Uncertainty Analysis Methods For Multi-Criteria Decision Analysis

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Abstract

Planning, design and operational decisions are made under complex circumstances of multiple objectives, conflicting interests and participation of multiple stakeholders. Selection of alternatives can be performed by means of traditional economics-based methods, such as benefit-cost analysis. Alternatively, analyses of decision problems, including water resource allocation problems, which involve trade-offs among multiple criteria, can be undertaken using multi-criteria decision analysis (MCDA). MCDA is used to assist decision makers (DMs) in prioritising or selecting one or more alternatives from a finite set of available alternatives with respect to multiple, usually conflicting, criteria.

In the majority of decision problems, MCDA is complicated by input parameters that are uncertain and evaluation methods that involve different assumptions. Consequently, one of the main difficulties in applying MCDA and analysing the resultant ranking of the alternatives is the uncertainty in the input parameter values (i.e. criteria weights (CWs) and criteria performance values (PVs)). Analysing the sensitivity of decisions to various input parameter values is, therefore, an integral requirement of the decision analysis process. However, existing sensitivity analysis methods have numerous limitations when applied to MCDA, including only incorporating the uncertainty in the CWs, only varying one input parameter at a time and only being applicable to specific MCDA techniques.

As part of this research, two novel uncertainty analysis approaches for MCDA are developed, including a distance-based method and a reliability based approach, which enable the DM to examine the robustness of the ranking of the alternatives. Both of the proposed methods require deterministic MCDA to be undertaken in the first instance to obtain an initial ranking of the alternatives. The purpose of the distance-based uncertainty analysis method is to determine the minimum modification of the input parameters that is required to alter the total values of two selected alternatives such that rank equivalence occurs. The most critical criteria for rank reversal to occur are also able to be identified based on the results of the distance-based approach. The proposed stochastic method involves defining the uncertainty in the input values using probability distributions, performing a reliability analysis by Monte Carlo Simulation and undertaking a significance analysis using the Spearman Rank Correlation Coefficient. The outcomes of the stochastic uncertainty analysis approach include a distribution of the total values of each alternative based upon the expected range of input parameter values. The uncertainty analysis methods are implemented using a software program developed as part of this research, which may assist in negotiating sustainable decisions while fostering a collaborative learning process between DMs, experts and the community. The two uncertainty analysis approaches overcome the limitations of the existing sensitivity analysis methods by being applicable to multiple MCDA techniques, incorporating uncertainty in all of the input parameters simultaneously, identifying the most critical criteria to the ranking of the alternatives and enabling all actors preference values to be incorporated in the analysis.

Five publications in refereed international journals have emerged from this research, which constitute the core of the thesis (i.e. PhD by Publication). The publications highlight how uncertainty in all of the input parameters can be adequately considered in the MCDA process using the proposed uncertainty analysis approaches. The methodologies presented in the publications are demonstrated using a range of case studies from the literature, which illustrate the additional information that is able to be provided to the DM by utilising these techniques. Publications 1 and 2 (Journal of Environmental Management and European Journal of Operational Research) demonstrate the benefits of the distance-based uncertainty analysis approach compared to the existing deterministic sensitivity analysis methods. In addition, the benefits of incorporating all of the input parameters in the uncertainty analysis, as opposed to only the CWs, are illustrated. The differences between global and non-global optimisation methods are also discussed. Publications 3 and 4 (Journal of Water Resources Planning and Management and Journal of Multi-Criteria Decision Analysis) present the stochastic uncertainty analysis approach and illustrate its use with two MCDA techniques (WSM and PROMETHEE). Publication 5 (Environmental Modelling & Software) introduces the software program developed as part of this research, which implements the uncertainty analysis approaches presented in the previous publications.

Despite the benefits of the approaches presented in the publications, some limitations have been identified and are discussed in the thesis. Based on these limitations, it is recommended that the focus for further research be on developing the uncertainty analysis methods proposed (and in particular the program, and extension of the program) so that it includes additional MCDA techniques and optimisation methods. More work is also required to be undertaken on the Genetic Algorithm optimisation method in the distance-based uncertainty analysis approach, in order to simplify the specification of input parameters by decision analysts and DMs.

Declaration

I, **Kylie Marie Hyde**, declare that the work presented in this thesis is, to the best of my knowledge and belief, original and my own work, except as acknowledged in the text, and that the material has not been submitted, either in whole or in part, for a degree at this or any other university.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

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Acknowledgements

The balance of personal life with doctoral research is a complex multi-criteria decision analysis problem. A doctoral candidate is forced to trade-off recreation time against time spent with a computer and a ceiling high stack of journal papers.

(Hajkowicz, 2000)

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Publications

The following publications and conference presentations have arisen from this research:

Journal Papers:

Hyde, K.M., Maier, H.R., (2006), Distance-Based and Stochastic Uncertainty Analysis for Multi-Criteria Decision Analysis in Excel using Visual Basic for Applications, Environmental Modelling & Software, In Press.

Hyde, K.M., Maier, H.R., Colby, C.B., (2006), New Distance-based Uncertainty Analysis Approach to Multi-Criteria Decision Analysis, European Journal of Operational Research, Under Review.

Hyde, K.M., Maier, H.R., Colby, C.B., (2005), A Distance-Based Uncertainty Analysis Approach to Multi-Criteria Decision Analysis for Water Resource Decision-making, Journal of Environmental Management, Vol 77, Iss 4, pp 278-290.

Hyde, K.M., Maier, H.R., Colby, C.B., (2004), Reliability-based approach to MCDA for water resources, Journal of Water Resources Planning and Management, Vol 130, Iss 6, pp 429-438.

Hyde, K.M., Maier, H.R., Colby, C.B., (2003), Incorporating Uncertainty in the PROMETHEE MCDA Method, Journal of Multi-Criteria Decision Analysis, Vol 12, Iss 4-5, pp 245-259.

Conference Papers:

Hyde, K.M., Maier, H.R., (2004), "Distance Based Uncertainty Analysis for Multi-Criteria Decision Analysis in Excel using Visual Basic for Applications", Mini Euro Conference 2004 – Managing Uncertainty in Decision Support Models, Coimbra, Portugal, 22 – 24 September.

Hyde, K.M., Maier, H.R., (2004), "Incorporating a Distance-based Uncertainty Analysis Approach to PROMETHEE", MCDM 2004 – New Paradigms for New Decisions, Whistler, Canada, 6 – 11 August.

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Glossary of Selected Acronyms and Notation

Acronyms

AHP	Analytic Hierarchy Process
ANN	Artificial Neural Network
BCA	Benefit Cost Analysis
CAM	Conflict Analysis Model
CGT	Cooperative Game Theory
CP	Compromise Programming
CTP	Composite Programming
CWs	Criteria Weights
DEA	Data Envelope Analysis
DISID	Displaced Ideal
DIVAPIME	Determination d'Intervalles de Variation pour les Parametres d'Importance des Methodes Electre
DM	Decision Maker
DSS	Decision Support System
DST	Dempster-Shafer Theory
EF	Ecological Footprint
EIA	Environmental Impact Assessment
EIS	Environmental Impact Statement
ELECTRE	Elimination and Choice Translating Reality (Elimination Et Choix Tradusiant la Réalité)
ESAP	Evaluation and Sensitivity Analysis Program
EVI	Expected Value of Information
EVPI	Expected Value of Perfect Information

GA	Genetic Algorithm
GAIA	Graphical Analysis for Interactive Assistance
GIS	Geographical Information System
GP	Goal Programming
GRAPA	Graphical Point Allocation
GRG2	Generalised Reduced Gradient Nonlinear Optimisation Method
GRS	Graphical Rating Scale
HDT	Hasse Diagram Technique
HIPRE	Hierarchical Preference Analysis Software
IMGP	Interactive Multiple Goal Programming
IOC	Importance Order of Criteria
JAS	Judgmental Analysis System
LCA	Life Cycle Assessment
LHS	Latin Hypercube Sampling
MACBETH	Measuring Attractiveness by a Categorical Based Evaluation Technique
MAS	Multi-Agent Systems
MADM	Multiple Attribute Decision Making or Modelling
MAUT	Multi-Attribute Utility Theory
MAVF	Multi-Attribute Value Function
MAVT	Multi-Attribute Value Theory
MCA	Multiple Criteria Analysis
MCDA	Multi-Criteria Decision Analysis
MCE	Multi-Criteria Evaluation
MCQA	Multi-Criterion Q Analysis
MCS	Monte Carlo Simulation
MDI	Minimum Discrimination Information
MESA	Matrix for the Evaluation of Sustainability Achievement

MEW	Multiplicative Exponent Weighting
MODM	Multi-Objective Decision Making
MODS	Multi-Objective Decision Support
NA	Not Applicable
NAIADE	Novel Approach for Imprecise Assessment and Decision Evaluations
NPV	Net Present Value
PC	Preference Cones
PROBE	Preference Robustness Evaluation
PROMETHEE	Preference Ranking Organisation METHod for Enrichment Evaluations
PROTR	Probabilistic Trade-off Development Method
PVs	Performance Values
PW	Present Worth
SAW	Simple Additive Weighting
SMAA	Stochastic Multiobjective Acceptability Analysis
SMAA-O	Stochastic Multicriteria Acceptability Analysis with Ordinal Criteria
SMART	Simple Multi-Attribute Rating Technique
SMARTER	SMART Exploiting Ranks
STEM	Step Method
SWT	Surrogate Worth Trade-Off
TBL	Triple Bottom Line
TOPSIS	Technique for Order Preference by Similarity to an Ideal Solution
UNK	Unknown
UTA	Utility Additive

VAS	Visual Analogue Scale
VBA	Visual Basic for Applications
VIP	Variable Interdependent Parameters
WA	Weighted Average
WLAM	Weighted Linear Assignment Method
WPM	Weighted Product Method
WSM	Weighted Sum Method
ZAPROS	Closed Procedures Near Reference Situations (abbreviation of Russian words)
Z-W	Zionts-Wallenius

Notation

d_e *or* L_2	Euclidean distance
d_m or L_1	Manhattan distance
d_k	Kullback-Leibler distance
LL_{xl} and UL_{xl}	lower and upper limits, respectively, of the PVs of each criterion for the initially lower ranked alternative
LL_{xh} and UL_{xh}	lower and upper limits, respectively, of the PVs of each criterion for the initially higher ranked alternative
LL_w and UL_w	lower and upper limits, respectively, of each of the CWs
M	total number of criteria
p	preference threshold
q	indifference threshold
$\dot{V}(a_y)opt$	modified total value of the initially lower ranked alternative obtained using the optimised parameters
$V(a_x)opt$	modified total value of the initially higher ranked alternative obtained using the optimised parameters
w_{mi}	initial CW of criterion *m*
w_{mo}	optimised CW of criterion *m*

x_{mnli}	initial PV of criterion *m* of initially lower ranked alternative *n*
x_{mnlo}	optimised PV of criterion *m* of initially lower ranked alternative *n*
x_{mnhi}	initial PV of criterion *m* of initially higher ranked alternative *n*
x_{mnho}	optimised PV of criterion *m* of initially higher ranked alternative *n*
$\Pi(a,b)$	outranking degree of every alternative *a* over alternative *b*
$\phi+$	leaving flow
$\phi-$	entering flow

Chapter 1
Introduction

1.1 Research problem background

1.1.1 Water resources

Water is arguably the world's most pressing resource issue, as there is no more important commodity or natural resource than water (Crabb, 1997). Human development depends on adequate water supplies, which is a fact that has driven the location of communities, the extent of agriculture, and the shape of industry and transportation. Between 1900 and 1995, world water use increased by a factor of six, which is more than double the rate of population growth during the same period (Schonfeldt, 1999). With only approximately 0.01% of all water on earth being renewable freshwater and available for use on a sustainable basis, there is increasing competition to obtain fresh water sources for agricultural, industrial and domestic purposes. Alternative water sources are therefore being sought, resulting in an increased demand for non-conventional water supplies, such as seawater desalination.

Floods, droughts, water scarcity and water contamination are among many water problems that are present today. The single biggest factor affecting present and future water supply in most countries is water quality, with Australia being no exception. For example, in South Australia (DWR, 2000):

- The River Murray is subject to increasing salt loads from interstate and from within South Australia, which will lead to significant increases in salinity in the coming decades, with salinity levels predicted to surpass the recommended Australian Drinking Water Guidelines.

- The Mount Lofty Ranges are subject to development that can pollute its streams. The area has also undergone substantial farm dam development and suffered extensive stream ecosystem degradation. Significant water quality impacts have also been measured in metropolitan water supply catchments.

- The level of development of many of South Australia's prescribed groundwater resources is approaching, or has reached, the sustainable level and usage has been limited to this level. In some cases, the sustainable limit has been exceeded, causing unacceptable rises in salinity and a reduction in groundwater levels.

The continual degradation of South Australia's existing water supplies is of concern, as water plays an important role in South Australia's economy, and future economic development within South Australia will be dependent on a reliable supply of water. South Australians use approximately 1,400 GL of water each year, of which 930 GL is used for irrigation (Schonfeldt, 1999). Non-traditional water resources make up the majority of unallocated water resources that are amenable to development in South Australia. The future water supply for South Australia may result from conjunctive use of seawater desalination, stormwater, wastewater, groundwater and surface water. Some users of water, particularly industrial and agricultural users, do not require drinking quality water for their purposes. Consequently, more emphasis will be placed on the provision of acceptable quality water for the intended use.

Further population growth, climate variability and regulatory requirements are increasing the complexity of the water resource allocation decision making process. To address the above concerns, a methodology is required which will enable water allocation decisions to be made in South Australia and world wide, alleviating the pressure on, and current unsustainable use of, existing water supplies.

1.1.2 Decision making

Economic and population growth world wide inevitably bring expansion and re-distribution of various water users and thus reduce the efficiency of existing water allocation policies, which creates the problem of identifying an optimal balance between the re-allocation of existing supplies and construction of new supply projects. A re-allocation of water may also cause numerous social, environmental, legal, cultural and equity changes. Simonovic *et al.* (1997) and Dunning *et al.* (2000) have noted that decisions regarding water resources allocation are often characterised by inadequate alternatives, uncertain consequences, complex interactions, participation of multiple stakeholders, conflicting interests and competing objectives that reflect different interests. As a result, water resource decisions generally involve large numbers of objectives, alternatives and criteria, which are tangible and intangible, as well as qualitative and quantitative (Raju and Pillai, 1999b). Schoemaker and Russo (1994) state that there are four general approaches to decision making ranging from intuitive to highly analytical. However, Dunning *et al.* (2000) believe that human decision makers (DMs) are ill equipped for making such complex decisions and, therefore, these decisions can rarely be solved with intuition alone (Ozernoy, 1992). Consequently, a formal approach to decision making is required.

The allocation of water supply sources has commonly been based on the fundamental objective of cost minimisation using the benefit cost analysis (BCA) approach, where trade-offs are predominantly made between cost and risk, and generally no attempt is made to find an optimal solution with regard to environmental, social and political perspectives. It has been shown that the introduction of the environmental and social dimensions can greatly affect the evaluation of alternative solutions (Georgopoulou *et al.*, 1997). Although many research efforts have attempted to equate dollar values to resource costs and benefits, to enable monetary analysis of alternatives to be undertaken, this remains complex and controversial (Flug *et al.*, 2000). The difficulties associated with the application of conventional BCA evaluation methodologies have led to a search for alternative analytical methods for project evaluation (Fleming, 1999; Munda *et al.*, 1994).

In the selection of a decision making method, an important consideration is that people's willingness to accept decisions, in great part, depends upon their perception that the process of arriving at a decision was rational. People are much more likely to accept a decision if they feel that they have been fairly treated during the decision making process. There is therefore a need to offer interested parties the opportunity to clearly present their own perspective in the decision making process.

Multi-criteria decision analysis (MCDA) is a methodology which enables preference information to be incorporated in the decision making process and is used throughout the world to aid making decisions with regard to a wide range of planning problems, including energy supply (Georgopoulou *et al.*, 1998; Siskos and Hubert, 1983), waste management (Karagiannidis and Moussiopoulos, 1997; Miettinen and Salminen, 1999), fisheries (Mahmoud and Garcia, 2000), forestry (Levy *et al.*, 2000a), agricultural land use (Levy *et al.*, 1998) and revegetation (Qureshi *et al.*, 1999).

Water resource planning is a typical area where MCDA techniques can be efficiently used to assist DMs in making optimal use of resources by (Flug *et al.*, 2000; Roy and Vincke, 1981; Vincke, 1983):

- Helping to identify critical issues;
- Attaching relative priorities to those issues;
- Selecting best compromise alternatives for further consideration; and
- Enhancing communication in the study of decision problems in which several points of view must be taken into consideration.

MCDA techniques have been applied to water resource systems internationally since the early 1970s (David and Duckstein, 1976; Roy *et al.*, 1992). However, the effectiveness of these tools in the decision making process is still disputed by researchers i.e. what their strengths and limitations are, whether they are valid tools and in what sense they are valid, and, most importantly, whether they help render better decisions (Goicoechea *et al.*, 1992).

In addition, uncertainty is ubiquitous in decision making. Each stage of the MCDA decision making process involves some form of uncertainty including: the selection of the method (Bouyssou, 1990), the choice of criteria, the assessment of the values of the criteria and the choice of weights (e.g. Janssen *et al.* (1990)). Lack of information is probably the most frequent cause of uncertainty according to Zimmerman (2000). The effective management of uncertainty is one of the most fundamental problems in decision making (Felli and Hazen, 1998).

1.2 Research problem statement

It is evident in the literature that MCDA is a decision making process that is able to assist DMs in gaining an enhanced insight into the various, often conflicting, aspects of a particular decision problem involving incommensurate criteria, while progressively building a solution. However, despite the many perceived benefits of applying the MCDA process and the plethora of aggregation techniques available to conduct MCDA from decades of research and development, it is well recognised in the literature that uncertainty remains a source of concern in the decision making process. The various sources of uncertainty cast significant doubt on the solutions obtained from the analysis. For example, the ranking of alternative options for water resource management problems obtained by applying MCDA has been found by numerous researchers to be dependent on the sources of uncertainty (e.g. Martin *et al.* (1999) and *Hobbs et al.* (1992)). Ranks are frequently given without any uncertainties or confidence intervals (Bertrand-Krajewski *et al.*, 2002). French (1995) identifies ten different sources of uncertainty which may arise in decision analysis. At one level there is uncertainty about the values assigned to the criteria. At another level there is uncertainty about the ability of the selected criteria to adequately represent the objectives of the analysis (i.e. problem structuring). In addition, there is uncertainty in the interpretation of results. Variability in each of these factors, individually and collectively, has the potential to affect the rankings of the alternatives.

Preliminary work on methods to overcome uncertainty (i.e. sensitivity and uncertainty analysis and its application to the field of MCDA) has been undertaken by various researchers (e.g. Barron and Schmidt (1988), Rios Insua and French (1991) and Janssen (1996)). However, the existing methods are deemed to be inadequate, as the sources of uncertainty are not satisfactorily taken into consideration. This conclusion is based upon the findings that:

- No method of uncertainty analysis is currently applicable to all MCDA techniques;
- Existing sensitivity analysis methods are not being applied to water resources case studies, perhaps due to their perceived complexity. Instead, the effect of the uncertainty on the ranking of the alternatives is generally ascertained by seemingly random modification of various input parameter values;

- Only one source of uncertainty is generally considered, and this is predominantly the preference values assigned to the criteria. Multi-parameter simultaneous variations are not undertaken; and

- Preference values of all actors are rarely included in the analysis; generally the values are averaged or aggregated.

Consequently, confidence cannot be placed in the current outcomes of the decision analysis process. Research is therefore required to improve the decision making process and thereby enable decisions regarding the feasibility and regional consequences of augmenting current water supplies by various other water sources to be made.

1.3 Research aim and objectives

The overall aim of the research project is stated as:

> *To develop and apply an improved MCDA methodology to enable water resources to be allocated efficiently considering social, environmental and economic implications, with known certainty in the decision outcomes and an understanding of the sensitivity of the ranking of the alternatives to uncertainty in the input parameters*.

The scope of this research, to achieve the overall aim, is defined by three primary and four secondary research objectives. Reference to these objectives is made throughout this thesis. Following is a statement of the objectives, and a discussion of their relevance is contained in Section 1.4:

I. Summarise the current knowledge regarding the various aspects of the MCDA process and identify any limitations of that process.

II. Development of an improved decision making approach, which will address the major shortcomings of the existing MCDA process identified in the literature. In particular, an uncertainty analysis methodology will be developed, which:

   a. Incorporates the preference information obtained from all actors involved in the decision analysis;

b. Enables multi-parameter variation of the input data to be incorporated into any sensitivity / robustness analysis, to quantify the uncertainty associated with any robustness of the rankings (which may or may not enable robust rankings to be obtained);

c. Identifies the most sensitive, and therefore most critical, input parameters to the decision outcome; and

d. Is applicable to a variety of MCDA techniques.

III. Apply and test the proposed approach on case studies, including existing water resources case studies in the literature.

1.4 Value of research

In broad terms, the expected outcomes of the research with regard to MCDA are:

- An improved uncertainty analysis methodology that will enable an understanding of the relative significance of various factors affecting the decision making process, including uncertainty in input parameters, to be gained; and

- Outcomes from the MCDA process that are less reliant on the initial assumptions that are made i.e. robust solutions.

The project will provide unbiased and independent research to obtain an integrated and holistic solution to the problem of decision making with environmental, social and economic criteria, and will quantify uncertainty in the decision making process. The significance of the proposed research lies in the advanced methodology that will be developed to provide additional insight to problems requiring formal decision analysis, in addition to quantifying the substantial amount of uncertainty currently involved in undertaking MCDA. The outcomes of this research may prevent the current, frequent practice, of discarding the results of a decision analysis due to confusion in the outputs, preferring instead to rely on cognitive decisions.

The research is also significant from a practical perspective, as there has been limited utilisation of the MCDA process to date in Australia (Proctor, 2001), in comparison to international standards, mainly due, perhaps, to research in this field being predominantly undertaken in Europe and the United States of America. A number of applications of MCDA have been reported in Australia, which are summarised in Table 1.1. Awareness of the applicability of MCDA to Australian water resources case studies will be enhanced through publication of the research findings in reputable and widely circulated Australian and international conferences and journals. In Australia, natural resource decision making is influenced by the release of a National Strategy for Ecologically Sustainable Development (Australian Commonwealth, 1992) and MCDA is consistent with the principles set out in the strategy.

Table 1.1 Some applications of MCDA reported in Australia

Reference	**Application**	**MCDA Technique**
Assim and Hill (1997)	Water management plans for the Murrumbidgee Irrigation Area and Districts	Not Available
Fleming (1999)	Water resource management, Northern Adelaide Plains, South Australia	EVAMIX
Lawrence *et al.* (2000)	Proposed water infrastructure developments in northern Queensland	Additive value function (using software 'Facilitator')
Qureshi and Harrison (2001)	Riparian vegetation options for the Scheu Creek Catchment in North Queensland	AHP
Deng *et al.* (2002)	Tourism attributes in Victorian parks	AHP
Proctor and Drechsler (2003)	Recreation and tourism activities in the Upper Goulburn Broken Catchment of Victoria	PROMETHEE (using software 'ProDecX')
Herath (2004)	Options for Wonga Wetlands management in the River Murray	AHP

Finally, as stated previously, water is an essential commodity and the way decisions are made for the future is imperative to the sustainability of the resource. Implementation of the methods developed during this research will enable improved planning of water resource allocation and water infrastructure development. However, the methods that have been developed and validated throughout this research to improve the MCDA process are not only applicable to water resource decision making, but any resource allocation problem which requires the inclusion of environmental, social and economic concerns in the analysis.

1.5 Organisation of thesis

The thesis is presented as a culmination of the research that has been undertaken and the subsequent papers that have been published. Figure 1.1 contains a flow chart of the subject matter presented in this thesis. Following is a description of the contents and purpose of each chapter in meeting the above stated research objectives (Section 1.3).

The thesis commences with an in depth analysis of the purpose of, and current thinking on, decision theory. **Chapter 2** provides a broad discussion on the subject, firstly reviewing the function of decision support and then evaluating a number of decision making methods that are available. The decision making method Multi-Criteria Decision Analysis (MCDA) is explored further, as it is concluded that this is the most appropriate decision making method for the purpose of water resource allocation decisions or any decision that requires inclusion of environmental, social and economic factors. Each stage of the MCDA process is discussed in detail with an emphasis on the perceived limitations and uncertainties within the process, with the aim of elucidating the rationale behind undertaking the research presented in this thesis. Chapter 2, therefore, addresses objective I of the thesis.

The main focus of this research, developed through review of the extensive literature on MCDA, as detailed in Chapter 2, is on how to incorporate all sources of uncertainty in the input parameters in the MCDA process to enable the DM to have confidence in, and an explicit understanding of, the outcomes of the decision analysis. Therefore, alternative methods and tools that have been proposed and are currently used to evaluate the sensitivity and uncertainty of the input parameters are discussed in **Chapter 3**.

Chapter 4 provides a description of two new methods that have been developed through this research to incorporate uncertainty in the decision making process and overcome the shortcomings of the existing sensitivity analysis methods discussed in Chapter 3. The philosophical, theoretical and mathematical basis of the two methods, which include a distance-based and stochastic method, is presented. In addition, a program developed in Visual Basic for Applications (VBA) to implement the two methods is described. The two methods and the associated program constitute the main contributions of the innovation arising from this research. Chapter 4 addresses the second objective of the thesis.

In **Chapter 5,** the proposed uncertainty analysis methods are compared to a number of the existing deterministic and stochastic sensitivity analysis methods contained in the literature (and described in Chapter 3) in order to demonstrate the shortcomings of these existing methods and the benefits of the proposed methods.

Five journal papers have been produced, and subsequently published (or been accepted for publication), based upon the information and research presented in Chapters 2 to 5. The papers demonstrate the contribution that has been made to the knowledge of the MCDA discipline through the research that has been undertaken. The papers, which have been peer reviewed and published in a range of well respected international journals, are summarised and presented in **Chapter 6.** It should also be noted that the findings of the research contained in this thesis has been presented at a range of Australian and International Conferences. All of the papers include applications of the methodology using case studies from the literature, therefore, Chapter 6 addresses objective III of the thesis.

Conclusions of this research, including the principal significance of the findings and the problems encountered, are included in **Chapter 7**. Recommendations relating to application of the uncertainty analysis methods and further directions of the research are also offered in Chapter 7.

At the conclusion of the main body of this document is **Chapter 8**, which contains an extensive reference list summarising the literature reviewed in this research, including books, theses, reports, journal papers and conference papers from around the world.

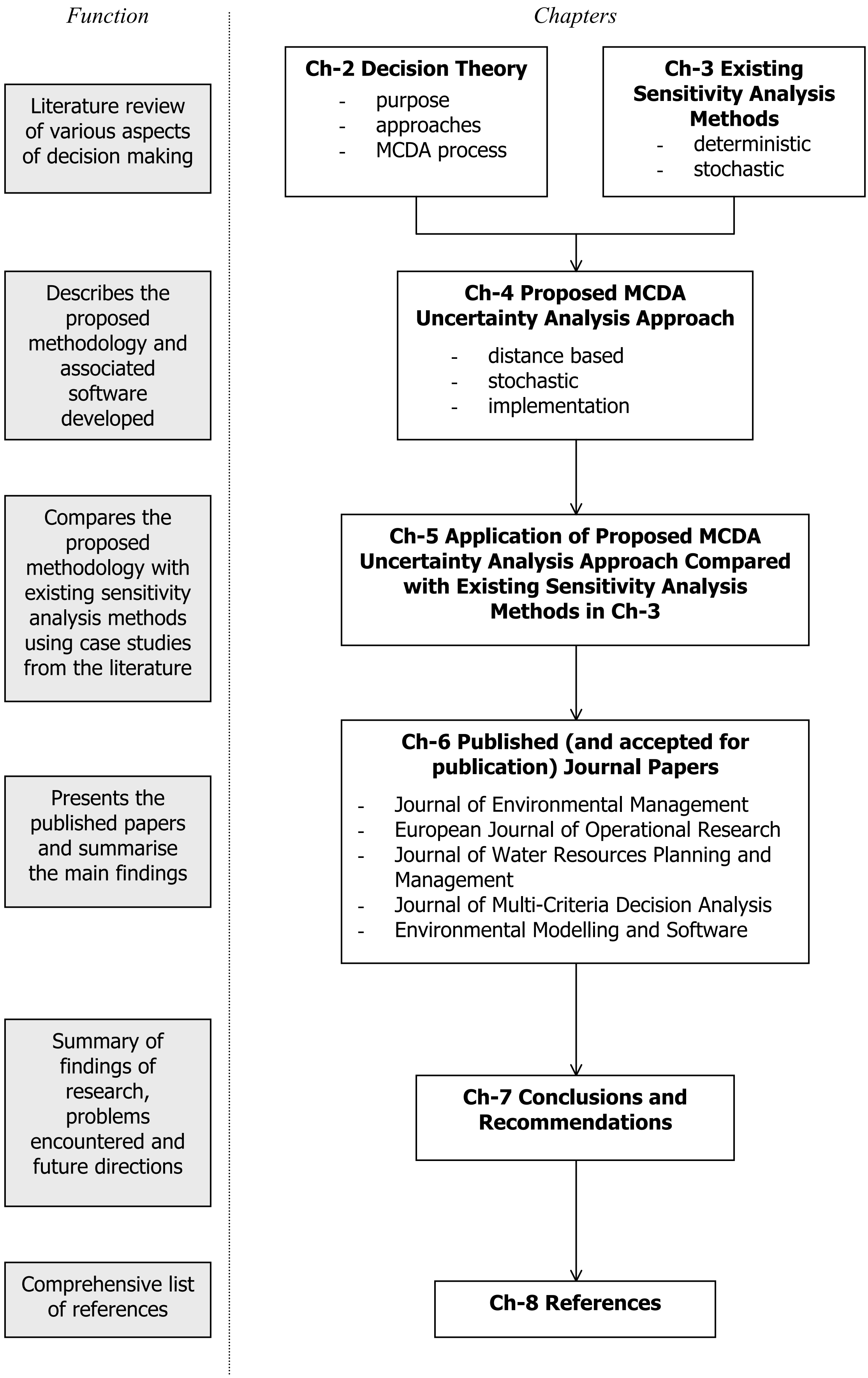


Figure 1.1 Flow chart summarising contents of thesis

Chapter 2
Decision Theory

2.1 Purpose of decision support

A decision represents the selection of one course of action from a set of alternatives. From a broad perspective, decision support relates to all forms of information generation and presentation that assist a decision maker (DM). One perspective of decision support is as a process that aids in making trade-offs, which are necessarily present in any decision problem, and the subsequent identification of an appropriate course of action (Hajkowicz, 2000). In this sense, only those techniques that deal with the concepts of efficiency, rationality and optimality in the decision process are included. These decision support methods do not merely present information rather they allow a DM to explore the impacts of their preferences within the confines of a given decision.

The concept of rationality provides the underlying theory and basis of decision support. Hollick (1993) defines a rational decision as "one that is consistent with the values, alternatives and information weighed by the individual or group making it". The purpose of decision support is to help make decisions more rational. This is an important difference to the commonly held notion of decision support systems being aimed at making 'better' decisions. Gough and Ward (1996) suggest that the concept of what constitutes a 'good' decision can be approached from one of two perspectives, namely substantive and procedural rationality. From the perspective of substantive rationality, a decision is considered good if the outcome of that decision is also good. Procedural rationality indicates that a decision is good if the procedure used to make the decision was also good.

Hajkowicz (2000) lists four criteria by which the quality of decision procedures may be judged. These include:

- *Transparency*

  Transparency of the process is one of the key requirements. Decisions involving multiple stakeholders, which lead to considerable gains or losses for certain parties, are likely to be closely scrutinised. By making explicit their preferences and reasons for selecting a particular course of action, DMs can place themselves in a more defensible position. Therefore, in group decision making and especially in evaluating public policy strategies, decisions need to be justified. With a transparent process the possible uncertainties and misinterpretations can be reduced both in communications and in combining the conflicting views. The stakeholders can therefore be assured that they are included in the process.

- *Understanding*

  Through using a more structured approach, DMs will generally gain a better understanding of the trade-offs and preferences involved in the problem.

- *Efficiency*

  Unstructured decision processes may be repetitive and cyclical with no clear gain.

- *Broad stakeholder participation*

  A good decision support framework will permit and / or encourage widespread involvement from members of the community. This helps render decision making more democratic and provides greater legitimacy to the decisions that are made.

Generally, when a decision model is used, the DM has acknowledged the existence of short-comings to their cognitive ability to make a sound decision. Decision support tools have direct and indirect value. The direct value of decision support is the prescribing of a decision alternative identified as optimal based on application of rational procedures. The indirect value results from the learning processes associated with using decision support. The 'black box' approach to decision making in which DMs merely provide preference information as inputs and receive evaluations of decision alternatives as outputs is likely to be of little procedural value. A more desirable approach to decision analysis involves close interaction between the DM, decision analyst and decision support model. Through this procedure, the DM can fully explore and understand the decision problem. It is important to reiterate that decision support models should not be relied upon to make the decisions, they are simply an aid that ensures full consideration is given to the most relevant facts

(Moss and Catt, 1996). Decision support models are used to further inform the DM about the decision problem.

2.2 Approaches to decision support

Models of decision support are aimed at improving human decision making processes. Decision support models use a variety of methods and tools, such as benefit cost analysis, environmental impact assessment, life cycle assessment, ecological footprint, agent modelling, triple bottom line and multi-criteria decision analysis, as part of the decision making process. These methods are described briefly below. These tools do not make the decision, they are merely tools designed to facilitate the decision making process. Each method attempts to present information in a reasoned, consistent and orderly way, amenable to interpretation by DMs (Joubert *et al.*, 1997). The type of tool selected depends on the decision being made and on the preference and capability of the DMs.

2.2.1 Benefit cost analysis

Benefit cost analysis (BCA) is a tool which enables DMs to assess the positive and negative effects of a set of alternatives by translating all impacts into a common measurement unit, usually monetary. This means that impacts that do not have a monetary value, such as environmental impacts, must be estimated in monetary terms. There are several ways to do this, such as estimating the costs of avoiding a negative effect (e.g. the cost of pollution control on an incinerator) or to establish how much individuals are willing to pay for an environmental improvement. Social impacts can also be evaluated in the same way. The main methods and approaches for valuing impacts in economic terms, as summarised by Lutz and Munasinghe (1994), include:

- Change in productivity;
- Loss of earnings;
- Defensive expenditures;
- Travel cost method;
- Wage differences;
- Property values;

- Avoidance cost;
- Replacement cost; and
- Contingent valuation method (which is a tool for estimating people's willingness to pay when there is no direct or indirect market for an effect i.e. benefits society receive from use of natural resources (Holland, 1997; Joubert *et al.*, 1997; Loomis, 2000)). For example, Hanley and Nevin (1999) applied contingent valuation in an attempt to evaluate residents' preferences over three proposed renewable energy options in Scotland. It aimed specifically to elicit monetary values for the environmental benefits and costs, as perceived by residents, for each renewable energy option. Raje *et al.* (2002) determine consumers' willingness to pay for water supply services.

The most commonly used evaluation method for comparing costs and benefits is the present worth (PW). The PW is derived from the net present value of costs (NPV_{costs}) subtracted from the net present value of benefits ($NPV_{benefits}$). Net present value of costs and benefits is determined using the discount rate, which indicates the rate at which dollar costs or benefits change in magnitude over time. On the completion of the analysis, the alternative with the greatest benefit and least cost is the preferred alternative, as BCA is concerned with ensuring that the benefits of decisions exceed the costs.

The BCA technique is one of the most widely applied methods for evaluating decision alternatives in a public policy setting (Hollick, 1993; Joubert *et al.*, 1997) and the general process for undertaking this methodology is (Joubert *et al.*, 1997):

1. Define the set of project alternatives;
2. Assess the impacts of each alternative;
3. Order the alternatives in terms of time;
4. Weight impacts of income distribution;
5. Convert the stream of weighted benefits and costs into a single net present value for each alternative; and
6. Perform a sensitivity analysis.

The benefit of BCA is that the results are presented in a clear manner, with all impacts summed up into one monetary figure. However, there is uncertainty involved in estimating the monetary value of environmental and / or social impacts and it also raises ethical issues (Holland, 1997; Morrissey and Browne, 2004). Environmental and resource problems have far-reaching economic and ecological aspects, which cannot always be encapsulated by a market system (Munda *et al.*, 1994). Therefore, the effects that are difficult to value are generally simply omitted from the analysis (Lutz and Munasinghe, 1994; Merkhofer, 1999). In addition, environmental decision making usually involves competing interest groups, conflicting objectives and different types of information and BCA is not a suitable decision aid for such a decision (Morrissey and Browne, 2004; Prato, 1999). The maximisation of economic efficiency is usually the overriding factor in a BCA at the expense of environmental and social criteria (Morrissey and Browne, 2004).

2.2.2 Environmental impact assessment

Environmental impact assessment (EIA) was developed in the 1950s and in January 1970, the USA had become the first country in the world to adopt the requirement of undertaking an EIA on major projects (Al-Rashdan *et al.*, 1999). EIA is a change-oriented, established, procedural tool (Finnveden and Moberg, 2005) that is mainly used for assessing the environmental impacts of projects and it is generally a site specific tool.

The purpose of the EIA process is to:

- Assess the impacts of a proposed activity on the environment before making the decision on whether to carry it out; and
- To develop and assess measures to avoid or minimise those impacts if it is decided to carry out the activity.

EIAs are undertaken in a variety of application areas such as large infrastructure projects including roads and railways, storage facilities, reservoirs and landfills (Janssen, 2001). For example, an EIA was undertaken on water quality deterioration caused by the decreased Ganges outflow and saline water intrusion in Bangledesh by Rahman *et al.* (2000). EIA is a potential information tool for decision support methods such as multi-criteria decision analysis (MCDA) (see Section 2.2.7) as is shown by Al-Rashdan *et al.* (1999) who utilise a combination of EIA and

MCDA to assess environmental problems in Jordan. MCDA is also often used to support EIAs in the Netherlands (Janssen, 2001).

2.2.3 Life cycle assessment

Life cycle assessment (LCA) is a holistic approach to environmental assessment which examines the environmental and social impacts of products, processes or services relative to each other across their entire life: from the production of raw materials, their processing, delivery, use and management of wastes (ISO 14040, 1997; Lundin and Morrison, 2002; Morrissey and Browne, 2004). The results of LCA create a perspective of the environment, which can broaden decision making beyond consideration of cost-effectiveness. A complete LCA study consists of the following four steps (Miettinen and Hamalainen, 1997):

1. Goal definition and scoping, which is the planning part of an LCA study;

2. Inventory analysis, where the material and energy balance of the system is calculated;

3. Impact assessment, consisting of classification, characterisation and valuation, where the potential environmental impacts of the system are evaluated; and

4. Improvement assessment, where a search for the most promising possibilities for reducing the environmental burden is conducted.

The application areas of LCAs are numerous, such as electricity generation (e.g. comparison of French coal power plants by Maurice *et al.* (2000) and electricity produced by waste incineration by Sonnemann *et al.* (2003)), transportation (e.g. comparison of different transportation fuel options by Tan *et al.* (2004)) and manufacturing (e.g. beverage packaging systems by Miettinen and Hamalainen (1997)). Few LCAs of water supply alternatives have been published. A LCA was carried out by Lundie *et al.* (2004) to examine the potential environmental impacts of Sydney Water Corporation's total operations. The aim of the study was to compare the relative sustainability of the operations under different planning scenarios, enabling consideration of environmental issues in parallel with financial, social and practical considerations in strategic planning. Another example in the literature is provided by Raluy *et al.*

(2004) where LCA was utilised to determine the environmental load of desalination technologies when integrated with different energy production systems. Australian Water Technologies Pty Ltd (2003) assessed the environmental impacts of alternative water supply augmentation options for the Eyre Peninsula region in South Australia using the LCA methodology. Three alternatives were compared in terms of the most relevant LCA indicators: energy consumption, global warming potential, eutrophication potential, photochemical oxidant formation potential, human toxicity potential and exotoxicity potential in the marine and terrestrial environments.

Applications of LCA demonstrate that it is a very powerful technique to calculate the total input and output flow of materials and energy from and to the environment during every step of a products life (Le Teno and Mareschal, 1998). LCA may be understood as a methodology for developing quantitative measures of global and regional potential environmental impacts of various options and, by definition, LCA only considers environmental issues. In reality there are also other issues (e.g. social, economic, political and technical) that cannot be ignored in any decision. Therefore, LCA should be seen in a broader context, as a tool that provides information on the environmental impacts for decision making (Miettinen and Hamalainen, 1997) and should not be used in isolation (Morrissey and Browne, 2004). This makes it a potential information tool for decision support methods such as MCDA (see Section 2.2.7). LCA is also limited as a decision support method because it has traditionally not been subject to public involvement (Morrissey and Browne, 2004).

2.2.4 Ecological footprint

The University of British Columbia's School of Community and Regional Planning developed the Ecological footprint (EF) in the early 1990s and over recent years the EF has become established as an important environmental indicator (McDonald and Patterson, 2004). The EF framework is a model that is based on acknowledging ecological limits and places less emphasis on the social and economic aspects of sustainability. The EF is a resource accounting and environmental education tool that inverts the traditional concept of carrying capacity (the population a given region could support) and instead seeks to determine what total area of land is required, regardless of where that land is located, to sustain a population, organisation or activity. The EF methodology assesses the land use by means of the sum of the areas that are necessary to dissipate emissions to the environment down to a natural concentration and to provide raw materials (e.g. yield of crop), energy (e.g. industrial energy yield) and infrastructure (e.g. area for the factory, raw materials and energy) (Brunner and Starkl, 2004). The EF measures human use of nature and aggregates human impact on the biosphere into one number - the bioproductive space occupied exclusively by a given human activity. This allows a comparison of nature's supply (or biocapacity) with humanity's demand (the Footprint, or consumption).

The EF concept has gained more significance since its first introduction, however, the main focus of EF studies has been on geographical entities (for example the EF of New Zealand undertaken by McDonald and Patterson (2004) and the EF of Australia undertaken by Simpson *et al.* (2000)) and more recently products and packaging systems (Lenzen *et al.*, 2003). Lenzen *et al.* (2003) calculated the EF for Sydney Water Corporation which gave valuable insights into the impacts associated with its operations and progress towards sustainability. Despite the benefits of the EF as an indicator of sustainability, Lenzen *et al.* (2003) found that a number of methodological issues limit the use of EF as a standalone tool. The inability of the EF to consider downstream impacts of the organisation's activities and the limited type of sustainability indicators capable of being included means that the EF will not be a true reflection of sustainability performance. EF can only be one input into an organisation's environmental planning and decision making processes.

2.2.5 Agent modelling

Agent based modelling is an artificial intelligence approach to simulating life-like situations. An agent is a self-contained entity that can be programmed to behave as intelligent beings to simple mechanical objects. A multi-agent model comprises agents and an environment, which both act and change in response to each other. Agents are the building blocks of an agent-based model and their complexities and abilities vary with the roles they perform within the model and with the characteristics of the object they imitate.

Multi-agent systems (MAS) enable models to be built which integrate human beings as an element of the ecosystem and can integrate socio-economic, ecological and spatial dynamics into one single model (Mathevet *et al.*, 2003). The applications of agent modelling technology are widespread and there is a large variation in the different types of agent-based models that have been used for modelling a large variety of different environmental and socio-economic systems. The use of modelling based on MAS for tackling natural resources and environment management issues is growing steadily (Barreteau *et al.*, 2001). For example, Barreteau *et al.* (2001) utilised MAS to assess the Senegal River irrigation systems and Feuillette *et al.* (2003) developed a MAS to take local and non-economic interactions into account when investigating the Kairouan water table in Central Tunisia.

Barreteau *et al.* (2001) envisage the use of MAS as a group decision support tool by providing a representation and a simulation of proposed scenarios of collective rules for common natural resource management. However, as the sole tool, they are limited as they are cumbersome and slow to develop and analysis of their results is still difficult (Barreteau *et al.*, 2001). In addition, there is no accepted way of combining different types of inputs and indicators (e.g. social, environmental, economic). The output could, however, be a potential information source for decision support methods such as MCDA (see Section 2.2.7).

2.2.6 Triple bottom line

Triple bottom line (TBL) focuses corporations, not just on the economic value they add, but also on their contribution to environmental and social values. The notion of reporting against the three components of economic, environmental and social performance is directly tied to the concept of sustainable development. TBL decision making has become an accepted approach to operationalising the intangible concepts of 'corporate social responsibility' and 'sustainability'. TBL focuses on data collection, analysis and decision making using economic, environmental and social performance information. Reporting on TBL aims to extend decision making and disclosure so that decisions explicitly take into consideration the impacts on natural and human capital, as well as financial capital.

A TBL report is therefore more than the presentation of the sum of environmental, social and economic / financial information. It must also seek to integrate this information to allow readers to understand the inter-relations and balance between the three dimensions from the standpoint of both processes (how decisions are made) and outcome (the results of decisions). The practicalities of TBL are still in development and organisations across the world are coming to terms with what this means to them, as well as how to actually measure performance. TBL reports have the same problems as agent based models with regard to how to combine the different types of inputs and indicators (e.g. social, environmental, economic), however, similar to agent modelling, the output of a TBL report may be a useful information source for decision support methods such as MCDA (see Section 2.2.7).

Some examples of companies in Australia producing public environmental reports, or TBL reports, include Sydney Water (who provides drinking water and wastewater services to people in NSW), BHP Billiton, Western Power and Telstra. However, these companies mainly undertake ex-post reporting.

2.2.7 Multi-criteria decision analysis

Multi-criteria decision analysis (MCDA) is a methodology used throughout the world to aid making decisions with regard to a wide range of planning problems, including: water resource management (Abrishamchi *et al.*, 2005; Duckstein *et al.*, 1994; Netto *et al.*, 1996; Ridgley and Rijsberman, 1994), wastewater management (Tecle *et al.*, 1988), energy supply (Georgopoulou *et al.*, 1998; Siskos and Hubert, 1983), waste management (Karagiannidis and Moussiopoulos, 1997; Miettinen and Salminen, 1999), fisheries (Mahmoud and Garcia, 2000), forestry (Levy *et al.*, 2000a), agricultural land use (Levy *et al.*, 1998) and revegetation (Qureshi *et al.*, 1999).

MCDA provides a way to systematically structure and analyse complex decision problems (Mustajoki *et al.*, 2004). The philosophical bases of a multi-criteria approach are to provide insight into the nature of the conflicts among objectives and reach consensus among stakeholders, rather than eliminating the conflicts. The main benefits are often related to the process and the increased problem understanding that it creates (Hamalainen and Salo, 1997). MCDA improves the ability of DMs to explore and assess trade-offs between the achievements of alternatives and to analyse their impacts on different stakeholders (Mysiak *et al.*, 2005).

The benefits of MCDA are (Morrissey and Browne, 2004; Mustajoki *et al.*, 2004; Mysiak *et al.*, 2005):

- It provides a systematic approach to evaluate policy options and helps enhance the mutual understanding and consensus between stakeholders;
- A mixture of qualitative and quantitative information can be incorporated thereby not requiring the assignment of monetary values to ecological services and avoiding some of the ethical and practical shortcomings of BCA. MCDA therefore goes beyond the evaluation of purely economic consequences and allows non-economic criteria to be assessed on an equal basis;
- Account can be taken of the preferences of the various stakeholder groups with conflicting objectives; and
- It facilitates public participation and collaborative decision making.

The operational differences between BCA and MCDA are threefold (Joubert *et al.*, 1997):

- BCA reduces problems to a single dimension objective function (real net present value). In contrast, MCDA explicitly introduces several criteria, each representing a particular dimension of the problem or point of view;
- In BCA, typically all impacts and expressed preferences are converted to common units (money). In order to have common units of comparison, MCDA rates or ranks alternatives on a preference scale for each criterion and weights the criteria, thereby avoiding the need to convert to monetary units; and
- Conventionally, BCA only attempts to make trade-offs between the dimensions of the problem explicit within its sensitivity analysis, while under MCDA the trade-offs between different stakeholders and criteria are a focus of the analysis.

Water resource planning is undoubtedly a typical area where MCDA techniques can be efficiently used to help DMs make optimal use of resources (Netto *et al.*, 1996). By taking several individual and often conflicting criteria into account in a multi-dimensional way, MCDA will lead to more robust decision making rather than optimising a single dimensional objective function (such as BCA) (Morrissey and Browne, 2004).

Despite the perceived benefits of MCDA, there are also numerous shortcomings, which include:

- There are a multitude of MCDA techniques available but it is often not clear to the DM which method is most appropriate for a particular decision making situation;
- Limited real-world applications of MCDA are reported in the literature; and
- Incorporation of the preferences of multiple stakeholders can be difficult with the majority of MCDA techniques and a lot of this information is subsequently 'lost' from the analysis.

2.3 Selection of decision support method

When linking tools and decision context, some aspects of the context will influence the choice of tool, whereas others will influence how the tool is used. For example, LCA is traditionally used for products and EIA is traditionally used for projects. Another aspect which determines the choice of tool is what types of impacts the DM is interested in. Different stakeholders may also find different tools appropriate for different situations. For example, it is well known that, in particular in urban water management, practitioners may generally be reluctant to base their decisions on integrated decision aid methods that go beyond cost based techniques (Brunner and Starkl, 2004).

Opinions on the 'best' method to use for a particular decision making situation are divergent and all of the methods described in Section 2.2 have their strengths and weaknesses. All methods presented have a role to play in ensuring that environmental considerations are included at different stages of the planning process. MCDA is, however, regarded to be of considerable potential value to water resource issues and therefore has been selected as the decision making method to utilise in this research. It should be noted that the literature on MCDA is scattered and does not sit entirely within any of the academic disciplines (psychology, civil engineering, management science, operational research and natural resources) (Al-Shemmeri *et al.*, 1997a), therefore, an extensive review of the literature has been undertaken and a consolidation of the findings is contained in the sections below.

2.4 Definition of MCDA terminology

One issue that causes considerable misunderstanding and confusion in the field of MCDA is the absence of a consistent and unified terminology (Bana e Costa *et al.*, 1997). In decision analysis, a variety of terms appear in the literature and are used in slightly different ways by different authors and often the terms are used interchangeably (MacCrimmon, 1973). Acronyms, which essentially have the same meaning, include:

- MCA – multiple criteria analysis;
- MODS – multiple objective decision support;
- MADM – multiple attribute decision making or modelling;
- MCE – multi-criteria evaluation;
- MCDA – multiple criteria decision aid or multi-criteria decision analysis; and
- MCDM – multiple criteria decision making or modelling.

The term 'multi-criteria decision analysis' (MCDA) will be used in this thesis to refer to techniques that have the following characteristics:

- A finite number of alternative plans or options;
- A set of criteria by which the alternatives are to be judged; and
- A method for ranking the alternatives based on how well they satisfy the criteria.

The terms criterion and attribute, and action and alternative are used interchangeably throughout the literature. To avoid any ambiguity, the terms criterion and alternative will be used henceforth in this thesis. Useful general definitions of the terms used frequently in this thesis are:

- A 'criterion' is a tool allowing comparison of alternatives according to a particular point of view (Bouyssou, 1990). Hobbs *et al.*

(1992) state that a criterion is a physical, biological, economic or other characteristic of the alternatives that the actors consider important;

- ‘Actors’ comprise individuals, groups of individuals, institutions and administrative authorities which influence directly or indirectly the decision making process through their priorities and value systems (Georgopoulou *et al.*, 1997);

- A ‘stakeholder’ is any individual or group which can affect or is affected by a decision that is made;

- ‘Alternatives’ are the courses of action which can be pursued and which will have outcomes measured in terms of the criteria (Corner *et al.*, 2001);

- An ‘objective’ is defined as a statement describing the desired future state of reality (Hajkowicz, 2000); and

- An ‘aggregation method’ is the algorithm for the synthesis of the MCDA input data (Belton and Pictet, 1997) i.e. MCDA technique.

2.5 MCDA process

The structuring and framing of a decision situation is a constructive and learning process, which seeks to build a ‘more-or-less’ formal representation of the decision. The MCDA process integrates the objective environmental components of the decision context, with the subjective and context-dependent points of view, concerns or objectives, in such a way that the value-systems of actors or stakeholders are made explicit (Bana e Costa *et al.*, 1997). The structuring phase is often considered to be the most important step of the decision making process in order to arrive at an accepted compromise solution (Corner *et al.*, 2001; Georgopoulou *et al.*, 1998). Frequently, however, much more emphasis is given to the mechanics of a particular MCDA method than this initial step of the procedure (Georgopoulou *et al.*, 1997).

MCDA typically involves several key stages, which are collectively referred to as the MCDA process. Numerous authors have identified stages of the MCDA process and whilst minor differences exist, most descriptions of the MCDA process distinguish similar stages and it is generally the order of the stages and the level of detail that differs. As an example, the key elements of the decision process, as recommended by Lawrence *et al.* (2001), are summarised in Table 2.1. Lawrence *et al.* (2001) define the process as iterative and state that it may be far from sequential and continuous.

The main stages of the MCDA process, as defined for the purposes of this thesis, are shown on Figure 2.1, and are discussed in the following sections.

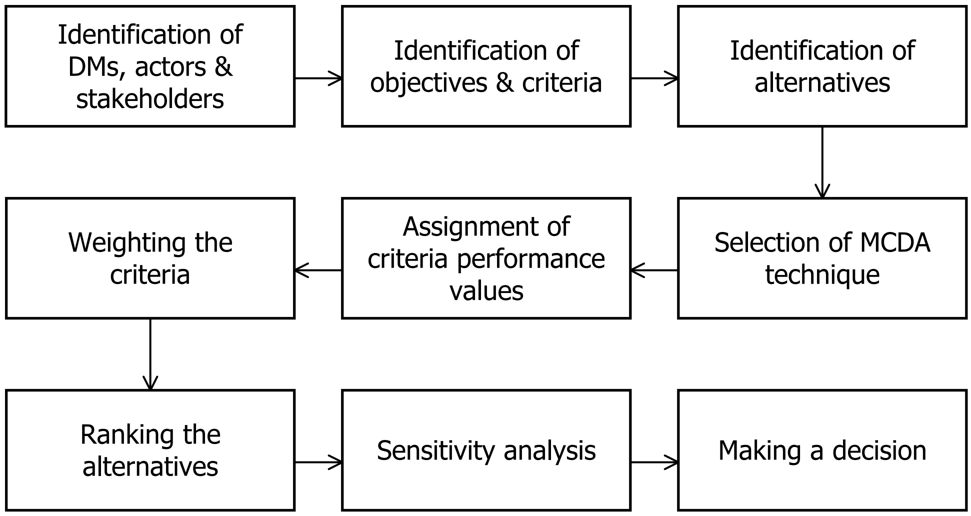


Figure 2.1 Summary of the MCDA process

2.5.1 Identification of decision makers, actors and stakeholders

An important aspect of the decision making process in a democratic society is the question of 'who decides?' Increasing attention has been given to incorporating public participation into the decision making process. The advantages of allowing public involvement in decision making have been well documented and such participation often strives for wider community understanding and therefore sanctioning of the decision concerned (Proctor and Drechsler, 2003).

Table 2.1 The key elements of the MCDA process

NOTE:
This table is included on page 28 of the print copy of the thesis held in the University of Adelaide Library.

Source: Lawrence *et al.* (2001)

The stakeholders consist of all the different people associated with the planning and decision process. In the beginning of the process, all stakeholders should be identified and it be explicitly determined who should participate in the planning process (i.e. actors), in which phases, and to what extent. Keeping the actors informed from the beginning will increase the probability of a successful decision process (Lahdelma *et al.*, 2000). Actors include policy makers, planners, administrators and others and are generally selected to be representative of the stakeholders of the particular decision problem. The process of selection of actors should be open and transparent. The number of actors varies with each decision problem, depending on factors such as the time and resources available and the perceived level of importance of the decision. Depending on their interests, the actors will stand up for different alternatives and objectives, thus creating competition and conflicts based on misunderstanding, opposing interests and different values. For successful planning and decision making, it is important to identify the true points of views of actors (Lahdelma *et al.*, 2000). Only after all the points of views of actors are recognised is it possible to identify the criteria necessary for decision making. Thus, the criteria come from the actors involved in the process (i.e. the criteria are context dependent).

2.5.2 Identification of objectives and criteria

An important component of the MCDA process is the articulation of the intent and definition of the decision criteria. The criteria are designed to compare and assess each of the alternatives and therefore must relate to the overall objective of the decision making task. Criteria are essential components of MCDA since they form the basis for the evaluation of the considered alternatives. Criteria and objectives will have the most significant impact on the final ranking of alternatives as they determine the information inputs to the MCDA model (Hajkowicz *et al.*, 2000). Hokkanen and Salminen (1994) believe that the determination of criteria, which are understood and accepted by all actors, is a central, and difficult, problem in MCDA.

According to Jacquet-Lagreze and Siskos (2001), four types of criteria are used in MCDA:

1. Measurable criteria;
2. Ordinal criteria;
3. Probabilistic criteria; and
4. Fuzzy criteria.

The set of criteria selected should be complete without being redundant (i.e. all major aspects are taken into consideration), while at the same time the number of criteria should be as small as possible and a double-counting of the impacts avoided (Georgopoulou *et al.*, 1998). Bouyssou (1990), Georgopoulou *et al.* (1997), and Al-Kloub *et al.* (1997) all concur that a consistent family of criteria, in order to assist the proper evaluation of potential alternatives, must be:

- Legible (i.e. contain a sufficiently small number of criteria so as to provide a basis for discussion, allowing the assessment of inter-criteria information necessary for the implementation of an aggregation procedure);
- Operational / understandable / measurable (i.e. considered by all DMs as a sound basis for the continuation of the decision aid study);
- Exhaustive / complete (i.e. contain every important point of view);
- Monotonic (i.e. the partial preferences modelled by each criterion have to be consistent with the global preferences expressed on the alternatives);
- Non-redundant (i.e. criteria should not be double counted); and
- Minimal / essential (i.e. unnecessary criteria should not be included).

The criteria against which the merits of alternatives are assessed depend on the project and the defined objectives. The selection process of the decision criteria must involve discussions with the actors as well as studying the physical system at hand (Hamalainen *et al.*, 2001). This step in the process may require several iterations of discussions. Project evaluation may have to consider technical, economic, financial, legal, environmental, social, political, risk and sustainability aspects of performance. Kheireldin and Fahmy (2001) describe an 'inductive approach' to select the criteria, which starts with an inventory of all features of the alternatives and then these features are grouped and aggregated in such a way that a set of key evaluation criteria is developed. Simonovic and Bender (1996) propose a model for determining evaluation criteria which uses grounded theory from the social sciences to build an objective structure which is capable of representing the interests of all parties involved.

As actors must weight criteria at some stage in the MCDA process, it is advisable to limit the number of criteria to a manageable size. Assessing the relative importance of individual criteria amidst a large number of criteria can easily exceed the cognitive abilities of the actors (Hajkowicz *et al.*, 2000). Bouyssou (1990) also believes that the number of criteria should be restricted because of the cognitive limitations of the human mind and because of the need to gather the necessary information for each alternative. There appears to be a general rule of thumb that the number of criteria for a decision analysis should not exceed 10 or 12 (Belton and Vickers, 1990; Bouyssou, 1990; Proctor, 2001) although some studies have used more (see Appendix A which summarises applications of MCDA in the literature and includes the number of criteria used in each case study, where available).

2.5.3 Identification of alternatives

Generating alternatives is a very important stage in the structuring of a decision problem (Ozernoy, 1984). Zionts (1983) states that the process of determining the set of alternatives may require more effort than choosing among the alternatives. Hajkowicz *et al.* (2000) found that very little research has been undertaken on techniques and processes for identifying alternatives, however, some suggested methods for identifying alternatives are (Merkhofer, 1999):

- Group participation and structured brainstorming;
- Strategy tables; and

- Progressively, as information is introduced and analysed throughout the decision procedure.

With strategy tables, different mechanisms or types of actions that might be considered are listed as columns in a matrix (Merkhofer, 1999), as shown in Table 2.2. Alternative strategies are developed by selecting compatible combinations of actions from the different columns. The strategy table facilitates the identification and construction of alternatives for complex decisions by encouraging a systematic, comprehensive consideration of options (Merkhofer, 1999).

Table 2.2 Sample strategy table for identifying alternatives

Water Source	Storage	Treatment	Energy
Groundwater	Reservoir	Desalination	Solar
Seawater	Tank		Wind
Wastewater	Aquifer		Grid
Surface water			Diesel

Whatever method is used to arrive at a set of alternatives, the selection of these alternatives is critically important to the success of the decision analysis. The alternatives should be defined explicitly and stated clearly. The number of alternatives is highly situational and may vary between any discrete number and infinity (Munda *et al.*, 1994). It is recommended to start with a relatively coarsely defined set of alternatives, in order to be representative. The alternatives need to include a sufficient diversity of options so that any potentially feasible alternative may be found either within this set, or by interpolation between elements of this set (Stewart and Scott, 1995).

Computer programs can assist in formulating alternatives by generating all possible combinations of a set of factors. A long list of alternatives may be reduced to a manageable set by eliminating those that do not satisfy an initial screening criterion (Goicoechea *et al.*, 1992; Hajkowicz *et al.*, 2000; Ozernoy, 1984; Royal Assessment Commission, 1992; Stewart and Scott, 1995). The main purpose of screening is to remove inferior alternatives from the feasible set, so that the remaining alternatives can be investigated in more detail (Rajabi *et al.*, 2001). Screening criteria are a key element in identifying feasible alternatives.

Klauer *et al.* (2002) also suggest that it is important to include a variety of alternatives which reflect the preferences of all stakeholders. Choosing some alternatives that only reflect the view of some experts or interest groups is to be avoided in order to build trust among the stakeholders and to signal that the process opens up the possibility space to resolve the decision at hand.

2.5.4 Selection of MCDA technique(s)

MCDA techniques allow the DMs to understand the properties of different alternatives and the implication of their choices (Simonovic and Fahmy, 1999). Different behavioural scientists, operational researchers and decision theorists have proposed a variety of methods describing how a DM might arrive at a preference judgment when choosing among multiple criteria alternatives (Zanakis *et al.*, 1998). Dunning *et al.* (2000) state that many of the tools and techniques of MCDA have been motivated by problems regarding water resources and planning and therefore have been developed by workers in this field. General approaches to decision making range from intuitive to highly analytical (Schoemaker and Russo, 1994). MCDA has been evolving considerably since its origin during the 1960s and has been moving from optimisation methods to more interactive decision support tools (Bender and Simonovic, 2000). Some methods have many features in common while others are quite distinct (Ozernoy, 1987). The great majority of decision methods and decision support systems are based on quantitative evaluations (Larichev, 1998).

Before going into further detail on selecting a MCDA technique, some more information on the types of methods available will be provided.

<u>Classification of techniques</u>

The available MCDA techniques differ in the type of information they require, the methodology they are based on, the sensitivity tools they offer and the mathematical properties they verify (Mysiak *et al.*, 2005). MCDA has been evolving considerably since its birth in the 1960s. Divergent schools of thought have developed, emphasising different techniques and, more generally, different attitudes as to the way of supporting or aiding decision making (Mysiak *et al.*, 2005; Roy and Vanderpooten, 1997).

There is an abundance of MCDA methods, therefore, to deal with this richness, some kind of classification is necessary. According to Kangas *et al.* (2001a) and Guitouni and Martel (1998), there are three operational approaches to aggregation:

1. Value and utility theory (American School) i.e. Multi-attribute Utility Theory (MAUT), Multi-attribute Value Theory (MAVT), Weighted Sum Method (WSM) and Analytic Hierarchy Process (AHP);

2. Outranking (European School) i.e. Elimination and Choice Translating Reality (ELECTRE), Preference Ranking Organisation Method for Enrichment Evaluations (PROMETHEE) and Novel approach for imprecise assessment and decision evaluations (NAIADE); and

3. Interactive approaches (Martin *et al.*, 1999) like the Step Method (STEM) and Interactive Multiple Goal Programming (IMGP).

Outranking methods are characterised by the fact that the overall ranking of the alternatives is ultimately based on a pair-wise comparison of the alternatives with respect to each criterion using pair-wise preferences. The main difference between outranking methods on the one hand and MAUT on the other hand is that the latter assumes the existence of an overall value, or utility, that is to be maximised and governs all human decisions. Consequently, most of the work of a MAUT analysis consists of extracting the utility function from the actors' mind (Klauer *et al.*, 2002). General descriptions of the most commonly used discrete MCDA techniques are provided in Appendix B and have been categorised by outranking, value and utility theory, distance-based and verbal approaches.

There are many different views on how MCDA methods should be further subdivided (Hajkowicz *et al.*, 2000). Figure 2.2 displays a classification of MCDA techniques according to Hajkowicz *et al.* (2000), however, it should be noted that outranking methods are not included in this classification. Alternatively, Kheireldin and Fahmy (2001) state that the techniques for multi-criteria evaluation can be classified into four major categories: cardinal techniques, frequency techniques, scaling modelling, and mixed data. Harboe (1992) classified MCDA techniques according to when weights are assigned by the DMs (i.e. a-priori, a-posteriori and interactive). To the knowledge of Kaliszewski (2004), several MCDA method classifications have been proposed but they are all aimed at the MCDA research community and not at the potential MCDA user community. The complexity in selecting a technique is an area of MCDA where further research is required.

Figure 2.2 Classification of MCDA techniques according to Hajkowicz *et al.* (2000)

Selection of MCDA techniques

The availability of numerous methods to solve MCDA problems characterises both the flexibility and ambiguity of the MCDA approach (Mysiak *et al.*, 2005). The broader applications of the MCDA approach are hindered by the uncertainty of choosing one particular method among all those available (Mysiak *et al.*, 2005). Although several attempts have been made to facilitate the selection of the 'best' method for a decision situation, and experimental comparisons of discrete alternative MCDA methods have been carried out (see below), there is no universally agreed set of guidelines allowing the appropriate MCDA method to be selected for a given decision situation (Hersh, 1999; Mysiak *et al.*, 2005). Ozernoy (1987), Ozernoy (1992), Hwang and Yoon (1981) and Guitouni and Martel (1998) have outlined procedures for the selection of an appropriate MCDA method, however, often these procedures serve more as a tool for elimination rather than the selection of the 'right' method.

According to some researchers, the method to be used to solve a specific multi-criteria problem is itself a MCDA problem (Abrishamchi *et al.*, 2005). However, using another MCDA method to choose the most appropriate MCDA method can lead to a vicious circle (Guitouni and Martel, 1998; Ulengin *et al.*, 2000).

The selection of the most appropriate method for the decision making situation is, therefore, a very difficult task. A review of the literature by Topcu and Ulengin (2004) has found that analysts and researchers are seemingly incapable of making a proper selection of the most appropriate MCDA method, as generally they cannot justify their reason for choosing one MCDA method rather than another. The analyst usually selects a method developed by themselves, a method the analyst has most faith in, or a method the analyst is familiar with and has used before (Ozernoy, 1992; Ulengin *et al.*, 2000). Each method may potentially lead to different rankings, and the choice of a methodology is subjective and dependent on the pre-disposition of the DM (Mysiak *et al.*, 2005). In addition, in many cases, the conditions for use are not given (MacCrimmon, 1973), as has been found through the review of literature undertaken as part of this research (see Appendix A).

For any decision problem, there may be several possible methods and no obvious reason for choosing one over the other. There is little agreement as to whether particular methods are dominant in a general sense or dominant in a particular area of application. In many applications there is no strong basis for favouring a single MCDA method over other MCDA methods.

In addition to theoretical properties, practical applicability also plays an important role in the selection of an appropriate method for the problem to be solved (Miettinen, 2001). Intuitiveness and simplicity are two properties that are seen as extremely important if a decision support tool is to be realised for multiple DMs (Bender and Simonovic, 2000). Bana e Costa (1988) is convinced that many MCDA methods, although theoretically suitable, are subject to failure in interactive practical applications because of their lack of simplicity. Georgopoulou *et al.* (1997) have found that the complexity of certain MCDA techniques is the reason why in many decision making situations in Greece the simple "weighted average" technique is usually preferred. The problem of understandability of MCDA methods by the DMs has also been encountered in studies undertaken by Kangas *et al.* (2001b). Understanding requires simplicity, as DMs will generally rather live with problems they cannot solve than to accept solutions that they do not understand.

It is recommended by Hamalainen *et al.* (2001), Miettinen (2001), Lutz and Munasinghe (1994), Hipel (1992), Hajkowicz *et al.* (2000) and Evans (1984a) that the specific characteristics of the real world problem, including the personal traits of the DMs who will use the technique (i.e. the DM ability and / or desire to articulate various amounts and types of preference information), should be used to select the most appropriate MCDA method. This is because some methods are more suitable under particular conditions than others. Cognitive effort and aspects of learning are definitely two factors that must be considered when selecting a technique. Hobbs *et al.* (1992) believe that in selecting the method the user should be concerned with whether the method yields the information desired, its appropriateness to how the organisation make decisions, how easy it is to use and its validity. Hobbs *et al.* (1992) found that the comprehension of each method's concepts affects the users' perception of how easy it is to provide inputs. Kangas and Kangas (2005) believe that the choice of the best or most suitable method requires knowledge of many methods and consideration of case-study requirements. Compromises must often be made when selecting a method because versatile methods that enable deep analyses and complete exploitation of the available data are typically hard to use and understand. That is why simple and straightforward MCDA techniques are often needed in participatory approaches (Kangas and Kangas, 2005). Guitouni and Martel (1998) propose the following guidelines for selection of MCDA techniques:

1. Determine the stakeholders of the decision process. If there are many actors, one should think about group decision making methods.

2. Consider the DM 'cognition' (way of thinking) when choosing a particular preference elucidation method. Are they more comfortable with pair-wise comparisons than trade-offs, and vice versa?

3. Determine the decision problematic pursued by the DM (i.e. if the DM wants to obtain a ranking of the alternatives then a ranking method is appropriate).

4. Choose the MCDA technique that can properly handle the input information available and for which the DM can easily provide the required information; the quality and quantities of the information are major factors in the choice of the method.

5. The compensation degree of the MCDA method is an important aspect to consider and to explain to the DM. If the DM refuses any compensation, then many MCDA methods will not be appropriate.

6. The fundamental hypothesis of the method is to be met (verified) otherwise one should choose another method.

7. The decision support system (DSS) coming with the method is an important aspect to be considered.

As stated above, it is often difficult to ascertain which MCDA method is best suited to a given situation. Some recent studies show that it would be useful to utilise more than one MCDA technique (Bell *et al.*, 2001; Kangas *et al.*, 2001b; Noghin, 1997; Salminen *et al.*, 1998). If the methods do not agree, the DMs could be given the solutions from different methods with an explanation as to why they differ. Alternatively, Ulengin *et al.* (2000) present a framework called IDEA_{ANN}

which selects the most appropriate MCDA method for the DM using an Artificial Neural Network (ANN) approach.

A list of some of the software available to aid DMs in implementing various MCDA approaches is contained in Appendix C. Although computer packages have been found to be useful in supporting group decision, the extent to which software may be useful is sometimes compromised by the very sophistication of the packages, as software embodies the designers' rational model of group decision making and so prescribes the use of a method that may be inappropriate for the problem under consideration (Jessop, 2002). Levy *et al.* (1998) believes that alternative MCDA methods are too often marketed as competing products, rather than as complementary approaches for decision analysis. There is a great need for DSSs that allow practitioners the choice of tools for the particular problems that they are facing, since different techniques have their strengths and weaknesses.

Comparison of MCDA techniques

Many MCDA methods and associated commercial software packages (see Appendices B and C) have been developed over the years, but little is known about the relative merits of using different methods on similar problems. A literature search by Zanakis *et al.* (1998) revealed that a limited number of studies have been done in terms of comparing and integrating the different MCDA techniques. This could be because, as Olson *et al.* (1995) state, comparison across MCDA methods is not easy. Many authors dealing with this issue agree that one of the most important complications is that there is not a clear, objectively best, decision method in multiple criteria environments (Olson *et al.*, 1995). It is difficult or almost impossible to answer questions such as (Zanakis *et al.*, 1998):

- Which method is more appropriate for what type of problem?
- What are the advantages / disadvantages of using one method over another?
- Does a decision change when using different methods? If yes, why and to what extent?
- Does a particular decision method help render a better decision?

A summary of a number of the comparative studies published in the literature is contained in Table 2.3, in chronological order, stating the methods used and the main conclusions of the author(s) of the study. The common elements of published studies involve the search for convergent validity (i.e. that a common solution is found across techniques) and measurements comparing ease of use and understanding, time for completion, and the subject's confidence in each method (Corner and Buchanan, 1997). Raju *et al.* (2000) and Zanakis *et al.* (1998) also state that users may compare methods along additional criteria, such as perceived simplicity, trustworthiness, robustness and quality. Duckstein *et al.* (1982) compared MCDA techniques with regard to the following criteria: (i) type of data required (i.e. qualitative or quantitative); (2) consistency of results between methodologies; (iii) robustness of results with respect to changes in parameter values; (iv) ease of computation; and (v) the amount of interaction required between the DM and the decision analyst.

Table 2.3 Summary of a selection of studies comparing MCDA techniques

Reference	**MCDA Techniques Compared**	**Conclusion**
Duckstein *et al.* (1982)	ELECTRE, CP, MAUT	MAUT requires the most time for learning compared to ELECTRE and CP.
Brans *et al.* (1986)	PROMETHEE, ELECTRE III	PROMETHEE is more stable than ELECTRE III.
Karni *et al.* (1990)	ELECTRE, AHP, SAW	Rankings did not differ significantly between the approaches for three real life case studies.
Hobbs *et al.* (1992)	AHP, SAW, ELECTRE I, GP, additive utility functions, multiplicative utility functions	Which multi-criteria method is chosen can make a significant difference in the decision. No one method is consistently preferred by the users to others.
Goicoechea *et al.* (1992)	ELECTRE, AHP, MAVT	No significant differences across methods.
Larichev *et al.* (1993)	AHP, PC, ZAPROS, SMART	Similar results when there where distinct differences in the alternatives. SMART and AHP were both found to be easy to use.

Reference	**MCDA Techniques Compared**	**Conclusion**
Olson *et al.* (1995)	MAUT, AHP, ZAPROS	When alternatives are nearly equal in value, choice of method can result in different rankings of alternatives for even careful, consistent DMs. The underlying model does not seem to be as important a factor as the accuracy of the models' reflection of DM preference.
Zanakis *et al.* (1998)	SAW, MEW, AHP, ELECTRE, TOPSIS	As the number of alternatives increases the methods tend to produce dissimilar rankings and more rank reversals. The number of criteria had little affect.
Salminen *et al.* (1998)	ELECTRE III, PROMETHEE I, II, SMART	The difference from ELECTRE III solutions to PROMETHEE and SMART solutions is not great. Some indication that ELECTRE III had more functionality.
Bell *et al.* (2001)	Additive linear value function, additive non-linear value function, goal programming, ELECTRE I, fuzzy sets, linear utility function, non-linear utility function, Min Max Regret, stochastic dominance	Low predictive validities and intermethod correlations indicated that no single method can be used to identify the best alternative. The outcomes of the various methods often conflicted because each method frames the problem differently. Every method had its advocate and no one method was favoured by all participants in the experiment. Results also indicated that who applies the method and which method is used can strongly impact on results.
Kangas *et al.* (2001b)	MAVT, ELECTRE III, PROMETHEE II	Different methods gave somewhat different results although the planning problem analysed was the same.
Lerche *et al.* (2002)	HDT, PROMETHEE, NAIDE, ORESTE	The HDT was selected as the preferred method with PROMETHEE method close behind and well above its possible alternatives of NAIDE and ORESTE.

From the conclusions in Table 2.3, there appear to be contrasting opinions as to whether different MCDA techniques produce similar or dissimilar results. Gershon and Duckstein (1983) have a major criticism of MCDA methods, in that different techniques yield different results when applied to the same problem, under the same assumptions and by a single DM. Al-Shemmeri *et al.* (1997b) have also found that generally not all methods applicable to a specific decision situation generate similar solutions. Hobbs *et al.* (1992) found that which MCDA method is adopted can make a significant difference to the decision, in that the choice of method can affect the results as much, or more, than which person applies the method. Hersh (1999) and Munda *et al.* (1994) also agree that the use of different MCDA methods can lead to very different decisions. According to Mahmoud and Garcia (2000), choosing among MCDA methods to rank multiple criteria alternatives is critical not only because each method produces different rankings, but also because choosing a methodology is subjective, based upon the predisposition of the DM. Comparative studies reviewed by Karni *et al.* (1990) indicated that different algorithms, variable scaling factors and use of criteria weights (CWs) lead to different outcomes. Other researchers have argued the opposite, namely that, given a type of problem, the solutions obtained by different MCDA methods are essentially the same (Goicoechea *et al.*, 1992; Karni *et al.*, 1990; Larichev *et al.*, 1993; Salminen *et al.*, 1998).

Differences in the performance of methods can arise not only because of differences in the way the methods process information, but also because of the order in which they are applied and peculiarities in the software design (Hobbs *et al.*, 1992). The results of comparative studies are also dependent on how close the relative attainment of the alternatives is i.e. many studies (such as Aldag and Power (1986) and Goicoechea *et al.* (1992)) that have found little difference in analysis outcomes have had distinctly different choices.

None of the large variety of MCDA methods can be claimed to be superior to others in every aspect and can be considered as appropriate for all decision making situations (Kangas and Kangas, 2005; Miettinen, 2001; Ulengin *et al.*, 2000). Any MCDA method cannot be considered as a tool for discovering an 'objective truth'. Such models should function as an aid to the user to learn more about the problem and solutions to reach the ultimate decision.

2.5.5 Assignment of performance values

A value (or score) must be assigned to each alternative indicating its performance in relation to each criterion, which will be referred to as criteria performance values (PVs). There are three types of possible measurement scales: (i) ordinal, (ii) interval and (iii) ratio. An ordinal scale provides information on order only. An interval scale provides a measure of the difference between two alternatives, but does not indicate actual magnitude. A ratio scale has a natural origin (zero value) and provides a measure of both difference and magnitude. The terms qualitative and ordinal are used interchangeably and quantitative is reserved for interval and ratio scales. There are several techniques for determining the criteria PVs ranging from qualitative judgments of an expert to sophisticated mathematical models (Kheireldin and Fahmy, 2001). McLaren and Simonovic (1999) recommend the following considerations for including expert opinion in sustainable decision making:

- Expert opinion should never be substituted for reliable, replicable quantitative data;
- Where expert opinion is used in conjunction with other qualitative or quantitative data, it should be expressly stated and the source of each impact estimate included; and
- Wherever possible, more than one expert should be consulted.

The assessment of the data plays a crucial role in the decision analysis, as the results obtained by application of a MCDA method are strongly related to the actual values assigned to these input parameters (Wolters and Mareschal, 1995). A particular criterion PV is not fixed or known exactly and is affected by the following three phenomena according to Roy *et al.* (1992):

1. Imprecision (because of the difficulty of determining it, even in the absence of random fluctuation);
2. Indetermination (since the method of evaluation results from a relatively arbitrary choice between several possible definitions); and
3. Uncertainty (since the value involved varies with time).

Other researchers also agree that assigning values to criteria is a difficult process. For example, in all of the real-life applications that Miettinen and Salminen (1999) have been involved in, it has always been impossible to define accurate values for all of the criteria. Mareschal (1986) also states that many criteria are difficult to quantify and cannot easily be reduced to a single figure. McLaren and Simonovic (1999) state that quantitative data is crucial for good decision making, but it is important to consider the origin and reliability of the data. There are many reasons why data can be uncertain, such as variability in conditions (e.g. soil or groundwater), data measurement failures and large dominating external factors such as climate change or economic growth (Klauer *et al.*, 2002). If an expert assesses PVs, their own subjectivity could also introduce bias and uncertainties (Bertrand-Krajewski *et al.*, 2002). Qualitative data have traditionally been considered inferior to quantitative data (McLaren and Siminovic, 1999). According to Xu *et al.* (2001), it is generally thought that ordinal performance information is less demanding on the expert than cardinal information, because the latter requires accurate evaluation of the performances of the alternatives on the given criteria, which is usually inaccurate, unreliable or even unavailable, especially in an uncertain environment. However, uncertainties linked to both the calculation of PVs and their use in decision-aid tools have been rarely accounted for, as can be seen in the tables in Appendix A.

2.5.6 Standardisation of criteria performance values

A key benefit of MCDA is that it can handle performance measures in different units. However, before applying some quantitative evaluation ranking methods, such as weighted summation or discordance analysis, it is necessary for all criteria PVs to be reduced to a comparable or standardised basis (Hajkowicz *et al.*, 2000; Royal Assessment Commission, 1992). Standardisation is intended to eliminate the effects of scale that would otherwise introduce a weighting.

Various standardisation methods have been proposed in the literature, such as the additive constraint, ratio-scale properties and interval scale property method (Kheireldin and Fahmy, 2001). The most commonly adopted standardisation methods adjust criterion scores based on their distance from a maximum and / or minimum value, as summarised in Table 2.4. Other techniques use an ideal point instead of the minimum or maximum value. The ideal point is a value that represents the best possible or most desired outcome for a given criterion (Hajkowicz *et al.*, 2000). Alternative methods of standardisation include division by the sum of the values, division by an ideal or target value and vector normalisation (Royal Assessment Commission, 1992).

Table 2.4 Common methods for linear standardisation of performance measures in the effects table

Standardisation Method	For positive criteria (where a higher value is better)	For negative criteria (where a lower value is better)
1	$s_{ij} = \dfrac{x_{ij}}{\left(\sum_{i=1}^{n} x^2{}_{ij}\right)^{1/2}}$	$s_{ij} = \dfrac{-x_{ij}}{\left(\sum_{i=1}^{n} x^2{}_{ij}\right)^{1/2}}$
2	$s_{ij} = \dfrac{x_{ij}}{x_j \max}$	$s_{ij} = \dfrac{x_j \min}{x_{ij}}$
3	$s_{ij} = \dfrac{x_{ij} - x_j \min}{x_j \max - x_j \min}$	$s_{ij} = \dfrac{x_j \max - x_{ij}}{x_j \max - x_j \min}$

where:

s_{ij} = the standarisation value for x_{ij};

x_{ij} = the score indicating the performance of alternative *i* against alternative *j*

Source: Hajkowicz *et al.* (2000), Hersh (1999), Royal Assessment Commission (1992)

In standardisation of criteria PVs, it is important to consider whether it is necessary to transform the data according to a certain utility function. A utility function represents the way in which a DM derives utility from a particular criterion. The most commonly applied utility function has a linear form. Linear utility is often assumed due to a lack of knowledge about the DM's true preferences. Commonly used utility functions include linear, concave, convex and parabolic functions (Hajkowicz *et al.*, 2000).

There is no obvious reason for selecting one standardisation method over another. Since the objective of the standardisation is to enable comparisons between criteria originally measured on quite different scales, the method that appears best suited to do this has to be determined on a case-by-case basis. In some situations the choice of method may have little impact on the final results. In other situations it could have a significant effect (Royal Assessment Commission, 1992).

The final ranking may therefore be dependent on the type of standardisation method applied (Janssen, 1996).

2.5.7 Weighting the criteria

Purpose

The factors that influence a decision are typically given different priorities, which represent the relative importance of each criterion. Most MCDA methods require some measure of relative importance to be attached to the criteria. Such information can be expressed in various forms such as: lexicographic orders, minimum requirements, aspiration levels (goals, targets) as well as specific numerical weights (Janssen *et al.*, 1990; Royal Assessment Commission, 1992). These values are then used by the MCDA aggregation model in subsequent evaluation of the alternatives (Hajkowicz *et al.*, 2000).

Assigning weights to the criteria is possibly the most valuable aspect of MCDA because it allows different views and their impact on the ranking of alternatives to be explicitly expressed (Royal Assessment Commission, 1992) and, in addition, the weighting process increases problem understanding (Hamalainen and Salo, 1997). The relative importance of the selected criteria is not equally perceived by all people (Georgopoulou *et al.*, 1998), which means that the actors give different levels of relative importance to the various criteria, naturally favouring the ones expressing their own points of view, thus generating conflicts (Bana e Costa, 1986). The criteria weights (CWs), therefore, make explicit those areas which may ultimately require possible trade-off solutions and thus they provide a greater focus for a complex decision problem (Proctor and Drechsler, 2003).

Techniques

The deep complexity of eliciting information from humans has been noted by many psychologists and researchers in decision making (Larichev *et al.*, 1993), therefore, ideally, the CWs should be derived through close interaction between the actors and the decision analyst (Hajkowicz, 2000). Development of a structured approach for assigning CWs consistently is desirable for solving practical MCDA problems (Yeh *et al.*, 1999b). Many techniques for the determination of CWs have been proposed (Al-Kloub *et al.*, 1997; Hajkowicz *et al.*, 2000; Hobbs *et al.*, 1992; Kheireldin and Fahmy, 2001; Moshkovich *et al.*, 1998; Yeh *et al.*, 1999b). Some methods are based on sound theory, while others use simplified heuristic approaches (Moshkovich *et al.*, 1998). Hobbs (1980) has undertaken a survey of some of the available weighting methods and this list has been extended to incorporate other existing weighting methods, as found in the literature reviewed, and is contained in Appendix D.

The classification, selection and comparison of the numerous weighting techniques available are discussed below.

Classification of techniques

There is an extensive range of weighting methods available and, not surprisingly, there has been a variety of ways of classifying these techniques suggested in the literature. Nijkamp *et al.* (1990) and Mousseau (1995) separate the weighting techniques into classes of methods which involve direct estimation of CWs and indirect estimation of CWs. Direct methods require an explicit statement of the relative importance of each criterion from the DM. Such statements can be recorded in qualitative or quantitative ways. Requiring a DM to distribute a fixed number of percentage points amongst the criteria is an example of a direct weighting method (Hajkowicz *et al.*, 2000).

Indirect weighting methods estimate CWs based on simulated or real decision behaviours. They generally require the actors to rank or score a set of alternatives against a set of evaluative criteria. Using various techniques such as multiple linear regression analysis, it is possible to implicitly derive weights for the criteria (Hajkowicz *et al.*, 2000). There are also different techniques based on trade-off analysis for indirectly extracting the actors' preference system (Georgopoulou *et al.*, 1998).

Alternatively, from the extensive literature on CW determination, Schoemaker and Waid (1982) separate the techniques into statistical versus subjective approaches. The former tend to be based on regression analysis, using holistic judgments. The subjective approach revolves around decomposed judgments, which are often unrepresentative, but possibly simpler and more refined. Alternative weighting schemes also differ in terms of the type of safeguards in place to reduce judgmental inconsistency (Schoemaker and Waid, 1982). Ma *et al.* (1999) classify weighting methods into subjective and objective approaches. Subjective approaches select CWs based on preference information of attributes given by the DM which includes the eigenvector method (Saaty, 1977), weighted least square method and the Delphi method. The objective approaches determine CWs based on objective information and they include principle element analysis, entropy method (Hwang and Yoon, 1981) and multiple objective programming.

An alternative way to specify information about CWs is to establish intervals for the individual variation of CWs. In addition, the actor may be able to give certain linear relations, which express partial information about marginal substitution rates between the criteria. The ordinal approach is where the actor is requested to estimate only the rank order of the criteria (Fernandez *et al.*, 1998).

Selection of technique

Due to the large number of weighting techniques available, selection of an appropriate method is a difficult task. Bottomley *et al.* (2000) believe that the selection of a method of elicitation generally has been considered somewhat arbitrary. There are no obvious reasons given in the literature for selecting one method over another and as Hamalainen and Salo (1997) state, if researchers have not been able to make it clear which is the best method of assigning CWs, then they are likely to remain unclear to the actors as well. This is a similar problem as encountered when selecting which MCDA technique or PV standardisation technique to utilise, as discussed in Sections 2.5.4 and 2.5.6, respectively.

The direct assignment of weights to the criteria is the simplest approach for deriving preferences (Georgopoulou *et al.*, 1998). However, methods that ask actors to choose CWs directly generally do not guarantee that the CWs are theoretically valid. The assignment of numbers to ordered estimates reduces the reliability of measurements, because by assigning numbers, a subjective quantitative scale is constructed which can never be precise (Larichev, 1992). Direct estimation of the relative importance of criteria by assigning a value to each criterion, or by allocating a fixed number of points among the criteria, proves to be a very difficult task for the actors (Janssen, 1996).

There are several arguments in the literature favouring rank order methods:

- Ranking methods are easier and possibly more reliable than methods that require judgments sufficient to specify ratios of CWs (Eckenrode, 1965). Ordinal input is less complex and therefore it is expected to more accurately reflect DM preference (Moshkovich *et al.*, 2002).

- Actors may be unavailable or unwilling to provide more than ordinal information (Barron and Barrett, 1996).

- If the decision is being made by a group, they may be able to agree on the ranking of the criteria but not on precise CWs.

- Evaluations generated by rank order methods correlate more highly with those generated by more precise numerical methods than do evaluations generated by the equal weights method (Barron and Barrett, 1996).

Simos (1990) concluded that the method chosen to elicit CWs should be simple and comprehensible to all involved in the process, as a method that was easily understood would have more credibility than other more complex, less easily understood weighting techniques (Rogers and Bruen, 1998b). Schoemaker and Waid (1982) believe that the choice of method is itself a multi-criteria one, involving ease of use, mean performance, dispersion, normative justification and trustworthiness. Levy *et al.* (1998) state that the particular weighting method used depends on the nature of the criteria, the amount of information available and the preferences of the DM.

In spite of the MCDA structure which is common to most approaches based on prior articulation of preferences, it needs to be recognised that the notion of preference is made operational by quite dissimilar mathematical representations in each MCDA approach (Bana e Costa *et al.*, 1997). The way of formalising the relative importance of each criterion differs from one aggregation model to another (Bana e Costa *et al.*, 1997; Mousseau, 1995; Roy and Mousseau, 1996). There should be consistency between the aggregation procedure used and the questions asked of the actors in order to elicit a set of CWs (Munda *et al.*, 1994). The interpretation of the CWs is different for a compensatory MCDA method (e.g. MAUT) compared to a non-compensatory system (e.g.

ELECTRE) (Bana e Costa *et al.*, 1997) and, therefore, the method used must be selected accordingly (i.e. it is theoretically incorrect to use the same CWs with different MCDA methods). However, it can be argued that in real-life decision making situations the DMs cannot fully understand how each method deals with CWs and therefore, in practice, CWs do not depend very much on the decision model (Hokkanen *et al.*, 1998).

Compensatory approaches

In compensatory methods, the CWs amount to being substitution rates, allowing differences in preferences, as they relate to different criteria, to be expressed on the same scale i.e. that the weights be proportional to the relative value of unit changes in their attribute value functions (Hobbs, 1980; Poyhonen and Hamalainen, 2000). These parameters are in fact scaling constants needed for the cardinal criteria-functions to be commensurate in some way. In other words, if $CW_1 = 2$ and $CW_2 = 4$, actors must be indifferent between the change in $V_1(X_1)$ of 1 and a change in $V_2(X_2)$ of 0.5. This condition also implies that weights are on a "ratio level of measurement". That is, $CW_1 = 2$ and $CW_2 = 4$ means that a unit change in $V_1(X_1)$ must be half as valuable as a unit change in $V_2(X_2)$.

Thus, in these approaches, CWs have no absolute or intrinsic meaning and there is no sense in attempting to derive them without knowledge of the criterion or its value function. If the value trade-offs are done properly and address the question of how much of one specific criterion is worth how much of another specific criterion, the insights from the analysis are greatly increased (Bana e Costa *et al.*, 1997).

The AHP assumes that actors take the set of alternatives explicitly into account when they assess the CWs. Value theory based methods assume that actors give preference statements about the CWs so that they reflect the criteria ranges (Poyhonen and Hamalainen, 2000). Value theory based weighting methods include SMART (von Winterfeldt, 1986), SWING (von Winterfeldt, 1986) and SMARTER (Barron and Barrett, 1996).

Non-compensatory methods

Within non-compensatory methods (i.e. outranking methods, where the aggregation procedures are based on concordance and discordance), the notion of importance arises in a very different manner. The CWs used are not constants of scale, but are simply a measure of the relative importance of the criteria involved (Rogers and Bruen, 1998b). Two concepts occur here: a coefficient of importance, which is analogous to the number of voters defending the particular point of view, and a veto threshold, which is analogous to the importance of someone with veto power in a collective decision situation.

Bottomley and Doyle (2001) believe that there are fundamental differences between methods of elicitation and would caution practitioners against arbitrarily selecting a method. Below is a summary of the type of CWs required for particular MCDA methods (Mousseau, 1995; Roy and Mousseau, 1996):

- Scaling constants in MAUT;
- Intrinsic weights in PROMETHEE methods;
- Intrinsic weights combined with veto thresholds in ELECTRE methods; and
- Eigenvectors of a pair-wise comparison matrix in the AHP method.

Comparison of techniques

There have been many studies comparing differences in techniques used to estimate CWs (Moshkovich *et al.*, 2002; Olson *et al.*, 1995) and a number of these are summarised in Table 2.5.

Based on the variety of different outcomes from the comparative studies that have been undertaken, it is difficult to know which technique or techniques produce 'better' results. Although it is relatively easy to compare two or more techniques on criteria such as efficiency, determining which techniques lead to higher degrees of consensus within a group is more challenging (Shirland *et al.*, 2003). Aloysius *et al.* (2006) state that many studies have not consistently shown that any one preference elicitation technique is objectively superior to all other techniques. Butler *et al.* (1997) have also found that experimental studies have revealed numerous sources of inconsistencies rather than a single, superior assessment technique. Olson *et al.* (2000) have found that the method used for elicitation of CWs influenced the results to a greater extent than did the underlying MCDA approach. Bottomley and Doyle

(2001) state that many studies have compared different methods of eliciting CWs, however, these studies constitute a loose body of knowledge from which few clear findings have emerged.

Table 2.5 A selection of comparative studies of criteria weighting methods

Reference	**Weighting Methods Compared**	**Conclusion**
Eckenrode (1965)	Informal indirect methods: ranking, rating, three versions of paired comparisons and a method of successive comparisons	There were no significant differences in the sets of CWs derived by any of the methods, but it was found that ranking was by far the most efficient method.
Hobbs (1980)	Indifference trade-off and rating methods	Methods resulted in significantly different CWs. The results of the subsequent MCDA analysis also differed.
Schoemaker and Waid (1982)	Multiple regression (MR), analytic hierarchies (AH), direct trade-offs (DT), point allocations (PA), unit weighting (UW)	The various methods yielded significantly different CW estimates, both with respect to means and standard deviations. The methods also differed in variance. It was concluded that the appropriateness of various methods remains an open question, as it may vary across subjects and tasks.
Simos (1990)	A range of weight selection techniques	Found very little consistency between them and therefore believed that this lack of consistency between different approaches made the process of criterion weighting the weak link within the decision aid process.

Reference	Weighting Methods Compared	Conclusion
Fischer (1995)	Direct importance weights, trade-off weights, swing weights	There is systematic discrepancy between CWs inferred from trade-offs and CWs inferred from direct judgments of criteria importance. Trade-off judgments gave greater weight to the most important criterion than did direct importance ratings or swing weight assessments.
Barron and Barrett (1996)	Rank sum, reciprocal of ranks, rank order centroid weights, equal weights	Rank order centroid weights are useful, usable, efficacious weights whose average performance is excellent in absolute terms and is superior to that of previously proposed rank-based surrogate weights (i.e. rank sum and reciprocal of ranks).
Leon (1997)	SMART, SMARTS, GRAPA	A high level of congruence between the CWs and the ranking of the alternatives indicated to Leon (1997) that both SMART and GRAPA are acceptable techniques for weight elicitation.
Jia *et al.* (1998)	Equal weighting of all criteria, two methods for using judgments about the rank ordering of weights and a method for using the ratios of weights	The marginal density function over each CW for the simulation study had a beta distribution. The results suggested that ratio weights were either better than rank order weights or tied with them. The rank-order-centroid method favoured the rank-sum method.
Rogers and Bruen (1998b)	Three existing methods of criterion weighting by Hokkanen and Salminen (1994), Simos (1990) and Mousseau (1995)	All three methods vary in complexity and all have their drawbacks. The first two methods are simple and straightforward, yet lack a firm methodological basis.

Reference	**Weighting Methods Compared**	**Conclusion**
Bottomley *et al.* (2000)	Point allocation, direct rating	CWs elicited by direct rating were more reliable than those elicited by point allocation. The subjects of the experiment preferred direct rating to point allocation and the CWs were more stable for direct rating than point allocation.
Svensson (2000)	Visual analogue scale (VAS), graphic rating scale (GRS), five-point verbal descriptor scale (VDS-5)	High level of stability in the VDS-5 and GRS assessments imply two scales are superior to the VAS. VDS-5 and GRS assessments were also consistent.
Bell *et al.* (2001)	Point allocation, hierarchical point allocation, swing weighting / AHP, trade-off weighting	Participants recommended using revised CWs more than any single weighting method. Different approaches yielded different CWs.
Bottomley and Doyle (2001)	Direct rating with Max100 and Min10	The CWs elicited using Max100 were more internally consistent than direct rating. In turn, direct rating was more reliable than Min10.
Poyhonen and Hamalainen (2001)	AHP, direct point allocation, SMART, SWING weighting and trade-off weighting	This study found that the CWs differ because DMs choose their responses from a limited set of numbers. The consequences are that the spread of CWs and the inconsistencies among the preference statements become dependent on the number of criteria present in the comparison. The results also show that the DMs tend to interpret the numbers in a different way than what value theory assumes.

Uncertainty

A great deal of behavioural research has focused on the correct procedure to assess CWs. Many authors have identified the allocation of CWs as a major shortcoming of the MCDA process (Morrissey and Browne, 2004), predominantly due to the uncertainty in the elicitation of the values. The opinions of researchers are mixed as to whether the uncertainty in the CWs has an impact on the ranking of the alternatives. There is evidence in many situations that varying the specific CWs assessed for separate criteria often does not change the selection of the most preferred alternative. However, the situation may not be the same for decisions where differences between alternatives are small, as Moshkovich *et al.* (1998), Jia *et al.* (1998) and Barron and Schmidt (1988) found that slight differences in CWs can lead to reversals in the ranking of alternatives.

Subjectivity of values

The necessity to obtain complicated judgments from actors concerning CWs in the application of some of the MCDA methods is one of the most difficult parts of the decision aid process (Larichev and Moshkovich, 1995; Roy *et al.*, 1986), but is considered by many researchers, including Mahmoud and Garcia (2000), to be the most important step.

In theory, the CWs are specified by the actors and enter the analysis as well-defined constants. A problem arises though in the specification of values for the CWs for a number of reasons. Firstly, however values are chosen for CWs, there can be no certainty that they are the correct ones, as weighting is the major judgmental phase of the MCDA process (Hajkowicz *et al.*, 2000). Statements of preference made about the relative values of the CWs are subject to cognitive and other biases. People can often agree on verbal definitions for an object, but have greater difficulty in numerical estimation of the same concept (Moshkovich *et al.*, 1998). It is therefore evident that the specification of CWs is not an easy task and several papers indicate that subjects make essential errors in quantitative measurement of CWs. The research has shown that the CWs assigned by the subjects are not reliable and stable information (Larichev, 1992).

Not only may actors find it difficult to provide precise figures about their preferences, but preferences may change as the decision aid process evolves (Dias and Climaco, 2005). In addition, the method used to obtain

CWs may itself be chosen from a number of more or less equally plausible contenders (Jessop, 2004). Moreover, the procedures used to elicit the values of the CWs may require more time and patience than some actors can spare (Dias and Climaco, 2005).

Selection of technique

There are two issues with regard to the methods for determining CWs: first, do different methods yield different CWs? Second, if yes, do such differences matter in terms of prediction (Schoemaker and Waid, 1982)? Experiments and previous research show that different techniques for deriving CWs may lead to different results (Moshkovich *et al.*, 1998; Olson *et al.*, 1995; Poyhonen and Hamalainen, 2001). Mareschal (1988), Barron and Barrett (1996) and Salminen *et al.* (1998) concur that the CWs are highly dependent on the elicitation method. In addition, there is no agreement as to which weight elicitation technique produces more accurate results since the 'true' target CWs remain unknown (Barron and Barrett, 1996; Miettinen and Salminen, 1999). Jia *et al.* (1998) state that because different techniques can lead to different decisions, it is important to determine which approach gives the best results under different circumstances. Jia *et al.* (1998) found that there is a direct trade-off between accuracy of techniques and effort. Therefore, although there is no shortage of weighting methods, there is only limited information as to their reliability and validity (Bottomley and Doyle, 2001).

Multiple actors

When the decision analysis concerns only one DM, the mathematical incorporation of the preference weights into the decision making problem is relatively straightforward. When it concerns more than one DM, the process becomes more complex and controversial (Proctor, 2001). Decision making groups can range from cooperative, with very similar goals and outlooks, to antagonistic, with diametrically opposed objectives (Davey and Olson, 1998). If group members have different viewpoints, a method of aggregating preferences and reconciling differences is needed. A major reason for difficulties in the search for a compromise solution has been the lack of a procedure by which all actors involved in the MCDA process could present their views (Georgopoulou *et al.*, 1997). It is important that all actors feel that their points of view are taken into account in the decision analysis process (Miettinen and Salminen, 1999).

Leyva-López and Fernández-González (2003) distinguish two main general approaches, which use a multi-criteria decision aid technique for aggregating group preferences:

1. The group is asked to agree on the alternatives, criteria, PVs, CWs and any remaining parameters before the MCDA technique provides a ranking. This method performs well when the assessments of parameters coming from the different actors do not show strong divergences, or when the final ranking is sufficiently robust to handle them.

2. A group consensus is only needed for defining the set of potential alternatives. Actors define their own criteria and parameter values and then the MCDA method is used to obtain a personal ranking. Next, each actor is considered as a separate criterion, and the preferential information contained in its particular ranking is aggregated in a final collective ordering.

In traditional group decision making, not all of the CWs of the different actors have been taken into account, but instead, some average, median or some other aggregate measure of CW has been used with sensitivity analysis (Hokkanen *et al.*, 2000; Miettinen and Salminen, 1999). For example, in the decision support software Web-HIPRE, the preferences of group members are aggregated using the arithmetic mean method (Mustajoki *et al.*, 2004). Also, in the analysis undertaken by Hokkanen and Salminen (1994), the final CW was obtained through majority (i.e. the final weighting value for a given criterion was the value assigned to it by the largest number of the 45 actors).

However, these techniques may result in a weight vector and ranking preferred by no-one, as in group decision situations, the opinions and preferences of the actors frequently diverge (Dias and Climaco, 2005). Reducing the vector of quantitative CWs from each actor into a single CW vector may lose important trade-off information related to the outcomes of the analysis under extreme weightings. Moreover, actors with CWs that are very different from the calculated averages are most likely to disagree to such a technocratic enforcement of a 'consensus' and may not wish to participate any further in the decision analysis process.

Proctor and Drechsler (2003) state that there is no agreement in the literature on how to reduce such weight variability among actors. Simos

(1990) believes that average weightings or weightings obtained by majority should not be used. Instead, that the weightings for all actors should be compiled and the extreme values for these weightings be used within a sensitivity analysis. The overall rankings delivered by these weightings within the chosen MCDA technique can then be analysed and a final ranking agreed to (Hokkanen and Salminen, 1994). Leyva-López and Fernández-González (2003) state that there are not many approaches which solve the group ranking problem with multiple criteria in an acceptable way.

2.5.8 MCDA technique specific parameters

A number of the existing MCDA techniques require additional parameters (to the CWs and PVs) to be defined. For example, when using outranking methods (such as PROMETHEE and ELECTRE), the assessment of the alternatives is based on what are called pseudo-criteria. Pseudo-criteria are formed using two different threshold values, the indifference (q) and preference (p) threshold, which describe the priority difference between the criterion values of two alternatives. If the difference with regard to a criterion is less than the indifference threshold, the alternatives are considered to be indifferent with regard to that criterion. If the difference is larger than the preference threshold, the alternative that is regarded better with respect to the criterion in question is considered to be better without any doubt. If the difference is larger than the indifference threshold, but less than the preference threshold, priority between alternatives is uncertain. ELECTRE enables the user to also set what is called the veto threshold for the criteria. If an alternative performs so badly in regard to one criterion that the difference exceeds the veto threshold, even good criteria with regard to other criteria will not suffice to compensate such a great deficiency (Kangas *et al.*, 2001a; Kangas and Kangas, 2005; Miettinen and Salminen, 1999; Ostanello, 1983).

The assignment of the threshold values, however, is not an easy task (Salminen *et al.*, 1998). In addition, the aggregation methods may produce different results for different threshold values (Zanakis *et al.*, 1998), which of course leaves open the question of which value should be selected by the user of the MCDA method. For example, values of discrimination thresholds are to be given by the DM when using the ELECTRE MCDA techniques and the choice can be very subjective (Rogers and Bruen, 2000). It is difficult to fix directly their values and to have a clear global understanding of the implications of these values in terms of the output of the model (Mousseau *et al.*, 2000). Roy *et al.* (1986) also state that both the choice of threshold functions and that of the numerical values characterising it contain a certain amount of arbitrariness. There are some types of input for which it is too easy to supply meaningless, but numerically usable values. Hobbs *et al.* (1992) found in an experiment that the participants did not understand the concordance and discordance concepts in ELECTRE, yet they were able to supply the values of those indices when requested, which does not render confidence in the method's output.

As the assignment of the threshold values is usually a difficult task, some researchers have developed methods to assist in determining appropriate values. Mousseau *et al.* (2000) have proposed to infer values of these parameters from examples of decisions supplied by the DM. ELECTRE Tri Assistant implements this methodology in a way that requires much less cognitive effort from the DM. Rogers and Bruen (2000) also report that methods for linking thresholds in a logical way to physical, physiological and behavioural aspects of problems are under active development (see Rogers and Bruen (1998a)). Kangas *et al.* (2001b) have determined the values of p and q in their applications by percentage limits, calculated as percent of the range of variation in criterion values within the strategy alternatives.

In the outranking MCDA technique, PROMETHEE, the function relating the difference in performance to preference is called the generalised criterion function, and it is selected by the DM. The preference functions according to the PROMETHEE algorithms are used to compute the degree of preference associated with the best action in pair-wise comparisons. Six types of generalised criterion functions (Figure 2.3) have been suggested by Brans *et al.* (1986) with the aim of realistically modelling the DMs' preference, which gradually increases from indifference to strict preference, and to facilitate the inclusion of the inherent uncertainty in the criteria PVs in the decision analysis process.

The generalised criterion functions take into account the deviations and the scales of the criteria. These functions take on different forms, depending on the use of the indifference (q) and preference (p) thresholds, assigning a value between 0 and 1 (preference) to all differences of evaluations between two alternatives, and this, for every criterion. The *q* threshold is the value below which the difference between two scenario evaluations is considered non-significant. The *p* threshold is the value from which that same difference is considered significant (Martin *et al.*, 1999).

The six types of generalised criterion functions, illustrated in Figure 2.3, are described below (Brans *et al.*, 1986).

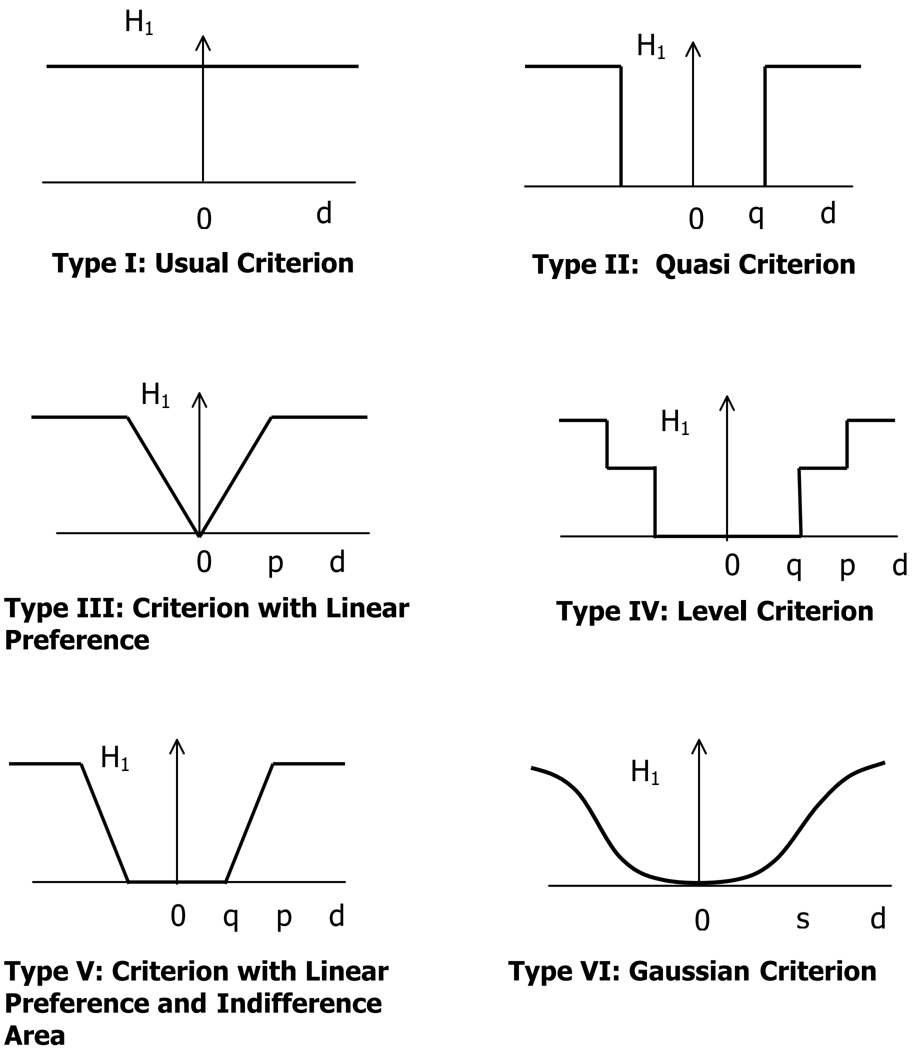


Figure 2.3 PROMETHEE generalised criterion functions

Type I: Usual Criterion

$$H(d) = \begin{cases} 0 \text{ if } d = 0 \\ 1 \text{ if } d \neq 0 \end{cases}$$

In this case there is indifference between the two alternatives, *a* and *b*, if and only if *f(a)* = *f(b)*; as soon as the two evaluations are different, the DM has a strict preference for the alternative having the greatest evaluation.

Type II: Quasi Criterion

$$H(d) = \begin{cases} 0 \text{ if } -q \leq d \leq q \\ 1 \text{ if } d < -q \text{ or } d > q \end{cases}$$

The two alternatives are indifferent to the DM as long as the difference between their evaluations (i.e. *d*) does not exceed the indifference threshold *q*; if this is not the case, there is strict preference. *q* is the greatest value of the difference between two evaluations, below which the DM considers the corresponding alternatives as indifferent.

Type III: Criterion with Linear Preference

$$H(d) = \begin{cases} d \,/\, p \text{ if } -p \leq d \leq p \\ 1 \text{ if } d < -p \text{ or } d > p \end{cases}$$

As long as *d* is lower than *p*, the preference of the DM increases linearly with *d*. If *d* becomes greater than *p*, there is a strict preference situation. When the DM identifies some criterion to be of that type, the DM has to determine the value of the preference threshold which is the lowest value of *d* above which the DM considers that there is strict preference of one of the corresponding alternatives.

Type IV: Level Criterion

$$H(d) = \begin{cases} 0 \text{ if } |d| \leq q \\ \tfrac{1}{2} \text{ if } q < |d| \leq p \\ 1 \text{ if } p < |d| \end{cases}$$

In this case, an indifference threshold q and a preference threshold p are simultaneously defined. If d lies between q and p, there is a weak preference situation.

Type V: Criterion with Linear Preference and Indifference Area

$$H(d) = \begin{cases} 0 \text{ if } |d| \leq q \\ (|d| - q)/(p - q) \text{ if } q < |d| \leq p \\ 1 \text{ if } p < |d| \end{cases}$$

In this case, the DM considers that their preference increases linearly from indifference to strict preference between the two thresholds q and p.

Type VI: Gaussian Criterion

$$H(d) = 1 - \exp\{-d^2/2\sigma^2\}$$

The Gaussian criterion only requires the determination of s, which is done easily according to the experience obtained with the Normal distribution in statistics.

2.5.9 Ranking the alternatives

By the application of an MCDA aggregation method, a ranking of the alternatives from best to worst can be achieved. The two major types of orderings that can be established include a complete order and a partial order (Hajkowicz *et al.*, 2000). The 'best' solution at the completion of a MCDA is the solution which provides the desirable cross-section of trade-offs amongst the criteria (Simonovic *et al.*, 1997). A robust solution is one that has acceptable impacts for the majority of the criteria (Simonovic *et al.*, 1997). Since, as identified in the sections above, uncertainties are present, great care has to be taken when the results of a MCDA are interpreted.

2.5.10 Sensitivity analysis

The above sections have reiterated that decision analysis contains numerical inputs that are not completely certain (French *et al.*, 1998). Uncertainty arises from judgmental estimates, ambiguity and imprecision of meaning and numerical accuracy of calculations. The output of a decision aid depends critically on data input. It therefore becomes necessary to explore the effect of inevitable uncertainties on the selection or ranking of the decision to be made (Jessop, 2004; Rios Insua, 1990) and understand the relationship between changes in those values and subsequent changes in model output. This relationship constitutes the underpinning of a class of analytic procedures collectively referred to as sensitivity analysis (Felli and Hazen, 1998).

Sensitivity analysis refers broadly to any analytic method designed to quantify the impact of parametric variation on model output (Felli and Hazen, 1998). The aim of sensitivity analysis is to ascertain how much the uncertainty in the output of a model is influenced by the uncertainty in its input factors. Sensitivity analysis can be a very powerful tool because it reveals the strengths and weaknesses of the analysis (Royal Assessment Commission, 1992). A comprehensive decision analysis requires extensive sensitivity or robustness analysis (Belton and Hodgkin, 1999). However, effective, comprehensive and useful sensitivity analysis is quite difficult (Larichev, 1998).

Sensitivity analysis of a result is most often studied parameter by parameter (Dias and Climaco, 2000; Roy and Vincke, 1981). A rough sensitivity analysis often occurs where the CWs are modified in a more or less arbitrary way and the changes in the results are examined. This procedure is often ad hoc, inadequate, incomplete and unsatisfactory and can quickly become time-consuming and very expensive (Levy *et al.*, 2000b; Mareschal, 1988). The results obtained also often cast a rather confusing light on the decision (Roy and Vincke, 1981). Many authors have, however, suggested techniques for modelling uncertainty and imprecision (French *et al.*, 1998) which are mainly focused on the assessment and influence of the CWs (Wolters and Mareschal, 1995). This is inadequate, as various other aspects of the multi-criteria problem (i.e. PVs, standardisation method, criteria weighting method, evaluation method) may have an effect on the ranking of the alternatives. For example, Moshkovich *et al.* (1998) found that criteria importance influences the result less than the performance values assigned.

Further discussion on the commonly used sensitivity analysis methods is contained in Chapter 3.

2.5.11 Making a decision – consensus

In environmental problems, the views of stakeholders are typically conflicting and the need for transparent methods to settle any differences is evident (Mustajoki *et al.*, 2004). Exploring the issues from different perspectives brings insight and fosters communication, allowing the actors to reach a decision. As a tool for conflict management, MCDA is an important evaluation method, which has demonstrated its usefulness in many environmental management problems (Munda *et al.*, 1994). MCDA techniques cannot solve all conflicts, but can help to provide more insight into the nature of these conflicts by providing systematic information and ways to arrive at political compromises in cases of divergent preferences by making the trade-offs in a complex situation more transparent to DMs (Munda *et al.*, 1994).

Acceptance of the decision by all stakeholders is highly important to the successful realisation of a sustainable solution (Rijsberman and van de Ven, 2000). Since the objective of MCDA is to assist in the decision making process, presentation of the results in a form that is easily understood by the DM is extremely important. Pictet *et al.* (1994) state that it is the last three stages in decision aid process (i.e. results representation, interpretation, and recommendation) that are, at least in practice, among the most important ones.

Ray and Triantaphyllou (1999) found that there has been little work done on how one can evaluate the conflict of different rankings for the same set of alternatives. Karni *et al.* (1990) states that a consensus ranking may be obtained in one of two ways:

1 Output consensus: find a unified ranking from the individual rankings; or

2 Input consensus: find a unified set of weights from the individual weights, using the arithmetic or geometric mean weights, and

then derive a unique consensus ranking based on the unified weights.

Alternatively, Bender and Simonovic (1997) suggest that distance metrics can be used to assess the degree of consensus among DMs. The degree of consensus indicates the relative strength of ranking and five measures for a degree of consensus can be found in Bender and Simonovic (1997) i.e. the highest discrepancy measure checks whether any DMs disagree on the distance metric value of an alternative. The two DMs who disagree most vehemently are chosen to represent the consensus measure.

Instead, Mareschal (1986) use Kendall's *W* concordance index to determine a global measure of the concordance between the rankings given by the different actors:

$$W = \frac{12}{m^2(n-1)n(n+1)} \sum_{j=1}^{n} \left(R_j - \frac{m(n+1)}{2} \right)^2$$ **Equation 2.1**

with

$$R_j = \sum_{l=1}^{m} r_l(a_j)$$ **Equation 2.2**

where:

W =a number between 0 and 1

m = the number of actors

n = the number of alternatives

Spearman's coefficient of correlation is also used by Mareschal (1986) between the ranking given by an expert and the average ranking given by the others to estimate which actors contribute to discordance and to suggest modification to the set of actors.

Chapter 3
Existing Sensitivity Analysis Methods

3.1 Introduction

Sensitivity analysis is a fundamental concept in the effective use and implementation of decision models, whose purpose is to assess the stability of an optimal alternative under changes in input parameters, the impact of the lack of controllability of certain input parameters, and the need for precise estimation of input parameter values (Evans, 1984b). Conducting a sensitivity analysis is often insightful, however, approaches to sensitivity analysis are generally 'ad hoc rules of thumb' tailored to a particular decision analysis method being used (Isaacs, 1963). With few exceptions, the informal approach is typically one-dimensional, considering sensitivity to one or, at most two data inputs or set of assumptions at a time, the remaining data being taken as fixed (Rios Insua and French, 1991). This single criterion approach can be misleading, as it ignores the potential interaction that can result from simultaneous manipulations of multiple input parameters (Butler *et al.*, 1997).

Despite only informal sensitivity analysis approaches generally being performed in applications of MCDA reported in the literature (see Appendix A), numerous formal sensitivity analysis methods have been proposed in the literature for application with MCDA methods. The formal sensitivity analysis methods can be classified in a variety of ways, which can aid in understanding the applicability of a specific sensitivity analysis method to a particular decision model and analysis objective. Frey and Patil (2002) classify sensitivity analysis methods as:

(1) Mathematical:

Mathematical methods assess the sensitivity of a model output to the range of variation of an input and typically involve calculating the output using a few values of an input that represent the possible range of the input.

(2) Statistical:

Statistical methods involve running simulations in which inputs are drawn from probability distributions and the effect of variance in inputs on the output distribution is assessed. Depending on the method, one or more inputs are varied at a time.

(3) Graphical:

Graphical methods give representation of sensitivity in the form of graphs, charts or surfaces. Generally, graphical methods are used to give visual indication of how an output is affected by variation in inputs.

Alternatively, sensitivity analysis methods have been partitioned into four categories in the health economics literature (Felli and Hazen, 1999): (i) simple sensitivity analysis, (ii) threshold analysis, (iii) analysis of extremes and (iv) probabilistic sensitivity analysis. Other classifications focus on the capability, rather than the methodology, of a specific technique (Saltelli *et al.*, 2000).

13 formal sensitivity analysis methods have been selected (based on detail available and type of method) to be described in this chapter to provide an overview of the methods available, while references for a number of other existing sensitivity analysis techniques are included for completeness. For the purposes of the thesis, the methods described have been classified as deterministic and stochastic methods, which is comparable to the classification of Frey and Patil (2002) i.e. mathematical and statistical, respectively. A summary of the methods described in this chapter is contained in Table 3.1 for deterministic methods and Table 3.2 for stochastic methods. Some extensions to specific MCDA methods and software packages have also been proposed in the literature, which are briefly described in Section 3.4.

Each sensitivity analysis method described in this thesis has its own key assumptions and limitations. Some methods provide more information regarding the nature of the sensitivity than others. Rios Insua and French (1991) list several properties that should be included in a sensitivity analysis tool. It should:

- Provide a meaningful measure of sensitivity, which might draw the DMs' attention quickly to issues of sensitivity;
- Take into account the most recent information and not introduce extraneous elements;
- Enable the identification of critical judgments;
- Enable the identification of competitors of the highest ranked alternative that are adjacent to the potentially optimal solution[1]; and
- Be easy to implement.

Where possible, these properties have been used to assess the sensitivity analysis methods described in this chapter.

3.2 Deterministic sensitivity analysis methods

This section describes, in chronological order, ten deterministic sensitivity analysis methods that predominantly use distance-based techniques[2]. Table 3.1 summarises the deterministic methods described in this Section, and a number of other methods that have not been described in detail in this Section. The input parameters that are included in the sensitivity analysis, and the MCDA method that the sensitivity analysis method(s) are applicable to, are also included in Table 3.1.

[1] A concept introduced by Rios Insua and French (1991), which refers to searching for alternatives that may share optimality with the highest ranked alternative.

[2] Details of other deterministic sensitivity analysis methods can be found in the following references: Fishburn *et al.* (1968), Bana e Costa (1988), French (1992) and Felli and Hazen (1998).

Table 3.1 Summary of selected deterministic sensitivity analysis methods utilised with MCDA

Sensitivity Analysis Method	Summary	CWs only		PVs only		MCDA technique(s) applied to	Distance Metric (where applicable)	Described In:
		Single	All	Single	All			
Isaacs (1963)	Sensitivity to subjective probability estimates	x	✓*	x	x	WSM	Euclidean	See reference
Roy *et al.* (1986)	Compare outputs using different sets of CWs and thresholds	x	✓#	x	x	ELECTRE III	NA	See reference
Barron and Schmidt (1988)	Changes in CWs that result in equal ranking of alternatives with highest ranked alternative	x	✓	x	x	Additive MAVT	Euclidean, Kullback-Leibler	Chapter 3
Mareschal (1988)	Weight stability intervals	✓	x	x	x	Additive utility functions, PROMETHEE	NA	Chapter 3
Soofi (1990)	Logarithmic information discrepancy	x	✓	x	x	MAVT	Kullback-Leibler	See reference
Rios Insua and French (1991)	Simultaneous variation of data to find possible competitors of best solution	x	✓	x	x	WSM	Euclidean, Chebyshev	Chapter 3
Wolters and Mareschal (1995)^ - Method 1 - Method 2 - Method 3	(1) Sensitivity of ranking to changes in evaluations of all alternatives on certain criteria; (2) influence of specific changes in certain PVs; (3) minimum modification of CWs required to make an alternative ranked first	x	✓ (3)	✓ (2)	✓ (1)	PROMETHEE, Additive MCDM methods	(3) City block distance metric (i.e. Manhattan Distance)	Chapter 3

Note:
* method states it is estimation of 'probability' estimation error but methodology is still relevant if replace probability with weights

also apply method to preference and veto thresholds

^ numbers in brackets refer to either Method 1, Method 2 or Method 3

Table 3.1 Continued …

Sensitivity Analysis Method	Summary	CWs only		PVs only		MCDA technique(s) applied to	Distance Metric (where applicable)	Described In:
		Single	All	Single	All			
Janssen (1996)	Minimum modification of CWs required to make an alternative ranked first	x	✓	x	x	WSM	Euclidean	Chapter 3
Triantaphyllou and Sanchez (1997)	(1) smallest changes in CWs which can alter ranking of alternatives; (2) how critical PVs are in ranking of alternatives	✓	x	✓	x	WSM, WPM, AHP	NA	Chapter 3
Ringuest (1997)	Determine change in CWs for different alternative to be ranked first, and whether CW rank order changes	x	✓	x	x	WSM, MAUT	Manhattan and Tchebycheff	Chapter 3
Guillen *et al.* (1998)	Proportion in which CWs must be modified to change the ranking between 2 alternatives	x	✓	x	x	WSM	NA	Chapter 3
Proll *et al.* (2001)	Finds 'nearest competitors' of current optimal alternative	x	✓	x	x	Additive methods	Manhattan, Euclidean and Tchebycheff	Chapter 3
Shepetukha and Olson (2001)	Systematically explore entire region within CW bounds	x	✓	x	x	WSM, MAUT	NA	See reference
Jessop (2004)	(1) choose alternative which maximises number of times it is best; (2) maximise difference between best and second best; (3) minimise difference between chosen alternative and best under different CWs	x	✓	x	x	WSM, MAUT	NA	Chapter 3

3.2.1 Barron and Schmidt (1988)

Barron and Schmidt (1988) recommended two procedures to accomplish a sensitivity analysis when utilising multi-attribute value models. These are an entropy-based and a least squares procedure to determine the CWs sufficient to equate, or reverse by a prescribed amount, the overall additive multi-attribute value of any pair of mutually dominated alternatives:

1. *Entropy-based procedure:*

   When the initial CWs are all equal, the method computes the 'most nearly equal CWs' that promote the second ranked alternative by an amount Δ, where Δ is the difference in the total values of two alternatives.

   The entropy-based (i.e. Kullback-Leibler distance) procedure has two limitations: (i) the procedure seeks 'nearly equal' weights, and (ii) the weight computation requires that the objective function be solved iteratively.

2. *"Least squares" procedure:*

   Determines the set of CWs that is 'closest' to the given set of CWs which promotes the inferior alternative so that its total value exceeds that of the first alternative by an amount Δ, where Δ is the difference in the total values of two alternatives. The concept of 'close' is operationalised by the minimum squared deviation principle (i.e. L_2-metric or Euclidean Distance). As part of this method the objective function is formulated as a quadratic programming problem.

   The "least squares" sensitivity analysis method proposed by Barron and Schmidt (1988) commences with some initial values for the CWs, which result in a most preferred alternative (i.e. the alternative with the highest total value). For each other non-dominated alternative, in turn, a new CW set is found which has the effect that this new alternative has a total value equal to that of the most preferred alternative. The frequency with which alternatives are found to be best is a guide to the sensitivity of the result and to the nomination of a preferred alternative. New CWs are found which are in some sense closest to the originals while permitting rank adjustment. The methods rely heavily on the initial specification of the CWs (Jessop, 2004). Barron and Schmidt (1988) utilise the software package GINO (General INteractive Optimizer) to solve the objective function, however, no information is provided on this software package or how the method may be formulated and solved with an alternative software package (it should be noted that adequate equations are provided by Barron and Schmidt (1988) so that the methods could be operationalised if the reader has the necessary skills and knowledge).

Both sensitivity analysis methods proposed by Barron and Schmidt (1988) are limited by only considering the uncertainty in the CWs and they are only shown to be applicable to the additive multi-attribute value model. Intervals that the CWs can be varied between (i.e. constraints) are discussed but they are not included in the formulation and critical criteria are not identified in the methodology. A case study is undertaken by Barron and Schmidt (1988) to illustrate the proposed methodology, however, the only results presented in the paper are the modified CWs and adequate discussion is not included on how the DM would utilise the results obtained (i.e. the Euclidean distance is not presented and neither is the difference between the original CWs and the changed CWs).

3.2.2 Mareschal (1988)

Mareschal (1988) proposes a sensitivity analysis method whereby stability intervals for the weights of different criteria are defined. These consist of the values that the weight of one criterion can take without altering the results given by the initial set of CWs, all other CWs being kept constant. In other words, the method investigates what the ranking of the alternatives becomes when all of the CWs are kept constant, except those for one criterion (say x_i), which may be increased or decreased from their initial value. In order to keep the modified set of CWs normalised, all of the CWs need to be adjusted to ensure that only the importance of the CW being assessed relative to the other criteria is modified. Details of the formulation of the stability intervals can be found in Mareschal (1988).

Mareschal (1988) also extends the notion of a stability interval in order to study the total impact on the ranking of the alternatives when the total subset of the CWs is modified. Three types of stability are assessed:

1. *Full stability:* in this situation, the position of each alternative in the ranking of alternatives is of interest to the DM. Full stability is defined as the absence of any modification in the whole structure.

2. *Partial stability:* in many practical cases, the DM only has to select one or a small number of alternatives and is only interested in the highest ranked alternatives. What is important is that the position of these alternatives remains stable and it does not matter whether the worst alternatives are sensitive to CW changes. So the notion of stability concerns the stability of only a part of the ranking of alternatives.

3. *Subset stability:* if the DM only wants to eliminate the worst alternatives and to obtain a subset of good alternatives, with no additional information about these alternatives, the subset stability is defined as the stability of this set.

Mareschal (1988) states that the construction of such stability intervals is possible for a wide range of MCDA methods, including additive utility functions and outranking methods. The CW sensitivity analysis proposed by Mareschal (1988) only determines the stability of a ranking of alternatives i.e. boundaries are derived within which the values of (combinations of) the CWs are allowed to vary without modifying the ranking. This provides sufficient information on the ranking itself, however, it does not give insight into the way the ranking is changed if these boundaries are exceeded. The method is also limited, as it only considers the sensitivity of the CWs and not the combined sensitivity with the PVs. Another shortcoming is that the focus of the methodology is on changing the weight of one criterion at a time.

The example decision problem presented by Mareschal (1988) demonstrates how the methodology can be utilised. Information is provided on how to interpret the results, however, the most critical criteria are not identified.

3.2.3 Rios Insua and French (1991)

Rios Insua and French (1991) have developed a conceptual framework for sensitivity analysis in MCDA with a discrete set of alternatives which allows simultaneous variation of judgmental data (i.e. CWs). The methods proposed by Rios Insua and French (1991) allow inferior alternatives to be discarded from the analysis and the competitors of a current optimal alternative to be found. Moreover, the methods identify the 'smallest' changes necessary in the parameters before a significant change in the ranking of the alternatives occurs. This is achieved by using either the Euclidean distance metric or the Chebyshev distance metric. Constraints are able to be included in the formulation, such as lower and upper bounds of the CWs.

Rios Insua and French (1991), as part of their methodology, also calculate a 'sensitivity index' (r):

$$r = \frac{\Delta}{\delta}$$ **Equation 3.1**

where:

$$\Delta = -0.5\sum w_i^{\,2} + \sum \omega_i w_i$$ **Equation 3.2**

$$\delta = \sqrt{-2\Delta + \sum \omega_i^{\,2}}$$ **Equation 3.3**

where:

w_i = the initial CWs

ω_i = the estimates of the CWs

If $r = 1$ then the alternative is completely insensitive to changes in CWs and if $r = 0$ then the alternative is completely sensitive to changes in CWs.

In addition, different types of graphical representation of the output of the sensitivity analysis are discussed by Rios Insua and French (1991), including principal components analysis and star graphs. Further details of the proposed approaches can be found in Rios Insua and French

(1991). The limitation of the methodology is that only uncertainty in the CWs is considered and it is only shown to be applicable to the WSM.

Rios Insua and French (1991) illustrate the framework by way of an example decision problem. Limited information is provided in the paper on how the problem was solved (i.e. the NAG LP routine E04MBF and E04NAF). Information is provided on how to interpret the results, including which alternative is the clearest competitor of the initially highest ranked alternative, however, the most critical criteria are not identified and the modified CWs are not presented.

3.2.4 Wolters and Mareschal (1995)

Three types of sensitivity analysis methods are presented by Wolters and Mareschal (1995):

1. *To determine the sensitivity of a ranking to changes in the data of all alternatives on certain criteria*.

   This form of sensitivity analysis method is important in case uncertainties are present in certain criterion PVs. A set of scenarios has to be defined in order to incorporate the uncertainties;

2. *To determine the influence of changes in PVs of a specific alternative on certain criteria*.

   This form of sensitivity analysis is important if uncertainties arise in the PVs of just one alternative, which is studied by iteration. For example, the method can determine how much the value of one criterion has to be reduced by for a selected alternative to be ranked first; and

3. *To determine the minimum modification of the CWs required to make a specific alternative ranked first by exploring the total weight space while taking into account specific requirements on the variations of the weights.*

   A linear objective function and a number of constraints are derived. The DM is able to state that the CWs of certain criteria are more likely to change than others. Considerations such as the relative importance of two criteria remaining constant or a set of criteria keeping the same relative importance are taken into account by adding additional constraints to the linear programming model. Weight intervals specified by the DM can also be taken into consideration. The objective function measures the distance between the initial CWs and the modified ones. The minimum weight modification that is determined enables a proximity ranking to be defined. Thus, it can be studied which alternative is closer (and consequently more likely) to being ranked first, given an initial set of CWs.

The Wolters and Mareschal (1995) approach is equivalent to minimising the L_1-metric (i.e. city block or Manhattan distance metric), subject to constraints, but without the assumption of equal weights. The analyses are focused on and elaborated for the PROMETHEE methods, however, Wolters and Mareschal (1995) believe that the methods are generally applicable to additive MCDA methods, including additive utility theory. The limitation of the methodologies presented is that the uncertainty in the CWs and PVs is not considered simultaneously.

A numerical example is presented by Wolters and Mareschal (1995) to illustrate the methodology, however, no information is provided on the software or program utilised. Apart from this, the example problem does demonstrate how the sensitivity analysis method can be utilised and the output information that is provided to the DM to aid in making a decision (e.g. Manhattan distance and modified CWs).

3.2.5 Janssen (1996)

A procedure is described by Janssen (1996) to estimate certainty intervals for CWs and PVs. Within a certainty interval, the ranking of two alternatives is not sensitive to changes in PVs or CWs.

A method of 'halving' is introduced which provides the DM with an indication of the degree of sensitivity of a certain outcome ($A_a > A_b$) to changes in the value of a certain CW or a certain PV. For example, an initial CW is selected and it is tested whether the original ranking holds. If the ranking does not hold, an additional CW is tested which is in the middle between the initial test CW and the original CW. This process is repeated. By undertaking this methodology, Janssen (1996) states that the DM is informed on how sensitive the results of the MCDA are to changes in the input parameters.

This method is limited, as it focuses on the input parameter of one criterion and assumes that the ratio between the other input parameters remains unaltered. The method also assesses the uncertainty in the CWs and PVs separately. Janssen (1996) does, however, note that the method of 'halving' can be used for any multi-criteria method.

Alternatively, an algorithm has been developed by Janssen (1996) which uses the method of 'halving' to find the set of CWs with the smallest Euclidean distance from the original set of CWs that reverses the ranking of the alternatives being assessed. The procedure does not guarantee that the turning point with the minimum distance is found. The solution may be a local optimum and not a global one. How this optimisation is undertaken is not described by Janssen (1996) and it also does not consider the uncertainty in the PVs.

The sensitivity analysis methods described above are illustrated by Janssen (1996) using a nuclear power plant siting decision problem and the Evamix method to initially rank the alternatives. The certainty intervals of the CWs where rank reversal will not occur between pairs of alternatives is presented to the DM, along with the minimum Euclidean distance and the associated modified set of CWs for where rank reversal will occur between selected pairs of alternatives.

3.2.6 Triantaphyllou and Sanchez (1997)

The method proposed by Triantaphyllou and Sanchez (1997) involves performing a sensitivity analysis on the weights of the decision criteria and the PVs of the alternatives expressed in terms of the decision criteria. Separate methods are proposed for three MCDA methods (weighted sum model (WSM), weighted product model (WPM) and analytic hierarchy process (AHP)). The method described in this section is based upon the WSM.

The minimum quantity that a CW needs to be changed to reverse the current ranking can be calculated for each pair of alternatives for each criterion by:

$$\delta_{1,1,2} = \frac{(P_2 - P_1)}{(x_{2,1} - x_{1,1})} \qquad \textbf{Equation 3.4}$$

where:

P_2 = ranking of alternative 2

P_1 = ranking of alternative 1

$x_{2,1}$ = performance value of criterion 1 of alternative 2

$x_{1,1}$ = performance value of criterion 1 of alternative 1

The following condition must be satisfied for the new weight to be feasible:

$$\delta_{1,1,2} \leq w_1 \qquad \textbf{Equation 3.5}$$

Sometimes there may not be a feasible value, as it may be impossible to reverse the existing ranking by making changes to the current weight of the criterion.

The modified weight of the first criterion is therefore:

$$w_1{*} = w_1 - \delta_{1,1,2} \qquad \textbf{Equation 3.6}$$

The percentage change in the weights can be defined as:

$$\%w = \frac{w_1{*}}{w_1} \times 100 \qquad \textbf{Equation 3.7}$$

The critical criterion is defined as the smallest relative value of $\%w$ in all rows that is related to the best alternative, however, it can also be found for all alternatives.

The threshold value R (in %) by which the performance measure of an alternative, in terms of criterion c_m, denoted as P_i, needs to be modified so that the ranking of alternatives a_n and a_p will be reversed is given as follows:

$$R_{n,p} = \frac{(P_n - P_p)}{w_m} \times \frac{100}{x_{m,n}} \qquad \textbf{Equation 3.8}$$

Furthermore, the following condition should also be satisfied for the threshold value to be feasible:

$$R_{n,p} \leq 100 \qquad \textbf{Equation 3.9}$$

The limitations of this method are that the approach depends on the MCDA method applied, the sensitivity of the CWs and the PVs are assessed independently and only one input is varied at a time. Despite these shortcomings, the critical criteria are identified, which has not been a part of the methodology in the sensitivity methods already presented in this chapter. In addition, the notion of relative and absolute 'smallest' changes in the input parameters is introduced by Triantaphyllou and Sanchez (1997).

3.2.7 Ringuest (1997)

Two measures of sensitivity analysis are presented by Ringuest (1997):

1. An alternative is considered insensitive if the weights which yield a different alternative as best are "not close" to the weights which led to the original best alternative.

2. A decision is considered insensitive if, in addition to satisfying the criterion above, the rank order implied by the weights, which led to the original best solution, must be altered for a different alternative to be preferred.

The solutions are obtained by solving two linear programs which minimise the L_1 (i.e. city block or Manhattan distance metric) and L_∞ (i.e. Tchebycheff metric) metrics, subject to a number of linear constraints. It should be noted that in the L_p-metric, the effect of P is to place more or less emphasis on the relative contribution of the individual deviations.

The larger the value of P chosen, the greater the emphasis given to the largest of the deviations forming the total. Ultimately, when $P = \infty$, the largest of the deviations completely dominates the distance measure. The values $P = 1$ and $P = 2$ are also commonly used. When $P = 1$ the deviations are simply summed over all criteria. When $P = 2$ the metric measures the shortest geometric distance between two points, a straight line, and is referred to as the Euclidean distance.

The Ringuest (1997) approach is an extension of the Barron and Schmidt (1988), Rios Insua and French (1991) and Wolters and Mareschal (1995) sensitivity analysis methods. The emphasis of the Ringuest (1997) method is on rank reversals, in addition to "closeness" of changes in the input parameters. Ringuest (1997) includes constraints, such as the CWs are to have the same order as the original CWs and the total values are constrained so that they are in the same rank order. Ringuest (1997) state that the formulation can be solved by any linear programming package. The most important criteria are identified, based on the rankings of the CWs.

The limitation of this sensitivity analysis method is that is only considers the sensitivity in the CWs. The formulation is also only described for the multi-attribute value model. The only output provided from the numerical example is the modified CWs and the comparison of the original and changed rankings of the CWs when constraints were not included in the formulation.

3.2.8 Guillen *et al.* (1998)

Guillen *et al.* (1998) proposed an index that allows the DM to "determine the robustness of the preference between two alternatives". The index is defined as the proportion by which the DM must modify the CWs to change the preferences between two alternatives. The robustness index can be calculated for each pair of alternatives using the following equation:

$$r(a_1, a_2) = \frac{w_1 \times (x_{1,1} - x_{1,2}) + \ldots + w_m \times (x_{1,n} - x_{2,n})}{w_1 \times |(x_{1,1} - x_{1,2})| + \ldots + w_m \times |(x_{1,n} - x_{2,n})|} \qquad \textbf{Equation 3.10}$$

$r(a_1, a_2)$ takes its value in the interval {-1,1}. a_1 dominates a_2 on all criteria when $r(a_1, a_2)$=1.

The CWs required to reverse a ranking can be calculated using the following equation:

$$w_1^* = if\left(x_{1,1} > x_{1,2}, w_1 - w_1 \times r(a_1,a_2), w_1 + w_1 \times r(a_1,a_2)\right)$$ **Equation 3.11**

The limitations of this method are that it only compares two alternatives at a time, is only applicable to one MCDA model (the general additive preference model), is only concerned with the CWs, and critical CWs are not identified as all weights are adjusted by an equal proportion dependent on the initial value. In addition, the sum of the weights at the commencement of the analysis equals one, however, following the adjustments of the weights based on the robustness measure attained the sum of the weights is no longer equal to one. This would seem to be a fundamental flaw in the method.

3.2.9 Proll *et al.* (2001)

Proll *et al.* (2001) suggest a framework for sensitivity analysis using distance tools, through which immediate contenders for optimality are detected. The process involves:

1. Ranking the alternatives;
2. Considering the first ranking alternative as optimal;
3. Finding least changes in parameters leading to the first ranked alternative being outranked by other alternatives; and
4. Ranking alternatives according to minimum distance, where the Manhattan (L_1), Euclidean (L_2), and Tchebycheff (L_∞) distance metrics are utilised.

This methodology is based on the framework proposed by Rios Insua and French (1991). The main focus of the work undertaken by Proll *et al.* (2001) was to improve on the algorithm and the implementation in order to reduce the computational load. The implementation proposed performs a local optimisation first and only performs a global optimisation if necessary. Proll *et al.* (2001) have developed a simulated annealing method which exploits the structure of the problem constraints to ensure that random neighbours of the current point are always feasible. Proll *et al.* (2001) highlight the limitations and implications of the implementation such as potentially only identifying local optima.

No application of the proposed methodology is presented in the paper, only a summary of results where the improvements to the sensitivity algorithm were tested on decision problems from the literature. Although the modifications appear to reduce the computation time, no discussion is provided on the outputs that are provided to the DM and only the uncertainty in the CWs is considered. The MCDA methods that the methodology is applicable to is also not discussed.

3.2.10 Jessop (2004)

Given the factors which may make CW determination problematic, Jessop (2004) believes that it is natural to seek to justify the selection of an alternative by showing that it is insensitive to CW imprecision. Three views are tested by Jessop (2004) on a real data set, each in an attempt to identify optimal alternatives:

1. Choose the alternative which maximises the number of plausible scenarios in which it is best (sensitivity). Given some initial set of CWs, a preferred alternative is identified. Each other alternative in turn has its performance set equal to that of the preferred alternative by some suitable adjustment of the CWs. The number of these problems for which an alternative has the highest total value is taken as a measure of performance. A characteristic of this method is that, as a result of each optimisation, it is not necessarily the case that the alternative whose performance is being addressed is the best performer. It should be noted that this method is the same as that proposed by Barron and Schmidt (1988) (see Section 3.2.1);

2. Maximise the difference between the best and second best ranked alternatives (robustness). For each alternative, CWs are chosen such that the alternative has a higher total value than all other alternatives. The superiority of the alternative may be defined either by the difference between its total value and the total value of all the others or between its total value and the total value of the second best alternative; and

3. Minimise the difference between a chosen alternative and the best alternative under different weighting schemes (risk aversion) i.e. a minimax regret criterion. For each of the optimisations made for the superiority analyses (number 2 above), the total value of all but one alternative will be sub-optimal. The alternative for which the maximum degree of sub-optimality is minimised may be seen as the preferred alternative.

All of the methodologies described and tested by Jessop (2004) only consider the uncertainty in the CWs, only utilise the WSM MCDA technique and do not identify critical criteria.

3.2.11 Summary

The majority of the deterministic sensitivity analysis methods discussed in this thesis seek to determine what the smallest changes in the input parameters are for an alternative to outrank the initial highest ranked alternative. Another common feature is the use of distance metrics. The main differences are in the way that the decision problem is formulated and the information that is presented to the DM.

More specifically, the Rios Insua and French (1991) sensitivity analysis method and the third method presented by Wolters and Mareschal (1995) are, in general, extensions of the method proposed by Barron and Schmidt (1988), even though Rios Insua and French (1991) state that the Barron and Schmidt (1988) analyses do not proceed along the way that they have described. The predominant difference is that constraints are able to be incorporated in the formulation in the Rios Insua and French (1991) and Wolters and Mareschal (1995) methodologies. In addition, Rios Insua and French (1991) and Wolters and Mareschal (1995) utilise the distance metrics obtained to determine the likelihood that an alternative may be outranked by changes in the CWs, whereas Barron and Schmidt (1988) only consider the changed CWs. All of the methods do not determine the critical criteria. The approach proposed by Ringuest (1997) is also of the same type as the methods proposed by Barron and Schmidt (1988), Rios Insua and French (1991), and Wolters and Mareschal (1995). However, Ringuest (1997) not only includes constraints on the CWs, but also on the total value of the alternatives and critical criteria are able to be identified.

The main focus of the work undertaken by Proll *et al.* (2001) was to improve on the algorithm suggested by Rios Insua and French (1991) and the implementation in order to reduce the computational load. Of the deterministic methodologies presented in this chapter, this is the only one that discusses the implementation of the method in detail. In addition, Proll *et al.* (2001) highlight the limitations and implications of the implementation such as potentially only identifying local optima.

Sensitivity analysis methods that are not explicitly based on distance measures have also been developed, including the approach developed by Bana e Costa (1988). In addition, the second sensitivity analysis method presented by Jessop (2004) is quite distinct to the other deterministic methods discussed in this chapter, whereby the difference between the first and second ranked alternatives is maximised.

Another different class of sensitivity analysis methods described in Section 3.2 is that of determining certainty intervals, as proposed by Janssen (1996) and Mareschal (1988). The only similarity between the approach proposed by Triantaphyllou and Sanchez (1997) and those of Mareschal (1988) and Janssen (1996) is that they all only allow one parameter to vary at a time, while the remainder are fixed.

3.3 Stochastic sensitivity analysis methods

An alternative approach to deterministic sensitivity analysis in the MCDA context is through simulation. A commonly used strategy to implement these types of sensitivity analyses is the Monte Carlo Simulation (MCS) approach, which generally requires the DM to specify percentage ranges which indicate how much PVs and CWs can vary (Hajkowicz *et al.*, 2000). There have been significantly fewer stochastic sensitivity analysis methods proposed in the literature, compared to the deterministic methods, consequently, only three stochastic sensitivity analysis methods have been selected to be presented in this section and a summary of these methods is contained in Table 3.2[3].

[3] Details of other stochastic sensitivity analysis methods can be found in the following references: Critchfield and Willard (1986), Helton (1993), and Mareschal (1986).

Table 3.2 Summary of selected stochastic sensitivity analysis methods

Sensitivity Analysis Method	CWs only		PVs only		MCDA technique(s) applied to:
	Single	All	Single	All	
Janssen (1996)	✓	✓	✓	x	EVAMIX, WSM
Butler *et al.* (1997)	x	✓	x	x	MAUT, AHP
Jessop (2002)	x	✓	x	x	WSM

3.3.1 Janssen (1996)

Janssen (1996) introduces a procedure to analyse systematically the sensitivity of the ranking of alternatives to overall uncertainty in CWs and PVs. The sensitivities of rankings of alternatives to overall uncertainty in PVs and CWs are analysed using a MCS approach. The DM is asked to estimate the maximum percentage that actual values may differ from the PVs or set of CWs. A random generator is used to translate this information into a large number of PV and CW sets around the original PVs or CWs. Rankings are then determined for all PVs or CWs. The PVs and CWs are assumed to be normally distributed. The results of the analysis are presented in a probability matrix, which represents the probability that alternative *i* ranks at position *n*.

The methodology has been applied to a transportation example, using the MCDA WSM approach. The results are discussed to indicate the information provided to the DM following the analysis. For example, Janssen (1996) states that the ranking of two alternatives is considered sufficiently certain if the difference between the weighted sums of these alternatives exceeds the arbitrarily set value of 0.2.

The limitations of the sensitivity analysis methods proposed by Janssen (1996) are that the potential uncertainty in all of the input parameters is not taken into consideration simultaneously and only one type of distribution is utilised to represent the uncertainty in the input parameter values. Also, no information is provided on how the methodology is implemented.

3.3.2 Butler *et al.* (1997)

Butler *et al.* (1997) suggest a methodology which utilises MCS to vary all of the CWs of a MCDA model simultaneously. In addition, the method investigates the impact of varying the functional form of the multi-attribute aggregation. Butler *et al.* (1997) state that the simulation approach is applicable to any MCDA method that utilises CWs in an aggregative scheme (e.g. AHP, MAUT).

Three general classes of simulation models were presented by Butler *et al.* (1997) that offer assistance in the evaluation of CWs for MCDA models:

1. *Random CWs* which require no weight assessments (uniform distribution on (0,1)) to explore the entire domain of possible weight combinations;

2. *Random rank order CWs* which requires an importance ranking of criteria, as the relative importance ranking of criteria may be less controversial than the exact magnitude of the CWs. Rank ordering is where the highest ranked criterion is the one the DM would most prefer to increase from the worst to the best level of performance. The rank order weights on the measures is maintained, but the weights are otherwise generated at random; and

3. *Response distribution CWs* where the assessed CWs are treated as means of probability distributions of responses and CWs are then generated from these distributions. The idea is to consider the assessed CWs as responses obtained from a distribution of possible responses. The additive model uses gamma distributions and the multiplicative model uses beta distributions to simulate the CWs.

Output statistics of the ranking results are mode, minimum, 25th percentile, 50th percentile, 75th percentile, maximum, mean, and standard deviation. Cumulative ranking distribution figures are also produced, as well as a figure displaying the range of rankings for each alternative. The limitation of the sensitivity analysis methods proposed by Butler *et al.* (1997) is that only the sensitivity in the CWs is considered and the relative impact of the CWs is not assessed.

A coal power plant site selection problem is used by Butler *et al.* (1997) to demonstrate the proposed methodology. 5,000 random independent trials are undertaken to obtain the results. However, this is the only information provided on how the simulations were conducted.

3.3.3 Jessop (2002)

Jessop (2002) proposes a methodology which models the effect of uncertainty in the CWs via MCS. The process starts with the idea that all criteria are equally important (i.e. each CW has a value of 1 divided by the number of criteria). It is suggested by Jessop (2002) that the initial uncertainty may be modelled with a uniform distribution, with limits ranging from about zero to approximately twice the initial value. The simulated ranks of the alternatives are ordered according to the mean simulated rank.

The method is limited by only considering the uncertainty in the CWs and that no correlation in the CWs is considered if the initial CWs are not equally ranked. However, it is acknowledged by Jessop (2002) that the method could be extended to take uncertainty in the PVs into account.

The methodology is demonstrated by Jessop (2002) by applying it to an example decision problem which involved prioritisation of an IT budget. No information is provided on how the simulation was undertaken i.e. what program was utilised and how many iterations were completed to arrive at the results presented. The WSM MCDA technique was applied to obtain the rankings of the alternatives, but it is not stated by Jessop (2002) whether the methodology is applicable to other MCDA techniques.

3.3.4 Summary

The three stochastic methods described in this section all utilise MCS to simultaneously assess the impact of the uncertainty of the CWs on the decision problem. Only the method proposed by Janssen (1996) enables consideration of the impact that the uncertainty of the PVs may have on the results of the decision analysis and none of the methods analyse the impact of the uncertainty of all of the input parameters concurrently. The approach proposed by Jessop (2002) is very similar to that of Janssen (1996) and only Butler *et al.* (1997) extends the basic idea of how the

CWs may be incorporated in the analysis, by varying the level of information required to define the distributions.

3.4 Extensions of existing MCDA techniques

Most MCDA methods do not take into account the uncertainty of the data they analyse. Therefore, some authors have proposed alterations to some MCDA methods as opposed to using standalone sensitivity methods, (as described in Sections 3.2 and 3.3), which are described below.

3.4.1 PROMETHEE

Mareschal (1986)

Mareschal (1986) define the notion of stochastic MCDA to take into account uncertainty in the PVs when utilising the outranking MCDA aggregation method, PROMETHEE. Joint distributions are proposed to be utilised for the PVs, however, in practice it is recognised that generally only the marginal distributions are known for the experts' evaluations and the exact joint distribution is not known. Mareschal (1986) therefore proposes a methodology that requires the calculation of the differences of the marginal distributions. Mareschal (1986) states that it is possible to compute the expected values of the flows, as flows are linear combinations of the preference functions.

D'Avignon and Vincke (1988)

D'Avignon and Vincke (1988) propose a multi-criteria procedure which transforms distributive evaluations of alternatives, according to DM's preferences, in order to progress to a ranking of these alternatives. The procedure consists, for each couple of alternatives, to construct a distributive preference degree with respect to each criterion and a distributive outranking degree over all criteria. These distributive outranking degrees are then explored in order to rank the alternatives, totally or partially. The method was developed as the uncertainty of consequences and the imprecision of data often imply the use of probability distributions to characterise the evaluation of each alternative with respect to each criterion. As the method is concerned with pair-wise comparisons of alternatives it is classified in the set of outranking methods and has been included in the section on PROMETHEE, although it draws on the concepts of both the PROMETHEE and ELECTRE outranking methods.

Le Teno and Mareschal (1998)

An extension of the PROMETHEE framework was introduced by Le Teno and Mareschal (1998) to cope with interval values of the criteria PVs. As part of the proposed approach, it is suggested that a decision matrix of worst bounds only and a decision matrix of best bounds only be considered. They can either be lower or upper bounds, depending on whether the criterion is to be maximised or minimised. Instead of one net flow for each alternative, there are four possible values obtained using this approach. An interval contains more information than a single value, but no assumption is made on the distribution of values or sub-intervals within its bounds. The user, however, is still required to choose generalised criterion functions for each of the criteria, in addition to the associated threshold values. The uncertainty in the CWs is not taken into consideration, which is another limitation of this approach.

ProDecX

ProDecX is a software program (in development), which utilises the PROMETHEE MCDA technique (Proctor and Drechsler, 2003). In ProDecX, for each criterion, the weights are sampled from the weights given by the actors. Given the various CWs from the different actors, the software determines the mean and standard deviation of the net flux for each alternative. The standard deviation of the net flux is a very important indicator of whether there is consensus on the rank order of the alternatives or not. The smaller the standard deviation compared to the differences between the average net fluxes of two alternatives, the more conclusive the ranking i.e. the higher the consensus (Proctor and Drechsler, 2003). The uncertainty in the PVs, generalised criterion functions and associated threshold values are not considered in the program, which is a limiting factor.

Klauer *et al.* (2002)

Since PROMETHEE in its basic version is not able to process uncertain information, an extension to the method has been developed by Klauer *et al.* (2002). To take uncertainty into account, in addition to capturing correlations, probability distributions of the differences between the performances of the alternatives are determined for all pairs of alternatives. The uncertainty in the CWs is taken into account by a number of steps. In the first step, the CWs are not fixed and instead a large number (i.e. 1,000) of CWs are randomly drawn. For each CW combination, the PROMETHEE method is performed and a rank order of the alternatives is determined. Subsequently, for each alternative, the frequencies of the alternative being rank 1, rank 2 etc. is calculated. In the second step, the variability of the weights is partially restricted by fixing the ratios of some of the weights. Yet again, the methodology only takes the uncertainty of the CWs into consideration and no provision is made to assess the uncertainty of the other input parameter values.

3.4.2 ELECTRE

Miettinen and Salminen (1999)

Miettinen and Salminen (1999) propose a method to describe to the actors what kind of weighting ranks a specific alternative as the best in the minimum-procedure for the discrete MCDA technique ELECTRE III. The aim is to clarify the decision problem to the actors by providing different weighting vectors describing the acceptability of each alternative in different types of situations. It is a non-convex problem and therefore may have several local optima. Whether the optimisation ends up with a local or a global optimum depends highly on the selected starting values. This can be partly overcome by solving the problem with several different starting weights and selecting the best as the final solution. On the other hand, methods of global optimisation could be employed. However, Miettinen and Salminen (1999) believe that this would increase the computational costs and complexity. It should be noted that the method does not remove the need to perform sensitivity analysis on the threshold values or the PVs.

Leyva-López and Fernández-González (2003)

Leyva-López and Fernández-González (2003) propose a method called ELECTRE-GD, which is a natural extension of the ELECTRE III approach to collaborative group decision making, using a genetic algorithm for exploiting the fuzzy outranking relation. The proposed method is not limited to ELECTRE, as it can be used with any method based on building a fuzzy preference relation. As part of the method there exists a DM with authority for establishing consensus rules and priority information on the set of group members. ELECTRE-GD works with the natural heuristic used by collaborative groups for making reasonable or consensus agreements, based on universally accepted majority rules combined with the necessary observance of significant minorities. Details of the methodology are contained in Leyva-López and Fernández-González (2003).

Nowak (2004)

Nowak (2004) introduces a method to employ the concepts of thresholds in a stochastic case. The ranking of the alternatives is obtained by distillation procedures proposed in ELECTRE III. The evaluations of alternatives with respect to criteria are expressed in the form of probability distributions, however, only single values are used for the CWs and the thresholds.

3.4.3 Multi-attribute utility theory

Fischer *et al.* (2000)

Fischer *et al.* (2000) develop a family of models addressing preference uncertainty in multi-attribute evaluation (i.e. RandMAU), a family of additive (RandAUF) and multiplicative (RandMUF) random weights multi-attribute utility models. In RandMAU models, preference uncertainty is represented as random variation in both the weighting parameters governing trade-offs among criteria and the curvature parameters governing single-attribute evaluations.

3.5 Discussion

Each approach to uncertainty and sensitivity analysis has its advantages and disadvantages. For a given analysis problem, the available approaches should be considered and the approach that seems most appropriate for the problem should be selected. This selection should take into account the nature of the decision model, the type of uncertainty and sensitivity analysis results desired, the cost of modifying and / or evaluating the model, the human cost associated with mastering and implementing the technique, and the time period over which an analysis must be performed (Helton, 1993).

Although there are a number of sensitivity analysis methods proposed in the literature, they do not appear to have been utilised in applications of MCDA, as summarised in Appendix A. Generally, either the informal approach is utilised or no sensitivity analysis is conducted at all. Ten deterministic and three stochastic sensitivity analysis methods have been described in this chapter to provide an overview of the types of sensitivity analysis methods that are available (and it should be noted that although it is a comprehensive selection, this is not a complete list of available methods[4]). Despite the methods presented having some fundamental differences, the methods all have the same aims of providing general, yet formal, approaches to sensitivity analysis that attempt to identify any possible competitors of a current best alternative.

The majority of methods presented in this chapter, however, do not explicitly provide a meaningful measure of sensitivity, which might draw the DMs' attention quickly to issues of sensitivity. Generally, only the modified input parameters are presented, with limited discussion of how the DM would interpret the results. In addition, only some of the methods (e.g. Triantaphyllou and Sanchez (1997) and Ringuest (1997)) enable, or discuss, the identification of critical criteria i.e. the criteria that have the most impact on the ranking of the alternatives.

However, the main disadvantages of the existing sensitivity analysis methods described in this chapter include:

(i) They generally involve systematically varying one or more of the CWs over their entire ranges while the remaining parameters are fixed, and hence important combined effects of changes in the CWs and the PVs cannot be determined;

(ii) The methods are predominantly developed for a specific MCDA technique, therefore, if various MCDA methods are used in the analysis, a range of sensitivity analysis methods must be utilised;

(iii) The majority of the methods disregard the correlation between the CWs in their analysis; and

[4] Alternative paradigms for analysing uncertainty, which are not discussed here, are Bayesian methods, Fuzzy sets, Dempster-Shafer reasoning and Entropy.

(iv) The CWs, and hence criteria, that are most critical to determining the ranking of the alternatives in the analysis are not identified.

Sensitivity analyses undertaken using the procedures presented in this chapter would therefore be incomplete and unsatisfactory.

Despite the methods presented in this chapter spanning over 17 years of research, the methods have been cited a limited number of times in other journal papers, as is summarised in Table 3.3. In the instances when the journal papers are cited, this is generally in discussions regarding sensitivity analysis, prior to introducing a 'new' sensitivity analysis methodology and not for the purposes of utilising the 'existing' method.

The only application of a sensitivity analysis method that has been described in this chapter has been found in a paper by Ulengin *et al.* (2001). Ulengin *et al.* (2001) assessed a decision problem using the MCDA techniques PROMETHEE I and II, where nine alternative water crossings were being assessed using ten criteria. Although it is not referenced, it appears that the sensitivity analysis method of Mareschal (1988) was utilised by Ulengin *et al.* (2001) to conduct their robustness analysis in order to improve the confidence in the results and to justify the alternatives. This is thought to be the case because intervals of the weights of the fundamental criteria for which the first rank of the complete pre-order among alternatives does not change, provided all other factors remain unchanged, were computed.

Bana e Costa (1988) believes that many methods, although theoretically suitable, are subject to failure in interactive practical applications because of their lack of simplicity. Despite the simplicity of the general concepts of the proposed sensitivity analysis approaches, which have been presented in this chapter, the presentation of the methodologies is quite complex and confusing, mainly due to the large number of equations with undefined parameters, which could be one reason why the methods have seemingly not been implemented.

Table 3.3 Number of citations of sensitivity analysis methods presented in Chapter 3

Method / Reference	**Number of Times Article Cited on Web of Science (July 2005)[1]**	**Application Example in Journal Paper**
Barron and Schmidt (1988)	18	Assess cities
Mareschal (1988)	24	Location of hydroelectric power plant
Rios Insua and French (1991)	34	Floodplain management options
Wolters and Mareschal (1995)	10	Assess alternative heat exchanger networks
Triantaphyllou and Sanchez (1997)	15	Numerical example
Ringuest (1997)	3	Numerical example
Guillen *et al.* (1998)	UNK	Numerical example
Proll *et al.* (2001)	2	Numerical example
Jessop (2004)	1	Selection of council investment project
Janssen (1996)	UNK	Siting nuclear power plants
Butler *et al.* (1997)	14	Siting coal power plants
Jessop (2002)	2	Prioritisation of an IT budget

Note: (1) Web of science indexes a wide range of journal articles in all fields. It is an electronic resource that is published by the Institute for Scientific Information.

(2) UNK = unknown

Rios Insua and French (1991) state that one of the favourable properties of a sensitivity analysis is that it should be easy to implement. However, little discussion is provided by the authors on how to implement the methods that are described in this chapter and what software has been used to undertake the example decision problems used to illustrate the proposed methodologies. This is potentially another reason why the methods have not been implemented.

Based on the findings in this chapter, it is concluded that there is a need to develop sensitivity analysis approaches for MCDA that address the shortcomings outlined above. The details of the proposed approaches developed as part of this research to overcome these limitations are given in Chapter 4.

Chapter 4
Proposed MCDA Uncertainty Analysis Approach

4.1 Introduction

Uncertainty is a source of complexity in decision making and there are various forms of uncertainty that may arise in MCDA from imprecision to ambiguity or lack of clarity. As described in Section 2.5, at one level there is uncertainty about the scores that appear in the effects table i.e. PVs. There is also uncertainty surrounding the assignment of CWs. At another level there is uncertainty about the ability of the selected criteria to adequately represent the objectives that the DM is trying to achieve, uncertainty in model assumptions and uncertainty in the interpretation of results. Variability in all these factors has the potential to affect the rankings of the alternatives of decision problems (Royal Assessment Commission, 1992).

Although the magnitude of the effects of uncertainty in the input parameter values may vary, it is natural for the DM to seek a justification for a recommendation which explicitly addresses these uncertainties. A number of authors have discussed the need for, and value of, more comprehensive and systematic sensitivity analysis (Hodgkin *et al.*, 2005). This is confirmed by the results of the literature review of the existing sensitivity analysis methods contained in Chapter 3. The existing sensitivity analysis methods are generally deemed to be inadequate, mainly because it may be the case that a decision is insensitive to the variations of some parameters in a set individually, but sensitive to their simultaneous variation (Felli and Hazen, 1998), which the existing methods do not take into consideration.

Two uncertainty analysis approaches have been developed (distance-based and stochastic) as part of this research, which are the principal contributions of this work. The main aim of the proposed uncertainty analysis methods is to find any competitors of a current optimal alternative. However, the purpose of the proposed methods is not necessarily to arrive at a different recommendation, but rather to provide a more secure basis from which the recommendation can be made.

Differences in CWs or PVs matter little unless the resulting rank order of alternatives also differs.

The proposed uncertainty analysis methods are described in the subsequent section and the overall process is illustrated in Figure 4.1. Deterministic MCDA is undertaken as the first stage in the proposed approach in order to structure the decision problem and obtain an initial rank order for all of the alternatives. As shown in Figure 4.1, the user is able to select between the two proposed uncertainty analysis approaches (distance-based and stochastic) to provide further information to the DM about the robustness of the initial ranking of the alternatives.

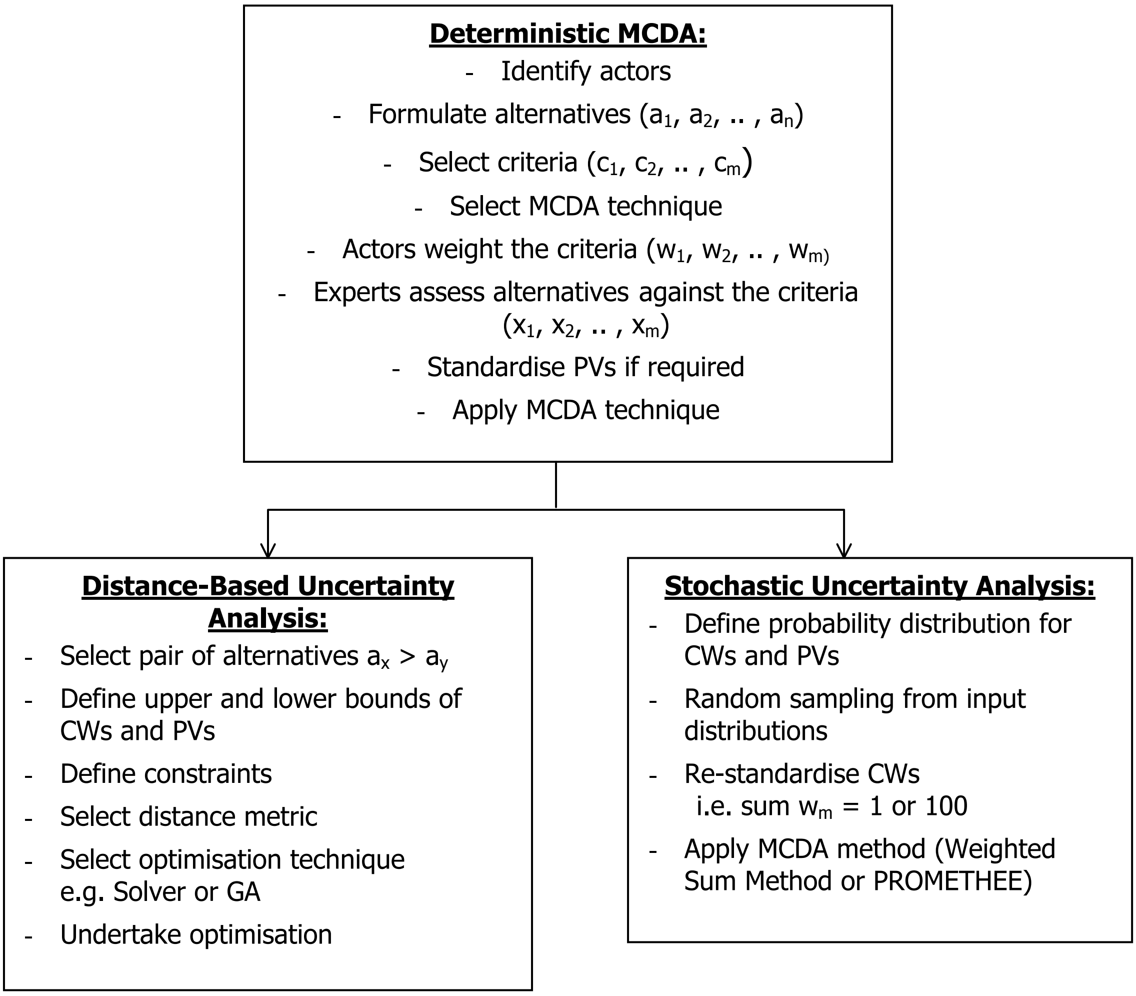


Figure 4.1 MCDA approach with proposed uncertainty analysis methods

Some of the benefits and advancements of the proposed uncertainty analysis approach to keep in mind while reading about the methods are:

- The limitations of the single criterion sensitivity analysis approaches are alleviated by evaluating all possible combinations of input parameters in the proposed approaches;

- Most MCDA models are concerned with refinements of the evaluation steps rather than addressing the decision making process itself and how participation of all relevant stakeholders in the decision making process may be improved (Morrissey and Browne, 2004). The suggested procedure addresses this problem by providing an opportunity to consider the whole range of the specified individual CWs;

- The proposed uncertainty analysis methods address uncertainty in the PVs as well as the CWs. This is important, as most research in the field has focused on accurate estimation of the CWs. A consistent finding across studies undertaken by Olson *et al.* (1998) is that not only are accurate CW estimates important, but also the alternative scores (i.e. PVs) on the criteria;

- Only few multi-criteria methods have been developed to cope with uncertainty. Given that the widely used outranking MCDA method PROMETHEE, in its basic versions, is not able to process uncertain information, an extension of this method that is able to incorporate uncertainty in the PVs and CWs is developed as part of the proposed approach;

- It is easier for each actor and / or expert to provide constraints or bounds on the variables than to find the most 'correct' value for them, which is facilitated in the proposed approaches;

- Exploring the rankings of the alternatives that are known to remain the same, despite the imprecision, (the 'robust' conclusions) provides information to drive the actors' discussions forward towards a consensus (Dias and Climaco, 2005);

- The proposed uncertainty analysis methods are not restricted to particular MCDA approaches;

- The critical input parameters to rank reversal of the alternatives are able to be identified in the proposed approaches which provides direction for further analysis and data collation, if required; and

- Correlation between the CWs is able to be incorporated in the stochastic uncertainty analysis approach.

4.2 Deterministic MCDA

Deterministic MCDA is performed as the first stage of the proposed approach to determine the total values of the alternatives and hence the ranking of each alternative for each actor's set of CWs. The method undertaken is as described in Sections 2.5.1 to 2.5.9, but is summarised here for completeness. Initially, actors are generally selected to be representative of the stakeholders of the particular decision problem. The number of actors varies with each decision problem depending on factors such as the time and resources available and the perceived level of importance of the decision. The decision analysis situation is then translated into a set of alternatives and appropriate criteria must also be chosen to enable information about these alternatives to be collected. The alternatives and criteria are generally developed by the actors under the guidance of the decision analyst. The CWs are elicited from the actors using one of a variety of available techniques. The PVs that are assigned to each criterion for each alternative may be obtained from models (e.g. groundwater level change), available data (e.g. groundwater salinities) or by expert judgment based on previous knowledge and experience. The type of value assigned to each criteria PV may be quantitative (e.g. the groundwater may rise by 0.5 m) or qualitative (e.g. the groundwater rise may be 'medium').

An existing MCDA technique, such as a value focused approach (e.g. Weighted Sum Method (WSM) (Janssen, 1996)) or an outranking method (e.g. PROMETHEE (Brans *et al.*, 1986)), is then utilised to determine the total value of each alternative for the assigned input parameters. The objective of the process is to rank the alternatives in the order of preference (e.g. Rank 1, most preferred alternative to Rank *n*, least preferred alternative), which is based on the total value of each of the alternatives. In group decision making it is assumed that the actors agree on the criteria and the direction of the preferences. However, the differences between the individual CWs may in some cases be large. In traditional group decision making, not all of the CWs have been taken into account, but instead, some average or median CW has been used with sensitivity analysis (Miettinen and Salminen, 1999), as discussed in Section 2.5.7. In this methodology, it is proposed that a total value is obtained by considering each actor's set of CWs when more than one actor is involved in the decision making process.

Although the ranking of alternatives is obtained, no information is provided to the DM with regard to how likely it is that a reversal of the rankings of the alternatives will occur with a change in input parameters (i.e. CWs and PVs). Therefore, following the completion of the deterministic analysis, the following uncertainty analysis approaches can be undertaken.

4.3 Distance-based uncertainty analysis approach

4.3.1 Concept

The alternatives that are immediate contenders for being ranked first are the ones that are of real interest to the decision analyst. These candidates for the highest ranking position can be detected using distance-based tools (Proll *et al.*, 2001). Vincke (1999) defines the concept of robustness to express the fact that a solution, obtained for one scenario of data and one set of values for the parameters of the method, is 'far or not' from another solution, obtained for another scenario of data and another set of values for the parameters of the method. Hence, the concept of robustness will inevitably be based on a notion of distance or dissimilarity between solutions.

As discussed in Section 3.2, various distance-based sensitivity analysis methods have been proposed in the literature. The method proposed in this thesis, however, extends the methods presented by researchers including Barron and Schmidt (1988), Rios Insua and French (1991), Wolters and Mareschal (1995) and Ringuest (1997). The aim of the proposed distance-based uncertainty analysis method is to find the nearest competitors of the current highest ranked alternative and is achieved by identifying the 'smallest' changes necessary in the input parameters before a change in the ranking of the alternatives occurs. The main advancement of the proposed method in comparison to the existing deterministic sensitivity analysis methods is the simultaneous variation of all of the input parameters.

In some decision situations, one alternative will always be superior to another, regardless of the values the input parameters take. In this case, the ranking of the alternatives is robust, as it is insensitive to the input parameters. However, in many instances, this is not the case, and a number of different combinations of the input parameters will result in rank equivalence. By determining the smallest overall change that needs to be made to the input parameters (i.e. CWs and PVs) in order to achieve rank equivalence, the robustness of the ranking of two alternatives (a_x and a_y) is obtained.

This concept is illustrated in Figure 4.2 for a simple two-dimensional example. In Figure 4.2 the criteria PVs of the lower ranked alternative ($PV_{1,y}$, $PV_{2,y}$), which result in a total value of $V(a_y)$, are given by point Y and the criteria PVs of the higher ranked alternative ($PV_{1,x}$, $PV_{2,x}$), which result in a total value of $V(a_x)$, are given by point X. In this example, all combinations of PV_1 and PV_2 on the curved line labelled $V(a_y)opt = V(a_x)opt$ will modify the total values of alternative y and alternative x so that rank equivalence occurs between the two alternatives. Consequently, the robustness of the ranking of alternatives x and y is given by the shortest distance between point Y and the $V(a_y)opt = V(a_x)opt$ line and point X and the $V(a_y)opt = V(a_x)opt$ line, which are labelled d_1 and d_2 respectively, which is combined into a single distance measure, d. If this distance is large, then more substantial changes need to be made to the input parameters in order to achieve rank equivalence, and the ranking of the two alternatives is relatively insensitive to input parameter values (i.e. robust). Conversely, if this distance is small, minor changes in the input parameters will result in rank equivalence, and the ranking of the alternatives is sensitive to input parameter values (i.e. not robust). As the proposed approach identifies the combination of input parameters that is the shortest distance from the original parameter set, the input parameters to which the rankings are most sensitive are also identified.

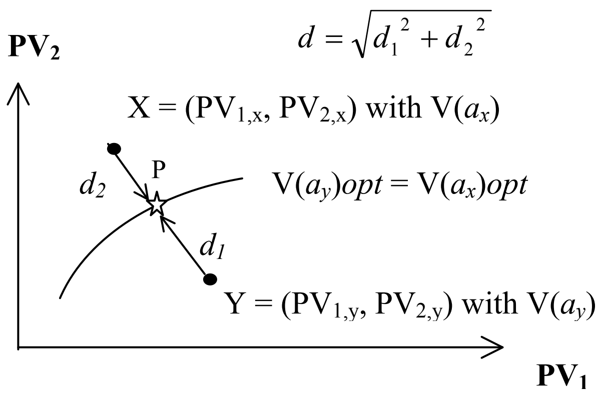


Figure 4.2 2D Concept of proposed distance-based uncertainty analysis approach

4.3.2 Formulation

As stated above, the purpose of the proposed distance-based uncertainty analysis approach is to determine the minimum modification of the MCDA input parameters (i.e. CWs and PVs) that is required to alter the total values of two selected alternatives (e.g. a_x and a_y) such that rank equivalence occurs. The minimum modification of the original input parameters is obtained by translating the problem into an optimisation problem and exploring the feasible input parameter ranges. The objective function minimises a distance metric, which provides a numerical value of the amount of dissimilarity between the original input parameters of the two alternatives under consideration and their optimised values. Optimised refers to the set of input parameters that is the smallest distance from the original parameter set, such that when the optimised set is used, the total values of the two alternatives being assessed are equal. The Euclidean Distance, d_e, is one of the most commonly used distance metrics (Barron and Schmidt, 1988; Isaacs, 1963; Ringuest, 1997; Rios Insua and French, 1991) and is therefore included in the methodology. However, other distance metrics such as the Manhattan Distance, d_m, and the Kullback-Leibler Distance (Barron and Schmidt, 1988; Jessop, 2004; Soofi, 1990), d_k can also be used.

The objective function for each of these distance metrics is defined as:

Min $d_e = \sqrt{\sum_{m=1}^{M} \left(w^{\#}{}_{jmi} - w^{\#}{}_{jmo}\right)^2 + \left(x^{\#}{}_{mnli} - x^{\#}{}_{mnlo}\right)^2 + \left(x^{\#}{}_{mnhi} - x^{\#}{}_{mnho}\right)^2}$ **Equation 4.1**

or

Min $d_m = \sum_{m=1}^{M} \left|\left(w^{\#}{}_{jmi} - w^{\#}{}_{jmo}\right)\right| + \left|\left(x^{\#}{}_{mnli} - x^{\#}{}_{mnlo}\right)\right| + \left|\left(x^{\#}{}_{mnhi} - x^{\#}{}_{mnho}\right)\right|$ **Equation 4.2**

or

Min $d_k = \sum_{m=1}^{M} w^{\#}{}_{jmo} \ln \frac{w^{\#}{}_{jmo}}{w^{\#}{}_{jmi}} + x^{\#}{}_{mnlo} \ln \frac{x^{\#}{}_{mnlo}}{x^{\#}{}_{mnli}} + x^{\#}{}_{mnho} \ln \frac{x^{\#}{}_{mnho}}{x^{\#}{}_{mnhi}}$ **Equation 4.3**

(Note: # refers to the standardised values of these parameters (see Equation 4.9).)

Equations 4.1 – 4.3 are subject to the following constraints:

$\sum_{m=1}^{M} w_{j_{mi}} = \sum_{m=1}^{M} w_{j_{mo}}$ **Equation 4.4**

$V(a_y)opt = V(a_x)opt$ **Equation 4.5**

$LL_{xl} \le x_{mnlo} \le UL_{xl}$ for *m* = 1 to *M* **Equation 4.6**

$LL_{xh} \le x_{mnho} \le UL_{xh}$ for *m* = 1 to *M* **Equation 4.7**

$LL_w \le w_{j_{mo}} \le UL_w$ for *m* = 1 to *M*, for actor *j* and $LL_W > 0$ **Equation 4.8**

where:

w_{jmi} = the initial CW of criterion *m* of actor *j*

w_{jmo} = the optimised CW of criterion *m* of actor *j*

x_{mnli} = the initial PV of criterion *m* of initially lower ranked alternative *n*

x_{mnlo} = the optimised PV of criterion *m* of initially lower ranked alternative *n*

x_{mnhi} = the initial PV of criterion *m* of initially higher ranked alternative *n*

x_{mnho} = the optimised PV of criterion *m* of initially higher ranked alternative *n*

d_e = the Euclidean Distance

d_m = the Manhattan Distance

d_k = the Kullback-Leibler Distance

M = the total number of criteria

$V(a_y)opt$ = the modified total value of the initially lower ranked alternative obtained using the optimised parameters

$V(a_x)opt$ = the modified total value of the initially higher ranked alternative obtained using the optimised parameters

LL_{xl} and UL_{xl} = the lower and upper limits, respectively, of the PVs of each criterion for the initially lower ranked alternative

LL_{xh} and UL_{xh} = the lower and upper limits, respectively, of the PVs of each criterion for the initially higher ranked alternative

LL_w and UL_w = the lower and upper limits, respectively, of each of the CWs for the selected actor's CWs

It should be noted that there is only one term for the CWs in Equations 4.1 to 4.3 because the CWs are common to all alternatives.

To ensure that the scale of the input parameters does not influence the optimisation, the values used in the distance metric (i.e. Equations 4.1 – 4.3) are standardised using the following formula:

$$x^{\#}_{mnli} = \frac{x_{mnli}}{\sigma_{Xm}}$$ **Equation 4.9**

where:

$x_{mnli}^{\#}$ = the standardised initial PV of criterion m of initially lower ranked alternative n

x_{mnli} = the initial PV of criterion m of initially lower ranked alternative n

σ_{Xm} = the standard deviation of the set of PVs of criterion m

This formula is also applied to the other parameters in Equations 4.1 – 4.3, respectively. It should be noted in the use of Equation 4.9 that if there is only one set of actor CWs then this formula is not able to be utilised as a standard deviation cannot be calculated. In this situation, non-standardised CWs are utilised.

The objective function (Equations 4.1 to 4.3) is subject to a number of constraints, including the total sum of the 'optimised' CWs has to equal the total sum of the original CWs (Equation 4.4). The modified total value of the initially lower ranked alternative (a_y) must also be equal to the modified total value of the initially higher ranked alternative (a_x) (Equation 4.5). The total values of the alternatives are determined using the selected MCDA technique (e.g. WSM or PROMETHEE) with the optimised values of the input parameters.

The expected ranges that the input parameters can be varied over to obtain a reversal in ranking of the selected alternatives (i.e. $a_y > a_x$) are constraints of the objective function (Equations 4.6 – 4.8). Specification of the minimum and maximum values of the input parameters of the pair of alternatives represents the potential uncertainty and variability in the assignment of these values in the initial stage of the decision analysis process. The range of values (i.e. upper and lower bounds) that are specified for each PV of the selected alternatives represent the set of possible values for that variable, which can either be based upon knowledge of the experts or the data that are available. The feasible range of CWs is defined to represent the expected variability in the preference values due to the subjective and ambiguous nature of the values elicited. The CW ranges can be defined by either the DM or actors or, alternatively, actual ranges of the available data can be utilised (i.e. the minimum and maximum values of the CWs elicited from the actors involved in the decision process). The actors can therefore define limits of the variation of the input parameters that they are able to accept as reasonable, according to their preference model (Bana e Costa, 1986). In the situation where the experts or actors are confident in the original input parameter values, the lower and upper bounds of the particular parameter would be equal to the original input parameter value. For example, this may be particularly relevant for the situation where qualitative data ranges (e.g. High to Low, where 1 equals High and 5 equals Low) are used for a particular criterion PV.

While the exact magnitude of the CWs may be called into question, the relative importance ranking of the criteria may be less contentious, as obtaining the rank order information is often easier and subject to less error than assessing numerical weights (Butler *et al.*, 1997). Therefore, the CW rank orders are able to be preserved while generating CWs as part of the methodology, which places substantial restrictions on the domain of the possible CWs.

If the PROMETHEE method is the selected MCDA technique for analysis of the particular decision problem, the original method requires that the generalised criterion functions be selected for each criterion by the DMs, and the associated threshold values defined (see Section 2.5.5). Defining the generalised criterion functions is not required in the proposed methodology due to the way in which uncertainty is taken into consideration in the definition of the input values using the ranges of values described above (Equations 4.6 to 4.8). The proposed method of incorporating the uncertainty in the PVs is less subjective than assigning generalised criterion functions, as determining the upper and lower values of the range of uncertainty (c.f. thresholds for generalised criterion functions) is more intuitive for the actors or experts as actual data are often available and the values that have to be chosen have a physical meaning. A Type I generalised criterion function (i.e. Level or Usual Criterion) must be assigned to each of the criteria to enable the preference functions between alternatives to be established, which is an essential component of the outranking MCDA methodology. However, if the DM did want to select generalised criterion functions, the proposed methodology can be extended to include variation of the threshold values as part of the optimisation process. This is because another element of uncertainty, which has been described in Section 2.5.5, is the specification of method specific parameters, such as the preference and indifference thresholds.

4.3.3 Optimisation

In order to obtain the robustness of the ranking of each pair of alternatives (i.e. a_x and a_y) for each actor's set of CWs, the optimisation problem given by Equations 4.1 to 4.9 needs to be solved. This can be achieved using a number of optimisation techniques. In this thesis, the performance of two types of optimisation algorithms is compared, namely a gradient method and an evolutionary-based optimisation algorithm.

Generalised reduced gradient (GRG2) nonlinear optimisation method

The Generalised Reduced Gradient (GRG2) nonlinear optimisation method can be used to solve the objective function (Equations 4.1 to 4.3) by changing the CWs and PVs within their specified ranges, subject to the defined constraints (Equations 4.4 – 4.8). GRG2 works by first evaluating the function and their derivatives at a starting value of the decision vector and then iteratively searches for a better solution using a search direction suggested by the derivatives (Stokes and Plummer, 2004). The search continues until one of several termination criteria are met. Among these are:

(i) The optimality criteria have been met to within a specified tolerance;

(ii) The difference between the objective values at successive points is less than some tolerance for a specified number of consecutive iterations;

(iii) A default or user-specified iteration limit or time has been exceeded; or

(iv) A feasible point cannot be found or a feasible non-optimal point has been obtained, but a direction of improvement cannot be found.

If no solution can be identified, the DM can be confident that the ranking of the two alternatives is robust (i.e. that no changes in the CWs or PVs between the specified ranges will result in a reversal of the ranking).

Random numbers are generated between the specified input parameter ranges for the CWs and PVs to be used as the starting values of the input parameters for the optimisation. GRG2 is not a global optimisation algorithm, therefore, to increase the chances of finding global or near-global optima, the optimisation is repeated a number of times using different randomly generated starting values. This aims to minimise the impact that the starting values have on the outcome of the analysis. A non-feasible (NF) outcome occurs when any of the constraints are violated and a not applicable (NA) result occurs if the alternative to be optimised is initially ranked higher than its paired alternative.

Genetic algorithm (GA)

The difficulties associated with using mathematical optimisation techniques have contributed to the development of alternative approaches, such as evolutionary-based algorithms. The increasing interest in a biologically motivated adaptive system for solving optimisation problems (Chang and Chen, 1998) has resulted in evolutionary-based algorithms which include: genetic algorithms, memetic algorithms, particle swarm, ant-colony systems and shuffled frog leaping (Elbeltagi *et al.*, 2005).

The Genetic Algorithm (GA), which is an heuristic iterative search technique that attempts to find the best solution in a given decision space based on a search algorithm that mimics Darwinian evolution and survival of the fittest in a natural environment (Goldberg, 1989), was the first evolutionary-based technique introduced in the literature. GAs have been described as being one of the most promising techniques in that domain and have therefore received a great deal of attention regarding their potential for optimising complex systems (Chang and Chen, 1998). GAs have been used successfully to solve complex combinatorial optimisation problems, such as optimising simulation models, fitting nonlinear curves to data, solving systems of nonlinear equations and machine learning (Chang and Chen, 1998). In the water resources field, GAs have been applied to a variety of problems including the design and maintenance of water distribution systems (Dandy and Engelhardt, 2001; Savic and Walters, 1997), optimal designs of groundwater remediation systems (Chan Hilton and Culver, 2000), generating the trade-off curve between minimum total treatment cost and reliability (Vasquez *et al.*, 2000) and streamflow and sediment yield estimates (Muleta and Nicklow, 2005). Stewart *et al.* (2004) developed a GA to solve a nonlinear combinatorial optimisation problem (i.e. Goal Programming (GP)) and applied it to a land use planning problem in the Netherlands. Mirrazavi *et al.* (2001) examined the use of GAs as a tool for the solution and analysis of multi-objective programming models (i.e. GP models). Leyva-López and Fernández-González (1999) presented a GA for improving the quality of a decision when a fuzzy outranking relation is exploited. Based on their demonstrated ability to reach near optimum solutions to large problems, GAs have been selected to solve the objective function defined by Equations 4.1 to 4.9.

One of the reasons for the extensive use of GAs is their ability to exploit the information accumulated about an initially unknown search space in order to bias subsequent searches into useful subspaces (Herrera *et al.*, 1998). An advantage GAs have over traditional optimisation techniques (such as GRG2) is that they do not require the use of gradient information, only the value of the fitness function itself. Another advantage of GAs is that they search from a population of points, investigating several areas of the search space simultaneously, and therefore have a greater chance of finding the global optimum.

However, a potential disadvantage is that constraints are unable to be incorporated specifically in the formulation of the GA, therefore, they are included in the objective function and multiplied by penalty values to discourage the selection of infeasible solutions by decreasing their fitness. The previously defined objective functions (Equations 4.1 to 4.3) are therefore reformulated and two penalty methods are incorporated in the approach, defined as the additive penalty method (Equation 4.10) and the exponential penalty method (Equation 4.11), respectively:

$$\text{Min } P_1 \times \left| \sum_{m=1}^{M} w_{jmi} - \sum_{m=1}^{M} w_{jmo} \right| + (P_2 \times d) + P_3 \times \left| V(a_x)opt - V(a_y)opt \right| \qquad \textbf{Equation 4.10}$$

or

$$\text{Min } e\left(- \frac{P_1}{\left| \sum_{m=1}^{M} w_{jmi} - \sum_{m=1}^{M} w_{jmo} \right|} \right) + (P_2 \times d) + e\left(- \frac{P_3}{\left| V(a_x)opt - V(a_y)opt \right|} \right) \qquad \textbf{Equation 4.11}$$

Equations 4.10 and 4.11 are subject to the constraints given by Equations 4.4 – 4.8. P_1, P_2 and P_3 are penalty values that are user defined (they should be the same order of magnitude as the denominator for the exponential penalty method) and *d* is the distance metric utilised (i.e. d_e, d_m or d_k).

<u>Discussion</u>

Both of the optimisation methods presented here to solve the objective function have their advantages and disadvantages. The main advantage of using GRG2 is its speed of arriving at a solution. However, its disadvantage is that because it is a gradient method, the chances of a local solution being obtained are high. The advantage of the GA is that it is a global search technique, however, it can take a longer time to converge compared with the GRG2. The processing time of the optimisation methods is dependent on the complexity of the decision problem being assessed (i.e. how many alternatives and criteria are involved in the decision problem and the 'robustness' of the ranking of the alternatives). The size of the population and the number of generations selected have been found to have the most significant impact on the time required for the GA to complete its analysis. A trade-off is therefore required between the amount of time to undertake the analysis and the level of certainty that the minimum distance has been obtained.

4.3.4 Interpretation of results

The output of the proposed uncertainty analysis method is the minimum distance metric for each pair of alternatives, which can be summarised in a matrix. A non-feasible or a very large value of the distance metric between two alternatives informs the DM that one alternative will predominantly be superior to another, regardless of the input parameter values selected between the specified ranges. Conversely, if the distance is small, slight changes in the input parameters will result in rank equivalence and the ranking of the alternatives can therefore be concluded as being sensitive to the input parameter values. The decision making process can be improved considerably by identifying critical input parameters and then re-evaluating more accurately their values. The most critical criteria can be identified by examining the relative and absolute change in input parameter values:

Absolute $\Delta\, x_{mln} = x_{mnlo} - x_{mnli}$ or Absolute $\Delta\, w_{jm} = w_{jmo} - w_{jmi}$ **Equation 4.12**

$$\left.\begin{array}{l}\text{Relative } \Delta\, x_{mln} = \dfrac{x_{mnlo} - x_{mnli}}{x_{mnli}} \times 100 \ \% \\ \quad \text{or} \\ \text{Relative } \Delta\, w_{jm} = \dfrac{w_{jmo} - w_{jmi}}{w_{jmi}} \times 100 \ \% \end{array}\right\} \quad \textbf{Equation 4.13}$$

It should be noted that Equations 4.12 and 4.13 can also be used to determine the most critical PVs of the initially higher ranked alternative. The input parameters that exhibit the smallest relative change in value to achieve rank equivalence between two alternatives are most critical to the reversal in ranking. The results provide the DM with further information to aid in making a final decision, including information on the most critical input parameters while simultaneously varying each of the input parameters.

4.3.5 Practical considerations

A number of practical considerations are highlighted in this section that need to be contemplated when utilising the proposed distance-based uncertainty analysis approach:

- Selection of the distance metric

  Several distance metrics have been proposed in the formulation of the distance-based uncertainty analysis approach (Equations 4.1 – 4.3) and it should be noted that the methodology is not limited to these distance metrics. Users of the methodology should also be aware that different results may be obtained by using different distance metrics. One way of mitigating against this in practice is by using several distance metrics if people are concerned about the impact that the distance metric has on the analysis (Rios Insua and French, 1991).

- Incorporation of actors' CWs

  The methodology is only able to utilise one set of actor CWs at a time. When a large number of actors are involved in the decision analysis, it may be impractical to undertake the methodology for all of the actors' CWs. The way in which the analysis will proceed is dependent on the particular decision situation. However, it is envisaged that the methodology will be used to undertake the uncertainty analysis for specific actors who are uncertain of their CWs. Alternatively, the DM may directly reject some sets of CWs as too extreme, or the DM may consider sets of CWs which result in different alternatives as being optimal.

- Optimisation method

  Two different optimisation methods have been proposed as examples of how the objective function may be solved. It is impractical in the thesis to have demonstrated all possible algorithms for the many different classes of problem which may arise. Therefore, a general-purpose, robust method has been proposed and various alternative optimisation methods may be utilised if appropriate.

- Interpretation of the results

  The determination of whether or not two sets of input parameters are similar and whether the distance metric obtained for pairs of alternatives is close or not, is subjective and dependent on the decision problem. However, values of distance give a good indication of relative robustness of alternatives for each decision problem.

4.4 Stochastic uncertainty analysis approach

4.4.1 Concept

It is sometimes not sufficient to just ask "how far?", as in the proposed distance-based approach. Another concern is "how likely and to what effect?" A shortcoming of the distance-based approach, described in Section 4.3, is that it does not consider the likelihood that the input parameter values are changed by a certain amount. If sufficient information is available to define probability distributions for likely values of each input parameter, the stochastic uncertainty analysis approach can be used to obtain additional information on the likelihood of alternatives achieving a particular ranking. However, specification of appropriate probability distributions is difficult in most cases and particularly when PVs are qualitative or only on an ordinal scale (Royal Assessment Commission, 1992).

A number of stochastic uncertainty analysis procedures have been presented in the literature by researchers including Janssen (1996), Butler *et al.* (1997) and Jessop (2002), as described in Section 3.3. The proposed stochastic approach for analysing decisions is summarised in Figure 4.3, which extends the existing approaches. As with the distance-based methodology, the likely range of values for each input parameter is able to be included in the analysis as part of the proposed approach, as opposed to the seemingly most prevalent method, where only a single, often subjective and imprecise, value is assigned to each CW and PV. The proposed methodology selects the input parameters at random using a computer simulation program so that the entire domain of possible CW and PV combinations can be explored in an efficient manner. The determination of how dependent a solution is on the various, generally uncertain, input values also becomes an integral part of the MCDA process in the suggested methodology. The proposed approach, which involves defining the uncertainty in the input values, performing a reliability analysis and undertaking a significance analysis, is described in detail below. As shown in Figure 4.1, this methodology can follow on from the deterministic analysis, described in Section 4.2.

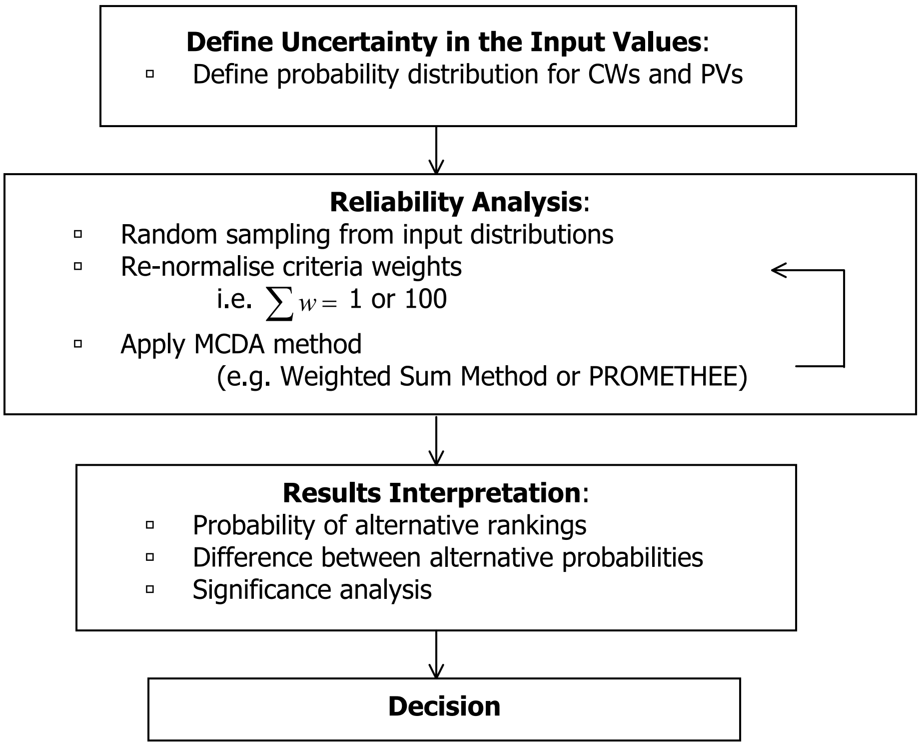


Figure 4.3 Steps in the proposed stochastic uncertainty analysis approach

4.4.2 Formulation

The first stage of the stochastic uncertainty analysis approach is to define the uncertainty in the input parameter values.

<u>Define uncertainty in the criteria weights (CWs)</u>

In the situation where a relatively large number of actors have been included in the decision process, the CWs of the actors can be considered as a representative sample of the CWs of a population of stakeholders. These preference values can therefore be portrayed by a probability distribution which is fitted to the actual CWs, ensuring that all of the information obtained from the actors is explicitly incorporated in the decision making process. The distributions also represent the actors' uncertainty about their own preferences. Goodness of fit statistics are used to determine how representative the fitted distributions are of the actual sets of CWs (using the Kolmogorov-Smirnov test for parameters with fewer than 30 measurements and the chi-square test for the parameters with more than 30 measurements (Sonnemann *et al.*, 2003)).

It is possible that when the actors are defining the weights for each criterion a particular pattern may emerge. For example, large weights for one criterion may always be associated with low weights for another criterion. A correlation analysis is therefore undertaken as part of the methodology to determine if there is a relationship between the available sets of CWs. The results of the correlation analysis are incorporated in the distributions to ensure that sampling from the CW probability distributions represents the actual assignment of CWs by actors. It is also important to note that the distributions of the CWs should be truncated such that they have a lower bound of zero and an upper bound of the total sum of the CWs, which is generally one or 100 (Rietveld and Ouwersloot, 1992).

In the circumstance where a small number of actors is involved in the decision making process, and there are consequently insufficient CWs available to fit a representative distribution (e.g. fewer than 10 actors), either a normal or uniform distribution of CWs can be utilised to enable uncertainty and subjectivity in the CWs to be incorporated in the analysis. The probability distribution that is easy to support in the absence of further information is the uniform distribution, where all the CWs are equally probable (Rietveld and Ouwersloot, 1992). It is often said that a uniform distribution of CWs represents a position which may be seen as neutral and, as such, provides a suitable start for an assessment (Jessop, 2004). To characterise the uniform distributions, upper and lower bounds of the CWs need to be defined using either actor specified limits or bounds based upon the actual CWs available. The analysis therefore becomes somewhat more analogous to traditional sensitivity analysis, where the behaviour of the ranking of alternatives is explored within the expected range of CWs.

It is not possible with the current proposed methodology to maintain the rank order of the CWs while the weights are otherwise generated at random. However, the CWs generated can be tested to determine how many of the sets of CWs do preserve their criteria rank order. The vectors of CWs that maintain the original rank order are then utilised in the analysis. This places substantial restrictions on the domain of possible CWs that are consistent with the actors' judgment of criteria importance, however, the results from the rank order simulation may provide more meaningful results.

Define uncertainty in the criteria performance values (PVs)

The uncertainty, imprecision and variability in the quantitative PVs can be represented by continuous probability distributions such as uniform or normal. Distributive evaluations incorporate the variability and the imprecision of assessments given by the experts. Distributions could represent the uncertainty of the evaluations of the alternatives on different criteria (due to the imprecision of the measuring tool and / or lack of knowledge of the consequences of the alternatives), as well as fluctuations among evaluations (D'Avignon and Vincke, 1988).

A range of values (i.e. upper and lower bounds for uniform distributions) must be assigned to each PV representing the set of possible values for that variable, which can either be based upon knowledge of the experts or the data that are available. The corresponding distribution characterises the likelihood that the appropriate value to use for this parameter falls in the various subsets of the defined range. In the case where PVs are assessed using qualitative measures and converted to integer scales (e.g. a scale of 1 to 5, where 1 represents 'low' and 5 represents 'high'), the incorporation of uncertainty and variability in the analysis becomes more challenging. A discrete uniform distribution can be utilised for qualitative PVs, which is characterised by defining upper and lower limits of each PV (e.g. a groundwater rise of 'medium' may be between 'medium low' and 'medium high', or in an integer scale between 2 and 4). The problem may therefore arise that such a 'wide' range of values is deemed inappropriate for the particular criterion. If this situation occurs and the expert is satisfied that the deterministic (or integer) value adequately represents the performance of the criterion, then this value can be utilised and no distribution defined.

The methodology categorises information about the PVs into two different levels:

(i) A relatively high level of data is modelled by a normal distribution, leaving the determination of a mean value and a standard deviation; and

(ii) A low level of knowledge is modelled by a uniform distribution, which only needs a minimum and a maximum value, defining an overall range for the parameter value.

4.4.3 Reliability analysis

Following characterisation of the uncertainty, subjectivity and variability in the CWs and PVs, reliability analysis is undertaken to determine the most probable ranking of each of the alternatives based upon the expected range of possible input values for each CW and PV. The approach involves utilising existing MCDA techniques to determine the total value of each alternative, however, the advancement is in the application of Monte Carlo simulation (MCS) (Kottegoda and Rosso, 1997; Yen, 2002) to enable repeated application of the selected MCDA method, with the range of possible input values defined in the problem formulation phase. MCS refers to the traditional technique for using random or pseudo-random numbers to sample values from a probability distribution. MCS techniques are entirely random, which means that a given sample may fall anywhere within the range of the input distribution. However, samples are more likely to be drawn from areas of the distribution which have higher probabilities of occurrence. Two particularly appealing features of MCS are the full coverage of the range of each input variable and the ease with which an analysis can be implemented. Another reason why MCS has been adopted in the methodology is that it is an alternative to using joint distributions over the parameters for multi-parametric problems, such as MCDA, as joint distributions can be quite cumbersome (Felli and Hazen, 1998).

Three key stages are involved in the proposed reliability analysis process:

(i) The input values (i.e. CWs and PVs) are randomly sampled from their respective probability distributions, assuming the CW and PV distributions are independent, while maintaining the correlation structure of the CWs;

(ii) The randomly drawn vector of CWs is normalised, as the sum of all elements of the weight vector must equal the original total sum of the CWs (e.g. 1 or 100) (Janssen, 1996); and

(iii) The selected MCDA technique is applied to determine the total value of each alternative for that realisation (i.e. with the randomly drawn vector of PVs and normalised CWs). The MCS continues until convergence occurs (e.g. until the sample values approximate the input parameter distributions), or until a specified number of realisations are completed.

The proposed approach is applicable to existing MCDA techniques, such as value focused approaches (e.g. WSM (Janssen, 1996)) and outranking methods (e.g. PROMETHEE (Brans *et al.*, 1986)).

Because the accuracy of the statistics obtained from MCS depends on the number of simulations performed, a large number of simulations are required for an uncertainty problem with a large number of parameters (Manache and Melching, 2004). To determine how many realisations are sufficient to analyse the uncertainty of model output, the mean and standard deviation of the output statistics are calculated and plotted after each realisation (Yu *et al.*, 2001).

4.4.4 Interpretation of results

Realisation of MCSs provides DMs with far more information than a single estimate. Repeated simulation of a decision model enables the DM to estimate critical long-term probabilities while allowing all problem parameters of interest to vary according to their distributions (Felli and Hazen, 1998). The output statistics, distributions and correlation among input and output variables allow the estimation of the uncertainty in the model output and the identification of the parameters and input variables to which the output is most sensitive (Manache and Melching, 2004).

The results of the reliability analysis, therefore, provides the DM with valuable information, including distributions of the total values for a single alternative or the difference between values for competing alternatives. Knowledge of the likelihood of the total value over the entire range of possible input values enables the DM to better assess the risk of an adverse outcome or select an alternative based upon the likelihood that its total value will exceed that of its competitor by a specified amount. The probability that an alternative *n* receives rank *r*, based on all probable criteria input parameters, is also available to the DM, in order to assess the robustness of a solution.

If the uncertainty in the performances of the alternatives is large, it becomes difficult to clearly decide which of the alternatives is superior and by how much. The basic rationale in the probabilistic comparison of two alternatives is that, the smaller the overlap between the two membership functions, the larger the preference for the superior alternative (Rietveld and Ouwersloot, 1992). The mean and standard deviation of each of the cumulative distributions are therefore also presented to the DM.

To analyse the similarity in the overall value scores of the alternatives, statistical analysis is conducted. The non-parametric Wilcoxon Rank Sum Test (or Mann-Whitney Test) is able to inform the DM whether one output distribution is 'better' than another (Kottegoda and Rosso, 1997). The method begins by assembling the total values from two alternatives, a_x and a_y, into a single set of size $N = n_{ax} + n_{ay}$. These total values are then rank-ordered from lowest (rank #1) to highest (rank #N) with tied ranks included where appropriate. If there are total values that are tied for ranks, each receives the average of the ranks. Once the values have been sorted in this manner, the rankings are returned to the sample a_x and a_y. If the two populations have the same distribution, then the sum of the ranks of the first sample and that in the second sample should be close to the same value. A z value for the null hypothesis that the two distributions are the same is determined using the following equations:

$$\mu_{ax} = \frac{n_{ax}(N+1)}{2}$$ **Equation 4.14**

$$\sigma = \sqrt{\frac{n_{ax} n_{ay}(N+1)}{12}}$$ **Equation 4.15**

$$z_{ax} = \frac{(T_{ax} - \mu_{ax}) \pm 0.5}{\sigma}$$ **Equation 4.16**

where:

T_{ax} = the sum of the n_{ax} ranks in group a_x

T_{ay} = the sum of the n_{ay} ranks in group a_y

T_{axay} = the sum of the N ranks in groups a_x and a_y combined

Note: correction for continuity: -0.5 when $T_{ax} > \mu_{ax}$ and +0.5 when $T_{ax} < \mu_{ax}$

In all instances, z_{ax} and z_{ay} will have the same absolute value and opposite signs. Critical values for the Wilcoxon Rank Sum test are contained in Table 4.1.

Table 4.1 Critical values of +/- *z* for the Wilcoxon Rank Sum test

Level of Significance for *a*				
Directional Test				
0.05	0.025	0.01	0.005	0.0005
Non-Directional Test				
--	0.05	0.02	0.01	0.001
$z_{critical}$				
1.645	1.960	2.326	2.576	3.291

The approach adopted here is similar to that used by Olson *et al.* (1995) and De Kort *et al.* (2006), where the Wilcoxon Rank Test was used to analyse the similarity in the overall value scores of the alternatives developed by each actor using each decision making technique included in the study.

The purpose of MCDA methods is to help the DM understand the problem and progressively build a solution. It is therefore interesting to know the influence of the input parameters on the obtained solution (Vincke, 1999). In a first iteration, rather basic range and distribution assumptions can be used to determine which input variables dominate the behaviour of the output. Often, most of the variation of the output will be caused by a relatively small subset of the input variables. Once the most important input variables are identified, resources can be concentrated on characterising their uncertainty further.

The sensitivity contribution of the model parameters to the model output can be quantified by various measures (Manache and Melching, 2004). These measures are based on regression and correlation analyses applied to the original parameter and output values or to their rank-transformed values. Linear regression measures are effective when the relation between model input and output is approximately linear ($R^2 \sim 1$). When nonlinearity between model input and output is present, nonlinear regression models can be used or some transformation on the data can be applied. One such approach is the rank transformation method, where the original values of the input parameters and the model output are replaced by their rankings (Manache and Melching, 2004).

Pearson's *r* and non-parametric correlations (Kendall's *tau* and Spearman's *R*) are frequently used to measure differences between pairs of sets of weights or evaluations (Hobbs *et al.*, 1992). To facilitate interpretation of the results of the stochastic uncertainty analysis approach introduced here, significance analysis can be used to identify the relative contribution that each input parameter (i.e. each CW and PV) has in determining the total value of an alternative. The most significant inputs to the decision analysis can be determined using the Spearman Rank Correlation Coefficient (Kottegoda and Rosso, 1997):

$$R = 1 - \left(\frac{6\sum_{i=1}^{d} D_i^{\,2}}{d\left(d^2 - 1\right)} \right)$$

Equation 4.17

where:

d = total number of data points (i.e. MCS realisations)

D_i = difference between ranks (i.e. Rank of total value of alternative n for data point i (i.e. $\sum_{m=1}^{M} w_m x_{m,n}$ – Rank of the input parameter, w_m or x_{mn}, for data point i))

M = total number of criteria

m = criterion number

w_m = CW of criterion m

x_{mn} = PV of criterion m and alternative n

The correlation coefficient is calculated between each input parameter (i.e. CW and PV) and the total value of each alternative using the data obtained from the reliability analysis. The input values and total alternative values are ranked within each data set, with the highest value obtaining a ranking of one, if it is a maximising criterion. An example is shown in Table 4.2 for PVs. The same analysis applies to CWs. The value of R always lies between –1 and +1, where a value of –1 or +1 indicates perfect association between the parameters, the plus sign occurring for identical rankings and the minus sign occurring for reverse rankings. When R is close to zero, it is concluded that the variable under consideration (i.e. a particular CW or PV) does not have a significant impact on the ranking of the alternative. Once the most important input parameters are identified from the significance analysis, resources can be concentrated on characterising their uncertainty if further analysis is required to arrive at a final decision.

Table 4.2 Spearman Rank Correlation Coefficient example calculation (d = 4)

PV, Criterion 2, Alt 1		Alt 1 Total Values		D_d
Value	**Rank** ($r_d\, x_{2,1}$)	**Value** ($w_2 x_{2,1}$)	**Rank** ($r_d\, w_2 x_{2,1}$)	($r_d\, x_{2,1}$ - $r_d\, w_2 x_{2,1}$)
10	1	110	2	-1
7	4	68	4	0
8	3	113	1	2
9	2	88	3	-1
$R = 1 - \left[\frac{6 \times (1+0+4+1)}{4 \times (16-1)}\right]$				

4.5 Discussion

Two uncertainty analysis approaches have been proposed which provide the benefits of:

- Allowing all expected uncertainty and subjectivity in the CWs and PVs to be incorporated in the analysis;
- Jointly varying the CWs and PVs;
- Allowing all actors' preferences to be included in the analysis by fitting distributions to the data;
- Including any correlations between the CWs in the analysis;
- Being applicable to multiple MCDA techniques;
- Providing information on the relative importance of the inputs;
- Not requiring the actors to specify generalised criterion functions and the associated parameters when utilising the PROMTHEE MCDA method; and
- Not requiring posterior sensitivity analysis to be undertaken, as the uncertainty analysis methods are incorporated within the decision making process.

The choice of uncertainty analysis method may depend on the amount of data available and the output required by the DM. Trade-offs are required between the computation time of the uncertainty analysis approaches and certainty that the near global optimum solution has been obtained. The processing time of the uncertainty analysis methods is dependent on the complexity of the decision problem that is being assessed (i.e. how many alternatives and criteria are involved in the decision problem and the 'robustness' of the ranking of the alternatives).

4.6 Implementation of proposed uncertainty analysis approach

4.6.1 Introduction

A program has been developed as part of this research to enable implementation of the proposed uncertainty analysis approaches described in Sections 4.3 and 4.4. The program presented and described in this section has been designed to provide practical support for public decisions in conflict situations where environmental and socio-economic effects are to be considered e.g. to aid the decision making process for MCDA problems. Weistroffer and Narula (1997) believe that it is desirable for a decision support system (DSS) to have the following characteristics:

(i) Capture and reflect the thinking process of the DM;

(ii) Support multiple decision processes and several decision styles;

(iii) Be easy and convenient to use and not require extensive training;

(iv) Help DMs to structure situations and the initial stages of resolutions;

(v) Allow a DM to adopt the system as they gain experience in the DSS's capabilities; and

(vi) Be user-friendly.

Simonovic and Bender (1996) state that important characteristics of a DSS for sustainable management of water resources includes accessibility, flexibility, facilitation, learning, interaction and ease of use. Alexouda (2005) has also found that the user interface of a DSS can influence its acceptance by the user. Therefore, the user's skills, needs and expectations have been considered in the design and implementation of the user interface of the program developed as part of this research.

These desired characteristics have been achieved through:

(i) Utilising Microsoft Excel as the development environment, as it is a software package that a large majority of people are familiar with;

(ii) Incorporating multiple MCDA techniques; and

(iii) Designing the forms so that they step through the entire MCDA process, as described in Section 2.5.

A detailed description of the program is provided below and flow charts of the structure of the program are included in Appendix E.

It should be noted that there have been numerous specially developed computer packages that support the application of MCDA methods and a list of some of the available software is included in Appendix C in an effort to alert potential DMs to the range of tools that are already available. This list extends and updates previous overviews of MCDA software undertaken by Buede (1992), Buede (1996), Weistroffer and Narula (1997) and Siskos and Spyridakos (1999). These software packages may fall into three groups:

(i) Commercially available software packages;

(ii) Software packages developed primarily for research purposes; and

(iii) Programs written for experimental implementation and testing of new MCDA techniques.

The program developed as part of this research falls into category number three.

Trial versions of a large number of existing MCDA computer packages are available for download from the internet, however, the purchase of the software of some of the most popular MCDA methods is prohibitive for people who may not be familiar with MCDA and if uncertainty exists about which method is most applicable for the particular decision problem(s) to be assessed. In addition, the majority of the software presented in Appendix C only includes one MCDA technique, therefore, if multiple techniques are required to be utilised, it becomes a very expensive process. Also, if people would like to use different methods, they have to familiarise themselves with different software environments. These factors may limit the uptake of the MCDA process by potential new users. Saltelli *et al.* (1999) also note that global sensitivity analysis is still largely absent or rudimentary in commercial packages for decision analysis. A sensitivity analysis method can be termed as global if it allows all the input factors to vary over their range of uncertainty (Saltelli *et al.*, 1999).

Selecting the most appropriate software package for a specific application is not a trivial task (Ozernoy, 1988). Often mentioned is the observation that, given a choice, DMs prefer relatively unsophisticated MCDA DSSs (Aloysius *et al.*, 2006). It would be useful to develop a more detailed classification of the listed packages and the degree of desirability of their features to aid in the selection of an appropriate piece of software, rather than the basic details presented in Appendix C. However, such classification would potentially be of temporary value, as new packages are being developed and existing packages are being constantly upgraded. Various attempts at reviewing software packages have been reported including the review of Decision Lab 2000 by Geldermann and Zhang (2001), the review of DEFINITE by Anderson (2002) and the comparison and evaluation of Expert Choice, Criterium, Logical Decision, VIMDA and VISA by Zapatero *et al.* (1997).

4.6.2 Program description

The program developed as part of this research is written in Visual Basic for Applications (VBA), which is the programming language incorporated in Microsoft Excel. The advantage of using Microsoft Excel as a development environment is that it provides capabilities that allow for analysis and manipulation of the data and visualisation of the results. In addition, as stated above, Microsoft Excel is familiar, not to mention readily available, to a large majority of people. Consequently, using the program does not necessitate becoming familiar with a new software environment. Help files are included throughout the program, which provide theoretical information on the analysis that is implemented, and information on how to use the program itself. The structure, methodology and use of the program are illustrated in Figure 4.4 and described in the sections below.

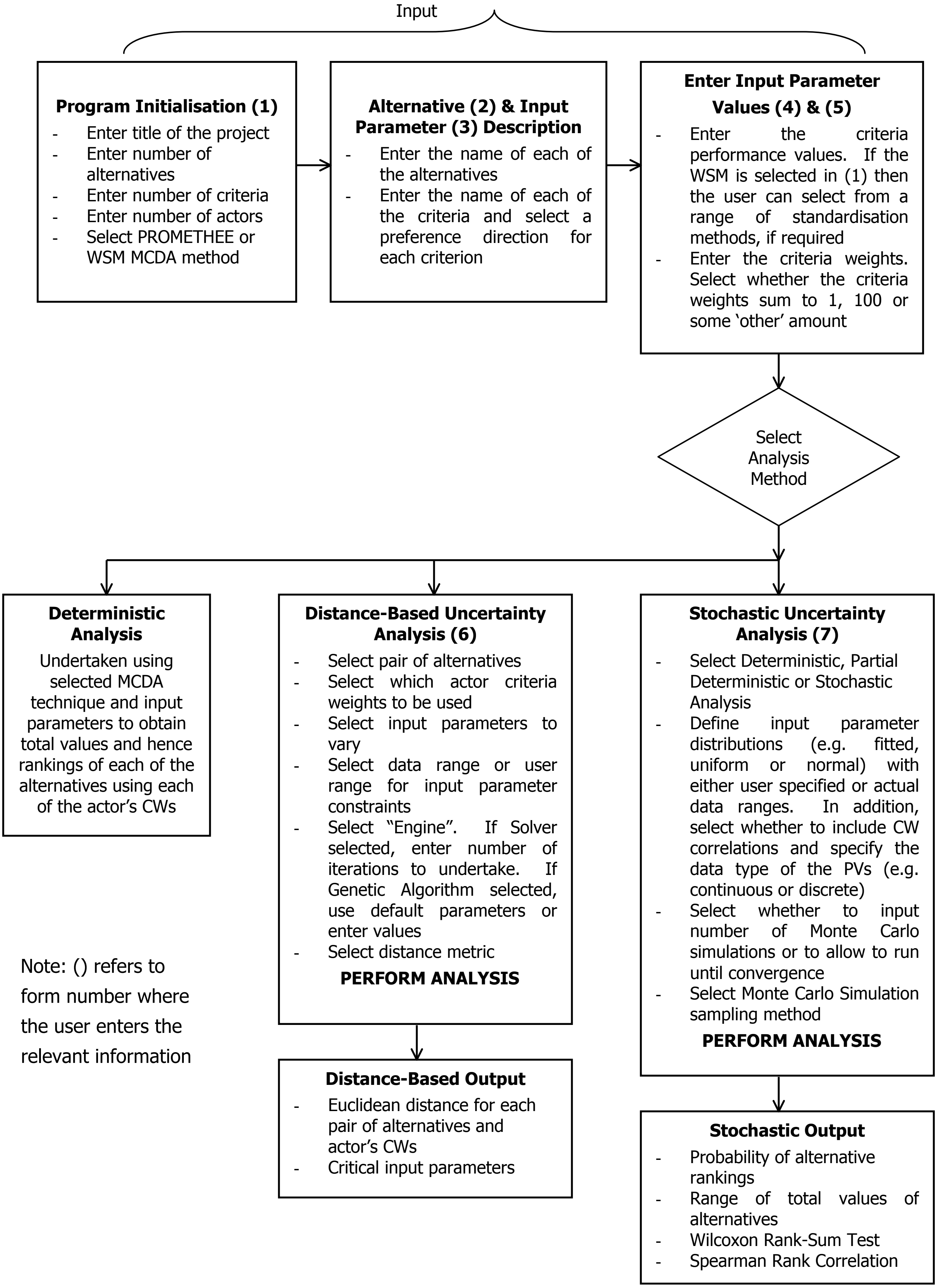


Figure 4.4 Program structure

Decision analysis formulation

Program initialisation

After starting the program, a form appears (Figure 4.5) which asks the user if they would like to start a new decision problem or open an existing one. Following the selection of one of these options and pressing the continue button (i.e. forward arrow), the Program Initialisation Form, which is shown in Figure 4.6, is displayed. This form enables the decision analysis problem to be defined by the user, including entering the number of alternatives, criteria and actors. The methods available to determine these inputs are described in Sections 2.5.1, 2.5.2 and 2.5.3, respectively. The program is able to assess decision problems with a maximum number of 30 alternatives and 24 criteria, which, based on the discussion in Section 2.5.2[5], should satisfy the requirements of most decision making situations.

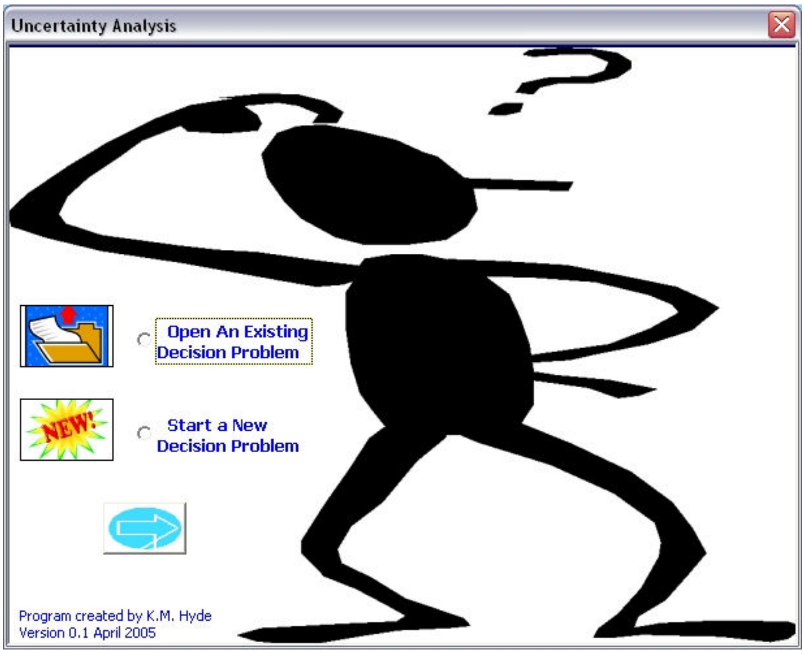


Figure 4.5 Example of MCDA uncertainty analysis initial choice form

[5] The final part of this section discusses the cognitive abilities of the DM and that "there appears to be a general rule of thumb that the number of criteria for a decision analysis should not exceed 10 or 12".

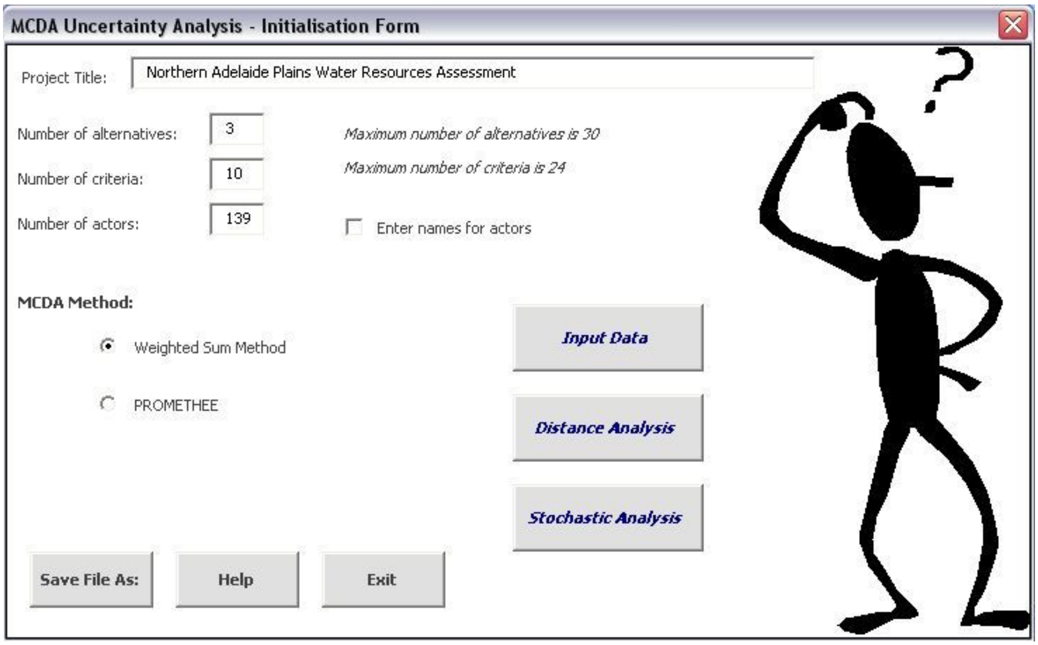


Figure 4.6 Example of MCDA uncertainty analysis initialisation form

The user is also required to select one of the two available MCDA techniques (i.e. Weighted Sum Method (WSM) or PROMETHEE) on the Program Initialisation Form (Figure 4.6), which are utilised to determine the total value of each alternative for the assigned input parameters. As discussed in Section 2.5.4, the WSM is associated with the Utility and Value Theory MCDA classification scheme, while PROMETHEE is an outranking methodology and thereby from an alternative 'school of thought'.

WSM was selected as one of the methods to include in the program as its simplicity means that it is commonly used by practitioners (Butler and Olson, 1999; Ringuest, 1997). The WSM involves calculating an appraisal score for each alternative (V(a_n)) by multiplying each criterion PV ($x_{m,n}$) by its appropriate CW (w_m), followed by summing the weighted scores for all criteria as follows (Janssen, 1996):

$$V(a_n) = \sum_{m=1}^{M} w_{jm} x_{mn}$$ **Equation 4.18**

where:

m = the criterion number

M = the total number of criteria

n = the alternative number

j = the actor

Alternatively, the basic PROMETHEE methods build a valued outranking relation. The preference function associated with each criterion gives the degree of preference, expressed by the DM, for alternative *a* with respect to alternative *b* on criterion x_j. Further details of the PROMETHEE method are contained in Appendix B and Brans *et al.* (1986). As described in Section 4.3.2, it should be noted that Level I generalised criterion functions are utilised in the proposed uncertainty analysis approaches for each of the criteria, therefore, the user does not need to select the generalised criterion functions or the associated thresholds. This is because uncertainties associated with the criteria PVs are considered elsewhere in the proposed uncertainty analysis approaches. However, it should also be reiterated that if the PROMETHEE method is utilised and the user would like to use one of a number of generalised criterion functions defined by Brans *et al.* (1986) in the deterministic analysis, this is also possible in the proposed program, as shown in Figure 4.7. This flexibility in the program enables a range of analyses to be undertaken, such as comparison of the results of the proposed uncertainty analysis approaches which only utilise the Level 1 generalised criterion functions, with, for example, existing case studies that have utilised a number of the commonly used generalised criterion functions.

For a new decision problem, the user must save the file as a unique workbook before continuing (by pressing the Save File As button), which enables it to be opened and utilised again, if required, following the completion of the analysis.

Alternative and input parameter description

A description of each alternative and criterion can be entered in respective forms following program initialisation. The preference direction (i.e. minimise or maximise) for each of the criteria must also be selected by the user on the Criteria Descriptions and Preference Direction Form, as shown in Figure 4.8. It should be noted that the program has been developed based on the assumption that consensus has been reached with regard to the preference direction for each of the criteria, as only one preference direction is able to be entered for each criterion.

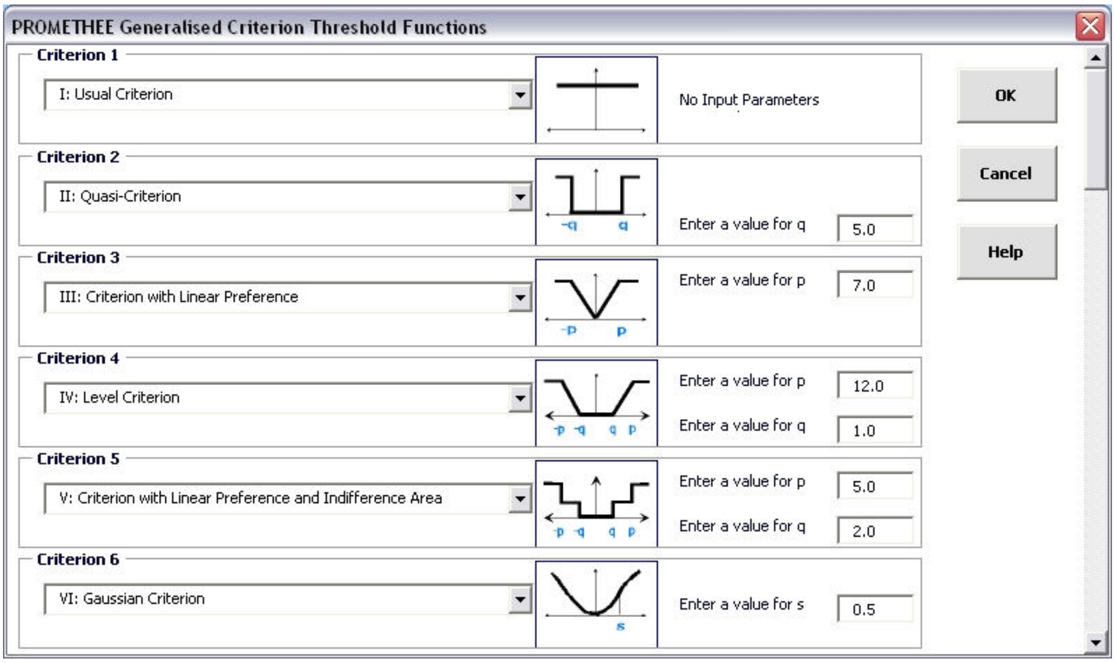


Figure 4.7 Example of the PROMETHEE generalised criterion functions form

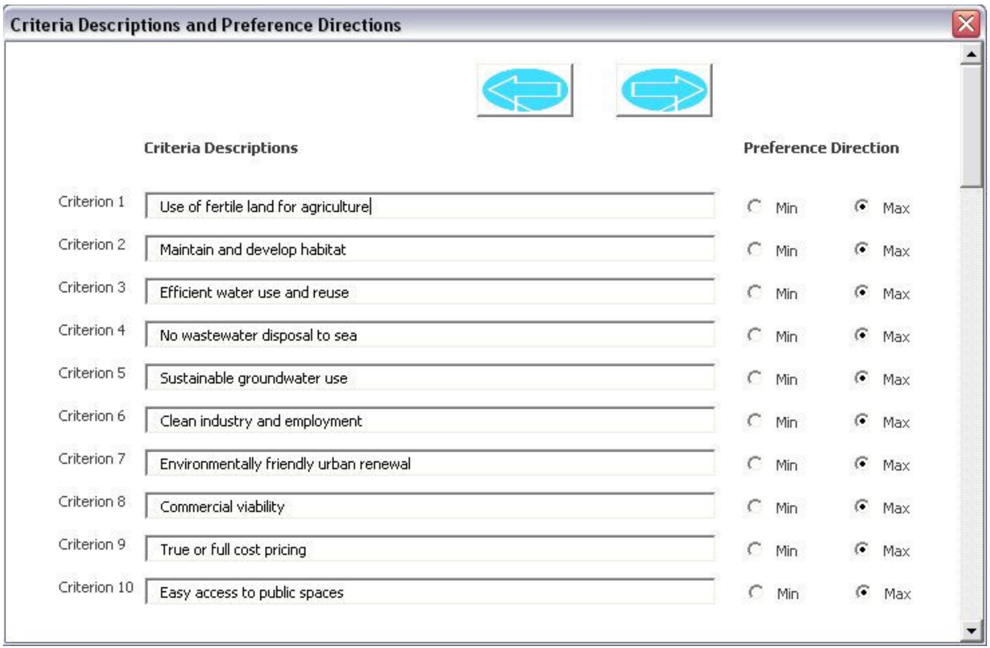
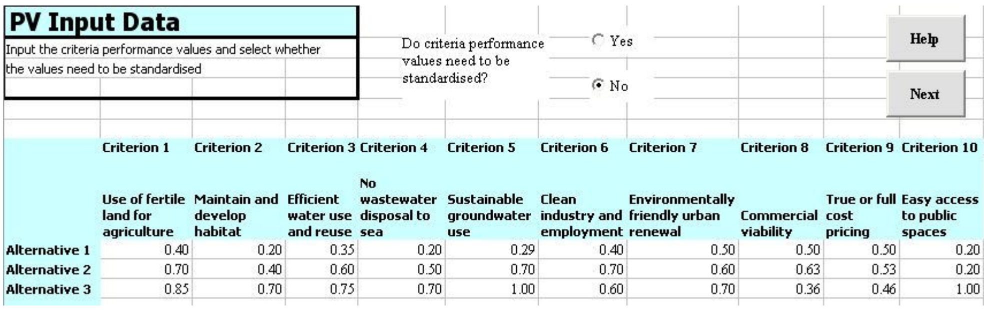


Figure 4.8 Example of the criteria descriptions and preference directions form

Input parameter values

The next step in the decision analysis process is to assess the alternatives by the criteria that have previously been defined and elicit the preferences from the actors. Once the relevant data have been obtained, by the methods described in Sections 2.5.5 and 2.5.7, the PVs and CWs can be entered, or copied from existing files, by the user into the spreadsheets available (see Figure 4.9 for an example of the PV Input Data worksheet). It should be noted that the PVs are required to be standardised to commensurable units when the WSM is used. To accomplish this, two standardisation methods are available for use (methods 2 and 3 contained in Table 2.4) if the WSM is the selected MCDA technique and if the data are entered in incommensurable units. The only additional information required on the CWs form is the total sum of the CWs.


PV Input Data

Input the criteria performance values and select whether the values need to be standardised

Do criteria performance values need to be standardised?

Yes

No

Help

Next

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6	Criterion 7	Criterion 8	Criterion 9	Criterion 10
	Use of fertile land for agriculture	Maintain and develop habitat	Efficient water use and reuse	No wastewater disposal to sea	Sustainable groundwater use	Clean industry and employment	Environmentally friendly urban renewal	Commercial viability	True or full cost pricing	Easy access to public spaces
Alternative 1	0.40	0.20	0.35	0.20	0.29	0.40	0.50	0.50	0.50	0.20
Alternative 2	0.70	0.40	0.60	0.50	0.70	0.70	0.60	0.63	0.53	0.20
Alternative 3	0.85	0.70	0.75	0.70	1.00	0.60	0.70	0.36	0.46	1.00

Figure 4.9 Example of the performance value input data worksheet

<u>**Decision analysis**</u>

The user is asked to save the input data that have been entered, as described above, before continuing with the decision analysis process. A selection is then able to be made by the user between undertaking deterministic analysis, distance-based uncertainty analysis or stochastic uncertainty analysis, as shown in Figure 4.10.

DETERMINISTIC ANALYSIS

If the Deterministic Analysis button is pressed by the user on the form shown in Figure 4.10, the traditional decision analysis methodology used to determine the total values of the alternatives, and hence the ranking of each alternative for each set of actor's CWs, using the selected MCDA technique, is undertaken, as described in Sections 2.5.9 and 4.2. A ranking of the alternatives is obtained for each of the actors' CWs in addition to the total value of each of the alternatives, which is displayed in tabular and graphical form. It should be noted that the PROMETHEE II method has been programmed so that the total flows are both presented as normalised and un-normalised. This is so that results can be compared with those obtained from other studies previously undertaken, or potentially future studies, as results of some studies such as Mareschal (1988) do not normalise the flows (i.e. by dividing by (number of alternatives – 1)). In contrast, the software DecisionLab 2000 presents the normalised flows.

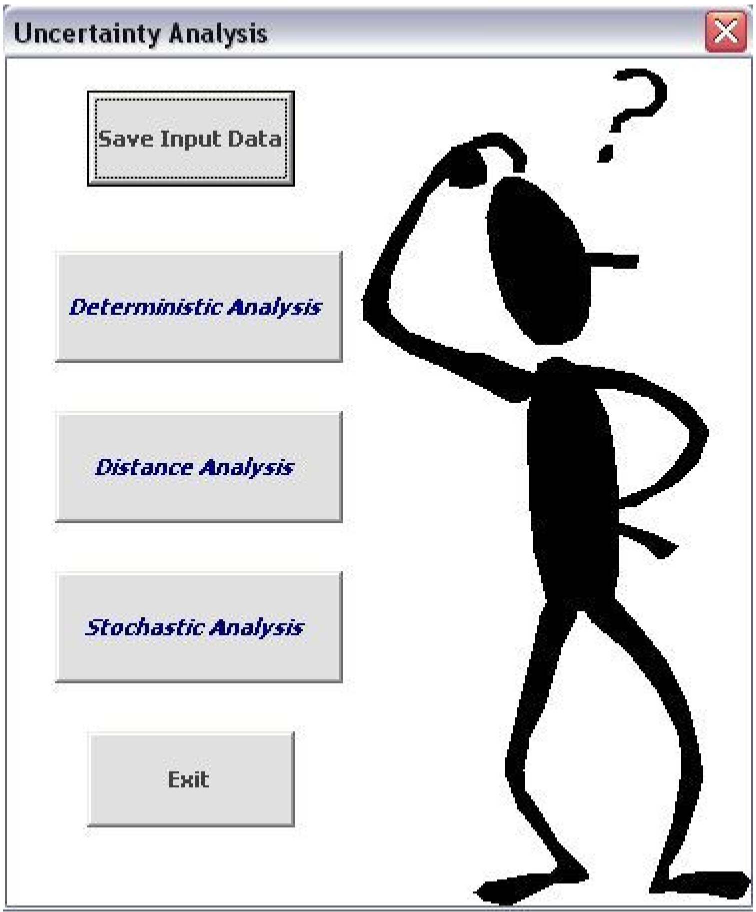


Figure 4.10 Example of the choice of uncertainty analysis method form

Following the deterministic analysis, the user may choose to execute either the distance-based uncertainty analysis methodology or the stochastic uncertainty analysis approach using the same form as shown in Figure 4.10.

DISTANCE-BASED UNCERTAINTY ANALYSIS APPROACH

The methodology of the distance-based uncertainty analysis approach utilised in the program is contained in Section 4.3.

If the user decides to click on the distance-based uncertainty analysis button shown on the form in Figure 4.10, the Distance-based Uncertainty Analysis user form is displayed, which is shown in Figure 4.11. As can be seen on this form, the user of the program has a number of parameters to input and selections to make before the distance-based uncertainty analysis can be run. The user must enter the pair of alternatives that the approach is applied to. In addition, the actors' CWs that are to be utilised for the analysis must also be specified. The main advantage of the proposed distance-based methodology is the ability to simultaneously vary the CWs and PVs within expected ranges of uncertainty. However, flexibility is incorporated into the program by allowing the user to select that only the CWs or only the PVs are varied, while the other parameters remain fixed (Figure 4.11).

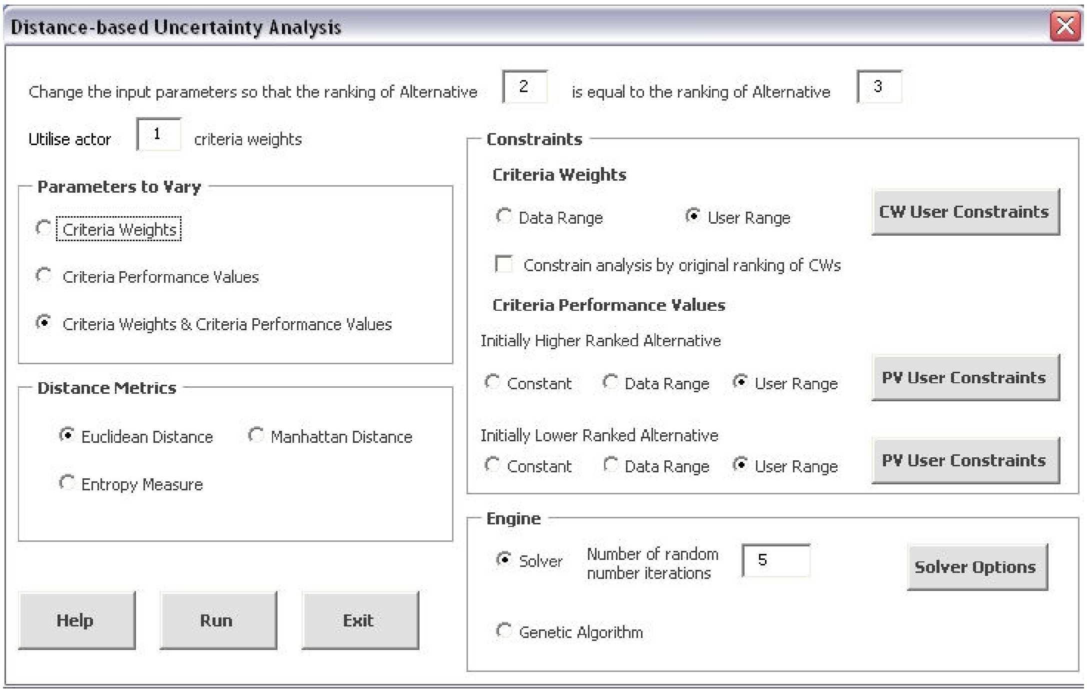


Figure 4.11 Example of the distance-based uncertainty analysis form

The user must also select one of three available distance metrics: Euclidean Distance, Manhattan Distance and Entropy Measure, which are described in Section 4.3.2. Rios Insua and French (1991) state that the insight brought to the DMs by identifying the nearest point at which the ranking changes in the Euclidean Distance metric is of different quality to that brought by the Manhattan Distance. It is observed by Rios Insua and French (1991) that distances tend to 'favour' some regions and the results of the analysis are metric dependent. Different results were obtained when using the Euclidean distance and the Chebyshev distance in the example undertaken by Rios Insua and French (1991). One way of mitigating this is by using several distances when undertaking the analysis.

As discussed in Section 4.3.2, the range of each input parameter also needs to be specified by the user, which defines the feasible range that each parameter is able to be varied between in order to achieve a reversal in ranking (i.e. $a_y > a_x$) (Equations 4.6 to 4.8). The values can either be based upon knowledge of experts (e.g. select the User Range button on the form shown in Figure 4.11 and enter the data, as shown in Figure 4.12) or the data that are available (e.g. select the Data Range button on the Distance-based Uncertainty Analysis form (Figure 4.11), which uses the minimum and maximum input values for each criterion based on the values specified).

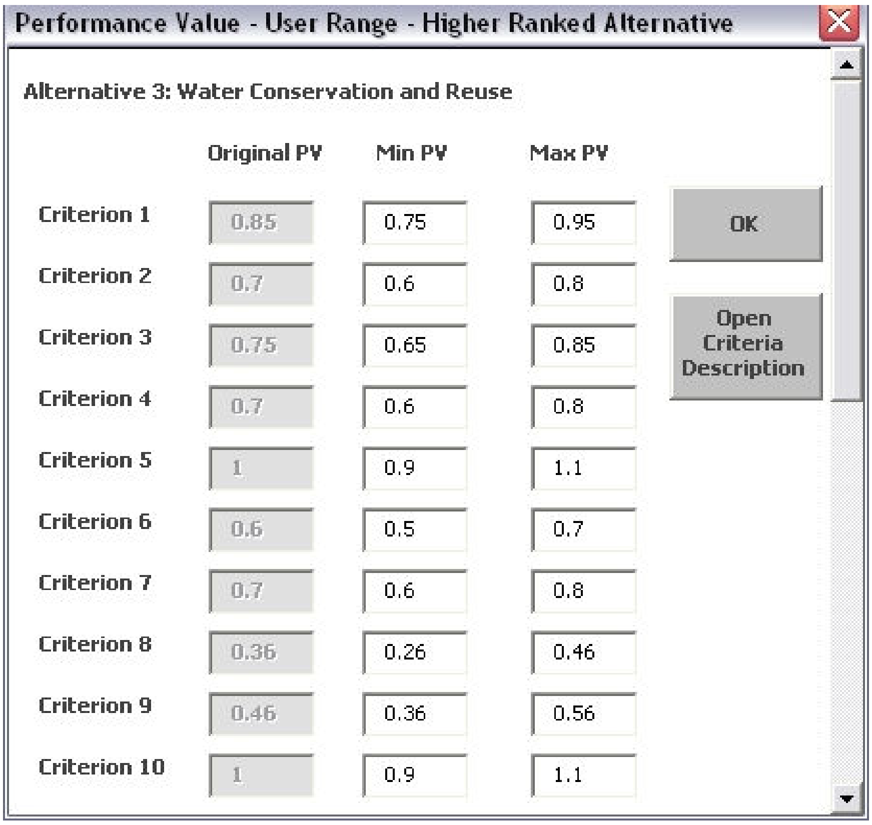


Figure 4.12 Example of the form where user defined PV ranges for distance-based uncertainty analysis are entered

As stated in the description of the methodology in Section 4.3.2, the user is also able to elect to restrict the optimisation by maintaining the original CW rank order. This is incorporated into the program by adding constraints into the optimisation, based on a binomial number check, as demonstrated in Table 4.3. In this instance, the optimised CWs of iteration *z* would not be accepted, as the CWs do not maintain the original rank order due to the rank order violation of CW2 and CW1, as shown by the shaded cells in Table 4.3.

Table 4.3 Example of how the program maintains CW rank order

		CW1	**CW2**	**CW3**	**CW4**	**CW5**
Original CWs		8	6	10	4	12
Iteration *z*, Optimised CWs		7	8	9	6	10
		If CW_y > CW_x Then 1 otherwise 0				
Original CWs						
		C_x				
		CW1	**CW2**	**CW3**	**CW4**	**CW5**
	CW1	-				
	CW2	0	-			
C_y	**CW3**	1	1	-		
	CW4	0	0	0	-	
	CW5	1	1	1	1	-
Iteration z, Optimised CWs						
		C_x				
		CW1	**CW2**	**CW3**	**CW4**	**CW5**
	CW1	-				
	CW2	1	-			
C_y	**CW3**	1	1	-		
	CW4	0	0	0	-	
	CW5	1	1	1	1	-

In order to obtain the robustness of the ranking of each pair of alternatives (i.e. a_x and a_y) for each actors' set of CWs, the optimisation problem given by Equations 4.1 – 4.8 in Section 4.3.2 must be solved. Two 'engines' are available in the program for selection by the user to minimise the objective function: Solver and Genetic Algorithm (GA). Solver is a Microsoft Excel Add-In Function, which is based upon the

Generalised Reduced Gradient (GRG2) nonlinear optimisation method. A number of parameters must be selected and defined before using Solver and the form shown in Figure 4.13 is displayed by clicking the Solver Options button. The user may either use the default Solver Options and parameter values, which appear in the form, or input their own. The feasibility tolerance ("precision" option in Solver) controls how accurately a constraint must be satisfied. The fractional change tolerance ("convergence" in Solver) specifies the amount by which the objective value must differ from (on a relative basis) its previous value in a specified number of iterations in order for the algorithm to continue (Stokes and Plummer, 2004). Central differences are more accurate than forward differences but require twice as many function evaluations (Stokes and Plummer, 2004). Information which may aid the selection of these parameters is included in the Help file.

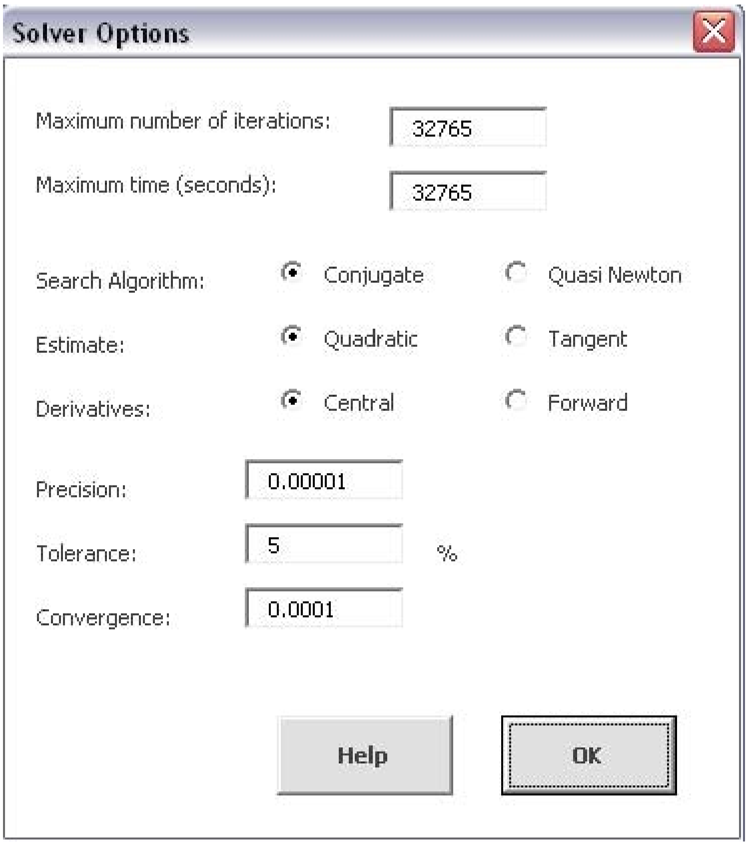


Figure 4.13 Example of the Solver input parameters form

The user must also specify the number of random number iterations on the distance-based uncertainty analysis user form. This is because Solver is dependent on the starting values and the only way to determine whether a local or global optimum has been achieved is to start from a user specified number of starting points. Excel's binomial pseudo-random number generator (i.e. RANDBETWEEN function) was used in the program to assign starting values automatically for each random iteration. By performing this operation repeatedly, sufficiently varied starting values can be obtained. When the same solution is produced from each iteration, there can be more confidence that a global minimum has actually been obtained. When different solutions are obtained from different starting values, it is necessary to explore the objective function more completely and undertake a global optimisation using the GA approach.

The main advantage of using GRG2 (i.e. Solver) is its speed of arriving at a solution, however, its disadvantage is that because it is a gradient method, the chances of a local solution being obtained are high.

Alternatively, as discussed in Section 4.3.3, the user may elect to use a GA, which is a heuristic iterative search technique that attempts to find the best solution in a given decision space based on a search algorithm that imitates Darwinian evolution and survival of the fittest in a natural environment (Goldberg, 1989). Five main parameters affect the performance of GAs: population size, number of generations, crossover rate, mutation rate and penalty function values (Raju and Kumar, 2004). The parameters that must be selected and defined before using the GA are contained on the form shown in Figure 4.14, which appears after the user presses the GA Options button on the distance-based uncertainty analysis form[6]. The user may either use the default GA Options and parameter values or input their own. Information that may aid the selection of these parameters is included in the help file. A description of the way in which a GA works, which also provides some information concerning the parameters in the GA Options form, is contained below.

When utilising a GA, the decision space is referred to as the environment, the potential solutions to the optimisation problem are called chromosomes and the total number of chromosomes is called the population size. The standard GA method, which is incorporated in the program and described below, is illustrated in Figure 4.15. The population of chromosomes in the GA used in this program is generated randomly using an integer / real number scheme, as opposed to a binary

[6] Note: this button is not shown in Figure 4.11, but the button appears when the GA toggle button is clicked, instead of the Solver toggle button (which is activated in Figure 4.11). Both GA and Solver cannot be selected at the same time.

scheme. In real-value coding, there is no discretisation of the decision variable space. In early GAs, strings were composed of binary bits, however, researchers such as Chang and Chen (1998), Wardlaw and Sharif (1999) and Setnes and Roubos (2000) have observed that a real-coded GA performs better in terms of efficiency and precision compared to a binary-coded GA when applied to multidimensional, high-precision or continuous problems. Whenever a parameter is binary coded, there is the danger that the reduced level of precision does not represent parameter values that produce the best solution values, unless decision variables can only take on discreet values.

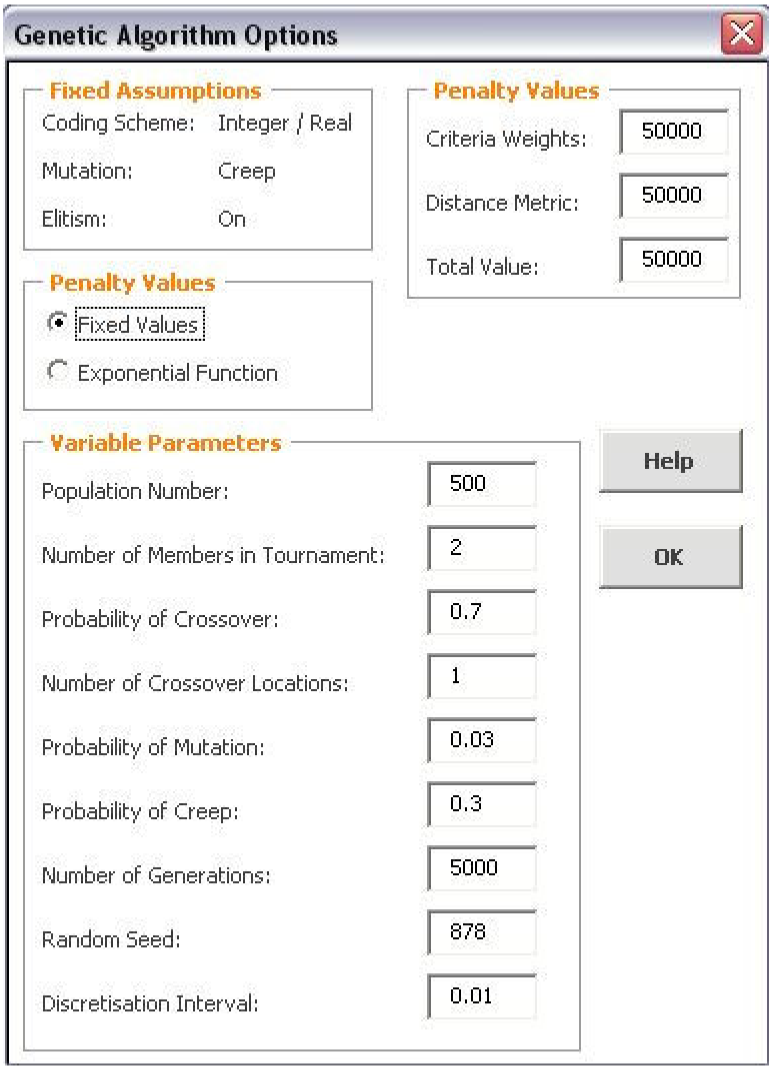


Figure 4.14 Example of the Genetic Algorithm input parameters form

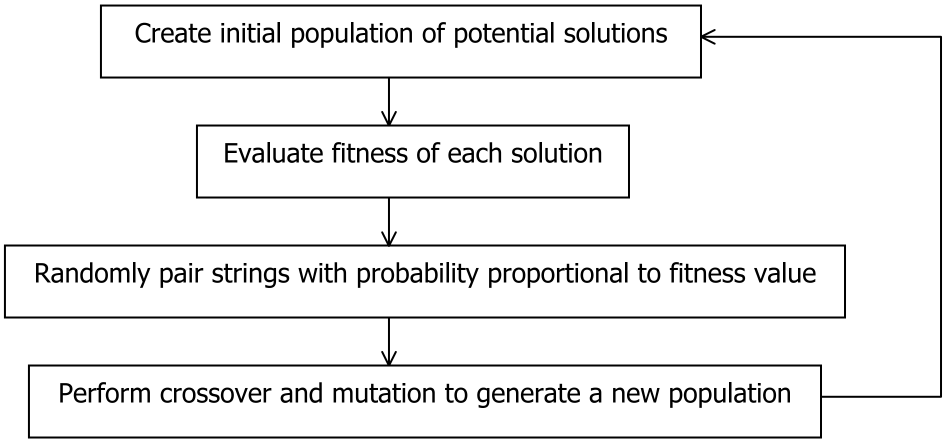


Figure 4.15 The process of a standard Genetic Algorithm

The iterations of the optimisation process are called generations and the GA proceeds by evaluating the best sets of chromosomes in the population at each generation. These sets of chromosomes are found by evaluating the objective function (Equation 4.10 or Equation 4.11) for each chromosome in the population and by using this objective function value to indicate the fitness of the chromosomes. The chromosomes in a population compete with each other for survival, based on their fitness levels, and more fit individuals are given a higher probability of mating and reproducing and hence influencing the following generations. Through competition for survival, the population evolves to contain high-performing chromosomes.

Goldberg and Deb (1990) compared various selection schemes and indicated a preference for the tournament selection scheme. The GA used in this program uses tournament selection, where two (or other specified number of members in a tournament) chromosomes from the population are paired off at random, and the "fitter" of the two chromosomes survives, and the other chromosome is eliminated. Next, members of the parent pool, which consist of the winners of the tournaments, are paired up at random and have the opportunity to exchange information via a process called crossover. The probability that a pair of strings will exchange information is referred to as the probability of crossover, usually in a range of 0.5 – 1.0 (Ahmed and Sarma, 2005; Elbeltagi *et al.*, 2005). More crossover points are used if there are a larger number of decision variables (i.e. input parameters), which gives a greater possibility for offspring to be different from their parents.

In order to ensure sufficient exploration of the decision space, the value of some of the integers in a chromosome are changed at random in a process called mutation. Whether mutation of a particular integer occurs is governed by the probability of mutation. Creep mutation will mutate a decision variable either up or down by a single increment of the discretisation interval. An integer / real GA can only be crossed at decision variable locations, therefore, if the mutation rate is not very high there is little opportunity for the search space to be explored (Dandy and Engelhardt, 2001). However, a small mutation rate of less than 0.1 is usually used in case studies reported in the literature (Elbeltagi *et al.*, 2005).

The chromosomes obtained after the application of the processes of selection, crossover and mutation (i.e. the children) become the parents in the next generation and the process is repeated until a predefined stopping condition is met (e.g. the completion of a fixed number of iterations, such as the number of generations specified). Elitism is employed in the GA used in this program, which ensures that the fittest member of a generation is guaranteed to survive the selection process in the next generation. This makes certain that there is no reduction in fitness from one generation to the next. For a more detailed description of GAs, the reader is referred to Goldberg (1989).

As described in Section 4.3.3, constraints are unable to be incorporated specifically in the formulation of the GA, therefore, they are included in the objective function and multiplied by penalty values to discourage the selection of infeasible solutions by decreasing their fitness. Two penalty methods (i.e. Fixed Values and Exponential Function) are included in the program and can be selected by the user on the GA Options form (see Figure 4.14). The penalty values can therefore either be constant or vary with generation. A variable penalty function means that the penalty is very lenient at the start of the algorithm, but grows progressively harsher as the algorithm runs (i.e. it uses a multiplier that is an exponential function of the total constraint violation and / or generation number). The exponential function used in the program is based upon the function proposed by Carlson *et al.* (1996).

Cieniawski *et al.* (1995) identified constraint handling as a shortcoming of GAs. The experience of Chan Hilton and Culver (2000) is that to incorporate constraints into GAs using the standard method of adding penalty functions requires substantial fine-tuning for each problem solved. The amount of effort or total number of GA searches required to determine reasonable penalty weights is an important component of the overall efficiency of GAs. If a set of penalties is too harsh, then the few solutions found that do not violate constraints quickly dominate the mating pool and yield sub-optimal solutions. A penalty that is too lenient can allow infeasible solutions to flourish as they can have higher fitness values than feasible solutions. The main difficulty in applying penalty functions is that they are problem dependent.

Another perceived disadvantage of GAs is that there are a large number of parameters that are required to be defined, which therefore renders the GA as being difficult to use, especially when used by practitioners who have little knowledge on how to set these parameters. A summary of parameters used in GAs in various case studies presented in the literature is contained in Table 4.4 to provide some information on the range of parameter values that are commonly utilised. From Table 4.4, it can be seen that many of the papers do not contain complete information on the parameters that were utilised to undertake the GA analysis. There is also little consistency between the values selected, therefore, it is recommended that values be trialled for each individual case that is assessed. This can make GAs computationally intensive, especially in determining the best combination of crossover and mutation probabilities. In addition, large population sizes and large numbers of generations increase the likelihood of obtaining a global optimum solution, but substantially increase processing time (Elbeltagi *et al.*, 2005). However, Deb (2000) states that fixing the correct population size is an important factor for proper working of a GA and use the simple formula of: $N = 10 \times n$ where N is the population size and n is the number of variables.

The GA also has advantages including that it can handle difficult problems that have large nonlinear search spaces. Their principal advantage over many other optimisation techniques is the use of a population of solutions that simultaneously searches various parts of the solution space. This greatly reduces the likelihood of convergence on a local minimum solution. Another advantage is the fact that any nonlinear, integer, logical or discontinuous objective function or constraint can be included in the optimisation. The major disadvantage of GAs is the lengthy computer time associated with the need to carry out many simulations (Dandy and Engelhardt, 2001). Mirrazavi (2001) have found that the fitness function value of the first integer solution is important and usually has an effect on the time taken to solve the model to optimality.

The Solver trial number or the GA generation number, respectively, is shown in the Excel task bar during the analysis so that the user may monitor the progress of the program. Following completion of the specified number of iterations or generations, the user is asked if they would like to view the output or perform another analysis. The output of the program is discussed in Section 4.3.4.

Table 4.4 GA input parameters used in case studies in the literature

Reference	Application	Probability of crossover	Probability of mutation	Number of generations	Initial population	Tournament size	Termination Criterion	Real or binary coded	Penalty method
Chang and Chen (1998)	Flood control reservoir model	0.90	0.010	10	100	UNK	10^{th} generation	Compares real and binary	UNK
Herrera *et al.* (1998)	Numerical experiments	0.60	0.005	5,000	61	UNK	$5{,}000^{th}$ generation	Real	UNK
Chan Hilton and Culver (2000)	Granular activated carbon treatment design	0.85	0.011	350	200	UNK	UNK	Binary	Compare additive & multiplicative penalty method
	Air stripping tower design	0.85	0.028	100	150	UNK	UNK	Binary	Compare additive & multiplicative penalty method
Burn and Yulianti (2001)	Waste load allocation	0.80	0.050	200	40	One quarter of the population of strings	UNK	Binary	Penalty coefficient method
Dandy and Engelhardt (2001)	Rehabilitation of water pipes	0.70	0.010	UNK	488	8	UNK	Binary	UNK
Hsiao and Chang (2002)	Optimal groundwater management	0.5 - 1.0	1/population	22 - 43	50 - 100	UNK	User-defined stopping criterion or maximum number of generations achieved	Binary	UNK
Milutin and Bogardi (2002)	Derive the optimal release distribution within a multi-reservoir water supply system	0.75	0.005	100	30	UNK	100^{th} generation	Binary	UNK
Maier *et al.* (2003)	Optimising mutual information of ecological data clusters	0.6 – 0.9	0.001 – 0.1	5,000	30	30	$5{,}000^{th}$ generation	Binary	UNK

Reference	**Application**	**Probability of crossover**	**Probability of mutation**	**Number of generations**	**Initial population**	**Tournament size**	**Termination Criterion**	**Real or binary coded**	**Penalty method**
Raju and Kumar (2004)	Irrigation planning	0.6 – 1.0	0.01 – 0.12	200	50	UNK	200^{th} generation	Real	Penalty function
Ahmed and Sarma (2005)	Optimal operating policy of a multipurpose reservoir	0.6 – 1.0	0.005 – 0.2	5,000	3 times the length of string	UNK	$5{,}000^{th}$ generation	Real	Method proposed by Deb (2000)
Almasri and Kaluarachchi (2005)	Nitrate concentration of groundwater	0.90	0.080	300	300	UNK	0.01	Real	Penalty values
Chan Hilton and Culver (2005)	Groundwater remediation	0.90	0.031	150	32	UNK	UNK	Binary	UNK
Elbeltagi *et al.* (2005)	Construction	0.80	0.008	500	200	UNK	10 consecutive generation cycles or the objective function reached its known target	Real	UNK
Muleta and Nicklow (2005)	Streamflow and sediment yield estimates	UNK	0.20	75	150	UNK	75^{th} generation	Real	UNK

STOCHASTIC UNCERTAINTY ANALYSIS APPROACH

The methodology of the stochastic uncertainty analysis approach utilised in the program is contained in Section 4.4.

If the user decides to undertake the stochastic uncertainty analysis by clicking the respective button shown on the form in Figure 4.10, the Stochastic Uncertainty Analysis user form is displayed, which is shown in Figure 4.16. Flexibility is a key component of the program, with the user having many options to assess the various sources of input parameter uncertainty using the stochastic uncertainty analysis approach. The user may either select partial deterministic (distributions for either CWs or PVs only) or full stochastic analysis (distributions for both CWs and PVs) on the Stochastic Uncertainty Analysis user form (Figure 4.16).

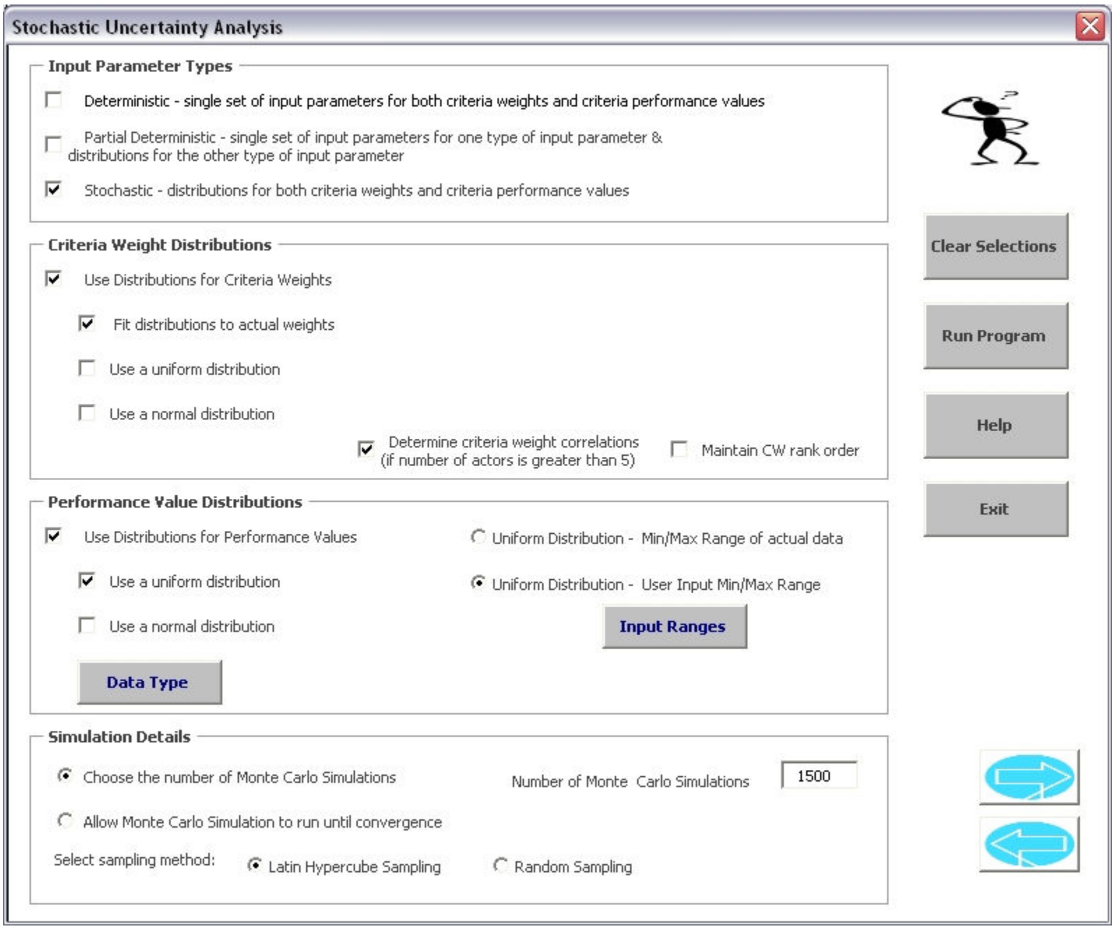


Figure 4.16 Example of stochastic uncertainty analysis form

The user must then choose the distribution type for the input parameter values selected to be varied (i.e. CWs and / or PVs). With regard to the CWs, the user has the option to fit a distribution to the actual CWs, ensuring that all of the information obtained from the actors is explicitly incorporated in the decision making process. The commercial Microsoft Excel @Risk add-in program developed by Palisade (2000) is used to fit distributions to the data and goodness of fit statistics are reviewed to determine how representative the fitted distributions are of the actual sets of CWs elicited from the actors. Alternatively, the user can select either a normal or uniform distribution to enable uncertainty and subjectivity in the CWs to be incorporated in the analysis. The user must then also characterise the distributions, by defining the upper and lower bounds of the CWs using either actor specified limits or bounds based upon the actual CWs available. Selection of the bounds based upon the actual CWs option for the upper and lower limits is only possible when there is more than one actor involved in the decision process. If a normal distribution is selected to represent the uncertainty in the CWs, then the mean and standard deviation are required to be specified to enable the distribution to be characterised.

The user can also elect to undertake a correlation analysis, with the results incorporated in the distributions, to ensure that sampling from the CW probability distributions represents the actual assignment of CWs by actors. The program utilises the tool available in Microsoft Excel to undertake the correlation analysis (i.e. Correlation tool in Data Analysis). The user may also elect to constrain the CWs by maintaining the original rank order of the CWs. This constraint is not able to be included directly into the MCS sampling, therefore, the program performs a check after the completion of the simulation to determine how many sets of CWs conform to the constraint and maintain the original CW rank order. It is these sets of CWs that are then utilised to determine the total value of the alternatives.

The uncertainty, imprecision and variability in the quantitative PVs can also be represented by continuous probability distributions, such as uniform or normal. A range of values (i.e. upper and lower bounds for uniform distributions) must be assigned to each PV by the user, representing the set of possible values for that variable, which can either be based upon knowledge of the experts or the data that are available. A discrete uniform distribution can be utilised for qualitative PVs, which is characterised by defining upper and lower limits of each PV (e.g. a groundwater rise of 'medium' may be between 'medium low' and 'medium high', or in an integer scale between 2 and 4). The user must also specify whether each criterion belongs to a discrete or continuous data type by clicking on the Data Type button.

Hajkowicz (2000) found that often the variables will be selected under a normal distribution, increasing the likelihood that they will be closer to the original value. However, even though the choice of the distribution for generating random values may be somewhat arbitrary, Barron and Barrett (1996) observed, through experiences with various distributions, that they do not produce any qualitative differences in the results. Helton (1993) also found that sensitivity results are generally less dependent on the actual distributions assigned to the input variables than they are on the ranges chosen for the variables.

Following the definition of the input parameter distributions, the MCS must be characterised by the user. The user is able to elect whether the MCS will run until convergence of the input distributions or until the number of user specified iterations is completed. The user may also choose whether Random Sampling or Latin Hypercube sampling[7] is utilised. These two sampling techniques are included in the program as they are the techniques utilised by the selected add-in program, @Risk (Palisade, 2000). In addition, Random Sampling and Latin Hypercube sampling are two of the most widely utilised sampling techniques. Several studies have shown that under various conditions Latin Hypercube sampling results in more stable estimates than Random Sampling (Helton, 1993) and it is known to generate representative samples more efficiently (Yu *et al.*, 2001). For models with high computational requirements, Manache and Melching (2004) recommend using the Latin Hypercube sampling technique, which provides the flexibility of MCS with less computational load. Latin Hypercube sampling was used by Felli and Hazen (1998) to increase the rate of convergence of simulated quantities. They allowed each simulation to run until the percentage change in the mean value of all simulated quantities remained stable at less than 0.75%.

[7] Latin Hypercube sampling is a stratified sampling approach that efficiently estimates the statistics of an output. The probability distribution of each basic variable is subdivided into *N* ranges with an equal probability of occurrence (1/*N*). Random values of the basic variable are simulated such that each range is sampled just once. The order of selection from the ranges is randomised and the model is executed *N* times with a random combination of basic variable values from each range for each basic variable.

An efficient sampling scheme that reduces the number of samples required for each iteration can significantly improve the computational efficacy of the stochastic optimisation procedure (Kalagranam and Diwekar, 1997). Kalagranam and Diwekar (1997) found that Hammersley sampling (based on quasi-random sequences) requires far fewer samples as compared to other conventional techniques (such as Latin Hypercube sampling) to approximate the mean and variance of distributions. Hammersley sampling has not been utilised in this program, but could be incorporated in future work, as it may be useful in problems with a large number of criteria to reduce computation time.

Information is provided in the Help file to aid the user in making the selections required on the form, which is especially relevant if the user is unfamiliar with MCS. The Microsoft Excel @Risk Add-in program (Palisade, 2000) is used to undertake the MCS. The Run Program button must be pressed to undertake the analysis. If all of the information required has not been entered on the form, a dialogue box appears requesting that the particular missing information be entered by the user before continuing (see for example Figure 4.17). The progress of the stochastic uncertainty analysis may be monitored by reading the text which is displayed in the task bar of Microsoft Excel, as the program will display which part of the method it is currently executing / performing.

Figure 4.17 Example of an error message when utilising the stochastic uncertainty analysis program

Probabilistic sensitivity analysis provides a mechanism for the DM to directly examine output distributions, such as the distribution for a single alternative or the difference between distributions for some pair of competing alternatives. Knowledge of the likelihood of each total value (or difference in total value) over the entire range of possible input values enables the DM to better assess the risk of an adverse outcome, or, in the case of difference in total values between two competing alternatives, select an alternative based on the likelihood that its total value will exceed that of its competitors by some specified amount. The predictions of the stochastic model are also presented by using the mean and the standard deviation of an appropriate probability density function of the results (Rauch, 1998). Further details of the output of the program are contained in Section 4.4.4 and are illustrated in Section 5.6.

Chapter 5
Comparison of Proposed MCDA Uncertainty Analysis Approach with Existing Sensitivity Analysis Methods

5.1 Introduction

Having discussed the background literature regarding MCDA in Chapters 2 and 3 and proposed an approach to overcome some of the limitations of the application of MCDA in Chapter 4, the aims of this chapter are to:

- *Demonstrate* the limitations of existing sensitivity analysis methods applicable to MCDA;
- *Illustrate* the benefits of the proposed uncertainty analysis methods; and
- *Validate* the program developed as part of this research, where possible.

In order to achieve these aims, the proposed uncertainty analysis methods presented in Chapter 4 are compared with a selection of the existing sensitivity analysis methods described in Chapter 3, as summarised in Table 5.1. The comparisons are undertaken by utilising the example decision problems that were originally used to demonstrate the respective existing sensitivity analysis methods. The principal reasons why these existing sensitivity analysis methods were selected for comparison in this chapter are:

- The selected sensitivity analysis methods are representative of the range of available existing sensitivity methods;
- The MCDA technique utilised to demonstrate the existing sensitivity analysis methods in the example case study is available in the program that has been developed as part of this research. For example, the sensitivity analysis methodology proposed by Janssen (1996) in Section 3.3.1 was not able to be used as a basis for comparison as the case study was undertaken using a

methodology called EVAMIX, which is not included in the program developed; and

- The required data are available to undertake the case study presented in the original paper. For instance, an example decision problem was not undertaken to demonstrate the sensitivity analysis method presented in Proll *et al.* (2001). In addition, the Wolters and Mareschal (1995) method described in Section 3.2.4 was not able to be assessed because not all of the criteria PV data were included in the paper by Wolters and Mareschal (1995).

Only limited aspects of the proposed uncertainty analysis methodology, and hence the program, are examined in this chapter. It should also be noted that the examples presented in this chapter are for illustrative purposes, as the goal of this chapter is to demonstrate the applicability of the proposed methodology, rather than to give a complete detailed analysis of the examples utilised. In addition, the sensitivity analysis methods are presented in this chapter in the same order as shown in Table 5.1.

Table 5.1 Summary of sensitivity analysis methods presented and compared in Chapter 5

Reference	MCDA Method	Reference in Chapter 3	Proposed Approach
Mareschal (1988)	PROMETHEE	Section 3.2.2	Distance-based (Section 4.3)
Rios Insua and French (1991)	WSM	Section 3.2.3	
Ringuest (1997)	WSM	Section 3.2.7	
Guillen *et al.* (1998)	WSM	Section 3.2.8	
Butler *et al.* (1997)	WSM	Section 3.3.2	Stochastic (Section 4.4)

As an additional note, limited comparisons of sensitivity analysis methods have been provided in the literature thus far. One of the largest studies was an evaluation of ten sensitivity analysis methods by Frey and Patil (2002). The study found that no single sensitivity analysis method was clearly superior to all others, with each method having its own key assumptions and limitations and own demands regarding the time and effort needed to apply the method and interpret the results. It was concluded by Frey and Patil (2002) that two or more methods, preferably with different foundations, should be used to increase the confidence that the identification of key inputs is robust. However, this would be difficult and time consuming for most decision problems and therefore it is not thought to be a practical or ideal solution. The findings of Frey and Patil (2002) emphasise the need for more general sensitivity analysis methods, which is the basis of the work undertaken as part of this research. The benefits of the methods introduced in this thesis are demonstrated in the sections below via comparisons with existing sensitivity analysis methods.

5.2 PROMETHEE, Mareschal (1988) sensitivity analysis & distance-based uncertainty analysis

5.2.1 Background to case study

Mareschal (1988) demonstrates the proposed stability interval sensitivity analysis method described in Section 3.2.2 by assessing a decision problem where four possible locations to build a hydroelectric plant are proposed. For this purpose, the DM selected four criteria and the input data utilised are contained in Table 5.2. It should also be noted that criterion 2 was maximised while the remaining criteria were minimised. Equal CWs were assumed across the criteria.

Table 5.2 Input parameter values in example decision problem assessed by Mareschal (1988)

Criterion	CW	Generalised Criterion Function	Threshold Values	Performance Values			
				Alt1	Alt2	Alt3	Alt4
C1: manpower cost	1	II: Quasi	q = 10	80	65	83	52
C2: power	1	V: Linear*	q = 0, p = 30	90	58	60	72
C3: maintenance cost	1	IV: Level	q = 1, p = 6	5.4	9.7	7.2	2.0
C4: villages to evacuate	1	I: Usual	-	8	1	4	3

Note: * As there is a *q* value defined in the paper, this criterion has been defined as a linear criterion with preference and indifference area, however, as the value of *q* is 0, it would probably be more appropriate to label it as the Type III criterion with linear preference (see Section 2.5.5 for more information on the generalised criterion functions).

5.2.2 Problem formulation

Deterministic analysis

The PROMETHEE II MCDA approach was utilised by Mareschal (1988) to obtain the total flows of each of the alternatives based upon the input data in Table 5.2. Deterministic analysis was also undertaken using the program developed as part of this research to verify the output of the program. The MCDA technique PROMETHEE was utilised with the criteria PVs, CWs and generalised criterion functions and thresholds provided by Mareschal (1988). As mentioned previously in Section 4.3.2, generalised criterion functions are not required to be specified for each of the criteria in the proposed uncertainty analysis approaches, therefore, deterministic analysis was repeated as part of this research using Level I generalised criterion functions for each criterion.

Uncertainty analysis

The aim of the uncertainty analysis undertaken by Mareschal (1988) is to find stability intervals for the CWs, which consist of the values that the weight of one criterion can take without altering the ranking of alternatives obtained using the initial set of weights, all other weights being kept constant. The methodology described in Section 3.2.2 and Mareschal (1988) was utilised to undertake this analysis and the results presented here for this approach are those obtained by Mareschal (1988).

The proposed distance-based uncertainty analysis method (Section 4.3) has similarities with the stability interval method proposed by Mareschal (1988). Therefore, analysis using the proposed distance-based uncertainty analysis method is also conducted to enable a comparison to be carried out between the two methods. Two scenarios were assessed using the approach presented in this thesis: (1) vary the CWs only, and (2) vary both the CWs and PVs.

The program developed as part of this research was used to undertake the distance-based uncertainty analysis. The various inputs required to be specified include:

- Upper and lower limits of the input parameter values (i.e. uncertainty interval);

  No information was provided by Mareschal (1988) on the uncertainty associated with the CWs or the PVs, therefore, upper and lower limits that define the feasible range for the CWs and PVs (Equations 4.6 – 4.8) were assumed for the purposes of undertaking the analysis, and are contained in Table 5.3 (for the PVs of the two highest ranked alternatives only). The assumption is based on providing a wide range for the variation of the input parameters and that the range is as equal as possible across the input parameters so as to not bias the results.

- Distance metric; and

  The Euclidean Distance was selected, as it is one of the most commonly used distance metrics.

- Optimisation method.

  The optimisation of the objective function (Equation 4.1) of the proposed uncertainty analysis approach for the case study was undertaken using the Microsoft Excel Add-In Solver Function and the default Solver options. 50 random starting values for the input parameters were used for each pair of alternatives to sufficiently vary the starting values, with the aim of increasing the chances of finding near globally optimal solutions.

5.2.3 Results

Deterministic analysis

The overall total flows obtained by Mareschal (1988) using the PROMETHEE II MCDA technique are contained in Table 5.4. It is apparent that Alternative 4 is the highest ranked alternative, followed by Alternatives 2, 1 and 3 respectively. The results obtained using the program developed as part of this research are the same as the results obtained by Mareschal (1988), therefore, confirming the validity of the deterministic PROMETHEE element of the program[8].

Table 5.3 Upper and lower limits for the input parameters used in the distance-based uncertainty analysis of the Mareschal (1988) case study

Criterion	CW		Alt 4 PVs		Alt 2 PVs	
	LL	UL	LL	UL	LL	UL
C1	0.1	3.0	42.0	62.0	55.0	75.0
C2	0.1	3.0	62.0	82.0	48.0	68.0
C3	0.1	3.0	0.1	4.0	7.7	11.7
C4	0.1	3.0	1.0	5.0	0.01	3.0

Note: LL = lower limit and UL = upper limit

The deterministic results obtained by assigning the Level 1 generalised criterion functions to each of the criteria, using the program developed as part of this research, are also contained in Table 5.4. These results were confirmed using the commercial software package Visual Decision (2000), providing further evidence of the validity of the program developed. The main difference between the rankings obtained with the different generalised criterion functions is the rank reversal of Alternatives 2 and 1. This result indicates that the ranking of the second and third ranked alternatives is not stable.

Table 5.4 Overall total flows obtained by Mareschal (1988) and by using Level 1 generalised criterion functions for each criterion

Generalised Criterion Functions	Result	Alt 1	Alt 2	Alt 3	Alt 4
As defined by Mareschal (1988) (see Table 5.2)	**Total flow**	-0.475	0.117	-1.208	1.566
	Rank	3	2	4	1
Level 1 (Usual Criterion) for each criterion	**Total flow**	0.000	-0.500	-1.500	2.000
	Rank	2	3	4	1

[8] It should be noted that to obtain the same results as Mareschal (1988), the Level III type generalised criterion function was utilised for criterion 2 in the program.

The large difference in total flow between the two highest ranked alternatives would potentially give the DM confidence that Alternative 1 is the 'best' alternative and that the ranks would be reasonably stable. However, it is still advisable to undertake uncertainty analysis to confirm this notion, the results of which are described below.

Uncertainty analysis

Mareschal (1988) approach

The weight stability intervals to maintain full stability of the rankings of the alternatives (contained in Table 5.4) obtained by Mareschal (1988) through using the methodology described in Section 3.2.2, are summarised in Table 5.5. These intervals consist of the values that the weight of one criterion can take without altering the results given by the initial set of CWs, all other CWs being kept constant. From Table 5.5 it is evident that the rankings of the alternatives are most sensitive to the weights assigned to criterion 2 and criterion 4, as these have the smallest stability intervals.

Table 5.5 Weight stability intervals determined by Mareschal (1988) for full stability of the ranking of the alternatives

NOTE:
This table is included on page 157 of the print copy of the thesis held in the University of Adelaide Library.

For partial stability, the CW intervals within which Alternative 4 remains the highest ranked alternative have been determined by Mareschal (1988) and are presented in Table 5.6. The pairs of alternatives considered to arrive at the results were: (Alt 4, Alt 1), (Alt 4, Alt 2) and (Alt 4, Alt 3). From these results, Mareschal (1988) concluded that Alternative 4 has quite a stable position in the ranking and that the CWs have to be significantly varied to modify this position. However, as stated in Section 3.2.2, a limitation of the method is that it does not inform the DM of what will happen to the ranking of the alternatives once the stability intervals are exceeded.

Table 5.6 Weight stability intervals determined by Mareschal (1988) for partial stability of the ranking of the alternatives where Alt 4 remains the highest ranked alternative

NOTE:
This table is included on page 158 of the print copy of the thesis held in the University of Adelaide Library.

Proposed distance-based uncertainty analysis approach

The results obtained using the proposed distance-based uncertainty analysis approach are summarised in Table 5.7, where the Euclidean Distance is provided for one alternative outranking another alternative when the CWs are allowed to vary simultaneously within the expected range of uncertainty (e.g. the Euclidean Distance for Alternative 2 outranking Alternative 4 is 1.698). The initially lower ranked alternatives are listed in the leftmost column of Table 5.7 in rank order. There is only a minor difference between the Euclidean distances obtained for the analysis of the pairs of alternatives: (Alt 1, Alt 4) and (Alt2, Alt4). These results therefore inform the DM that although there is quite a significant difference in total flows between the alternatives (see Table 5.4), based on the Euclidean Distances obtained, it is difficult to say that Alternative 2 is 'better' than Alternative 1 and vice versa. No feasible changes in CWs were able to be identified which would result in rank equivalence between Alternatives 4 and 3, which informs the DM that the ranking of Alternatives 4 and 3 is robust. Based on the magnitude of the Euclidean Distance the ranking of Alternative 4 is robust when only the CWs are considered in the uncertainty analysis.

The optimised CWs and associated relative changes obtained when determining how robust the ranking of Alternative 4 is in comparison with the remaining alternatives are contained in Table 5.8. From these data, the most critical CWs can be identified and are: CW1 for Alternatives 4 and 2 to obtain rank equivalence and CW3 for Alternatives 4 and 1 to achieve equal ranking, as they exhibit the smallest relative change. It is evident, however, that quite significant changes are required in the majority of the CWs for either Alternative 2 or Alternative 1 to outrank Alternative 4.

Table 5.7 Euclidean distances obtained by using the proposed distance-based uncertainty analysis approach, simultaneously varying CWs, Mareschal (1988) case study

NOTE:
This table is included on page 159 of the print copy of the thesis held in the University of Adelaide Library.

Table 5.8 Optimised CWs obtained from distance-based uncertainty analysis for alternatives outranking Alternative 4, varying CWs only, Mareschal (1988) case study

NOTE:
This table is included on page 159 of the print copy of the thesis held in the University of Adelaide Library.

It is often the case, however, that uncertainty in the criteria PVs can have an impact on the ranking of the alternatives, therefore, additional analysis has been undertaken using the proposed approach by varying the CWs and PVs simultaneously. The optimised input parameter values which result in Alternative 4 outranking Alternative 2 are contained in Table 5.9. A Euclidean Distance of 0.367 was obtained from this analysis, which is considerably less than that obtained when varying the CWs only (i.e. d_e = 1.698). The results therefore demonstrate that much smaller changes in input parameters will result in a reversal of ranking between pairs of alternatives when both CWs and PVs are incorporated in the analysis, as shown in Table 5.9. This is an important outcome, as it is generally only the variability in the CWs that is considered when a sensitivity analysis is undertaken, which, in this particular instance, would lead the DM to believe that the rankings of the alternatives using the initial CWs was robust.

The most significant input parameters to the ranking of the alternatives can also be determined from the results obtained, by examining the relative change between the optimised values and the original values. From Table 5.9 it can be seen that the most critical input parameters are CW1, CW3, PV3 Alt 2 and PV1 Alt 2. These results therefore illustrate that it is not only the CWs that have the most impact on the results of a decision analysis.

Table 5.9 Optimised CWs and PVs for Alternative 2 to outrank Alternative 4, Mareschal (1988) case study

NOTE:
This table is included on page 160 of the print copy of the thesis held in the University of Adelaide Library.

5.2.4 Discussion

When only the uncertainty in the CWs is taken into consideration the results of the Mareschal (1988) sensitivity analysis method and the proposed distance-based uncertainty analysis approach both arrive at the conclusion that Alternative 4 has quite a stable position in the ranking and that the CWs have to be altered significantly to modify this position. The Mareschal (1988) approach, however, has a number of limitations including:

- The method does not provide insight into the way the ranking of the alternatives is changed if the CW stability boundaries identified are exceeded;
- The focus of the methodology is on changing the weight of one criterion at a time; and
- The method only considers the sensitivity of the CWs and not the combined sensitivity with the PVs.

As stated in the previous chapters, only varying one input parameter at a time, or one type of input parameter (i.e. CWs), is not adequate to gain a complete understanding of the impact that changes in the input parameter values may have on the ranking of the alternatives. It is often the case that uncertainty in the criteria PVs can have an impact on the ranking of the alternatives, but the approach proposed by Mareschal (1988) does not consider the uncertainty in the PVs. To be able to jointly vary the CWs and PVs with the proposed distance-based uncertainty analysis approach provides the DM with valuable information regarding the stability of the ranking of the alternatives that is not able to be provided by Mareschal (1988).

The results of the case study using the proposed distance-based uncertainty analysis approach demonstrated that both the CWs and PVs have an impact on the ranking of the alternatives through:

- A smaller Euclidean Distance being obtained when the CWs and PVs were varied simultaneously compared to when only the CWs were included in the analysis; and
- Some of the PVs being identified as the most critical inputs to cause a reversal in ranking between Alternatives 4 and 2.

The results of the proposed approach also demonstrate that the complete rankings and the difference between the total flows should not be relied upon when selecting an optimal alternative. The case study therefore demonstrates the benefits of the proposed uncertainty analysis approach and the additional information that is provided to the DM compared with the Mareschal (1988) methodology.

5.3 WSM, Rios Insua and French (1991) sensitivity analysis method & distance-based uncertainty analysis approach

5.3.1 Background to case study

Rios Insua and French (1991) illustrate the sensitivity analysis methodology presented in Section 3.2.3 with a floodplain management problem in Dallas, Texas. Four criteria were defined to assess eight alternatives and the criteria PVs utilised are contained in Table 5.10. The CWs were fixed, as shown in Table 5.10, and only one set of CWs was provided.

Table 5.10 Input parameter values in floodplain management decision problem assessed by Rios Insua and French (1991)

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
CWs	10	7	3	5	6	8	9	4	2	1
Performance Values										
Alt1	6	6	8	8	8	4.3	3.3	1	4	6
Alt2	5	5	7	7	7	5.9	4	1	4	7
Alt3	8	2	3	4	4	7.4	5.6	5.3	2	4
Alt4	7	8	6	6	6	2.7	1	1	6	1
Alt5	4	4	4	5	5	6.6	4.7	1	3	5
Alt6	6	7	5	2	2	8	2.8	8	7	2
Alt7	4	3	3	4	4	7.7	2.7	8	1	8
Alt8	1	3	1	1	1	1	8	6	8	2

5.3.2 Problem formulation

Deterministic analysis

The WSM MCDA approach was utilised by Rios Insua and French (1991) to obtain the total values of each of the alternatives based upon the input data in Table 5.10. Deterministic analysis was also undertaken using the program developed as part of this research (Section 4.6) to verify the output of the program.

Distance-based uncertainty analysis

Rios Insua and French (1991) assess the sensitivity of the decision problem using the methodology presented in Section 3.2.3. The aim of the Rios Insua and French (1991) approach is to identify the 'smallest' changes necessary in the CWs before a significant change in the ranking of the alternatives occurs. To enable the analysis to be undertaken, Rios Insua and French (1991) assumed upper and lower bounds of the CWs, which are summarised in Table 5.11.

Table 5.11 Upper and lower limits for the input parameters used in the distance-based uncertainty analysis of the Rios Insua and French (1991) case study

Criteria	CW*		PVs Alt 1		PVs Alt 6	
	LL	UL	LL	UL	LL	UL
C1	5	20	3	9	3	9
C2	4	10	3	9	4	10
C3	0	15	5	11	2	8
C4	5	10	5	11	1	7
C5	0	10	5	11	0.01	5
C6	5	10	1.3	7.3	5	11
C7	5	15	0.3	6.3	0.01	5.8
C8	3	5	0.01	4	5	11
C9	0	3	1	7	4	10
C10	0	5	3	9	0.01	5

Note: * CW upper and lower limits (i.e. UL & LL) are as published in Rios Insua and French (1991). Upper and lower limits of the PVs have been assumed to enable the analysis to be undertaken using the distance-based uncertainty analysis method developed in this research.

The distance-based uncertainty analysis approach presented in this thesis was used to provide a basis of comparison with the Rios Insua and French (1991) sensitivity analysis method. The upper and lower bounds of the CWs in Table 5.11 were used in the analysis with the proposed approach. The Euclidean Distance was also selected to enable comparison with the Rios Insua and French (1991) results. The program developed as part of this research was utilised to undertake the uncertainty analysis. Solver was selected as the 'engine' to solve the objective function and 50 trials were undertaken to minimise the impact of the starting values on the solution of the optimisation problem.

Initially, the analysis was undertaken by only simultaneously varying all of the CWs. To illustrate the benefits of the proposed distance-based uncertainty analysis approach, further analysis was undertaken by varying the CWs and PVs concurrently. The upper and lower limits of the PVs, which represent the expected uncertainty in the input parameter values, therefore, need to be defined to enable the analysis to be undertaken. The upper and lower limits of the PVs of the two highest ranked alternatives, assumed for the purpose of conducting the analysis, are contained in Table 5.11. The upper and lower limits of the PVs of the remaining alternatives have not been included, however, they have been assigned on the same basis as those of Alternatives 1 and 6 (i.e. that the upper limit (UL) is the original PV + 3 and the lower limit (LL) is the original PV – 3).

5.3.3 Results

Deterministic analysis

The overall total values, and the corresponding rank order, obtained by Rios Insua and French (1991) using the WSM and the values in Table 5.10 are presented in Table 5.12. From these results it is evident that the current optimal alternative is Alternative 1. However, there is little difference between the total value of the second ranked alternative, Alternative 6, and Alternative 1. Some form of sensitivity analysis is therefore required to determine the robustness of the ranking of the alternatives.

Table 5.12 Overall total values obtained by Rios Insua and French (1991) in rank order

Alternative	A1	A6	A2	A3	A4	A5	A7	A8
Total Value	296.1	293.2	285.2	275.8	257.6	245.1	236.9	167
Rank	1	2	3	4	5	6	7	8

The deterministic results obtained using the program developed as part of the research presented in this thesis are the same as those obtained by Rios Insua and French (1991), confirming the validity of the deterministic component of the program using the WSM.

Uncertainty analysis

Rios Insua and French (1991) approach

The results of the sensitivity analysis undertaken by Rios Insua and French (1991) are shown in Table 5.13, with the Euclidean Distances presented for each of the alternatives to outrank the highest ranked alternative (Alternative 1). Based on the results in Table 5.13, Rios Insua and French (1991) concluded that the decision problem is sensitive to changes in CWs. The clearest competitor of Alternative 1 is Alternative 6, as can be seen by the results in Table 5.13, as it has the smallest Euclidean Distance (i.e. 0.25).

Proposed distance-based uncertainty analysis approach

The same results were obtained by using the proposed distance-based uncertainty results when only the CWs are simultaneously varied within the expected range of uncertainty, as is shown in Table 5.13. The only difference lies in the result for Alternative 4 outranking Alternative 1, as a slightly different Euclidean Distance is obtained using the proposed distance-based uncertainty analysis approach compared to the value reported in Rios Insua and French (1991). This disparity in results is probably due to the inability of Solver to find a globally optimal solution.

Table 5.13 Euclidean distances for the highest ranked alternative compared with the other alternatives, Rios Insua and French (1991) case study

NOTE:
This table is included on page 165 of the print copy of the thesis held in the University of Adelaide Library.

The optimised CWs obtained when utilising the proposed distance-based uncertainty analysis approach for Alternative 6 to outrank Alternative 1 are contained in Table 5.14. The 'changed' CWs obtained by Rios Insua and French (1991) are only reported in the paper for the Chebyshev Distance, therefore, the optimised CWs are not able to be compared. From the results in Table 5.14, it is evident that the most critical criteria for rank reversal to occur are criteria 1, 2, 6 and 7, as these criteria have the smallest relative change when compared with the original CWs. The finding by Rios Insua and French (1991) that the decision problem is extremely sensitive to CW changes is also demonstrated in Table 5.14. It is also interesting to note that the changes are small enough such that the original rank order of the CWs is maintained without incorporating this as a constraint in the formulation of the optimisation problem.

Table 5.14 Changes in CWs for Alternative 6 to outrank Alternative 1 obtained using the proposed distance-based uncertainty analysis approach and altering CWs only, Rios Insua and French (1991) case study

NOTE:
This table is included on page 166 of the print copy of the thesis held in the University of Adelaide Library.

As discussed in Section 2.5, the uncertainty in the criteria PVs can also have a significant impact on the ranking of the alternatives, however, the Rios Insua and French (1991) sensitivity analysis method does not take this form of uncertainty into account. The CWs and PVs have therefore been simultaneously varied between the expected bounds of uncertainty using the proposed distance-based uncertainty analysis approach. The Euclidean Distances obtained using this methodology are summarised in Table 5.13 and the optimised CWs and PVs for Alternative 6 to outrank Alternative 1 are contained in Table 5.15.

The Euclidean Distance for Alternative 6 to outrank Alternative 1 when considering uncertainty in all of the input parameters is 0.70, which is larger than the Euclidean Distance obtained when only the uncertainty in the CWs is taken into consideration. This is an unusual result, as the remainder of the Eudlidean Distances in Table 5.13 are smaller when the PVs are included in the uncertainty analysis. Analysis of the optimised input parameters in Table 5.15 indicates that significantly larger changes are required to be made to the PVs of Alternative 6 compared to the other input parameters (i.e. CWs and Alt 1 PVs) for Alternative 6 to outrank Alternative 1 due to the large relative changes (i.e. seven of the 10 criteria PVs of Alternative 6 have a relative change of over 20%). In contrast, the most critical input parameters are those that exhibit the smallest relative change (i.e. CW1, CW6, CW7, CW8 and PV 4 Alt 1).

Table 5.15 Optimised CWs and PVs for Alternative 6 outranking Alternative 1 using the proposed distance-based uncertainty analysis approach, Rios Insua and French (1991) case study

NOTE:
This table is included on page 167 of the print copy of the thesis held in the University of Adelaide Library.

5.3.4 Discussion

The results of the decision problem assessed by Rios Insua and French (1991) found that Alternative 1 was the highest ranked alternative of the eight potential floodplain management options. Due to the small difference in the total values of the two highest ranked alternatives, it would be difficult for the DM to confidently select Alternative 1 over Alternative 6. Therefore, the sensitivity of the ranking of the alternatives to changes in the input parameters has been assessed using the Rios Insua and French (1991) sensitivity analysis method and the distance-based uncertainty analysis method presented in this thesis.

When the sensitivity of the results of the decision analysis were assessed by altering the CWs using the distance-based uncertainty analysis methods proposed in this thesis and by Rios Insua and French (1991), it was found that the ranking of the two highest ranked alternatives was sensitive to the values assigned to the CWs. Based on this finding, the DM would then have the opportunity to revisit the weights assigned to the criteria, in particular those that were identified as being most critical, and revise any if required. The limitation of this approach is that no information is provided to the DM with regard to the impact on the ranking of the alternatives if there is some uncertainty surrounding the PVs assigned to each of the criteria.

The proposed distance-based uncertainty analysis approach allows the simultaneous variation of CWs and PVs and the results of this analysis indicate that the ranking of Alternatives 1 and 6 is more robust when considering the changes in all of the input parameters as opposed to only the CWs (as relatively large changes in the PVs of Alternative 6 are required for rank reversal to occur). However, it is interesting to observe that the ranking of the remaining alternatives with respect to Alternative 1 becomes less stable when the uncertainty of all of the input parameters are considered, as is indicated by the smaller Euclidean Distances (for example, the Euclidean Distance obtained when Alternative 3 is compared to Alternative 1 is 0.92 when all of the input parameters are considered, whereas a Euclidean Distance of 2.04 is obtained when only the CWs are taken into account).

On the basis of the results of the uncertainty analysis, when all of the input parameters are included in the analysis, it is not able to be stated with any certainty that Alternative 1 is the ‘best’ alternative. The DM would be advised to review the input parameter values, in particular those that have been identified as being most critical to the ranking of the alternatives, prior to making a final decision. The decision analysis problem would then be re-evaluated with any revised input parameter values and potentially smaller uncertainty intervals (i.e. upper and lower limits) if more information is obtained.

5.4 WSM, Ringuest (1997) sensitivity analysis & distance-based uncertainty analysis

5.4.1 Background to case study

Ringuest (1997) used a hypothetical numerical decision problem to demonstrate the sensitivity analysis method that is discussed in Section 3.2.7. A DM had to choose from three alternatives, each evaluated on four criteria and Table 5.16 provides a summary of the input data utilised. Ringuest (1997) assumed that the values for each criterion have been scaled on the interval (0,1) and that the CWs have been assessed using an appropriate methodology, such as the Simple Multi-attribute Rating Technique (SMART).

Table 5.16 Input parameter values in example decision problem assessed by Ringuest (1997)

Criteria	Performance Values			CWs
	Alt 1	Alt 2	Alt 3	
1	0.8	0.7	0.7	0.25
2	0.6	0.9	0.4	0.40
3	0.7	0.5	0.9	0.20
4	0.6	0.7	0.7	0.15
Overall Value	**0.67**	**0.74**	**0.62**	

5.4.2 Problem formulation

Deterministic analysis

The WSM MCDA approach was utilised by Ringuest (1997) to obtain the total values of each of the alternatives based upon the input data in Table 5.16. Deterministic analysis was also undertaken using the program developed as part of this research to verify the output of the program.

Distance-based uncertainty analysis

Ringuest (1997) analysed the sensitivity of the total values of the rankings of the alternatives to changes in the relative CWs by solving the L_1 and L_∞ problems, as described in Section 3.2.7.

The distance-based uncertainty analysis method developed as part of this research (Section 4.3) was used to demonstrate the benefits of the proposed approach compared with the sensitivity analysis method presented by Ringuest (1997). Two scenarios were assessed using the approach presented in this thesis: (1) vary the CWs only, and (2) vary both the CWs and PVs. No information was provided by Ringuest (1997) on the uncertainty associated with the CWs or the PVs. Upper and lower limits that define the feasible range for the CWs and PVs were therefore assumed to provide a reasonably wide range of uncertainty in which to vary the parameters between, and are contained in Table 5.17.

Table 5.17 Upper and lower limits for the input parameters used in the distance-based uncertainty analysis of the Ringuest (1997) case study

Criteria	CW Bounds		PV Bounds					
			Alt 1		Alt 2		Alt 3	
	LL	UL	LL	UL	LL	UL	LL	UL
C1	0.05	0.45	0.60	1.00	0.50	0.90	0.50	0.90
C2	0.20	0.60	0.40	0.80	0.70	1.10	0.20	0.60
C3	0.01	0.40	0.50	0.90	0.30	0.70	0.70	1.10
C4	0.01	0.35	0.40	0.80	0.50	0.90	0.50	0.90

The program developed as part of this research was used to undertake the distance-based uncertainty analysis. The various inputs required to be specified include:

- Upper and lower limits of the input parameter values (i.e. uncertainty interval);

  No information was provided by Ringuest (1997) on the uncertainty associated with the CWs or the PVs, therefore, upper and lower limits that define the feasible range for the CWs and PVs (Equations 4.6 – 4.8) were assumed for the purposes of undertaking the analysis. These are contained in Table 5.17 (for the PVs of the two highest ranked alternatives only). The assumption is based on providing a wide range for the input parameters to vary between and that the range is as equal as possible across the input parameters so as to not bias the results.

- Distance metric; and

  The Euclidean Distance was selected, as it is one of the most commonly used distance metrics.

- Optimisation method.

  The optimisation of the objective function (Equation 4.1) of the proposed uncertainty analysis approach for the case study was undertaken using the Microsoft Excel Add-In Solver Function and the default Solver options. 50 random starting values for the input parameters were used for each pair of alternatives to sufficiently vary the starting values, with the aim of increasing the chances of finding near globally optimal solutions.

5.4.3 Results

Deterministic analysis

Combining the values in Table 5.16 resulted in overall values of 0.67 for Alternative 1, 0.74 for Alternative 2 and 0.62 for Alternative 3. Thus, the DM would prefer Alternative 2. There is little difference between the total values of each of the alternatives, therefore, further information is required on the robustness of the rankings to aid in the decision making process.

The results of the MCDA using the program developed as part of this research were the same as those obtained by Ringuest (1997).

Uncertainty analysis

Ringuest (1997) approach

Ringuest (1997) initially utilised the sensitivity analysis approach described in Section 3.2.7 to determine how much the weight vector must change to make the second ranked alternative, Alternative 1, the preferred alternative. The L_1 and L_∞ solutions obtained by Ringuest (1997) are presented in Table 5.18. Both solutions result in CWs that are quite different from the original CWs. The L_1 solution implies that criterion 3 is the most important, as it has the highest CW, while the L_∞ solution implies that criterion 1 is the most important. The rankings of the CWs inferred by both the L_1 and L_∞ solutions differ from those implied by the original CWs. When Ringuest (1997) added constraints which required the CWs to maintain the original rank order, the model yielded no feasible results. It should be noted that the Euclidean (L_2) and Manhattan distances (L_1) provided in Table 5.18 were calculated using the program developed as part of this research, as no distances were provided in the results of the paper by Ringuest (1997).

Table 5.18 Results obtained by Ringuest (1997) for CWs only, Alternative 1 greater than Alternative 2

Criteria	Original CWs	Optimised CWs	
		L_1	L_∞
C1	0.25	0.277	0.350
C2	0.40	0.255	0.300
C3	0.20	0.318	0.300
C4	0.15	0.150	0.050
Euclidean Distance		**0.189**	**0.200**
Manhattan Distance		**0.290**	**0.400**
Total values of Alternatives		**Alt 1 = Alt 2 = Alt 3 = 0.69**	**Alt 1 = Alt 2 = 0.700, Alt 3 = 0.67**

Note: in the Ringuest (1997) formulation, the constraints include that the total value of Alternative 1 will be greater than the total value of Alternative 2, but also that the total value of Alternative 1 will be greater than the total value of Alternative 3. The proposed distance-based uncertainty analysis approach only incorporates the constraint that the total value of Alternative 1 will be greater than the total value of Alternative 2. The results presented in this table also have no constraints placed on the preference order of the CWs.

Ringuest (1997) also investigated how much the weight vector must change to make Alternative 3 (the third ranked alternative under the original CWs) the preferred alternative and the solutions are contained in Table 5.19. These results also indicate that the modified CWs are quite different from the original CWs and the solutions imply that rank reversals of the CWs are necessary for Alternative 3 to become the preferred alternative.

Table 5.19 Results obtained by Ringuest (1997) for CWs only, Alternative 3 greater than Alternative 2

NOTE:
This table is included on page 173 of the print copy of the thesis held in the University of Adelaide Library.

Proposed distance-based uncertainty analysis approach

The results obtained by utilising the proposed distance-based uncertainty analysis method for Alternatives 1 and 3 to outrank the highest ranked alternative, Alternative 2, based upon simultaneously varying the CWs, are contained in Table 5.20. Slightly different CWs were obtained using the proposed approach compared to those obtained by Ringuest (1997), however, similar Euclidean Distances resulted, which indicates that there are multiple solutions to the decision problem.

Table 5.20 Distance-based uncertainty analysis solutions and bounds, altering CWs only, Ringuest (1997) case study

NOTE:
This table is included on page 173 of the print copy of the thesis held in the University of Adelaide Library.

As discussed in Section 2.5, the uncertainty in the criteria PVs can also have an impact on the ranking of the alternatives, however, the Ringuest (1997) sensitivity analysis method does not take this form of uncertainty into account. The CWs and PVs have therefore been simultaneously varied between the expected ranges of uncertainty using the proposed distance-based uncertainty analysis approach. The results from this analysis for Alternative 1 to outrank the highest ranked alternative, Alternative 2, based upon simultaneously varying the CWs and PVs, are contained in Table 5.21.

A significantly smaller Euclidean Distance is obtained when both the CWs and PVs are incorporated in the uncertainty analysis, which indicates that only minor changes in the CWs and PVs are required for rank reversal to occur. The most critical input parameters can also be obtained by reviewing the results of Table 5.21 and determining the relative difference between the original input parameter values and the optimised values, as detailed in Section 4.3.4. In this particular decision problem, the PVs have the greatest impact on the ranking of Alternatives 1 and 2, as they exhibit the smallest relative change (in comparison to the CWs), therefore highlighting the importance of incorporating the PVs in any uncertainty analysis.

Table 5.21 Distance-based uncertainty analysis solutions, Alternative 1 outrank Alternative 2, altering CWs and PVs, Ringuest (1997) case study

5.4.4 Discussion

The results of the decision problem assessed by Ringuest (1997) found that Alternative 2 was the highest ranked alternative of the three potential options. Due to the small difference in the total values of the three alternatives, it would be difficult for the DM to confidently select Alternative 2 over Alternatives 1 and 3. Therefore, the sensitivity of the ranking of the alternatives to changes in the input parameters has been assessed using the Ringuest (1997) sensitivity analysis method and the distance-based uncertainty analysis method presented in this thesis.

The methodology presented by Ringuest (1997) to assess the impact of variations in the CWs on the ranking of the alternatives provides some valuable information to the DM. An alternative is considered insensitive by Ringuest (1997) if the CWs which are required for a different alternative to be preferred are "not close" to the original CWs and the rank order implied by the original CWs must be altered for any other alternative to become preferred. The presence of CW rank reversals in the results obtained by Ringuest (1997) implies, without ambiguity, that the solution is insensitive. In the absence of rank reversals, a further distinction between sensitive and insensitive solutions is based on the closeness of the new CWs to the original CWs as measured by the L_P-metric.

The comparison of the method presented by Ringuest (1997) and the uncertainty analysis approach proposed in this thesis revealed that when considering the CWs only, there are different solutions to the problem which result in similar distance measures. However, the results obtained require rank reversals of the CWs. If the DM is confident in the original rank order of the CWs, the results would suggest that the ranking of the alternatives is robust, as there is no combination of CWs that will result in reversal of the ranking of the alternatives while maintaining the original rank order of the CWs.

Despite its benefits, the methodology proposed by Ringuest (1997) does not address the uncertainty that may also be present in the PVs. Conducting the uncertainty analysis by simultaneously varying the CWs and PVs illustrated how small variations in all of the input parameters result in a different conclusion than when the CWs are the only input parameters that are included in the uncertainty analysis (i.e. d_e = 0.18

when varying CWs only and d_e = 0.082 when varying all input parameters simultaneously). It is also interesting to note that the CW rank order is maintained when all of the input parameter values are varied simultaneously, which does not occur when only the CWs are considered in the analysis. In addition, by assessing the relative change of the input parameter values to determine the most critical criteria, it is evident that the PVs have more impact on the ranking of the alternatives, as they exhibit the smallest relative change(s). These results further highlight the importance of incorporating the uncertainty in the PVs in the decision analysis.

In summary, if only the CWs were considered in the sensitivity analysis, the DM would conclude that the ranking of the alternatives was robust and Alternative 2 would be the preferred alternative. When all of the input parameters are included in the sensitivity analysis, a different outcome is obtained and it would be concluded that the ranking of the alternatives is not robust and that further work is required to reduce the uncertainty in the input parameter values prior to reassessment of the decision problem.

5.5 WSM, Guillen *et al.* (1998) sensitivity analysis & distance-based uncertainty analysis

5.5.1 Background to case study

A simple hypothetical case study taken from Guillen *et al.* (1998) is used to illustrate the existing sensitivity analysis method described in Section 3.2.8 and the benefits of the proposed distance-based uncertainty analysis approach. As part of this case study, four alternatives are assessed by three criteria and one set of CWs. The decision analysis matrix for this case study, including the total values ($V(a_n)$) for each of the alternatives using the WSM, is contained in Table 5.22.

5.5.2 Problem formulation

Deterministic analysis

The WSM MCDA approach was utilised by Guillen *et al.* (1998) to obtain the total values of each of the alternatives based upon the input data in Table 5.22. Deterministic analysis was also undertaken using the program developed as part of this research to verify the output of the program.

Table 5.22 Input parameter values in example decision problem assessed by Guillen *et al.* (1998)

NOTE:
This table is included on page 177 of the print copy of the thesis held in the University of Adelaide Library.

Distance-based uncertainty analysis

Guillen *et al.* (1998) analysed the sensitivity of the total values of the rankings to changes in the CWs by utilising the methodology described in Section 3.2.8.

The distance-based uncertainty analysis approach presented in this thesis was selected for comparison with the Guillen *et al.* (1998) sensitivity analysis method. The first scenario undertaken involved varying the CWs only, however, the second part of the analysis involved simultaneously varying the CWs and PVs. Table 5.23 contains the upper and lower limits that were assumed for the CWs and the PVs of each of the alternatives to enable the analysis to be undertaken, as no information on the uncertainty of the input parameters was provided by Guillen *et al.* (1998). The basis of the assumed limits was to provide a wide interval for the input parameters to be varied between to supply information to the DM on the impact that changes to these input parameters have on the ranking of the alternatives.

Table 5.23 Upper and lower bounds of input parameters for analysis of Guillen *et al.* (1998) case study

NOTE:
This table is included on page 177 of the print copy of the thesis held in the University of Adelaide Library.

The analysis was undertaken using the program developed as part of this research. Solver was selected as the optimisation method and 50 iterations were undertaken for each pair of alternatives in order to sufficiently vary the starting values. The Euclidean Distance was the chosen distance metric, as it is the most commonly used distance metric.

5.5.3 Results

Deterministic analysis

Combining the values in Table 5.22 results in overall values of 600 for Alternative A, and 598 for Alternatives B, C and D, respectively. The differences between the total values of the four alternatives do not discriminate between any of them, and therefore, further information is required for the DM to make a decision on which alternative to select.

The same results are obtained using the program developed as part of this research, as those by Guillen *et al.* (1998), therefore confirming the validity of the deterministic component of the program described in Section 4.2.

Uncertainty analysis

Guillen et al. (1998) approach

The minimum alteration to each of the CWs needed using the Guillen *et al.* (1998) approach so that the current ranking of Alternatives B, C and D will be reversed with respect to Alternative A is determined using Equation 3.10 (Section 3.2.8). The corresponding values of the robustness index are quite different, namely r(Alt A, Alt B) = 0.003, r(Alt A, Alt C) = 0.20, and r(Alt A, Alt D) = 1.0. From these values, it is evident that there is almost indifference between Alternatives A and B due to the small value of r, and dominance of Alternative A over Alternative D, as r is equal to one, meaning that no changes to the CWs will result in a rank reversal between these two alternatives. The changed CWs required for Alternatives B and C to equal the total value of Alternative A are contained in Table 5.24, as calculated using Equation 3.8 by Guillen *et al.* (1998). For example, as the PV of criterion 1 of Alternative A is greater than that of Alternative B, the change in weight of Criterion 1 required for Alternative B to equal Alternative A is 1 – 1x0.003 = 0.997.

Table 5.24 Changed CWs based on Guillen *et al.* (1998) robustness values

NOTE:
This table is included on page 179 of the print copy of the thesis held in the University of Adelaide Library.

Proposed distance-based uncertainty analysis approach

The results obtained from utilising the proposed distance-based uncertainty analysis approach to determine the changes in the CWs required for one alternative to outrank another are summarised in Table 5.25. The same results were obtained for all of the iterations of the optimisation using different starting values, which indicates that the solution space is not very complex. The attainment of a small Euclidean Distance (d_e) for analysis of the ranking of Alternatives A and B signifies that only small changes in the CWs are required for rank equivalence between the two alternatives, which indicates that the ranking of these two alternatives is not very robust. No feasible changes in CWs were able to be found which would result in rank equivalence between Alternatives A and D, which informs the DM that the ranking of Alternatives A and D is robust.

Table 5.25 Optimised CWs using proposed distance-based uncertainty analysis approach, Guillen *et al.* (1998) case study

NOTE:
This table is included on page 179 of the print copy of the thesis held in the University of Adelaide Library.

One of the benefits of the proposed approach is the ability to identify the input parameters which have the most impact on the ranking of the alternatives. For example, the most critical CWs identified using the proposed approach, as shown in Table 5.25, are CW1 for Alternatives A and B to obtain rank equivalence and CW3 for the ranking of Alternatives A and C to be reversed. These parameters are identified as they have the smallest relative change between the original and optimised value.

An advancement of the Guillen *et al.* (1998) sensitivity analysis methodology is the ability to incorporate the uncertainty in the PVs in the proposed uncertainty analysis approach. The results of utilising the proposed distance-based uncertainty analysis approach to determine the changes in the CWs and PVs required for Alternative B to outrank Alternative A are summarised in Table 5.26. A larger Euclidean Distance is obtained compared to the scenario when only the CWs are incorporated in the uncertainty analysis, however, as can be seen in Table 5.26, only relatively small changes in the input parameters are required for Alternative B to outrank Alternative A when all input parameters are included simultaneously.

Table 5.26 Optimised CWs and PVs using proposed distance-based uncertainty analysis approach, Guillen *et al.* (1998) case study

5.5.4 Discussion

Based on the deterministic results of the simple hypothetical case study assessed by Guillen *et al.* (1998) using the WSM, the DM was not able to differentiate between the four alternatives. Sensitivity analysis was therefore required to be undertaken to provide more information to the

DM on the robustness of the ranking of the alternatives. Investigation of the effect that changes in CWs would have on the ranking of pairs of alternatives by applying the method proposed by Guillen *et al.* (1998) and the distance-based uncertainty analysis approach proposed in this thesis, found that the two methods were in agreement in terms of which of the alternatives would equal the highest ranked alternative with the smallest change in CWs (i.e. Alternative B). However, closer inspection of the results illustrates the shortcomings of the existing sensitivity analysis method, as discussed below.

The changes in CWs required to obtain rank equivalence between pairs of alternatives, determined using the Guillen *et al.* (1998) robustness measures, as shown in Table 5.24, illustrate how the total sum of the changed CWs does not equal the original total sum of the CWs. This is a fundamental flaw in the methodology, as the CWs should be re-normalised following the analysis to maintain the original preference structure.

The Euclidean Distance (d_e) obtained when assessing the impact that uncertainty of the CWs has on the ranking of the alternatives using the proposed approach also provides relatively consistent information with that provided by the Guillen *et al.* (1998) robustness measure (e.g. large r values correspond to large d_e values or non-feasible results). However, optimised CWs for rank equivalence between Alternatives A and D were able to be obtained using the proposed approach, indicating that although large changes are required to achieve rank equivalence, it is incorrect to state that no changes to the CWs can be made to reverse the ranking, as was found by the Guillen *et al.* (1998) method.

The results also show that significantly larger required changes in the CWs are identified by the Guillen *et al.* (1998) method for pairs of alternatives to achieve rank equivalence compared to the proposed approach, due to the CWs having to be changed by the same relative amount in the Guillen *et al.* (1998) method.

Following application of the Guillen *et al.* (1998) sensitivity analysis method, the DM is still left wondering whether any uncertainty in the PVs will also have an impact on the ranking of the alternatives. By undertaking the proposed distance-based uncertainty analysis approach, and varying all input parameters simultaneously between their expected ranges of uncertainty, it is evident that the PVs do have an influence on the ranking of the alternatives and should be considered when performing uncertainty analysis. Based on the results obtained, it would be recommended to the DM that the input parameter values should be reassessed, in particular the ones identified as most critical, prior to re-evaluating the decision problem, as the ranking of the alternatives is not robust.

5.6 WSM, Butler *et al.* (1997) sensitivity analysis & stochastic uncertainty analysis approach

5.6.1 Background to case study

The case study utilised by Butler *et al.* (1997) to illustrate the simulation sensitivity analysis approach described in Section 3.3.2 involved the selection of a site for a coal power plant. Six criteria were used to assess the 13 alternative sites, and the input parameter values used in the analysis are contained in Table 5.27.

5.6.2 Problem formulation

Deterministic analysis

The WSM MCDA approach was utilised by Butler *et al.* (1997) to obtain the total values of each of the alternatives based upon the input data in Table 5.27. Deterministic analysis was also undertaken using the program developed as part of this research to verify the output of the program.

Stochastic uncertainty analysis

The methodology presented in Section 3.3.2 and Butler *et al.* (1997) was utilised to assess the sensitivity of the ranking of the 13 sites to changes in the CWs. A completely random weighting scheme was initially applied to the CWs provided in Table 5.27 by Butler *et al.* (1997) and 5,000 independent trials were undertaken. A rank order simulation was also conducted, where the CWs were generated while preserving their rank order.

A comparison is undertaken between the proposed stochastic uncertainty analysis approach presented in Section 4.4 and the sensitivity analysis approach of Butler *et al.* (1997). The analysis of three scenarios using the proposed uncertainty analysis method were chosen to be performed, as follows:

(i) Varying CWs only with no restrictions on CW rank order;

(ii) Varying CWs only with restrictions on CW rank order; and

(iii) Varying CWs and PVs simultaneously, with no restrictions on CW rank order.

Table 5.27 Input parameter values in example decision problem assessed by Butler *et al.* (1997)

	C1: Cost	C2: Air quality	C3: Site biology	C4: Socio-economic	C5: Impact on fish	C6: Line biology
CW	0.52	0.19	0.17	0.07	0.03	0.02
Performance Values						
Alt1	1.0000	0.7331	0.7400	0.8234	0.7211	0.4375
Alt2	0.9167	0.4088	0.7600	0.7831	0.7548	0.5750
Alt3	0.9333	0.5333	0.9650	0.7380	0.7188	1.0000
Alt4	0.8500	0.9539	0.9300	0.9295	0.7188	1.0000
Alt5	0.9833	0.9211	0.9300	0.7569	0.7188	0.1413
Alt6	0.8333	0.9737	0.9300	0.8748	0.8577	1.0000
Alt7	0.9333	0.0001	0.9300	0.9250	0.5969	0.8500
Alt8	0.9333	0.6833	0.9650	0.9160	0.6328	1.0000
Alt9	0.9000	0.0001	1.0000	0.5360	0.5558	1.0000
Alt10	0.5333	0.8092	0.0001	0.9385	0.7188	0.1957
Alt11	0.4000	0.2700	0.9300	0.6588	0.7188	0.8500
Alt12	0.2833	0.6667	0.9300	0.1450	0.7188	1.0000
Alt13	0.4667	0.8882	0.9000	0.9340	0.9000	0.6750

No information was provided by Butler *et al.* (1997) on the uncertainty of the input parameter values, therefore, uniform distributions were selected to represent the uncertainty in the input parameter values and the assumed upper and lower limits for the CWs and the PVs of the two highest ranked alternatives are contained in Table 5.28. The upper and lower limits of the remaining alternatives have not been included in Table 5.28, however, they have been assigned based on the same assumption as those of Alternatives 4 and 5 (i.e. that the upper limit (UL) is the original PV + 0.2 and the lower limit (LL) is the original PV – 0.2, but it should be noted that the LL cannot be less than zero). An example of the uniform distributions utilised is contained in Figure 5.1.

Table 5.28 Upper and lower limits for the input parameters used to define the uniform distributions for the proposed stochastic uncertainty analysis, Butler *et al.* (1997) case study

Criterion	CWs		PVs Alt 5		PVs Alt 4	
	LL	UL	LL	UL	LL	UL
C1	0.42	0.62	0.7833	1.1833	0.6500	1.0500
C2	0.09	0.29	0.7211	0.1211	0.7539	1.1539
C3	0.07	0.27	0.7300	1.1300	0.7300	1.1300
C4	0.01	0.17	0.5569	0.9569	0.7295	1.1295
C5	0.01	0.13	0.5188	0.9188	0.5188	0.9188
C6	0.01	0.12	0.0413	0.3413	0.800	1.200

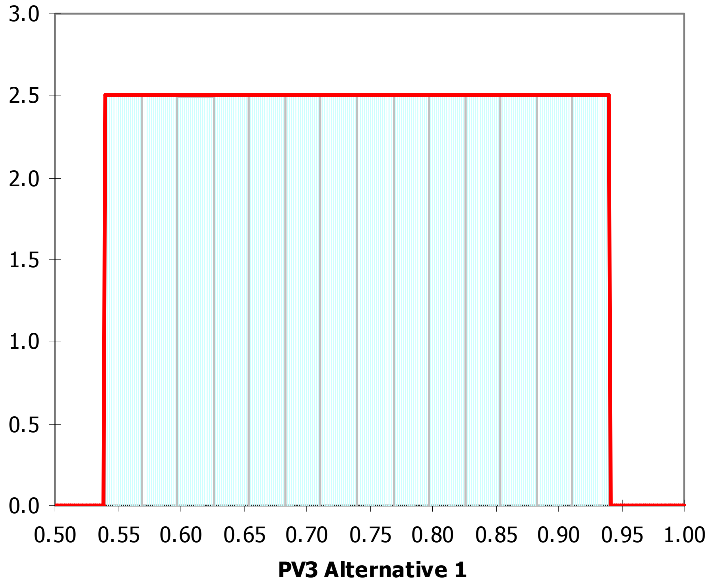


Figure 5.1 Uniform distribution for PV1 Alternative 3, Butler *et al.* (1997) case study

Analyses were performed using the program developed as part of this research, as described in Section 4.6.2. Latin Hypercube Sampling was utilised to sample the input parameters from their distributions and 5,000 simulations were undertaken so as to be able to compare the results with the analysis undertaken by Butler *et al.* (1997).

5.6.3 Results

Deterministic analysis

Combining the input parameter values in Table 5.27 results in overall total values as presented in Figure 5.2 and Table 5.29. From these results it is evident that the DM would prefer Alternative 5, which has an overall total value of 0.9218. However, it is clearly demonstrated in Figure 5.2 that there is little difference in the total values of the highest ranked alternatives. The DM therefore requires further information to enable a confident selection between the alternatives to be made or to be provided direction in identifying any further information that is required.

The same total values and associated rankings are obtained using the program developed as part of this research, as those by Butler *et al.* (1997), thereby validating the deterministic portion of the program described in Section 4.6.2.

NOTE:
This figure is included on page 185 of the print copy of the thesis held in the University of Adelaide Library.

Figure 5.2 Total values of alternatives obtained using WSM for the Butler *et al.* (1997) case study

Table 5.29 Total values and associated rank order obtained using WSM with input parameter values provided by Butler *et al.* (1997)

NOTE:
This table is included on page 186 of the print copy of the thesis held in the University of Adelaide Library.

Stochastic uncertainty analysis

Butler et al. (1997) approach

The results of the stochastic analysis undertaken by Butler *et al.* (1997) with completely random CWs are contained in Table 5.30. The output consists of the minimum, maximum, mean and standard deviation of the ranks of each of the alternatives obtained from the 5,000 simulations. Butler *et al.* (1997) found that Alternative 6 was the top ranked alternative in 50% of the simulations and that Alternatives 4, 8 and 13 also performed well. When comparing the mean rank with the original rank order, Alternatives 5 and 13 display the greatest change (i.e. Alternative 5 is originally the highest rank alternative when deterministic analysis is undertaken, whereas when the CWs are altered on a random basis, Alternative 5 has a mean rank of 7.22), while only minor differences are noted for the remaining alternatives. This result demonstrates that the ranking of Alternative 5 is dependent on the weights that are assigned to the criteria.

Table 5.30 Results of stochastic analysis undertaken by Butler *et al.* (1997) with completely random CWs

The range of possible rankings for the alternatives is much narrower when the rank order of criteria is imposed in the simulation. As stated above, the random simulation results of Butler *et al.* (1997) suggest that Alternative 13 is a good choice as a site for a power plant. However, once rank order is enforced in the simulation, the performance of Alternative 13 drops considerably. Butler *et al.* (1997) found that Alternatives 4, 5 and 6 were consistent top performers. Alternatives 1, 3 and 8 also appeared to be superior when compared with the remainder of the alternatives. Note that these results have not been included in the thesis as they were not presented by Butler *et al.* (1997).

Proposed stochastic uncertainty analysis approach

The results from utilising the proposed stochastic uncertainty analysis approach and varying the CWs simultaneously are contained in Table 5.31. A number of scenarios were undertaken in an attempt to emulate the analysis undertaken by Butler *et al.* (1997). The random simulation results using the proposed stochastic uncertainty analysis suggest that Alternatives 4 and 6 are good choices as sites for a power plant. A comparison of the mean ranks obtained by simultaneously varying the CWs randomly using the Butler *et al.* (1997) approach and the proposed stochastic uncertainty analysis approach is contained in Figure 5.3. From this figure it is evident that similar results are obtained, thereby verifying the stochastic component of the program developed as part of this research.

NOTE:
This figure is included on page 188 of the print copy of the thesis held in the University of Adelaide Library.

Figure 5.3 Comparison of mean ranks obtained by using the Butler *et al.* (1997) and proposed stochastic uncertainty analysis approach when randomly varying the CWs

Once the range within which the CWs are able to be varied is reduced and rank order of the CWs is enforced in the simulation, the performance of the alternatives changes considerably, as can be seen from the results presented in Table 5.31[9]. When the CW rank order is maintained, the rank order of the alternatives preserves the original rank order obtained using deterministic CWs (Table 5.29), with the exception of rank reversals of Alternatives 6 and 4 (ranked 2nd and 3rd) and Alternatives 13 and 9 (ranked 9th and 10th), as can be seen in Figure 5.4. These results confirm the results obtained by Butler *et al.* (1997).

[9] It should be noted that the proposed stochastic uncertainty analysis approach is able to include correlations of the CWs, but it is not able to specifically incorporate the constraint of maintaining the CW rank order. The simulation is therefore undertaken as would occur with randomly generating the CWs, and then the sets of CWs that do not maintain the rank order are discarded from the analysis (refer to Section 4.4.2).

NOTE:
This figure is included on page 189 of the print copy of the thesis held in the University of Adelaide Library.

Figure 5.4 Comparison of mean ranks for various scenarios using the proposed stochastic uncertainty analysis approach, Butler *et al.* (1997) case study

Even though the analysis has been constrained by the rank order of the CWs, it is still difficult to distinguish between Alternatives 4, 5 and 6 (the highest ranked alternatives), therefore further analysis would be required for the DM to select a preferred alternative. In addition, any uncertainty that may be present in the PVs has not been considered.

As has been established throughout the thesis, it is also important to consider the uncertainty in the PVs when assessing a decision problem using MCDA, which is not able to be undertaken with the Butler *et al.* (1997) approach. The results of utilising the proposed stochastic uncertainty analysis approach and simultaneously varying the CWs and PVs is contained in Table 5.32. The output of the analysis using the program developed as part of this research also provides the DM with the range of possible values that each alternative may attain, as illustrated in Figure 5.5. Figure 5.5 also illustrates the probability of each alternative obtaining a total value less than or equal to any variable value. An alternative method for presenting the results of the analysis is provided in Table 5.33, where the DM can see the probability that an alternative will obtain a ranking of 1 – 13.

Table 5.31 Results of the proposed stochastic uncertainty analysis approach altering CWs only, Butler *et al.* (1997) case study

NOTE:
This table is included on page 190 of the print copy of the thesis held in the University of Adelaide Library.

Table 5.32 Results of stochastic analysis with random CWs and PVs, Butler *et al.* (1997) case study

NOTE:
This table is included on page 191 of the print copy of the thesis held in the University of Adelaide Library.

Similar results are obtained when the uncertainty in the PVs are included in the uncertainty analysis. From all of the results presented, it is evident that it is equally probable that Alternatives 4, 5, 6 and 8 are contenders to be selected as the 'best' alternative when all uncertainty in the input parameters is taken into consideration. Further information may therefore be required by the DM to enable uncertainty in the input parameters to be reduced. The benefit of the proposed approach is that the most critical input parameters are able to be identified using the significance analysis, as discussed in Section 4.4.4. The results of the significance analysis for Alternative 5 are shown in Figure 5.6 where it can be seen that the most critical input parameters are the PV of criterion 1, the CW of criterion 6 and the PV of criterion 2, which demonstrates that PVs can have an impact on the ranking of the alternatives.

Table 5.33 Probability matrix that Alternative *m* obtains rank *r*, Butler *et al.* (1997) case study

NOTE:
This table is included on page 192 of the print copy of the thesis held in the University of Adelaide Library.

NOTE:
This figure is included on page 193 of the print copy of the thesis held in the University of Adelaide Library.

Figure 5.5 Cumulative frequency distribution for the results of alternatives when CWs and PVs are simultaneously varied, Butler *et al.* (1997) case study

NOTE:
This figure is included on page 193 of the print copy of the thesis held in the University of Adelaide Library.

Figure 5.6 Spearman rank correlation coefficients for Alternative 5, when CWs and PVs are simultaneously varied, Butler *et al.* (1997) case study

5.6.4 Discussion

The deterministic results of the decision problem to locate a site for a coal power plant would suggest to the DM that Alternative 5 was the 'best' alternative. Incorporating random uncertainty in the CWs into the analysis suggests to the DM that the ranking of Alternative 5 is not robust and further analysis is required. Obtaining similar results for the proposed stochastic uncertainty analysis approach as those obtained by

Butler *et al.* (1997) allow confidence to be placed in the program that has been developed as part of this research.

Based on the results of the simulation analysis by Butler *et al.* (1997) and the proposed stochastic uncertainty analysis approach, when CW rank order is imposed on the analysis, it appears that the original recommendation of Alternative 5 cannot be made confidently, as there is a number of alternatives that are also in contention to be the highest ranked alternative. Similar results are also obtained when the uncertainty of PVs is incorporated in the analysis. Further analysis would therefore be required by the DM to differentiate between these alternatives and to make a final selection, which can be based upon the results of the significance analysis.

5.7 Summary

Five existing uncertainty analysis approaches, including four deterministic and one stochastic approach, have been compared to the proposed uncertainty analysis approaches in this chapter. Case studies previously presented to demonstrate the existing uncertainty analysis approaches have been utilised. The main findings of this chapter are:

- The program developed as part of this research has been verified by comparing the results obtained from existing sensitivity analysis methods; and

- More information is provided to the DM when using the proposed uncertainty analysis approaches and direction is provided for further clarifying the input data to enable more analysis if required using the proposed approaches.

The case study applications demonstrate the versatility of the proposed approaches, as they can be applied to any decision problem and can be utilised with multiple MCDA approaches. It should be noted that none of the case studies which the existing uncertainty analysis method was applied to had multiple DMs involved in the decision analysis, therefore, another benefit of how the proposed methods can be applied in this instance was not able to be demonstrated.

Chapter 6
Published Journal Papers

Five journal papers and three conference papers have been written, submitted and accepted for publication on all aspects of the proposed MCDA uncertainty analysis approaches, which have been described in Chapter 4. The focus of this chapter is on the journal papers and a summary of these papers is contained in Table 6.1, including title of journal paper, title of journal, contents of paper, case studies utilised and milestone dates. The publication process has been valuable for obtaining feedback on the proposed approaches from the reviewers and verification of the validity of the research that has been undertaken, as the papers have all been accepted for publication in a range of reputable international journals.

The choice of journals was generally based on the desire to expose the techniques to fields in which MCDA is not a 'traditional' decision analysis methodology (e.g. Journal of Water Resources Planning and Management), but to also present the methodologies to the MCDA research community (e.g. Journal of Multi-Criteria Decision Analysis).

This chapter contains a section on each of the published (and accepted for publication) papers, which includes:

- A statement of authorship;
- A summary of the aims and findings; and
- How the papers relate (i) to each other and (ii) to the research presented in Chapters 2 to 5 of the thesis.

The journal papers are contained in Appendix F.

Table 6.1 Summary of journal papers (published or accepted for publication)

Title of Paper	**Journal**	**Summary of Content**	**Example Decision Problem(s)**	**Milestone Dates**
A Distance-Based Uncertainty Analysis Approach to Multi-Criteria Decision Analysis for Water Resource Decision-making	Journal of Environmental Management	- distance based uncertainty analysis approach - vary CWs simultaneously - GRG2 optimisation method - compare with existing sensitivity analysis techniques - WSM MCDA approach is used in example decision problems	Hypothetical numerical example by Guillen *et al.* (1998) Irrigation planning alternatives by Raju *et al.* (2000) Groundwater management problem by Duckstein *et al.* (1994)	Submitted: 27 April 2004 Comments received from reviewers: 15 Sept. 2004 Submitted in revised form: 20 Oct. 2004 Accepted: 7 June 2005
New Distance-based Uncertainty Analysis Approach to Multi-Criteria Decision Analysis	European Journal of Operational Research	- distance based uncertainty analysis approach - vary CWs and PVs simultaneously - GRG2 and GA optimisation methods - PROMETHEE MCDA approach is used in example decision problem	Selection of locations for small hydroelectric plants by Mladineo *et al.* (1987)	Submitted: 16 Nov. 2004 Comments received from reviewers: 11 May 2005 Submitted in revised form: 1 Nov 2005 Accepted: TBA

Title of Paper	**Journal**	**Summary of Content**	**Example Decision Problem(s)**	**Milestone Dates**
Reliability-based Approach to MCDA for Water Resources	Journal of Water Resources Planning and Management	- stochastic uncertainty analysis approach - vary CWs and PVs simultaneously - WSM MCDA approach is used in example decision problems	Irrigation planning alternatives by Raju *et al.* (2000) Sustainable water resource allocation study by Fleming (1999)	Submitted: 22 Aug. 2003 Comments received from reviewers: 7 Jan. 2004 Submitted in revised form: 12 Feb. 2004 Accepted: 15 April 2004
Incorporating Uncertainty in the PROMETHEE MCDA Method	Journal of Multi-Criteria Decision Analysis	- stochastic uncertainty analysis approach - vary CWs and PVs simultaneously - PROMETHEE MCDA approach	Renewable energy scenario selection by Georgopoulou *et al.* (1998)	Submitted: 17 Feb. 2004 Comments received from reviewers: 8 Nov. 2004 Submitted in revised form: 11 Nov. 2004 Accepted: 30 Nov. 2004
Distance-Based and Stochastic Uncertainty Analysis for Multi-Criteria Decision Analysis in Excel using Visual Basic for Applications	Environmental Modelling & Software	- distance-based and stochastic uncertainty analysis approach - vary CWs and PVs simultaneously - demonstrate program developed to undertake uncertainty analysis methods	Sustainable water resource allocation study by Fleming (1999)	Submitted: 6 Jan. 2005 Comments received from reviewers: 11 May 2005 Submitted in revised form: 26 July 2005 Accepted: 5 Aug. 2005

6.1 Publication 1

Title: A Distance-Based Uncertainty Analysis Approach to Multi-Criteria Decision Analysis for Water Resource Decision-making

Authors: Hyde, K.M., Maier, H.R., Colby,C.B.

Publication details: Journal of Environmental Management, 2005, Vol. 77, Iss 4, pp 278-290

6.1.1 Statement of authorship

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Developed methodology, performed analysis, interpreted data, wrote manuscript and acted as corresponding author.

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Maier, H.R.

Aided in development of methodology, data interpretation and manuscript evaluation.

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Manuscript evaluation.

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6.1.2 Discussion

This journal paper is an extension of the conference paper:

> Hyde, K.M., Maier, H.R., Colby, C.B., (2003), "The Applicability of Robustness Measures to Water Resources Decision-making", MODSIM Conference Proceedings, International Congress on Modelling and Simulation, Integrative Modelling of Biophysical, Social and Economic Systems for Resource Management Solutions, Townsville, Australia, July 14 – 17.

An invitation was received from the conference session organiser to be involved in a special edition of the Journal of Environmental Management. Publication in the journal was seen as an opportunity to present the research undertaken to an audience that may not typically be aware of MCDA. Of the 133 'applications' of MCDA contained in Appendix A, only 10 of the 'applications' have been published in the Journal of Environmental Management, as is summarised in Table 6.2. A basic summary of MCDA is therefore presented in the journal paper, as some readers may not be familiar with the methodology and the process involved with undertaking MCDA.

The majority of multi-criteria methods require the definition of quantitative weights for the criteria, in order to assess the relative importance of the different criteria (as discussed in Section 2.5.7 of the thesis). Publication 1 highlights how CWs are subjective, ambiguous and imprecise in nature and that they should not be utilised in MCDA as well defined constants, which is often the case. Despite the uncertainty in the CWs, a review of the literature has identified that many applications of MCDA do not undertake sensitivity analysis to determine the impact that changes in the CWs have on the ranking of the alternatives (see Appendix A and Table 6.2). If sensitivity analysis is undertaken, it is generally carried out by arbitrarily varying a few of the input parameters individually. Even though numerous sensitivity analysis methods are presented in the literature, it is rare that a formal approach is applied. The implication is that it may be difficult for a consensus to be reached if uncertainty exists in the outcomes of the decision analysis. Sensitivity analysis has therefore been identified as a shortcoming of the MCDA approach. To demonstrate this shortcoming, a summary of some of the existing sensitivity analysis methods and their limitations are provided in

Publication 1, while a more extensive review is contained in Chapter 3 of the thesis.

Table 6.2 Examples of applications of MCDA in the Journal of Environmental Management

Reference	Application	MCDA Technique	Sensitivity Analysis
Almasri and Kaluarachchi (2005)	Optimal management of nitrate concentration of groundwater	Importance order of criteria (IOC)	Not undertaken
Khalil *et al.* (2005)	Selection of hydrothermal pre-treatment conditions of waste sludge destruction	PROMETHEE & GAIA	Not undertaken
Herath (2004)	Incorporating community objectives in improved wetland management	AHP	Change CWs (not a formal process)
Huth *et al.* (2004)	Assess rain forest growth results	MAVT and exploratory analysis	Not undertaken
Randall *et al.* (2004)	Compare alternatives for the long-term management of surplus mercury	AHP	Change CWs (not a formal process)
Tzeng (2002)	Strategies for improving air quality in Tapei	CP (VIKOR) and TOPSIS	CW stability intervals
Qureshi and Harrison (2001)	Compare Riparian vegetation options	AHP	Not undertaken
Martin et al. (2000)	Development of leasable minerals in a forest	MAVF	Not undertaken
Hokkanen *et al.* (2000)	Cleaning polluted soil	SMAA-2	SMAA-2
Ozelkan and Duckstein (1996)	Identify the satisfactory water resources projects being designed at the Austrian part of the Danube	PROMETHEE I, II, GAIA, MCQA I, II, III, CP, CGT	Not undertaken

The aim of Publication 1 is to present an uncertainty analysis methodology that is able to overcome the limitations of existing sensitivity analysis methods. A simplified version of the distance-based uncertainty analysis approach described in Section 4.3 of the thesis is introduced in Publication 1, which simultaneously varies the CWs within their expected range of uncertainty. The purpose of the methodology is to provide more information to the DM on the robustness of the ranking of the alternatives than is provided by existing sensitivity analysis techniques available for application with MCDA. In Publication 1, the distance-based uncertainty analysis approach is compared to two existing sensitivity analysis methods (described in Section 3.2.6 and Section 3.2.8 of the thesis) by utilising three case studies from the literature. The results contained in Publication 1 demonstrate the benefits of the proposed method, including how the critical criteria can be identified, which may direct any further analysis of the CWs provided by the actors. By incorporating the uncertainty in all of the CWs in the analysis, the DM and actors can be confident of the results that are obtained.

The limitation of the methodology presented in Publication 1 is that doubt surrounding the ranking of the alternatives remains, as uncertainty in the criteria PVs is not taken into consideration. In addition, a non-global optimisation method is utilised to solve the objective function, which means that in a complex decision space, the DM cannot be certain that the minimum distance-metric has been found.

6.2 Publication 2

Title: New Distance-based Uncertainty Analysis Approach to Multi-Criteria Decision Analysis

Authors: Hyde, K.M., Maier, H.R., Colby,C.B.

Publication Details: European Journal of Operational Research, Under Review, 2006[10].

[10] This paper is still currently under review, and as detailed in Table 6.1, changes addressing the reviewers concerns have been completed and the revised paper has been sent back to the editor of the journal.

6.2.1 Statement of authorship

<u>Hyde, K.M. (Candidate)</u>

Developed methodology, performed analysis, interpreted data, wrote manuscript and acted as corresponding author.

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<u>Maier, H.R.</u>

Aided in development of methodology, data interpretation and manuscript evaluation.

Signed: ………………………………………. Date: …………………….

<u>Colby, C.B.</u>

Manuscript evaluation.

Signed: ………………………………………. Date: …………………….

6.2.2 Discussion

The results presented in Publication 1 demonstrate the benefits of considering the uncertainty of the CWs simultaneously in the uncertainty analysis. However, it is evident from the literature (as discussed in Section 2.5.5 of the thesis) that the criteria PVs do not enter the MCDA as well defined constants and should therefore be considered in the uncertainty analysis. It is also recognised in the literature that the PVs assigned to the criteria, through expert judgment or models, can have an impact on the ranking of the alternatives. The review of the existing deterministic sensitivity analysis methods presented in Section 3.2 of the thesis demonstrates that few methods have been developed which assess the uncertainty of the PVs. A summary of these findings is also presented in Publication 2.

The aim of Publication 2 is, therefore, to extend the methodology presented in Publication 1 by incorporating all input parameters in the proposed distance-based uncertainty analysis approach, which is a significant contribution to the MCDA field. The benefits of incorporating all of the input parameters in the uncertainty analysis, compared to only the CWs (as presented in Publication 1), are discussed in Publication 2. The case study aids in demonstrating the DMs' enhanced understanding of the robustness of the ranking of the alternatives when the proposed approach is utilised.

Another aim of Publication 2 is to demonstrate how an evolutionary algorithm, such as genetic algorithms (GAs), can be utilised to solve the objective function of the proposed distance-based uncertainty analysis approach. The rationale behind utilising an evolutionary algorithm is that it is a global optimisation technique, meaning that it searches from a population of points, compared to an optimisation method such as GRG2, which requires the use of a gradient fitness function and therefore may become trapped in local optima. The two suggested optimisation methods (GRG2 and GA) described in Section 4.3.3 of the thesis are compared in Publication 2 using a case study from the literature. The results demonstrate that for simple solution spaces the GRG2 is able to find the minimum distance metric, but when all of the input parameters are included in the uncertainty analysis, the GA is more reliably able to arrive at a minimum robustness measure.

In addition, the distance-based uncertainty analysis methodology was demonstrated using the outranking PROMETHEE MCDA technique, which illustrates the applicability of the proposed methodology for various MCDA techniques, as the methodology was applied using the WSM MCDA approach in Publication 1. Outranking MCDA methods, such as PROMETHEE, require additional input parameters to be defined, including generalised criterion functions and their associated threshold values (see Section 2.5.8 of the thesis). These functions and values are also difficult to assign and may therefore result in further uncertainty in the decision analysis. The benefit of applying the proposed approach when utilising the PROMETHEE MCDA technique is that Level 1 generalised criterion functions are assigned to each of the criteria to enable the outranking to be undertaken (and therefore the thresholds are not required to be defined). It is considered that the uncertainty in the input parameters is adequately taken into consideration by allowing the parameters to vary between an uncertainty interval that is defined by the DM, experts and / or actors. This has been verified in Publication 2 by comparing the results of the decision analysis in the case study from the literature with the results of utilising the proposed approach.

Despite the benefits, it is acknowledged that there are some difficulties associated with the proposed distance-based uncertainty analysis methodology presented in Publications 1 and 2, including:

- Pair-wise comparisons of alternatives are required to be undertaken, which can be quite time consuming if it is deemed necessary that all comparisons need to be performed, and especially if there are a considerable number of alternatives;

- The analysis can only be undertaken for one actor's set of CWs at a time. If there are a large number of actors involved in the decision analysis, and in particular they are all uncertain of the CWs they have provided, this methodology may not be appropriate;

- The GRG2 optimisation method initially presented in Publication 1 (but also utilised in Publication 2) is not a global optimisation method, therefore, it is not appropriate for use with complex decision problems (such as when there are a large number of criteria), as it is not guaranteed that a near-global solution will be obtained; and

- The GA has been proposed as an alternative optimisation method to the GRG2 in Publication 2. The benefit of the GA is that it is able to arrive at near-global optimum solutions to the objective function, however, the results are dependent on the GA specific input parameters and it may be difficult for people who are not familiar with GAs to assign these parameters.

6.3 Publication 3

Title: Reliability-based Approach to MCDA for Water Resources

Authors: Hyde, K.M., Maier, H.R., Colby,C.B.

Publication Details: Journal of Water Resources Planning and Management, 2004, Vol 130, Iss 6, pp 429-438

6.3.1 Statement of authorship

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Maier, H.R.

Aided in development of methodology, data interpretation and manuscript evaluation.

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6.3.2 Discussion

Publications 1 and 2 have presented and demonstrated the benefits and shortcomings of applying the proposed distance-based uncertainty analysis methodology, which is able to incorporate the uncertainty of all of the input parameters simultaneously and provide a robustness measure which indicates the changes in input parameters required for a reversal in ranking of two alternatives. Publication 3 sought to overcome the limitations of the distance-based approach with a stochastic uncertainty analysis methodology (presented in Section 4.4 of the thesis), which also aims to extend the existing stochastic sensitivity analysis methods discussed in Section 3.3 of the thesis.

The Journal of Water Resources Planning and Management was selected for submission of Publication 3, as there have been limited 'applications' of MCDA published in this journal, as shown in Table 6.3. Publication in the Journal of Water Resources Planning and Management also enables exposure of the water resources community to the proposed new approach.

Table 6.3 Examples of applications of MCDA in the Journal of Water Resources Planning and Management

Reference	Application	MCDA Technique	Sensitivity Analysis
Abrishamchi *et al.* (2005)	Urban water supply alternatives in Iran	CP	Two sets of CWs for each DM and two sets of PVs
Rossi *et al.* (2005)	Drought mitigation measures	NAIADE	Not undertaken
Flug *et al.* (2000)	Resources and flow alternatives presented in the EIS for the Glen Canyon Dam	Weighted Average Method	Change CWs
Netto *et al.* (1996)	Design a long-term water supply system	ELECTRE III	Two sets of CWs
Duckstein *et al.* (1994)	Groundwater resources management problem	CP, ELECTRE III, MAUT, UTA	Not undertaken
Tecle *et al.* (1988)	Wastewater management option	CP, ELECTRE I, cooperative game theory	Two sets of CWs for CP and 21 pairs of threshold values for ELECTRE I

As with Publications 1 and 2, Publication 3 also discusses the uncertainty present in the MCDA process and the limitations of existing sensitivity analysis methods to overcome these aspects of uncertainty. The main aim of Publication 3 is to introduce a stochastic uncertainty analysis approach that is able to incorporate all input parameters in the uncertainty analysis simultaneously (as described in Section 4.4 of the thesis). The principal difference between the distance-based uncertainty analysis approach and the stochastic uncertainty analysis approach is that all of the alternatives are included in the uncertainty analysis at the same time when the stochastic approach is utilised, compared to pair-wise comparison of alternatives in the distance-based approach. In addition, the purpose of the stochastic approach is to determine the most probable ranking of each of the alternatives based upon the expected distribution of possible input values for each CW and PV.

Conflict resolution is an important part of the group decision making process since it aids the smooth transition towards a compromise solution. The effective and efficient engagement of actors in the decision analysis process is an emerging issue and actors who are involved need to be satisfied that their input to the process will yield returns. One of the most significant contributions of the stochastic uncertainty analysis approach presented in Publication 3 is the ability to fit distributions to the CWs elicited from the actors, which enables all actors' preferences to be included in the decision analysis. In addition, the CWs obtained from the fitted distribution are considered to be representative of the preferences of all stakeholders, rather than just the actors involved in the decision analysis. This is an advancement of existing methods, which generally only use an average of the CWs when multiple actors are involved in the decision analysis. As discussed in Section 2.5.7 of the thesis, this results in a large loss of information and may lead to potential difficulties in gaining a consensus on the decision outcomes. The benefits of fitting distributions to the CWs are demonstrated in Case Study 2 in Publication 3.

Another important contribution of the stochastic uncertainty analysis approach, as shown in Publication 3, is the ability to identify the critical input parameters by using the Spearman Rank Correlation Coefficient. In the situation where one alternative cannot be considered 'better' than another alternative, when uncertainty in all of the input parameters is taken into consideration, the critical input parameters can direct further studies, if required, to reduce the uncertainty on the input parameter values that are deemed to be of most importance to the ranking of the alternatives.

6.4 Publication 4

Title: Incorporating Uncertainty in the PROMETHEE MCDA Method

Authors: Hyde, K.M., Maier, H.R., Colby,C.B.

Publication Details: Journal of Multi-Criteria Decision Analysis, 2003, Vol 12, Iss 4-5, pp 245-259

6.4.1 Statement of authorship

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Data interpretation and manuscript evaluation.

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Colby, C.B.

Manuscript evaluation.

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6.4.2 Discussion

Publication 4 also presents the stochastic uncertainty analysis approach contained in Publication 3, however, as with Publication 2, the main aim is to demonstrate the use of the approach with an alternative MCDA technique, PROMETHEE. The proposed extension to the outranking PROMETHEE MCDA methodology is illustrated by applying it to a case study from the literature. The benefits of incorporating all input parameter values in the analysis is demonstrated by only performing the uncertainty analysis considering uncertainty in the CWs and then comparing the results to when all input parameters are incorporated in the analysis.

A limitation of Publication 4 is that it does not compare the deterministic results when various generalised criterion functions are utilised and when only level 1 generalised criterion functions are assigned in the proposed approach.

6.5 Publication 5

Title: Distance-Based and Stochastic Uncertainty Analysis for Multi-Criteria Decision Analysis in Excel using Visual Basic for Applications

Authors: Hyde, K.M., Maier, H.R.

Publication Details: Environmental Modelling & Software, 2005, In Press

6.5.1 Statement of authorship

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Maier, H.R.

Data interpretation and manuscript evaluation.

Signed: ……………………………………….. Date: ……………………..

6.5.2 Discussion

A culmination of the work that has been undertaken during the research project is incorporated in this journal paper, which has been accepted for publication in Environmental Modelling & Software. The main aim of Publication 5 is to present the program that has been developed to implement the uncertainty analysis methods described in Publications 1 to 4. A description of the program and its capabilities is also contained in Section 4.6 of the thesis. The program has been designed to provide practical support for public decisions in conflict situations where environmental and socio-economic effects are to be considered e.g. to aid the decision making process for MCDA problems.

Many DSSs have been developed, as summarised in Publication 5 and in Appendix C of the thesis. Trial versions of a large number of existing MCDA computer packages are available for download from the internet, however, the purchase of the software of some of the most popular MCDA methods is quite prohibitive for people who may not be familiar with MCDA and if uncertainty exists about which method is most applicable for the particular decision problem(s) to be assessed. In addition, the majority of the software presented in Appendix C only includes one MCDA technique, therefore, if multiple techniques are required to be utilised, purchasing existing software packages becomes a very expensive process. Also, if people would like to use different methods, they have to familiarise themselves with different software environments. These factors may limit the uptake of the MCDA process by potential new users. Taking these factors into consideration, one of the aims of the program developed as part of this research was to provide a user-friendly program that has the ability to undertake MCDA utilising a selection of MCDA techniques.

The use of the program is demonstrated by applying it to a case study from the literature. A limitation of the program presented in Publication 5 (and in Section 4.6 of the thesis) is it was not developed according to a formal software engineering process. It should be noted that the validity of the program has been demonstrated in Chapter 5 of the thesis, however, it is envisaged that significant improvements in the efficiency, reliability, maintainability and extendability of the program could be achieved by re-writing the software.

Features of the program (and hence the methodology) that could be further investigated and developed to:

- Include more MCDA techniques. Based on the review of the literature (see Section 2.5.4) it is evident that more research also needs to be undertaken on the classification of MCDA approaches and providing information to DMs and actors such that the approaches do not appear as 'black box' approaches, leaving them to question the results of the analysis;
- Enable distributions and uncertainty intervals to be assigned to the generalised criterion function input parameters when the PROMETHEE method is utilised, as the current methodology only used Level 1 generalised criterion functions (which do not require any specific parameters to be assigned);
- Allow CW rank order to be incorporated in the sampling of the distributions in the stochastic uncertainty analysis approach (instead of after the sampling has occurred);
- Allow the DMs to specify information such as: (i) the relative importance of two criteria must remain constant, (ii) some CWs are more likely to change than others, and (iii) a certain CW is more likely to increase than to decrease; and
- Include alternative optimisation techniques in the distance-based uncertainty analysis approach.

Chapter 7
Conclusions and Recommendations

This chapter details the major findings of the thesis and discusses the conclusions relating to the objectives of the thesis detailed in Section 1.3.

7.1 Decision theory

Typically, Benefit Cost Analysis (BCA) has been used to evaluate water resource management decision alternatives in Australia, which is designed to examine the economic efficiency of a project. Recent conceptual thinking about sustainability in developing water resources relates to handling risk and preventing adverse conditions. However, where performance measures other than cost and risk are considered to be important, multi-criteria decision analysis (MCDA) may be regarded as a preferable methodology, as it is a complementary tool to other methodologies such as BCA, life cycle assessment (LCA) and ecological footprint (EF).

It has also become necessary to recognise all significant interests and consider a full range of options for sound, comprehensive water management to be achieved. Stakeholder participation cannot succeed in improving decision making if it occurs in an unstructured and ad hoc fashion and it is widely recognised that MCDA is able to provide the required structure to overcome this problem. Beyond MCDA's numerical output, insight gained from the process of working through the method is a primary benefit of MCDA. Such insight becomes an invaluable negotiating aid towards a widely acceptable, and accepted, compromise solution.

As discussed in Chapter 2 of the thesis, the purpose and strength of formal decision-aiding procedures, such as MCDA methods, is to help improve the quality of decisions by:

- Making decision making more explicit, rational and efficient;
- Providing a framework which improves the DM's understanding of a decision problem and the trade-offs involved between competing objectives;

- Facilitating the integration of a wide range of viewpoints and expertise from a variety of disciplines by helping stakeholders articulate and apply their values to the decision problem rationally and consistently, as it is an intuitively appealing decision making process which DMs find logical;
- Contributing uniformity to problem structure and evaluation perspective so that everyone entrusted to make choices or decisions can be assured of viewing all relevant facets of a problem;
- Providing a diversity of evaluation techniques which enable the use of quantitative and qualitative data in any measurement units;
- Providing a framework for integrating and synthesising large amounts of complex evaluative data;
- Making the problem very simple or elaborate to suit a particular application and the needs of the DMs;
- Increasing confidence in the decision; and
- Documenting the decision analysis process, making the decision process more transparent.

Due to these benefits, MCDA was considered to be the formal decision analysis approach that would be most applicable to assist in assessing water resource decision problems.

7.2 MCDA process

The first objective of the thesis was to *summarise the current knowledge regarding the various aspects of the MCDA process and identify any limitations of that process*. The MCDA process generally follows the sequence of: (1) identifying DMs (final decision makers), actors (people involved in the decision analysis process) and stakeholders (anyone who might be affected by the decision); (2) selecting criteria; (3) defining alternatives; (4) choosing an MCDA technique; (5) weighting the criteria; (6) assessing the performance of values against the criteria; (7) transforming the criteria performance values to commensurable units, if required; (8) applying the selected MCDA technique; (9) performing sensitivity analysis; and (10) making the final decision.

An extensive review of the literature regarding MCDA was undertaken as part of this research on each stage of the MCDA process, which is contained in Section 2.5 of the thesis. Despite the recognised benefits of undertaking MCDA to assess a decision problem, a number of limitations of the MCDA process were identified during the review of the literature, including:

- Confusion remains as to which MCDA technique is most applicable for a particular decision making situation. Perhaps as a result of this, justification of why certain MCDA methods have been applied to a particular case study is predominantly not disclosed in the literature. In addition, no consensus has been reached on whether one MCDA method produces different results to another MCDA method when assessing the same decision problem;

- A number of methods for eliciting CWs are also available and there is no consensus on whether one method is more appropriate in a given decision situation. In addition, in a group decision making situation, preference values of multiple DMs (i.e. CWs) are generally averaged or aggregated to enable a final decision to be obtained, which results in a significant loss of information and potential difficulty in reaching a consensus; and

- Sensitivity analysis is either not undertaken or a formal method is not applied. It is generally only the CWs that are altered arbitrarily and uncertainty present in the PVs assigned to the criteria is not considered.

Sensitivity analysis is a required element of the MCDA process, as it has been established that the input parameters to MCDA techniques are subjective and incomplete. CWs are subjective because personal opinions vary on which criteria are important and which are not. PVs are incomplete because knowledge and pertinent information on the criteria may be scarce, subject to change, or unknown. Numerous formal sensitivity analysis methods have been presented in the literature and a selection of deterministic and stochastic sensitivity analysis methods are explored in Chapter 3. It is concluded that these methods are unable to adequately take uncertainty in the input parameters into account. The lack of adequate sensitivity analysis methods is considered a fundamental shortcoming of the MCDA process, which is why it is the focus of the research work that has been undertaken.

7.3 Proposed MCDA uncertainty analysis approaches

The second objective of this thesis was to *develop an improved decision making approach that addresses the major shortcomings of the existing MCDA process identified in the literature*. Applied research has therefore been conducted to acquire new knowledge that will aid in the making of decisions, in particular with regard to water resources. As a consequence, this thesis addresses the research gaps in uncertainty analysis by presenting two approaches (distance-based and stochastic) in Chapter 4 of the thesis to overcome the identified existing shortcomings by:

- Allowing all of the available, albeit uncertain and subjective, information to be incorporated in the decision analysis concurrently;
- Jointly assessing the uncertainty in the CWs and PVs;
- Providing an opportunity to consider the whole range of the specified individual CWs;
- Extending the MCDA outranking technique, PROMETHEE, such that it is able to incorporate uncertainty in the PVs and CWs and generalised criterion functions are not required to be defined;
- Being non-MCDA technique specific;
- Identifying the most critical input parameters to the ranking of the alternatives;
- Including any correlations between the CWs in the analysis; and
- Not requiring posterior sensitivity analysis to be undertaken.

It should be noted that analysis of the other aspects, which result in uncertainty in the output (i.e. selection of criteria weighting and MCDA methods) are also important, but has not been the focus of this thesis.

The proposed distance-based uncertainty analysis approach determines the minimum modification of the MCDA input parameters (i.e. CWs and PVs) that is required to alter the total values of two alternatives such that rank equivalence occurs. The minimum modification of the original input parameters is obtained by translating the problem into an optimisation problem and exploring the feasible input parameter ranges. The objective function minimises a distance metric, which provides a numerical value of the amount of dissimilarity between the original input parameters of the two alternatives under consideration and their optimised values. Two optimisation methods have been presented in the thesis to solve the objective function: the GRG2 nonlinear optimisation method and the genetic algorithm (GA).

The proposed stochastic uncertainty analysis approach enables distributions to be defined for each of the input parameters, which represent the set of possible values for each variable. Reliability analysis is then undertaken to determine the most probable ranking of each of the alternatives based upon the expected range of possible input values for each CW and PV. The approach involves utilising existing MCDA techniques to determine the total value of each alternative, however, one of the advancements is the application of Monte Carlo simulation (MCS) to enable repetition of the selected MCDA method with the range of possible input values. The use of MCS in the stochastic uncertainty analysis approach provides DMs with far more information than a single estimate.

The two uncertainty analysis methods are implemented by a program that has been developed in Microsoft Excel using Visual Basic for Applications (VBA), as part of this research. Flexibility has been incorporated in the program to enable various scenarios to be undertaken and it currently supports two MCDA techniques: WSM and PROMETHEE. Help files are incorporated to aid in the utilisation of the program and to inform the user of various technical aspects of the program. A user who is familiar with other Windows applications should easily be able to use the program.

The proposed uncertainty analysis methods have been compared with some existing formal sensitivity analysis methods in Chapter 5. This chapter not only demonstrates the benefits of the proposed approaches and the advantages over the existing methods, but has also served to validate the program that has been developed.

7.4 Published papers

The research work presented in this thesis has been accepted and acknowledged by the MCDA and water resources research communities through publication of five journal papers in internationally renowned journals and presentation at three Australian and international conferences. The journals were selected so as to expose the research undertaken to a wide audience.

The third objective of the research was to *apply and test the proposed approaches to case studies in the literature*. The case studies presented in the published papers demonstrate how MCDA and the proposed methodology are not only applicable to water resource decision making, but to a range of problems where selection between discrete alternatives is required.

Some important conclusions derived from the published papers are presented below:

- Publication 1 demonstrates the benefits of the proposed distance-based uncertainty analysis approach compared to existing sensitivity analysis methods, including the benefits of simultaneously varying the input parameters, compared to only one input parameter at a time;

- Publication 2 establishes the significance of incorporating the uncertainty associated with PVs in the distance-based uncertainty analysis approach, in addition to the uncertainty in the CWs;

- Publication 2 illustrates the differences between global (i.e. GA) and non-global (i.e. GRG2) optimisation techniques for solving the objective function in the proposed distance-based uncertainty analysis approach. For a complex decision space, it is concluded that a GA has more chance of arriving at a robust solution;

- Publication 3 presents the stochastic uncertainty analysis approach and the case study demonstrates the benefits of being able to fit a distribution to the CWs elicited from the actors involved in the decision analysis process, compared to using an average CW;

- Publication 4 demonstrates the applicability of the stochastic uncertainty analysis approach with the outranking PROMETHEE MCDA technique and how assigning generalised criterion functions to each of the criteria is not required due to uncertainty in the PVs being represented by probability distributions;

- Publication 5 introduces the program developed as part of this research to implement the two proposed uncertainty analysis methods. This demonstrates how expensive, 'black box' software is not required to be purchased for MCDA to be undertaken; and

- Each of the case studies performed in the published papers demonstrates how the proposed uncertainty analysis methods are able to determine the most significant input parameters in the decision analysis.

7.5 Limitations and recommendations for further research

Due to time restrictions and other constraints, the proposed uncertainty analysis approaches were not able to be formally trialled in a real decision making situation whereby the practicalities of the proposed methodologies and associated program would have been tested.

Despite the benefits, it is acknowledged that there are some limitations associated with the proposed distance-based uncertainty analysis methodology including:

- Pair-wise comparisons of alternatives are required to be undertaken, which can be quite time consuming if it is deemed necessary that all comparisons need to be performed, and especially if there are a considerable number of alternatives;

- The analysis can only be undertaken for one actor's set of CWs at a time. If there are a large number of actors involved in the decision analysis, and in particular they are all uncertain of the CWs they have provided, this methodology may not be appropriate;

- The GRG2 optimisation method is not a global optimisation method, therefore, it is not appropriate for use with complex decision problems (such as when there are a large number of criteria), as it is not guaranteed that a near-global solution will be obtained; and

- The GA has been proposed as an alternative optimisation method to the GRG2. The benefit of the GA is that it is able to arrive at near-global optimum solutions to the objective function, however, the results are dependent on the GA specific input parameters and it may be difficult to assign these parameters for people who are not familiar with GAs.

It is recommended that aspects of the program (and hence the methodology) could be further investigated and developed to:

- Include additional MCDA techniques;
- Enable distributions and uncertainty intervals to be assigned to the generalised criterion function input parameters when the PROMETHEE method is utilised;
- Allow CW rank order to be incorporated in the sampling of the distributions in the stochastic uncertainty analysis approach instead of after the sampling has occurred;
- Allow the DMs to specify information such as: (i) the relative importance of two criteria must remain constant, (ii) some CWs are more likely to change than others, and (iii) a certain CW is more likely to increase than to decrease; and
- Include alternative optimisation techniques in the distance-based uncertainty analysis approach.

Chapter 8
References

Abrishamchi, A., Ebrahimian, A., Tajrishi, M., Marino, M. A. (2005). Case Study: Application of Multicriteria Decision Making to Urban Water Supply. *Journal of Water Resources Planning and Management*. Vol 131, Iss 4, pp 326-335.

Abu-Taleb, M. F., Mareschal, B. (1995). Water resources planning in the Middle East: application of the PROMETHEE V multicriteria method. *European Journal of Operational Research*. Vol 81, Iss 3, pp 500-511.

Ahmed, J. A., Sarma, A. K. (2005). Genetic algorithm for optimal operating policy of a multipurpose reservoir. *Water Resources Management*. Vol 19, Iss 2, pp 145-161.

Aldag, R. J., Power, D. J. (1986). An empirical assessment of computer-assisted decision analysis. *Decision Sciences*. Vol 17, Iss 3, pp 572-588.

Alexouda, G. (2005). A user-friendly marketing decision support system for the product line design using evolutionary algorithms. *Decision Support Systems*. Vol 38, Iss 4, pp 495-509.

Alho, J. M., Kangas, J. (1997). Analyzing Uncertainties in Experts' Opinions of Forest Plan Performance. *Forest Science*. Vol 43, Iss 4, pp 521-528.

Alho, J. M., Kangas, J., Kolehainen, O. (1996). Uncertainty in Expert Predictions of the Ecological Consequences of Forest Plans. *Applied Statistics*. Vol 45, Iss 1, pp 1-14.

Al-Kloub, B., Al-Shemmeri, T., Pearman, A. (1997). The role of weights in multi-criteria decision aid, and the ranking of water projects in Jordan. *European Journal of Operational Research*. Vol 99, Iss 2, pp 278-288.

Almasri, M. N., Kaluarachchi, J. J. (2005). Multi-criteria decision analysis for the optimal management of nitrate contamination of aquifers. *Journal of Environmental Management*. Vol 74, Iss 4, pp 365-381.

Aloysius, J. A., Davis, F. D., Wilson, D. D., Taylor, A. R., Kottemann, J. E. (2006). User acceptance of multi-criteria decision support systems: The impact of preference elicitation techniques. *European Journal of Operational Research*. Vol 169, Iss 1, pp 273-285.

Al-Rashdan, D., Al-Kloub, B., Dean, A., Al-Shemmeri, T. (1999). Environmental impact assessment and ranking the environmental projects in Jordan. *European Journal of Operational Research*. Vol 118, Iss 1, pp 30-45.

Al-Shemmeri, T., Al-Kloub, B., Pearman, A. (1997a). Computer aided decision support system for water strategic planning in Jordan. *European Journal of Operational Research*. Vol 102, Iss 3, pp 455-472.

Al-Shemmeri, T., Al-Kloub, B., Pearman, A. (1997b). Model choice in multicriteria decision aid. *European Journal of Operational Research*. Vol 97, Iss 3, pp 550-560.

Anand Raj, P. A. (1995). Multicriteria methods in river basin planning - a case study. *Water Science and Technology*. Vol 31, Iss 8, pp 261-272.

Anand Raj, P. A., Kumar, D. N. (1996). Ranking of river basin alternatives using ELECTRE. *Hydrological Sciences*. Vol 41, Iss 5, pp 697-713.

Anand Raj, P. A., Nagesh Kumar, D. (1998). Ranking multi-criterion river basin planning alternatives using fuzzy numbers. *Fuzzy Sets and Systems*. Vol 100, Iss 1-3, pp 89-99.

Anderson, M. (2002). Software Review: DEFINITE. *Journal of Multi-Criteria Decision Analysis*. Vol 11, Iss 6, pp 345-346.

Anderson, R. M., Hobbs, B. F., Koonce, J. F., Locci, A. B. (2001). Using decision analysis to choose phosphorus targets for Lake Erie. *Environmental Management*. Vol 27, Iss 2, pp 235-252.

Ascough, J. C., Deer-Ascough, L. A., Shaffer, M. J., Hanson, J. D. (1996). *Subjective evaluation of decision support systems using multiattribute decision making (MADM)*. First International Conference on Multiple Objective Decision Support Systems

(MODSS) for Land, Water and Environmental Management: Concepts, Approaches and Applications. Honolulu, Hawaii. pp 269-280.

Assim, F., Hill, C. (1997). *Multi-criteria analysis - a tool for wetland management decision making*. 41st Annual Conference of the Australian Agricultural and Resource Economics Society. Broadbeach, Gold Cost, Queensland.

Australian Commonwealth. (1992). *National strategy for ecologically sustainable development*. Australian Government Publishing Service.

Australian Water Technologies Pty Ltd. (2003). *Life cycle assessment of future water supply options for the Eyre Peninsula, South Australia*. SA Water. 29.

Bana e Costa, C. A. (1986). A multicriteria decision aid methodology to deal with conflicting situations on the weights. *European Journal of Operational Research*. Vol 26, Iss 1, pp 22-34.

Bana e Costa, C. A. (1988). A methodology for sensitivity analysis in three-criteria problems: a case study in municipal management. *European Journal of Operational Research*. Vol 33, Iss 2, pp 159-173.

Bana e Costa, C. A., Stewart, T. J., Vansnick, J. C. (1997). Multicriteria decision analysis: Some thoughts based on the tutorial and discussion sessions of the ESIGMA meetings. *European Journal of Operational Research*. Vol 99, Iss 1, pp 28-37.

Bana e Costa, C. A., Vansnick, J. C. (1997). Applications of the MACBETH Approach in the framework of an additive aggregation model. *Journal of Multi-Criteria Decision Analysis*. Vol 6, Iss 2, pp 107-114.

Barda, O. H., Dupuis, J., Lencioni, P. (1990). Multicriteria location of thermal plants. *European Journal of Operational Research*. Vol 45, Iss 2-3, pp 332-346.

Bardossy, A., Duckstein, L. (1992). Analysis of a karstic aquifer management problem by fuzzy composite programming. *Water Resources Bulletin*. Vol 28, Iss 1, pp 63-73.

Barreteau, O., Bousquet, F., Attonaty, J.-M. (2001). Role-playing games for opening the black box of multi-agent systems: method and lessons of its application to Senegal River Valley irrigated systems. *Journal of Artificial Societies and Social Simulation*. Vol 4, Iss 2.

Barron, F. H., Barrett, B. E. (1996). Decision quality using ranked attribute weights. *Management Science*. Vol 42, Iss 11, pp 1515-1523.

Barron, H., Schmidt, P. (1988). Sensitivity analysis of additive multiattribute value models. *Operations Research*. Vol 36, Iss 1, pp 122-127.

Bell, M. L., Hobbs, B. F., Elliott, E. M., Ellis, H., Robinson, Z. (2001). An evaluation of multi-criteria methods in integrated assessment of climate policy. *Journal of Multi-Criteria Decision Analysis*. Vol 10, Iss 5, pp 229-256.

Bella, A., Duckstein, L., Szidarovszky, F. (1996). A multicriterion analysis of the water allocation conflict in the upper Rio Grande Basin. *Applied Mathematics and Computation*. Vol 77, Iss 2-3, pp 245-265.

Belton, V., Ackermann, F., Shepherd, I. (1997). Integrated support from problem structuring through to alternative evaluation using COPE and VISA. *Journal of Multi-Criteria Decision Analysis*. Vol 6, Iss 3, pp 115-130.

Belton, V., Hodgkin, J. (1999). Facilitators, decision makers, D.I.Y. users: Is intelligent multicriteria decision support for all feasible or desirable? *European Journal of Operational Research*. Vol 113, Iss 2, pp 247-260.

Belton, V., Pictet, J. (1997). A framework for group decision using a MCDA model: sharing, aggregating or comparing individual information. *Revue des systemes de decision*. Vol 6, Iss 3, pp 283-303.

Belton, V., Vickers, S. (1990). *Use of a simple multi-attribute value function incorporating visual interactive sensitivity analysis for multiple criteria decision making*. In: Readings in Multiple Criteria Decision Aid. Ed. C. A. Bana e Costa. Springer-Verlag. pp 319-334.

Bender, M. J., Simonovic, S. P. (1997). Consensus as the measure of sustainability. *Hydrological Sciences*. Vol 42, Iss 4, pp 493-500.

Bender, M. J., Simonovic, S. P. (2000). A fuzzy compromise approach to water resource systems planning under uncertainty. *Fuzzy Sets and Systems*. Vol 115, Iss 1, pp 35-44.

Bertrand-Krajewski, J. L., Barraud, S., Bardin, J.-P. (2002). Uncertainties, performance indicators and decision aid applied to stormwater facilities. *Urban Water*. Vol 4, Iss 2, pp 163-179.

Beynon, M. (2002). DS/AHP method: a mathematical analysis, including an understanding of uncertainty. *European Journal of Operational Research*. Vol 140, Iss 1, pp 148-164.

Beynon, M., Cosker, D., Marshall, D. (2001). An expert system for multi-criteria decision making using Dempster Shafer theory. *Expert Systems with Applications*. Vol 20, Iss 4, pp 357-367.

Beynon, M., Curry, B., Morgan, P. (2000). The Dempster-Shafer theory of evidence: an alternative approach to multicriteria decision modelling. *Omega*. Vol 28, Iss 1, pp 37-50.

Borcherding, K., Eppel, T., von Winterfeldt, D. (1991). Comparison of weighting judgments in multiattribute utility measurement. *Management Science*. Vol 37, Iss 12, pp 1603-1619.

Bottomley, P. A., Doyle, J. R. (2001). A comparison of three weight elicitation methods: good, better, and best. *Omega*. Vol 29, Iss 6, pp 553-560.

Bottomley, P. A., Doyle, J. R., Green, R. A. (2000). Testing the reliability of weight elicitation methods: Direct rating versus point allocation. *Journal of Marketing Research*. Vol 37, Iss 4, pp 508-513.

Bouyssou, D. (1990). *Building criteria: a prerequisite for MCDA*. In: Readings in multiple criteria decision aid. Ed. C. A. Bana e Costa. Springer-Verlag. pp 58-80.

Brans, J. P., Macharis, C., Kunsch, P. L., Chevalier, A., Schwaninger, M. (1998). Combining multicriteria decision aid and system dynamics for the control of socio-economic processes. An iterative real-time procedure. *European Journal of Operational Research*. Vol 109, Iss 2, pp 428-441.

Brans, J. P., Mareschal, B. (1990). *The PROMETHEE methods for MCDM; the PROMCALC, GAIA and BANKADVISOR software*. In: Readings in multiple criteria decision aid. Ed. C. A. Bana e Costa. Springer-Verlag.

Brans, J. P., Vincke, P., Mareschal, B. (1986). How to select and how to rank projects: the PROMETHEE method. *European Journal of Operational Research*. Vol 24, Iss 2, pp 228-238.

Briggs, T., Kunsch, P. L., Mareschal, B. (1990). Nuclear waste management: an application of the multicriteria PROMETHEE methods. *European Journal of Operational Research*. Vol 44, Iss 1, pp 1-10.

Brunner, N., Starkl, M. (2004). Decision aid systems for evaluating sustainability: a critical survey. *Environmental Impact Assessment Review*. Vol 24, Iss 4, pp 441-469.

Buede, D. M. (1992). Overview of the MCDA Software Market. *Journal of Multi-Criteria Decision Analysis*. Vol 1, Iss 1, pp 59-61.

Buede, D. M. (1996). Second Overview of the MCDA Software Market. *Journal of Multi-Criteria Decision Analysis*. Vol 5, Iss 4, pp 312-316.

Burn, D. H., Yulianti, J. S. (2001). Waste-load Allocation Using Genetic Algorithms. *Journal of Water Resources Planning and Management*. Vol 127, Iss 2, pp 121-129.

Butler, J., Jia, J., Dyer, J. (1997). Simulation techniques for the sensitivity analysis of multi-criteria decision models. *European Journal of Operational Research*. Vol 103, Iss 3, pp 531-546.

Butler, J., Olson, D. L. (1999). Comparison of centroid and simulation approaches for selection sensitivity analysis. *Journal of Multi-Criteria Decision Analysis*. Vol 8, Iss 3, pp 146-161.

Carlson, S. E., Shonkwiler, R., Sani, A. F., Aral, M. M. (1996). *Annealing a genetic algorithm over constraints*. www.math.gatech.edu/~shenk/constrainedGA.pdf.

Chan Hilton, A. B., Culver, T. B. (2000). Constraint handling for genetic algorithms in optimal remediation design. *Journal of Water Resources Planning and Management*. Vol 126, Iss 3, pp 128-137.

Chan Hilton, A. B., Culver, T. B. (2005). Groundwater remediation design under uncertainty using genetic algorithms. *Journal of Water Resources Planning and Management*. Vol 131, Iss 1, pp 25-34.

Chang, F. H., Chen, L. (1998). Real-coded genetic algorithm for rule-based flood control reservoir management. *Water Resources Management*. Vol 12, Iss 3, pp 185-198.

Choi, D. J., Park, H. (2001). Analysis of water privatization scenarios in Korea with multi-criteria decision making techniques. *Journal of Water Supply Research and Technology - AQUA*. Vol 50, Iss 6, pp 335-352.

Choo, E. U., Schoner, B., Wedley, W. C. (1999). Interpretation of criteria weights in multicriteria decision making. *Computers and Industrial Engineering*. Vol 37, Iss 3, pp 527-541.

Cieniawski, S. E., Eheart, J. W., Ranjithan, S. (1995). Using genetic algorithms to solve a multiobjective groundwater monitoring problem. *Water Resources Research*. Vol 31, Iss 2, pp 399-409.

Corner, J. L., Buchanan, J., Henig, M. (2001). Dynamic decision problem structuring. *Journal of Multi-Criteria Decision Analysis*. Vol 10, Iss 3, pp 129-141.

Corner, J. L., Buchanan, J. T. (1997). Capturing decision maker preference: experimental comparison of decision analysis and MCDM techniques. *European Journal of Operational Research*. Vol 98, Iss 1, pp 85-97.

Corner, J. L., Kirkwood, C. W. (1991). Decision analysis applications in the operations research literature, 1970 - 1989. *Operations Research*. Vol 39, Iss 2, pp 206-219.

Crabb, P. (1997). *Impacts of Anthropogenic Activities, Water Use and Consumption on Water Resources and Flooding*. Central Queensland University Publishing Unit [for Environment Australia, Dept. of the Environment]: Rockhampton, Queensland.

Critchfield, G. C., Willard, K. E. (1986). Probabilistic analysis of decision trees using Monte Carlo simulation. *Medical decision making*. Vol 6, Iss 1, pp 85-92.

Dandy, G. C., Engelhardt, M. (2001). Optimal Scheduling of Water Pipe Replacement using Genetic Algorithms. *Journal of Water Resources Planning and Management*. Vol 127, Iss 4, pp 214-223.

Davey, A., Olson, D. L. (1998). Multiple Criteria Decision Making Models in Group Decision Support. *Group Decision and Negotiation*. Vol 7, Iss 1, pp 55-75.

David, L., Duckstein, L. (1976). Multi-criterion ranking of alternative long-range water resources systems. *Water Resources Bulletin*. Vol 12, Iss 4, pp 731-754.

D'Avignon, G. R., Vincke, P. (1988). An outranking method under uncertainty. *European Journal of Operational Research*. Vol 36, Iss 3, pp 311-321.

De Keyser, W., Peeters, P. (1996). A note on the use of PROMETHEE multicriteria methods. *European Journal of Operational Research*. Vol 89, Iss 3, pp 457-461.

De Kort, I. A. T., Booij, M. J. (2006). Decision making under uncertainty in a decision support system for the Red River. *Environmental Modelling & Software*. In Press.

Deb, K. (2000). An efficient constraint handling method for genetic algorithms. *Computer methods in applied mechanics and engineering*. Vol 186, Iss 2-4, pp 311-338.

Delgado, M. G., Sendra, J. B. (2004). Sensitivity analysis in multicriteria spatial decision-making: a review. *Human and Ecological Risk Assessment*. Vol 10, Iss 6, pp 1173-1187.

Deng, J., King, B., Bauer, T. (2002). Evaluating natural attractions for tourism. *Annals of Tourism Research*. Vol 29, Iss 2, pp 422-438.

Diakoulaki, D., Koumoutsos, N. (1991). Cardinal ranking of alternative actions: extension of PROMETHEE method. *European Journal of Operational Research*. Vol 53, Iss 3, pp 337-347.

Dias, L., Climaco, J. (2005). Dealing with imprecise information in group multicriteria decisions: a methodology and a GDSS architecture. *European Journal of Operational Research*. Vol 160, Iss 2, pp 291-307.

Dias, L., Costa, J. P., Climaco, J. N. (1998). A parallel implementation of the PROMETHEE method. *European Journal of Operational Research*. Vol 104, Iss 3, pp 521-531.

Dias, L., Mousseau, V., Figueira, J., Climaco, J. (2002). An aggregation/disaggregation approach to obtain robust conclusions with ELECTRE TRI. *European Journal of Operational Research*. Vol 138, Iss 2, pp 332-348.

Dias, L. C., Climaco, J. N. (2000). Additive aggregation with variable interdependent parameters: the VIP analysis software. *Journal of the Operational Research Society*. Vol 51, Iss 9, pp 1070-1082.

Duckstein, L., Gershon, M., McAniff, R. (1982). Model selection in multiobjective decision making for river basin planning. *Advances in Water Resources*. Vol 5, Iss 3, pp 178-184.

Duckstein, L., Treichel, W., Magnouni, S. E. (1994). Ranking of groundwater management alternatives by multicriterion analysis. *Journal of Water Resources Planning and Management*. Vol 120, Iss 4, pp 546-565.

Dunning, D. J., Ross, Q. E., Merkhofer, M. W. (2000). Multiattribute utility analysis for addressing Section 316(b) of the Clean Water Act. *Environmental Science & Policy*. Vol 3, Iss 1, pp S7-S14.

DWR. (2000). *State Water Plan - Volume 2 South Australia's Water Resources*. Department for Water Resources: Adelaide.

Dyer, J., Edmunds, T., Butler, J., Jia, J. (1998). A multiattribute utility analysis of alternatives for the disposition of surplus weapons-grade plutonium. *Operations Research*. Vol 46, Iss 6, pp 749-762.

Dyer, J. S., Fishburn, P. C., Steuer, R. E., Wallenius, J., Zionts, S. (1992). Multiple criteria decision making, multiple attribute utility theory: the next ten years. *Management Science*. Vol 38, Iss 5, pp 645-654.

Easley, R. F., Valacich, J. S., Venkataramanan, M. A. (2000). Capturing group preferences in a multicriteria decision. *European Journal of Operational Research*. Vol 125, Iss 1, pp 73-83.

Eberhard, R., Joubert, A. (2001). *An evaluation of alternative water supply augmentation and water demand management options for the City of Cape Town using multi-criteria decision analysis*. University of Cape Town. 12.

Eckenrode, R. T. (1965). Weighting Multiple Criteria. *Management Science*. Vol 12, Iss 3, pp 180-192.

Edwards, W. M. (1977). How to Use Multiattribute Utility Measurement for Social Decision Making. *IEEE Transactions on Systems, Man and Cybernetics*. Vol 7, Iss, pp 326-340.

Elbeltagi, E., Hegazy, T., Grierson, D. (2005). Comparison among five evolutionary-based optimization algorithms. *Advanced Engineering Informatics*. Vol 19, Iss 1, pp 43-53.

Evans, G. W. (1984a). An overview of techniques for solving multiobjective mathematical problems. *Management Science*. Vol 30, Iss 11, pp 1268-1282.

Evans, J. R. (1984b). Sensitivity analysis in decision theory. *Decision Sciences*. Vol 15, pp 239-247.

Felli, J. C., Hazen, G. B. (1998). Sensitivity analysis and the expected value of perfect information. *Medical decision making*. Vol 18, Iss 1, pp 95-109.

Felli, J. C., Hazen, G. B. (1999). A Bayesian approach to sensitivity analysis. *Health Economics*. Vol 8, Iss 3, pp 263-268.

Fernandez, F. R., Marmol, A., Puerto, J. (1998). *Improving weighting information in interactive decision procedures. A visual guide*. XIVth International Conference on Multiple Criteria Decision Making. Charlottesville, Virginia. pp 260-271.

Feuillette, S., Bousquet, F., Le Goulven, P. (2003). SINUSE: a multi-agent model to negotiate water demand management on a free access water table. *Environmental Modelling & Software*. Vol 18, Iss 5, pp 413-427.

Figueira, J., Roy, B. (2002). Determining the weights of criteria in the ELECTRE type methods with a revised Simos' procedure. *European Journal of Operational Research*. Vol 139, Iss 2, pp 317-326.

Finnveden, G., Moberg, A. (2005). Environmental systems analysis tools - an overview. *Journal of Cleaner Production*. Vol 13, Iss 12, pp 1165-1173.

Fischer, G. W. (1995). Range sensitivity of attribute weights in multiattribute value models. *Organizational behavior and human decision processes*. Vol 62, Iss 3, pp 252-266.

Fischer, G. W., Jia, J. M., Luce, M. F. (2000). Attribute conflict and preference uncertainty: The RandMAU model. *Management Science*. Vol 46, Iss 5, pp 669-684.

Fishburn, P. C. (1967). Methods of estimating additive utilities. *Management Science*. Vol 13, pp 435-453.

Fishburn, P. C., Murphy, A. H., Isaacs, H. H. (1968). Sensitivity of decisions to probability estimation errors: a reexamination. *Operations Research*. Vol 16, Iss 1-3, pp 254-267.

Fleming, N. (1999). *Sustainability and water resources management for the Northern Adelaide Plains, South Australia*. Department of Civil and Environmental Engineering, The University of Adelaide.

Fleming, N. S., Daniell, T. M. (1996). *Matrix for evaluation of sustainability achievement (MESA) an aid to water resources project evaluation*. 23rd Hydrology and Water Resources Symposium. Hobart, Australia. pp 521-527.

Flug, M., Seitz, H. L. H., Scott, J. F. (2000). Multicriteria decision analysis applied to Glen Canyon Dam. *Journal of Water Resources Planning and Management*. Vol 126, Iss 5, pp 270-276.

Foltz, J. C., Lee, J. G., Martin, M. A., Preckel, P. V. (1995). Multiattribute assessment of alternative cropping systems. *American Journal of Agricultural Economics*. Vol 77, Iss 2, pp 408-420.

French, S. (1992). Mathematical programming approaches to sensitivity calculations in decision analysis. *Journal of the Operational Research Society*. Vol 43, Iss 8, pp 813-819.

French, S. (1995). Uncertainty and Imprecision: Modelling and Analysis. *The Journal of the Operational Research Society*. Vol 46, Iss 1, pp 70-79.

French, S., Simpson, L., Atherton, E., Belton, V., Dawes, R. M., Edwards, W., Hamalainen, R. P., Larichev, O. I., Lootsma, F., Pearman, A., Vlek, C. (1998). Problem formulation for multi-criteria decision analysis: report of a workshop. *Journal of Multi-Criteria Decision Analysis*. Vol 7, Iss 5, pp 242-262.

Frey, H. C., Patil, S. R. (2002). Identification and review of sensitivity analysis methods. *Risk Analysis*. Vol 22, Iss 3, pp 553-578.

Geldermann, J., Zhang, K. (2001). Software Review: Decision Lab 2000. *Journal of Multi-Criteria Decision Analysis*. Vol 10, Iss 6, pp 317-323.

Geneletti, D. (2005). Formalising expert opinion through multi-attribute value functions: An application in landscape ecology. *Journal of Environmental Management*. Vol 76, Iss 3, pp 255-262.

Georgopoulou, E., Lalas, D., Papagiannakis, L. (1997). A multicriteria decision aid approach for energy planning problems: the case of renewable energy option. *European Journal of Operational Research*. Vol 103, Iss 1, pp 38-54.

Georgopoulou, E., Sarafidis, Y., Diakoulaki, D. (1998). Design and implementation of a group DSS for sustaining renewable energies exploitation. *European Journal of Operational Research*. Vol 109, Iss 2, pp 483-500.

Georgopoulou, E., Sarafidis, Y., Mirasgedis, S., Zaimi, S., Lalas, D. P. (2003). A multiple criteria decision-aid approach in defining national priorities for greenhouse gases emissions reduction in the energy sector. *European Journal of Operational Research*. Vol 146, Iss 1, pp 199-215.

Gershon, M., Duckstein, L. (1983). Multiobjective approaches to river basin planning. *Journal of Water Resources Planning and Management*. Vol 109, Iss 1, pp 13-28.

Gershon, M., Duckstein, L., McAniff, R. (1982). Multiobjective River Basin Planning with Qualitative Criteria. *Water Resources Research*. Vol 18, Iss 2, pp 193-202.

Giupponi, C., Mysiak, J., Fassio, A., Cogan, V. (2004). MULINO-DSS: a computer tool for sustainable use of water resources at the catchment scale. *Mathematics and Computers in Simulation*. Vol 64, Iss 1, pp 13-24.

Goicoechea, A., Stakhiv, E. Z., Li, F. (1992). Experimental evaluation of multiple criteria decision models for application to water resources planning. *Water Resources Bulletin*. Vol 28, Iss 1, pp 89-102.

Goldberg, D. E. (1989). *Genetic algorithms in search, optimization and machine learning*. Addison-Wesley: Reading.

Goldberg, D. E., Deb, K. (1990). *A comparitive analysis of selection schemes used in genetic algorithms*. In: Foundation of Genetic Algorithms. Ed. Morgan Kaufman. pp 69-93.

Gough, J. D., Ward, J. C. (1996). Environmental decision-making and lake management. *Journal of Environmental Management*. Vol 48, Iss 1, pp 1-15.

Goumas, M., Lygerou, V. (2000). An extension of the PROMETHEE method for decision making in fuzzy environment: Ranking of alternative energy exploitation projects. *European Journal of Operational Research*. Vol 123, Iss 3, pp 606-613.

Guillen, S. T., Trejos, M. S., Canales, R. (1998). *A robustness index of binary preferences*. XIVth International Conference on Multiple Criteria Decision Making. Charlottesville, Virginia. pp 105-113.

Guitouni, A., Martel, J. (1998). Tentative guidelines to help choosing an appropriate MCDA method. *European Journal of Operational Research*. Vol 109, Iss 2, pp 501-521.

Haastrup, P., Maniezzo, V., Mattarelli, M., Mazzeo Rinaldi, F., Mendes, I., Paruccini, M. (1998). A decision support system for urban waste management. *European Journal of Operational Research*. Vol 109, Iss 2, pp 330-341.

Hajeeh, M., Al-Othman, A. (2005). Application of the analytical hierarchy process in the selection of desalination plants. *Desalination*. Vol 174, Iss 1, pp 97-108.

Hajkowicz, S. (2000). *An evaluation of multiple objective decision support for natural resource management*. Department of Geographical Sciences and Planning, University of Queensland.

Hajkowicz, S. (2002). *Regional Priority Setting in Queensland: A multi-criteria evaluation framework*. CSIRO Land and Water. Consultancy Report for the Queensland Department of Natural Resources and Mines. 34.

Hajkowicz, S., Young, M., Wheeler, S., MacDonald, D., Young, D. (2000). *Supporting decisions: understanding natural resource management assessment techniques*. CSIRO Land and Water. 136.

Hamalainen, R. P., Kettunen, E., Ehtamo, H. (2001). Evaluating a framework for multi-stakeholder decision support in water resources management. *Group Decision and Negotiation*. Vol 10, Iss 4, pp 331-353.

Hamalainen, R. P., Salo, A. A. (1997). The issue is understanding the weights. *Journal of Multi-Criteria Decision Analysis*. Vol 6, Iss 6, pp 340-343.

Hanley, N., Nevin, C. (1999). Appraising renewable energy developments in remote communities: the case of the North Assynt Estate, Scotland. *Energy Policy*. Vol 27, Iss 9, pp 527-547.

Haralambopoulos, D. A., Polatidis, H. (2003). Renewable energy projects: structuring a multi-criteria group decision-making framework. *Renewable Energy*. Vol 28, Iss 6, pp 961-973.

Harboe, R. (1992). Multiobjective Decision Making Techniques for Reservoir Operation. *Water Resources Bulletin*. Vol 28, Iss 1, pp 103-110.

Heilman, P., Davis, G., Lawrence, P., Hatfield, J. L., Huddleston, J. (2002). *The Facilitator - an open source effort to support multiobjective decision making*. iEMS 2002 Integrated Assessment and Decision Support. Lugano Switzerland. pp 253-258.

Helton, J. C. (1993). Uncertainty and sensitivity analysis techniques for use in performance assessment for radioactive waste disposal. *Reliability Engineering and System Safety*. Vol 42, pp 327-367.

Herath, G. (2004). Incorporating community objectives in improved wetland management: the use of the analytic hierarchy process. *Journal of Environmental Management*. Vol 70, Iss 3, pp 263-273.

Herrera, F., Lozano, M., Verdegay, J. L. (1998). Tackling Real-Coded Genetic Algorithms: Operators and Tools for Behavioural Analysis. *Artificial Intelligence Review*. Vol 12, Iss 4, pp 265-319.

Hersh, M. A. (1999). Sustainable decision making: the role of decision support systems. *IEEE Transactions on Systems, Man, And Cybernetics Part C: Applications and Reviews*. Vol 29, Iss 3, pp 395-408.

Hill, M. J., Braaten, R., Veitch, S. M., Lees, B. G., Sharma, S. (2005). Multi-criteria decision analysis in spatial decision support: the ASSESS analytic hierarchy process and the role of quantitative methods and spatially explicit analysis. *Environmental Modelling & Software*. Vol 20, Iss 7, pp 955-976.

Hipel, K. W. (1992). Multiple objective decision making in water resources. *Water Resources Bulletin*. Vol 28, Iss 1, pp 3-12.

Hobbs, B. F. (1980). A comparison of weighting methods in power plant siting. *Decision Sciences*. Vol 11, pp 725-737.

Hobbs, B. F., Chankong, V., Hamadeh, W., Stakhiv, E. Z. (1992). Does choice of multicriteria method matter? An experiment in water resources planning. *Water Resources Research*. Vol 28, Iss 7, pp 1767-1779.

Hobbs, B. F., Horn, G. T. F. (1997). Building public confidence in energy planning: a multimethod MCDM approach to demand-side planning at BC gas. *Energy Policy*. Vol 25, Iss 3, pp 357-375.

Hobbs, B. F., Meier, P. M. (1994). Multicriteria methods for resource planning: an experimental comparison. *IEEE Transactions on Power Systems*. Vol 9, Iss 4, pp 1811-1817.

Hodgkin, J., Belton, V., Buchanan, I. (1998). *Intelligent user support in multicriteria decision support*. XIVth International Conference on Multiple Criteria Decision Making. Charlottesville, Virginia. pp 284-295.

Hodgkin, J., Belton, V., Koulouri, A. (2005). Supporting the intelligent MCDA user: A case study in multi-person multi-criteria decision support. *European Journal of Operational Research*. Vol 160, Iss 1, pp 172-189.

Hokkanen, J., Lahdelma, R., Miettinen, K., Salminen, P. (1998). Determining the implementation order of a general plan by multicriteria method. *Journal of Multi-Criteria Decision Analysis*. Vol 7, Iss 5, pp 273-284.

Hokkanen, J., Lahdelma, R., Salminen, P. (1999). A multiple criteria decision model for analyzing and choosing among different development patterns for the Helsinki cargo harbor. *Socio-Economic Planning Sciences*. Vol 33, Iss 1, pp 1-23.

Hokkanen, J., Lahdelma, R., Salminen, P. (2000). Multicriteria decision support in a technology competition for cleaning polluted soil in Helsinki. *Journal of Environmental Management*. Vol 60, Iss 4, pp 339-348.

Hokkanen, J., Salminen, P. (1994). *The choice of a solid waste management system by using the ELECTRE III decision-aid method*. In: Applying multiple criteria aid for decision to environmental management. Ed. M. Paruccini. Kluwer Academic Publishers. pp 111-153.

Hokkanen, J., Salminen, P. (1997a). Choosing a solid waste management system using multicriteria decision analysis. *European Journal of Operational Research*. Vol 98, Iss 1, pp 19-36.

Hokkanen, J., Salminen, P. (1997b). ELECTRE III and IV Decision Aids in an Environmental Problem. *Journal of Multi-Criteria Decision Analysis*. Vol 6, Iss 4, pp 215-226.

Hokkanen, J., Salminen, P. (1997c). Locating a Waste Treatment Facility by Multicriteria Analysis. *Journal of Multi-Criteria Decision Analysis*. Vol 6, Iss 3, pp 175-184.

Holland, A. (1997). The foundations of environmental decision-making. *International Journal of Environment and Pollution*. Vol 7, Iss 4, pp 483-496.

Hollick, M. (1993). *Introduction to project evaluation*. Longman Cheshire: Melbourne.

Hsiao, C.-T., Chang, L.-C. (2002). Dynamic optimal groundwater management with inclusion of fixed costs. *Journal of Water Resources Planning and Management*. Vol 128, Iss 1, pp 57-65.

Huth, A., Drechsler, M., Kohler, P. (2004). Multicriteria evaluation of a simulated logging scenarios in a tropical rain forest. *Journal of Environmental Management*. Vol 71, Iss 4, pp 321-333.

Hwang, C. L., Yoon, K. (1981). *Multiple attribute decision making: methods and applications: a state-of-the-art survey*. Springer-Verglag.

Isaacs, H. H. (1963). Sensitivity of decisions to probability estimation errors. *Operations Research*. Vol 11, pp 536-552.

ISO 14040. (1997). *Environmental Management - life cycle assessment - principles and framework*. International Organization for Standardization.

Jaber, J. O., Mohsen, M. S. (2001). Evaluation of non-conventional water resources supply in Jordan. *Desalination*. Vol 136, Iss 1-3, pp 83-92.

Jacquet-Lagreze, E. (1990). *Interactive assessment of preferences using holistic judgments. The PREFCALC system*. In: Readings in Multiple Criteria Decision Aid. Ed. C. A. Bana e Costa. Springer-Verlag. pp 335-350.

Jacquet-Lagreze, E., Siskos, Y. (2001). Preference disaggregation: 20 years of MCDA experience. *European Journal of Operational Research*. Vol 130, Iss 2, pp 233-245.

Janssen, R. (1996). *Multiobjective Decision Support for Environmental Management*. Kluwer Academic Publishers: Netherlands.

Janssen, R. (2001). On the use of multi-criteria analysis in environmental impact assessment in the Netherlands. *Journal of Multi-Criteria Decision Analysis*. Vol 10, Iss 2, pp 101-109.

Janssen, R., Goosen, H., Verhoeven, M. L., Verhoeven, J. T. A., Omtzigt, A. Q. A., Maltby, E. (2005). Decision support for integrated wetland management. *Environmental Modelling & Software*. Vol 20, Iss 2, pp 215-229.

Janssen, R., Herwijnen, M. (1992). How to use the DEFINITE demonstration disks.

Janssen, R., Nijkamp, P., Rietveld, P. (1990). *Qualitative multicriteria methods in the Netherlands*. In: Readings in multiple criteria decision aids. Ed. C. A. Bana e Costa. Springer-Verlag. pp 383-409.

Janssen, R., van Herwijnen, M. (1989). *Graphical Decision Support Applied to Decisions Changing the Use of Agricultural Land*. In: Lecture Notes in Economics and

Mathematical Systems. Ed. P. Korhonen, A. Lewandowski and J. Wallenius. Springer-Verlag. pp 293-302.

Jarke, M., Jelassi, M. T., Shakun, M. F. (1987). MEDIATOR: Towards a Negotiation Support System. *European Journal of Operational Research*. Vol 31, Iss 3, pp 314-334.

Jessop, A. (2002). Prioritisation of an IT budget within a local authority. *Journal of the Operational Research Society*. Vol 53, Iss 1, pp 36-46.

Jessop, A. (2004). Sensitivity and robustness in selection problems. *Computers & Operations Research*. Vol 31, Iss 4, pp 607-622.

Jia, J., Fischer, G. W., Dyer, J. S. (1998). Attribute weighting methods and decision quality in the presence of response error: A simulation study. *Journal of Behavioral Decision Making*. Vol 11, Iss 2, pp 85-105.

Joubert, A., Leiman, A., de Klerk, H., Katua, S., Aggenbach, J. (1997). Fynbos (fine bush) vegetation and the supply of water: a comparison of multi-criteria decision analysis and cost-benefit analysis. *Ecological Economics*. Vol 22, Iss 2, pp 123-140.

Kalagranam, J. R., Diwekar, U. M. (1997). An Efficient Sampling Technique for Off-line Quality Control. *Technometrics*. Vol 39, Iss 3, pp 308-319.

Kaliszewski, I. (2004). Out of the mist - towards decision-maker-friendly multiple criteria decision making support. *European Journal of Operational Research*. Vol 158, Iss 2, pp 293-307.

Kangas, A., Kangas, J., Pykalainen, J. (2001a). Outranking methods as tools in strategic natural resources planning. *Silva Fennica*. Vol 35, Iss 2, pp 215-227.

Kangas, J., Kangas, A. (2005). Multiple criteria decision support in forest management - the approach, methods applied, and experiences gained. *Forest Ecology and Management*. Vol 207, Iss 1-2, pp 133-143.

Kangas, J., Kangas, A., Leskinen, P., Pykalainen, J. (2001b). MCDM Methods in Strategic Planning of Forestry on State-Owned Lands in Finland: Applications and Experiences. *Journal of Multi-Criteria Decision Analysis*. Vol 10, Iss 5, pp 257-271.

Kangas, J., Store, J., Leskinen, P., Mehtatalo, L. (2000). Improving the quality of landscape ecological forest planning by utilising advanced decision-support tools. *Forest Ecology and Management*. Vol 132, Iss 2-3, pp 157-171.

Karagiannidis, A., Moussiopoulos, N. (1997). Application of ELECTRE III for the integrated management of municipal solid wastes in the Greater Athens Area. *European Journal of Operational Research*. Vol 97, Iss 3, pp 439-449.

Karni, R., Sanchez, P., Tummala, V. M. R. (1990). A comparative study of multiattribute decision making methodologies. *Theory and Decision*. Vol 29, pp 203-222.

Keefer, D. L., Kirkwood, C. W., Corner, J. (2004). Perspective on Decision Analysis Applications, 1990-2001. *Decision Analysis*. Vol 1, Iss 1, pp 4-22.

Keeney, R. L., Raiffa, H. (1976). *Decisions with Multiple Objectives, Preferences and Value Trade Offs*. Wiley: New York.

Keeney, R. L., Wood, E. F. (1977). An illustrative example of the use of multiattribute utility theory for water resource planning. *Water Resources Research*. Vol 13, Iss 4, pp 705-712.

Khalil, W. A., Shanableh, A., Rigby, P., Kokot, S. (2005). Selection of hydrothermal pre-treatment conditions of waste sludge destruction using multicriteria decision-making. *Journal of Environmental Management*. Vol 75, Iss 1, pp 53-64.

Kheireldin, K., Fahmy, H. (2001). Multi-criteria approach for evaluating long term water strategies. *Water International*. Vol 26, Iss 4, pp 527-535.

Kiss, L. N., Martel, J., Nadeau, R. (1994). ELECCALC - an interactive software for modelling decision maker's preferences. *Decision Support Systems*. Vol 12, Iss 4-5, pp 311-326.

Klauer, B., Drechsler, M., Messner, F. (2002). *Multicriteria analysis under uncertainty with IANUS - method and empirical results*. UFZ Centre for Environmental Research. Discussion Paper. 25.

Korhonen, P. (1988). A visual reference direction approach to solving discrete multiple criteria problems. *European Journal of Operational Research*. Vol 34, Iss 2, pp 152-159.

Korhonen, P., Larichev, O. I., Mechitov, A., Moshkovich, H., Wallenius, J. (1997). Choice behaviour in a computer-aided multiattribute decision task. *Journal of Multi-Criteria Decision Analysis*. Vol 6, Iss 4, pp 233-246.

Kottegoda, N. T., Rosso, R. (1997). *Statistics, Probability, and Reliability for Civil and Environmental Engineers*. McGraw-Hill: Singapore.

Lahdelma, R., Hokkanen, J., Salminen, P. (1998). SMAA - Stochastic multiobjective acceptability analysis. *European Journal of Operational Research*. Vol 106, Iss 1, pp 137-143.

Lahdelma, R., Miettinen, K., Salminen, P. (2005). Reference point approach for multiple decision makers. *European Journal of Operational Research*. Vol 164, Iss 3, pp 785-791.

Lahdelma, R., Salminen, P. (2001). SMAA-2: Stochastic Multicriteria Acceptability Analysis for Group Decision Making. *Operations Research*. Vol 49, Iss 3.

Lahdelma, R., Salminen, P. (2002). Psuedo-criteria versus linear utility function in stochastic multi-criteria acceptability analysis. *European Journal of Operational Research*. Vol 141, Iss 2, pp 454-469.

Lahdelma, R., Salminen, P., Hokkanen, J. (1999). *Combining stochastic multiobjective acceptability analysis and DEA*. In: Integrating Technology and Human Decisions: Global Bridges and the 21st Century. Ed. D. K. Despotis and C. Zopounidis. New Technologies Publications. pp 629-632.

Lahdelma, R., Salminen, P., Hokkanen, J. (2000). Using multicriteria methods in environmental planning and management. *Environmental Management*. Vol 26, Iss 6, pp 595-605.

Lahdelma, R., Salminen, P., Hokkanen, J. (2002). Locating a waste treatment facility by using stochastic multicriteria acceptability analysis with ordinal criteria. *European Journal of Operational Research*. Vol 142, Iss 2, pp 345-356.

Lamy, F., Bolte, J., Santelmann, M., Smith, C. (2002). Development and evaluation of multiple-objective decision-making methods for watershed management planning. *Journal of the American Water Resources Association*. Vol 38, Iss 2, pp 517-529.

Larichev, O. I. (1992). Cognitive validity in design of decision-aiding techniques. *Journal of Multi-Criteria Decision Analysis*. Vol 1, pp 127-138.

Larichev, O. I. (1998). *Problems of measurement in decision analysis*. XIVth International Conference on Multiple Criteria Decision Making. Charlottesville, Virginia. pp 343-356.

Larichev, O. I. (2001). Ranking multicriteria alternatives: the method of ZAPROS III. *European Journal of Operational Research*. Vol 131, Iss 3, pp 550-558.

Larichev, O. I., Moshkovich, H. M. (1995). ZAPROS-LM - A method and system for ordering multiattribute alternatives. *European Journal of Operational Research*. Vol 82, Iss 3, pp 503-521.

Larichev, O. I., Moshkovich, H. M., Mechitov, A. I., Olson, D. L. (1993). Experiments comparing qualitative approaches to rank ordering of multiattribute alternatives. *Journal of Multi-Criteria Decision Analysis*. Vol 2, Iss 1, pp 5-26.

Larichev, O. I., Olson, D. L., Moshkovich, H. M., Mechitov, A. J. (1995). Numerical vs cardinal measurements in multiattribute decision making: how exact is enough? *Organizational behavior and human decision processes*. Vol 64, Iss 1, pp 9-21.

Lawrence, P., LeGrand, J., Jeffreys, I. (2002). *Integrating spatial models and decision support systems for improved catchment planning*. Riversymposium. Brisbane, Australia.

Lawrence, P., Robinson, J., Eisner, R. (2001). *A decision environment: going beyond a decision framework to improve the effectiveness of decision making in natural resource management*. MODSIM 2001: International congress on modelling and simulation. The Australian National University, Canberra. pp 1613-1618.

Lawrence, P., Shaw, R., Lane, L., Eisner, R. (2000). *Participatory multiple objective decision making processes: emerging approaches with new challenges*. Watershed Management and Operations Management. Ft Collins, Colorado.

Le Teno, J. F., Mareschal, B. (1998). An interval version of PROMETHEE for the comparison of building products' design with ill-defined data on environmental quality. *European Journal of Operational Research*. Vol 109, Iss 2, pp 522-529.

Leleur, S., Salling, K. B., Jensen, A. V. (2004). *Modelling Decision Support and Uncertainty for Large Transport Infrastructure Projects: the CLG-DSS Model of the Oresund Fixed Link*. MUDSM: Managing Uncertainty in Decision Support Models. Coimbra, Portugal.

Lenzen, M., Lundie, S., Bransgrove, G., Charet, L., Sack, F. (2003). Assessing the Ecological Footprint of a Large Metropolitan Water Supplier: Lessons for Water Management and Planning towards Sustainability. *Journal of Environmental Planning and Management*. Vol 46, Iss 1, pp 113-141.

Leon, O. G. (1997). On the death of SMART and the birth of GRAPA. *Organizational behavior and human decision processes*. Vol 71, Iss 3, pp 249-262.

Lerche, R., Bruggemann, P., Sorensen, L., Carlsen, O., Nielsen, J. (2002). A comparison of partial order technique with three methods of multi-criteria analysis for ranking of chemical substances. *Journal of Chemical Information and Computer Sciences*. Vol 42, Iss 5, pp 1086-1098.

Leskinen, P., Kangas, A. S., Kangas, J. (2004). Rank-based modelling of preferences in multi-criteria decision making. *European Journal of Operational Research*. Vol 158, Iss 3, pp 721-733.

Levy, J. K., Hipel, K. W., Kilgour, D. M. (1998). *Sustainability indicators for multiple criteria decision making in water resources: an evaluation of soil tillage practices using Web-HIPRE*. XIVth International Conference on Multiple Criteria Decision Making. Charlottesville, Virginia. pp 433-444.

Levy, J. K., Hipel, K. W., Kilgour, D. M. (2000a). Using environmental indicators to quantify the robustness of policy alternatives to uncertainty. *Ecological Modelling*. Vol 130, Iss 1-3, pp 79-86.

Levy, J. K., Hipel, K. W., Kilgour, D. M. (2000b). Web-based multiple criteria decision analysis: web-HIPRE and the management of environmental uncertainty. *INFOR*. Vol 38, Iss 3, pp 221-244.

Leyva-Lopez, J. C., Fernandez-Gonzalez, E. (1999). A genetic algorithm for deriving final ranking from a fuzzy outranking relation. *Foundations of computing and decision sciences*. Vol 24, Iss 1, pp 33-47.

Leyva-Lopez, J. C., Fernandez-Gonzalez, E. (2003). A new method for group decision support based on ELECTRE III methodology. *European Journal of Operational Research*. Vol 148, Iss 1, pp 14-27.

Liu, D., Stewart, T. J. (2004). Object-oriented decision support system modelling for multicriteria decision making in natural resource management. *Computers & Operations Research*. Vol 31, Iss 7, pp 985-999.

Loomis, J. B. (2000). Environmental Valuation Techniques in Water Resource Decision Making. *Journal of Water Resources Planning and Management*. Vol 126, Iss 6, pp 339-344.

Lotfi, V., Stewart, T. J., Zionts, S. (1992). An aspiration-level interactive model for multiple criteria decision-making. *Computers and Operations Research*. Vol 19, Iss 7, pp 671-681.

Lundie, S., Peters, G. M., Beavis, P. C. (2004). Life Cycle Assessment for Sustainable Metropolitan Water Systems Planning. *Environmental Science & Technology*. Vol 38, Iss 13, pp 3465-3473.

Lundin, M., Morrison, G. M. (2002). A life cycle assessment based procedure for development of environmental sustainability indicators for urban water systems. *Urban Water*. Vol 4, Iss 2, pp 145-152.

Lutz, E., Munasinghe, M. (1994). Integration of environmental concerns into economic analyses of projects and policies in an operational context. *Ecological Economics*. Vol 10, Iss 1, pp 37-46.

Ma, J., Fan, Z., Huang, L. (1999). A subjective and objective integrated approach to determine attribute weights. *European Journal of Operational Research*. Vol 112, Iss 2, pp 397-404.

MacCrimmon, K. R. (1973). *An overview of multiple objective decision making*. In: Multiple Criteria Decision Making. Ed. J. Cochrane and M. Zeleny. University of South Carolina Press. pp 18-44.

Macharis, C., Springael, J., De Brucker, K., Verbeke, A. (2004). PROMETHEE and AHP: The design of operational synergies in multicriteria analysis. Strengthening PROMETHEE with ideas of AHP. *European Journal of Operational Research*. Vol 153, Iss 2, pp 307-317.

Mahmoud, M. R., Fahmy, H., Labadie, J. W. (2002). Multicriteria siting and sizing of desalination facilities with geographic information system. *Journal of Water Resources Planning and Management*. Vol 128, Iss 2, pp 113-120.

Mahmoud, M. R., Garcia, L. A. (2000). Comparison of different multicriteria evaluation methods for the Red Bluff diversion dam. *Environmental Modelling & Software*. Vol 15, Iss 5, pp 471-478.

Maier, H. R., Radbone, L., Finkemeyer, T., Hume, T., Butcher, M., Goonan, P. (2003). *Optimising the Mutual Information of Ecological Data Clusters using Genetic*

Algorithms. 28th Hydrology and Water Resources Symposium. Wollongong, Australia. pp 89-96.

Manache, G., Melching, C. S. (2004). Sensitivity Analysis of a Water-Quality Model Using Latin Hypercube Sampling. *Journal of Water Resources Planning and Management*. Vol 130, Iss 3, pp 232-242.

Mander, S., Gough, C. (2002). *Multi criteria evaluation of low carbon energy technologies*. iEMS 2002 Integrated Assessment and Decision Support. Lugano Switzerland. pp 157-162.

Mareschal, B. (1986). Stochastic multicriteria decision making and uncertainty. *European Journal of Operational Research*. Vol 26, Iss 1, pp 58-64.

Mareschal, B. (1988). Weight stability intervals in multicriteria decision aid. *European Journal of Operational Research*. Vol 33, Iss 1, pp 54-64.

Mareschal, B., Brans, J. P. (1988). Geometrical representations for MCDA. *European Journal of Operational Research*. Vol 34, Iss 1, pp 69-77.

Martin, N. J., St Onge, B., Waaub, J. P. (1999). An integrated decision aid system for the development of Saint Charles River alluvial plain, Quebec, Canada. *International Journal of Environment and Pollution*. Vol 12, Iss 2-3, pp 264-279.

Martin, W. E., Bender, H. W., Shields, D. J. (2000). Stakeholder objectives for public lands: ranking of forest management alternatives. *Journal of Environmental Management*. Vol 58, Iss 1, pp 21-32.

Mathevet, R., Bousquet, F., Le Page, C., Antona, M. (2003). Agent-based simulations of interactions between duck population, farming decisions and leasing of hunting rights in the Camargue (Southern France). *Ecological Modelling*. Vol 165, Iss 2-3, pp 107-126.

Maurice, B., Frischknecht, R., Coelho-Schwirtz, V., Hungerbuhler, K. (2000). Uncertainty analysis in life cycle inventory. Application to the production of electricity with French coal power plants. *Journal of Cleaner Production*. Vol 8, Iss 2, pp 95-108.

McDonald, G. W., Patterson, M. G. (2004). Ecological Footprints and interdependencies of New Zealand regions. *Ecological Economics*. Vol 50, Iss 1-2, pp 49-67.

McLaren, R. A., Siminovic, S. P. (1999). Data needs for sustainable decision making. *International Journal of Sustainable Development World Ecology*. Vol 6, Iss, pp 103-113.

Mendoza, G. A., Prabhu, R. (2003). Qualitative multi-criteria approaches to assessing indicators of sustainable forest resource management. *Forest Ecology and Management*. Vol 174, Iss 1-3, pp 329-343.

Merkhofer, M. W. (1999). *Assessment, refinement and narrowing of options*. In: Tools to aid environmental decision making. Ed. V. H. Dale, English, M.R. Springer. pp 231-284.

Merkhofer, M. W., Conway, R., Anderson, R. G. (1997). Multiattribute utility analysis as a framework for public participation in siting a hazardous waste management facility. *Environmental Management*. Vol 21, Iss 6, pp 831-839.

Miettinen, K. (2001). *Some methods for nonlinear multi-objective optimization*. Evolutionary multi-criterion optimization: first international conference. Zurich, Switzerland. pp 1-20.

Miettinen, K., Hamalainen, R. P. (1997). How to benefit from decision analysis in environmental life cycle assessment (LCA). *European Journal of Operational Research*. Vol 102, Iss 2, pp 279-294.

Miettinen, K., Lahdelma, R., Salminen, P. (1999). *SMAA-O - stochastic multicriteria acceptability analysis with ordinal criteria*. University of Jyvaskyla. Reports of the Department of Mathematical Information Technology, Series B, Scientific Computing.

Miettinen, K., Salminen, P. (1999). Decision-aid for discrete multiple criteria decision making problems with imprecise data. *European Journal of Operational Research*. Vol 119, Iss 1, pp 50-60.

Milutin, D., Bogardi, J. J. (2002). *Performance criteria for multiunit reservoir operation and water allocation problems*. In: Risk, reliability, uncertainty and robustness of water resources systems. Ed. J. J. Bogardi and Z. W. Kundzewicz. Cambridge University Press. pp 199-208.

Miranda, J. I. (2001). Multicriteria analysis applied to the sustainable agriculture problem. *International Journal of Sustainable Development World Ecology*. Vol 8, pp 67-77.

Mirrazavi, S. K., Jones, D. F., Tamiz, M. (2001). A comparison of genetic and conventional methods for the solution of integer goal programmes. *European Journal of Operational Research*. Vol 132, Iss 3, pp 594-602.

Mladineo, N., Margeta, J., Brans, J. P., Mareschal, B. (1987). Multicriteria ranking of alternative locations for small scale hydro plants. *European Journal of Operational Research*. Vol 31, Iss 2, pp 215-222.

Morrissey, A. J., Browne, J. (2004). Waste management models and their application to sustainable waste management. *Waste Management*. Vol 24, Iss 3, pp 297-308.

Moshkovich, H. M., Mechitov, A. I., Olson, D. L. (2002). Ordinal judgments in multiattribute decision analysis. *European Journal of Operational Research*. Vol 137, Iss 3, pp 625-641.

Moshkovich, H. M., Schellenberger, R. E., Olson, D. L. (1998). Data influences the result more than preferences: some lessons from implementation of multiattribute techniques in a real decision task. *Decision Support Systems*. Vol 22, Iss 1, pp 73-84.

Moss, R., Catt, D. C. (1996). The application of decision theory to sustainable management of an upland Scottish Estate. *Journal of Applied Statistics*. Vol 23, Iss 2-3, pp 211-231.

Mousseau, V. (1995). *Elicitating information concerning the relative importance of criteria*. In: Advances in Multicriteria Analysis. Ed. P. Pardalos, Y. Siskos and C. Zopounidis. Kluwer Academic. pp 17-43.

Mousseau, V., Figueira, J., Dias, L., Gomes da Silva, C., Climaco, J. (2003). Resolving inconsistencies among constraints on the parameters of an MCDA model. *European Journal of Operational Research*. Vol 147, Iss 1, pp 72-93.

Mousseau, V., Figueira, J., Naux, J. P. (2001). Using assignment examples to infer weights for ELECTRE TRI method: some experimental results. *European Journal of Operational Research*. Vol 130, Iss 2, pp 263-275.

Mousseau, V., Slowinski, R. (1998). Inferring an ELECTRE TRI model from assignment examples. *Journal of Global Optimization*. Vol 12, Iss 2, pp 157-174.

Mousseau, V., Slowinski, R., Zielniewicz, P. (1999). *ELECTRE TRI 2.0a Methodological Guide and User's Manual*. LAMSADE, Universite Paris-Dauphine.

Mousseau, V., Slowinski, R., Zielniewicz, P. (2000). A user-oriented implementation of the ELECTRE-TRI method integrating preference elicitation support. *Computers & Operations Research*. Vol 27, Iss 7-8, pp 757-777.

Muleta, M. K., Nicklow, J. W. (2005). Decision support for watershed management using evolutionary algorithms. *Journal of Water Resources Planning and Management*. Vol 131, Iss 1, pp 35-44.

Munda, G. (1995). *Multicriteria evaluation in a fuzzy environment: Theory and applications in ecological economics*. Physica-Verlag: Heidelberg, Germany.

Munda, G., Nijkamp, P., Rietveld, P. (1994). Qualitative multicriteria evaluation for environmental management. *Ecological Economics*. Vol 10, Iss 2, pp 97-112.

Mustajoki, J., Hamalainen, R. P., Marttunen, M. (2004). Participatory multicriteria decision analysis with Web-HIPRE: a case of lake regulation policy. *Environmental Modelling & Software*. Vol 19, Iss 6, pp 537-547.

Mysiak, J., Giupponi, C., Fassio, A. (2002). *Decision support for water resouce management: an application example of the MULINO DSS*. iEMS 2002 Integrated Assessment and Decision Support. Lugano Switzerland. pp 138-143.

Mysiak, J., Giupponi, C., Rosato, P. (2005). Towards the development of a decision support system for water resource management. *Environmental Modelling & Software*. Vol 20, Iss 2, pp 203-214.

Narasimhan, R., Vickery, S. K. (1988). An experimental evaluation of articulation of preferences in multi criterion decision-making (MCDM) methods. *Decision Sciences*. Vol 19, Iss 4, pp 880-888.

Netto, O. C., Parent, E., Duckstein, L. (1996). Multicriterion design of long-term water supply in southern France. *Journal of Water Resources Planning and Management*. Vol 122, Iss 6, pp 403-413.

Nijkamp, P., Rietveld, P., Voogd, H. (1990). *Multicriteria evaluation in physical planning*. North Holland: Amsterdam.

Nijkamp, P., Vos, J. B. (1977). A multicriteria analysis for water resource and land use development. *Water Resources Research*. Vol 13, Iss, pp 513-518.

Noghin, V. D. (1997). Relative Importance of Criteria: a Quantitative Approach. *Journal of Multi-Criteria Decision Analysis*. Vol 6, Iss 6, pp 355-363.

Nowak, M. (2004). Preference and veto thresholds in multicriteria analysis based on stochastic dominance. *European Journal of Operational Research*. Vol 158, Iss 2, pp 339-350.

Offringa, G., de Wet, G. (1996). A quality of life-based decision support model for the determination of water research priorities. *Water SA*. Vol 22, Iss 4, pp 359-371.

Olson, D. L., Mechitov, A., Moshkovich, H. (1998). *Multicriteria decision aid techniques: some experimental conclusions*. XIVth International Conference on Multiple Criteria Decision Making. Charlottesville, Virginia. pp 357-368.

Olson, D. L., Mechitov, A., Moshkovich, H. (2000). *Learning aspects of decision aids*. Multiple criteria decision making in the new millenium: proceedings of the 15th international conference on multiple criteria decision making. Ankara, Turkey. pp 41-48.

Olson, D. L., Moshkovich, H. M., Schellenberger, R., Mechitov, A. I. (1995). Consistency and accuracy in decision aids: experiments with four multiattribute systems. *Decision Sciences*. Vol 25, Iss 6, pp 723-748.

Onta, P. R., Gupta, A. D., Harboe, R. (1991). Multistep planning model for conjunctive use of surface and groundwater resources. *Journal of Water Resources Planning and Management*. Vol 117, Iss 6, pp 662-678.

Opricovic, S., Tzeng, G. H. (2004). Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*. Vol 156, Iss 2, pp 445-455.

Ostanello, A. (1983). *Outranking Methods*. International Summer School on Multiple Criteria Decision Making Methods, Applications and Software. Acireale, Italy. pp 41-60.

Ozelkan, E. C., Duckstein, L. (1996). Analysing water resources alternatives and handling criteria by multi criterion decision techniques. *Journal of Environmental Management*. Vol 48, Iss 1, pp 69-96.

Ozernoy, V. M. (1984). *Generating alternatives in multiple criteria decision making problems: a survey*. Decision making with multiple objectives: proceedings of the sixth international conference on multiple-criteria decision making. Ohio, USA. pp 322-330.

Ozernoy, V. M. (1987). *A framework for choosing the most appropriate discrete alternative multiple criteria decision-making method in decision support systems and expert systems*. Toward interactive and intelligent decision support systems: proceedings of the seventh international conference on multiple criteria decision making. Kyoto, Japan. pp 56-64.

Ozernoy, V. M. (1988). Some issues in designing an expert system for multiple criteria decision making. *Acta Psychologica*. Vol 68, Iss 1-3, pp 237-253.

Ozernoy, V. M. (1992). Choosing the best multiple criteria decision-making method. *INFOR*. Vol 30, Iss 2, pp 159-171.

Palisade. (2000). @Risk - Risk Analysis Add-In for Microsoft Excel. 4.5.2 - Professional Edition.

Paschetta, E., Tsoukias, A. (2000). A Real-World MCDA Application: Evaluating Software. *Journal of Multi-Criteria Decision Analysis*. Vol 9, Iss 5, pp 205-226.

Pictet, J., Maystre, L. Y., Simos, J. (1994). *Surmeasure: an instrument for representation and interpretation of ELECTRE and PROMETHEE methods results*. In: Applying multiple criteria aid for decision to environmental management. Ed. M. Paruccini. Kluwer Academic Publishers. pp 291-304.

Podinovski, V. V. (1999). A DSS for multiple criteria decision analysis with imprecisely specified trade-offs. *European Journal of Operational Research*. Vol 113, Iss 2, pp 261-270.

Poyhonen, M., Hamalainen, R. P. (2000). There is hope in attribute weighting. *INFOR*. Vol 38, Iss 3, pp 272-282.

Poyhonen, M., Hamalainen, R. P. (2001). On the convergence of multiattribute weighting methods. *European Journal of Operational Research*. Vol 129, Iss 3, pp 569-585.

Prato, T. (1999). Multiple attribute decision analysis for ecosystem management. *Ecological Economics*. Vol 30, Iss 2, pp 207-222.

Proctor, W. (2001). *Valuing Australia's Ecosystem Services Using a Deliberative Multi-criteria Approach*. European Society for Ecological Economics, Frontiers 1 Conference: Fundamental Issues of Ecological Economics. Cambridge, England.

Proctor, W., Drechsler, M. (2003). *Deliberative Multi-criteria Evaluation: A case study of recreation and tourism options in Victoria Australia*. European Society for Ecological Economics, Frontiers 2 Conference. Tenerife.

Proll, L. G., Salhi, A., Insua, D. R. (2001). Improving an optimization-based framework for sensitivity analysis in multi-criteria decision-making. *Journal of Multi-Criteria Decision Analysis*. Vol 10, Iss 1, pp 1-9.

Qureshi, M. E., Harrison, S. R. (2001). A decision support process to compare Riparian vegetation options in Scheu Creek catchment in North Queensland. *Journal of Environmental Management*. Vol 62, Iss 1, pp 101-112.

Qureshi, M. E., Harrison, S. R., Wegener, M. K. (1999). Validation of multicriteria analysis models. *Agricultural Systems*. Vol 62, Iss 2, pp 105-116.

Rahman, M., Hassan, M. Q., Islam, M. S. (2000). Environmental impact assessment on water quality deterioration caused by the decreased Ganges outflow and saline water intrusion in south-western Bangladesh. *Environmental Geology*. Vol 40, Iss 1-2, pp 31-40.

Rajabi, S., Hipel, K. W., Kilgour, D. M. (1999). Water supply planning under interdependence of actions: theory and application. *Water Resources Research*. Vol 35, Iss 7, pp 2225-2235.

Rajabi, S., Hipel, K. W., Kilgour, D. M. (2001). Multiple criteria screening of a large water policy subset selection problem. *Journal of the American Water Resources Association*. Vol 37, Iss 3, pp 533-546.

Raje, D. V., Dhobe, P. S., Deshpande, A. W. (2002). Consumer's willingness to pay more for municipal supplied water: a case study. *Ecological Economics*. Vol 42, Iss 3, pp 391-400.

Raju, K. S., Duckstein, L. (2004). Integrated application of cluster and multicriterion analysis for ranking water resources planning strategies: a case study in Spain. *Journal of Hydroinformatics*. Vol 6, Iss 4, pp 295-307.

Raju, K. S., Duckstein, L., Arondel, C. (2000). Multicriterion analysis for sustainable water resources planning: a case study in Spain. *Water Resources Management*. Vol 14, Iss 6, pp 435-456.

Raju, K. S., Kumar, D. N. (1998). *MCDMGSS: A group decision support system for multicriterion analysis*. International Conference on System Dynamics. Kharagpur, India. pp 67-73.

Raju, K. S., Kumar, D. N. (2004). Irrigation Planning using Genetic Algorithms. *Water Resources Management*. Vol 18, Iss 2, pp 163-176.

Raju, K. S., Pillai, C. R. S. (1999a). Multicriterion decision making in performance evaluation of an irrigation system. *European Journal of Operational Research*. Vol 112, Iss 3, pp 479-488.

Raju, K. S., Pillai, C. R. S. (1999b). Multicriterion decison making in river basin planning and development. *European Journal of Operational Research*. Vol 112, Iss 2, pp 249-257.

Raluy, R. G., Serra, L., Uche, J., Valero, A. (2004). Life-cycle assessment of desalination technologies integrated with energy production systems. *Desalination*. Vol 167, Iss 1, pp 445-458.

Randall, P., Brown, L., Deschaine, L., Dimarzio, J., Kaiser, G., Vierow, J. (2004). Application of the analytic hierarchy process to compare alternatives for the long-term management of surplus mercury. *Journal of Environmental Management*. Vol 71, Iss 1, pp 35-43.

Rauch, W. (1998). Problems of decision making for a sustainable development. *Water Science and Technology*. Vol 38, Iss 11, pp 31-39.

Ray, T. G., Triantaphyllou, E. (1999). Procedures for the evaluation of conflicts in rankings of alternatives. *Computers & Industrial Engineering*. Vol 36, Iss 1, pp 35-44.

Ridgley, M. A., Rijsberman, F. R. (1994). Multicriterion Analysis and the Evaluation of Restoration Policies for a Rhine Estuary. *Socio-Economic Planning Sciences*. Vol 28, Iss 1, pp 19-32.

Rietveld, P., Ouwersloot, H. (1992). Ordinal data in multicriteria decision making, a stochastic dominance approach to siting nuclear power plants. *European Journal of Operational Research*. Vol 56, Iss 2, pp 249-262.

Rijsberman, M. A., van de Ven, F. H. M. (2000). Different approaches to assessment of design and management of sustainable urban water systems. *Environmental Impact Assessment Review*. Vol 20, Iss 3, pp 333-345.

Ringuest, J. L. (1997). Lp-metric sensitivity analysis for single and multi-attribute decision analysis. *European Journal of Operational Research*. Vol 98, Iss 3, pp 563-570.

Rios Insua, D. (1990). *Sensitivity Analysis in Multi-objective Decision Making*. Springer-Verlag: Berlin.

Rios Insua, D., French, S. (1991). A framework for sensitivity analysis in discrete multi-objective decision-making. *European Journal of Operational Research*. Vol 54, Iss 2, pp 176-190.

Rogers, M., Bruen, M. (1998a). Choosing realistic values of indifference, preference and veto thresholds for use with environmental criteria within ELECTRE. *European Journal of Operational Research*. Vol 107, Iss 3, pp 542-551.

Rogers, M., Bruen, M. (1998b). A new system for weighting environmental criteria for use within ELECTRE III. *European Journal of Operational Research*. Vol 107, Iss 3, pp 552-563.

Rogers, M., Bruen, M. (2000). Using ELECTRE III to choose route for Dublin Port motorway. *Journal of Transportation Engineering*. Vol 126, Iss 4, pp 313-323.

Romero, C., Rehman, T. (1987). Natural resource mangement and the use of multiple criteria decision-making techniques: a review. *European Review of Agricultural Economics*. Vol 14, Iss 1, pp 61-89.

Rossi, G., Cancelliere, A., Giuliano, G. (2005). Case Study: Multicriteria Assessment of Drought Mitigation Measures. *Journal of Water Resources Planning and Management*. Vol 131, Iss 6, pp 449-457.

Rowe, M. D., Pierce, B. C. (1982). Sensitivity of the weighted summation decision method to incorrect application. *Socio-Economic Planning Sciences*. Vol 16, Iss 4, pp 173-177.

Roy, B. (1990). *The outranking approach and the foundations of ELECTRE methods*. In: Readings in multiple criteria decision aids. Ed. C. A. Bana e Costa. Springer-Verlag. pp 155-183.

Roy, B. (1991). The outranking approach and the foundations of ELECTRE methods. *Theory and Decision*. Vol 31, Iss 1, pp 49-73.

Roy, B., Bouyssou, D. (1985). *An example of comparison of two decision-aid models*. In: Multiple Criteria Decision Methods and Applications. Ed. J. Spronk and G. Fandel. pp 361-381.

Roy, B., Mousseau, V. (1996). A theoretical framework for analysing the notion of relative importance of criteria. *Journal of Multi-Criteria Decision Analysis*. Vol 5, Iss 2, pp 145-159.

Roy, B., Present, M., Silhol, D. (1986). A programming method for determining which Paris metro stations should be renovated. *European Journal of Operational Research*. Vol 24, Iss 2, pp 318-334.

Roy, B., Slowinski, R., Treichel, W. (1992). Multicriteria Programming of Water Supply Systems for Rural Areas. *Water Resources Bulletin*. Vol 28, Iss 1, pp 13-31.

Roy, B., Vanderpooten, D. (1997). An overview on "The European school of MCDA: Emergence, basic features and current works". *European Journal of Operational Research*. Vol 99, Iss 1, pp 26-27.

Roy, B., Vincke, P. (1981). Multicriteria analysis: survey and new directions. *European Journal of Operational Research*. Vol 8, Iss 3, pp 207-218.

Royal Assessment Commission. (1992). *Multi-criteria analysis as a resource assessment tool*. Research Paper. 49.

Saaty, T. L. (1977). A scaling method for priorities in hierarchical structures. *Journal of Mathematical Psychology*. Vol 15, Iss, pp 234-281.

Saaty, T. L. (1980). *The Analytic Hierarchy Process*. McGraw Hill: New York.

Salminen, P., Hokkanen, J., Lahdelma, R. (1998). Comparing multicriteria methods in the context of environmental problems. *European Journal of Operational Research*. Vol 104, Iss 3, pp 485-496.

Salo, A. A. (1995). Interactive decision aiding for group decision support. *European Journal of Operational Research*. Vol 84, Iss 1, pp 134-149.

Salo, A. A., Hamalainen, R. P. (1995). Preference programming through approximate ratio comparisons. *European Journal of Operational Research*. Vol 82, Iss 3, pp 458-475.

Salo, A. A., Hamalainen, R. P. (1997). On the Measurement of Preferences in the Analytic Hierarchy Process. *Journal of Multi-Criteria Decision Analysis*. Vol 6, Iss 6, pp 309-319.

Salo, A. A., Hamalainen, R. P. (2001). Preference Ratios in Multiattribute Evaluation (PRIME) - Elicitation and Decision Procedures under Incomplete Information. *IEEE Transactions on Systems, Man and Cybernetics*. Vol 31, Iss 6, pp 533-545.

Saltelli, A., Chan, K., Scott, E. M. (2000). *Sensitivity Analysis*. John Wiley and Sons: West Sussex, England.

Saltelli, A., Tarantola, S., Chan, K. (1999). A role for sensitivity analysis in presenting the results from MCDA studies to decision makers. *Journal of Multi-Criteria Decision Analysis*. Vol 8, Iss 3, pp 139-145.

Savic, D. A., Walters, G. A. (1997). Genetic Algorithms for Least-Cost Design of Water Distribution Networks. *Journal of Water Resources Planning and Management*. Vol 123, Iss 2, pp 67-77.

Schoemaker, P. J. H., Russo, J. E. (1994). *A pyramid of decision approaches*. In: Decision theory and decision analysis: trends and challenges. Ed. S. Rios. Kluwer Academic Publishers. pp 53-78.

Schoemaker, P. J. H., Waid, C. C. (1982). An experimental comparison of different approaches to determining weights in additive utility models. *Management Science*. Vol 28, Iss 2, pp 182-196.

Schonfeldt, C. (1999). *Future Water Resources for South Australia*. www.atse.org,au/publications/focus/focus-schonfeldt.html.

Setnes, M., Roubos, H. (2000). GA-Fuzzy Modeling and Classification: Complexity and Performance. *IEEE Transactions on Fuzzy Systems*. Vol 8, Iss 5, pp 509-522.

Shafike, N. G., Duckstein, L., Maddock, T. (1992). Multicriterion analysis of groundwater contamination management. *Water Resources Bulletin*. Vol 28, Iss 1, pp 33-43.

Sharifi, M. A., Rodriguez, E. (2002). Design and development of a planning support system for policy formulation in water resources rehabilitation: the case of Alcazar De San Juan District in Aquifer 23, La Mancha, Spain. *Journal of Hydroinformatics*. Vol 4, Iss 3, pp 157-175.

Shepetukha, Y., Olson, D. L. (2001). Comparative analysis of multiattribute techniques based on cardinal and ordinal inputs. *Mathematical and Computer Modelling*. Vol 34, Iss 1-2, pp 229-241.

Shirland, L. E., Jesse, R. R., Thompson, R. L., Iacovou, C. L. (2003). Determining attribute weights using mathematical programming. *Omega*. Vol 31, Iss 6, pp 423-437.

Simon, U., Bruggemann, R., Pudenz, S. (2004). Aspects of decision support in water management - example Berlin and Potsdam (Germany) II - improvement of managment strategies. *Water Research*. Vol 38, Iss 19, pp 4085-4092.

Simonovic, S. P., Bender, M. J. (1996). Collaborative planning-support system: an approach for determining evaluation criteria. *Journal of Hydrology*. Vol 177, Iss 3-4, pp 237-251.

Simonovic, S. P., Burn, D. H., Lence, B. (1997). Practical sustainability criteria for decision-making. *International Journal of Sustainable Development World Ecology*. Vol 4, Iss 4, pp 231-244.

Simonovic, S. P., Fahmy, H. (1999). A new modelling approach for water resources policy analysis. *Water Resources Research*. Vol 35, Iss 1, pp 295-304.

Simos, J. (1990). *Evaluer L'Impact sur L'Environment*. Presses Polytechniques et Universitaires Romandes: Lausanne.

Simpson, R. W., Petroeschevsky, A., Lowe, I. (2000). An Ecological Footprint Analysis for Australia. *Australian Journal of Environmental Management*. Vol 7, Iss 1, pp 11-18.

Siskos, J., Hubert, P. (1983). Multi-criteria analysis of the impacts of energy alternatives: a survey and a new comparative approach. *European Journal of Operational Research*. Vol 13, Iss 3, pp 278-299.

Siskos, J., Spyridakos, A. (1999). Intelligent multicriteria decision support: overview and perspectives. *European Journal of Operational Research*. Vol 113, Iss 2, pp 236-246.

Solymosi, T., Dombi, J. (1986). A method for determining the weights of criteria: the centralized weights. *European Journal of Operational Research*. Vol 26, Iss 1, pp 35-41.

Sonnemann, G. W., Schuhmacher, M., Castells, F. (2003). Uncertainty assessment by a Monte Carlo simulation in a life cycle inventory of electricity produced by a waste incinerator. *Journal of Cleaner Production*. Vol 11, Iss 3, pp 279-292.

Soofi, E. S. (1990). Generalized entropy-based weights for multiattribute value models. *Operations Research*. Vol 38, Iss 2, pp 362-363.

Stam, A., Sun, M., Haines, M. (1996). Artificial neural network representations for hierarchical preference structures. *Computers & Operations Research*. Vol 23, Iss 12, pp 1191-1201.

Stewart, T. J., Janssen, R., van Herwijnen, M. (2004). A genetic algorithm approach to multiobjective land use planning. *Computers & Operations Research*. Vol 31, Iss 14, pp 2293-2313.

Stewart, T. J., Scott, L. (1995). A scenario-based framework for multicriteria decision analysis in water resources planning. *Water Resources Research*. Vol 31, Iss 11, pp 2835-2843.

Stokes, S., Plummer, J. (2004). Using spreadsheet solvers in sample design. *Computational statistics & data analysis*. Vol 44, Iss 3, pp 527-546.

Svensson, E. (2000). Comparison of the Quality of Assessments Using Continuous and Discrete Ordinal Rating Scales. *Biometrical Journal*. Vol 42, Iss 4, pp 417-434.

Takeda, E. (2001). A method for multiple pseudo-criteria decision problems. *Computers & Operations Research*. Vol 28, Iss 14, pp 1427-1439.

Tan, R. R., Culaba, A. B., Purvis, M. R. I. (2004). POLCAGE 1.0 - a possibilistic life-cycle assessment model for evaluating alternative transportation fuels. *Environmental Modelling & Software*. Vol 19, Iss 4, pp 907-918.

Tecle, A. (1992). Selecting a multicriterion decision making technique for watershed resources management. *Water Resources Bulletin*. Vol 28, Iss 1, pp 129-140.

Tecle, A., Fogel, M., Duckstein, L. (1988). Multicriterion selection of wastewater management alternatives. *Journal of Water Resources Planning and Management*. Vol 114, Iss 4, pp 383-398.

Topcu, Y. I., Ulengin, F. (2004). Energy for the future: An integrated decision aid for the case of Turkey. *Energy*. Vol 29, Iss 1, pp 137-154.

Triantaphyllou, E. (1999). Reduction of pairwise comparisons in decision making via a duality approach. *Journal of Multi-Criteria Decision Analysis*. Vol 8, Iss 6, pp 299-310.

Triantaphyllou, E., Sanchez, A. (1997). A sensitivity analysis approach for some deterministic multi-criteria decision-making methods. *Decision Sciences*. Vol 28, Iss 1, pp 151-194.

Tzeng, G. H., Lin, C. W., Opricovic, S. (2005). Multi-criteria analysis of alternative-fuel buses for public transportation. *Energy Policy*. Vol 33, Iss 11, pp 1373-1383.

Tzeng, G. H., Tsaur, S. H., Laiw, Y. D., Opricovic, S. (2002). Multicriteria analysis of environmental quality in Taipei: public preferences and improvement strategies. *Journal of Environmental Management*. Vol 65, Iss 2, pp 109-120.

Ulengin, F., Topcu, Y. I., Sahin, S. O. (2000). *An artificial neural network approach to multicriteria model selection*. International conference on multiple criteria decision making. Ankara, Turkey. pp 101-110.

Ulengin, F., Topcu, Y. I., Sahin, S. O. (2001). An integrated decision aid system for Bosphorus water-crossing problem. *European Journal of Operational Research*. Vol 134, Iss 1, pp 179-192.

Ulvila, J. W., Seaver, D. A. (1982). Decision analysis for water resources planning. *Omega*. Vol 10, Iss 2, pp 185-194.

Vaidya, O. S., Kumar, S. (2006). Analytic hierarchy process: An overview of applications. *European Journal of Operational Research*. Vol 169, Iss 1, pp 1-29.

van Huylenbroeck, G. (1995). The conflict analysis method: bridging the gap between ELECTRE, PROMETHEE and ORESTE. *European Journal of Operational Research*. Vol 82, Iss 3, pp 490-502.

Vasquez, J., Maier, H. R., Lence, B., Tolson, B. A., Foschi, O. (2000). Achieving water quality system reliability using genetic algorithms. *Journal of Environmental Engineering*. Vol 126, Iss 10, pp 954-962.

Vetschera, R. (1994). McView: An integrated graphical system to support multi-attribute decisions. *Decision Support Systems*. Vol 11, Iss 4, pp 363-371.

Vincke, P. (1983). *Multiattribute utility theory as a basic approach*. International Summer School on Multiple Criteria Decision Making Methods, Applications and Software. Acireale, Italy. pp 27-40.

Vincke, P. (1999). Robust solutions and methods in decision-aid. *Journal of Multi-Criteria Decision Analysis*. Vol 8, Iss 3, pp 181-187.

Visual Decision. (2000). Decision Lab. 1.01.0386.

von Winterfeldt, D., Edwards, W. (1986). *Decision analysis and behavioral research*. Cambridge University Press: New York.

Vuk, D., Kozelj, B., Mladineo, N. (1991). Application of multicriterional analysis on the selection of the location for disposal of communal waste. *European Journal of Operational Research*. Vol 55, Iss 2, pp 211-217.

Wardlaw, R., Sharif, M. (1999). Evaluation of genetic algorithms for optimal reservoir system operation. *Journal of Water Resources Planning and Management*. Vol 125, Iss 1, pp 25-33.

Weistroffer, H. R., Narula, S. C. (1997). The state of multiple criteria decision support software. *Annals of Operations Research*. Vol 72, Iss 1, pp 299-313.

White, D. J. (1990). A bibliography on the applications of mathematical programming multiple-objective methods. *Journal of the Operational Research Society*. Vol 41, Iss 8, pp 669-691.

Wolters, W. T. M., Mareschal, B. (1995). Novel types of sensitivity analysis for additive MCDM methods. *European Journal of Operational Research*. Vol 81, Iss 2, pp 281-290.

Xu, X., Martel, J.-M., Lamond, B. F. (2001). A multiple criteria ranking procedure based on distance between partial preorders. *European Journal of Operational Research*. Vol 133, Iss 1, pp 69-80.

Yeh, C. H., Deng, H., Pan, H. (1999a). Multi-Criteria Analysis for Dredger Dispatching under Uncertainty. *The Journal of the Operational Research Society*. Vol 50, Iss 1, pp 35-43.

Yeh, C. H., Willis, R. J., Deng, H., Pan, H. (1999b). Task oriented weighting in multi-criteria analysis. *European Journal of Operational Research*. Vol 119, Iss 1, pp 130-146.

Yen, B. C. (2002). *System and component uncertainties in water resources*. In: Risk, reliability, uncertainty and robustness of water resources systems. Ed. J. J. Bogardi and Z. W. Kundzewicz. Cambridge University Press. pp 133-142.

Yu, C. (2002). A GP-AHP method for solving group decision-making fuzzy AHP problems. *Computers & Operations Research*. Vol 29, Iss 14, pp 1969-2001.

Yu, P. S., Yang, T. C., Chen, S. J. (2001). Comparison of uncertainty analysis methods for a distributed rainfall-runoff model. *Journal of Hydrology*. Vol 244, Iss 1, pp 43-59.

Zanakis, S. H., Solomon, A., Wishart, N., Dublish, S. (1998). Multi-attribute decision making: a simulation comparison of selected methods. *European Journal of Operational Research*. Vol 107, Iss 3, pp 507-529.

Zapatero, E. G., Smith, C. H., Weistroffer, H. R. (1997). Evaluating Multi-Attribute Decision Support Systems. *Journal of Multi-Criteria Decision Analysis*. Vol 6, Iss 4, pp 210-214.

Zhu, X., Dale, A. P. (2000). Identifying opportunities for decision support systems in support of regional resource use planning: an approach through soft systems methodology. *Environmental Management*. Vol 26, Iss 4, pp 371-384.

Zhu, X., Dale, A. P. (2001). JavaAHP: A web-based decision analysis tool for natural resource and environmental management. *Environmental Modelling & Software*. Vol 16, Iss 3, pp 251-262.

Zimmerman, H.-J. (2000). An application-oriented view of modeling uncertainty. *European Journal of Operational Research*. Vol 122, Iss 2, pp 190-198.

Zionts, S. (1983). *Multiple criteria mathematical programming: an overview and several approaches*. International Summer School on Multiple Criteria Decision Making Methods, Applications and Software. Acireale, Italy. pp 85-128.

Appendix A

Applications of MCDA

A number of surveys of MCDA applications are available in the literature, including:

- Romero and Rehman (1987) review 150 applications of MCDA techniques to planning and management problems in fisheries, forestry, water and land resources;
- White (1990) gives a bibliography on applications of multiple-objective methods which use no *a priori* explicit value function and are complex enough to require mathematical programming aids;
- Corner and Kirkwood (1991) produced a survey of applications of decision analysis that appeared in major English language operations research journals and other closely related journals from 1970 through 1989;
- Keefer *et al.* (2004) follow on from the survey of Corner and Kirkwood (1991) by providing an extensive review of decision analysis applications published from 1990 through 2001 in major English language operations research and closely related journals;
- Delgado and Sendra (2004) review different procedures of sensitivity analysis in GIS based MCDA projects; and
- Vaidya and Kumar (2006) present a literature review of 150 applications of the MCDA tool, AHP.

A survey of applications has also been undertaken as part of this research to provide a guide to relevant source material for practitioners facing a situation where decision analysis might potentially be applicable. In addition, the tables below (A1 and A2) show for each case study, where possible, the alternatives and criteria used to construct the effects table and the particular analytical technique(s) used. The tables have been separated into case studies that either do or do not involve water resource management decision problems. The case studies are presented in chronological order in each of the tables. The tables have been completed with the information available in the reference documents and where the data is not available, UNK appears in the cell. Where data is not required, NA appears in the cell. Further information on the case studies can be obtained from the cited documents.

Table A1 Applications of MCDA to water resource management decision problems

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
David and Duckstein (1976)	Alternative water resource systems to meet long-range goals	ELECTRE	Simplicity, usefulness shown in previous practical applications	5	12	1	Assign a certain number of points out of a maximum of 20	Quantitative criteria based on previous studies. Qualitative criteria on a five scale range (e.g. excellent to bad)	NA	Examined the influence of the threshold values on the outcome
Keeney and Wood (1977)	Water resource development plans	MAUT	UNK	5	12	1	Rank order of CWs, then trade-offs	From a previous paper. Used a 1-100 scale	NA	Increase cost and keep all other factors constant. Increase and decrease two CWs while remaining CWs are held constant

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Gershon *et al.* (1982)	Alternative river basin strategies	ELECTRE I & ELECTRE II	Allow for a ranking of a discrete set of systems with qualitative data	25	13	UNK	UNK	UNK	NA	Scales altered such that the same scale interval is used for all criteria and apply equal weights
Ulvila and Seaver (1982)	Meeting water supply needs	Additive MAUT	UNK	3	7	3	Trade-off	Experts	Scale 0 – 100	Weights altered
Onta *et al.* (1991)	Conjunctive use of surface and groundwater	CP	UNK	6	6	1	UNK	Simulation models	UNK	CWs altered, change in PVs for the optimal alternative
Rios Insua and French (1991)	Floodplain management	WSM	UNK	8	10	1	UNK	UNK	NA	Distance based approach for uncertainty in the CWs
Bardossy and Duckstein (1992)	Karstic aquifer management	Fuzzy CP	UNK	6	6	1	Fuzzy ordering technique	Data available	UNK	Not undertaken
Ridgley and Rijsberman (1994)	Evaluation of restoration policies for a Rhine Estuary	AHP (Expert Choice Software)	Ease of use & understanding by non-experts, ability to conduct sensitivity analysis & the nature of the computer software	7	17	UNK	No individual or collective weight sets were elicited or obtained	Models and qualitative scales	UNK	Changes in CWs based on 6 different viewpoints. Use of high and low cost estimates

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Abu-Taleb and Mareschal (1995)	Evaluate and select from a variety of potentially feasible water resources development options in Jordan	PROMETHEE V	UNK	42	18	1	UNK – based on previous study	Approximations	NA	Interval of CWs for which ranking does not change
Anand Raj (1995)	Most suitable planning of the reservoirs for development of the Krishna river basin	ELECTRE I & ELECTRE II	Requires only an interval scale, discrete alternatives	27	6	UNK	UNK	Reports	NA	Uniform CWs, 7 different sets of threshold values
Anand Raj and Kumar (1996)	Most suitable planning of the reservoirs for development of the river basin	ELECTRE I & ELECTRE II	Problem includes criteria with both quantitative and qualitative data and discrete alternatives	24	18	UNK	UNK	UNK	NA	Scales, thresholds and CWs altered
Netto *et al.* (1996)	Design a long-term water supply system	ELECTRE III	UNK	8	13	4	4 level scale (none, low, medium and high importance)	Experts, simulation models	NA	2 sets of CWs
Offringa and de Wet (1996)	Determination of low risk, applied water research priorities	SMART	Suited requirements best	UNK	55	4	Swing and ratio weighting	Scale of 0 to 100	UNK	UNK
Al-Kloub *et al.* (1997)	Rank water projects in Jordan	PROMETHEE	Software driven, user-friendly, provides direct interpretation of parameters, sensitivity analysis of results	72	24	UNK	Pair-wise comparison matrix (JAS software)	Experts	NA	Change CWs

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Al-Shemmeri *et al.* (1997a)	Variety of potentially feasible water resource development options in Jordan	PROMETHEE	Software driven, user-friendly, provides direct interpretation of parameters and the authors have considerable experience in using this technique	72	24	UNK	Pair-wise comparison matrix (JAS software)	Experts	NA	Change CWs and view changes in ranking of alternatives. Identify CW stability intervals
Bender and Simonovic (1997)	Alternative water resources development solutions	CP	UNK	6	8	6	Directly assigned by DM's	UNK	UNK	Not undertaken
Joubert *et al.* (1997)	Expansion of water provision to the greater Cape Town area	WSM (comparison with BCA)	Simplest approach	5	6	UNK	Swing weights	Scale of 0 – 100 assigned by stakeholders	UNK	Not undertaken – suggest assessing ranks with respect to PVs and CWs
Anand Raj and Kumar (1998)	Most suitable planning of reservoirs	Fuzzy MCDA	Demonstrating methodology developed	7	8	3	Actors assign fuzzy numbers between range of 0 - 10	Experts assign fuzzy numbers between range of 0 - 10	NA	Not undertaken
Levy *et al.* (1998)	Determine the best soil tillage strategy to ensure the sustainability of water resources systems in Ontario, Canada	Web-HIPRE	UNK	3	10	UNK	Direct weighting, value function, SMART, SWING, pair-wise	UNK	Convert to a value of 0 to 1	Not undertaken

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Al-Rashdan *et al.* (1999)	Selecting wastewater projects	PROMETHEE	User friendly, internal report selected PROMETHEE as the best method	12	15	1	Pair-wise comparison matrix (JAS software)	Calculated or subjective scale of 1 - 5	NA	Change CWs, weight stability intervals, types of threshold functions, threshold parameters
Martin *et al.* (1999)	Land use planning and management of the alluvial plain of Saint Charles River, Quebec	PROMETHEE (PROMCALC and GAIA)	Allows for individual as well as group DMs	8	11	12	Choice of Simos game cards or distribution of 100 points over criteria set	GIS	NA	GAIA
Rajabi *et al.* (1999)	Best combination of long-term water supply options	GP	Validity, acceptability, ease of use	12	7	UNK	Rate of penalty for unit deviation from the goal of each criterion obtained from interviews and estimations	Estimated based on expert judgment	UNK	Change of CWs
Raju and Pillai (1999a)	Selection of the best alternative in irrigation development	MAUT, STOPROM-2	DM interrogated in probabilistic terms to assess utility function for MAUT; STOPROM-2 is a probabilistic model where evaluations are real random variables following normal distribution function	5	8	2	Keeney & Raffia method for MAUT, AHP for STOPROM-2	Experts	Scale 0 – 100	16 combinations of scaling constants for each expert's case for MAUT, Orthogonal array for STOPROM-2
Flug *et al.* (2000)	Resources and flow alternatives presented in the EIS for the Glen Canyon Dam	Weighted Average	Most widely used technique	9	7	UNK	Survey - rank	Expert rating of 1 to 9	NA	Change CWs

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Lawrence *et al.* (2000)	Prioritise water infrastructure developments	Additive value function (Facilitator software)	Using software developed by authors	12	36	25	Importance order of criteria	Scoring system of 0 (max neg impact) to 10 (max positive impact) by experts	NA	Not undertaken
Choi and Park (2001)	Analysis of water privatisation scenarios in Korea	WSM	Most widely used method	3	25	4 groups	Ratio method	Interview with DM's, 5 scales from –2 (very negative) to 2 (very positive)	NA	Not undertaken
Eberhard and Joubert (2001)	Alternative water supply augmentation and water demand management options for the City of Cape Town	WSM (VISA software)	Simple method	13	19	1	Swing weights	0 – 100 scale, estimated and calculated	UNK	Three different weight sets
Jaber and Mohsen (2001)	Evaluation and selection of potential non-conventional water resources supply	AHP	Simplicity, used for many applications	4	5	UNK	Directly assigned (value of 1 to 9)	UNK	NA	Not undertaken
Kheireldin and Fahmy (2001)	National water strategy analysis for Egypt	CP	UNK	4	14	UNK	Rating method	Mathematical simulation model	Interval scale property method	6 sets of CWs
Lamy *et al.* (2002)	Watershed restoration planning	WSM	Few input requirements from the DM, it is flexible and easy to interpret	20	28	UNK	Trade-offs	GIS	UNK	Not undertaken

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Mahmoud *et al.* (2002)	Siting of desalination facilities	MAUT	UNK	7	3	1	UNK	Calculated in GIS, subjectively assigned	UNK	Not undertaken
Sharifi and Rodriguez (2002)	Evaluation of policies for aquifer management	WSM and ELECTRE II (DEFINITE software was used)	UNK	10	4	13	Direct rating (divide 100 points between criteria) and AHP	Estimated	Row-max and interval standardisation methods	Not undertaken
Raju and Duckstein (2004)	Alternative irrigation strategies	ELECTRE III	Ability to incorporate the fuzzy nature of decision making by using thresholds	12	10	1	Choose any intermediate values to minimise subjectivity in the weights	Scale of 0 – 100 used to rate the criteria (100 for highly important and 0 for unsatisfactory)	NA	Analysis performed with and without thresholds. Various combinations of thresholds used
Simon *et al.* (2004)	Water management strategies	HDT	Non-compensatory	9	4	0	NA	Simulation model	NA	UNK
Abrishamchi *et al.* (2005)	Urban water supply alternatives in Iran	CP	Easy to be explained and understood by the DMs	8	9	2	Five level scale (very low, low, medium, high, very high). Numerical values were substituted for the qualitative ratings of the weights	Two criteria were measured quantitatively and rest of criteria estimated subjectively and then substituted for numerical values	NA	Two sets of CWs for each DM and two sets of PVs

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Hajeeh and Al-Othman (2005)	Select the most appropriate desalination technology	AHP	Commonly utilised & due to lack of quantitative data	4	7	'Several'	Pair-wise comparisons	Experts were consulted	NA	Not undertaken
Rossi *et al.* (2005)	Drought mitigation measures	NAIDE	Discrete multi-criteria method. Allows either crisp, stochastic or fuzzy PVs	8	12	5	Preferences of each stakeholder with respect to each alternative on a scale of extremely bad to perfect	8 quantitative criteria from simulation model and 4 qualitative criteria measured on a scale of extremely bad to perfect (9 levels)	NA	Not undertaken
Comparison of Methods										
Duckstein *et al.* (1982)	River basin planning	ELECTRE, CP, MAUT	UNK	25	13	1	UNK	UNK	UNK	Parameter changes
Gershon and Duckstein (1983)	River basin planning	ELECTRE, CP, CGT, MAUT	Techniques have been shown to provide useful results for water resource management problems and represent substantially different approaches	25	13	UNK	Pair-wise comparisons	Experts	UNK	Scales and CWs are changed (i.e. CWs to equal weights), L_p distance measure is varied in CP
Tecle *et al.* (1988)	Wastewater management option	CP, ELECTRE I, CGT	UNK	15	12	1	UNK	UNK	UNK	2 sets of CWs for CP, 21 pairs of threshold values for ELECTRE I

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Goicoechea *et al.* (1992)	Water supply study to propose alternatives plans for meeting the long-range water supply needs of Washington DC	MATS-PC, EXPERT CHOICE, ARIADNE, ELECTRE	UNK	10	10	21	Ranking of 1 (worst) to 7 (best) for subjective criteria weights	Eight criteria on a 1 – 7 subjective scale, two criteria on data available	UNK	Not undertaken
Hobbs *et al.* (1992)	Compare alternative plans for meeting the long-range water supply needs of the Washington DC metro area	Holistic, GP, ELECTRE I, AHP, Additive Value Functions, Multiplicative utility functions	Compare methods that are widely used and represent divergent philosophies	10	10	21	Direct rating, indifference trade-off questions & probabilistic equivalence method	7 criteria with subjective ratings	UNK	Not undertaken
Shafike *et al.* (1992)	Groundwater contamination management	CP, ELECTRE II, MCQA II	UNK	15	3	0	Equal weights and variable selected set of weights	Groundwater model	NA	Not undertaken
Tecle (1992)	Selection of MCDA technique for watershed resources management problems	AHP, CTP, CP, CGT, DISID, ELECTRE, ESAP, GP, MAUT, MCQA, PROTR, Z-W, STEM, SWT, PROMETHEE	Authors familiarity with methods	15	24, four groups of criteria	UNK	Assigned a CW between 1 and 5	Subjective scale with a value ranging from 1 to 10, where 1 represents the worst case	NA	84 sets of possible combinations of the CWs
Duckstein *et al.* (1994)	Groundwater resources management problem	CP, ELECTRE III, MAUT, UTA	Investigate which is the most applicable method for groundwater management problems	13	3	1	UNK	Groundwater model	NA	Not undertaken

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Bella *et al.* (1996)	Water allocation conflict in the Upper Rio Grande Basin	ELECTRE III, CP	CP is a simple transparent approach, it requires little input from the DM. ELECTRE III allows flexibility provided by fuzzy outranking	30	18	1	UNK	Data obtained from a report	UNK	Different values for the balancing factor (p) in CP are used. Equal CWs
Ozelkan and Duckstein (1996)	Identify the satisfactory water resources projects being designed at the Austrian part of the Danube	PROMETHEE I, II, GAIA, MCQA I, II, III, CP, CGT	UNK	12	33	3	A total of 200 points are used for each weight set, and weights ranging from 1 to 30 are assigned to each criteria, 30 being very important	Verbal criteria are evaluated on a 5 point scale from very good to not satisfactory	Linear value function transformation	Not undertaken
Raju and Kumar (1998)	Select the best reservoir configuration, India	PROMETHEE II, EXPROM-2, CP	UNK	8	6	2	Based on a scale of 1 to 10	UNK	NA	Parameter changes
Raju and Pillai (1999b)	Select the best reservoir configuration, India	ELECTRE-II, PROMETHEE-II, AHP, CP, EXPROM-2	Different approaches	8	6	1	Scale of 0-10	Calculated and expert opinion	UNK	CWs and the dependent parameters of each method i.e. thresholds
Mahmoud and Garcia (2000)	Management alternatives for the operation of a diversion dam	WA, ELECTRE II, AHP, PROMETHEE II, CP	Wide application to different problems in the water resources area	11	8	1	Equal CWs (to be assigned by DM later when MCDA method selected)	Experts	Two methods presented in paper. Does not state which one actually used	Different CWs tested

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Raju *et al.* (2000)	Sustainable water resources study of an irrigation area	PROMETHEE-II, EXPROM-2, ELECTRE-III, ELECTRE IV, CP	UNK	7	10	3 groups	UNK	Experts (A = high / very cheap. E = very poor / very low)	NA	Change thresholds, criterion functions, distillation coefficients, CWs

Table A2 Applications of MCDA to non-water resources decision problems

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Nijkamp and Vos (1977)	Alternative land reclamation projects	Concordance Analysis	UNK	5	12	UNK	Interval scale and then average of all actors CWs	Publications and technical investigations	UNK	Not undertaken
Siskos and Hubert (1983)	Energy alternatives	ELECTRE III	UNK	6	11	4	CWs first given for groups of criteria and then each of the groups was weighted	Triangular fuzzy numbers	NA	Not undertaken
Brans *et al.* (1986)	Hydroelectric power station location	PROMETHEE I and II	UNK	6	6	1	Equal weights	UNK	NA	Small deviations on thresholds. 300 random variations of the thresholds of generalized criteria with linear preference and indifference area
Mareschal (1986)	Power-plant projects	PROMETHEE	UNK	4	3	6	Equal CWs	Experts assigned values of 1 to 5	NA	Not undertaken

Citation	**Application**	**MCDA Technique(s)**	**Why Chose MCDA Technique(s)**	**# Alts**	**# Criteria**	**# DMs**	**CW Method**	**PV Method**	**PV Standardisation Method**	**Sensitivity Analysis**
Roy *et al.* (1986)	Renovation of Paris metro stations	ELECTRE III	Seemed to be the best suited to this type of multi-criterion decision analysis	224	7	Not small	Order criteria, comparison of criteria, range of variation of CWs assigned	Surveys, observations, qualitative scales, experts judgment	NA	5 variables chosen (including threshold values and CWs) corresponding to 32 sets of values
Mladineo *et al.* (1987)	Location of small scale hydro plants	PROMETHEE I and II (use XPROM 2 software)	UNK	6	9	1	UNK	Calculations & measurements for quantitative criteria and estimate for qualitative	NA	Not undertaken
Bana e Costa (1988)	Comparing alternative settlement patterns for Sintra, Portugal	WSM	UNK	5	3	1	No CWs assigned	Estimated, calculated	(Max – value) / (Max – Min)	TRIDENT & OUTWEIGH methods to explore the CWs space
Barron and Schmidt (1988)	Selection amongst US cities	MAV	UNK	15	9	1	Equal CWs	Based on a composite index from fairly 'objective' components	Rescaled so that between 0 and 1	What CWs equate the highest ranked alternative with the remaining alternatives
Mareschal (1988)	Location of hydro-electric power plant	PROMETHEE II	UNK	4	4	1	Equal CWs	UNK	NA	Stability intervals of CWs
Mareschal and Brans (1988)	Car selection	PROMETHEE II and GAIA	Demonstrate GAIA	28	6	1	Equal CWs	UNK	NA	GAIA
Janssen and van Herwijen (1989)	Changing use of agricultural land	WSM, Ideal Point Method	UNK	118	24	UNK	Direct estimation, Saaty procedure	UNK	Standardised between 0 and 100	Not undertaken

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Briggs *et al.* (1990)	Nuclear waste management	PROMETHEE I and II	Obtain a complete ordering of the alternatives, difference in PVs should be included in the analysis, independent on the scales chosen for representing the various criteria, easily understood by the DM	27	4	1	Assigned equal weights. Doesn't say how decided on generalised criterion functions or initial thresholds	Models, assumptions	Normalised to a maximum of 1	Thresholds of preference functions and CWs – random alterations i.e. weight of each criterion is doubled to two, while the other criteria keep their weight equal to one
Barda *et al.* (1990)	Thermal power plant siting	ELECTRE III	UNK	24	14	3	Reflect attitude of the DMs – arbitrary, value of 1 to 7	Calculations, data available, expert opinion	NA	With and without veto thresholds
Vuk *et al.* (1991)	Location for the disposal of communal waste	PROMETHEE I & II (software PROMCALC was utilised)	UNK	5	15	1	UNK	Ordinal scale – values obtained empirically & expert estimation	NA	GAIA – weight stability intervals

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Hokkanen and Salminen (1994)	Choice of a solid waste management system	ELECTRE III	Discriminating power of the criteria is fuzzy	11	8	45	Scale criteria from 1 to 7, 7 being most important. Secondly, give number 1 to least important criterion and then base the other importance value on how many times more important they appeared than the least important criterion	Calculated based on data available, expert questionnaire, estimations	NA	Change threshold values and CWs by certain steps
Foltz *et al.* (1995)	Cropping systems	MAUT	UNK	72	19	1	Rank order	Model output	UNK	Not undertaken
van Huylenbroeck (1995)	Purchase of a tractor	CAM (combines preference function approach of ELECTRE and PROMETHEE with the conflict analysis test of ORESTE)	Demonstrating methodology	8	10	1	Rank order	Previous information and subjective analysis	NA	Change preference functions, different rank order sets of CWs
Wolters and Mareschal (1995)	Heat exchanger networks	PROMETHEE	UNK	10	10	1	UNK	UNK	NA	Methods as described in paper and Chapter 3 of thesis
Alho *et al.* (1996)	Suitability of various forest plans for Black Grouse (valued game bird)	AHP	Used with forest management previously	10	5	15	Pair-wise comparisons	Model	UNK	Regression analysis on CWs

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Ascough *et al.* (1996)	Assess two potential graphical user interfaces for a DSS	WSM	UNK	2	21	UNK	Pair-wise comparisons	Actors assign values in a questionnaire on a range of 1 to 10. Average value was then used	NA	Not undertaken
Fleming and Daniell (1996)	Assessment of disposal basin options for Woolpunda groundwater interception scheme	MESA	UNK	3	24	UNK	UNK	Environmental impact statement	UNK	Not undertaken
Janssen (1996)	Nuclear power plant location	Evamix	UNK	9	15	1	Expected value method used to transform qualitative information on CWs into quantitative CWs	8 of the criteria with qualitative scale on range of 1 to 5	UNK	MCS analysis of CWs and PVs as described in reference and Chapter 3
Al-Shemmeri *et al.* (1997b)	Selection of an MCDA technique for water development projects	PROMETHEE	UNK	16	24	1	JAS software	Subjective scale of 1 – 10 or rated as 1 or 0	NA	Not undertaken
Butler *et al.* (1997)	Site selection for a coal power plant	MAUT	Provides a logical and tractable means to make trade-offs among conflicting objectives	13	6	1	UNK	UNK	UNK	Simulation using random CWs, rank order CWs and response distribution CWs

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Georgopoulou *et al.* (1997)	Renewable energy sources	ELECTRE III	Widely applied in environmental and energy management problems	8	15	8	Criteria ranked by order of decreasing importance then descending numerical values were assigned following the Simos (1990) approach	Quantitative values calculated or obtained from recent studies and publications	NA	Exclusion of the criterion of visual amenity, less strict criteria thresholds, exclusion of the criterion of safety in covering peak load demand
Hokkanen and Salminen (1997c)	Location of a waste treatment facility	PROMETHEE I and II (PROMCALC and GAIA software)	Outranking methods have been developed for situations where it is not possible to acquire exact preference information from the actors. Get some insight into their applicability to real problems	4	14	28	Evaluate the importance of each criteria on a scale of 1 to 7 and median of CWs used	Estimations, 10 point scales, measurements	NA	Weight stability intervals using GAIA
Karagiannidis and Moussiopoulos (1997)	Municipal solid waste management	ELECTRE III	Includes some aspects that are often neglected by other methods and yields relatively stable results	5	24	UNK	Indifference and preference thresholds quantified by researchers. 2 sets of CWs used – same magnitude to criteria of same nature	Expert consultation	NA	Random change in CWs

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Hokkanen and Salminen (1997a)	Choosing a solid waste management system	ELECTRE III	Proved useful decision aids in various real applications, able to handle imprecise data, number of DMs does not limit methodology, quick and easy to use, need as little preference information as possible	22	8	113	(1) assign CWs ranging from 1 to 7, (2) assign number 1 to least important criterion and then base the other importance values on how many times more important they appeared than the least important criterion. Final CWs based on majority	Calculations based on available data, expert estimation	NA	Changes in threshold values and CWs
Merkhofer *et al.* (1997)	Siting a hazardous waste management facility	MAUT	UNK	5	7	9	UNK	Experts and stakeholders	NA	Not undertaken
Dyer *et al.* (1998)	Disposition of surplus weapons-grade plutonium	MAUT (Logical Decisions software package)	UNK	13	9	UNK	Trade-offs	Experts	UNK	Varying CWs (single CW only and random simulation of CWs), form of the multi-attribute utility function (i.e. additive versus multiplicative) and assumptions in the base-case analysis

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Georgopoulou *et al.* (1998)	Renewable energy technologies	PROMETHEE I and II	One of the most known and widely applied outranking methods. Method follows a transparent computational procedure that can be widely understood by DMs	6	8	11	Hierarchical ranking of criteria	Combination of qualitative and quantitative values based on data available	NA	Stricter indifference and preference thresholds were assigned to all economic criteria
Haastrup *et al.* (1998)	Urban waste management	NAIADE	UNK	5	5	10	Linguistic variables	UNK	UNK	Not undertaken
Lahdelma *et al.* (1998)	General plan application	SMAA	Illustrating methodology developed and presented in paper	6	4	UNK	None – method determines range of weights which supports preference of each alternative	Values and their uncertainties – doesn't say how these values were determined	UNK	Not undertaken
Hokkanen *et al.* (1999)	Different options for developing Helsinki Harbour	SMAA	Method where no preference information is required from DMs	13	11	UNK	Set of favourable weight vectors	Scale estimation based on expert evaluation, calculations, available data	UNK	Assessment of weight space using SMAA
Podinovski (1999)	Options to reduce air pollution caused by coal-fired thermal power plants	WSM (use DAM software)	UNK	11	10	1	UNK	Model	UNK	Visual sensitivity which shows continuously changing results of analysis when one of the trade-off ranges is being changed by a scroll bar

Citation	**Application**	**MCDA Technique(s)**	**Why Chose MCDA Technique(s)**	**# Alts**	**# Criteria**	**# DMs**	**CW Method**	**PV Method**	**PV Standardisation Method**	**Sensitivity Analysis**
Yeh *et al.* (1999a)	Dredger dispatching	Fuzzy MAUT	Demonstrating method developed	12	5	1	5 scale linguistic values from very unimportant to very important	5 scale linguistic values from very low to very high	NA	UNK
Hokkanen *et al.* (2000)	Finalists for competition on cleaning polluted soil	SMART, ELECTRE III, SMAA-2	UNK	9	5	UNK	AHP	Interval of values by experts for 4 criteria, value taken directly from contract offers for cost criterion	UNK	SMAA-2 (uniform distribution of CWs and PVs)
Kangas *et al.* (2000)	Landscape ecological forest planning	Bayesian analysis	UNK	10	6	1	AHP	Results of optimisation calculations & expert opinion	AHP	CWs of two criteria
Martin *et al.* (2000)	Development of leasable minerals in a forest	MAVF	UNK	6	4	3	Swing weights	Mid value splitting technique was used to elicit the attribute value functions	UNK	Not undertaken
Paschetta and Tsoukias (2000)	Selection of GIS software	ELECTRE TRI	Presence of ordinal information requiring 'repeated sorting' of the alternatives	6	6	4	UNK	UNK	NA	Not undertaken

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Rogers and Bruen (2000)	Selecting best route for section of motorway in Dublin	ELECTRE III	Permits a general ordering of alternatives, even in cases where individual pairs of options cannot be directly compared because of insufficient information to distinguish between them. Able to use mix of both quantitative and qualitative data	8	7	3	Questionnaire based method in which pair-wise voting is used to gauge the relative importance of criteria	Models, estimations, qualitative and quantitative	NA	Two adjusted sets of CWs were tested
Anderson *et al.* (2001)	Setting phosphorus targets for Lake Erie	MAVT	UNK	4	6	2	Trade-off method, which required respondents to express explicitly how much of the range of one criteria they were willing to give up to receive a specified improvement in another. Also asked to spread 100 points among the criteria in proportion to their relative importance	Model, expert estimation	UNK	CWs by applying confidence intervals and one aspect of model structure
Miranda (2001)	Compare existing farming systems	WSM	Intuitive, will lead to a unique choice	4	30	3	UNK	UNK	NA	Not undertaken

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Qureshi and Harrison (2001)	Compare Riparian vegetation options	AHP and DEFINITE Software	UNK	4	12	5	Pair-wise comparisons by survey	Experts by subjective estimates (scales 1 to 3 and −1 to −3)	5 methods available in DEFINITE software	Not undertaken
Shepetukha and Olson (2001)	Siting a new facility	SMART and WSM	UNK	7	4	1	Swing weighting & centroid weighting	3 quantitative criteria and one qualitative given a value of poor to great	NA	Equal weights and greater dispersion in weights
Ulengin *et al.* (2001)	Selecting most suitable water crossing structure	PROMETHEE I and PROMETHEE II	Easier to use than ELECTRE, threshold values have a significant meaning, more stable than ELECTRE, easily used in spreadsheets, better performance than AHP, user-friendly, applied successfully in real-life planning problems	9	10	19	Normalised values of centrality of cognitive map	1 – 9 scale where 1 = low and 9 = extremely high	NA	Compute intervals of the CWs for which the first rank of the complete pre-order among alternatives does not change, all other factors remaining unchanged
Bertrand-Krajewski *et al.* (2002)	Most appropriate cleansing frequency for a stormwater facility	ELECTRE II & ELECTRE III	UNK	4	4	1	UNK	Simulation model	NA	Changes in threshold values. Assess uncertainty in the PVs when using ELECTRE III

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Jessop (2002)	Prioritisation of an IT budget	WSM	Simple, transparent	34	6	9	A value up to 6	Value of 0 (worst) – 5 (high)	UNK	Test changes to CWs randomly. MCS with uniform CWs
Lahdelma *et al.* (2002)	Location for a waste treatment facility	SMAA-O	UNK	4	17	0	UNK	Experts assigned ordinal scales for each criterion	NA	Put more weight on criteria with shared ranks
Lawrence *et al.* (2002)	Land management scenarios	WSM (Facilitator software)	UNK	4	7	UNK	Importance order of criteria	Score within the range 0 to 10 (with 10 representing max benefit or min impact) by experts	NA	Equal importance of CWs, equal and greatest importance to the environmental and social factors and less importance to the economic issues and assigning the greatest importance to the economic issues
Mander and Gough (2002)	Wind energy	WSM	Simple, transparent	6	10	UNK	Participants are asked to weight the criteria according to their perspective and the perceived importance of the criteria	Information provided and expert opinion	UNK	Not undertaken

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Tzeng (2002)	Strategies for improving air quality in Tapei	CP (VIKOR) and TOPSIS	UNK	9	7	15	UNK	Expert evaluation by using linguistic values and transformed by scaling into numbers	NA	CW stability intervals
Georgopoulou *et al.* (2003)	Potential CO_2 reduction measures	ELECTRE Tri	Allows the introduction of thresholds, provides the possibility of assigning potential alternatives into pre-defined categories	37	5	10	Three different sets regarding the relative importance coefficients were formulated (economic, environment orientated, realistic)	Data available	NA	Increased values of both thresholds by +10%, +30% and +50% and decreased values of both thresholds by –10%, -30% and –50%
Haralambopoulos and Polatidis (2003)	Renewable energy project selection	PROMETHEE II	The outranking method PROMETHEE is considered to be well suited for energy decision problems including ease of use and decreased complexity	4	5	4	Didn't apply a formal method	Expert judgment	NA	Not undertaken
Proctor and Drechsler (2003)	Recreation and tourism options in Victoria	PROMETHEE (ProDecX)	UNK	5	15	UNK	Citizen's Jury – split 100 cannelini beans between criteria	Experts with a range of quantitative and qualitative criteria	NA	Systematically varied CWs and PVs with ranges identified
Herath (2004)	Incorporating community objectives in improved wetland management	AHP	UNK	5	3	221 (3 groups)	Survey of pair-wise comparisons	UNK	UNK	Change CWs

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Huth *et al.* (2004)	Assess rain forest growth results	MAVT and exploratory analysis	UNK	64	18	UNK	UNK	Simulations and literature	UNK	Not undertaken
Jessop (2004)	Selection of a project for investment by a council	UNK	UNK	34	6	9	Rank criteria and equal CWs	5 point scale from worst to best	NA	MCS with uniform distribution of CWs, all CWs set to 0.17
Nowak (2004)	Computer development projects	ELECTRE III	UNK	10	4	1	Assume values which sum to 1	The evaluation of alternatives with respect to criteria are expressed in the form of probability distributions	NA	UNK
Randall *et al.* (2004)	Compare alternatives for the long-term management of surplus mercury	AHP (Expert Choice Software)	Widely used	11	13	UNK	Equal CWs & pair-wise comparisons	Reports, EIS, workshops	UNK	Change CWs for 12 cases
Topcu and Ulengin (2004)	Renewable energy project selection	PROMETHEE I & II	Algorithm that is not explained	7	5	1	UNK	UNK	NA	CWs (but does not describe how undertaken)
Almasri and Kaluarachchi (2005)	Optimal management of nitrate concentration of aquifers	IOC	Conceptually simple and easy to program	4	14	UNK	UNK	Modelling	Normalised	Not undertaken

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Geneletti (2005)	Viability of the remnant patches of natural ecosystems	MAVF	UNK	Applied to a spatial map	3	1	Swing technique	Expert opinion	UNK	None undertaken
Hodgkin *et al.* (2005)	Potential future for a daycare centre	MAVT (VISA & AVID software)	Demonstrate software	9	9	4	UNK	UNK	UNK	Principle components analysis
Khalil *et al.* (2005)	Hydrothermal conditions for sludge destruction	PROMETHEE & GAIA	UNK	48	7	1	Equal weights	Experiments, all quantitative data	NA	Not undertaken
Comparison of Methods										
Roy and Bouyssou (1985)	Siting of a nuclear power plant	MAUT, ELECTRE III	UNK	9	6	6	Lottery comparisons, indices of importance	Experts	UNK	100 different sets of parameters
Narasimhan and Vickery (1988)	Determine salaries	AHP and Z-W	Differ substantially in the way preferences are articulated and could be implemented on a microcomputer	UNK	6	28	Trade-offs	UNK	UNK	Not undertaken
Karni *et al.* (1990)	Evaluating bank branches	AHP, SAW, ELECTRE, WLAM	UNK	10	18	3	AHP	Bank database	UNK	Not undertaken
Larichev *et al.* (1993)	Job selection	ZAPROS, AHP, preference cones	UNK	30	5	28	UNK	UNK	UNK	UNK
Hobbs and Meier (1994)	Hypothetical portfolios of resources for the year 2010	Holistic, additive value functions, GP	UNK	12	12	12	Direct selection of weights on a scale of 0-100, trade-off weighting	UNK	UNK	Not undertaken
Olson *et al.* (1995)	Job selection	Logical Decision, DECAID, Expert Choice, ZAPROS	UNK	8	4	21	Direct, trade-offs	UNK	UNK	Not undertaken

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Corner and Kirkwood (1997)	Experiment using production planning problem	SMART, Z-W, NAIVE	UNK	7	3	84	Equal CWs, rank order sum CWs, SMART	UNK	UNK	UNK
Hobbs and Horn (1997)	Electricity utility planning	Holistic, additive value function	UNK	20	9	16	Ratio-questioning-swing-weighting hybrid and trade-off assessment	Expert judgment, models	A base range was selected for each criterion to be used as the denominator. The worst value was subtracted from the PV and the result divided by the base range	Not undertaken
Hokkanen and Salminen (1997b)	Choosing a solid waste management system in two regions	ELECTRE III and ELECTRE IV	UNK	22 (1) & 11 (2)	8	113 (1) & 45 (2)	Each DM defined CWs (no details provided). Final CWs were determined by way of majority	UNK – detailed in unpublished reference	NA	Equal CWs, discordance removed
Moshkovich *et al.* (1998)	Selection of new faculty members	SMART and ZAPROS	ZAPROS was selected because it is based on an additive value function, CWs are not directly elicited, preference is elicited through pair-wise comparisons	45	6	2	Rank order of criteria, weights were assigned in accordance with rank order	Evaluated on a 10 point scale	NA	Not undertaken
Salminen *et al.* (1998)	Land use planning problem	ELECTRE III, PROMETHEE I, II and SMART	Number of DMs is large, possibility to obtain preference information is limited, number of criteria is great	3	10	54	Equal values for the thresholds. CW method not provided.	UNK	NA	UNK

Citation	Application	MCDA Technique(s)	Why Chose MCDA Technique(s)	# Alts	# Criteria	# DMs	CW Method	PV Method	PV Standardisation Method	Sensitivity Analysis
Easley *et al.* (2000)	Allocation of sales territory to existing sales staff in a drug company	AHP, hierarchical pair-wise voting procedure, folded normal AHP, ordinal pair-wise voting method	UNK	5	UNK	20	Ordinal and ratio scaled	UNK	UNK	UNK
Bell *et al.* (2001)	Climate policy alternatives	Additive linear value function, additive non-linear value function, GP, ELECTRE I, fuzzy sets, linear utility function, non-linear utility function, Min Max Regret, Stochastic Dominance, holistic	Useful for evaluating discrete alternatives, can be conveyed to those unfamiliar with decision analysis and have been widely applied elsewhere	7	6	21	Point allocation, hierarchical point allocation, swing weighting / AHP, trade-off weighting	Values and min and max ranges, from model outputs	UNK	MCS using probabilistic inputs. Linear programming based methods to incorporate ranges of CWs
Kangas *et al.* (2001b)	Forest management	MAVT (HIPRE), ELECTRE III, PROMETHEE II	ELECTRE III and PROMETHEE II were chosen to test the applicability of outranking methods to forestry. ELECTRE III was chosen because ELECTRE II is an older version.	6	17	4	UNK	UNK	Converted to relative partial values, which were scaled on a fixed interval between 0 and 1 (100%)	Multi-attribute value function was changed. Three different threshold combinations were tested using ELECTRE
Lahdelma and Salminen (2002)	Randomly generated test problems	SMAA and SMAA-3	The methods are based on an additive utility function and an outranking model	4, 6, 8 10, 12	2, 3, …, 12	UNK	NA	UNK	UNK	UNK

Citation	**Application**	**MCDA Technique(s)**	**Why Chose MCDA Technique(s)**	**# Alts**	**# Criteria**	**# DMs**	**CW Method**	**PV Method**	**PV Standardisation Method**	**Sensitivity Analysis**
Opricovic and Tzeng (2004)	Mountain climbing destination	TOPSIS, VIKOR	UNK	3	2	1	Equal weights	Subjective scale of 1 to 5 for criterion 1 and quantitative value for criterion 3	Linear normalisation and vector normalisation	Not undertaken
Tzeng *et al.* (2005)	Alternative fuel buses for public transport	TOPSIS, VIKOR	UNK	12	11	4	AHP, average CWs utilised	Expert evaluation by questionnaires and previous data	UNK	Not undertaken

Appendix B

Description of MCDA techniques

As discussed in Section 2.5.4 of this thesis, there is an abundance of MCDA techniques available and the existing MCDA techniques differ in the type of information they require, the methodology they are based on, the sensitivity tools they offer and the mathematical properties they verify (Mysiak *et al.*, 2005). Divergent schools of thought have developed, emphasising different techniques and, more generally, different attitudes as to the way of supporting or aiding decision making (Mysiak *et al.*, 2005; Roy and Vanderpooten, 1997). Some of the techniques from the divergent schools of thought are described below and references are provided for more information to be obtained if required. A useful summary of MCDA techniques, and the input information they require, is contained in Guitouni and Martel (1998).

B1 Outranking techniques

Outranking methods represent the European school of thought among MCDA techniques. Many different outranking methods have been presented, some of them with various versions for different kinds of decision-aid purposes. The outranking approach to MCDA builds a relation, called an outranking relation, given the information available. It is a MCDA model that uses various mathematical functions to indicate the degree of dominance of one alternative over another. Outranking methods facilitate comparison between alternatives by ascribing initial weights to decision criteria, then varying these weights as part of a sensitivity analysis, if their exact value is not known. Comparison between alternatives proceeds on a pair-wise basis with respect to each decision criterion, and establishes the degree of dominance or outranking of one option over another. The result is a ranking of the various alternatives (Rogers and Bruen, 1998a).

Outranking MCDA methods do not look for a pareto-optimal solution, but aid the decision process by ranking alternatives. Criteria in outranking, unlike value focused approaches, are non-compensatory which means that a poor performance of an alternative on a criterion cannot be compensated by a greater performance on another. The basic principle of outranking is that, providing that alternative *a* performs better than alternative *b* on a majority of criteria and that there is no criterion such that *b* is strongly better than *a*, then *a* will be preferred over *b*. The assessment of the alternatives is based on what are called pseudo-criteria. Pseudo-criteria are formed using two different threshold values, the indifference and preference threshold, that describe the priority difference between the criterion values of two alternatives. If the difference with regard to a criterion is less than the indifference threshold, the alternatives are considered to be indifferent in regard to that criterion. If the difference is larger than the preference threshold, the alternative that is regarded better with respect to the criterion in question is considered to be better without any doubt. If the difference is larger than the indifference threshold, but less than the preference threshold, priority between alternatives is uncertain.

Of outranking methods, the PROMETHEE and ELECTRE methods have been applied widely and are described in more detail below. In addition, the NAIADE method and the Hasse Diagram Technique are also described.

B1.1 ELECTRE

Concordance and discordance analysis are used in the ELECTRE methods. The idea of ELECTRE is to choose alternatives which are (i) dominant on most of the criteria and (ii) do not cause an unacceptable level of discontent for any one criterion. The output of ELECTRE is an outranking relationship. The various versions of the ELECTRE methods are detailed in Table B1 and each is quite distinct from the other in terms of data required and output produced (Rogers and Bruen, 1998a).

Table B1 ELECTRE methods

<table>
<tr><th>Method</th><th>Designed for Situations:</th></tr>
<tr><td>ELECTRE I</td><td rowspan="2">Selection problems</td></tr>
<tr><td>ELECTRE IS</td></tr>
<tr><td>ELECTRE TRI</td><td>Sorting problems</td></tr>
<tr><td>ELECTRE II</td><td rowspan="3">Ranking problems
ELECTRE II is an older version where an abrupt change from indifference to strict preference is assumed instead of pseudo-criteria
The main difference between III & IV is that the relative importance indices for the different criteria are not applied to the latter</td></tr>
<tr><td>ELECTRE III</td></tr>
<tr><td>ELECTRE IV</td></tr>
</table>

Source: Kangas *et al.* (2001a)

ELECTRE IS and ELECTRE III have succeeded ELECTRE I and II (Roy, 1990; Roy, 1991). The characteristics of each method are summarised in Table B2. Along with the other pseudo criteria, ELECTRE enables the user to also set what is called the veto thresholds for the criteria. If an alternative performs so badly in regard to one criterion, that the difference exceeds the veto threshold, even good criteria with regard to other criteria will not suffice to compensate so great a deficiency (Kangas and Kangas, 2005).

Table B2 Main characteristics of the ELECTRE methods

ELECTRE	I	IS	II	III	IV	A
Kind of criteria	True Criteria	-	True Criteria	Pseudo Criteria	Pseudo Criteria	-
Possibility to consider indifference and / or preference thresholds	No	Yes	No	Yes	Yes	Yes
Necessity to quantify relative importance of criteria	Yes	Yes	Yes	Yes	No	Yes
Additional preference information	Weights Concordance Discordance levels	-	Weights Concordance Discordance levels	Weights	-	-
Outranking relation	Deterministic	-	Deterministic	Fuzzy	Deterministic, strong weak	Fuzzy
Final results	Kernel	Kernel with consistency and connected indices	Partial ranking	Partial ranking	Partial ranking	An assignment to pre-defined categories

Source: Ostanello (1983); Karagiannidis and Moussiopoulos (1997)

ELECTRE TRI is a multi-criteria sorting method and details on the ELECTRE TRI method can be found in Mousseau *et al.* (1999) and Mousseau *et al.* (2000). This method requires the elicitation of preferential parameters (i.e. weights and thresholds) in order to construct a preference model. Due to the difficulty experienced by DMs in assigning the preferential parameters, Mousseau and Slowinski (1998), Mousseau *et al.* (2000) and Mousseau *et al.* (2001) present a procedure aimed at reducing the cognitive effort of the DM by inferring the CWs on the basis of assignment examples (i.e. from holistic information on the DMs judgments). Dias *et al.* (2002) also present a new approach to elicit an ELECTRE TRI model in a way that integrates the preference elicitation phase and the construction of robust conclusions. Further work has been undertaken on the ELECTRE TRI method by Mousseau *et al.* (2003) to propose two algorithms for solving inconsistencies among constraints on the parameters.

The general ELECTRE methodology can be described by the following (Takeda, 2001):

Consider n alternatives $a_i = 1, 2, \ldots, n$ and let

$$A = \{ a_i \}$$

Let $g_1, g_2, \ldots, g_m$ be m-criteria. Thus each alternative a_i is characterised by a multi-criteria outcome denoted by a vector:

$$(g_1(a_i), g_2(a_i), \ldots g_m(a_i))$$

It is assumed that the DM prefers larger to smaller values for each criterion. In the presence of imprecision, it is often reasonable to admit that if a positive difference $g_k(a_i) - g_k(a_j)$ is small, a_i and a_j are regarded as indifferent. To make it possible, the concept of a semi-criterion is introduced. By introducing an indifference threshold q_k, strict preference P_k and indifference I_k are defined as:

$$P_k\,(a_i,a_j) \text{ iff } g_k\,(a_i) - g_k\,(a_j) > q_k$$

$$I_k(a_i,a_j) \text{ iff } |\; g_k\,(a_i) - g_k\,(a_j)| \leq q_k$$

In order to avoid an abrupt change from strict preference to indifference, two thresholds, an indifference threshold q_k and a preference threshold p_k, are introduced: when the positive difference $g_k(a_i) - g_k(a_j)$ is sufficiently small, that is, $g_k(a_i) - g_k(a_j) \leq q_k$, a_i and a_j are considered indifferent. To have strict preference, it is necessary that the positive difference $g_k(a_i) - g_k(a_j)$ be sufficiently large, that is $g_k(a_i) - g_k(a_j) > p_k$. The case where $q_k < g_k(a_i) - g_k(a_j) \leq p_k$ is insufficiently large, that is, $g_k(a_i)- g_k(a_j)> p_k$. The case where $q_k < g_k(a_i)\leq p_k$ is interpreted as a hesitation between indifference and strict preference. It is called a weak preference and denoted by $W_k(a_i, a_j)$. This concept allows the ambiguity inherent in the presence of imprecision, uncertainty or indetermination, to be apprehended.

Thus P_k, W_k and I_k are defined as:

$P_k(a_i, a_j)$ iff $g_k(a_i) - g_k(a_j) > p_k$

$W_k(a_i, a_j)$ iff $q_k < g_k(a_i) - g_k(a_j) \leq p_k$

$I_k(a_i, a_j)$ iff $| g_k(a_i) - g_k(a_j)| \leq q_k$

Then g_k (a) is called a pseudo-criterion.

For each pseudo-criterion g_k, a monocriterion outranking relation $c_k(a_i, a_j)$ is defined in the outranking relation method as follows:

$P_k(a_i, a_j)$: $c_k (a_i, a_j) = 1$ and $c_k(a_j, a_i) = 0$

$W_k(a_i, a_j)$: $c_k(a_i, a_j) = 1$ and $0<c_k(a_j, ak) < 1$

$I_k(a_i, a_j)$: $c_k(a_i, a_j) = 1$ and $c_k(a_j, a_i) = 1$

Using the weights w = $w(k)$, the concordance index $C(a_i, a_j)$ is defined as follows:

$$C(a_i, a_j) = \sum_{k=1}^{m} w_k c_k (a_i, a_j)$$

By introducing a veto threshold v_k for each criterion g_k, a discordance index $d_k(a_i, a_j)$ which rejects the assertion "a_i outranks a_j" is defined as:

If $g_k(a_j) - g_k(a_i) \leq p_k$ then $d_k(a_i, a_j) = 0$

If $p_k < g_k(a_j) - g_k(a_i) \leq v_k$ then $0 < d_k(a_i, a_j) <1$

If $g_k(a_j) - g_k(a_i) > v_k$ then $d_k(a_i, a_j) = 1$

In the final stage, the distillation method using a discrimination threshold function is used to rank alternatives in descending and ascending orders.

In summary, based upon ELECTRE III (Salminen *et al.*, 1998):

- Differences in criteria PVs are not taken into account totally; it does not matter how much a value of a criterion is better than that of another criterion;
- Uncertainty is dealt with by thresholds, which may be constant or proportional;
- With ELECTRE it is possible to decrease *c(a,b)* values by using discordance by defining veto thresholds for some or each criterion in order to decrease the compensation between the criteria values;
- Distillation for outranking degrees *S(a,b)* or 'min' procedure;
- The choice of s(λ) in distillation may influence the ranking;
- Rank reversal can occur with distillation;
- Generally a partial order is obtained.

B1.2 PROMETHEE

The PROMETHEE method uses positive and negative flows to rank scenarios. The positive outranking flow expresses how much each scenario outranks all the others and the negative flow expresses by how much each scenario is outranked by all the others. The best scenarios are those showing simultaneously high positive flows and small negative flows (Martin *et al.*, 1999).

Let A be a set of alternatives to rank or choose from. Assuming k criteria have been considered, for each alternative $a \varepsilon A$, $f_j(a)$ is the value of criterion j for alternative a. A ranking is performed in three steps (Mareschal and Brans, 1988):

<u>Step 1</u>: A preference function P_j is associated with each criterion j. $P_j(a,b)$ is calculated for each pair of alternatives. It varies from 0 to 1, starting at 0 if $f_j(a) = f_j(b)$ and increasing with $f_j(a) - f_j(b)$, to reach 1 when the difference is large enough. Various shapes can be used for P_j, depending on the situation modelled by criterion j. A weighting factor w_j is also attached to each criterion f_j. Weights indicate trade-offs between the criteria.

Step 2: The outranking degree Π(*a,b*) of every alternative *a* over alternative *b* is calculated. The higher Π(*a,b*) is, the more preferred alternative *a* is. The formulation is as follows:

$$\pi(a,b) = \frac{1}{W}\sum_{j=1}^{k} w_j P_j(a,b)$$

with

$$W = \sum_{j=1}^{k} w_j$$

The weights should be normalised so that (Brans *et al.*, 1998):

$$\sum_{j=1}^{k} w_j = 1$$

Step 3: Π(*a,b*) is the outranking degree of *a* relative to *b*. To get the "absolute" outranking power of alternative a the leaving flow is calculated as:

$$\phi^{+}(a) = \frac{1}{n-1}\sum_{b \varepsilon A, b \neq a} \pi(a,b)$$

The outranked power of alternative *a* is called the entering flow and is calculated as follows:

$$\phi^{-}(a) = \frac{1}{n-1}\sum_{b \varepsilon A, b \neq a} \pi(b,a)$$

Thus, a partial pre-order between alternatives is obtained from the intersection of the two rankings induced by ϕ+ and ϕ- (PROMETHEE I ranking). A complete pre-order is induced from the net flow of each alternative (PROMETHEE II), expressed as:

$$\phi(a) = \phi^{+}(a) - \phi^{-}(a)$$

The higher this value, the better alternative.

The associated PROMCALC software allows sensitivity analysis on the CWs and graphical investigation of the conflicts between the criteria based on

Principal Components Analysis by means of GAIA (Mareschal and Brans, 1988). The GAIA graphic is a projection of scenario performances in a space of *n* dimensions representing the *n* criteria, on the plane that preserves a maximum amount of information expressing the decision. Since it is a projection, some of the information expressing the decision is lost (Martin *et al.*, 1999).

In the phase of the overall, multi-stakeholder analysis, the points of view of the different actors are pooled. The global net flow is calculated as a weighted average of the individual net flows (Macharis *et al.*, 2004).

In summary (Salminen *et al.*, 1998):

- Differences in criteria PVs are not taken into account totally; it does not matter how much the preference threshold is exceeded;
- Uncertainty is dealt with by thresholds, which are constant;
- Additive model for credibility degrees $P_j(a,b)$;
- Rank reversal can occur;
- Partial order possible or complete order.

PROMETHEE methods may be applied when the following considerations are taken into account (De Keyser and Peeters, 1996):

- The DM can express his or her preferences between two alternatives on all the criteria on ratio scales;
- The DM can express the importance he or she attaches to the criteria on a ratio scale;
- The DM wants to take all criteria into account and is aware of the fact that the CWs are representing trade-offs;
- For all criteria the difference between evaluations must be meaningful;
- None of the possible differences between any of the criteria can give rise to discordance; and

- The DM knows exactly what can happen if one or more alternatives are added or deleted and is fully aware of the influences on the final decision.

EXPROM-2 is an extended version of PROMETHEE II (Diakoulaki and Koumoutsos, 1991), which is based on the notion of ideal and anti-ideal (maximum and minimum) solutions. The relative performance of one alternative over the other is defined by two preference indices, one by weak preference index (based on outranking) and the other by a strict preference index (based on the notion of ideal and anti ideal). The total preference index is taken as a measure of the intensity of preference of one alternative over the other for all criteria.

Goumas and Lygerou (2000) also extend PROMETHEE to deal with fuzzy input data and have named the method F-PROMETHEE. In the F-PROMETHEE method the performance of each scenario to each criterion is introduced as a fuzzy number. This comes from the fact that in most cases the input data cannot be defined within a reasonable degree of accuracy. Other parameters, expressing the opinion of the DM, such as the parameters of generalised criterion functions and the weighting factors, are considered as regular data with precise numerical values. The results of the calculations are in the form of fuzzy numbers.

Dias *et al.* (1998) discuss the application of parallel processing as a means of reducing computational time when performing robustness analysis of a decision obtained using the PROMETHEE MCDA technique. Several parallel programs were built and compared on a 16 processor computer and it was found that the reduction in the computer's response time was quite appreciable.

B1.3 Novel approach to imprecise assessment and decision environments

Novel approach to imprecise assessment and decision environments (NAIDE) was developed by Munda (1995) and is a discrete multi-criteria method particularly orientated to evaluate a finite number of alternatives for resources management and / or environmental protection. NAIADE allows either crisp, stochastic or fuzzy measurements of the PVs to be included. The NAIADE method is based on a two-phase algorithmic procedure (Rossi *et al.*, 2005):

- In the first phase a pair-wise comparison of alternatives is carried out, taking into account the intensity of preference of one alternative with respect to the others. The result is a partial ranking of the alternatives, allowing for incomparability relationships to hold. It should be noted that preferences of the DM in terms of weighting of the criteria are not accounted for within this first task, in order for the DM to identify non-dominated solutions without biases due to the relative importance of criteria.

- Because no weighting method of the criteria is assumed, the second phase of the procedure aims at identifying the solutions that can potentially gain higher consensus amongst stakeholders. This is a conflict minimisation method that, on the basis of the similarity of judgments of the different stakeholders towards alternatives, tries to identify coalitions that are most likely to be formed among groups of interest. Each coalition identifies its own preferred alternatives and in turn vetoes alternatives. Therefore, preferred alternatives can be identified on the basis of the consensus reached within each coalition, and which are thus most likely going to be realised.

Summarising, NAIADE can provide the following information (Haastrup *et al.*, 1998):

- Ranking of the alternatives according to the set of evaluation criteria (i.e. compromise solutions);

- Indications of the distance of the positions of the various interest groups (i.e. possibilities of convergence of interests or coalition formations); and

- Rankings of the alternatives according to actors' impacts or preferences.

B1.4 Hasse diagram technique

The Hasse Diagram Technique (HDT) is a partial order theory method (Simon *et al.*, 2004). The sorting of the scenarios is based on a simple $\leq$ comparison and no compensation among indicator values takes place. The result is depicted in a Hasse diagram (HD). The vertical arrangement of the scenarios represents their "overall" evaluation, with good solutions located at the bottom of the diagram. The horizontal arrangement of scenarios shows differences in their pattern of indicator values. Scenarios which are not connected with vertical lines are not comparable with each other because of antagonistic indicators, meaning that there is at least one pair of indicators in which one indicator is better evaluated on one scenario and worse in the other (the other indicator is evaluated the opposite way around). The indicators which are evaluated better represent the advantages of a scenario when compared with another one. The indicators which are evaluated worse represents the disadvantages of a scenario.

The exclusion of criteria weighting is a crucial step, as it is a tool to implement the participation of the stakeholders and their preferences to the evaluation process.

B2 Value / Utility systems

The value / utility system approach first aims to help the DM construct a partial utility function on each of the criteria in order to explain their system of preference. Secondly, the approach aggregates the partial utilities using designated trade-offs on the criteria in order to construct a global utility function. The two predominant aggregation methods for multi-attribute utility models are the additive and multiplicative forms.

B2.1 Multi-attribute value theory

In multi-attribute value theory (MAVT) the alternatives are evaluated with respect to each attribute and the attributes are weighted according to their relative importance. Assuming mutually preferentially independent criteria, an additive value function can be used to aggregate the component values.

Then, the overall value of the alternative is:

$$v(x) = \sum_{i=1}^{n} w_i v_i(x)$$

where:

n = the number of criteria

w_i = the weight of criteria *i*

$v_i(x)$ = the rating of alternative *x* with respect to attribute *i*

The sum of the weights is normalised to one and the component value functions $v_i()$ has values between 0 and 1. The weights w_i indicate the relative importance of criteria *i* changing from its worst level to its best level, compared to the changes in the other criteria.

The additive model is appropriate only when the DMs' preferences satisfy additive independence (Butler *et al.*, 1997). If additive independence is not satisfied, a multiplicative form can be used for aggregation. The additive model is a compensatory one: bad performance of an indicator can be compensated (to a certain extent) by good performance of another one. Fishburn (1967) provides 24 methods for assessing value functions.

A criticism of this approach is that it is rather difficult to implement from the actors' perspective, especially when a large number of criteria are involved (Barda *et al.*, 1990). Siskos and Hubert (1983) also believe that MAVT presents considerable operational complications, especially as far as assessment of probabilities and utilities attached to criteria are concerned.

B2.2 Weighted sum method

One of the most widely applied and most easily understood techniques is the weighted sum method (WSM) (Hajkowicz, 2002). Using weighted summation, the PVs are multiplied by the CWs and then summed for each alternative to obtain an overall performance score. WSM can only be used when quantitative weights information is available. A widely noted drawback of the WSM is the number of assumptions it requires. Rowe and Pierce (1982) and Hobbs (1980) list the assumptions of the WSM as follows:

- Criteria must have cardinal (interval or ratio) scales;
- There must be additive independence among the criteria;
- Value or utility function for criteria must be linear;
- Weights must be on a ratio scale; and
- Weights must reflect the relative importance of a unit change in the value (utility) function.

B2.3 Weighted product model

The weighted product model (WPM) is very similar to the WSM. The main difference is that instead of addition in the model there is multiplication. Each alternative is compared to the remaining alternatives by multiplying a number of ratios, one for each criterion. Each ratio is raised to a power equivalent to the relative weight of the corresponding criterion. In general, in order to compare alternatives A_p and A_q the following product has to be calculated (Triantaphyllou and Sanchez, 1997):

$$R\left(\frac{A_p}{A_q}\right) = \prod_{j=1}^{N}\left(\frac{a_{p,j}}{a_{q,j}}\right)^{W_j}$$

If the ratio R(A_p/A_q) is greater than or equal to 1, then the conclusion is that alternative A_p is more desirable than alternative A_q. The WPM is sometimes called dimensionless analysis because its structure eliminates any units of measure.

B2.4 Multi-attribute utility theory

Multi-attribute utility theory (MAUT) is a process derived from formal mathematical theory for making decisions that require balancing of competing objectives (Dunning *et al.*, 2000). MAUT can be viewed as an extension of value measurement, relating to the use of probabilities and expectations to deal with uncertainty. To translate the individual scores assigned to an alternative into a measure of the overall desirability of that alternative, the objectives and performance measures are used to construct utility functions. A utility function translates estimates of performance into a measure of the value or utility of that performance. It is an equation for aggregating the performance measures. The assignment of CWs specifies how DMs will want to make trade-offs among objectives. CWs must reflect the amount of change in one performance measure required to compensate for a specified change in others.

B2.5 Stochastic multi-criteria acceptability analysis

Stochastic Multi-criteria Acceptability Analysis (SMAA-method) has been developed by Lahdelma *et al.* (1998) for situations in which the use of DMs' preference information is not possible. Instead, the problem is described by typical weight vectors leading to each solution, taking into account the evident uncertainty embedded in the CWs. The criterion values are stochastic variables and are represented by probability distributions. For each alternative i, the set of weight vectors W_i is determined that makes the overall utility of alternative i greater than, or equal to, the utility of any other alternative. The values of each criterion for each alternative are stochastic variables with a joint density function. The SMAA-technique determines a stochastic acceptability index for each alternative, describing the variety of different valuations (weight combinations) that support the preference of that alternative.

SMAA is a family of methods rather than one individual technique. Different versions of SMAA have been developed for different kinds of decision problems and they are summarised in Table B3. SMAA methods were originally developed for discrete multi-criteria problems, where criterion data were uncertain or inaccurate, and where, for some reason, it was impossible to obtain accurate or any weight information from the DMs. SMAA methods are based on exploring the weight space in order to describe the valuations that would make each alternative the preferred one, or that would give a certain rank to an alternative, as in SMAA-2.

SMAA and SMAA-2 methods assume partial value / utility functions while SMAA-3 uses a double threshold model, as in some ELECTRE techniques (Kangas and Kangas, 2005).

Table B3 Summary of SMAA methods

Method	**Developers**	**Description**
SMAA	Lahdelma *et al.* (1998) and Hokkanen *et al.* (1999)	Is able to handle stochastic criteria values and arbitrarily shaped utility functions
SMAA-2	Lahdelma and Salminen (2001)	Extends the weight space analysis to all ranks. Identifies alternatives which are widely acceptable for the best ranks
SMAA-D	Lahdelma *et al.* (1999)	Uses Data Envelope Analysis (DEA) efficiency measure as a utility function
SMAA-3	Hokkanen *et al.* (1998)	Applies, instead of a utility function, the ELECTRE III outranking method with pseudo-criteria
SMAA-O	Miettinen *et al.* (1999)	Converts ordinal information into cardinal data by simulating all possible mappings between ordinal and cardinal scales that preserve the given rankings. As with the basic SMAA-method, the DMs unknown or partly known preferences are at the same time simulated by choosing weights randomly from appropriate distributions
Ref-SMAA	Lahdelma *et al.* (2005)	Is based on inverse analysis of the reference point space. The method generates random reference points from the reference point space and evaluates the decision alternatives based on an achievement function.

B2.6 Simple multi-attribute rating technique

Simple Multi-Attribute Rating Technique (SMART) is a decision-support method developed at the close of the 1960s and early 1970s in the field of multi-attribute utility theory (von Winterfeldt, 1986). SMART uses a linear additive model to estimate the value of each alternative. Initially dominated alternatives are eliminated (one alternative dominates another if its performance is at least as good as the dominated alternative on all criteria and better on at least one criterion). Single attribute utilities are then developed reflecting how well each alternative does on each criterion. Swing weighting is applied to determine weights for the linear additive model (see Appendix D). This operation begins with rank ordering criteria, considering their measurement scales. The DM is asked to compare two criteria, beginning with identifying which criterion would be most attractive to improve from the worst attainment considered to the best attainment considered. This provides a basis for rank ordering criteria. Then estimates of relative weights are obtained by comparing the most important criterion with each of the others, by asking the DM to assess how important the other criteria would be should the most important criterion be worth 100. Weights are obtained by normalising (sum the assessed values and divide each value by the sum). The last step of the swing weighting approach is to obtain values for each alternative using the sum of products of each weight multiplied by utility values for each alternative.

SMART shares many similarities with the basic ideas of AHP (see below), however, the central difference is that SMART does not use pair-wise comparisons (Kangas and Kangas, 2005). Direct rating in SMART means, for example, that criteria are directly assigned numerical values depicting their importance. When the importance of the individual criteria and the priorities of each of the alternatives with respect to each of the criteria have been determined, SMART can be used to perform the same computations as when using AHP.

B2.7 The analytic hierarchy process

The Analytic Hierarchy Process (AHP) originally developed by Saaty (1977, 1998) is a widely used MCDA technique (Kangas and Kangas, 2005) and it is sometimes classified as a MAUT approach (Dyer *et al.*, 1992). AHP has several advantages from the viewpoints of multiple-use and participatory planning. Using AHP, objective information, expert knowledge and subjective preferences can be considered together. Also, qualitative criteria can be included in the evaluation of alternative plans. AHP is based on the theory of ratio-scale estimation, and by using it, pair-wise comparisons of qualitatively expressed measures can be transferred into a ratio scale. In contrast, other related methods usually require criteria values to be quantitative and to be measured in ratio or interval scales.

There are, however, problems with AHP and many decision scientists have been critical of the method. Perhaps the two foremost problems with the application of AHP have been that the original comparison scale does not allow for the expression of any hesitation regarding the comparisons and that the AHP itself does not provide tools for in-depth analyses of the comparisons, particularly of the uncertainty inherent in the data. Furthermore, the number of comparisons increases rapidly as the number of alternatives and criteria increases. Large numbers of comparisons may be too costly and tedious.

To alleviate some of these problems, more advanced techniques for AHP have been developed (Hill *et al.*, 2005; Salo and Hamalainen, 1997; Stam *et al.*, 1996; Yu, 2002). Beynon *et al.* (2000), Beynon *et al.* (2001) and Beynon (2002) proposed a development of the traditional AHP, namely the DS / AHP method, which combines aspects of AHP with Dempster-Shafer Theory (DST) for the purpose of MCDA. The inclusion of DST allows the DM a greater level of control on the judgments made in comparison to standard AHP methods.

B3 Distance-based approaches

B3.1 Compromise programming

Compromise Programming (CP) is a distance-based technique designed to identify compromise solutions that are determined to be the closest, by some distance measure, to an ideal solution. The ideal solution is generally not feasible. One of the most frequently used measures of closeness is a family of weighted L_p metrics given as (Duckstein *et al.*, 1994; Raju *et al.*, 2000):

$$L_p(a) = \left[\sum_{j=1}^{J} w_j^P \left| \frac{f_j^* - f(a)}{M_j - m_j} \right|^P \right]^{\frac{1}{p}}$$

where:

$L_p(a)$ = L_p-metric for alternative a

$f_j(a)$ = value of criterion j for alternative a

M_j = maximum (ideal) value of criterion j in set A

m_j = minimum (anti ideal) value of criterion j in set A

f^*_j = ideal value of criterion j

w_j = weight of the criterion j

p = parameter reflecting the attitude of the DM with respect to compensation between deviations

For $p = 1$, all deviations from f^*_j are taken into account in direct proportion to their magnitudes meaning that there is full (weighted) compensation between deviations. For $2 \le p \le \infty$ the largest deviation has the greatest influence so that compensation is only partial (large deviations are penalised). For $p = \infty$, the largest deviation is the only one taken into account (min-max criterion) corresponding to zero compensation between deviations (perfect equity). In most decision analysis problems assessed using CP, only three points of the compromise set are calculated: p=1, 2 and ∞ (Shafike *et al.*, 1992).

The weights used in CP have normalised values corresponding to the values of the importance coefficients expressed in ELECTRE III and the scaling and weighting coefficients assessed in MAUT and UTA methods.

B3.2 Technique for order preference by similarity to the ideal solution

The fundamental premise of TOPSIS (technique for order preference by similarity to the ideal solution) is that the best alternative, say ith, should have the shortest Euclidean Distance from the ideal solution (made up of the best criteria regardless of alternative) and the furthest distance from the negative-ideal solution (Hwang and Yoon, 1981). The alternative with the highest relative closeness measure is chosen as best (Zanakis *et al.*, 1998).

The TOPSIS procedure consists of the following steps (Opricovic and Tzeng, 2004):

(i) Calculate the normalised decision matrix;

(ii) Calculate the weighted normalised decision matrix;

(iii) Determine the ideal and negative-ideal solution;

(iv) Calculate the separation measures, using the *n*-dimensional Euclidean Distance;

(v) Calculate the relative closeness to the ideal solution; and

(vi) Rank the preference order.

B4 Verbal decision analysis

B4.1 ZAPROS

The ZAPROS ('closed procedure near references situations') method allows one to construct a quasi-order of alternatives using only psychologically valid ways of eliciting information (Larichev, 1992). Only verbal (i.e. qualitative) measurements are used in all stages of ZAPROS. ZAPROS uses ranking rather than rating information, but the additive overall value rule is correct if there is an additive value function. In ZAPROS, the additive rule does not provide summation of values, but rather the means of obtaining pair-wise compensation between the components of two alternatives. Human preferences are obtained interactively and logical inconsistencies can be identified and the DM prompted for clarification in such instances (Larichev *et al.*, 1993). The output of ZAPROS is very approximate. Some alternatives could be incomparable. Alternatives only have ranks instead of exact quantitative evaluations and such approximate output may be more reliable.

The first version of ZAPROS was published in 1978. The second version gives the development of the original ideas. Both versions are based on the similar procedures of information elicitation from the DM (Larichev, 2001). The method ZAPROS-III uses the preference elicitation procedure proposed in the first version of the method.

Appendix C

MCDA decision support systems

MCDA research in the 1970s focused on theoretical foundations of MCDA. During the 1980s, emphasis shifted toward the implementation of MCDA techniques on computers with the aid of decision support systems (Dyer *et al.*, 1992). Research then focused on the user interface, on finding 'good' instead of 'optimal' solutions and on supporting the entire decision making process from problem structuring through solution implementation. As discussed in Section 4.6.1, a number of overviews of MCDA software have been presented in the literature including: Buede (1992), Buede (1996), Weistroffer and Narula (1997) and Siskos and Spyridakos (1999). A survey of some of the available DSSs has also been undertaken as part of this research and details are contained in Table C1 in alphabetical order of the name of the software.

Table C1 Summary of a selection of MCDA decision support systems

Software Name	MCDA Method Used	Developed By:	Web Site	Referenced In:	Demo Available	Limitations to Demo	Software Cost
AIM	Aspiration Levels	Professor Vahid Lofti, University of Michigan-Flint	UNK	Lotfi *et al.* (1992)	No	-	UNK
ASSESS	AHP	UNK	UNK	Hill *et al.* (2005)	No	-	UNK
AVID	MAVF	University of Stirling	Contact: jho@stir.ac.uk	Hodgkin *et al.* (2005)	No	-	UNK
CLG-DSS	BCA & MCDA	Leleur *et al.* (2004)	UNK	Leleur *et al.* (2004)	UNK	-	UNK
CONCORD I	ELECTRE I	UNK	UNK	Roy (1991)	UNK	-	UNK
CoOp	MAUT, AHP, ELECTRE	UNK	UNK	Salo (1995), Davey and Olson (1998)	No	-	UNK
Criterium Decision Plus 3.0	SMART, AHP, direct trade-offs	Info Harvest	http://www.infoharvest.com/ihroot/infoharv/products.asp#CDP30	Keeney and Raiffa (1976)	Yes	Limits model size to 20 data blocks	$695 US

Software Name	MCDA Method Used	Developed By:	Web Site	Referenced In:	Demo Available	Limitations to Demo	Software Cost
DECAID	MAUT	Pitz, G.	http://www.unc.edu/courses/2004fall/psyc/135/001/decaid.html	Olson *et al.* (1995), Larichev *et al.* (1995), Larichev (1998)	Yes	UNK	$0
DECIDE Wizard	PROMETHEE, ELECTRE	Christophe David	http://www.christophedavid.org/w/c/w.php/DECIDE/Wizard	UNK	Yes	No	Free for private use
Decision Analysis Module (DAM)	WSM	Podinovski, V. Warwick Business School, The University of Warwick, UK	UNK	Podinovski (1999), Dias and Climaco (2000)	UNK	UNK	UNK
DecideRight	SMART	Avantos Performance Systems	http://www.pcworld.com/downloads/file_description/0,fid,3971,00.asp Note: cannot find Avantos website	UNK	Yes	30 day trial	UNK
Decision Explorer	Ideas mapping	Banxia Software	http://www.banxia.com/demain.html	Mendoza and Prabhu (2003)	Yes	UNK	£295 (excl. VAT) academic rate
Decision Lab 2000	PROMETHEE I & II, GAIA	Visual Decision	http://homepages.ulb.ac.be/~bmaresc/PROMETHEE.htm www.visualdecision.com www.smg.ulb.ac.be	Geldermann and Zhang (2001)	Yes	Does not include the report generator and is limited in terms of number of actions, criteria and scenarios	$725 (academic) $1450 CA
Decision map	MAUA	UNK	UNK	Buede (1992)	UNK	-	UNK
Decision pad	Additive weighting	Apian Software	http://apian.com/downloads/	Weistroffer and Narula (1997)	Yes	UNK	$495 US

Software Name	MCDA Method Used	Developed By:	Web Site	Referenced In:	Demo Available	Limitations to Demo	Software Cost
DEFINITE (decisions on a finite set of alternatives)	5 different multi-criteria methods (e.g. WSM, ELECTRE), CBA	Institute for Environmental Studies, Free University of Amsterdam	http://www.falw.vu.nl/Onderzoeksinstituten/index.cfm?home_file.cfm?fileid=F24FDD9C-2DED-45B3-9B0355A242A9A706&subsectionid=602C4835-C246-41FA-8DD706E7084B0D06	Janssen and Herwijnen (1992), Qureshi *et al.* (1999), Royal Assessment Commission (1992), Janssen *et al.* (2005)	No	-	€ 1360
ELECCALC	ELECTRE II	Lamsade Softwares	UNK	Kiss *et al.* (1994)	UNK	-	UNK
ELECTRE IS	ELECTRE IS	Lamsade Softwares	http://www.lamsade.dauphine.fr/english/software.html#el2s	UNK	No	-	800 Euros
ELECTRE III - IV	ELECTRE III - IV	Lamsade Softwares	http://www.lamsade.dauphine.fr/english/software.html#el2s	Roy (1991)	Yes	UNK	800 Euros
ELECTRE TRI	ELECTRE TRI	Lamsade Softwares	http://www.lamsade.dauphine.fr/english/software.html#el2s	Mousseau *et al.* (2000)	Yes	UNK	800 Euros
Expert Choice 11	AHP	Forman, E.	http://www.expertchoice.com/	Olson *et al.* (1995)	Yes	Three levels in the hierarchy, seven objectives/criteria per cluster, eight alternatives, three participants, printing disabled	$475 US (academic) $2,750 US
EQUITY 3.2	MAUT	Enterprise LSE, The London School of Economics & Political Science, Catalyze	http://www.catalyze.co.uk/equity/equity.html	UNK	Yes	20 day evaluation	£1,600

Software Name	MCDA Method Used	Developed By:	Web Site	Referenced In:	Demo Available	Limitations to Demo	Software Cost
Facilitator	MAVT	Department of Natural Resources, Queensland	http://facilitator.sourceforge.net/	Heilman *et al.* (2002), Lawrence *et al.* (2002)	Yes	UNK	$0
GINO	General interactive optimiser	UNK	www.icaen.uiowa.edu/~ie166/Private/soft.html	Barron and Schmidt (1988)	Yes	UNK	$0
Hipre 3+ (Web Hipre)	AHP and MAVT (also SMART, SWING and SMARTER)	Helsinki University of Technology	http://www.sal.hut.fi/Downloadables/ http://www.hipre.hut.fi	Mustajoki *et al.* (2004), Levy *et al.* (2000b), Levy *et al.* (1998)	Yes	Maximum of three levels with three elements on each. Does not allow models to be saved	NA – mainly utilise web version
HIVIEW 3	MAVT	Catalyze	http://www.catalyze.co.uk/	Buede (1992)	Yes	20 day trial period	£850
IANUS	PROMETHEE	UFZ Centre for Environmental Research	UNK	Klauer *et al.* (2002)	UNK	-	UNK

Software Name	MCDA Method Used	Developed By:	Web Site	Referenced In:	Demo Available	Limitations to Demo	Software Cost
$IDEA_{ANN}$	Cognitive mapping, Even-Swap, Lexicographic, Lx semiorder, elimination by aspects, conjunctive, median ranking, majority, SMARTS, TOPSIS, AHP, AHP Rating, ELECTRE I, ELECTRE II, ELECTRE IV, PROMETHEE I & II, PRIAM, STEM, AIM, VIG, convex cones	Ulengin *et al.* (2000)	UNK	Ulengin *et al.* (2000)	UNK	-	UNK
IRIS	ELECTRE TRI	Luis Dias, Vincent Mousseau, José Figueira, João Clímaco, and Carlos Gomes Silva	http://www4.fe.uc.pt/lmcdias/iris.htm	Dias *et al.* (2002)	Yes	Limited problem sizes	1,000 Euros

Software Name	MCDA Method Used	Developed By:	Web Site	Referenced In:	Demo Available	Limitations to Demo	Software Cost
ISMAUT	MAUA	Department of Systems Engineering, University of Virginia	UNK	Buede (1992)	UNK	-	UNK
JavaAHP	AHP	CSIRO Tropical Agriculture, Queensland, Australia	http://chris.tag.csiro.au/JavaAHP/	Zhu and Dale (2000), Zhu and Dale (2001)	UNK	-	UNK
Joint Gains v1.1.4 2004	Negotiation Support in the internet, Method of Improving Directions	Raimo P. Hämäläinen	http://www.decisionarium.hut.fi/	UNK	UNK	-	$0
Lightyear	MAUA	Thoughtware	UNK	Buede (1992)	UNK	-	UNK
Logical Decision	MAUT, AHP	Smith, G.R. & Speiser, F.	http://www.logicaldecisions.com/	Keeney and Raiffa (1976), Larichev *et al.* (1995), Larichev (1998)	Yes	Cannot print or save	$504 US $310 US Academic
MACBETH	Additive value model	Carlos Bana e Costa, Jean-Marie DeCorte, Jean-Claude Vansnick	http://www.umh.ac.be/~smq/	Bana e Costa and Vansnick (1997)	Yes	-	$1,750 Euros

Software Name	MCDA Method Used	Developed By:	Web Site	Referenced In:	Demo Available	Limitations to Demo	Software Cost
MATS-PC	MAVT	C.A. Brown, D.P. Stinson, R.W. Grant (Bureau of Reclamation, Denver, Colorado)	UNK	Goicoechea *et al.* (1992)	UNK	-	UNK
MAUD	MAUA	Decision Analysis Unit, The London School of Economics	UNK	Buede (1992)	UNK	-	UNK
McView	Additive weighting	Rufolf Vetschera, Univ. Konstanz, Fac. Of Economics and Statistics	UNK	Vetschera (1994)	Program is no longer available	-	-
MEDIATOR	Utility function from PREFCALC	UNK	UNK	Jarke *et al.* (1987), Salo (1995), Davey and Olson (1998)	UNK	-	UNK

Software Name	MCDA Method Used	Developed By:	Web Site	Referenced In:	Demo Available	Limitations to Demo	Software Cost
MULINO decision support system (mDSS)	Pair-wise comparison, simple average weighting, additive linear value function, order-weighted averaging, TOPSIS, critical criterion method, borda rule, tornado diagram, macro criteria comparison	Mysiak *et al.* (2005)	http://www.feem.it/web/loc/mulino	Mysiak *et al.* (2002), Giupponi *et al.* (2004), Mysiak *et al.* (2005)	Yes	UNK	$0
MULTICRIT (MCDMGDSS)	ELECTRE I, ELECTRE II, PROMETHEE II, AHP, CP, MCQA, STOPROM-2, EXPROM-2	Department of Civil Engineering, SES College of Engineering, India	UNK	Raju and Pillai (1999a), Raju *et al.* (2000), Raju and Kumar (1998)	UNK	UNK	UNK
ProDecX	PROMETHEE	Centre for Environmental Research, Leipzig, Germany	UNK. Contact Martin.Drechsler@ufz.de	Proctor and Drechsler (2003)	No	-	UNK
PREFCALC	MAUA	Euro-Decision	UNK	Jacquet-Lagreze (1990), Buede (1992)	UNK	-	UNK

Software Name	MCDA Method Used	Developed By:	Web Site	Referenced In:	Demo Available	Limitations to Demo	Software Cost
Prime Decision	Preference Ratios in Multiattribute Evaluation and Preference Programming	Helsinki University of Technology	http://www.sal.hut.fi/Downloadables/	Salo and Hamalainen (2001)	Yes	UNK	$0 for academic use
PROBE	Additive hierarchical model for aggregating the classification of alternatives	CISED Consultores, Lisbon	UNK	Dias and Climaco (2000)	UNK	-	UNK
PROMCALC	PROMETHEE I & II & V	J.P. Brans and B. Mareschal	Note: Decision Lab 2000 is a more recent version mailto:jpbrans@vub.ac.be	Brans and Mareschal (1990), Brans *et al.* (1998)	UNK	-	$100 student rate
SMARTEDGE	MAUA	Haviland-Lee	UNK	Buede (1992)	UNK	-	UNK
SRF	ELECTRE	B. Roy and J. Figueira	http://www.lamsade.dauphine.fr/logiciel.html#srf	Figueira and Roy (2002)	UNK	-	UNK
Treeval	MAUA	Decision Analysis Group, University of Southern California	UNK	Buede (1992)	UNK	-	UNK
VIMDA	Reference directions	Numplan, Finland	UNK	Korhonen (1988), Korhonen *et al.* (1997)	UNK	-	UNK

Software Name	MCDA Method Used	Developed By:	Web Site	Referenced In:	Demo Available	Limitations to Demo	Software Cost
VIP Analysis and VIP-G	Additive MAUT/MAVT with imprecise information	Luis Dias and João Clímaco	http://www4.fe.uc.pt/lmcdias/english/vipa.htm	Dias and Climaco (2000), Dias and Climaco (2005)	Yes	None	$0
VISA	MAVT	Simul8	http://www.SIMUL8.com/products/visa.htm	Belton and Vickers (1990), Buede (1992), Belton *et al.* (1997), Hodgkin *et al.* (1998)	Yes	30 day trial period	$199 US
WINPRE	PAIRS and Preference Programming	Helsinki University of Technology	http://www.sal.hut.fi/Downloadables/	Salo and Hamalainen (1995), Hamalainen *et al.* (2001)	Yes	UNK	$0
WRC DSS	MAVT	Liu and Stewart (2004)	http://tjstew.sta.uct.ac.za/	Liu and Stewart (2004)	UNK	-	UNK
ZAPROS	ZAPROS II (Verbal Decision Analysis)	Oleg I.Larichev	UNK	Olson *et al.* (1995), Larichev (1998), Larichev (2001)	Yes	UNK	UNK

Note: websites and prices correct as at January 2005

Appendix D

Criteria weighting techniques

The purpose of a criteria weighting technique is to establish a set of cardinal or ordinal values which indicate the relative importance of each individual criterion. This information is then used in the selected aggregation method to evaluate the alternatives. Eliciting CWs is a crucial task in MCDA and there are a number of ways to accomplish it. All of the weighting methods present different features in terms of time needed, complexity, transparency etc. It must also be reinforced that distinct methods assessing CWs are designed for different aggregation rules. Choo *et al.* have found that CWs are often misunderstood and misused and that there is no consensus on their meaning (1999). Bottomley and Doyle (2001) state that although there is no lack of weighting methods, there is only limited information as to their reliability and validity, which has also been found during this research.

Following is a description of some of the weighting techniques available for use. As discussed in Section 2.5.7 of the thesis, Hajkowicz *et al.* (2000) classified criteria weighting techniques into quantitative and qualitative methods, Schoemaker and Waid (1982) separate the techniques into statistical versus subjective and Nijkamp *et al.* (1990) divide the weighting techniques into classes of methods which involve direct estimation of CWs and indirect estimation of CWs. In this appendix, the criteria weighting techniques have been separated into direct and indirect methods.

D1 Direct weighting techniques

Direct weighting methods require users to quantitatively state the relative "importance" of each criterion. The way the values for importance parameters are derived is defined independently of the aggregation rule in which these values will be used. Proceeding in this way, these methods are not able to ensure that information expressed in the DMs answers matches the use of this information in the MCDA approach (Mousseau, 1995). The difficulty with direct methods is that "importance" is ambiguously defined, and the definition users have in mind may have little to do with the trade-offs they are willing to make (Hobbs and Meier, 1994).

An example is *rating*, in which the user rates each criterions importance on a scale of say 0 to 10, with 10 being the most important. Other direct methods include *ratio questioning* and an approach called *Analytic Hierarchy Process* (Hobbs and Meier, 1994). Direct methods can assign weights to all criteria at once, but if there are many criteria, then a hierarchical approach is thought to be easier (Hobbs and Meier, 1994).

An example of the use of direct methods can be found in: Hokkanen and Salmimen (1994).

D1.1 Categorisation

Categorisation works similarly to the ranking method (see below), where the criteria are sorted into categories such as "high importance", "average importance" and "low importance" (with weights assigned of 3, 2 and 1) respectively (Al-Kloub *et al.*, 1997; Hobbs, 1980).

CWs from this method are on an ordinal level of measurement, as ratios of weights are arbitrarily fixed. With ordinal scales the ordering is significant, not differences in numbers or their ratios.

D1.2 Graphical weighting

Graphical techniques of criteria weighting rely on visual approaches to elicit weights from the DM. This avoids forcing the DM to express their preferences using numbers or pre-defined categories of importance. There are many variations on graphical weighting of criteria. One approach is to have a DM place a mark on a horizontal line. Criteria importance increases as the mark is placed further to the right end of the line. This approach enables DMs to express preferences in a purely visual manner (Hajkowicz *et al.*, 2000). Eckenrode (1965) undertakes the technique by presenting the criteria next to a continuous scale marked off in units from 0 to 10. The actor is then asked to draw a line from each criterion to any appropriate point on the value scale.

The manual technique called GRAPA (Graphical point allocation) is similar to dividing 100 points, except that the total number of points to be divided is 5J, where J is the number of criteria (Leon, 1997). The advantage of GRAPA is that the respondent can distribute 5J counters over a set of J columns labelled with the names of the criteria, providing a convenient response mode and visual display of the judgments as they are being made. Instructions make clear to respondents that the judgments must take ranges on each criteria PV into account.

D1.3 Paired comparisons

Paired comparisons is an ordinal method which involves the comparison of each criterion against every other criterion in pairs (Kheireldin and

Fahmy, 2001). It can be effective because it forces the DM to give thorough consideration to all elements of a decision problem. An advantage of the pair-wise comparisons technique is that the DM needs only to consider one pair at a time instead of simultaneous assessments of several items (Leskinen *et al.*, 2004).

A widely used form of cardinal paired comparisons is the analytic hierarchy process (AHP) (Al-Kloub *et al.*, 1997; Schoemaker and Waid, 1982). The method requires the DM to rate the importance of each criterion in its pair on a nine point scale, ranging from equal importance (1) to absolutely more important (9). Once all the paired comparisons have been made, eigenvalues are calculated to represent weights (Hajkowicz *et al.*, 2000). The original AHP method has been further developed by researchers including Alho *et al.* (1996) and Alho and Kangas (1997) by using the regression analysis of pair-wise comparisons data. Triantaphyllou (1999) introduces a dual formulation to obtaining preference values using pair-wise comparisons in order to reduce the number of pair-wise comparisons that are required to be undertaken by the DM.

Examples of the use of paired comparisons can be found in: Eckenrode (1965), Schoemaker and Waid (1982), Poyhonen and Hamalainen (2001), Bell *et al.* (2001) and Xu *et al.* (2001).

D1.4 Point allocation or fixed point scoring

The point allocation, or fixed point scoring (or Metfessel allocation), method requires the actor to distribute a fixed number of points (i.e. 100 points) among the various criteria so as to reflect their relative importance (Hobbs, 1980). The method is simple but lacks formal theory (Hwang and Yoon, 1981; Schoemaker and Waid, 1982).

In this technique, the DM is required to distribute a fixed number of points amongst the criteria. A higher point score indicates that the criterion has greater importance. Often percentages are used, as they are a measure with which many DMs are familiar. They key advantage of fixed point scoring is that it forces DMs to make trade-offs in a decision problem. Through fixed point scoring it is only possible to ascribe higher importance to one criterion by lowering the importance of another

(Hajkowicz *et al.*, 2000). It is an ordinal method because it is not known what the actor thinks is important.

Examples of the use of the point allocation method can be found in: Schoemaker and Waid (1982), Bottomley *et al.* (2000), Poyhonen and Hamalainen (2001) and Bell *et al.* (2001).

D1.5 Ranking method

Ranking is a necessary first step in most procedures for more precise weight elicitation (Barron and Barrett, 1996). Ranking is easier than most precise assignments. Ranking requires the DM to rank the criteria in order of importance (Hobbs, 1980; Kheireldin and Fahmy, 2001). The ranking method is less demanding than the rating method as it requires minimal information from the DM and is probably the easiest to handle conceptually. A drawback associated with ranking is that it will significantly limit the number of MCDA methods that can be applied (Hajkowicz *et al.*, 2000). This is because most MCDA methods require cardinal level data. Achieving a complete ranking may be difficult when a large number of criteria have to be ranked because the actor may lose their overview. A stepwise approach may be useful in this situation.

In order to use ordinal weights with cardinal ranking methods it is necessary to estimate cardinal weights from the ordinal information. There are several methods for determining approximate weights which make specific use of rank information. This can be achieved by either using the expected value method or by taking the naïve approach (Nijkamp *et al.*, 1990). Hobbs (1980) suggests that the least important criterion is assigned a weight of 1, the next lowest a 2 and so forth. To overcome the problem of eliciting weighting constants from the DM, an algorithm was developed by Kirkwood and Sarin (Foltz *et al.*, 1995). It requires that the ordering of the magnitudes of the weights on individual criteria be known, not their precise values. Alternatively, *rank sum* provides for weights which correspond to the ranks, normalised by dividing each by the sum of the ranks (Barron and Barrett, 1996; Jia *et al.*, 1998). The most important criterion has a weight of *n*/(sum of ranks) and the least important criterion, 1/(sum of ranks), where *n* is the total number of criteria. A similar approach gives relative weights based on the *reciprocal of the ranks*. That is, the non-normalised weights are 1, ½, , 1/*n*. Dividing through by the sum of these terms yields the final weights, which meet the restriction of summing to unity.

The *rank order centroid* weights is another surrogate weighting method where the weights are derived from a more systematic analysis of information implicit in the ranks (Barron and Barrett, 1996). This approach produces an estimate of weights that minimises the maximum error of each weight by identifying the centroid of all possible weights maintaining the rank order of objective importance (Butler and Olson, 1999). The centroid method is identical to the SMART method with the exception that weights are assessed based on the rank order of criteria importance (i.e. SMARTER). The centroid approach uses ordinal input information. The method is based on sounder input and is less subject to the errors introduced by inaccurate weight assessment (Hwang and Yoon, 1981). The approach is useful when: there are four or more criteria being considered, criteria are close in relative importance, and when there is limited time available for analysis.

An example of the use of these methods can be found in: Eckenrode (1965), Barron and Barrett (1996), Butler and Olson (1999), Shepetukha and Olson (2001).

D1.6 Rating technique

The rating technique obtains a score from a DM to represent the importance of each criterion (Kheireldin and Fahmy, 2001). Often numbers 1-5, 1-7 or 1-10 are used to indicate importance (Hobbs *et al.*, 1992). The rating method does not constrain the DM's responses as in the fixed point scoring method. It is possible to alter the importance of one criterion without adjusting the weight of another. This represents an important difference between fixed point scoring and the rating method (Hajkowicz *et al.*, 2000). The definition of importance actors may use may have little to do with the relative value of unit changes in criteria value functions, therefore the rating technique is an ordinal method. According to Hobbs (1980), the attractiveness of using the rating method lies in its ease of use.

An important difference between the rating method and the trade-off method is that the rating method can only be used when the criteria have been standardised, whereas for the trade-off method such an assumption is not necessary (Nijkamp *et al.*, 1990).

Max 100 is a rating technique where the most important criterion in the set is assigned a rating of 100, and then each other criterion is rated relative to it on a scale of 0 – 99 (Bottomley and Doyle, 2001). Min 100 is a rating technique where the least important criterion is assigned a rating of 10, then the other criteria are rated relative to the least important criterion on a scale with no specified upper-bound (Bottomley and Doyle, 2001).

Examples of the use of the rating method can be found in: Eckenrode (1965), Kheireldin and Fahmy (2001), Moss and Catt (1996) and Bottomley *et al.* (2000).

D1.7 Ratio questioning

Ratio questioning asks questions such as "What is the ratio of 'importance' of criteria x_i and x_j?" At least (n-1) such questions, involving each of the n criteria at least once, must be asked to establish a weight set (Hobbs, 1980).

An example of the use of ratio questioning can be found in: Hobbs and Horn (1997).

D1.8 SMART, SMARTS and SMARTER

A minor extension of directly judging ratios of weights was named SMART (for Simple MultiAttribute Rating Technique) by Edwards (1977). In SMART, 10 points are first given to the least inportant criterion. Then more points are given to the other criteria depending on the relative importance of their ranges. When using the SMART method, actors need to consider ranges as well as importance in judging ratios of weights, and hold ratios constant (Leon, 1997).

In 1994 Edwards and Barron presented a new rank weighting procedure intended to be an approximation to swing weights (Leon, 1997). They renamed SMART with swing weights, calling it SMARTS and proposed a rank weighting procedure which they named SMARTER (SMART Exploiting Ranks). In SMARTER, the weights are elicited with the centroid method of Solymosi and Dombi (1986) directly from the ranking of the alternatives. SMARTS requires the respondent to make judgments about hypothetical stimuli which are often difficult to make.

An example of the use of this weighting technique can be found in: Poyhonen and Hamalainen (2001).

D1.9 Swing weights

The Swing method is similar to the SMART procedure, but the procedure starts from the most important criterion, keeping it as the reference. The DM begins by rank ordering criteria in terms of their associated value ranges. Assuming that each criterion is at its worst possible level, the DM is asked which criterion they would most prefer to change from its worst level to its best level. The criterion chosen has the most important value range. Next, the DM is asked which criterion they would next most prefer to change from its worst to its best level. To quantify the relative value ranges, the DM next assigns a relative importance weight between 0 and 100. The criterion with the most preferred swing is most important and is assigned 100 points. Proceeding in this fashion, the DM rank orders the criteria and assigns relative importance weights to their value ranges. The final step in the Swing weight procedure is to normalise the relative importance of the weights (Jia et al., 1998).

Examples of the use of this method can be found in: Schoemaker and Waid (1982), Poyhonen and Hamalainen (2001) and Bell *et al.* (2001).

D1.10 Trade-off weighting

Keeney and Raiffa (1976) presented the trade-off method. The key idea of the procedure is to compare two alternatives described on two criteria (for the remaining criteria, both alternatives have identical values). Trade-off weighting asks users to state how much of one criterion they would be willing to give up to obtain a given improvement in another criterion (Hobbs and Meier, 1994; MacCrimmon, 1973). One alternative has the best outcome on the first and the worst outcome on the second criterion, the other has the worst on the first and the best on the second criterion. By choosing the preferred alternative out of the two the DM decides on the most important criterion. The critical step is the adjustment of the criteria outcome in order to yield indifference between the two alternatives. This is typically done by either worsening the chosen alternative in the preferred outcome, or improving the non-chosen alternative in the inferior outcome. Such indifferences have to be elicited for (n – 1) meaningfully selected pairs of alternatives (Al-Kloub *et al.*, 1997). If desired, several inconsistency checks can be carried out.

Nijkamp *et al.* (1990) state that practical applications show that respondents usually have great difficulties in giving point estimates of CWs in the way described above. Also, respondents sometimes do not like to express their priorities without knowing the implications.

An example of the use of trade-off weighting can be found in: Hobbs and Horn (1997).

D2 Indirect weighting techniques

In this section a number of methods for estimating weights in an indirect way are described (i.e. based on preference or indifference statements). Indirect methods explicitly integrate the MCDA method selected for use (Mousseau, 1995). The interaction with the DM is not based directly on the concept of importance but on indirect information from which information concerning the relative importance of criteria is inferred through the aggregation rule. Indirect weighting methods estimate weights based on simulated or real decision behaviours (Hajkowicz *et al.*, 2000). They generally require the DM to rank or score a set of alternatives against a set of evaluative criteria. Using various techniques, such as multiple linear regression analysis, as used in judgment analysis, it is possible to implicitly derive weights for the criteria (Hajkowicz *et al.*, 2000).

D2.1 DIVAPIME

DIVAPIME is a software package that supports an indirect weighting technique used to define a polyhedron of acceptable values for importance parameters in ELECTRE type methods (Mousseau, 1995). However, the part of the software concerning the elicitation of the importance coefficients may also apply to other MCDA methods that build one or several outranking relations, such as PROMETHEE.

The method determines a range of admissible values for the importance coefficients from a set of linear inequalities on these coefficients. The linear inequalities come from DM's answers to binary comparisons of fictitious alternatives. A range for each criterion weighting, rather than a single value, is deduced.

An example of the use of this method can be found in: Rogers and Bruen (1998b).

D2.2 Multiple regression method

The multiple regression method involves the DM being asked to provide overall (or holistic) evaluations of a set of alternatives. The response scale is interval. Using multiple regression, the relative importance of the independent variables is then estimated via traditional least squares (Hwang and Yoon, 1981; Schoemaker and Waid, 1982).

To obtain weights in an implicit manner judgment analysis can be used. In this method the DM is presented with criterion values for a set of real or hypothetical alternatives. The DM is asked to assign a utility score to each of the alternatives. Scales such as 1-10, 1-20 or 1-100 can be used. There should be a high degree of variability to assist the DM discern between alternatives. Multiple regression analysis is then conducted to determine the relative importance of each criterion in determining the DMs score. This can be a complex procedure and will typically require considerable time and effort from a DM (Hajkowicz *et al.*, 2000).

An example of the use of the method can be found in: Schoemaker and Waid (1982).

D2.3 Simos (1990)

The method proposed by Simos (1990) is an indirect technique for eliciting CWs based on a hierarchical ranking of criteria. Its main advantage is that it is less arbitrary than direct assignment of weights and much simpler than most indirect techniques and can be easily understood by DMs (Figueira and Roy, 2002; Georgopoulou *et al.*, 1998; Rogers and Bruen, 1998b). The name of each criterion is inscribed on a card and the DM puts the cards in order of importance. Blank cards are inserted to reinforce the rank differences. Descending numerical values are then assigned to the weights. The relative weight is then determined for each criterion.

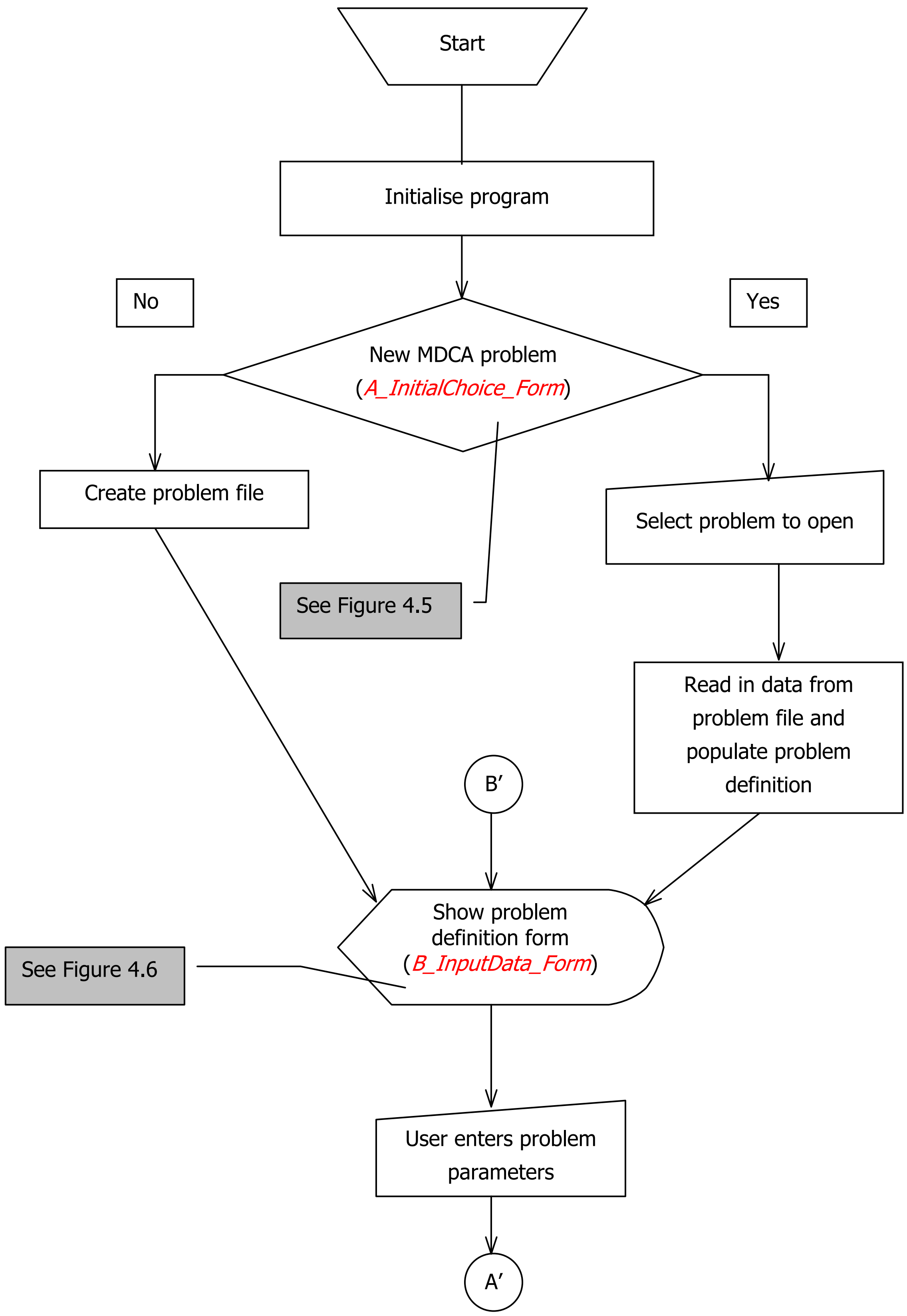
Examples of the use of the method can be found in: Georgopoulou *et al.* (1997) and Georgopoulou *et al.* (1998).

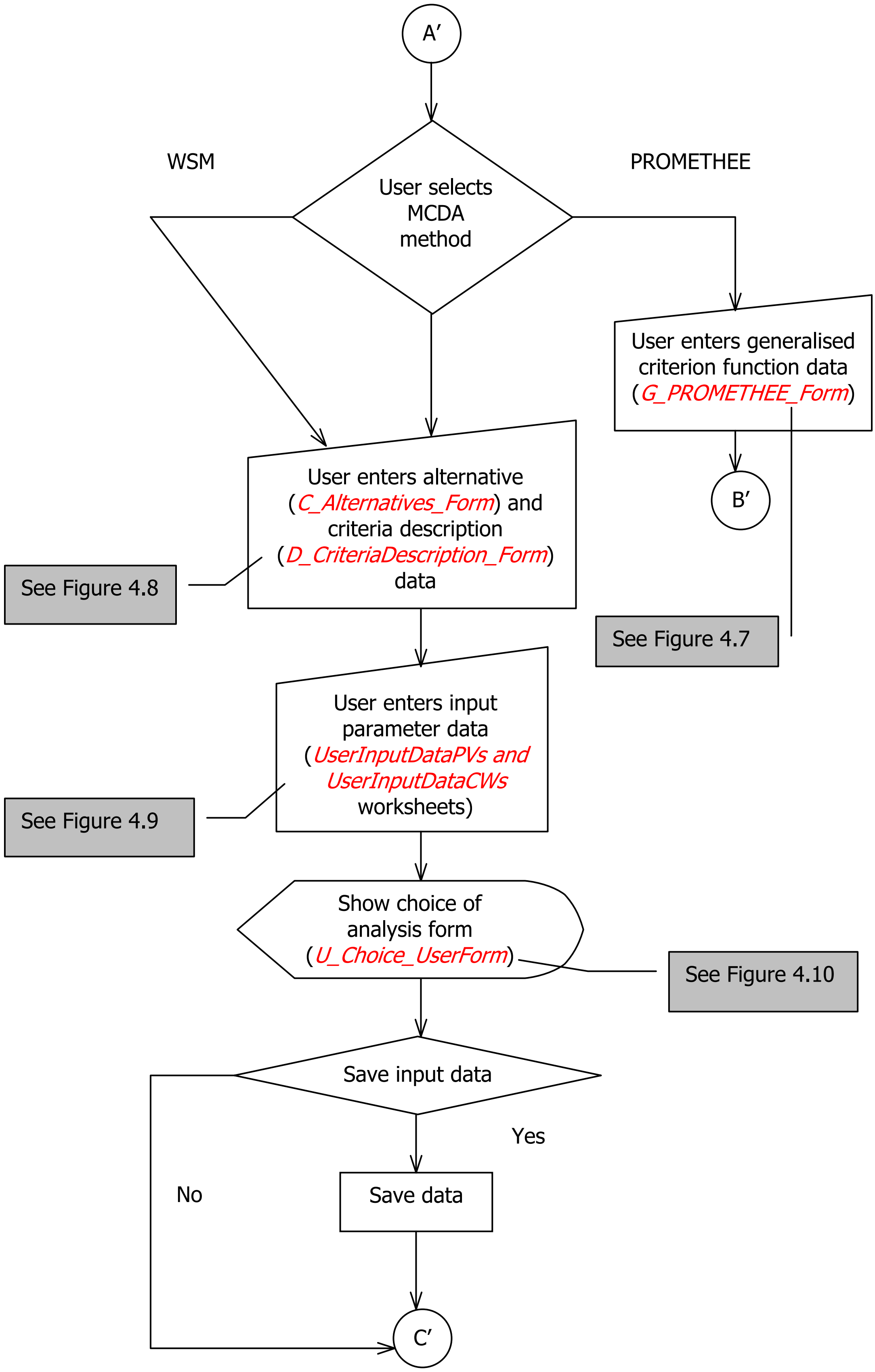
Figueira and Roy (2002) have found that the procedure recommended to convert the ranks into weights limits the set of the feasible weights because it determines automatically the ratio between the weight of the most important criterion and the weight of the least important criterion. A revised Simos procedure has been proposed which uses the Simos data collection method, but then asks the user to state how many times the last criterion is more important than the first one in the ranking (Figueira and Roy, 2002). A software program has been developed which uses this procedure (SRF), but there is only a French version of the program.

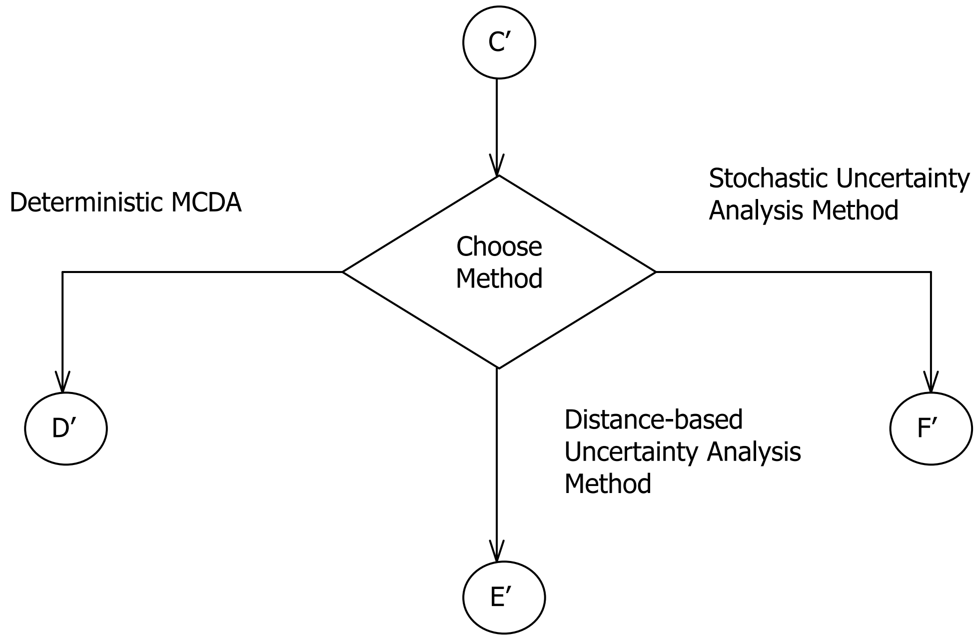
Appendix E

Structure of the VBA program

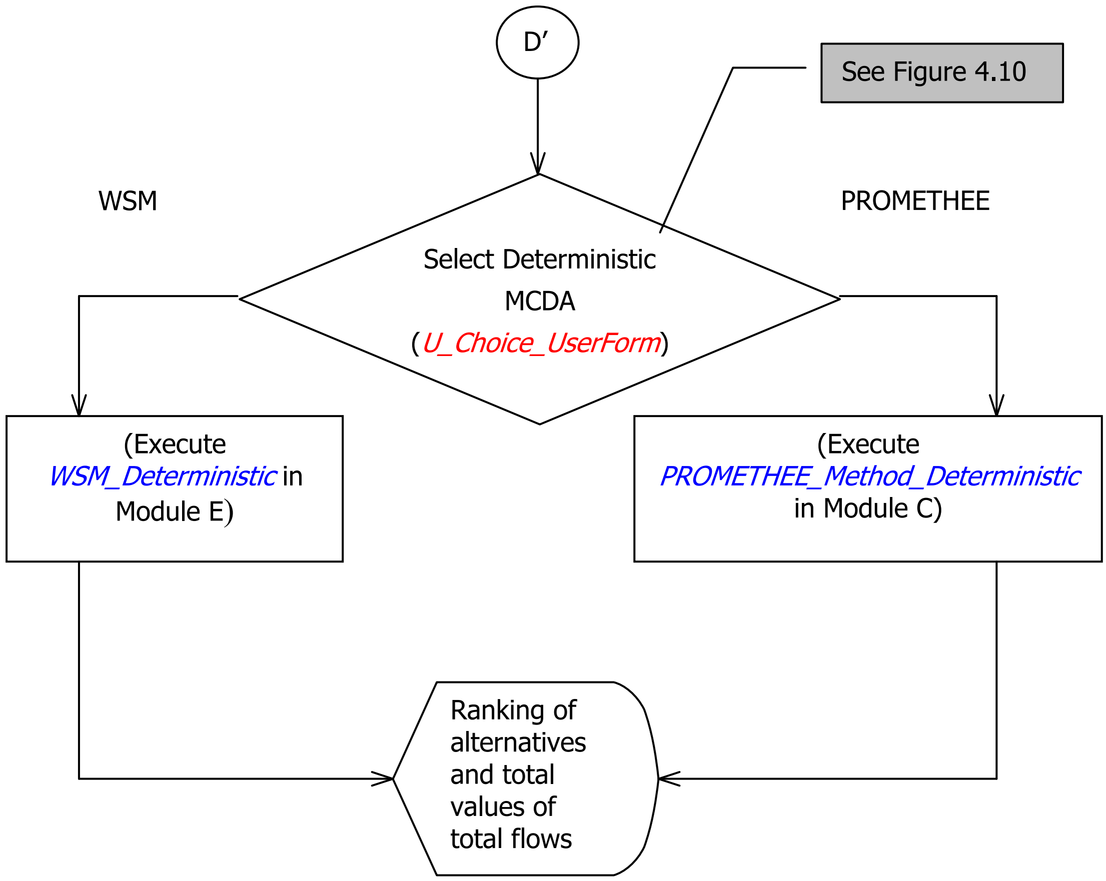
Layout A: Overall Program Structure



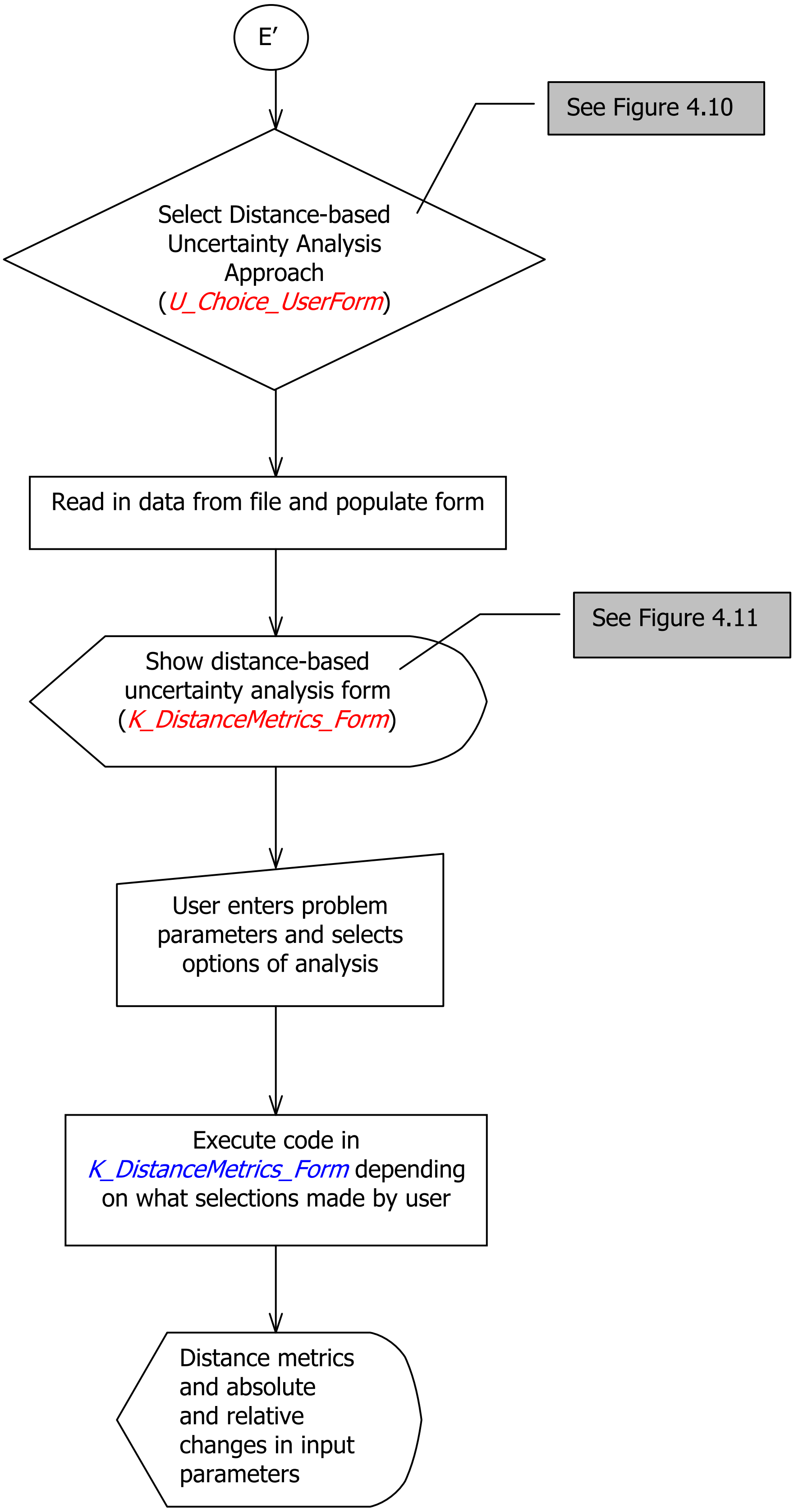

A′
WSM
User selects MCDA method
PROMETHEE
User enters generalised criterion function data (G_PROMETHEE_Form)
B′
See Figure 4.7
User enters alternative (C_Alternatives_Form) and criteria description (D_CriteriaDescription_Form) data
See Figure 4.8
User enters input parameter data (UserInputDataPVs and UserInputDataCWs worksheets)
See Figure 4.9
Show choice of analysis form (U_Choice_UserForm)
See Figure 4.10
Save input data
Yes
No
Save data
C′


C′
Deterministic MCDA
Choose Method
Stochastic Uncertainty Analysis Method
D′
Distance-based Uncertainty Analysis Method
F′
E′

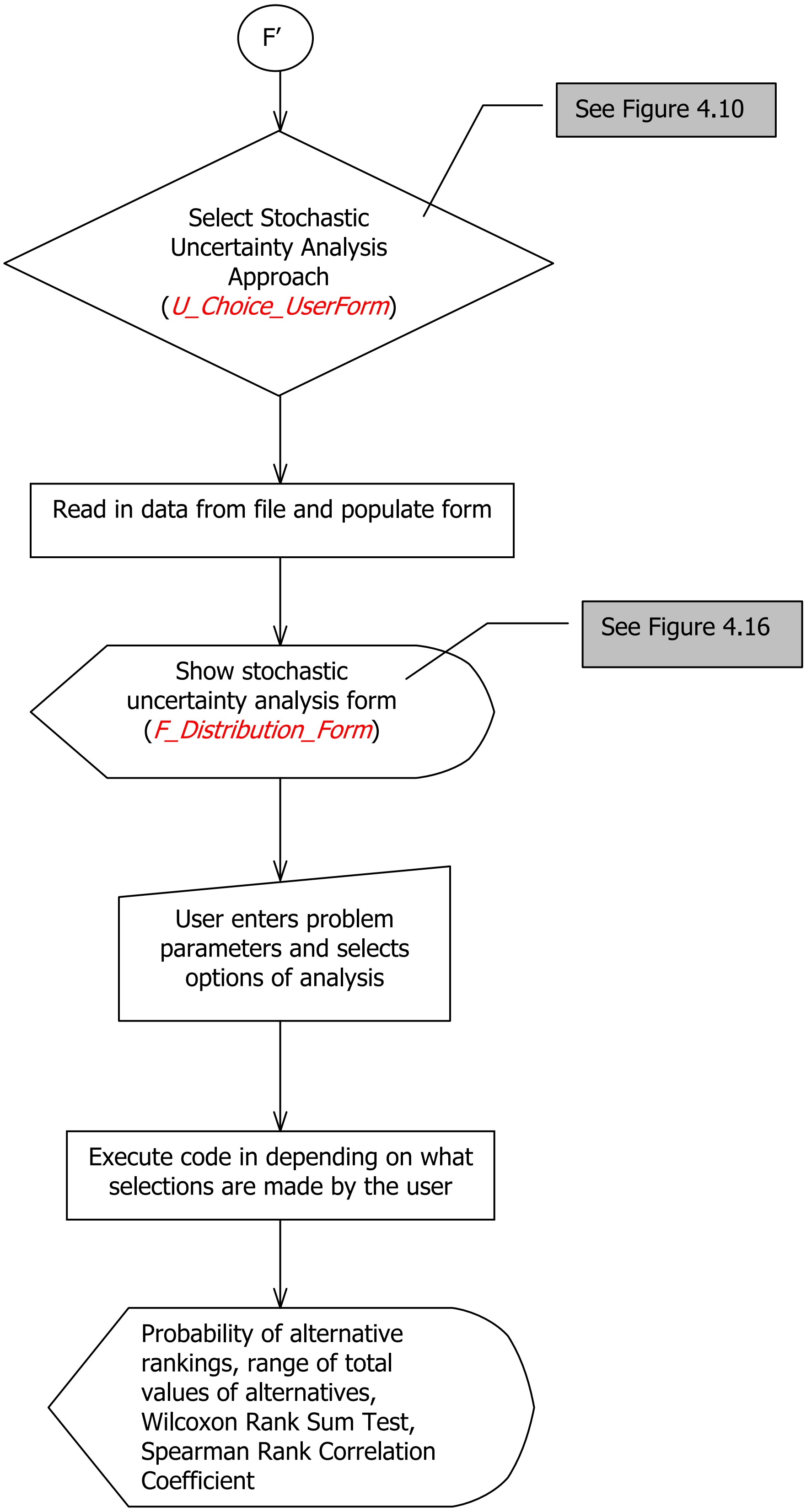
Layout D: Deterministic MCDA



Layout E: Distance-based Uncertainty Analysis Approach



Layout F: Stochastic Uncertainty Analysis Approach



Appendix F

Published, and accepted for publication, journal papers

Hyde, K. M., Maier, H. R., Colby, C. B. (2005) A distance-based uncertainty analysis approach to multi-criteria decision analysis for water resource decision-making. *Journal of environmental management*, v. 77 (4) pp. 278-290

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It is also available online to authorised users at:

http://dx.doi.org/10.1016/j.jenvman.2005.06.011

Hyde, Kylie M., Maier, Holger R., Colby, Christopher B., New distance-based uncertainty analysis approach to multi-criteria decision analysis
European Journal of Operational Research, Under Review, 2006

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ABSTRACT:

Analyses of water resource allocation problems, involving tradeoffs among multiple criteria, can be undertaken using multi-criteria decision analysis (MCDA). However, various sources of uncertainty exist in the application of MCDA methods, including the definition of criteria weights and the assignment of criteria performance values. Sensitivity analysis can be used to analyse the effects of these uncertainties on the ranking of alternatives, however, many existing methods have been found to have numerous limitations when applied to MCDA. In this paper, a distance-based method for uncertainty analysis is proposed, which enables the decision maker to examine the robustness of the ranking of the alternatives. The proposed method involves undertaking deterministic MCDA, distance metric optimisation and interpretation of results. The methodology is illustrated by applying it to a water resource allocation study previously undertaken in the literature using PROMETHEE and the performance of the two optimisation techniques, namely GRG2 and Genetic Algorithm, is compared. The results demonstrate the benefits of simultaneously considering the uncertainty in the criteria weights and the criteria performance values, as well as the advantages of utilising a Genetic Algorithm as the optimisation tool when the solution space of the decision problem is complex.

***Keywords:**. Decision analysis, Uncertainty, Euclidean Distance, Genetic Algorithms*

1. Introduction

Multi-criteria decision analysis (MCDA) is a formal approach that has been utilised to assist with the making of complex decisions, including water resource management decisions, for a number of decades (Anand Raj and Kumar, 1996; Choi and Park, 2001; David and Duckstein, 1976; Flug et al., 2000; Hajeeh and Al-Othman, 2005; Hobbs et al., 1992; Jaber and Mohsen, 2001; Kheireldin and Fahmy, 2001; Ridgley and Rijsberman, 1994). MCDA is widely used, as it facilitates stakeholder participation and collaborative decision making, does not necessarily require the assignment of monetary values to environmental or social criteria, and allows the consideration of multiple criteria in incommensurable units (i.e. combination of quantitative and qualitative criteria) (Hajkowicz, 2000). The MCDA process generally follows the sequence of: (i) identifying decision maker(s) (final decision makers, DMs), actors (people involved in the decision analysis process) and stakeholders (anyone who might be affected by the alternatives), (ii) selecting criteria, (iii) formulating alternatives, (iv) selecting an MCDA technique(s), (v) weighting the criteria, (vi) assessing the performance of alternatives against the criteria, (vii) transforming the criteria performance values (PVs) to commensurable units, if required, (viii) applying the selected MCDA technique(s) to obtain a

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ranking of the alternatives, (ix) performing sensitivity analysis, and (x) making the final decision.

Existing MCDA techniques include value focused approaches (e.g. Weighted Sum Method (WSM) (Janssen, 1996), and Analytic Hierarchy Process (AHP) (Saatay, 1977)) and outranking methods (e.g. PROMETHEE (Brans et al., 1986) and ELECTRE (Roy, 1991)). In general, the input parameters to existing MCDA techniques consist of criteria weights (CWs) and criteria PVs. It is generally recognised that weighting the criteria and assessing the performance of alternatives against the criteria are two of the most important, yet most challenging, aspects of applying the MCDA methodology. Due to the inherent difficulty in assigning and eliciting the input parameter values (discussed in Section 2), they are potential sources of considerable uncertainty in the analysis and it has been found that changes in the input parameter values may influence the outcomes of the decision analysis (Larichev and Moshkovich, 1995; Roy and Vincke, 1981). Despite this, the impact of the variability of the input parameters on the rankings of the alternatives has been largely overlooked in studies in which MCDA has been applied to water resources problems (see for example Al-Kloub et al., 1997; Connell Wagner, 2002; Duckstein et al., 1994; Flug et al., 2000; Gershon et al., 1982; Hajeeh and Al-Othman, 2005; Jaber and Mohsen, 2001; Kheireldin and Fahmy, 2001; Raju et al., 2000; Ulvila and Seaver, 1982). The effective incorporation, management and understanding of uncertainty in the input parameters remain the most fundamental problems in MCDA (Felli and Hazen, 1998).

Numerous sensitivity analysis methods designed to quantify the impact of parametric variation on MCDA output have been proposed in the literature (Barron and Schmidt, 1988; Guillen et al., 1998; Soofi, 1990; Wolters and Mareschal, 1995), however, generally only one input parameter is varied at a time, which is inadequate, as it may be the case that the ranking of the alternatives is insensitive to the variations of some parameters in a set individually, but sensitive to their simultaneous variation. To overcome some of the shortcomings of existing sensitivity analysis methods for MCDA, a distance-based approach for sensitivity analysis was proposed by Hyde et al. (2005). This method, however, along with other existing sensitivity analysis methods, has limitations because it focuses on the assessment and influence of the CWs only, which is further discussed in Section 3.

Few sensitivity analysis methods have been developed to assess the impact of PVs on the ranking of alternatives (Triantaphyllou and Sanchez, 1997), therefore, the impact of the uncertainty and variability in the criteria PVs is commonly disregarded. To overcome this limitation, a new distance-based approach is presented in Section 4 of this paper, which extends the method proposed by Hyde et al. (2005) by determining how sensitive the ranking of alternatives is to the simultaneous variation of all input parameters (e.g. CWs and PVs) over their expected range. A range of optimisation methods can be utilised to implement the proposed approach, and in this paper the performance of two different classes of optimisation algorithms is evaluated, including the gradient method used by Hyde et al. (2005) (i.e. GRG2 nonlinear optimisation method) and a global optimisation method (i.e. Genetic Algorithms, GAs). A case study undertaken by Mladineo et al. (1987) to assess alternative locations of hydro plants is utilised in Section 5 to demonstrate the benefits of altering all of the input parameters simultaneously, as opposed to only the CWs. The case study is also used to investigate the performance of each of the optimisation methods in effectively solving the objective function.

2. Uncertainty in the MCDA Input Data

The input data required by the majority of MCDA techniques is the assignment of criteria PVs by experts and the elicitation of CWs from actors. The PVs that are assigned to each criterion for each alternative may be obtained from models (e.g. streamflow losses), available data (e.g. power required) or by expert judgement based on previous knowledge and experience. The type of value assigned to each criteria PV may be quantitative (e.g. the power required may rise be 20 MW) or qualitative (e.g. the power required may be 'medium'). Providing precise figures for the criteria PVs is often difficult, as the alternatives being assessed are generally predicted future events. There may therefore be some imprecision, contradiction, arbitrariness and / or lack of consensus concerning the criteria PVs used in the analysis (Mousseau et al., 2003). The widely used MCDA outranking technique, PROMETHEE, has attempted to take this form of uncertainty into account by incorporating generalised criterion functions into the analysis. Six types of generalised criterion functions have been suggested by Brans et al. (1986) with the aim of realistically modelling the DMs' preference, which gradually increases from indifference to strict preference, and to facilitate the inclusion of the inherent uncertainty in the criteria PVs in the decision analysis process. However, the selection of an appropriate function and the associated thresholds for each criterion is a complex and ambiguous task for DMs and actors and, therefore, adds another element of uncertainty into the decision analysis process (Salminen et al., 1998).

The other type of input data required, CWs, indicate a criterion's relative importance and allow actors' views and their impact on the ranking of alternatives to be expressed explicitly. CWs are elicited by the decision analyst from the actors for each criterion using one of a variety of available techniques (see for example Hobbs (1980)) and, in theory, the CWs enter the analysis as well-defined constants. The specification of CWs, however, is not an easy task. Providing CWs is the major judgemental phase of the MCDA process (Hajkowicz et al., 2000) and research has shown that the CWs assigned by the actors are not reliable and stable information (Larichev, 1992). In addition, there is also often a large diversity of views among the actors involved in the analysis and these are predominantly insufficiently included in many decision analysis case studies reported in the literature (i.e. an average of the CWs obtained from the actors is utilised which can thereby leave actors sceptical of the result of the decision analysis). It should be noted that the inclusion of the generalised criterion functions in the MCDA technique PROMETHEE does not address the inherent imprecision and subjectivity of the CWs.

3. Existing Sensitivity Analysis Methods

To assess the extent the ranking of the alternatives is dependent on, and sensitive to, the input parameter estimates, sensitivity analysis is commonly used. Sensitivity analysis generally only involves altering the CWs, or if the PROMETHEE MCDA technique is utilised, the variation of generalised criterion functions or threshold values. However, this analysis is frequently incomplete and unsatisfactory, with values often altered arbitrarily, depending on the desired outcome.

Numerous sensitivity analysis methods, ranging from deterministic to stochastic, have been proposed in the literature that are applicable to existing MCDA techniques. The sensitivity analysis method proposed by Guillen et al. (1998) allows the DM to determine the robustness of the preference between two alternatives by calculating the proportion by which the DM must modify the CWs to change the ranking between two alternatives. Wolters and Mareschal (1995) have proposed an approach to determine the minimum modification of the CWs required to make a specific alternative ranked first, while taking into account specific requirements on the CW variations. Separate methods have been proposed by Triantaphyllou and Sanchez (1997) for three MCDA methods (WSM, weighted product model (WPM) and AHP) to determine the minimum quantity that a CW or PV, respectively, needs to be changed by to reverse the ranking for each pair of alternatives for each criterion. The foremost limitation of these methods, which is common with that of other existing methods, is that they only consider one data input at a time, with the remaining data inputs being fixed. The existing methodologies are also inadequate in the sense that they are only applicable to certain MCDA techniques.

Various distance based sensitivity analysis methods have also been proposed in the literature. Isaacs (1963), Fishburn et al. (1968), Evans (1984) and Schneller and Sphicas (1985) have utilised the Euclidean Distance to determine the sensitivity of decisions to probability estimation errors. Barron and Schmidt (1988) and Soofi (1990) use Euclidean Distances in problems where there is some imprecision in the CWs of an additive value function. Rios Insua and French (1991) introduce a framework for sensitivity analysis in multi-objective decision making within a Bayesian context and also utilise the Euclidean Distance and Chebyshev Distance. Alternatively, the TOPSIS method determines a solution utilising the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution, but it does not consider the relative importance of these distances (Hwang and Yoon, 1981; Opricovic and Tzeng, 2004). A distance-based approach for uncertainty analysis has been proposed by Hyde et al. (2005) which enables the uncertainty in the CWs to be taken into consideration in the MCDA process and is applicable to various MCDA techniques. The approach simultaneously varies the CWs within their expected range of uncertainty to find a set of CWs that are the minimum distance from the original set of CWs, which results in rank equivalence between a pair of alternatives. The most significant CWs are also identified in the proposed approach. The method, however, is limited because it does not simultaneously take the uncertainty of the criteria PVs into consideration. Each of the sensitivity methods described are therefore restricted in that they are either applicable to only one type of MCDA method, consider only one of the input parameters (i.e. CWs or PVs), or vary only one input parameter at a time, while the remaining parameters are kept constant.

Probabilistic sensitivity analysis approaches proposed by Critchfield and Willard (1986), Felli and Hazen (1998), Hyde et al. (2004) and Janssen (1996) are able to overcome some of the aforementioned shortcomings of the distance-based techniques for sensitivity analysis. These researchers have introduced methods to analyse systematically the sensitivity of the ranking of alternatives to overall uncertainty and changes in PVs and / or CWs during the decision-making process. For example, the diversity of the CWs has been captured by the methodology proposed by Hyde et al. (2004) where a distribution is fitted or assigned to the range of CWs elicited by the actors such that all actors' values are incorporated in the analysis. However, the stochastic methods generally require the assignment of appropriate probability distributions to the CWs and PVs, which might be difficult to accomplish in some situations due to a paucity of data. Consequently, a method is required which enables an understanding of the robustness of the ranking of the alternatives to be obtained in situations where insufficient information is available for distributions to be fitted to the input parameter values.

4. Proposed Approach

As discussed in Section 1, the proposed approach extends the uncertainty analysis method proposed by Hyde et al. (2005) through allowing simultaneous variation of all the input parameter values (i.e. CWs and PVs) and involves undertaking deterministic MCDA, followed by distance metric optimisation and interpretation of the results. The process is illustrated in Figure 1 and described in detail below.

4.1 Deterministic MCDA

Deterministic MCDA is performed as the first stage of the proposed approach to determine the total values of the alternatives and hence the ranking of each alternative for each actor's set of CWs. Actors are generally selected to be representative of the stakeholders of the particular decision problem. The number of actors varies with each decision problem depending on factors such as the time and resources available and the perceived level of importance of the decision. The decision analysis situation is translated into a set of alternatives and appropriate criteria must be chosen to enable information about these alternatives to be collected. The criteria generally have the most significant impact on the final ranking of alternatives, as they determine the information inputs to the MCDA model (Hajkowicz et al., 2000). The actors, under the guidance of the decision analyst, generally develop the alternatives and criteria. The CWs are elicited from the actors and the PVs are assigned to each criterion for each alternative.

An existing MCDA technique, such as a value focused approach (e.g. WSM (Janssen, 1996)) and outranking methods (e.g. PROMETHEE (Brans et al., 1986)), is then utilised to determine the total value of each alternative for the assigned input parameters. Generally, the objective of the DM is to rank the alternatives in order of preference (e.g. Rank 1, most preferred alternative to Rank *n*, least preferred alternative), which is based on the total value of each of the alternatives. Although the ranking of alternatives is obtained using the process outlined, no information is provided to the DM (or each of the actors) with regard to how likely it is that a reversal of the rankings of the alternatives will occur with a change in input parameters (i.e. CWs and PVs).

Deterministic MCDA:
- Identify actors
- Formulate alternatives $a_1, a_2, \ldots, a_n$
- Select criteria $c_1, c_2, \ldots, c_m$
- Select MCDA technique
- Actors weight the criteria, $w_1, w_2, \ldots w_m$
- Experts assess alternatives against the criteria
- Standardise PVs if required
- Apply MCDA technique

↓

Distance Metric Optimisation:
- Select pair of alternatives $a_x > a_y$
- Define upper and lower bounds of CWs and PVs
- Define constraints
- Select optimisation technique (e.g. Solver or GA)
- Undertake optimisation

↓

Interpret Results:
- Distance metric (e.g. Euclidean Distance) for each pair of alternatives
- Critical input parameters

Figure 1. Proposed Approach

4.2 Distance Metric Optimisation

Concept

The alternatives that are immediate contenders for being ranked first are of real interest to the decision analyst and can be identified using distance based tools (Proll et al., 2001). Vincke (1999) defines the concept of robustness to express the fact that a solution, obtained for one scenario of data and one set of values for the parameters of the method, is 'far or not' from another solution, obtained for another scenario of data and another set of values for the parameters of the method. Hence, the concept of robustness will inevitably be based on a notion of distance or dissimilarity between solutions. The aim of the proposed distance-based uncertainty analysis method is to find the nearest competitors of the current highest ranked alternative and is achieved by identifying the 'smallest' changes necessary in the input parameters before a change in the ranking of the alternatives occurs.

In some decision situations, one alternative will always be superior to another, regardless of the values the input parameters take. In this case, the ranking of the alternatives is robust, as it is insensitive to the input parameters. However, in many instances, this is not the case, and a number of different combinations of the input parameters will result in rank equivalence. By determining the smallest overall change that needs to be made to the input parameters (i.e. CWs and PVs) in order to achieve rank equivalence, the robustness of the ranking of two alternatives (a_x and a_y) is obtained. This concept is illustrated in Figure 2 for a simple two-dimensional example. In Figure 2, the criteria PVs of the lower ranked alternative ($PV_{1,y}$, $PV_{2,y}$), which result in a total value of $V(a_y)$, are given by point Y and the criteria PVs of the higher ranked alternative ($PV_{1,x}$, $PV_{2,x}$), which result in a total value of V(a_x), are given by point X. In this example, all combinations of PV_1 and PV_2 on the curved line labelled V(a_y)*opt* = V(a_x)*opt* will modify the total values of alternative *y* and alternative *x* so that rank equivalence occurs between the two alternatives. Consequently, the robustness of the ranking of alternatives *x* and *y* is given by the shortest distance between point Y and the V(a_y)*opt* = V(a_x)*opt* line and point X and the V(a_y)*opt* = V(a_x)*opt* line, which are labelled d_1 and d_2, respectively, and are combined into a single distance measure, *d*. If this distance is large, then more substantial changes need to be made to the input parameters in order to achieve rank equivalence, and the ranking of the two alternatives is relatively insensitive to input parameter values (i.e. robust). Conversely, if this distance is small, minor changes in the input parameters will result in rank equivalence, and the ranking of the alternatives is sensitive to input parameter values (i.e. not robust). As the proposed approach identifies the combination of input parameters that is the shortest distance from the original parameter set, the input parameters to which the rankings are most sensitive are also identified.

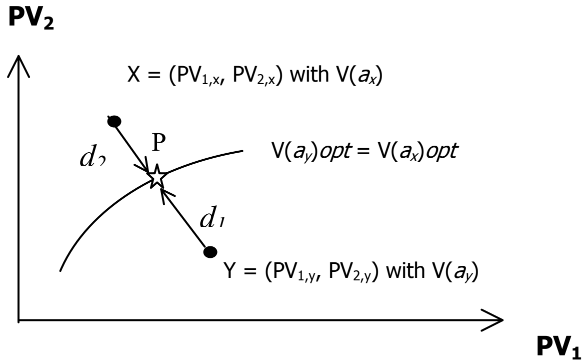

Figure 2. 2D Concept of Proposed Approach

Formulation

As stated above, the purpose of the proposed distance-based approach for uncertainty analysis is to determine the minimum modification of the MCDA input parameters (i.e. CWs and PVs) that is required to alter the total values of two selected alternatives (e.g. ax and ay) such that rank equivalence occurs. The minimum modification of the original input parameters is obtained by translating the problem into an optimisation problem. The objective function minimises a distance metric, which provides a single measure, indicating the amount of dissimilarity between the original input parameters of the two alternatives under consideration and their optimised values. Optimised refers to the set of input parameters that is the smallest distance from the original parameter set, such that when the optimised set is used, the total values of the two alternatives being assessed are equal. The Euclidean Distance, de, has been selected as the distance metric in this paper, as it is one of the most commonly used distance metrics in the literature (Barron and Schmidt, 1988; Isaacs, 1963; Rios Insua and French, 1991). However, other distance metrics such as the Manhattan Distance and relative entropy (i.e. Kullback-Leibler Distance) can also be used and the choice of an appropriate distance metric is dependent on the decision problem being assessed. When the Euclidean Distance is used, the objective function is defined as:

$$\text{Minimise } d_e = \sqrt{\sum_{m=1}^{M}\left(w^{\#}{}_{jmi} - w^{\#}{}_{jmo}\right)^2 + \left(x^{\#}{}_{mnli} - x^{\#}{}_{mnlo}\right)^2 + \left(x^{\#}{}_{mnhi} - x^{\#}{}_{mnho}\right)^2} \tag{1}$$

(Note: # refers to the standardised values of these parameters (see Equation 7).)

Subject to the following constraints:

$$\sum_{m=1}^{M} w_{jmi} = \sum_{m=1}^{M} w_{jmo} \tag{2}$$

$$V(a_y)opt = V(a_x)opt \tag{3}$$

$$LL_{xl} \le x_{mnlo} \le UL_{xl} \text{ for } m = 1 \text{ to } M \tag{4}$$

$$LL_{xh} \le x_{mnho} \le UL_{xh} \text{ for } m = 1 \text{ to } M \tag{5}$$

$$LL_{w} \le w_{jmo} \le UL_{w} \text{ for } m = 1 \text{ to } M, \text{ for chosen actor } j \text{ of } J \tag{6}$$

where

w_{jmi} is the initial CW of criterion *m* of actor *j*,

w_{jmo} is the optimised CW of criterion *m* of actor *j*,

x_{mnli} is the initial PV of criterion *m* of initially lower ranked alternative *n*,

x_{mnlo} is the optimised PV of criterion *m* of initially lower ranked alternative *n*,

x_{mnhi} is the initial PV of criterion *m* of initially higher ranked alternative *n*,

x_{mnho} is the optimised PV of criterion *m* of initially higher ranked alternative *n*,

d_e is the Euclidean Distance,

M is the total number of criteria,

$V(a_y)opt$ is the modified total value of the initially lower ranked alternative obtained using the optimised parameters,

$V(a_x)opt$ is the modified total value of the initially higher ranked alternative obtained using the optimised parameters,

LL_{xl} and UL_{xl} are the lower and upper limits, respectively, of the PVs of each criterion for the initially lower ranked alternative,

LL_{xh} and UL_{xh} are the lower and upper limits, respectively, of the PVs of each criterion for the initially higher ranked alternative, and

LL_w and UL_w are the lower and upper limits, respectively, of each of the CWs for the selected actor's CWs.

It should be noted that there is only one term for the CWs in Equation 1 because the CWs are common to all alternatives. In addition, if the PROMETHEE MCDA technique is utilised, the results are obtained without requiring the DM to specify generalised criterion functions for each of the criteria, as the uncertainty in the criteria PVs is taken into account by specifying upper and lower bounds (Equations 4 and 5).

To ensure that the scale of the input parameters does not influence the optimisation, the values used in the distance metric (i.e. Equation 1) are standardised using the following formula:

$$x^{\#}{}_{mnli} = \frac{x_{mnli}}{\sigma_{Xm}} \tag{7}$$

where

$x_{mnli}^{\#}$ is the standardised initial PV of criterion *m* of initially lower ranked alternative *n*,

x_{mnli} is the initial PV of criterion *m* of initially lower ranked alternative *n*,

σ_{Xm} is the standard deviation of the set of PVs of criterion *m*.

Equation 7 is also applied to the other parameters in Equation 1, respectively.

As is evident by the formulation above, the objective function (Equation 1) is subject to a number of constraints, including that the total sum of the 'optimised' CWs is required to equal the total sum of the original CWs (Equation 2). The modified total value of the initially lower ranked alternative (ay) must also be equal to the modified total value of the initially higher ranked alternative (ax) (Equation 3). The total values of the alternatives are determined using the selected MCDA technique (e.g. WSM or PROMETHEE) with the optimised values of the input parameters.

The expected ranges that the input parameters can be varied between to obtain a reversal in ranking of the selected alternatives (i.e. $a_y > a_x$) are also constraints of the objective function (Equations 4 - 6). Specification of the minimum and maximum values of the input parameters represents the potential uncertainty and variability in the assignment of these values in the initial stage of the decision analysis process. The range of values (i.e. upper and lower bounds) that is specified for each PV of the selected alternatives represents the set of possible values for that variable, which can either be based upon knowledge of experts or the data that are available. The feasible range of CWs is defined to represent the expected variability in the preference values due to the subjective and ambiguous nature of the values elicited and / or the diversity of views among the stakeholders. Either the DM or actors can define the CW ranges or, alternatively, actual ranges of the available data can be utilised (i.e. the minimum and maximum values of the CWs elicited from the actors involved in the decision process). In the situation where the experts or actors are confident in the original input parameter values, the lower and upper bounds of the particular parameter would be equal to the original input parameter. For example, this may be particularly relevant to the situation where qualitative data ranges (e.g. High to Low, where 1 equals High and 5 equals Low) are used for a particular criterion.

Optimisation

In order to obtain the robustness of the ranking of each pair of alternatives (i.e. a_x and a_y) for each actor's set of CWs, the optimisation problem given by Equations 1 – 7 needs to be solved. This could become computationally demanding if there are a considerable number of alternatives and / or a large number of actors, as a large number of comparisons would need to be undertaken. Generally, however, the analyst (and people involved in the decision analysis) is mainly concerned with the impact that the alternatives have on the highest ranked alternative. Therefore, for the majority of the time, only N-1 pairwise comparisons need to be undertaken. The decision problem can be solved using a number of optimisation techniques, including gradient methods (e.g. Generalised Reduced Gradient, GRG2) and evolutionary optimisation algorithms (e.g. GAs), for example. The GRG2 nonlinear optimisation method can be used to solve the objective function (Equation 1) by changing the CWs and PVs within their specified ranges, subject to the defined constraints (Equations 2 – 6). GRG2 works by first evaluating the function and its derivatives at a starting value of the decision vector and then iteratively searching for a better solution using a search direction suggested by the derivatives (Stokes and Plummer, 2004). The search continues until one of several termination criteria are met. If no solution can be found, the DM can be confident that the ranking of the two alternatives is robust (i.e. that no changes in the CWs or PVs between the specified ranges will result in a reversal of the ranking). GRG2 is not a global optimisation algorithm, therefore, to increase the chances of finding global or near-global optima, the optimisation should be repeated a number of times using different randomly generated starting values (which will be referred to as 'trials' for the remainder of the paper). This aims to minimise the impact that the starting values have on the outcome of the analysis.

Alternatively, the objective function can be solved by using optimisation algorithms that are more likely to find globally optimal solutions for complex problems, such as GAs, which are heuristic iterative search techniques that attempt to find the best solution in a given decision space based on a search algorithm that mimics Darwinian evolution and survival of the fittest in a natural environment (Goldberg, 1989). GAs have been used successfully to solve other complex combinatorial optimisation problems, including the design and maintenance of water distribution systems (Dandy and Engelhardt, 2001; Savic and Walters, 1997). The GA utilised in this research used a 'real' coding scheme, creep mutation and tournament selection. For a detailed description of GAs, the reader is referred to Goldberg (1989).

Constraints are unable to be incorporated directly in the formulation of the GA, therefore, they are included in the objective function and multiplied by penalty values to discourage the selection of infeasible solutions by decreasing their fitness. The previously defined objective function (Equation 1) is therefore reformulated and defined as:

$$\text{Minimise } P_1 \times \left| \sum_{m=1}^{M} w_{mi} - \sum_{m=1}^{M} w_{mo} \right| + (P_2 \times d_e) + P_3 \times \left| V(a_x)opt - V(a_y)opt \right| \tag{8}$$

subject to the constraints given by Equations 4 to 6. P_1, P_2 and P_3 are penalty values that are user defined and d_e is given by Equation 1. Penalty values are problem dependent and need to be determined using trial and error (Chan Hilton and Culver, 2000; Cieniawski et al., 1995). The amount of effort or total number of GA searches required to determine reasonable penalty values is an important component of the overall efficiency of the GA. If a set of penalties is too harsh, then the few solutions found that do not violate constraints quickly dominate the mating pool and yield sub-optimal solutions. A penalty that is too lenient can allow infeasible solutions to flourish as they can have higher fitness values than feasible solutions.

Both of the optimisation methods presented here to solve the objective function (i.e. GRG2 and GA) have their advantages and disadvantages. The main advantage of using GRG2 is its speed of arriving at a solution, however, its disadvantage is that because it is a gradient method, the chances of a local solution being obtained are high. The advantage of the GA is that it is a global search technique, however, it generally takes a longer time to converge compared with the GRG2. It should be noted, however, that the processing time of the optimisation methods is dependent on the complexity of the decision problem that is being assessed (i.e. how many alternatives and criteria are involved in the decision problem and the 'robustness' of the ranking of the alternatives). A trade-off is therefore required between the amount of time taken to perform the analysis and the level of certainty that the minimum distance has actually been obtained. An advantage GAs have over traditional optimisation techniques (such as GRG2) is that they do not require the use of a gradient fitness function, only the value of the fitness function itself. Another advantage of GAs is that they search from a population of points, investigating several areas of the search space simultaneously, and therefore have a greater chance of finding the global optimum. GAs do, however, require a large number of input parameters to be specified, which can take a considerable time to perfect for a particular decision problem and is therefore a limitation of this optimisation technique.

It should be noted that the approach presented in this paper is modular and can be customised to suit different situations by using alternative distance metrics and / or optimisation methods. In order to cater for the general case, the GRG2 and Genetic Algorithm optimisation approaches have been selected for this paper. However, if all of the constraints were linear (or bi-linear), the problem could be formulated as a quadratic program and more computationally efficient optimisation algorithms could be used.

4.3 Interpretation of Results

The output of the uncertainty analysis is the minimum value of the selected distance metric for each pair of alternatives, which can be summarised in a matrix. The distance obtained provides a relative measure of the robustness of the ranking of the alternatives. A non-feasible, or a very large, value of the distance metric between two alternatives informs the DM that one alternative will predominantly be superior to another, regardless of the input parameter values selected between the specified ranges. Conversely, if the distance is small, slight changes in the input parameters will result in rank equivalence and the ranking of the alternatives can therefore be concluded as being sensitive to the input parameter values.

The decision-making process can be improved considerably by identifying critical input parameters and then re-evaluating more accurately their values. The most critical criteria can be identified by examining the relative and absolute change in input parameter values:

$$\text{Absolute } \Delta\, x_{mln} = x_{mnlo} - x_{mnli} \qquad \text{or Absolute } \Delta\, w_{jm} = w_{jmo} - w_{jmi} \tag{9}$$

$$\text{Relative } \Delta\, x_{mln} = \frac{x_{mnlo} - x_{mnli}}{x_{mnli}} \times 100\ \% \quad \text{or Relative } \Delta\, w_{jm} = \frac{w_{jmo} - w_{jmi}}{w_{jmi}} \times 100\ \% \tag{10}$$

It should be noted that Equations 9 and 10 can also be used to determine the most critical PVs of the initially higher ranked alternative. The input parameters that exhibit the smallest relative change in value to achieve rank equivalence between two alternatives are most critical to the reversal in ranking. The method is therefore a useful tool as a starting point to direct negotiations and guide discussion with knowledge of the criteria that have the most impact. Hence, the results provide the DM with further information to aid in making a final decision, including information on the most critical input parameters obtained from their simultaneous variation.

5. Case Study

The proposed approach has been applied to a study undertaken by Mladineo et al. (1987) and details of the study, the problem formulation and the results are presented below. The methodology suggested by Hyde et al. (2005) is also utilised to illustrate the benefits of the approach proposed in this paper, which jointly varies all of the input parameter values in the decision analysis, as opposed to only considering the impact the CWs have on the rankings of the alternatives. In addition, the optimisation of the objective function is undertaken using the two techniques described in Section 4.2 (i.e. GRG2 and GA) to assess each of the approaches in terms of computational efficiency and their ability to find (near) global optimum solutions.

5.1 Background

Mladineo et al. (1987) utilised the PROMETHEE method to aid in the selection of locations of small hydro power plants in the River Cetina catchment, Croatia. One actor was involved in elaborating the six alternative locations for the hydro power plants and nine evaluation criteria, which cover a range of categories including land-use, environment, social and economic factors. The alternatives were evaluated using calculations and measurements for the measurable criteria, whereas for the non-measurable criteria were obtained by estimation based on verified descriptive value tables. The generalised criterion functions were selected so as to represent each criterion in the best possible way. A description of the criteria, the CWs, the generalised criterion functions and associated thresholds, the preference direction of each of the criteria and the criteria PVs used by Mladineo et al. (1987) for the MCDA are summarised in Table 1. The rankings of the alternatives from the MCDA undertaken by Mladineo et al. (1987), using the XPROM 2 computer package, are contained in Table 2. Altogether, Alternative 1 outranks all of the other proposals.

5.2 Problem Formulation Using Proposed Approach

As mentioned previously, generalised criterion functions are not required to be specified for each of the criteria in the proposed approach, therefore, deterministic analysis is repeated as part of this research using the MCDA technique PROMETHEE with the criteria PVs and CWs provided by Mladineo et al. (1987). Level I generalised criterion functions are used for each criterion to enable the outranking methodology to be undertaken. The feasible input parameter range for the CWs and the PVs must be specified, as defined by Equations 4 - 6. No information was provided by Mladineo et al. (1987) on the uncertainty associated with the criteria PVs or the CWs, therefore, the upper and lower limits of the input parameters were assumed for the purposes of this paper and the limits for the CWs and the PVs of the two highest ranked alternatives are included in Table 3.

Table 1
Description of the Criteria and the Associated Values Utilised by Mladineo et al. (1987)

Criterion	CW	Fn. Type*	Threshold Values	Pref. Dir^n	Alternative (Location)					
					1	2	3	4	5	6
Power of the plant (MW)	10	3	s = 0.5	Max	0.45	0.4	1.2	0.9	0.5	1.6
Approximate cost of the plant	10	2	p = 150, q = 4	Min	50	46	160	140	40	200
Access roads (km)	5	2	p = 12, q = 1	Min	5	4.6	16	14	4	20
Plant management and maintenance	8	2	p = 1.5, q = 0.2	Min	0.1	0.5	2.0	0.5	1.0	1.1
Geotechnical characteristics of the location (gradation)	5	1	p = 0, q =0	Max	4	4	3	3	4	4
Distance from the consumers' (consumption) centre (km)	10	2	p = 5, q = 2	Min	22	20	20	20	30	25
Streamflow losses (%)	5	3	s = 5	Min	11	5	12	22	1	1
Environmental impact (gradation)	8	1	p = 0, q = 0	Min	1	2	1	1	2	3
Possibility of assimilating the plant within the electroenergetic system of the country (gradation)	6	1	p = 0, q = 0	Min	1	1	2	2	3	2

Note *: 1 = Level criterion, 2 = Linear criterion, 3 = Gaussian criterion

Table 2
Complete Rankings of Alternatives

	Rank					
	1	2	3	4	5	6
Mladineo et al. (1987)						
	Alt 1	Alt 2	Alt 4	Alt 3	Alt 5	Alt 6
ϕ	0.3015	0.2187	-0.0454	-0.1160	-0.1568	-0.2022
Level 1 Generalised Criterion Functions						
	Alt 1	Alt 2	Alt 4	Alt 5	Alt 3	Alt 6
ϕ	0.2030	0.1911	0.0418	-0.0269	-0.1254	-0.2836

Table 3
Upper and lower limits of parameters for uncertainty analysis

Criterion	CWs			PVs Alt 1			PVs Alt 2		
	Original	**LL**	**UL**	**Original**	**LL**	**UL**	**Original**	**LL**	**UL**
1	10	7	13	0.45	0.1	0.75	0.40	0.1	0.7
2	10	7	13	50	40	60	46.00	40	52
3	5	2	8	5	1.6	8	4.6	1.6	7.6
4	8	5	11	0.1	0.01	0.4	0.5	0.2	0.8
5	5	2	8	4	1	7	4	1	7
6	10	7	13	22	16	26	20	16	24
7	5	2	8	11	7	15	5	2	8
8	8	5	11	1	0.05	4	2	0.05	5
9	6	3	9	1	0.05	4	1	0.05	4

Two scenarios were assessed in order to compare the approach presented in this paper with the methodology presented in Hyde et al. (2005): (1) vary the CWs only, and (2) vary both the CWs and PVs. The optimisation of these two scenarios was undertaken using the Microsoft Excel Add-In Solver Function, which uses the GRG2 non-linear optimisation method, and a GA. The main advantages of Solver are its wide availability and ease of use. Further information on Solver and the options available can be obtained from the help file of Microsoft Excel or Stokes and Plummer (2004). The Microsoft Excel binomial random number generator (i.e. the RANDBETWEEN function) was used to generate the random starting values of the input parameters. This operation was repeated a number of times (i.e. 500 trials) for each pair of alternatives to sufficiently vary the starting values with the aim of increasing the chances of finding near globally optimal solutions.

The performance of the GA is a function of a number of user-defined parameters, including penalty function values, mutation and crossover probabilities, generation and population numbers (Raju and Kumar, 2004). The optimal values of these parameters to be used during the analysis were obtained by considering suggested values published in the literature (Dandy and Engelhardt, 2001; Mirrazavi et al., 2001; Raju and Kumar, 2004). Sensitivity analysis was also undertaken by varying the size of the population (50 – 800) and the number of generations (100 – 1,000) to ascertain the impact these parameters have on the time taken for the analysis to be undertaken, in addition to the influence on the final value of the chosen distance metric (i.e. Euclidean Distance). The optimal parameters obtained, which were utilised in the analysis, are contained in Table 4. In addition, each of the penalty values (e.g. P_1, P_2, P_3 in Equation 8) were set to be 1,000, 1,500 and 3,500, respectively and the analysis was repeated with a number of random seeds. The GA was realised with a Visual Basic for Applications (VBA) implementation.

5.3 Results and Discussion

Deterministic Analysis

The complete rankings of the alternatives obtained using PROMETHEE with Level I generalised criterion functions for each of the criteria are contained in Table 2. The best performing alternative is Alternative 1, however, there is only a small difference in the total flow between Alternatives 1 and 2. It would therefore be difficult for the DM to confidently select an optimal alternative based on these results. The rankings using the Level 1 generalised criterion functions are slightly different to those obtained by Mladineo et al. (1987), shown in Table 2, with the most significant difference being the reversal in ranking between Alternatives 5 and 3. These results highlight the impact that the generalised criterion functions can have on the results of the decision analysis and the additional uncertainty that the generalised criterion functions can create due to the selection of the function and the associated thresholds required to be specified for each criterion.

Table 4
GA Input Parameters

Parameter	Value
m (population)	200
s (number of members per tournament)	8
P_c (probability of crossover)	0.8
n (number of crossover locations)	2
P_m (probability of mutation)	0.15
Pc (% probability that creep occurs)	0.5
Termination Criterion: Generations per run	500
Discretisation interval	0.25
Random Seed	368

Distance Metric Optimisation

As stated previously, one of the main benefits of the proposed approach is the ability for each actor (or groups of actors with similar preferences) to determine the impact that changes in the input parameter values will have on the outcome of the decision analysis, in particular in situations where an actor may be uncertain of the weights they have provided. The results of the proposed uncertainty analysis approach presented in this paper include a comparison of the robustness of the highest ranked alternative (Alternative 1) with the remainder of the alternatives, as space limitations restrict the presentation of the results for each combination of pairs of alternatives. Undertaking the full analysis would require a considerable number of calculations, however, with adequate computer resources and appropriate programming techniques it is possible to obtain the results within a reasonable and practical time frame.

GRG2 vs GA

The distance-based uncertainty analysis approach presented in this paper is demonstrated by applying it to the Mladineo et al. (1987) case study by using the GRG2 and GA optimisation techniques. The Euclidean Distances obtained that result in rank equivalence between the highest ranked alternative (Alternative 1) paired with the remaining alternatives by (i) varying the CWs only and (ii) simultaneously varying the CWs and PVs using both the (i) GRG2 and (ii) GA optimisation techniques described in Section 4.2 are summarised in Table 5. The initially lower ranked alternatives are listed in the leftmost column of Table 5 in rank order. The same values for the distance metric were obtained by the GRG2 when varying the CWs only, which indicates that the solution space is not very complex in this instance. No changes in CWs were able to be identified which would result in rank reversal of Alternative 1 and Alternatives 3 and 6, respectively, using both of the optimisation methods considered. There is minimal difference between the Euclidean Distances obtained, using the GA and GRG2 optimisation methods, for Alternatives 4 and 5 to outrank Alternative 1. The optimised CWs obtained from both of the optimisation methods for Alternative 4 to outrank Alternative 1, compared to the original CW values, are illustrated in Figure 3. Similar results are therefore obtained using either the GRG2 or the GA when incorporating only the CWs in the uncertainty analysis, which suggests that the DM could select either optimisation method to undertake this particular analysis.

Table 4
Comparison of Euclidean Distances of pairs of Alternatives for Actor 1 CWs, using GRG2 and GA and User Input Ranges for the Upper and Lower Limits of the CWs and PVs

	Rank	Alt.	CWs only		CWs and PVs	
			Solver	**GA**	**Solver**	**GA**
Initially lower ranked alternatives	**1**	**Alt 1**	-		-	-
	2	**Alt 2**	0.484	0.255	2.025	0.119
	3	**Alt 4**	5.283	5.274	2.459	0.104
	4	**Alt 5**	5.309	5.294	2.750	0.149
	5	**Alt 3**	NF	NF	2.564	0.146
	6	**Alt 6**	NF	NF	3.082	0.195

A different situation, however, occurs when the uncertainty analysis incorporates the simultaneous variation of the CWs and PVs. When utilising the GRG2 to solve the objective function, different final Euclidean Distances were obtained with each of the random starting values (i.e. for each trial) for each pair of alternatives, which indicates that the solution space is complex when more input parameters are included in the uncertainty analysis for this particular decision problem. This result also demonstrates that the starting values have a significant impact on the solution for this particular decision problem. Figure 4 illustrates the variation in Euclidean Distances obtained with different random starting positions in the solution space when the CWs and PVs are varied simultaneously using the GRG2 for Alternative 2 to outrank Alternative 1. The solution producing the minimum Euclidean Distance was designated the 'best' solution from the range of solutions obtained with random starting values, and these are the values presented in Table 5. Obtaining the different distances may undermine the confidence of the actors in the methodology if limited understanding of optimisation exists amongst the group of actors. Therefore, it is suggested that in an actual decision analysis situation, the best tool (or optimisation method) to solve the particular decision problem should be selected. For example, the Mladineo et al. (1987) decision problem has a complex decision space when all input parameters are incorporated in the analysis and therefore use of the GA would be more appropriate. The results obtained also demonstrate that when the solution space is complex, the chances of finding a near global solution are increased when utilising a GA, as smaller Euclidean Distances are obtained compared to the GRG2, as summarised in Table 5. This result is further highlighted in Figure 5, which illustrates the changes in the CWs that would result in, for example, Alternative 2 outranking Alternative 1, when the GA and GRG2 are utilised to find a solution to the objective function (it should be noted that these are only a subset of the results, and that the changes in the PVs have not been illustrated for the sake of clarity). It can be seen that the changes in CWs are so minor when the GA is utilised that the difference in the original CWs and the GA optimised values can hardly be distinguished.

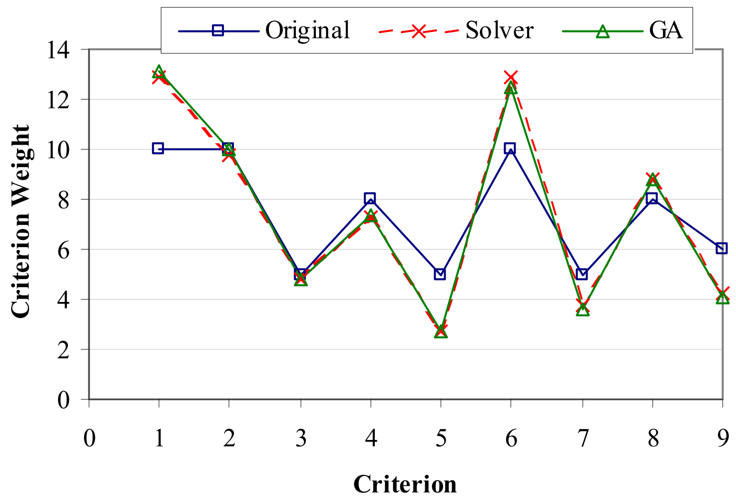

Figure 3. Comparison of optimised CWs using GA and GRG2 for Alternative 4 to outrank Alternative 1

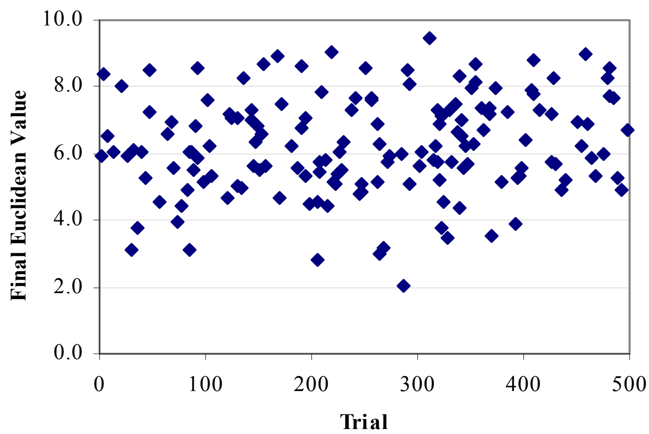

Figure 7. Final Euclidean Distances Obtained for Each Trial Number Using GRG2 and Comparing Alternatives 1 and 2 (CWs and PVs varied simultaneously)

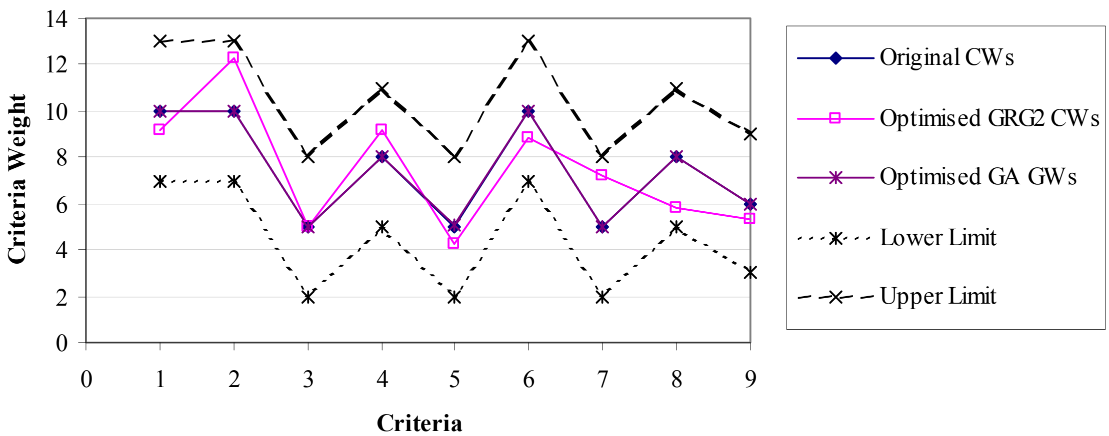

Figure 5. Comparison of Original CWs with the Optimised CWs using GRG2 and a GA while Varying CWs and PVs Simultaneously for Alternative 2 to Outrank Alternative 1

The time required to complete the uncertainty analysis using the two optimisation methods (GA and GRG2) is contained in Table 6. For this particular decision problem, the time taken to arrive at a solution is considerably shorter using the GA, compared to the GRG2 method. However, this is dependent on the GA input parameters selected (Table 4) and does not include the time taken to arrive at those final input parameters through a process of sensitivity analysis, as described in Section 5.2. The number of input parameters that have to be defined to undertake the GA is a limitation of this optimisation technique, in particular as the choice of optimal input parameters is problem dependent. As discussed above, however, one of the benefits of the GA is that there is a greater likelihood that a near globally optimum solution will be obtained.

Table 6
Time to complete simulation for Alternative 1 and 2

Computer Specifications	Optimisation Method	Trial / Generation Number	Time
Intel(R) Pentium(R), 1.46 GHz, 504 MB of RAM	GRG2	500	37 min & 18 secs
	GA	500	7 min & 6 secs

CWs vs CWs and PVs

The purpose of the analysis undertaken is to not only show the effect of the different optimisation methods on the results obtained, but also to determine the impact that inclusion of different input parameters in the uncertainty analysis has on the outcomes. A comparison between the Euclidean Distances in Table 5 shows that when incorporating all of the input parameters in the uncertainty analysis:

- Both GRG2 and GA were able to arrive at a solution for each pair of alternatives, whereas no changes in the CWs were able to be found which would result in the rank reversal of Alternatives 3 and 6;
- Smaller changes in the input parameters are required for rank reversal to occur when considering the changes in all input parameters, compared to only considering the uncertainty in the CWs, as demonstrated by the smaller Euclidean Distances obtained for the former case;
- Based on the GA results, Alternative 4 is more likely to outrank Alternative 1 when simultaneous changes in CWs and PVs are considered, compared to Alternative 2 being more likely to outrank Alternative 1 when only the CWs are incorporated in the analysis.

Only considering the CWs in the uncertainty analysis would therefore be likely to lead the DM to a different conclusion than when all of the input parameter values are incorporated. It is interesting to note that when all input parameters are incorporated in the uncertainty analysis, Alternative 3 has a smaller Euclidean Distance than Alternative 5, which indicates that the ranking of Alternative 5 is more robust than that of Alternative 3. As these are the two alternatives whose ranking reverses when Level 1 generalised criterion functions are used instead of the generalised criterion functions selected by Mladineo et al. (1987) (see Table 3), the results indicate that any uncertainty in the input parameters is adequately taken into consideration by utilising the upper and lower limits of the parameters without having to define generalised criterion functions for each of the criteria when utilising the PROMETHEE MCDA technique.

The benefits of the proposed approach can also be clearly demonstrated by considering the results of Alternatives 1 and 4, which are ranked first and third, respectively. In a traditional decision analysis approach, the large difference in the initial, deterministic, total flows of the two alternatives (i.e ϕ Alt 1 = 0.203 and ϕ Alt 4 = 0.042) would suggest to the DM that the ranking of the two alternatives is robust and therefore Alternative 4 would most likely be discarded from any further analysis. If the DM did decide to undertake further analysis and assess the impact that varying the CWs had on the ranking of the alternatives, the robustness of the ranking of Alternatives 1 and 4 would be confirmed, as it was found that large changes in the CWs are required for rank reversal to occur between the two alternatives (Euclidean Distance = 5.283 using the GRG2 and 5.274 using the GA). The Euclidean Distance obtained when undertaking the proposed uncertainty analysis approach, with all of the input parameters being taken into consideration simultaneously, however, implies that the ranking of Alternative 4, which is initially ranked 3rd, is not very robust when compared to the highest ranked alternative (Alternative 1), as it obtained the lowest Euclidean Distance (i.e. 0.104) when utilising the GA. This is a contradictory result to the findings of traditional methods of sensitivity analysis, demonstrating the additional insight that the proposed approach provides the DM.

Another benefit of the proposed uncertainty analysis approach is the capability of ascertaining the most critical input parameters which will cause rank reversal between a pair of alternatives, as discussed in Section 4.3. The critical input parameters are determined by comparing the original input parameter values with the optimised parameter values, and the criteria with the smallest relative changes are deemed to be the most critical. For example, when only the CWs are varied in the uncertainty analysis for Alternative 4 to outrank Alternative 1 using the GA optimisation method, the most critical input parameter is CW 3, as it has a relative change of 3% compared to CW 9, which has a relative change of 32%. When all of the input parameters are included in the uncertainty analysis, it is interesting to note that the smallest relative change required of some of the input parameters reduces to 1% and that a number of the most critical criteria are PVs, demonstrating that PVs may also have an impact on the outcomes of the decision analysis.

The outcomes of the application of the proposed approach to the decision problem originally assessed by Mladineo et al. (1987) not only clearly demonstrate the importance of varying all of the input parameters simultaneously, but that only minor changes in the input parameter values are required for rank equivalence to occur between most pairs of alternatives. Consequently, it is not possible, or appropriate, to say that one alternative is 'better' than the others, for this particular decision problem, with the assumed uncertainty of all of the CWs and PVs. Varying the CWs and PVs simultaneously provides the people involved with the decision analysis a complete understanding of the impact that uncertainty in all of the input parameters has on the ranking of the alternatives. This is an important contribution to the field of MCDA, as it is generally only the variability in the CWs that is considered when a sensitivity analysis is undertaken.

6. Summary and Conclusions

MCDA, and in particular PROMETHEE, is utilised extensively to assess many types of decision analysis problems, however, the uncertainty associated with the input parameter values is rarely, or ambiguously, considered. The proposed distance-based method for uncertainty analysis determines the parameter combinations that are critical in reversing the ranking of two selected alternatives, thereby allowing the DM to test the robustness of the decision outcomes to variations in the input data. The analysis is somewhat analogous to a traditional sensitivity analysis where the behaviour of the ranking of alternatives is explored within the expected range of CWs and PVs. The proposed approach, however, provides the benefits of jointly varying the CWs and PVs to obtain a single measure of robustness. The ability of the method to identify the most critical input parameters also provides the DM with valuable information, which can provide direction for further analysis, or confidence that a large change in the input parameters is required before a reversal in the ranking occurs.

Applying the proposed methodology to the case study undertaken by Mladineo et al. (1987) illustrates that different rankings can be obtained when different generalised criterion functions are used. Further analysis is required, however, to delineate whether it is the method that is the dominant factor in the change in rankings, or the uncertainty in the input data. The results of the proposed uncertainty analysis approach also demonstrate that the complete rankings and the difference between the total flows should not be relied upon when selecting an optimal alternative. It is also evident by applying the proposed approach to the case study that both the CWs and PVs have an impact on the ranking of the alternatives and therefore the uncertainty in all of the input parameter values should be considered in the decision analysis concurrently. Undertaking uncertainty analysis by varying the input parameters simultaneously between their expected ranges is essential to determine how robust the rankings of the alternatives are to the input parameter values. Only varying one input parameter at a time, or one type of input parameter (i.e. CWs), is not adequate to gain a complete understanding of the impact that changes in the input parameter values may have on the ranking of the alternatives.

In this paper, the performance of two types of optimisation algorithms is compared, namely gradient methods and GAs. Gradient methods can become trapped in local optima, which was found to be the case when the GRG2 non-linear optimisation method was applied to the case study, as different Euclidean Distances were obtained for each set of random starting values trialled when all of the input parameters were varied simultaneously. Obtaining different distance values may result in indecisiveness and undermine the actors' confidence in MCDA and the uncertainty analysis approach. By utilising a global optimisation technique, such as GAs, this difficulty can be overcome. The recommended approach is therefore to initially test the solution space by utilising the GRG2 optimisation method and if it is found to be complex, as with the Mladineo et al. (1987) case study, then the GA should be used to undertake the complete uncertainty analysis. The advantage of GAs is that they search from a population of points, investigating several areas of the search space simultaneously, and therefore have a greater chance of finding the global optimum. However, it can often take a long time to determine the most appropriate input parameters for the GA, compared with GRG2, and therefore a tradeoff is required between the amount of time to undertake the analysis and the level of certainty that the minimum distance metric has been obtained.

Only one set of CWs was utilised in the case study undertaken by Mladineo et al. (1987). In the situation where more than one actor is involved in the decision analysis, different rankings of the alternatives may be obtained due to the difference in preference values elicited from each of the actors. Therefore, one of the main benefits of the proposed approach is the ability for each actor (or groups of actors with similar preferences) to determine the impact that changes in the input parameter values will have on the outcome of the decision analysis, in particular in situations where an actor may be uncertain of the weights they have provided. The limitation of the proposed methodology is that not all actors' CWs can be assessed simultaneously, however, other methods are available to perform this form of uncertainty analysis, such as that presented in Hyde et al. (2004).

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References

Al-Kloub, B., Al-Shemmeri, T. and Pearman, A., 1997. The role of weights in multi-criteria decision aid, and the ranking of water projects in Jordan, European Journal of Operational Research 99, 278-288.

Anand Raj, P. A. and Kumar, D. N., 1996. Ranking of river basin alternatives using ELECTRE, Hydrological Sciences 41, 697-713.

Barron, H. and Schmidt, P., 1988. Sensitivity analysis of additive multiattribute value models, Operations Research 36, 122-127.

Brans, J. P., Vincke, P. and Mareschal, B., 1986. How to select and how to rank projects: the PROMETHEE method, European Journal of Operational Research 24, 228-238.

Chan Hilton, A. B. and Culver, T. B., 2000. Constraint handling for genetic algorithms in optimal remediation design, Journal of Water Resources Planning and Management 126, 128-137.

Choi, D. J. and Park, H., 2001. Analysis of water privatization scenarios in Korea with multi-criteria decision making techniques, Journal of Water Supply Research and Technology - AQUA 50, 335-352.

Cieniawski, S. E., Eheart, J. W. and Ranjithan, S., 1995. Using genetic algorithms to solve a multiobjective groundwater monitoring problem, Water Resources Research 31, 399-409.

Critchfield, G. C. and Willard, K. E., 1986. Probabilistic analysis of decision trees using Monte Carlo simulation, Medical decision making 6, 85-92.

Dandy, G. C. and Engelhardt, M., 2001. Optimal Scheduling of Water Pipe Replacement using Genetic Algorithms, Journal of Water Resources Planning and Management 127, 214-223.

David, L. and Duckstein, L., 1976. Multi-criterion ranking of alternative long-range water resources systems, Water Resources Bulletin 12, 731-754.

Duckstein, L., Treichel, W. and Magnouni, S. E., 1994. Ranking of groundwater management alternatives by multicriterion analysis, Journal of Water Resources Planning and Management 120, 546-565.

Evans, J. R., 1984. Sensitivity analysis in decision theory, Decision Sciences 15, 239-247.

Felli, J. C. and Hazen, G. B., 1998. Sensitivity analysis and the expected value of perfect information, Medical decision making 18, 95-109.

Fishburn, P. C., Murphy, A. H. and Isaacs, H. H., 1968. Sensitivity of decisions to probability estimation errors: a reexamination, Operations Research 16, 254-267.

Flug, M., Seitz, H. L. H. and Scott, J. F., 2000. Multicriteria decision analysis applied to Glen Canyon Dam, Journal of Water Resources Planning and Management 126, 270-276.

Gershon, M., Duckstein, L. and McAniff, R., 1982. Multiobjective River Basin Planning with Qualitative Criteria, Water Resources Research 18, 193-202.

Goldberg, D. E., 1989. Genetic algorithms in search, optimization and machine learning, Addison-Wesley, Reading.

Guillen, S. T., Trejos, M. S. and Canales, R., 1998. A robustness index of binary preferences. XIVth International Conference on Multiple Criteria Decision Making, Charlottesville, Virginia.

Hajeeh, M. and Al-Othman, A., 2005. Application of the analytical hierarchy process in the selection of desalination plants, Desalination 174, 97-108.

Hobbs, B. F., 1980. A comparison of weighting methods in power plant siting, Decision Sciences 11, 725-737.

Hobbs, B. F., Chankong, V., Hamadeh, W. and Stakhiv, E. Z., 1992. Does choice of multicriteria method matter? An experiment in water resources planning, Water Resources Research 28, 1767-1779.

Hwang, C. L. and Yoon, K., 1981. Multiple attribute decision making: methods and applications: a state-of-the-art survey, Springer-Verglag,

Hyde, K. M., Maier, H. R. and Colby, C. B., 2004. Reliability-Based Approach to Multicriteria Decision Analysis for Water Resources, Journal of Water Resources Planning and Management 130, 429-438.

Hyde, K. M., Maier, H. R. and Colby, C. B., 2005. A Distance-Based Approach to Undertaking Sensitivity Analysis of Criteria Weights for Water Resources Decision Making, Journal of Environmental Management In Press,

Isaacs, H. H., 1963. Sensitivity of decisions to probability estimation errors, Operations Research 11, 536-552.

Jaber, J. O. and Mohsen, M. S., 2001. Evaluation of non-conventional water resources supply in Jordan, Desalination 136, 83-92.

Janssen, R., 1996. Multiobjective Decision Support for Environmental Management, Kluwer Academic Publishers, Netherlands.

Kheireldin, K. and Fahmy, H., 2001. Multi-criteria approach for evaluating long term water strategies, Water International 26, 527-535.

Larichev, O. I., 1992. Cognitive validity in design of decision-aiding techniques, Journal of Multi-Criteria Decision Analysis 1, 127-138.

Larichev, O. I. and Moshkovich, H. M., 1995. ZAPROS-LM - A method and system for ordering multiattribute alternatives, European Journal of Operational Research 82, 503-521.

Mirrazavi, S. K., Jones, D. F. and Tamiz, M., 2001. A comparison of genetic and conventional methods for the solution of integer goal programmes, European Journal of Operational Research 132, 594-602.

Mladineo, N., Margeta, J., Brans, J. P. and Mareschal, B., 1987. Multicriteria ranking of alternative locations for small scale hydro plants, European Journal of Operational Research 31, 215-222.

Mousseau, V., Figueira, J., Dias, L., Gomes da Silva, C. and Climaco, J., 2003. Resolving inconsistencies among constraints on the parameters of an MCDA model, European Journal of Operational Research 147, 72-93.

Opricovic, S. and Tzeng, G. H., 2004. Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS, European Journal of Operational Research 156, 445-455.

Proll, L. G., Salhi, A. and Insua, D. R., 2001. Improving an optimization-based framework for sensitivity analysis in multi-criteria decision-making, Journal of Multi-Criteria Decision Analysis 10, 1-9.

Raju, K. S., Duckstein, L. and Arondel, C., 2000. Multicriterion analysis for sustainable water resources planning: a case study in Spain, Water Resources Management 14, 435-456.

Raju, K. S. and Kumar, D. N., 2004. Irrigation Planning using Genetic Algorithms, Water Resources Management 18, 163-176.

Ridgley, M. A. and Rijsberman, F. R., 1994. Multicriterion Analysis and the Evaluation of Restoration Policies for a Rhine Estuary, Socio-Economic Planning Sciences 28, 19-32.

Rios Insua, D. and French, S., 1991. A framework for sensitivity analysis in discrete multi-objective decision-making, European Journal of Operational Research 54, 176-190.

Roy, B. and Vincke, P., 1981. Multicriteria analysis: survey and new directions, European Journal of Operational Research 8, 207-218.

Roy, B., 1991. The outranking approach and the foundations of Electre methods, Theory and Decision 31, 49-73.

Saatay, T. L., 1977. A scaling method for priorities in hierarchical structures, Journal of Mathematical Psychology 15, 234-281.

Salminen, P., Hokkanen, J. and Lahdelma, R., 1998. Comparing multicriteria methods in the context of environmental problems, European Journal of Operational Research 104, 485-496.

Savic, D. A. and Walters, G. A., 1997. Genetic Algorithms for Least-Cost Design of Water Distribution Networks, Journal of Water Resources Planning and Management 123, 67-77.

Schneller, G. O. and Sphicas, G., 1985. On sensitivity analysis in decision theory, Decision Sciences 16, 399-409.

Soofi, E. S., 1990. Generalized entropy-based weights for multiattribute value models, Operations Research 38, 362-363.

Stokes, S. and Plummer, J., 2004. Using spreadsheet solvers in sample design, Computational statistics & data analysis 44, 527-546.

Triantaphyllou, E. and Sanchez, A., 1997. A sensitivity analysis approach for some deterministic multi-criteria decision-making methods, Decision Sciences 28, 151-194.

Ulvila, J. W. and Seaver, D. A., 1982. Decision analysis for water resources planning, Omega 10, 185-194.

Vincke, P., 1999. Robust solutions and methods in decision-aid, Journal of Multi-Criteria Decision Analysis 8, 181-187.

Wolters, W. T. M. and Mareschal, B., 1995. Novel types of sensitivity analysis for additive MCDM methods, European Journal of Operational Research 81, 281-290.

Hyde, Kylie M., Maier, Holger R., Colby, Chris B. (2004) Reliability-based approach to multicriteria decision analysis for water resources.
Journal of water resources, planning and management, v. 130 (6) pp. 429-438

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Journal of multi-criteria decision analysis v. 12 (4-5) pp. 245-259

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