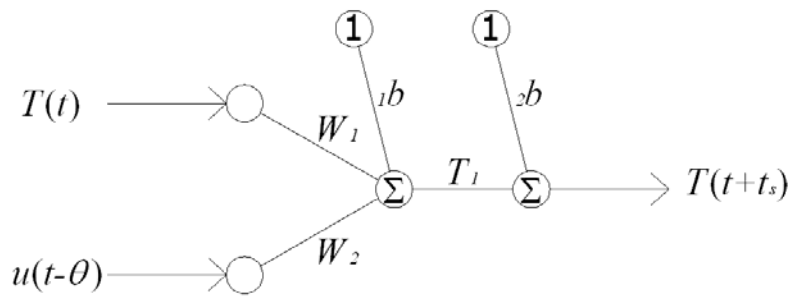


Figure 9. The structure of Dynamic Artificial Neural Network 1 (DANN1)

Table 2. Weights of DAAN1

$W_1$	$W_2$	$T$	${}_1b$	${}_2b$
1.3615	0.0014	0.7338	-0.1589	0.1166

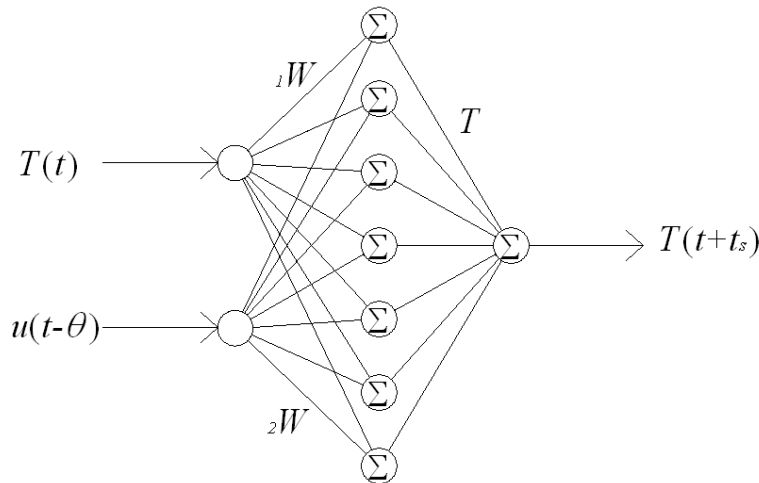


Figure 10. The structure of Dynamic Artificial Neural Network 2 (DANN2)

**Table 3. Weights of DAAN2**

$i$	${}_1W_i$	${}_2W_i$	$T_i$
1	0.8098	0.6867	0.7373
2	-0.9268	0.8452	-0.3106
3	0.1742	-0.1712	0.9616
4	0.9887	-0.1132	0.0388
5	0.0101	-0.2816	-0.0483
6	0.6968	0.6181	-0.6482
7	-0.9106	-0.7910	-0.3961

In checking, after the first stage, the previously estimated temperature was used to estimate the future temperature, or

$$\hat{T}_p(t+t_s) = \hat{F}_p(\hat{T}(t), u(t-\theta)) . \quad (29)$$

where  $\hat{T}$  is the estimated temperature and  $\hat{F}_p$  is an approximated ANN model of  $F_p$ . As a result, the modeling error returns to the checking process (error accumulation) and, if the model is not accurate enough, error accumulation leads to significant values of error after several stages of estimation [23, 24]. Checking results are shown in Fig.11. In checking, the mean of absolute error for DANN1 is 3.1585 °C and for DANN2 (transfer function) is 3.9876 °C. Considering the sampling time of 0.1, checking results are adequate. In the proposed predictive control approach, the estimation is performed only for 1.3 s . Least square of error leads to similar results in linear modeling [24].

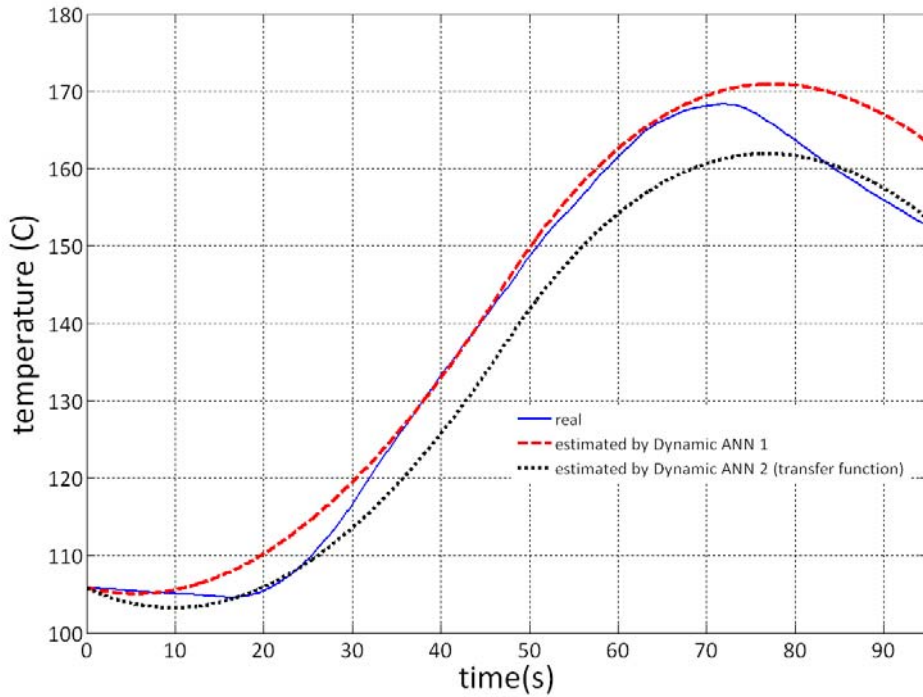


Figure 11. Checking results for DANN1 and DANN2

## XI. APPENDIX 2: EXPERIMENTAL SETUP

The components of the experimental setup are shown in Fig. 12.

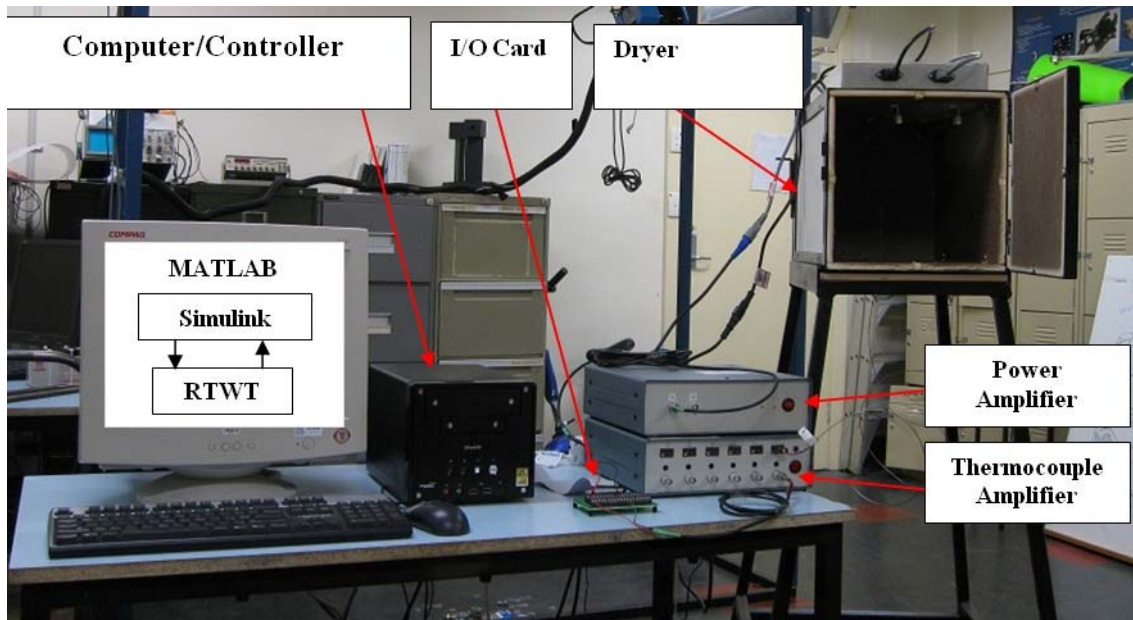


Figure 12. The main components of the experimental setup

The dryer has a halogen lamp [25] and a T-type thermocouple of 15 T 316 2000 D 75 T [26]. The walls of the dryer has been made with SUPERWOOL 607 insulation board [27] with the thickness of 20mm, and partially covered by steel sheets.

The approximate thermal conductivity of the fiber glass made insulation board varies with the temperature. The following table shows thermal conductivities vs. temperature of the insulation board:

**Table 4. Thermal conductivities vs. temperature of the insulation board**

Temperature(°C)	Thermal Conductivity( $W/mK$ )
400	0.08
600	0.12
800	0.16
1000	0.20

Real-Time Workshop [28] and Real-Time Windows Target (RTWT) toolboxes of MATLAB were employed to produce executable program of the Simulink model of the control system and connect the produced code to the hardware (the thermocouple and the lamp) through a HUMUSOFT MF624 Input/Output card [29]. Two amplifiers were also used to magnify the signals coming from the thermocouple to the computer and from the computer to the lamp.

## **XII. APPENDIX 3: IMC-BASED PID CONTROL SYSTEM DESIGN**

In order to design a PID controller based on internal model control (IMC) method, a linear model is needed. Through a system identification process, the following model is achieved (DANN2 in Appendix 1):

$$G(s) = \frac{0.0387}{1+152.812s} e^{-1.3s} \quad (30)$$

For a first order transfer function with a time delay  $G_p(s) = \frac{k_p}{1+\tau_p s} e^{-\theta s}$  (31), the

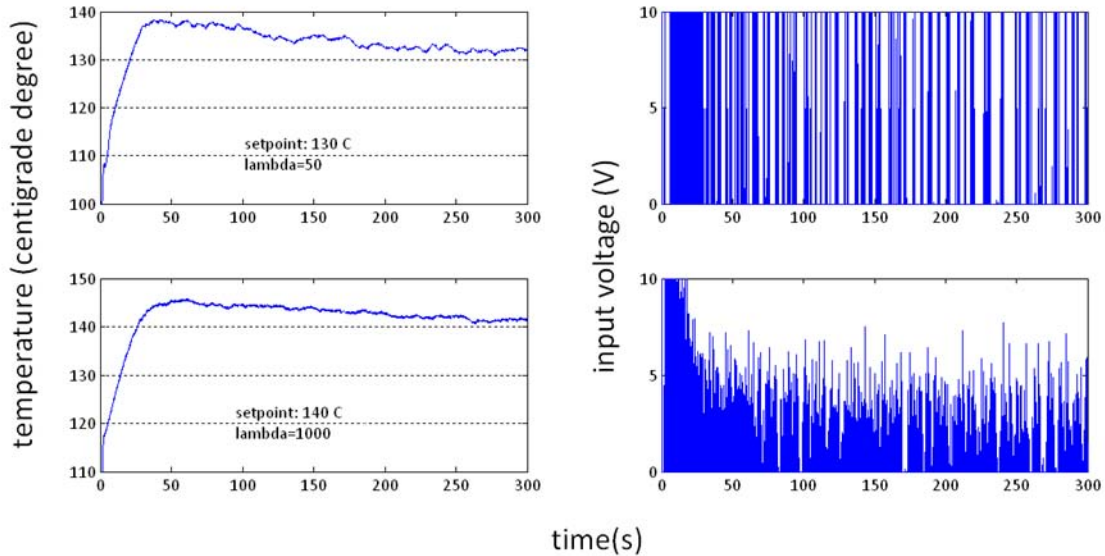
following PID controller is designed based on IMC technology [30]:

$$G_c(s) = \frac{(0.5\tau_p\theta)s^2 + (\tau_p + 0.5\theta)s + 1}{k_p(\lambda + 0.5\theta)s}, \quad (32)$$

where  $\lambda$  is the time constant of the controller. The smaller  $\lambda$ , the quicker convergence towards the reference, and the less robustness. So for the model shown in (30), the following controller was designed

$$G_c(s) = \frac{99.328 s^2 + 153.462 s + 1}{0.0387(\lambda + 0.65)s}. \quad (33)$$

Despite good results in simulation, in practice the designed PID controller does need lead to an adequate result, probably due to unmolded nonlinearities and the effects of discrete domain. Three sample experimental results with different values of  $\lambda$  are shown in Fig.13:



**Figure 13.** The temperature and input voltage to the dryer with the original IMC-based PID controllers

In order to improve the performance of the controllers, iterative tuning was performed. With

$$\lambda = 500, G_c(s) = 7.9204 + 5.1264s + 0.0516\frac{1}{s} \quad (34)$$

This controller was chosen for iterative tuning.

First, a performance function was defined in (35).

$$PF = \int_0^{t_f} |e(t)| dt + \rho_1 \int_0^{t_f} |u(t)| dt + \rho_2 \int_{t_s}^{t_f} |u(t) - u(t - t_s)| dt \quad (35)$$

where  $t_s$  and  $t_f$  are system sampling time and operation time (0.1s and 200s) and  $u(t)$  is the control input;  $\rho_1$  and  $\rho_2$  represents the importance of energy consumption and control input change in the performance function.

Initially, in simulation, the performance function was evaluated for three different sets of setpoints rather than one set (see Table 5) to result in a more realistic design/optimization. The initial temperature is 100°C. The sum of the values of performance function (SPF) for three different sets of setpoints (SPF) was calculated with  $\rho_1 = 0.0001$  and  $\rho_2 = 0.001$ , considering much higher values of control input compared to the error.

**Table 5. Different sets of reference (setpoints) for optimization purposes**

	$0 \leq t < 100$ s	$100 \text{ s} \leq t \leq 200$ s
Reference ( 1 <sup>st</sup> series)	120°C	170°C
Reference ( 2 <sup>nd</sup> series)	170°C	120°C
Reference (3 <sup>rd</sup> series)	150°C	150°C

After finding SPF for the initial set of control parameters (triple numbers shown in (34)) , a new set of parameters are derived:

$$p_n(i) = p_o(i)[1 + 0.1(2rand - 1)] \quad i=1,2,3 \quad (36)$$

where  $p_n$  and  $p_o$  represent new and original controller parameters respectively and  $rand$  represents a random number between 0 and 1.

The simulation was run with new control parameters, generated by (36), and SPF (the sum of the values of the performance function for three different series of setpoints) was calculated. Then the following sub-algorithm was employed:

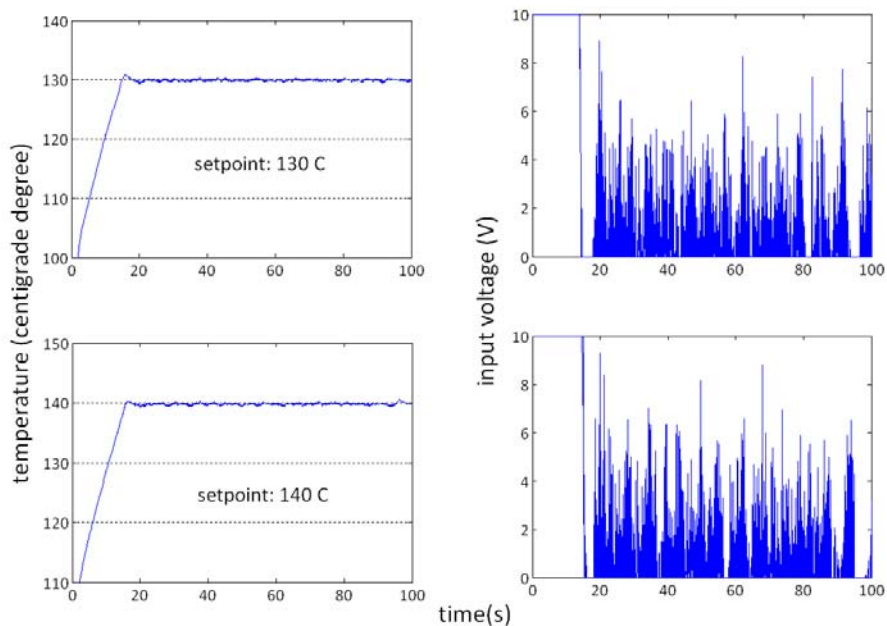


$$\begin{cases} p_{nn}(i) = p_n(i) - \eta \frac{SPF_n - SPF_o}{p_n(i) - p_o(i)} \\ p_o(i) = p_n(i) \\ p_n(i) = p_{nn}(i) \end{cases} \quad (37)$$

where subscripts of  $o$  and  $n$  stand for new and original. This algorithm was tried several times with  $\eta = 10^{-4}$ . In simulation, the initial value of SPF for original PID controller (shown in (34)) is 39.4544. The best controller achieved with a SPF of 17.3010 is

$$G_c(s) = 7.3022 + 1.7667s - 0.0075 \frac{1}{s} \quad (38)$$

Other values of  $\eta$ ,  $\rho_1$  and  $\rho_2$  were also tried. Authors are convinced there is no PID with the results significantly better than the proposed PID controller in (38). In comparison with simulation, in practice, the issue of undesirable substantial changes in control input is more serious and it can be decreased only at the price of higher error integral and overshoot. Some experimental results are shown in Fig.14 (comparable with Figs. 13 and 7):



**Figure 14. The temperature and input voltage to the dryer with the iteratively-tuned PID controller**

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# Conclusion

This thesis aims to broaden the knowledge about system dynamics applicable for control system design. Currently, mathematical models, input-output data and experts' knowledge in the form of linguistic expressions (fuzzy rules) are used as the sources of knowledge for control system designers. The project, presented here, was accomplished in the area of non-model-based control. In this area, there are questions about some cases of unexpectedly poor performance of control systems that cannot be answered adequately using the aforementioned forms of knowledge about system dynamics. Two of these questions were the motivation behind this research: Why does a non-model-based control system design approach lead to a repeating overshoot for one system and works satisfactorily for another (see Chapter 3)?, and in the case where the disturbance is not an issue, Why are feedforward-feedback control systems are so advantageous, or even essential, in controlling some systems while other systems can be controlled very well only with feedback controllers (see Chapter 4)? There are no explicit answers to these questions in the literature based on the three well-known forms of knowledge about system dynamics. It seemed new types of knowledge were needed about system dynamics to answer these questions convincingly.

In this thesis, the abovementioned questions were answered systematically. In Part 2, based on the concept of control inertia, the reason of the occurrence of unexpected repeating overshoots in some systems was explained. The proposed solution is fuzzy brakes: special fuzzy inference systems which modify the control input when the system response is in the vicinity of the reference. This study proved that fuzzy brakes do not make a stable closed-loop system unstable.

In Parts 3 and 4, the concepts of control equilibrium point, steady state control and generalized type zero (GTZ) systems were introduced. Based upon these concepts the necessity and the design method of feedforward control laws were addressed. According to the literature, if the disturbance is a main issue, a feedforward controller is useful; if not, the literature provides no certain way to define the applicability of a feedforward control law, and if applicable, and if so, how to derive it. These questions were addressed in this research. Based on the proposed concepts, a feedforward controller is applicable provided that the system is GTZ, and the steady state control law, which can be an artificial neural network, plays the main role in forming the feedforward controller. In Part 3, fuzzy controllers were designed as feedback controllers to be used together with steady state (feedforward) control laws. This part includes design and stability analysis of hybrid ‘fuzzy-steady state’ control systems for some case studies. As an improvement in fuzzy control, practical assumptions were used to prove the stability. Therefore, a non-model-based stability analysis method was proposed in fuzzy control. Part 4 shows that with the proposed feedforward control law (introduced in Part 3), for a wide class of process plants, the stability of feedforward-feedback control system was proved when the feedback controller is a gain with an arbitrarily high value. Chapters 7 and 8 explain and verify this control idea. However, two main issues may make this recent proposed methodology and its stability proof invalid or impractical: variable(s) coupled control output(s) and time delay. Both issues were discussed in detail and appropriate solutions were proposed in Chapters 9 and 10.

In summary, this research offers two more types of knowledge for control system designers. The characteristics of high control inertia and GTZ can define whether fuzzy brakes or steady state feedforward control laws are functional for system. These two (fuzzy brakes and steady state feedforward control laws) can improve the performance, decrease energy consumption

and change in control input if they are deliberately used. Design and stability discussion has been offered in detail in this thesis so as to make the proposed ideas applicable in practice.

However, this thesis is the first step in developing the aforementioned types of knowledge, and there still exists considerable room for further investigation in these areas. For example, in Part 2, the architecture and the mathematical structure of control systems with a fuzzy brake can be defined; however, further study is needed to find a way to precisely determine the values of the parameters of fuzzy brakes. The experimental test of the ideas of fuzzy brakes and control inertia can be advantageous in taking these ideas further. Parts 3 and 4 can also be completed through testing the developed methodologies on more complicated systems that have coupled variables (states) with the control output(s), time-delay and multiple control outputs and inputs.

# Appendices

## SYNOPSIS

This part includes four appendices of this thesis: two conference papers in their original format and a manuscript about the experiment reported in Chapter 10. The conference papers of Appendices 1~2 were written by the author of this thesis, cited in thesis chapters. The last appendix is the expanded version of the second appendix of Chapter 10. The equipment and the procedure of the experiment reported in Chapter 10 are explained in more detail in Appendix 3. The references of this appendix are references 25~29 of Chapter 10: the websites of hardware or software supplier companies.



## **Appendix 1**

### **Intelligent Modeling of MIMO Nonlinear Dynamic Process Plants for Predictive Control Purposes**

Presented in the 17th IFAC World Conference, Seoul, Korea, 6-11  
July 2008

Mohammadzaheri, M. and Chen, L. (2008) Intelligent Modelling of MIMO Nonlinear Dynamic Process Plants for Predictive Control Purposes. *Proceedings of the 17th World Congress, The International Federation of Automatic Control, Seoul, Korea, July 6-11, 2008.*

NOTE: This publication is included on pages 263-268 in the print copy of the thesis held in the University of Adelaide Library.

It is also available online to authorised users at:

<http://dx.doi.org/10.3182/20080706-5-KR-1001.02099>

## **Appendix 2**

### **Anti-overshoot Control of Model Helicopter's Yaw Angle with Combination of Fuzzy Controller and Fuzzy Brake**

International Conference on Intelligent and Advanced Systems, Kuala Lumpur, Malaysia, 25-28 November 2007

Mohammadzaheri, M. and Chen, L. (2008) Anti-overshoot control of model helicopter's yaw angle with combination of fuzzy controller and fuzzy brake. *2007 International Conference on Intelligent & Advanced Systems (ICIAS), Kuala Lumpur, Malaysia, 25-28, November 2007, pp. 99-103*

NOTE: This publication is included on pages 270-274. in the print copy of the thesis held in the University of Adelaide Library.

It is also available online to authorised users at:

<http://dx.doi.org/10.1109/ICIAS.2007.4658355>

## **Appendix 3**

### **Experimental Set Up and Implementation**

# Experimental Set Up and Implementation

## I. General Description

The main structure of the experimental setup is shown in Fig. 1 .

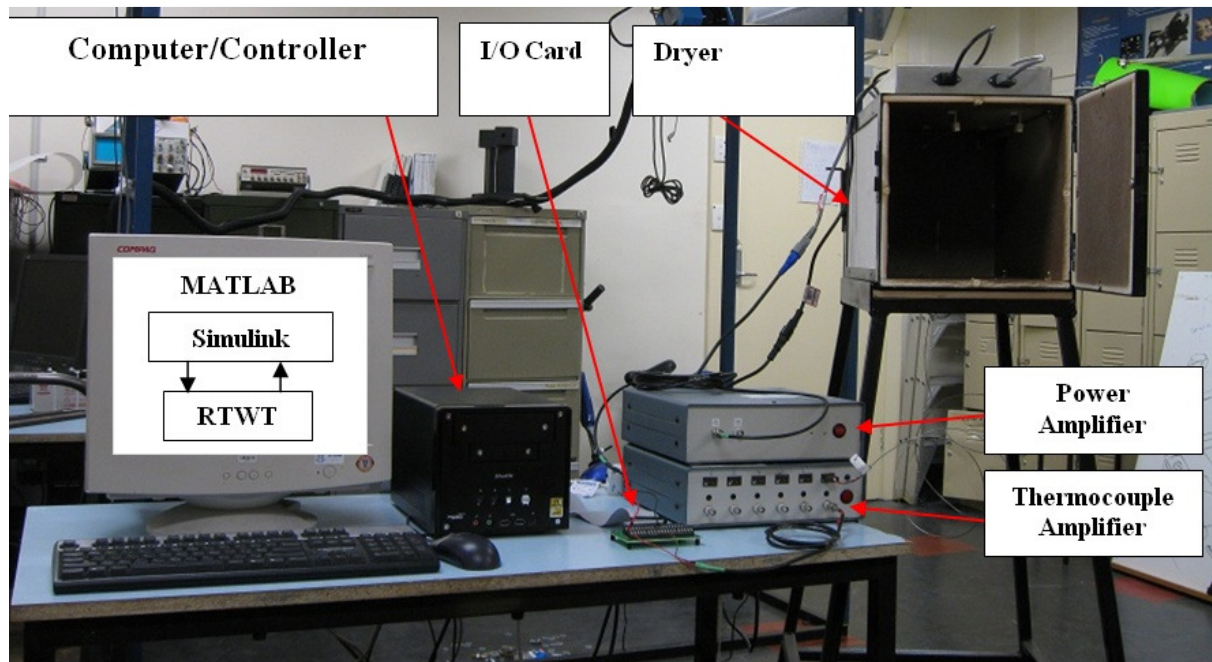


Figure 1. The dryer experimental setup

As shown in Fig.2, the infrared dryer has two radiation heat sources (lamps) and six temperature sensors (thermocouples). Both lamps and thermo-couples are arranged non-symmetrically to make more complicated modeling/control problems to test different methodologies. As a result of non-symmetry, the effect of any of lamps on any of temperature sensors is unique. That is, having two lamps and five sensors, ten single input-single output control problems, and ten double-input double-output control problems can be defined with this system, which may have different features because of various positions of sensors regard to lamps and furnace walls. All these double-input double-output control problems are coupled without a mathematical model to be used in control.

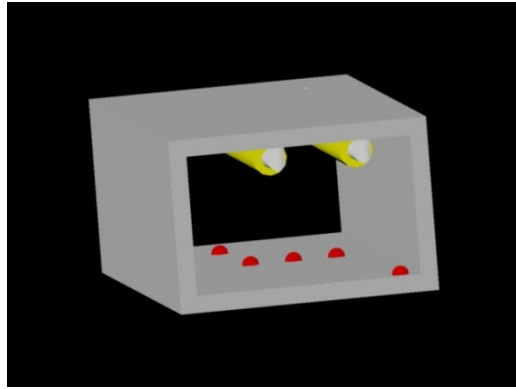


Figure 2. The arrangement of lamps and thermocouples in the dryer

The dryer body was built by insulation boards and steel sheets. T-type thermocouples with the accuracy of 1°C were selected as sensors. An amplifier increases the output voltage of thermocouples and direct this signal to a digital input-output card, connected to the computer, the control command is generated in the computer and is sent to the power control (poer amplifier) unit which change input voltage to the lamp(s) based on received signal for mthe computer. In the end, as actuators, a couple of special halogen lamps with their relevant equipment were needed.

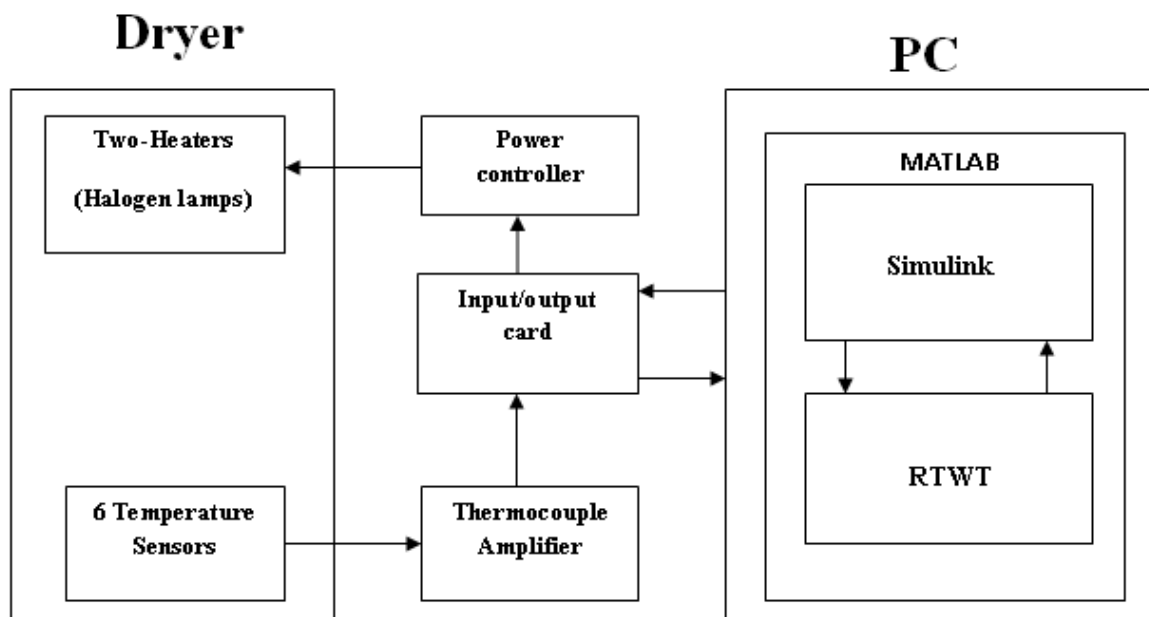
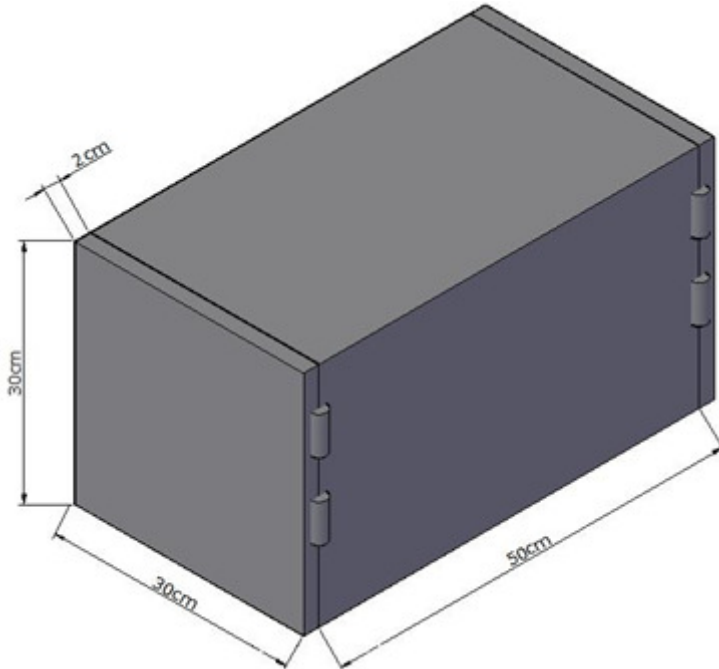


Figure 3. Connecting signals in the experimental setup

## II. Furnace Body

The dryer body is made by steel sheets and insulation board.



**Figure 4. The size of the dryer body**

Insulation boards are made from inorganic insulation glass fibres. They are bonded by thermosetting materials and manufactured in various thicknesses from 3mm up to 75mm. These boards are normally used for electrical insulation and heat protection.



**Figure 5. The walls of the dryer made by Insulation boards and steel sheets**



SUPERWOOL 607 insulation board with a thickness of 20mm made by Morgan Crucible Company in South Australia was used in this project.

### III. Thermocouples

A Thermocouple is generally utilized to convert heat into electrical power based on a discrepancy between electrical potential generated in two wires made by different metals that form the thermocouple. There is a nonlinear relationship between temperature change ( $\Delta T$ ) and output voltage (V) shown in equation 1

$$\Delta T = \sum_{n=0}^N a_n V^n, \quad (7)$$

where  $a_n$  is available for  $n$  from zero to nine for different thermocouples. Thermocouples are quick sensors (with very short time delays) and they are able to measure a wide range of temperatures. However, the accuracy of less than one Kelvin is hard to achieve in thermocouples.

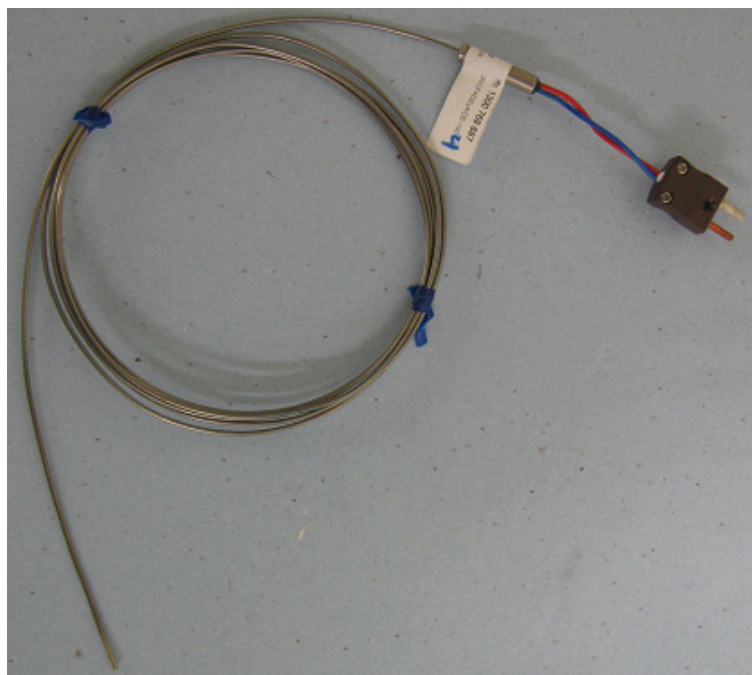


Figure 6. A type T thermocouple

Among different kinds of thermocouples, type T was selected for our application (see Table 2).

**Table 1: Thermocouple type T characteristics**

Type	Temperature range °C (continuous)	Temperature range °C (short term)	Tolerance class one (°C)	Tolerance class two (°C)
T	-185 to +300	-250 to +400	±0.5 between -40 °C & 125 °C ±0.004×T between 125 °C and 350 °C	±1.0 between -40 °C & 133 °C ±0.0075×T between 133 °C & 350 °C

#### IV. The Thermocouple Amplifier

In the thermocouple amplifier, the signal coming from thermocouple ( $43 \mu\text{V}/^\circ\text{C}$ ) is amplified to  $10 \text{ mV}/^\circ\text{C}$ . This signal is sent to the computer. Thermocouple amplifier type is AD595 and bought from ANALOG DEVICES Company. This amplifier has six input and output channels.



**Figure 7. Thermocouple Amplifier**

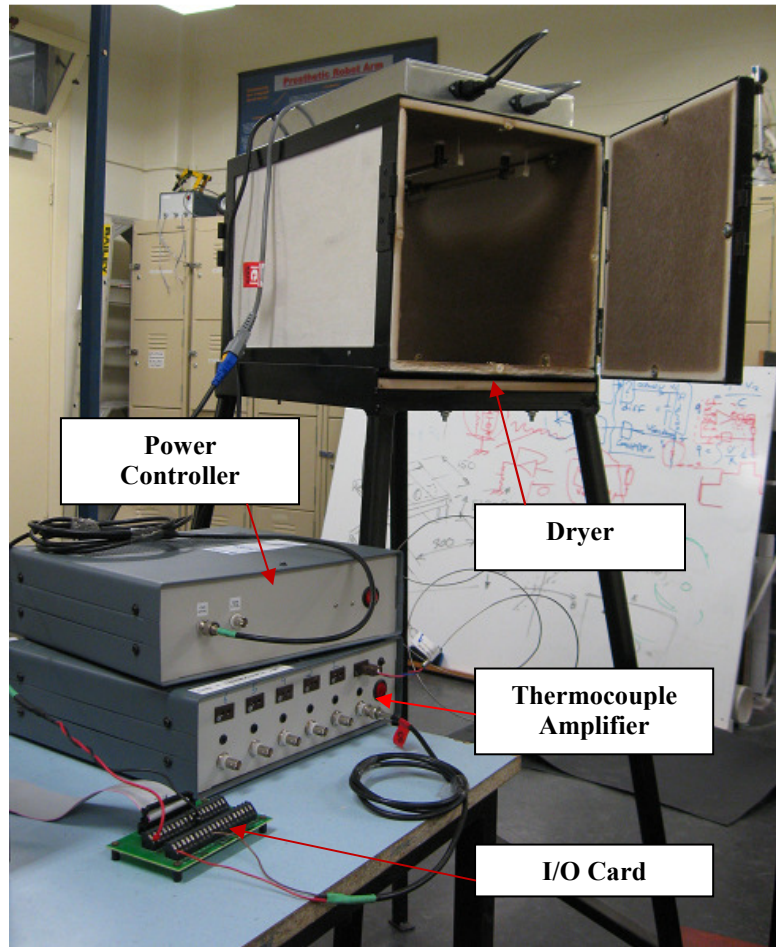
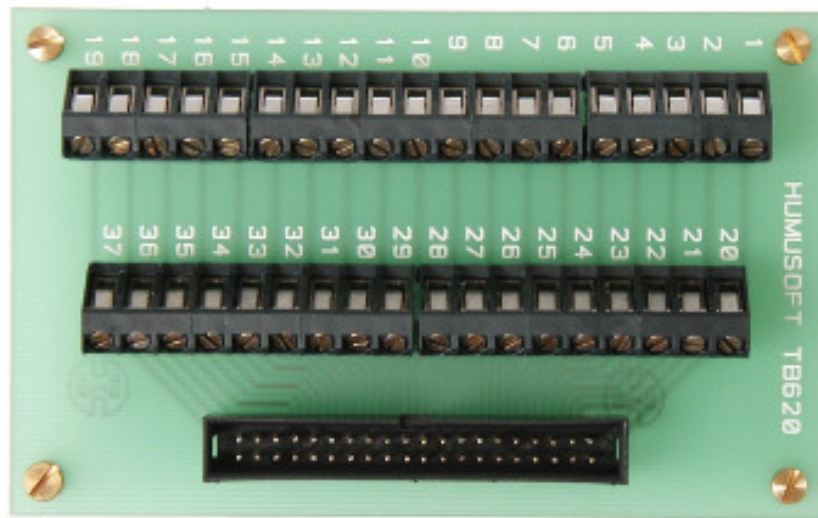


Figure 8. Thermocouple amplifier located between the I/O card and the dryer

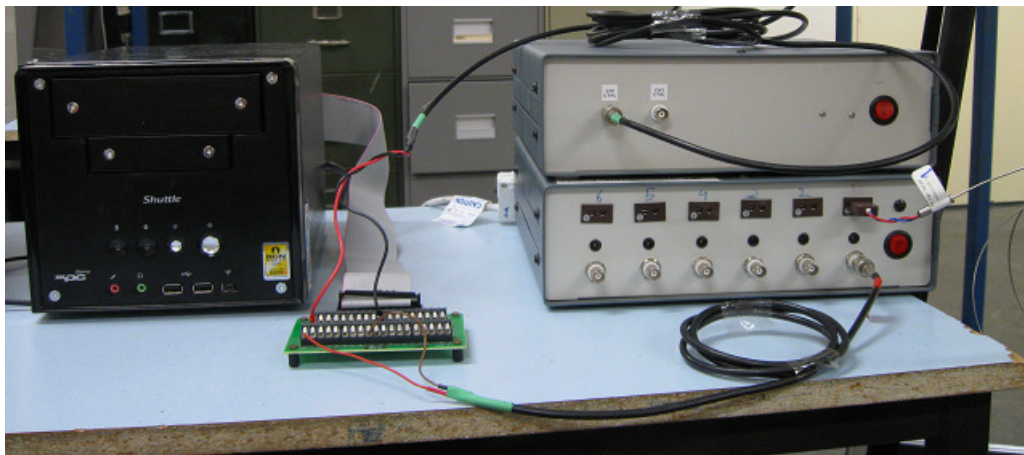
## V. Input/output Card

In this project, the selected I/O card converts analog signals coming from thermocouple amplifier to digital ones and send them to the computer. At the same time, the I/O card converts digital signals coming from computer to analog ones to send them to the power controller and then to the halogen lamp(s). I/O card type MF624 was selected which is compatible with RTWT toolbox of MATLAB software and specifically produced for thermal process. The MF 624 has a 8-channel 14-bit A/D converter with simultaneous sample/hold

circuit, eight independent 14-bit D/A converters, a 8-bit digital input port and a 8-bit digital output port, 4 quadrature encoder inputs and 5 time counters.



**Figure 9. MF 624 Multifunction I/O Card**



**Figure 10. The I/O card connects the computer to power and thermocouple amplifiers**

## **VI. The Power Controller Unit**

Power controllers provide circuit breaker functions such as protection of the load and wiring from overload conditions. In addition, power controller provide on/off control of the conduction of the load circuit and used to protect an AC wire harness against damage if the power controller experiences a short circuit failure caused by over-current or short circuiting.



**Figure 11. The power controller of dryer lamps**

The load (output) voltage of a power controller is adjusted by varying the time within each electrical half-cycle. AC electrical current and a control signals enter to a power controller; if electrical half-cycle is  $T$ , for a portion of  $T$ , the power controller is ON and let the current pass through and for the rest of electrical half-cycle is the power controller is OFF. This portion is defined by the control signal. We used FCAL/2 power controller manufactured by from UNITED AUTOMATION Company. In fact, our power control unit has two power controllers, for two lamps.

## **VII. Lamps**

There were two important factors for the radiation sources. They need respond to have a quick response to the control command (voltage coming from computer) and also they need to change the emitted heat flux continuously without harming. Halogen lamps could meet our criteria.

Among a variety of options, a 2kW halogen lamp was selected and purchased. Fig.12 and 13 show lamps and their support clamps.

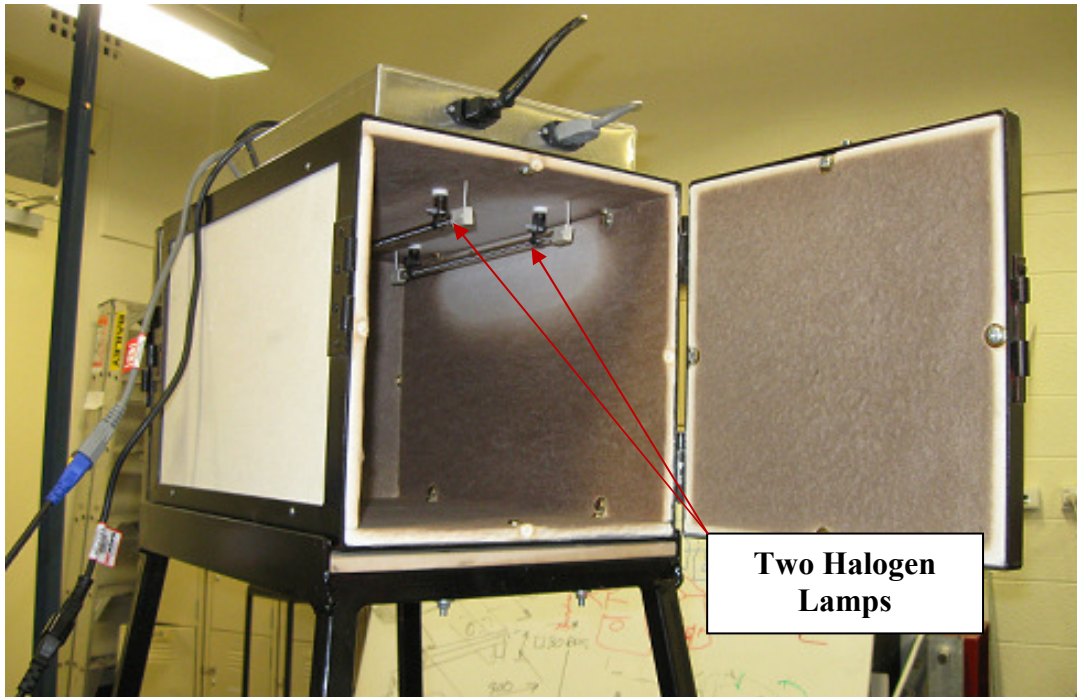


Figure 12. Halogen lamps in the dryer



Figure 13. Two halogen lamps with their clamps

## VIII. Some more implementation hints

- For the purpose of thermocouple calibration, we added a look-up table to the Simulink model. The input to this lookup table is the input signal from the thermocouple and its

input is the temperature. Required data for making the look-up table are collected using a standard calibrator.

- A low pass filter was added to the Simulink model to filter noisy output signal of the thermocouple.
- There are three critical steps in running a Simulink model in real time using RTWT. We need to “build” a C code based on the Simulink model, “connect” the generated code to hardware, and “run” the system.
- For RTWT application, the solver of the Simulink model must be “fixed step” one.