Nonlinear Dynamics Of Magnetic Bearing Systems

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Abstract

The aim of the work outlined in this thesis was to gain a deep insight into the effects of nonlinearities of magnetic bearings and the influence of time delays occurring in the feedback control path on their dynamic behaviour and performance. Emphasis was on stability analysis, bifurcation control, determination of stable operating conditions, prediction of bearing performance, and on aspects of nonlinear dynamic behaviour including bifurcations, coexistence of multiple solutions and complicated motions.

Magnetic bearings use magnetic forces to support various machine components. Because of the non-contact nature of suspension, this new bearing technology offers a number of significant advantages over conventional bearings such as rolling element and fluid film bearings. These advantages include elimination of the lubrication system, friction free operation, no wear, high rotor speed, adjustable dynamic properties, vibration control, on-line identification and fault diagnosis capabilities, and active health monitoring of rotordynamic systems.

Magnetic bearings have found their applications in turbomachinery, centrifuges, vacuum machinery, machine tool spindles, medical devices, robotics, high-speed drives, spacecraft equipment, contactless actuators, and vibration isolation, etc.

The stable operation of a magnetic bearing system can only be achieved by feedback control. A magnetic bearing system is basically composed of sensors, controllers,

amplifiers and electromagnets. All of these components are characterized by nonlinear behaviour and therefore the entire system is inherently nonlinear. However, in simulations of the dynamic behaviour of magnetic bearing systems, the nonlinearities were usually neglected for simplicity and the components of magnetic bearing systems were simplified to linear models. Moreover, many of control techniques currently used in magnetic bearing systems were generally designed by ignoring the nonlinearity of the magnetic forces and the nonlinear modelling of the sensors and actuators. The main reason for simplification was the intractability of the complexity of the actual model. In fact, the inherent nonlinear properties of magnetic bearing systems can lead to a dynamic behaviour of the rotor-magnetic bearing system that is distinctly different from that predicted using a simple linearized model. Therefore the nonlinearities should be taken into account. For certain cases, it may perhaps be theoretically possible to compensate partially certain nonlinearities at a cost of highly complex control strategies. However, nonlinearities originating from hardware and physical system limitations cannot be eliminated by software, and the control problem becomes very complicated due to the inherent nonlinearities associated with the electromechanical dynamics introduced into the magnetic bearing system.

The present thesis comprises 12 papers which were recently published in seven international journals. These papers contribute to study: 1) the effects of geometric coupling and nonlinear force relationships on the dynamic behaviour of magnetic bearings; 2) the effects of the nonlinear force relationship incorporating time delays or saturation of power amplifier (or saturation of magnetic material or limitation of the control current) on the dynamic behaviour and performance of magnetic bearings; 3) the effect of a combination of up to three components of nonlinearities on the dynamic behaviour and performance of magnetic bearings; 4) the nonlinear response of rotor-magnetic bearing systems under primary, sub-harmonic, and super-harmonic resonance conditions. These papers also contribute to develop: 1) an analytical technique (which is referred to as the matching method) for constructing an approximate solution to a periodically excited nonlinear-linear system; and 2) develop a nonlinear control approach from the bifurcation control point of view to stabilize a subcritical Hopf bifurcation, thereby extending the operational region.

It is found that in the presence of nonlinear magnetic forces, rotor-magnetic bearing systems may exhibit a variety of nonlinear behaviours including saddle-node bifurcations, Hopf bifurcations, jump phenomena, coexistence of multiple solutions, quasi-periodic and chaotic motions. For a rotor-magnetic bearing system with saturation constraints, it is shown that the forced response of the system may accept symmetric and asymmetric period-one solutions, subharmonic and chaotic solutions.

It is also found that the time delays occurring in the feedback control path may have a significant impact on the stability and dynamics of rotor-magnetic bearing systems. For a rotor supported by a two-pole magnetic bearing, it is shown that a Hopf bifurcation can occur when time delays pass certain values. Co-dimension two bifurcations, which result either from a non-resonant or resonant Hopf-Hopf interaction or from an interaction of a Hopf and a steady state bifurcation, are also found to be possible after the trivial fixed point loses its stability through Hopf bifurcations. Increasing the values of time delays not only can increase the peak amplitude of the forced response but also can shift the frequency-response curve to the right.

For a rotor-magnetic bearing system involving both geometric coordinate coupling and time delay, it is found that as the time delay increases beyond a critical value, the equilibrium position of the rotor motion becomes unstable via a Hopf bifurcation of multiplicity two and may bifurcate into two qualitatively different kinds of periodic motion. An interaction between the Hopf bifurcating periodic solutions and the external periodic excitation can induce primary and super-harmonic as well as sub-harmonic resonances in the neighbourhood of the Hopf bifurcation. The forced nonlinear response of the system may exhibit pitchfork bifurcations, Hopf bifurcations, symmetric and asymmetric phase-locked periodic motions, quasi-periodic motions, chaotic motions, and coexistence of two stable motions.

Statement of originality

To the best of my knowledge and belief, this thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, contains no material which has been published previously by any other person, except where otherwise referenced and cited.

If accepted for the award of the degree of Ph.D. in Mechanical Engineering, I consent that this thesis be made available for loan and photocopying.

Jin-Chen Ji

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Contents

Chapter 1

Introduction

One of the most innovative developments in the turbomachinery field involves the use of active magnetic bearings for rotor support. This new bearing technology provides significant improvements in the dynamic behavior and operation reliability as well as energy consumption by the bearings of rotor systems. The application of magnetic bearing technology has experienced substantial growth during the past two decades, since the First International Symposium on Magnetic Bearings was held in 1988 [Schweitzer, 1988]. Meanwhile, considerable research has been conducted to cover all aspects of magnetic bearings including sensing and control technology, modelling and identification, components and materials, and self-sensing techniques [Higuchi, 1990; Allaire, 1992; Schweitzer, Siegwart and Herzog, 1994; Matsumura, Okada, Fujita and Namerikawa, 1996; Allaire and Trumper, 1998; Schweitzer, Siegwart, Loesch and Berksun, 2000; Okada and Nonami, 2002; Lyndon and Trumper, 2004; Bleuler and Genta, 2006]. However, existing research on magnetic bearings leave many problems remaining to be solved. This thesis will address the problem of the effect of nonlinear properties and time delays on the nonlinear dynamics and dynamic stability of magnetic bearing systems.

This chapter is organized into five sections. Section 1 presents a brief introduction to magnetic bearings and the active topics of current research relevant to magnetic bearing technology. Research focus of this thesis is discussed in Section 2 through addressing the source of nonlinear properties of magnetic bearings, giving reasons for the research outlined in this thesis, and summarizing the aims of the research. Section 3 lists the 12 papers which comprise this thesis. Section 4 describes the organization of the thesis. Finally, Section 5 concludes the chapter by summarizing the main contributions of the completed research outlined in this thesis to the understanding of the nonlinear dynamics and dynamic stability of magnetic bearing systems.

1.1 Magnetic bearings and the active topics of relevant research

A bearing is a component used to reduce friction in a machine. Bearings may be classified broadly as radial bearings and thrust bearings according to the motions they allow. Alternatively, bearings may be grouped according to six common principles of operation: namely sliding bearings; rolling element bearings; jewel bearings; fluid bearings; flexure bearings and magnetic bearings.

Magnetic bearings use magnetic forces to support moving machinery without physical contact. Because of the non-contact nature of suspension, this new bearing technology offers a number of significant advantages over conventional bearings such as rolling element and fluid film bearings. These advantages include elimination of the lubrication system, very low friction, no wear, high rotor speed, adjustable dynamic properties, and on-line identification and fault diagnosis capabilities [Aenis, Knopf and Nordmann, 2002; Zhu, Robb and Ewins, 2003]. Magnetic bearings can offer high load-carrying capability by optimising system and material parameters including the bearing air gap, bearing material saturation flux, bearing surface area, number of bearing coil turns and amplifier power. Magnetic bearings can permit operation in extreme environments such as high temperatures [Mhango and Perryman, 2002], low temperatures [Ruffles, 2000] and vacuums [Schweitzer, 1992]. Magnetic bearing systems can provide the user with instantaneous peak values of all bearing currents and rotor positions, and can also display the lateral and axial velocity of the rotor from direct measurements. An advanced monitoring system incorporated in a magnetic bearing system not only can monitor system parameters such as vibration, electrical current, temperature and rotational speed, but can also analyse the unbalance by calculating its location and magnitude. The electronic controllers change bearing stiffness and damping properties, allowing for adjustments to system dynamics that affect resonant frequencies and reduce transmitted vibration. Magnetic bearings united into a rotor-bearing system may be used for synchronous disturbance control and vibration control [Knospe, Hope, Fedigan and Williams, 1995; Matsushita, Imashima and Okubo, 2002; Cole, Keogh and Burrows, 2002; Kasarda, Mendoza, Kirk and Wicks, 2004; Shi, Zmood and Qin, 2004]; vibration suppression and attenuation [Knospe and Tamer, 1997; Cole, Keogh and Burrows, 1998; Keogh, Cole and Burrows, 2002; Johnson, Nascimento, Kasarda and Fuller, 2003; Yu, Lin and Chu, 2007]; tracking a sinusoidal reference in the axial direction [Minihan, Lei, Sun, Palazzolo, Kascak and Calvert, 2003]; and for active health monitoring of rotordynamic systems [Mani, Quinn and Kasarda, 2006], to name just a few.

Magnetic bearings are now moving beyond promise into actual service in such applications as turbomachinery, centrifuges, vacuum machinery, machine tool spindles, medical devices, robotics, high-speed drives, spacecraft equipment, contactless actuators and vibration isolation. Magnetic bearings are also used in highprecision instruments and to support equipment in a vacuum, for example in flywheel energy storage systems. A flywheel in a vacuum has very low windage losses, but conventional bearings usually fail quickly in a vacuum due to poor lubrication.

The stable operation of a magnetic bearing system can only be achieved by feedback control. Conceptually, a typical active magnetic bearing is composed of four basic components; sensor, controller, power amplifier, and electromagnetic actuators. Figure 1 shows a block diagram of a magnetic bearing system. In particular, the non-contact position sensor is used to measure the position of the shaft. Then the controller converts sensor signals to the control signals and the power amplifier supplies the required currents to each of the actuator coils. Finally, the electromagnets generate the suspension and operating forces.

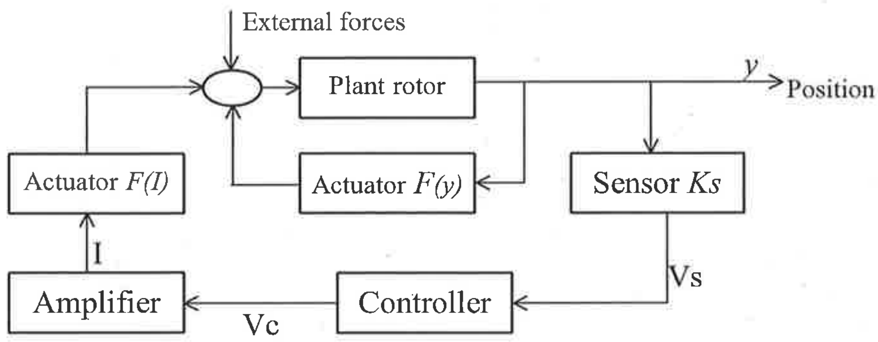

Figure 1 Block diagram of a simple magnetic bearing system

Figure 2 shows a single-degree-of-freedom magnetic system with a pair of opposed electromagnets (which is also called a two-pole magnetic bearing in the literature) in combination to provide magnetic attractive forces, where I_1 and I_2 represent the currents flowing in the coils, g_0 denotes the nominal air gap between the rotor and electromagnets, and x designates the displacement of the geometrical centre of the rotor from the centre of magnetic bearing. This simple model, without unnecessary complexity, represents a fundamental structure for many more complicated magnetic bearings. As a result, based on this model many researchers have designed control systems and self-sensing magnetic bearings as well as studied the stability and dynamics of simple rotor-bearing systems.

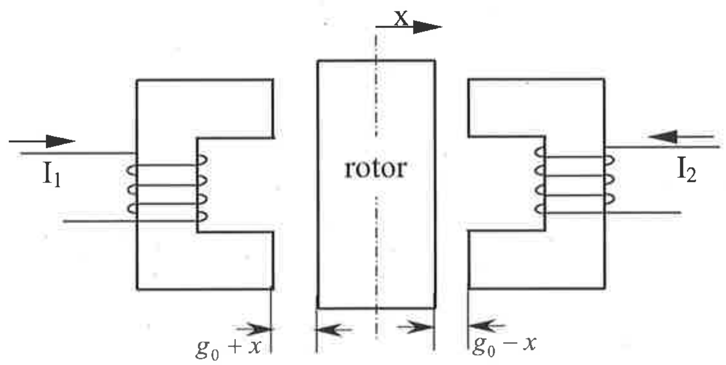

Figure 2 Schematic of the two-electromagnet magnetic bearing

A rotating machine with active magnetic bearings is commonly equipped with conventional bearings as a backup support system in the event of failure of magnetic bearings. The backup support system is usually referred to as auxiliary bearings or backup bearings in the literature. The auxiliary bearing system is designed to provide machine protection in the unlikely event of an electronic failure and power failure, which would cause loss of magnetic support and subsequent rotor delevitation. The loss of the magnetic bearing function during operation may lead to either a transient or persistent contact event between the auxiliary bearings and the magnetically suspended rotor. Subsequent interactions of the rotor and auxiliary bearings through a clearance may significantly influence the behaviour of the rotor through producing very large amplitude vibrations and high instantaneous loads. An important concern in this regard is the dynamic behaviour of the rotor when it is in contact with the auxiliary bearing. A deep understanding of the dynamics of the rotor contacting auxiliary bearings is essential to help design better auxiliary bearings.

The research relevant to magnetic bearings has received considerable interest from both research groups and industry engineers, since the First International Symposium on Magnetic Bearing Systems was held in Zurich, Switzerland in 1988 [Schweitzer, 1988]. The biannual International Symposium on Magnetic Bearings has established itself as the main forum of communication for the community of magnetic bearing researchers and development engineers.

The literature on magnetic bearings is huge and diverse, primarily due to a wide variety of fields in both theoretical research and practical applications. Hundreds of papers appear every year in academic journals, conference proceedings and technical reports. By summarizing the topics of interest for past international symposiums on magnetic bearings, the currently active topics of research on magnetic bearings, which cover all the aspects of research and applications, can be classified by specific subjects including: active magnetic bearings; passive magnetic bearings; superconductor magnetic bearings; micro bearings; magnetic actuators; new sensing and control technology; industrialization, safety and reliability aspects; modelling and identification; field experiences and case studies; components and materials; self-bearing (bearing less) motors; self-sensing (sensor less) techniques; and application of magnetic bearings for vibration control and online diagnosis.

Active magnetic bearings may be currently perceived as reaching a mature state. Their applications are steadily increasing in number while new application fields are emerging. It is expected that new fields in research will continue to appear to keep track with increasing numbers of applications. The work outlined in thesis is concerned with the nonlinear dynamics and dynamic stability of magnetic bearing systems.

1.2 Research focus of the thesis

As shown in Figure 1, a magnetic bearing system is basically composed of sensors, controllers, amplifiers and electromagnets. All of these components are characterized by nonlinear behaviour and therefore the entire system is inherently nonlinear. The most important nonlinearities are:

1). The nonlinear magnetic force to displacement and force to coil current relationships (or nonlinear force to magnetic flux relationship) of the electromagnets;

2). The geometric coupling between different electromagnets, which results in coupling between different orthogonal coordinate directions;

3). The nonlinear saturation of the ferromagnetic core material, which results in a flattening of the magnetization curve;

4). The hysteresis of the magnetic core material;

5). The saturation of the power amplifier and the limitation of the control current, which are caused by technical limitations of the power amplifier;

6). The unavoidable time delays in the controller and actuators, especially when the control algorithm is implemented on a digital signal processor;

7). The nonlinearity and the noise of the sensor system;

8). The nonlinearity of the coil inductance; and

9). The eddy current effect, the leakage and fringing effect, and the nonlinear magnetization B-H curve.

Although active magnetic bearings are nonlinear in nature, and their nonlinear properties can lead to a dynamic behaviour of rotor motion that is totally different from that predicted by a linear model, investigation of the effects of the nonlinearities on the stability and dynamics as well as the performance of magnetic bearings has only recently received the attention of the research community around the world.

There is a strong need for nonlinear dynamic analysis of magnetic bearing systems, at least for two main reasons. First, a fundamental scientific investigation of the effect of nonlinearities on the dynamic behaviour of magnetic bearings can provide valuable insight into system characteristics under various operating conditions, and predict the complicated dynamic behaviour of the system. Second, a precise parametric model of magnetic bearings is required for the controller design. The success of any magnetic bearing is highly dependent on the design of the controller that is used to control it. In turn, the controller relies heavily on *a priori* knowledge of the system dynamics. If the system model is not precisely known, the controller, which is designed for a specific purpose or aimed at compensation of a specific component of the nonlinearities, may fail to meet the performance requirements for the practical system. The work outlined in this thesis addresses many technical issues concerning the dynamic behaviour of active magnetic bearings so that they can be optimally designed for reliable and stable operation.

The overall aim of the work outlined in this thesis was to gain a deeper insight into the effects of the inherent nonlinearities of magnetic bearings and the influence of unavoidable time delays occurring in feedback control path on the stability and dynamic behaviour as well as performance of magnetic bearing systems. Emphasis was on stability analysis and local bifurcation control as well as on all aspects of nonlinear dynamic behaviour including bifurcations, co-existence of multiple solutions and amplitude-modulated motions.

Specific aims of the research were to:

1). investigate the effects of geometric coupling, and nonlinear magnetic force to displacement and force to coil current relationships, on the dynamic behaviour of magnetic bearings;

2). investigate the effects of the nonlinear electromagnetic force to displacement and force to coil current relationship incorporating time delays of the feedback control system or saturation of the magnetic material on the dynamic behaviour and performance of magnetic bearings;

3). investigate the effects of a combination of three components of nonlinearities (i.e, geometric coupling, nonlinear force to magnetic flux, and time delays occurring in the feedback control path) on the stability and dynamics as well as the performance of magnetic bearings;

4). determine the critical values of time delays associated with the controller and actuators and the parameter regimes for normal operation of magnetic bearings;

5). analyse the nonlinear response of rotor-magnetic bearing systems under primary, sub-harmonic and super-harmonic resonance conditions, explore the parameter regions for an existence of multiple solutions, and investigate local and global bifurcations as well as periodically- and chaotically-amplitude modulated response of rotor-bearing systems;

6). develop a nonlinear control approach from the bifurcation control point of view to stabilize a subcritical Hopf bifurcation, thereby creating a stable motion.

Modern dynamical systems theory, perturbation methods, and numerical simulations were applied to analyse the resultant nonlinear modelling of magnetic bearing systems. Experiments were conducted to validate the theoretical predictions on the influence of time delays presented in PD feedback controller on the stability and dynamics of a Jeffcott rotor system with an additional magnetic bearing located at the central disk.

1.3 Papers included in the thesis

The present thesis comprises 12 papers published in seven international journals in the fields of vibration control and nonlinear dynamics. The 12 papers are divided into four groups and each group of three papers composes a chapter in terms of the relevant topics of the papers. Most of the papers were published in 2003 through 2005, and a paper published in 2001 is also included for the sake of completeness. The author of this thesis was the prime author on all papers and the originator of all of the work.

For brevity, the 12 published papers will be referred to here as Paper i.j, where the first digit "*i*" indicates the chapter number and the second digit "*j*" represents the number of the paper included in the chapter. The paper number also indicates the number of the section composed of the paper. In particular, the 12 papers and their corresponding short running headlines for the chapters are list below:

Chapter 4: Nonlinear oscillations and local bifurcation control

Paper 4.1—Primary resonance response;

Ji, J.C., Hansen, C.H., 2001, Nonlinear oscillations of a rotor in active magnetic bearings. *Journal of Sound and Vibration*, vol.240(4), pp.599-612.

Paper 4.2—Super-harmonic resonance response;

Ji, J.C., Leung, A.Y.T., 2003, Nonlinear oscillations of a rotor-magnetic bearing system under super-harmonic resonance conditions. *International Journal of Non-linear Mechanics*, vol.38(6), pp.829-835.

Paper 4.3—Local bifurcation control;

Ji, J.C., Hansen, C.H., 2003, Local bifurcation control in a rotor-magnetic bearing system. *International Journal of Bifurcation and Chaos,* vol.13(4), pp.951-956.

Chapter 5: Periodic and chaotic motions of one-DOF nonlinear systems with saturation constraints

Paper 5.1—Piecewise linear oscillator;

Ji, J.C., 2004, Dynamics of a piecewise linear system subjected to a saturation constraint. *Journal of Sound and Vibration*, vol.271(3-5), pp.905-920.

Paper 5.2 —Nonlinear oscillator: Resonant response;

Ji, J.C., Hansen, C.H., 2004, Analytical approximation of the primary resonance response of a periodically excited piecewise nonlinear-linear oscillator. *Journal of Sound and Vibration*, vol.278(1-2), pp.327-342.

Paper 5.3—Nonlinear oscillator: Non-resonant response;

Ji, J.C., Hansen, C.H., 2004, Approximate solutions and chaotic motions of a piecewise nonlinear-linear oscillator. *Chaos, Solitons and Fractals*, vol.20(5), pp.1121-1133.

Chapter 6: Stability and dynamics of one-DOF nonlinear systems with time delays

Paper 6.1—Stability associated with a PD controller;

Ji, J.C., 2003, Stability and Hopf bifurcation of a magnetic bearing system with time delays. *Journal of Sound and Vibration*, vol.259(4), pp.845-856.

Paper 6.2 —Dynamics of a Jeffcott rotor-magnetic bearing system;

Ji, J.C., 2003, Dynamics of a Jeffcott rotor-magnetic bearing system with time delays. *International Journal of Non-linear Mechanics*, vol.38(9), pp.1387-1401.

Paper 6.3 —Stability associated with a PID controller;

Ji, J.C., 2003, Stability and bifurcation in an electromechanical system with time delays. *Mechanics Research Communications*, vol.30(3), pp.217-225.

Chapter 7: Stability and forced response of a two-DOF nonlinear system with time delay

Paper 7.1—Stability of the system without external excitations;

Ji, J.C., Hansen, C.H., 2005, Hopf bifurcation of a magnetic bearing system with time delay. *Transactions of the ASME Journal of Vibration and Acoustics*, vol.127(4), pp.362-369.

Paper 7.2—Forced phase-locked response;

Ji, J.C., Hansen, C.H., 2005, Forced phase-locked response of a nonlinear system with time delay after Hopf bifurcation. *Chaos, Solitons and Fractals,* vol.25(2), pp.461-473.

Paper 7.3—Super-harmonic resonances and non-resonant response;

Ji, J.C., Hansen, C.H., Li, X.Y., 2005, Effect of external excitations on a nonlinear system with time delay. *Nonlinear Dynamics*, vol.41(4), pp.385-402.

1.4 Organization of the thesis

In addition to three sections entitled *Introduction*, *Literature review*, *Conclusion and future work*, the main body of this thesis is composed of four chapters, each contributing to a specific topic relevant to this thesis. Each chapter is made up of three published papers, which cover different aspects of the topic of the chapter.

In particular, this thesis is organized into eight chapters. Chapter 2 reviews the recent research and developments in the nonlinear dynamics of magnetic bearing systems with an emphasis on dynamic stability and forced nonlinear response. Chapter 3 briefly describes the linkages between the 12 papers that are involved in this thesis. Chapter 4 studies the nonlinear oscillations and local bifurcation control of rotor-magnetic bearing systems involving nonlinear magnetic forces. Periodic and chaotic motions of one-DOF nonlinear systems with saturation constraints are discussed in Chapter 5 for both piecewise linear and nonlinear cases of magnetic forces. Chapter 6 considers the stability and dynamics of magnetic bearing systems under a PD and PID controller. Hopf bifurcations and the forced nonlinear response of a two-DOF nonlinear system with time delay are investigated in Chapter 7, based on the reduction of the infinite dimensional problem to the flow on a four-dimensional centre manifold. Chapter 8, the concluding chapter, summarises and discusses the work which has been completed in this thesis and also includes recommendations for further research in the area of the nonlinear dynamics of magnetic bearing systems.

1.5 Contributions of the thesis

Through 12 journal publications, this thesis has made significant contributions to a deep understanding of the nonlinear dynamics of magnetic bearing systems and local bifurcation control of a simple magnetic bearing system using a linear-plus-nonlinear feedback. The research work advances existing studies on the nonlinear modelling and dynamics of magnetic bearing systems in at least the following seven aspects; (a) development of more adequate models of magnetic bearing systems by considering two or three components of their nonlinear properties; (b) the judicious application of advanced mathematical techniques including modern dynamical theory to solve the resultant highly nonlinear differential equations governing the dynamics of rotor-bearing systems; (c) exploration of the complicated behaviour of magnetic bearing systems including the coexistence of two or three stable solutions and chaotic motions; (d) development of a linear-plus-nonlinear feedback control technique to stabilise a sub-critical Hopf bifurcation; (e) a deep understanding of the effect of time delays occurring in the feedback control path on the stability and dynamics of a magnetic bearing system; (f) a deep understanding of the effects of system parameters on the nonlinear response of magnetic bearing systems; (g) investigation of the primary and super-harmonic resonance response of an imbalanced rotor supported by magnetic bearings.

The innovation and novelty that characterize the research outlined in the thesis are listed below in more detail:

1) Five different mathematical models focusing on either a specific component of nonlinearites or a combination of up to three components of nonlinear properties of magnetic bearing systems have been developed. In particular, in studying the effect of actuators' saturation on the system dynamics, the essential nonlinear magnetic force generated by a two-pole magnet is modelled first by a piecewise linear function of displacement and later by a nonlinear-linear function of displacement. As a result, a periodically excited piecewise linear oscillator and a periodically excited piecewise nonlinear-linear oscillator are developed to study the dynamics and approximate solutions of the rotor-magnetic bearing systems. In studying the nonlinear resonance response of rotor-magnetic bearing systems with essential nonlinear magnetic forces, a set of two-degree-of-freedom (two-DOF) weakly nonlinear differential equations is developed by considering the nonlinear magnetic force and geometric coordinate coupling between two directions in an eight-pole magnetic bearing. Primary resonance response and super-harmonic resonance response are then examined by studying the first-order approximate solutions of the resultant nonlinear equations. In studying the effect of time delays occurring in a PD feedback controller on the

dynamic stability of a rotor supported by a two-pole magnetic bearing, a one-degree-of-freedom (one-DOF) nonlinear equation with time delay is developed in Chapter 6. In Chapter 7, a set of two-DOF nonlinear differential equations with time delay coupling in the nonlinear terms is developed to further study the influence of a time delay occurring in a PD feedback controller on the dynamic stability of a rotor supported by four-pole magnetic bearings. The developed model accounts for geometric coupling and nonlinear magnetic force as well as time delay. The analytical predictions and numerical results based on the developed models have provided a deep understanding of the nonlinear dynamics and stability of magnetic bearing systems. Some of the developed models have been accepted by the research community. For example, adopting the modelling of the effect of time delay on stability and dynamics that is developed in Chapter 6, Wang *et al* further studied the stability and bifurcation of a magnetic bearing system with time delay [Wang and Liu, 2005; Wang and Jiang, 2006].

2) Perturbation methods, modern dynamical system theory, bifurcation theory and numerical methods have been employed in combination to explore the nonlinear dynamic behaviour of magnetic bearing systems. By taking into account one or more components of nonlinear properties, the resulting mathematical modelling of the dynamics of magnetic bearing systems often incorporates highly nonlinear, non-autonomous differential equations. For these nonlinear dynamic systems, no exact analytical solutions can be found in the context of mathematics. Therefore, both approximately analytical and numerical solutions have been sought within the framework of nonlinear dynamics. Whenever applicable, the dynamic behaviour of magnetic bearing systems has been investigated at first option by suitable analytical

methods, since analytical methods can provide much more general information for the solutions and bifurcations than the numerical simulations. Then numerical simulations were employed to validate the analytical predictions, and to find periodic, quasi-periodic and chaotic responses of the rotor-magnetic bearing system.

3) An analytical procedure is developed for constructing an approximate solution to a periodically excited piecewise nonlinear-linear equation. The method involves combining an asymptotic expansion solution to the weakly nonlinear system and an exact solution to the linear system, then matching a set of continuity and periodicity conditions. The methodology developed can also be applicable to other types of non-smooth systems, which are characterized by different forms of equations of motion.

4) The effect of time delays occurring in the feedback control loop on the linear stability of a rotor supported by a two-pole magnetic bearing is investigated by analysing the associated characteristic transcendental equation. It is found that the trivial fixed point of the system can lose its stability through Hopf bifurcations when the time delay crosses certain critical values. Co-dimension two bifurcations are also found to be possible for the corresponding autonomous system. A Jeffcott rotor with an additional magnetic bearing located at the central disc is employed to investigate the effect of time delays on the nonlinear dynamic behaviour of the system. The primary resonance response is studied for its small nonlinear motions using the method of averaging. The effects of time delays and control gains as well as excitation amplitude on the amplitude of the steady-state response are investigated. Experiments are carried out to validate the theoretical predictions. The effect of time delays occurring in a proportional-integral-derivative (PID) feedback controller on

the linear stability of a two-pole magnetic bearing is also investigated by analysing the characteristic transcendental equation. It is found that codimension two bifurcations may result from non-resonant and resonant Hopf-Hopf bifurcation interactions.

5) The effect of time delays occurring in the feedback control path on the stability and dynamics of magnetic bearing systems has been further investigated by taking into account geometric coordinate coupling and nonlinear magnetic forces. It is found that as the time delay increases beyond a critical value, the equilibrium position of the rotor motion becomes unstable and may bifurcate into two qualitatively different types of periodic motion. The resultant Hopf bifurcation is associated with two coincident pairs of complex conjugate eigenvalues crossing the imaginary axis. Based on the reduction of the infinite dimensional problem to the flow on a four-dimensional centre manifold, the bifurcating periodic solutions are investigated using a perturbation method. An interaction between the Hopf bifurcating solutions and the periodic excitations is also studied in terms of primary resonance response, non-resonant response and secondary-resonance response.

6) Primary and super-harmonic resonance responses of magnetic bearing systems are investigated by using the method of multiple scales. It is shown that the steady state solutions may lose their stability by either a saddle-node bifurcation or Hopf bifurcation. The system exhibits typical dynamic behaviour of nonlinear systems including multiple coexisting solutions, jump phenomena, and sensitive dependence on initial conditions. Two or three stable solutions are found to be in co-existence in some combinations of system parameters.

7) A linear-plus-nonlinear feedback control scheme has been designed to stabilize the subcritical Hopf bifurcation in a rotor-magnetic bearing system, for which the linearized system possesses double zero eigenvalues. The addition of nonlinear terms to the original linear feedback control formulation is used to modify the coefficients of the nonlinear terms in the reduced normal forms. This type of local bifurcation control results in an extension of the stable operating regime, an enlargement of the attraction domain and performance improvement of magnetic bearings.

In brief summary, the work outlined in this thesis has provided considerable insight into the effects of nonlinearities and time delays on the nonlinear dynamic behaviour and stability of magnetic bearings by analysing their nonlinear dynamics using modern dynamical theory such as the centre manifold theory and the normal form method. The results of this research offer useful information for the design of magnetic bearings and the prediction of the nonlinear dynamic behaviour of rotor-magnetic bearing systems.

Chapter 2

Literature review

Since the First International Symposium on Magnetic Bearings was held in 1988 [Schweitzer, 1988], considerable research has been conducted to study all the aspects of magnetic bearings and their potential applications, which include dynamic modelling of magnetic bearing systems, nonlinear dynamics, feedback controller design, fault-tolerant control design, design of electromagnetic actuators, self-sensing magnetic bearings, nonlinear vibrations of a rotor contacting auxiliary bearings, and utilization of magnetic bearings for vibration control and online diagnosis as well as for identification of vibrational features from measurements. The literature relevant to magnetic bearings is huge and diverse primarily due to a wide variety of research topics arising from the aspects relevant to mechanical engineering, electrical engineering, electromagnetics and applied mathematics. Hundreds of papers in the area of magnetic bearings, including theoretical research and practical applications, appear every year in academic journals, conference proceedings and technical reports. However, there are still many technical issues remaining fully or partially unsolved. Nonlinear dynamics of magnetic bearing systems will be remaining an active topic of future research. This chapter will not cover all the aspects of research and applications relevant to magnetic bearings. For instance, there is no attempt to summarize many of the development originating in sensing and control system technology. The design of new magnetic bearings will not be discussed here either. Nevertheless, this chapter attempts to summarise the recent research on the nonlinear dynamics of magnetic bearings. Emphasis is given on the nonlinear dynamic behaviour and stability of a rotor supported by magnetic bearings in the presence of single or multiple components of nonlinearities which inherently exist in magnetic bearing systems. In an effort to make this literature review comprehensive and accessible to a wide audience, this review also provides background information on analytical methods, nonlinear vibrations resulting from a rotor contacting auxiliary bearings, and other active topics of research involving the nonlinear properties of magnetic bearing systems such as nonlinear self-sensing magnetic bearings and nonlinear control of magnetic bearings.

The writing of this chapter has proved to be a difficult task in part because the literature on magnetic bearings is growing rapidly every year and appearing in a large number of international journals and conferences, and because the subject has an interdisciplinary nature covering the mechanical engineering, electrical engineering and applied mathematics. Even in the context of the nonlinear dynamics of magnetic bearing systems, classification of published studies is also a formidable task, as this classification is inevitably biased toward the area which the present thesis focuses on. The present review is categorized into five main groups based on the research focuses and nonlinear modelling of rotor-magnetic bearing systems in existing studies, which are nonlinear vibrations of a rotor contacting auxiliary bearings; nonlinear dynamics of one-degree-of-freedom (one-DOF) nonlinear rotor-bearing systems; nonlinear dynamics of two-DOF nonlinear rotor-bearing systems; stability and dynamics of rotor-bearing systems with time delays; and other issues relevant to nonlinear magnetic bearing systems. Note that all the papers cited in this thesis were written in English. Some papers have been left out of this literature review because of their unavailability in preparation of this review.

The remaining part of this chapter is organized as follows: Section 1 briefly describes the analytical methods that have been used in analysis of the nonlinear dynamics of magnetic bearing systems. Section 2 briefly reviews the nonlinear vibrations of a rotor contacting auxiliary bearings that are essential to design backup bearing systems. Section 3 reviews the nonlinear dynamics of simple rotor-magnetic bearing systems for which the equations of motion are mathematically modelled by one-DOF nonlinear systems. Section 4 reviews the nonlinear dynamics of magnetic bearing systems whose mathematical modelling is governed by a set of two-DOF nonlinear systems. The effect of time delays on the stability and dynamics of rotor-magnetic bearing systems is discussed in Section 5. Section 6 concludes the chapter by presenting two emerging topics of research dealing with the nonlinear properties of magnetic bearings, which are nonlinear self-sensing magnetic bearings and nonlinear control of magnetic bearings.

2.1 Analytical methods

Analysis of dynamic behaviour has always been an important aspect in the design and assessment of rotor-bearing systems. Nonlinear rotor motion in rotating machinery is commonly caused by the nonlinear characteristics of supporting bearings. The bearings could be either conventional mechanical bearings (such as ball, journal, or fluid-film bearings) or magnetic bearings. Generally speaking, rotor-bearing systems may exhibit periodic, sub-harmonic and chaotic behaviour as well as period-doubling and Hopf bifurcations. For a rotor supported by rolling bearings, the nonlinear behaviour of the resultant rotor-bearing system results mainly from nonlinear Hertzian contact force, bearing clearances, and surface waviness [Harsha, 2006]. For a rotor supported by fluid film bearings, nonlinear hydrodynamic forces are a primary source of vibration and introduce the nonlinear dynamic behaviour of rotor motion [Shen et al, 2006].

For a rotor suspended by magnetic bearings, the nonlinear oscillations of rotor motion may result from either accidental contacts between the rotor and auxiliary bearings or the inherent nonlinearities of magnetic bearing systems. The clearance between the rotor and the inner race of the auxiliary bearing introduces a nonlinear dynamic feature after magnetic bearings fail. As discussed in Chapter 1, the inherently nonlinear properties of magnetic bearing systems are different from those of conventional bearings in terms of types and characteristics partially because the nonlinear magnetic forces are dependent on control currents or voltages (i.e. magnetic flux).

The characteristics of magnetic bearings are inherently nonlinear due to the prevalent nonlinearities. In a linear case study, the electromagnetic force is linearized about the operating point and considered to be a linear function of currents and air-gaps. The linearized magnetic forces may alternatively be expressed in terms of spring stiffness and damping, such as given by Tonoli and Bornemann [1998]; Kim and Lee [1999]; Ho, Yu and Liu [1999]; and Peel, Bringham and Howe [2000]. However, the linear relationship holds only locally and the linear behaviour of rotor motion can be achieved approximately only in small rotor deflections and small control currents. If the rotor deflections exceed half the gap the net magnetic force of an opposite pair of electromagnets differs more than 44% from its linear approximation [Skricka and

Markert, 2002]. Consequently, the performance of magnetic bearings may suffer rapid deterioration when the operation deviates from the equilibrium point. In practice, the nonlinear characteristics become quite significant for large control currents, large magnetic forces as well as small air-gaps [Chang and Tung, 1998; Kasarda, Marshall and Prins, 2007; Alasty and Shabani, 2006]. In order to fully utilize the capacity of a magnetic bearing, a nonlinear analysis is required to explore the nonlinear behaviour of a rotor suspended by magnetic bearing systems.

In comparison with the research on the nonlinear dynamic behaviour of a rotor supported by conventional mechanical bearings (which are either rolling element bearings or fluid-film bearings), research on the nonlinear dynamic behaviour of a rotor supported by magnetic bearings is far from intensive, mainly because the application of this new bearing technology is less extensive than the conventional mechanical bearings, though the use of magnetic bearings for turbomachinery has experienced substantial growth since the First International Symposium on Magnetic Bearings was held in 1988 [Schweitzer, 1988].

Fortunately, the growing engineering requirements of nonlinear analysis of the dynamic behaviour of magnetic bearing systems are paralleled by a notable advance in dynamical systems theory [Guckenheimer and Holmes, 1983; Wiggins, 1990], which permitted discovery, analysis and successive detection of several typical phenomena of the nonlinear range playing a fundamental role in the behaviour of many mechanical systems. Phrases like nonlinear resonances, bifurcations and chaos are by now well understood by many researchers in the community of mechanical engineering. Indeed, the application of modern dynamical systems theory to nonlinear mechanical systems represents one important achievement of recent years and a well-defined research field.

In modelling rotor-magnetic bearing systems in the presence of nonlinearity, the equations of motion governing the response of the suspended rotor are usually characterised by a set of either one- or two-DOF nonlinear differential equations with quadratic and cubic terms. The closed form of the solutions to these nonlinear differential equations cannot be found analytically, therefore either numerical integration solutions or approximate solutions obtained by using a perturbation method have been sought to study the nonlinear response of magnetic bearing systems.

In solving nonlinear differential equations, numerical integration schemes such as Runge-Kutta algorithm are commonly used to find periodic, quasi-periodic and chaotic solutions. While a numerical method such as the continuation method may be used to obtain unstable solutions.

There are currently many perturbation methods available for finding approximate periodic solutions for nonlinear systems. These asymptotic perturbation techniques include the averaging method [Hale, 1971], the method of multiple scales [Nayfeh and Mook, 1979], the harmonic balance method [Kim and Choi, 1997], the trigonometric collocation method [Chinta and Palazzolo, 1998], and an asymptotic perturbation method incorporating the harmonic balance method and the method of multiple scales [Maccari, 1998a; 1998b; 2000]. The perturbation method is employed to obtain a set of two or four averaged equations that determine the amplitudes and phases of the forced response of the rotor motion under primary resonances or secondary resonances. Floquet theory is utilized to study the local stability of periodic solutions [Hayashi, 1964; Rudiger, 1994]. Usually, a perturbation analysis is carried out up to the first-order approximation if the nonlinear systems involve cubic nonlinear terms only, since the higher-order approximate terms do not influence on the qualitative behaviour of the asymptotic solutions. On the other hand, if the systems involve both quadratic and cubic nonlinearities, the second-order approximate solutions are sought, as the quadratic nonlinearities cannot appear in the first-order approximate solutions.

In studying the effect of time delays on the dynamics and stability of rotor motion, the dynamic response of the rotor is mathematically modelled by either one- or two-DOF nonlinear differential equations with time delays. Such systems are usually referred to as the functional differential equations in the context of mathematics [Halanay, 1966; Hale, 1977; Hale and Verduyn Lunel, 1983]. The decomposition theory and centre manifold theorem are used to perform a reduction of an infinite dimensional nonlinear system to a set of two- or four-dimensional ordinary differential equations. Then a perturbation method is used to explore the bifurcating solutions and forced response of the system in the neighbourhood of Hopf bifurcations.

2.2 Nonlinear vibrations of a rotor contacting auxiliary bearings

In order to prevent physical interaction between the rotor and stator laminations of magnetic bearings and to provide rotor support in the event of bearing failure or during an overload situation, auxiliary bearings are usually incorporated a component of radial magnetic bearing systems. These auxiliary bearings also allow the rotor to temporarily run or safely coast down to a stop for maintenance purposes. The auxiliary bearings are also called "safety touch-down bearings", "back-up bearings" or "catcher bearings" in the literature. These bearings are usually ball bearings or carbon sleeves located on the stator. The clearance between the inner race of backup bearings and the rotor shaft is usually in the order of half the magnetic bearing clearance.

The loss of the magnetic bearing function during operation may lead to either a transient or persistent contact event between the auxiliary bearings and the magnetically suspended rotor. Subsequent interactions of the rotor and auxiliary bearings may significantly influence the behaviour of the rotor through producing very large amplitude vibrations and high instantaneous loads, even if the duration of contacts may be relatively short. In many applications such as space applications, safety is a major concern in the design of a rotor-magnetic bearing system. The rotor-bearing system is required to extend the operation of the rotor on auxiliary bearings by taking the maximum advantage of backup bearings and using backup bearings as true auxiliary bearings to provide support during critical situations in a safe manner. A deep understanding of the dynamics of the rotor drop phenomena is essential to help design better auxiliary bearings.

There are a number of theoretical and experimental studies in the literature concerned with the dynamics of rotors when they are in contact with the auxiliary bearings. These studies mainly focused on characterizing the transient response to determine the effects of the various bearing parameters, in particular friction and damping coefficients as well as stiffness.

Gelin et al. [1990] studied the transient dynamic behaviour of rotors landing on auxiliary bearings equipped in an industrial centrifugal compressor, while a Coulomb friction contact force was ignored in their numerical model. Ishii and Kirk [1991] and Kirk and Ishii [1993] developed a transient response technique for predicting the transient response during the rotor drop for a simple two-mass Jeffcott rotor system after magnetic bearings become inactive. They showed that an optimum damping could be chosen to prevent destructive backward whirl. Through experimental and theoretical investigation, Schmied and Pradetto [1992] reported on the vibration behaviour of a one-ton compressor rotor being dropped into the auxiliary bearings after magnetic bearings fail. Fumagalli et al. [1994] classified the touchdown process into four distinct phases of motion—free fall, impact, sliding and rolling—and investigated the influences of such parameters as air gap, friction coefficients and damping on the impact dynamics. Schweitzer et al. [1994] presented a good discussion of issues related to the touch-down dynamics of rotors on auxiliary bearings. Feeny [1994] explored the stability of cylindrical and conical whirls in a perfectly balanced and rigid rotor on rigid retainer bearings. Xie and Flowers [1994] presented a study on the steady state behaviour of a rotor-auxiliary bearing system and reported on its complex dynamic behaviour. Kirk et al. [1994a; 1994b] performed experimental rotor drop tests for balanced and unbalanced conditions, and developed a finite element code for the rotor and bearing system to perform stability analysis and unbalance response. Swanson, Kirk and Wang [1995] discussed experimental data for magnetically supported rotor drop initial transient response on ball and solid auxiliary bearings. Maslen and Barrett [1995] derived whirl conditions of a circularly isotropic rotor and catcher bearing support along with the test results of a commercial compressor rotor with bearings. Tessier [1997] described the development and delevitation tests of a flexible compressor rotor. Foiles and Allaire [1997] presented the nonlinear transient modelling of rotors during rotor drop on auxiliary bearings for two types of rotors; generator or turbine rotor and centrifugal compressor rotor. Chen et al. [1997] introduced the zero clearance auxiliary bearing. Experiments showed that the possibility of a backward whirl of a rotor could be reduced due to clearance elimination and damping. Ecker [1997] presented the steady state numerical results for a rigid rotor with imbalance on a catcher bearing fixed to the bearing housing.

Wang and Noah [1998] studied the dynamic response of a rotor landed on catcher bearings in a magnetically supported rotor, following postulated loss of power or overload of active magnetic bearings. They constructed an analytical model involving a disk, a shaft and auxiliary bearings on damped flexible supports and developed appropriate equations of the nonlinear dynamic system. The equations include a switch function to indicate contact or non-contact events and determine the existence of contact normal forces and tangential friction forces between the shaft and bearings. The fixed point algorithm (the shooting method) was used to obtain steady state periodic solutions of the unbalanced rotor for various parameters. It was observed that friction forces could cause both periodic and quasi-periodic large amplitude, full backward whirling. It was found that steady-state, periodic, quasi-periodic and chaotic co-existing solutions may occur for a given range of system parameters. The side forces tend to cause noncircular orbits and the rotor response becomes entangled displaying more complex patterns. Xie, Flowers, Feng and Lawrence [1999] used the harmonic balance method and direct numerical integration to study the steady-state responses of a rotor system supported by auxiliary bearings with a clearance. They discussed the influence of rotor imbalance, clearance, support stiffness and damping on the steady-state behaviour of the rotor motion. Bifurcation diagrams were used as a tool to examine the dynamic behaviour of the system as a function of the system parameters. They suggested that auxiliary bearing with low clearance, low support stiffness and high support damping tend to reject the development of multi-frequency and chaotic behaviour and provide the most favourable rotordynamic behaviour. By summarizing a number of studies concerned with auxiliary bearings, Kirk [1999] reviewed analytical techniques to predict rotor transient response and presented results of transient response evaluation of a full-size compressor rotor to illustrate some of the important parameters in the design for rotor drop.

Recently, Ji and Yu [2000] investigated the transient nonlinear dynamics of a high-speed unbalanced rigid rotor dropping onto rigid sliding bearings. They numerically studied the dynamics of the rotor in different regimes of the touchdown process—free fall, impact, sliding and rolling—and examined the influences of system parameters such as unbalance, air gap, coefficient of friction, and coefficient of restitution on the drop dynamics of the rotor. It was shown that when the unbalance is small, the resulting motion is also small. As the level of unbalance increases, the motion of the rotor becomes larger, so there is potential for damage to the rotor and backup bearings. Zeng [2002; 2003] studied numerically and experimentally the transient response of the rotor motion during the rotor drop when the rotor is supported by backup bearings. It was shown that the nonlinear rotor-backup bearing system would undergo irregular or chaotic motion at some rotating speeds. Under some conditions, the full clearance whirl motion of the rotor in backup bearings may happen, which may lead to damage to magnetic bearing systems. It was shown that the parameters of the backup device would be used to regulate the nonlinear resonances. Suitably large support damping, soft support stiffness and heavy support device mass could reduce the nonlinear resonance and hence avoid full clearance whirl motion of the rotor.

Sun, Palazzolo, Provenza and Montague [2004] presented numerical simulations of a rotor drop on catcher bearings in flywheel energy storage system. They developed a catcher bearing model which includes a Hertzian load-deflection relationship between mechanical contacts, speed-and-preload-dependent bearing stiffness due to centrifugal force, and a Palmgren's drag friction torque. The numerical results showed that friction coefficients, support damping and side loads are critical parameters to satisfy catcher bearing design objectives and prevent backward whirl. Later, Sun [2006a] presented numerical analysis for a rotor drop on catcher bearings and following thermal growths due to their mechanical rub using detailed catcher bearing and damper models. The catcher bearing model was determined based on its material, geometry, speed and preload using the nonlinear Hertzian load-detection formula. The thermal growths of bearing components during the rotor drop were approximated by using a one-dimensional thermal model. Then, Sun [2006b] predicted an estimated fatigue life of catcher bearing based on the Hertzian contact dynamic loads between bearing ball and races during touchdown. Numerical simulations for an energy-storage flywheel module revealed that a high-speed backward whirl significantly reduces the catcher bearing life and that an optimal damping lowers the catcher bearing temperature and increases the catcher bearing life.

It has been shown that most of studies have been performed from the perspective that the rotor will be shut down if one magnetic bearing fails. As a result, most of work has concentrated on the transient dynamic behaviour immediately following the failure of magnetic bearing. However, rotor mass loss, base excited motions and other abnormal operating conditions may lead to transient rotor motion of large amplitude and rotor-auxiliary bearing contacts, even if the magnetic bearing system continues to function. To actively return the rotor to a non-contacting state it is essential to determine the manner in which contact events affect the rotor vibration signals used for position control. Toward this aim, Keogh and Cole [2003a] developed an analytical procedure to assess the nature of rotor contact modes under idealized contacts when a magnetic bearing still retains a full control capability. Nonlinearities arising from contact and magnetic bearing forces were included in simulation studies involving rigid and flexible rotors to predict rotor response and evaluate rotor synchronous vibration components. It was shown that changes in synchronous vibration amplitude and phase induced by contact events cause existing controllers to be ineffective in attenuating rotor displacements. The widely used family of synchronous vibration controllers were found to be ineffective when persistent auxiliary bearing contact occurs. The findings were then used as a foundation for design of new controllers that are able to recover rotor position control under a range of contact cases. As such, Cole and Keogh [2003b] developed a method for robust control of synchronous vibration components that can maintain dynamic stability during interactions between the rotor and auxiliary bearings. The controllers were designed to minimize the severity and duration of contact and ensure that the rotor vibration returns to optimal levels, provided that sufficient control force capacity is available.

2.3 One-DOF nonlinear rotor-bearing systems

Active magnetic bearings use magnetic forces to support various machine components [Schwitzer, Bleuler and Traxler, 1994]. An active magnetic bearing consists of an electromagnetic actuator, position sensor, power amplifiers, and a feedback controller. Each actuator is composed of ferromagnetic component attached to the rotor and opposing pairs of stationary electromagnets (known as the stator). In simulations of the dynamic behaviour of magnetic bearing systems, nonlinearities were usually neglected for simplicity and the components of magnetic bearing systems were simplified to linear models. However, the nonlinear properties of magnetic bearings can lead to a behaviour of the rotor-bearing system that is distinctly different from that predicted using a simple linear model. It has been shown that the standard linear magnetic bearing model derived from linearization was too imprecise for the control design [Loesh, 2001]. Special attention is required to account for the nonlinear effects.

At an early stage of analysis, a two-pole, single-degree-of-freedom (single-DOF) magnetic bearing system has been used to study the nonlinear dynamics and stability of rotor motion. This simple model, as shown in Figure 2 of Chapter 1, without unnecessary complexity, represents a fundamental structure for many more complicated magnetic bearings. The equations of motion governing the dynamics of the magnetically suspended rotor by a two-pole magnetic bearing are mathematically modelled by one-DOF nonlinear systems. There are a number of studies in the literature that partially dealt with the problem of the nonlinear modelling and nonlinear dynamics of magnetic bearing systems by using one-DOF nonlinear systems.

Mohamed and Emad [1993] used numerical method to analyse the nonlinear oscillation of a rigid rotor in two radial active magnetic bearings. It was shown that the system undergoes Hopf bifurcation due to unstable periodic motion. However, only nonlinear force to magnetic flux relationship was considered along with the rotor gyroscopic effects, while other nonlinearities such as geometric coupling, hysteresis and saturation of magnetic material, and time delays of the feedback controller were neglected.

Laier and Markert [1995] carried out numerical simulation of nonlinear effects on magnetically suspended rotors. Jump phenomena were found to occur in the system. Springer, Schlager and Platter [1998] developed a nonlinear model including nonlinear magnetic force and magnetic saturation for radial magnetic bearing actuators. The transient vibration caused by impact forces was analysed using a numerical integration procedure. However, they did not consider geometric coupling, time delays occurring in the control system and limitations of the power amplifier and control current. Steinschaden and Springer [1999a] developed a simple nonlinear model containing only the nonlinear force to displacement and force to coil current relationship to investigate the dynamic behaviour of a radial active magnetic bearing system. It was shown that the simple model could exhibit symmetry breaking and period doubling bifurcations. However, the other important components of nonlinearity such as geometric coupling, hysteresis and saturation of the magnetic material, and time delays of the control system were neglected. Later, Steinschaden and Springer [1999b] studied the effects of saturation of the proportional-integral-derivative (PID) controller output and nonlinear magnetic force on the dynamic characteristics of a single-mass rotor by using numerical simulation. It was found that symmetry breaking and quasi-periodic solutions might take place for specific parameter sets. However, they did not take the geometric coupling and time delays of control system into account. Ji, Yu and Leung [2000] studied the bifurcation of rotor motion in the horizontal and vertical directions near the degenerate point of double–zero eigenvalue by using normal form method. The nonlinear magnetic force was expanded about the equilibrium point into a Taylor series of up to the third-order. It was shown that the vibratory behaviour in the vertical direction could be reduced on the centre manifold to the Bogdanov-Takens form. Saddle-node, Hopf and saddle-connection bifurcations were found in the reduced normal form equations. However, other nonlinearities such as geometric coupling, saturation of the power amplifier, hysteresis and saturation of the magnetic material, and time delays of the control system were neglected.

Ji [2004] developed a periodically forced single-DOF piecewise linear system subjected to a saturation constraint to study the dynamics of a rotor supported by a two-pole magnetic bearing with a proportional feedback control, in which the actuator is subject to saturation constraints. He simplified the magnetic force to be of a piecewise linear characteristic and focused on the determination of periodic motion with a doubly-entering saturation region per cycle. Symmetric period-one solutions were derived analytically and their stability characteristics were determined. Other kinds of solutions such as asymmetric, subharmonic and chaotic solutions as well as multiply-crossing saturation region per cycle periodic solutions, were found through numerical simulations to exist in the forced response of the system. Ji and Hansen [2004a] constructed an analytical approximate solution for the primary resonance response of a periodically excited nonlinear oscillator, which is characterized by a combination of a weakly nonlinear and a linear differential equation. The model was derived based on the nonlinear magnetic force of a two-pole magnetic bearing subjected to saturation constraints. Without eliminating the secular terms, a valid asymptotic expansion solution for the weakly nonlinear equation was analytically determined for the case of primary resonances. Then, a symmetric periodic solution for the overall system was obtained by imposing continuity and matching conditions. The stability characteristic of the symmetric periodic solution was examined by investigating the asymptotic behaviour of perturbations to the steady state solution. Later, Ji and Hansen [2004b] analytically constructed a global symmetric period-1 approximate solution for the non-resonant periodic response of a periodically excited piecewise nonlinear-linear oscillator. The approximate solutions were found to be in good agreement with the exact solutions that were obtained from the numerical integration of the original equations. In addition, the dynamic behaviour of the oscillator was numerically investigated with the help of bifurcation diagrams, Lyapunov exponents, Poincare maps, phase portraits and basins of attraction. The existence of sub-harmonic and chaotic motions and the coexistence of four attractors were observed for some combinations of the system parameters. Ji and Hansen [2005] applied a matching method and a modified averaging method to construct an approximate solution for the super-harmonic resonance response of a periodically excited nonlinear oscillator with a piecewise nonlinear-linear characteristic. The validity of the developed analysis was confirmed by comparing the approximate solutions with the results of direct numerical integration of the original equation.

2.4 Two-DOF nonlinear rotor-bearing systems

The aforementioned studies in Section 2.3 have greatly enhanced the understanding of the nonlinear dynamics of rotors supported by a two-pole magnetic bearing in presence of single or double components of nonlinearities. However, from a practical perspective, an advanced model to account for the geometric coordinate coupling appears to provide more appropriate results for nonlinear analysis of more complicated magnetic bearing systems such as four-pole pairs or eight-pole pairs magnetic bearings. The nonlinear modelling of rotor-magnetic bearing systems will be a two-DOF nonlinear system when the geometric co-ordinate coupling is taken into account.

Virgin, Walsh and Knight [1995] studied the effect of coordinate coupling due to the geometry of the pole arrangement on dynamic behaviour. Multiple coexisting solutions and fractal boundaries were obtained. However, they neglected all other important components of nonlinearities such as nonlinear magnetic force to displacement and force to coil current relationships, hysteresis and saturation of magnetic material, time delays of the control system, and limitations of the power amplifiers and control current. The effect of cross-coupling and nonlinear force to displacement and force to coil current relationships on the dynamics of a single-mass rotor-magnetic bearing system was numerically investigated by Chinta, Palazzolo and Kascak [1996]. Stable quasiperiodic vibration and period-two solutions were found. Unfortunately, they employed a very simple four-pole magnetic bearing model and neglected other important nonlinearities such as saturation of magnetic material, time delays of the control system, and limitations of the power amplifier and control current. Later, Chinta and Palazzolo [1998] derived the equations of motion of a two-DOF mass in a magnetic bearing with geometric coupling between the horizontal and vertical components of rotor motion. They studied the nonlinear forced response by using both imbalance force and non-imbalance harmonic force. Stable periodic motion of the forced response was obtained by numerical integration and the approximate method of trigonometric collocation, while the unstable motion was obtained by the collocation method. The local stability of periodic motions and bifurcation behaviour were obtained by Floquet theory. The system parameters such as rotor speed, imbalance eccentricity, forcing amplitude, rotor weight and geometric coupling were investigated to find regimes of nonlinear behaviour such as jumps and sub-harmonic motion. It was found that the motion of a rotor in magnetic bearing may undergo cyclic-fold bifurcation (saddle-node bifurcation) with an increase of the forcing amplitude and undergo period-doubling bifurcation with an increase of frequency. The period-two and period-four subharmonic motions were also found to exist in the forced response.

Ji and Leung [2000] studied the primary resonance response of a rigid rotor-magnetic bearing system by using a perturbation method. It was shown that the steady state response became unstable either via saddle-node bifurcations or via Hopf bifurcations. Ji and Hansen [2001] investigated the nonlinear response of a rotor supported by active magnetic bearings under both primary and internal resonances. The method of multiple scales was used to obtain four averaged equations that describe the modulation of the amplitudes and phases of vibrations in the horizontal and vertical directions. It was shown that the steady state solutions may lose their stability by either saddle-node bifurcations or Hopf bifurcations. In the regime of multiple coexisting solutions, two stable solutions were found. However, they did not consider saturation and hysteresis of the magnetic material, time delays of the control system, and limitations of the power amplifier and control current. Later, based on the model that was developed by Ji and Hansen [2001], Ji and Leung [2003] studied the super-harmonic resonance response of the rigid rotor-magnetic bearing system. It was shown that the steady-state superharmonic periodic solutions may lose their stability by either saddle-node or Hopf bifurcations. The system may exhibit many typical characteristics of the behaviour of nonlinear dynamical systems such as multiple coexisting solutions, jump phenomena and sensitive dependence on initial conditions. The effects of the feedback gains and imbalance eccentricity on the nonlinear response of the system were studied. Ho, Liu and Yu [2003] studied the effect of a thrust active magnetic bearing on the stability and bifurcation of a rotor-magnetic bearing rotor system using a component mode synthesis method. They focused on the influence of nonlinearities on the stability and bifurcation of periodic motion of the rotor-bearing system subjected to the influences of both journal and thrust magnetic bearings and mass eccentricity. The periodic motions and their stability margins were obtained by using the shooting method and path-following technique. It was found that the thrust magnetic bearing and mass eccentricity of rotor may cause the spillover of system nonlinear dynamics and degradation of the stability and bifurcation of periodic motion.

By using the asymptotic perturbation method, Zhang and Zhan [2005] investigated nonlinear oscillations and chaotic dynamics of a rotor-magnetic bearing system with eight-pole legs and the time-varying stiffness. The stiffness of magnetic bearings was assumed to be the time varying in a periodic form. The resulting dimensionless equations of motion for the rotor-magnetic bearing system with the time-varying stiffness in the horizontal and vertical directions were a two-DOF nonlinear system with quadratic and cubic nonlinearities and parametric excitation. The asymptotic perturbation method was used to obtain the averaged equations in the case of primary parametric resonance and subharmonic resonance. It was found that there existed period-3, period-4, period-6, period-7, period-8, quasiperiodic and chaotic oscillations in the rotor-magnetic bearing system with the time-varying stiffness. The numerical results explored the phenomena of multiple solutions and the soft-spring type and the hardening-spring type in the nonlinear frequency-response curves for the rotor-magnetic bearing system. Zhang, Yao and Zhan [2006] then numerically investigated the Shinikov type multi-pulse chaotic dynamics for the rotor-magnetic bearing system, based on the same model developed in Zhang and Zhan [2005]. A new jumping phenomenon was shown to exist in the forced response of the rotor-magnetic bearing system with the time-varying stiffness.

Amer and Hegazy [2007] studied the nonlinear dynamic behaviour of a rigid rotor supported by active magnetic bearings without gyroscopic effects. The vibration of the rotor was modelled by coupled second-order nonlinear ordinary differential equations with quadratic and cubic nonlinearities. The steady-state response and stability of the system were studied numerically by applying the method of multiple scales and the frequency response function method. Different shapes of chaotic motion were found to exist by using phase-plane method. The system parameters were shown to have different effects on the nonlinear response of the rotor. Multiple-valued solutions, jump phenomena, hardening and softening nonlinearity were found to occur in steady-state response. Inayat-Hussain [2007] numerically investigated the response of an imbalanced rigid rotor supported by active magnetic bearings. Nonlinearity arising from electromagnetic force-coil current and force-air gap relationships, and the effects of geometrical cross-coupling were incorporated in the mathematical model of the rotor-bearing system. The response of the rotor was observed to exhibit a rich variety of dynamic behaviour including synchronous, sub-synchronous, quasi-periodic and chaotic vibrations. It was shown that the transition from synchronous rotor response to chaos was via a torus bearkdown route. With an increase of the rotor imbalance magnitude, the synchronous rotor response was found to undergo a secondary Hopf bifurcation resulting in quasi-periodic vibration.

2.5 Stability and dynamics associated with time delays

A magnetic bearing system is inherently unstable and thus a feedback control must be employed to stabilize the system. Time delays occurring in the feedback control loop are unavoidable especially in modern digital control systems, even though the control decision process is carried out very quickly. Time delays may have a profound impact on the stability and dynamics of a rotor-magnetic bearing system. There are two sources of time delays in the digital controller loop. First, the A/D and D/A conversions take time. The sample and hold devices introduce a delay of a half sampling period. The second source of delay is the controller computation. The amount of the computation delay depends upon how the inputs and outputs are synchronized in the controller implementation algorithm. Another source of time delay may be the reaction of the electromagnetic actuators to the control decisions.

Ji [2003a] investigated the effect of time delays occurring in a PID feedback controller on the linear stability of a balanced rotor supported by a two-pole magnetic bearing. It was found that the trivial fixed point of rotor motion may lose its stability through Hopf bifurcations when the time delay crosses certain critical values. Co-dimension two bifurcations resulting form non-resonant and resonant Hopf-Hopf interactions were also found to exist in the system. He [Ji, 2003b] also studied the effect of time delays occurring in the proportional-derivative (PD) feedback control loop on the linear stability of a simple magnetic bearing system by analysing the associated characteristic transcendental equation. It was found that a Hopf bifurcation may take place in the autonomous system when time delays pass certain values. The direction and stability of the Hopf bifurcation were determined by applying the normal form method and constructing a center manifold. It was shown that a bifurcation of codimension two may occur through a Hopf and a steady state bifurcation interaction. Ji [2003c] examined the effect of time delays presenting in a PD feedback controller on the nonlinear dynamic behaviour of a Jeffcott rotor with an additional magnetic bearing locating at the central disc. For the corresponding autonomous system, linear stability analysis was performed by constructing a center manifold. It was found that the trivial solution may lose its stability through either a single or double Hopf bifurcation. For the non-autonomous system, the primary resonance response was studied for its small non-liner motions using the method of averaging. The effects of time delays and control gains as well as excitation amplitude on the amplitude of the steady-state response were investigated both theoretically and experimentally. It was shown that the steady state response may exhibit saddle-node and Hopf bifurcations. Increasing the values of time delays tend to increase the peak amplitude of the response and shift the frequency-response curve to the right. Large time delays may induce instability of the system.

Based on the model developed by Ji [2003b], Wang and Liu [2005] further investigated the stability of the magnetic bearing system with time delays by analysing the distribution of the roots of the associated characteristic equation. It was found that Hopf bifurcation occurs when the delay passes through a sequence of critical values. The explicit algorithm for determining the direction of the Hopf bifurcations and the stability of bifurcating periodic solutions were derived by using the theory of normal form and center manifold. Later, Wang and Jiang [2006] reported on the multiple stabilities of the magnetic bearing system with time delays. They performed the centre manifold reduction and normal form computation for simple zero singularity and carried out detailed bifurcation analysis. Some numerical simulations were also presented to illustrate the results found.

Ji and Hansen [2005b] studied the influence of a time delay occurring in a PD feedback controller on the dynamic stability of a rotor suspended by magnetic bearings by taking geometric coordinate coupling into account. The equations of motion governing the response of the rotor were derived to be a set of two-DOF nonlinear differential equations with time delay coupling in the nonlinear terms. It was found that as the time delay increases beyond a critical value, the equilibrium position of rotor motion becomes unstable and may bifurcate into two qualitatively different kinds of periodic motion. The resultant Hopf bifurcation of multiplicity two was found to be associated with two coincident pairs of complex conjugate eigenvalues crossing the imaginary axis. Based on the reduction of the infinite dimensional problem to the flow on a four-dimensional centre manifold, the bifurcating periodic solutions were obtained using a perturbation method. Ji and Hansen [2005c] considered the forced dynamic behaviour of the corresponding nonlinear non-autonomous system in the neighbourhood of the Hopf bifurcation of multiplicity two with the aid of the decomposition theorem and centre manifold theorem. As a result of the interaction between the Hopf bifurcating periodic solutions and the external periodic excitation, primary resonances may occur in the forced response of the system when the forcing frequency is close to the Hopf bifurcating periodic frequency. The method of multiple scales was used to obtain four first-order ordinary differential equations that determine the amplitudes and phases of the phase-locked periodic solutions. The first-order approximations of the periodic solutions were found to be in excellent agreement with those obtained by direct numerical integration of the delay-differential equation. It was also found that the steady state solutions of the nonlinear non-autonomous system may lose their stability via either a pitchfork or Hopf bifurcation. It was shown that the primary resonance response may exhibit symmetric and asymmetric phase-locked periodic motions, quasi-periodic motions, chaotic motions and coexistence of two stable motions. Based on the behaviour of solutions to the four-dimensional system of ordinary differential equations, Ji, Hansen and Li [2005] investigated the effect of external excitations on the dynamic behaviour of the corresponding non-autonomous system following the Hopf bifurcation of the trivial equilibrium of the corresponding autonomous system. It was shown that the interaction between the Hopf bifurcating solutions and the high level excitations may induce non-resonant or secondary resonance response, depending on the ratio of the frequency of bifurcating periodic motion to the frequency of external excitation. The first-order approximate periodic solutions for the non-resonant and super-harmonic resonance response were observed to be in good agreement with those obtained by direct numerical integration of the delay differential equation. It was found that the non-resonant response may be either periodic or quasi-periodic. It was shown that the super-harmonic resonance response may exhibit periodic and quasi-periodic motions as well as a co-existence of two or three stable motions.

2.6 Other issues relevant to nonlinear magnetic bearings

This section is to provide an introductory familiarity with two merging topics of research in which the nonlinear properties of magnetic bearings should be (and have been) taken into account, which are nonlinear self-sensing magnetic bearings and nonlinear control techniques. Consideration of nonlinearity in the dynamic model for self-sensing magnetic bearings can capture their full potential capacities for which nonlinearity plays an essential role to enhance robustness. The linear feedback controllers designed based on linearized model cannot be effective across the entire work region, as highly nonlinear properties of magnetic bearings may diminish the performance of magnetic bearing systems when the operations departs from the equilibrium point. Nonlinear control techniques should be (and have been) designed to account for the nonlinear properties of magnetic bearing systems.

2.6.1 Nonlinear self-sensing magnetic bearings

The self-sensing (sensorless) magnetic bearing is a special kind of magnetic bearing by using the same structure as both an actuator and a sensor. Self-sensing magnetic bearings use measurement of voltage and current in electromagnets to estimate the position of a magnetically levitated rotor. By estimating position in this manner, explicit proximity sensors are eliminated. The position information is deduced from the electromagnetic interaction between the stator and rotor. There are basically two classes of approach to self-sensing magnetic bearings, namely the self-sensing magnetic bearing with a linear controller, and the self-sensing magnetic bearing with modulation method. The main advantages of self-sensing magnetic bearings include the reduction of the manufacturing costs, elimination of hardware complexity, simplification of the assembly and maintenance of the magnetic bearing systems, and provision of a more compact design and integration design of the rotor-bearing system with higher natural frequencies. Self-sensing magnetic bearings have attracted a lot of attention from the research community, since the problem was first reported in 1990 [Vischer and Bleuler, 1990]. For example, Mizuno and Bleuler [1995] studied the cancellation control of static load and sinusoidal disturbance in self-sensing magnetic bearings by applying a geometric approach. Based on the assumption that the switching frequency of the amplifier changes linearly with the gap between the electromagnet and the suspended object, Mizuno, Ishii and Araki [1998] analysed the dynamic characteristics of a hysteresis amplifier for designing new circuits.

One of the obstacles confronting self-sensing technology is the nonlinearity associated with operation of the actuator in its magnetic saturation regime. This problem is especially important in high specific capacity magnetic bearings. Development of a nonlinear model will greatly extend the operating range of self-sensing bearings, as the linear behaviour of magnetic bearings can be achieved approximately locally only in small range of small rotor deflections and small control currents.

Skricka and Markert [2001] explored the effects of cross-axis sensitivity and coordinate coupling on self-sensing by using nonlinear magnetic reluctance models. It was shown that a self-sensing method based on single magnet models might result in larger errors in the estimated position. Thus, they suggested that a precise model including nonlinearities of geometric coupling and saturation of the magnetic material need to be developed to predict precisely the behaviour of active magnetic bearings. Later, Skricka and Markert [2002a; 2002b] further studied two aspects of the integration of electromagnetic bearings by considering the nonlinearity of magnetic force. The nonlinear magnetic force was compensated by software integrated in the digital controller. The rotor position was identified from the electric state variables directly at the power amplifiers. The realisation of linear magnetic force was achieved by software using control methods instead of pre-magnetization currents.

Recently, Maslen, Montie and Iwasaki [2006] developed a linear periodic model of the magnetic bearing system to predict more acceptable levels of robustness. The essential features of the nonlinearity were retained in their model by linearization along a periodic trajectory. A linear time-invariant model derived by linearizing system at a fixed equilibrium point, which is widely used in modelling self-sensing magnetic bearings, was found to be potential inaccurate for general nonlinear self-sensing magnetic bearings.

2.6.2 Nonlinear control techniques

One obstruction to more widespread industrial application of magnetic bearings is the high sensitivity of the control system to parametric uncertainties and bearing nonlinearities [Knospe, 2007; Hung, Albritton and Xia, 2003]. Due to the intractability of the complexity of the actual model, many of control techniques for active magnetic bearings currently used were generally designed by ignoring the nonlinearity of magnetic force and the nonlinear effects of the sensors and actuators. The feedback control design were typically designed by using a linearised model of the system, but highly nonlinear properties of the bearing can limit the performance of the overall system. The classical approach for magnetic bearing controller design was to perform a generalized Taylor series linearization about a nominal equilibrium point. Because of the abundant research in the linear control theory, linear controllers have been applied to magnetic bearing systems extensively. For example, Cho [1993] investigated the application of sliding mode control to stabilise an electromagnetic suspension system for use in vibration isolation platforms and magnetic bearings. Setiawan, Mukherjee and Maslen [2001; 2002] studied synchronous sensor runout and unbalance compensation for magnetic bearing systems. Thibeault and Smith [2002] used bounds on sensitivity and complementary sensitivity to deduce achievable robustness and performance limits for a magnetic bearing system to achieve good robustness and performance. Hu, Lin, Jiang and Allaire [2005] developed a systematic control design approach for magnetic bearing systems that are subject to both input and state constraints.

The linear feedback system designs based on linearizing the dynamic equations about the equilibrium point are not valid across the entire working region, because the controller performance may suffer rapid deterioration when the operation deviates from the equilibrium point. In order to maximize magnetic bearing capabilities where the nonlinearity may play a crucial role, the control system needs to properly compensate for the nonlinear dynamics of magnetic bearing systems. For some cases, it may perhaps be theoretically possible to compensate partially certain nonlinearities at a cost of highly complex control strategies [De Queiroz, Dawson and Suri, 1998; Li, 1999; Hong and Langari, 2000]. However, nonlinearities originating from hardware and physical system limitations cannot be eliminated by software, and the control problem becomes very complicated due to the inherent nonlinearities associated with the electromechanical dynamics introduced into the magnetic bearing system.

Recently, many nonlinear control techniques have been designed to account for the nonlinear magnetic bearing model. Lei, Palazzolo, Na and Kascak [2000] developed a unique control approach for prescribed large motion control using magnetic bearings in a proposed active stall control test rig. They employed nonlinear fuzzy logic control to the nonlinear magnetic bearing model, which involves nonlinear B-H curve, Ampere's law and Maxwell stress tensor. Schroder, Green, Grum and Fleming [2001] demonstrated a convenient method for automating a tuning process to produce an optimal design. The magnetic circuit dynamics included in the modelling of the nonlinear characteristics of magnetic bearings. It was found that the optimised controllers removed a nonlinear high-to-low-frequency coupling effect. Yeh, Chung and Wu [2001] proposed a sliding control scheme to deal with the nonlinear, uncertain dynamics of magnetic bearing systems. The model characterized both the main electromechanical interaction and the secondary electromagnetic effects such as flux leakage, fringing fluxes and finite core permeance. The controller consisted of two parts: the nominal control part that linearizes the nonlinear dynamics, and the robust control part that provides robust performance against the uncertainties.

Hung, Albritton and Xia [2003] designed a nonlinear control system for a magnetic journal bearing using a combination of feedback linearization and backstepping concepts. The derived equations of motion included flux linkage, electromagnetic dynamics, and magneto-mechanical dynamics, as well as state variable model. Ji and Hansen [2003] developed a linear-plus-nonlinear feedback control to stabilize an unstable Hopf bifurcation in a rotor-magnetic bearing system, for which the linearizied system possesses double zero eigenvalues. The addition of nonlinear terms was used to modify the coefficients of the nonlinear terms in the reduced normal forms. It was found that the feedback control incorporating certain quadratic terms renders the Hopf bifurcation supercritical, thereby extending the operation region of magnetic bearing systems.

Chapter 3

Linkages between publications

The primary objective of the research described in the following chapters is to gain a deep understanding of the effect of the nonlinear properties of magnetic bearings and time delays occurring in the feedback control path on the stability and dynamics of a rotor supported by magnetic bearings. This thesis comprises 12 published papers on the stability, local bifurcation control and nonlinear dynamics of magnetic bearing systems. The 12 papers are categorized into four groups, with each group contributing to a chapter. Specifically, the nonlinear oscillations and local bifurcation control of rotor-magnetic bearing systems are presented in Chapter 4, where the essential nonlinear magnetic force is considered in modelling of the nonlinear properties of magnetic bearings. In Chapter 5, the effect of saturation constraints of the electromagnetic actuators on the dynamics of magnetic bearing systems is discussed in terms of periodic and chaotic motions of nonlinear systems. The effect of time delays occurring in the feedback control path on the stability and dynamics of a rotor supported by a two-pole magnetic bearing is discussed in Chapter 6 using a model of one-degree-of-freedom (one-DOF) nonlinear systems with time delays. By including the geometric coupling of magnetic forces, a more complicated model is developed in Chapter 7 for studying the influence of a time delay occurring in a PD feedback controller on the dynamic stability and dynamics of a rotor supported by four-pole magnetic bearings. In the presence of geometric coordinate coupling and time delay, the equations of motion governing the response of the rotor are a set of two-DOF nonlinear differential equations with time delay coupling in the nonlinear terms. The stability and forced response of the system are then studied in Chapter 7 based on the developed model. In subsequent sections, the linkages between the 12 published papers are described in more detail.

In Chapter 4 entitled "Nonlinear oscillations and local bifurcation control", Paper 4.1 and Paper 4.2 discuss the primary and super-harmonic resonance response of a rotor-magnetic bearing system. Paper 4.3 presents a linear-plus-nonlinear feedback control procedure to stabilise a subcritical Hopf bifurcation occurring in a simple magnetic bearing system. In particular, a two-DOF nonlinear system with cubic nonlinearities is developed to study the effect of essential nonlinear magnetic force on the response of the system in Paper 4.1. Depending on the relationships of natural frequencies and forcing frequency, primary, sub-harmonic and super-harmonic resonances may occur in the forced response. Paper 4.1 studies the primary resonance response of the system and Paper 4.2 examines the super-harmonic resonance response. It is found that the forced response of the system may exhibit a variety of nonlinear behaviour including bifurcations, jump phenomena and coexistence of multiple solutions. For a rotor-magnetic bearing system with a proportional-derivative (PD) feedback controller, the corresponding autonomous system may demonstrate saddle-node bifurcation and subcritical Hopf bifurcation when feedback gains are near a degenerate point of double-zero eigenvalues. A linear-plus-nonlinear feedback controller is thus developed in Paper 4.3 to stabilise a subcritical Hopf bifurcation. The addition of quadratic terms is used to modify the coefficients of the nonlinear terms in the reduced normal forms.

A magnetic bearing is required to provide a larger suspension force to support the rotor when the rotor undergoes an unwanted larger amplitude motion. However, physical limitations of either power amplifiers or ferromagnetic core material prevent the forcing increasing beyond some practical limit, which leads to saturation phenomena. An occurrence of saturation constraints may lead to a poor dynamic behaviour of magnetic bearing systems and degrade the control system's performance. For a magnetic bearing system with saturation constraints, an in-depth knowledge of the system response can be of prime importance in designing the control system and to avoid unacceptable levels of vibrations. In Paper 5.1, the essential magnetic force under normal operating conditions is simplified by its linear approximate form, whereas the magnetic force with saturation constraints is mathematically of a piecewise linear characteristic. The dynamics of magnetic bearing systems is then governed by a periodically forced single-degree-of-freedom (single-DOF) piecewise linear system. It is found that the system may accept symmetric and asymmetric period-one solutions, subharmonic and chaotic solutions. In Paper 5.2, without simplification the magnetic force with saturation constraints is mathematically characterized by a piecewise nonlinear-linear function. Under the weakly nonlinear magnetic force and saturation constraints, the equations of motion governing the dynamics of a rotor suspended in a single-DOF magnetic bearing is mathematically modelled by a combination of a weakly nonlinear and a linear differential equation. An approximate solution for the primary resonance response of the resultant periodically excited nonlinear-linear oscillator is analytically constructed without eliminating the secular terms. Paper 5.3 develops an analytical technique for constructing a global symmetric period-one approximate solution for the non-resonant periodic response of the oscillator, and also presents numerical investigation on bifurcation and subharmonic as well as chaotic motions.

A magnetic bearing system is inherently unstable and thus a feedback controller must be employed to stabilize the system. Time delays occurring in the feedback control loop are unavoidable even though the control decision process is carried out very quickly. Such time delays can have a significant impact on the stability of the system. The effect of time delays on the stability and dynamics of a rotor supported by two-pole magnetic bearing is studied in Chapter 6, which is entitled "Stability and dynamics of one-DOF nonlinear systems with time delays". In particular, a one-DOF nonlinear system with time delay is developed in Paper 6.1, where the critical value of time delay is also determined and the stability of trivial equilibrium is studied by construction of a centre manifold. Paper 6.2 studies the dynamics of a Jeffcott rotor-magnetic bearing system with time delays. The model considered is a Jeffcott rotor with an additional magnetic bearing located at the central disc. For simplicity, the coupling effects of magnetic forces between the two coordinate axes are neglected. As such, the equations of motion are decoupled into two components of the rotor motion. The effect of time delays on the primary resonance response is analytically and experimentally studied. In both Papers 6.1 and 6.2, the feedback control system is assumed to generate a current that is proportional to the rotor displacement and velocity, i.e., a proportional-derivative (PD) controller. Paper 6.3 studies the stability of a two-pole magnetic bearing in which the feedback controller is a proportional-integral-derivative (PID) controller. A third-order delay differential equation is developed as the equation of motion. It is shown that resonant and non-resonant Hopf bifurcations may appear as the time delay reaches certain value.

A more sophisticated model is developed in Chapter 7 by taking into account geometric coordinate coupling in four-pole magnetic bearings. First of all, in order to study the influence of a time delay occurring in a PD feedback controller on the dynamic stability of a rotor suspended by two magnetic bearings, Paper 7.1 develops a set of two-DOF nonlinear differential equations with time delay coupling in the nonlinear terms. Modern dynamical theory and a perturbation method are then used to study the stability and bifurcation of the corresponding autonomous system. It is found that as the time delay increases beyond a critical value, the equilibrium position of the system may bifurcate into two different kinds of periodic motions through a Hopf bifurcation of multiplicity two for the corresponding autonomous system. The presence of external excitation in the corresponding non-autonomous system may induce a complex forced response in the neighbourhood of Hopf bifurcation. Depending on the ratio of the forcing frequency and the frequency of Hopf bifurcation, the forced response of the system may exhibit primary resonances, secondary resonances and non-resonant motions. Interactions between the external periodic excitation and Hopf bifurcating periodic solutions are studied in Paper 7.2 and Paper 7.3 under primary resonances and non-resonances as well as secondary resonances. In particular, Paper 7.2 studies the primary resonance response when the forcing frequency is close to the frequency of Hopf bifurcation. The non-resonant response and super-harmonic resonance response are investigated in Paper 7.3, when the forcing frequency is close to one-third of the Hopf bifurcation frequency and is well separated from any possible value which may result in any resonances. It is found that the periodic excitation has a significant effect on the dynamic behaviour of the system in the neighbourhood of Hopf bifurcation of the trivial equilibrium.

Chapter 4

Nonlinear oscillations and local bifurcation control

This chapter aims to study the primary and super-harmonic resonance response of a rotor suspended by large air-gap magnetic bearings and to develop a linear-plus-nonlinear feedback control method to stabilize an unstable Hopf bifurcation existing in a simple magnetic bearing system. This chapter is composed of three papers, which will be regarded as three sections and referred to here as Paper 4.1—Primary resonance response; Paper 4.2—Super-harmonic resonance response; and Paper 4.3—Local bifurcation control.

In a typical magnetic bearing system, the essential nonlinearity results from the magnetic force to displacement and force to coil current relationships of the electromagnets. The nonlinear magnetic force is even more pronounced in a large air-gap magnetic bearing system. It is thus of scientific interest to study the effect of nonlinear magnetic forces on the nonlinear dynamic response of a shaft suspended by large air-gap magnetic bearings. Toward this end, Paper 4.1 and Paper 4.2 consider the primary and super-harmonic resonance response of a rotor-magnetic bearing system. In particular, a two-degree-of-freedom (two-DOF) nonlinear system with cubic non-linearities is developed to investigate the effect of essential nonlinear magnetic force on the nonlinear response of the system in Paper 4.1. Depending on the relationships of natural frequencies and forcing frequency, primary, sub-harmonic and super-harmonic resonances may occur in the forced response. Paper 4.1 studies the primary resonance response of the system and Paper 4.2 examines the super-harmonic resonance response. It is found that the forced response of the rotor-magnetic bearing system may exhibit a variety of nonlinear behaviour including bifurcations, jump phenomena and coexistence of multiple solutions.

It has been shown that a rotor-magnetic bearing system with a proportional-derivative (PD) feedback controller may exhibit rich bifurcation behaviour near a degenerate point of double-zero eigenvalue [Ji, Yu and Leung, 2000]. The unstable periodic orbit created by a subcritical Hopf bifurcation grows in amplitude as the perturbation parameters vary until it collides with the saddle point. This subcritical Hopf bifurcation may lead to large penalties in the performance of magnetic bearings. Thus a linear-plus-nonlinear feedback control scheme is designed to stabilize the subcritical Hopf bifurcation in Paper 4.3. The addition of quadratic terms is used to modify the coefficients of the nonlinear terms in the reduced normal forms, thereby rendering the Hopf bifurcation supercritical.

CHAPTER 4

Paper 4.1: Nonlinear Oscillations of a Rotor in Active Magnetic Bearings

Authorship: J.C. Ji, C.H. Hansen

Published in: Journal of Sound and Vibration, vol.240, 2001, pp.599-612.

Short running headline: Primary resonance response

Statement of Authorship

Jin-Chen Ji (Candidate)

Performed theoretical analysis, carried out numerical simulations, interpreted data, wrote manuscript and acted as corresponding author.

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Professor Colin Hansen (Supervisor)

Supervised development of work, helped in data interpretation and manuscript evaluation, and provided editorial comments and suggestions.

Signed .... *Date*..........................

J.C. Ji and C.H. Hansen (2001) Non-Linear Oscillations of a Rotor in Active Magnetic Bearings.
Journal of Sound and Vibration, v. 240 (4), pp. 599–612, March 2001

NOTE: This publication is included in the print copy of the thesis held in the University of Adelaide Library.

It is also available online to authorised users at:

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CHAPTER 4

Paper 4.2: Nonlinear Oscillations of a Rotor-Magnetic Bearing System under Super-Harmonic Resonance Conditions

Authorship: J.C. Ji, A.Y.T. Leung

Published in: International Journal of Non-Linear Mechanics, vol.38, 2003, pp.829-835.

Short running headline: Super-harmonic resonance response

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Supervised development of work, helped in data interpretation and manuscript evaluation, and provided editorial comments and suggestions.

Signed .... *Date*..........................

J.C. Ji and A.Y.T. Leung (2003) Non-linear oscillations of a rotor-magnetic bearing system under superharmonic resonance conditions.
International Journal of Non-Linear Mechanics, v. 38 (6), pp. 829–835, September 2003

Paper 4.3: Local Bifurcation Control in a Rotor-Magnetic Bearing System

Authorship: J.C. Ji, C.H. Hansen

Published in: International Journal of Bifurcation and Chaos, vol.13, 2003, pp.951-956.

Short running headline: Local bifurcation control

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Signed .... *Date*..........................

J.C. Ji and C.H. Hansen (2003) Local Bifurcation Control in a Rotor-Magnetic Bearing System.
International Journal of Bifurcation and Chaos, v. 13 (4), pp. 951-956, April 2003

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Chapter 5

Periodic and chaotic motions of one-DOF nonlinear systems with saturation constraints

This chapter studies the effect of saturation constraints of the electromagnetic actuators on the dynamics of a rotor supported by a two-pole magnetic bearing in terms of periodic and chaotic motions.

A magnetic bearing is required to provide a larger suspension force necessary to support the rotor when the rotor undergoes an unwanted larger amplitude motion. However, the saturation phenomena of magnetic forces resulting from the physical limitations of either power amplifiers or ferromagnetic core material prevent the forcing increasing beyond some practical limit. An occurrence of saturation constraints may lead to a poor dynamic behaviour of magnetic bearing systems and degrade their performance. For a magnetic bearing system with saturation constraints, an in-depth knowledge of the system response can be of prime importance in designing the control system and to avoid unacceptable levels of vibrations. This chapter is organised into three sections including three papers, namely Paper 5.1—Piecewise linear oscillator; Paper 5.2—Nonlinear oscillator: Resonant response; and Paper 5.3—Nonlinear oscillator: Non-resonant response.

In Paper 5.1, the essential magnetic force under normal operating conditions is simplified by its linear approximate form, whereas the magnetic force with saturation constraints is mathematically of a piecewise linear characteristic. The dynamics of a rotor suspended by a two-pole magnetic bearing is then governed by a periodically excited single-degree-of-freedom (single-DOF) piecewise linear system. It is found that the system may accept symmetric and asymmetric period-one solutions, sub-harmonic and chaotic solutions. In Paper 5.2, without simplification the magnetic force with saturation constraints is mathematically characterized by a piecewise nonlinear-linear function. Under the weakly nonlinear magnetic force and saturation constraints, the equations of motion governing the dynamics of a rotor suspended in a single-DOF magnetic bearing are mathematically modelled by a combination of a weakly nonlinear and a linear differential equation. An approximate solution for the primary resonance response of the resultant periodically excited nonlinear-linear oscillator is analytically constructed without eliminating the secular terms. Paper 5.3 develops an analytical technique for constructing a global symmetric period-one approximate solution for the non-resonant periodic response of the oscillator, and also presents numerical results of bifurcations and subharmonic as well as chaotic motions.

CHAPTER 5

Paper 5.1: Dynamics of a Piecewise Linear System Subjected to a Saturation Constraint

Authorship: J.C. Ji

Published in: Journal of Sound and Vibration, vol.271, 2004, pp.905-920.

Short running headline: Piecewise linear oscillator

Statement of Authorship

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J.C. Ji (2004) Dynamics of a piecewise linear system subjected to a saturation constraint.
Journal of Sound and Vibration, v. 271 (3/5), pp. 905–920, April 2004

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It is also available online to authorised users at:

http://dx.doi.org/10.1016/S0022-460X(03)00759-4

CHAPTER 5

Paper 5.2: Analytical Approximation of the Primary Resonance Response of a Periodically Excited Piecewise Nonlinear-Linear Oscillator

Authorship: J.C. Ji, C.H. Hansen

Published in: Journal of Sound and Vibration, vol.278, 2004, pp.327-342.

Short running headline: Nonlinear oscillator: Resonant response

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Supervised development of work, helped in data interpretation and manuscript evaluation, and provided editorial comments and suggestions.

Signed .... *Date*..........................

J.C. Ji and C.H. Hansen (2004) Analytical approximation of the primary resonance response of a periodically excited piecewise non-linear–linear oscillator.
Journal of Sound and Vibration, v. 278 (1/2), pp. 327–342, November 2004

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It is also available online to authorised users at:

http://dx.doi.org/10.1016/j.jsv.2003.10.022

CHAPTER 5

Paper 5.3: Approximate Solutions and Chaotic Motions of a Piecewise Nonlinear-Linear Oscillator

Authorship: J.C. Ji, C.H. Hansen

Published in: Chaos, Solitons and Fractals, vol.20, 2004, pp.1121-1133.

Short running headline: Nonlinear oscillator: Non-resonant response

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Professor Colin Hansen (Supervisor)

Supervised development of work, helped in data interpretation and manuscript evaluation, and provided editorial comments and suggestions.

Signed .... *Date*..........................

J.C. Ji and C.H. Hansen (2004) Approximate solutions and chaotic motions of a piecewise nonlinear–linear oscillator.
Chaos, Solitons & Fractals, v. 20 (5), pp. 1121–1133, June 2004

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Chapter 6

Stability and dynamics of one-DOF nonlinear systems with time delays

This chapter is aimed at investigating the effect of time delays occurring in the feedback control path on the stability and dynamics of a rotor supported by a two-pole magnetic bearing. This chapter comprises three papers, which will be referred to here as Paper6.1—Stability associated with a PD controller; Paper 6.2—Dynamics of a Jeffcott rotor-magnetic bearing system; and Paper 6.3—Stability associated with a PID controller.

A magnetic bearing system is inherently unstable and thus a feedback controller must be employed to stabilize the system. Time delays occurring in the feedback control loop are unavoidable especially in modern digital control systems, even though the control decision process is carried out very quickly. The sources of delays in a magnetic bearing system may be the A/D and D/A conversions, the controller computation, and the reaction of the electromagnetic actuators to the control decisions. It is widely accepted that large time delays can destabilize steady-state solutions while small ones cannot influence the qualitative behaviour of solutions. In existing studies, many researchers tended to ignore time delays in their models where they thought time delays were small. However, such common practice would not be true without any qualification. In fact, time delays may have a significant impact on the stability and dynamics of a rotor-magnetic bearing system.

In this chapter, a one-DOF nonlinear system with time delay is developed in Paper 6.1, where the critical value of time delay is also determined and the stability of the trivial equilibrium is studied by the construction of a centre manifold. Paper 6.2 studies the dynamics of a Jeffcott rotor-magnetic bearing system with time delays. The model considered is a Jeffcott rotor with an additional magnetic bearing located at the central disc. For simplicity, the coupling effects of magnetic forces between the two coordinate axes are neglected. As such, the equations of motion are decoupled into two components of rotor motion. The effect of time delays on the primary resonance response is analytically and experimentally studied. In Papers 6.1 and 6.2, the feedback control system is assumed to generate a current that is proportional to the rotor displacement and velocity, i.e., a proportional-derivative (PD) controller. Paper 6.3 examines the stability of a two-pole magnetic bearing in which the feedback controller is a proportional-integral-derivative (PID) controller. A third-order delay differential equation is developed as the equation of motion. It is shown that resonant and non-resonant Hopf bifurcations may appear as the time delay reaches certain value.

CHAPTER 6

Paper 6.1: Stability and Hopf Bifurcation of a Magnetic Bearing System with Time Delays

Authorship: J.C. Ji

Published in: Journal of Sound and Vibration, vol.259, 2003, pp.845-856.

Short running headline: Stability associated with a PD controller

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Signed .... *Date*..........................

J.C. Ji (2004) Stability and Hopf Bifurcation of a Magnetic Bearing System With Time Delays.
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It is also available online to authorised users at:

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CHAPTER 6

Paper 6.2: Dynamics of a Jeffcott Rotor-Magnetic Bearing System with Time Delays

Authorship: J.C. Ji

Published in: International Journal of Non-Linear Mechanics, vol.38, 2003, pp.1387-1401.

Short running headline: Dynamics of a Jeffcott rotor-magnetic bearing system

Statement of Authorship

Jin-Chen Ji (Candidate)

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Signed .... *Date*..........................

J.C. Ji (2003) Dynamics of a Jeffcott rotor-magnetic bearing system with time delays. *International Journal of Non-Linear Mechanics, v. 38 (9), pp. 1387–1401, November 2003*

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CHAPTER 6

Paper 6.3: Stability and Bifurcation in an Electromechanical System with Time Delays

Authorship: J.C. Ji

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Short running headline: Stability associated with a PID controller

Statement of Authorship

Jin-Chen Ji (Candidate)

Performed theoretical analysis, carried out numerical simulations, interpreted data, wrote manuscript and acted as corresponding author.

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Chapter 7

Stability and forced response of a two-DOF nonlinear system with time delay

This chapter develops a more complicated model to study the influence of a time delay occurring in a PD feedback controller on the dynamic stability and dynamics of a rotor supported by four-pole magnetic bearings. In the presence of geometric coordinate coupling and time delay, the equations of motion governing the response of the rotor are a set of two-degree-of-freedom (two-DOF) nonlinear differential equations with time delay coupling in the nonlinear terms. The stability and forced response of the system are studied using the developed model in three papers, namely Paper 7.1—Stability of the system without external excitations; Paper 7.2—Forced phase-locked response; and Paper 7.3—Super-harmonic resonances and non-resonant response.

In particular, Paper 7.1 develops a set of two-DOF nonlinear differential equations with time delay coupling in the nonlinear terms by taking into account geometric coordinate coupling in four-pole magnetic bearings. Modern dynamical theory and a perturbation method are then used to study the stability and bifurcation of the corresponding autonomous system. It is found that as the time delay increases beyond a critical value, the equilibrium position of the corresponding autonomous system may bifurcate into two different kinds of periodic motions through a Hopf bifurcation of multiplicity two.

The presence of external excitation in the corresponding non-autonomous system may induce a complex forced response in the neighbourhood of Hopf bifurcation. Depending on the ratio of the forcing frequency and the frequency of Hopf bifurcation, the forced response of the rotor-magnetic bearing system may exhibit primary resonances, secondary resonances and non-resonant motions. Paper 7.2 studies the primary resonance response when the forcing frequency is close to the frequency of Hopf bifurcation. The non-resonant response and super-harmonic resonance response are investigated in Paper 7.3, where the forcing frequency is close to one-third of the Hopf bifurcation frequency and is well separated from any possible value which may result in any resonances. It is found that the external excitation has a significant effect on the dynamic behaviour of the rotor-magnetic bearing system in the neighbourhood of Hopf bifurcation of the trivial equilibrium.

CHAPTER 7

Paper 7.1: Hopf Bifurcation of a Magnetic Bearing System with Time Delay

Authorship: J.C. Ji, C.H. Hansen

Published in: Transactions of the ASME Journal of Vibration and Acoustics, vol.127, 2005, pp.362-369.

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Statement of Authorship

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Professor Colin Hansen (Supervisor)

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J.C. Ji and C.H. Hansen (2004) Hopf Bifurcation of a Magnetic Bearing System with Time Delay.
Journal of Vibration and Acoustics, v. 127(4), pp. 362-369, December 2004

CHAPTER 7

Paper 7.2: Forced Phase-Locked Response of a Nonlinear System with Time Delay after Hopf Bifurcation

Authorship: J.C. Ji, C.H. Hansen

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CHAPTER 7

Paper 7.3: Effect of External Excitations on a Nonlinear System with Time Delay

Authorship: J.C. Ji, C.H. Hansen, X.Y. Li

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Statement of Authorship

Jin-Chen Ji (Candidate)

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Helped in data interpretation and manuscript evaluation, and provided editorial comments and suggestions.

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J.C. Ji, C.H. Hansen and X. Li (2005) Effect of External Excitations on a Nonlinear System with Time Delay.
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Chapter 8

Conclusion and future work

As stated in Chapter 1, the primary aim of the work outlined in this thesis was to gain a deep understanding of the stability and nonlinear dynamics of magnetic bearing systems incorporating one or more components of nonlinear properties. The research work presented in this thesis has successfully achieved the anticipated objectives.

8.1 Concluding remarks

The characteristics of magnetic bearings are inherently nonlinear due to the prevalent nonlinearities of magnetic bearing components. A linear dynamic analysis using the linearized model is not appropriate for the entire work region, as the linear relationship holds only locally and the linear behaviour of rotor motion can be achieved approximately only in small rotor deflections and small control currents. In order to fully utilize the capacity of a magnetic bearing, a nonlinear analysis is required to explore the nonlinear behaviour of a rotor suspended by magnetic bearing systems.

This thesis comprises 12 papers which were recently published in seven international journals in the fields of vibration control and nonlinear dynamics. These papers highlighted the influences of the nonlinear properties of magnetic bearings and the effects of time delays of the feedback control loops on the dynamic behaviour and stability of rotor-bearing systems incorporating magnetic bearings.

The main outcomes of the thesis are listed below:

1). A thorough understanding of the effects of geometric coupling and nonlinear magnetic force to displacement and force to coil current relationships on the dynamic behaviour and performance of magnetic bearings;

2). A complete understanding of the effects of the nonlinear electromagnetic force to displacement and force to coil current relationship incorporating time delays of the control system or saturation of the power amplifier on the dynamic behaviour and performance of magnetic bearings;

3). A deep understanding of the stability and dynamics of magnetic bearing systems under a combination of three components of nonlinearities: namely geometric coupling; nonlinear force to magnetic flux; and time delays in the feedback controller;

4). A rigorous determination of the critical values of time delays associated with controller and actuators in two-pole and four-pole magnetic bearings with a proportional-derivative controller;

5). An understanding of the mechanism of instability of the trivial equilibrium of magnetic bearings involving time delay presented in a feedback control loop;

6). A thorough understanding of the interaction of the external excitation and bifurcating solutions that immediately follows from Hopf bifurcation of the trivial equilibrium of the corresponding autonomous systems of an unbalanced rotor vertically supported by two four-pole magnetic bearings;

7). Development of appropriate models of magnetic bearing systems necessary for analysing the effects of one or more components of nonlinear properties on the stability and dynamics of magnetic bearing systems;

8). Development of an analytical procedure for locating approximate solutions for the resonant and non-resonant response of a periodically excited nonlinear-linear oscillator;

9). Examination of the nonlinear response of rotor-magnetic bearing systems under primary, sub-harmonic and super-harmonic resonance conditions;

10). Exploration of the existence of multiple solutions, local and global bifurcations, periodically- and chaotically-amplitude modulated responses of rotor-magnetic bearing systems;

11). Development of a linear-plus-nonlinear feedback technique from the bifurcation control point of view to stabilize unstable Hopf bifurcations, thereby extending the operational region of magnetic bearing systems near linear stability boundaries;

12). Provision of valuable information on the dynamic stability and nonlinear dynamics for online detection of malfunctions of magnetic bearing systems at an early stage of their development before they become catastrophic for the operation of magnetic bearings;

13). Prediction of the occurrence of undesirable instabilities and bifurcations as well as the complex dynamic response of magnetic bearing systems;

14). Successful application of modern dynamical theory and perturbation methods to analyse the resultant highly nonlinear differential equations.

This thesis has attempted to develop more reliable and sophisticated models encompassing a combination of two and three components of typical nonlinearities, which are capable of representing specific nonlinearities of magnetic bearings. The analytical and experimental studies based on these developed models have led to insights on the effects of the combination of two and three components of nonlinearities and the influence of time delays occurring in the feedback control path on the stability and observed dynamics of magnetic bearing systems. The research outlined in this thesis has provided fundamental concepts of how the time delays, nonlinear magnetic forces, geometric coordinate coupling and saturation constraints can lead to instability and complex dynamic behaviour of magnetic bearing systems.

8.2 Future work

Although significant efforts have been made to gain a deep understanding of the stability and nonlinear dynamics of magnetic bearing systems, there are many challenging problems remaining unsolved. The following discussion presents some of the key aspects that will drive the future research on the nonlinear dynamics and nonlinear modelling of magnetic bearing systems.

Nonlinear modelling of magnetic bearing systems is very challenging because of their highly individually nonlinear nature and complexity. It has been shown that an accurate rotor-magnetic bearing system model with suitable uncertainty descriptions is of critical importance in applying advanced control techniques [Li, Lin, Allaire and Luo, 2006], where the high frequency dynamics were treated as uncertainties. An accurate model plays an important role in the control design and dynamic analysis of rotor-magnetic bearing systems due to the complexity involved. This thesis has dealt with the most important nonlinearities predominant in magnetic bearing systems. It should be noted that magnetic bearing systems may encounter less important nonlinearities, such as the nonlinearity of the coil inductance, the nonlinearity of the sensor system, and the nonlinearities resulting from the eddy current effect, the leakage and fringing effect, as discussed in Chapter 1. A subject of future research would be towards the development of reliable and comprehensive models of complicated magnetic bearings with multiple components of nonlinearities, which enable the treatment of nonlinearities in large groups which include the less important nonlinearities. A preferred system modelling should include a flexible rotor, magnetic bearings, sensors, amplifier dynamics and digital controllers. The development of a comprehensive model of magnetic bearings is a formidable task if all components of nonlinearities are included. Whether or not such a model is possible remains unknown, as some components of nonlinearities are still far from being fully understood and have not yet been accurately identified. Indeed, given both strong and weak nonlinearities involved in magnetic bearings, such a model, if developed, may be either too hard to be analytically tractable or too complex to be useful. On the other hand, it is not too difficult to envisage improvements in existing models developed in this thesis by an inclusion of one or two components of less important nonlinearities.

The nonlinear magnetic characteristics of high-Tc superconductor and the permanent magnet system are not fully understood either, although Hikihara, Adachi, Moon and Ueda [1999] reported on the dynamic behaviour of a flywheel rotor suspended by

HTSC magnetic bearing and showed a gyroscopic motion under a hysteretic suspending force between HTSCs and permanent magnets.

In addition to studying the nonlinear dynamics of magnetic bearings using complex models, it is of interest to examine the dynamic behaviour of magnetic bearing systems after single or multiple poles fail. This issue has not been significantly pursued in literature from the nonlinear dynamics point of view, although existing studies have addressed this issue from the controller design point of view by developing fault-tolerant control schemes using the linearised magnetic forces and linear system models [Maslen et al., 1999; Chen, 1999; Sahinkaya, Cole, Keogh and Burrows, 2000; Cole and Burrows, 2001; Na and Palazzolo, 2000; Na, Palazzolo and Provenza, 2002; Na, 2004]. System faults can be broadly classified as either internal or external to the magnetic bearing control system [Cole, Keogh, Sahinkaya and Burrows, 2004]. The principal objective of fault-tolerant control is to provide uninterrupted control and high load capacity for continuous operation of the bearing when power amplifiers or coils suddenly fail. Failure of a single system component can give rise to destructive rotor dynamic behaviour. In the case of single or multiple coil failures, the other coils are required to produce the desired force necessary for suspension when some coils in a magnetic bearing fail suddenly. Relatively large increase in currents and flux densities would then be required to maintain stability and similar dynamic properties before and after a failure occurs. As a result, nonlinearities become strongly significant for large currents, large magnetic forces as well as small air gaps, while linearization about the rotor equilibrium position and nominal perturbation current is valid only for small coil control current variations under constant bias current and small rotor displacements. An understanding of the transient response of a rotor supported by magnetic bearings with one (or more) failed pole(s) would definitely provide useful information for the detection and control compensation needed to alleviate the effect of undesirable pole failures, thereby eliminating the possible occurrence of severe damage of the whole magnetic bearing systems. The transient response from normal operation to fault-tolerant control with some coils failed is also of interest. It is conjectured that the transient response of the orbit of rotor would become elliptic due to asymmetric position stiffness of the failed bearings.

Another promising direction of future research appears to be bifurcation control and anti-control of magnetic bearing systems. Bifurcation control and anti-control deal with modification of system bifurcative characteristics by a designed control input [Chen, Moiola and Wang, 1999; 2000; Chen, Hill and Yu, 2003]. Typical objectives of bifurcation control and anti-control include delaying the onset of an inherent bifurcation, stabilizing an unstable solution, introducing a new bifurcation at a preferable parameter value, and optimising the system performance near a bifurcation point. It has been shown that even a simple magnetic bearing system is a rich source of bifurcation phenomena. Saddle-node bifurcations, pitchfork bifurcations and Hopf bifurcations have been found to exist in the nonlinear response of magnetic bearing systems. Unstable bifurcations are unlikely to be of use as they can lead magnetic bearing systems to harmful or even catastrophic situations. In these troublesome cases, unstable bifurcations should be either delayed in their occurrence or eliminated if possible. For example, saddle-node bifurcations may lead to jump and hysteresis phenomena and unstable bifurcations may lead to a divergent dynamic response. Control of such bifurcations not only can significantly improve the performance of magnetic bearing systems, but also can extend the operation regimes. Conventionally, a proportional-differential (PD) or proportional-integral-differential (PID) feedback controller is used to stabilize the system. Magnetic bearing systems have a significant advantage over other physical systems, as nonlinear feedback strategies can be easily implemented. User-specified controller gains will allow for more flexibility in evaluating the transient and steady state response characteristics.

The idea of bifurcation control has bee proposed in Chapter 4 for stabilizing a sub-critical Hopf bifurcation in a simple magnetic bearing system, so that undesirable unstable behaviour of the system can be prevented. Some possible topics of future research would be directed towards the control of saddle-node bifurcation and pitchfork bifurcation as well as Neimark bifurcation which commonly exist in the forced nonlinear response of magnetic bearing systems.

One interesting application of anti-control of bifurcation would be the creation of a stable Hopf bifurcation in a magnetic bearing system at some preferred parameter values. Creating Hopf bifurcations can be viewed as designing limit cycles with specified oscillatory behaviour into a system. A rotor-magnetic bearing system with a fault is generally a complicated nonlinear system, whose behaviour is complex, including quasi-periodic and chaotic vibrations. Oscillatory behaviour will aid in effective fault diagnosis. The introduction of stable amplitude-modulated motion may serve as a warning signal of an impending failure for magnetic bearings. The controlled system will then exhibit quasi-periodic motions at some preferred values of the system parameters. Anti-control of bifurcation can also be used to modify phase-locked response of a magnetic bearing system for intelligent maintenance. The synchronized response may not only be used to extract dynamic features for intelligent maintenance, but would also be used to assess the equipment performance and to detect degradation. For example, measurement of the modified dynamics can be used for on-line monitoring of the response, which will provide useful information for fault diagnosis and maintenance of magnetic bearings. Pole failures, usually caused by power amplifier failure or coil short circuit, can be catastrophic for magnetic bearings. The occurrence of these failures result in a significant change in the measured dynamics prior to catastrophic failure and measurement of this can be used as a predictive tool. Due to the response synchronization, only a few sensors will be needed to measure the signals required for the identification of a failure. It is expected that control and anti-control of bifurcation would be viable techniques for improving and optimising the performance of magnetic bearing systems.

Besides the research on conventional magnetic bearings, another topic of future research will be focusing on modelling and understanding of the nonlinear dynamics of a rotor supported by self-sensing magnetic bearings. Self-sensing magnetic bearing is a special kind of magnetic bearing working without external position sensors. The position information required by the controller is deduced from the air gap dependent properties of the electromagnets. The main advantage of self-sensing magnetic bearings is the reduction of manufacturing costs. Self-sensing bearings have a number of features that make them interesting for solving technical problems. The absence of a position sensor simplifies the construction, the assembly and maintenance of the magnetic bearing system. Two essential methods are known for the self-sensing operation, namely self-sensing magnetic bearing with a linear controller, and self-sensing magnetic bearing with modulation method. The former method extracts the position information from the coil currents. The controller adjusts the voltage over the coils and stabilizes the levitation of the rotor. The latter method is based on generating a position signal from the air gap dependence of the coil impedance.

One of the important obstacles confronting self-sensing technology is the nonlinearity associated with operation of the actuator in its magnetic saturation regime. This problem is especially important in high specific capacity magnetic bearings (high load capacity to weight ratio). The topics of future research relevant to self-sensing magnetic bearings will include the development of a nonlinear theoretical model capable of accurately predicting magnet bearing performance and of precisely deriving control signals, and a thorough understanding of the effect of nonlinearities on the estimation of the rotor position for self-sensing magnetic bearings.

The rapid development of sensing and control technology and further understanding of the nonlinear dynamic behaviour of magnetic bearing systems would definitely lead to the design of more reliable and efficient magnetic bearing systems for many new application fields. A wide application of magnetic bearing systems would no doubt stimulate increasing research interest in the nonlinear dynamics of magnetic bearing systems.

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