

Boundary-layer Flows in non-Newtonian Fluids

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Abstract

We examine the boundary-layer flow of generalised Newtonian fluids. A specific member of this class of non-Newtonian fluids, namely the Ostwald-de Waele or power-law fluid, is studied in some detail. We show, through the numerical solution of the governing equations, that this empirical model of fluids encountered in physical and industrial situations is of limited benefit when considering the boundary-layer flow of such a fluid. We then develop and employ a Carreau viscosity model in the same context and show that the numerical marching scheme has better convergence behaviour than was the case for power-law fluids. We also investigate the boundary-layer flow of a Newtonian fluid over a thin film of non-Newtonian fluid, described by a Carreau fluid model, by focusing specifically on similarity-type solutions.

Signed Declaration

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to Paul Peter Dabrowski and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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List of Publications

Publications in Refereed Journals

1. Denier, J. P. & **Dabrowski, P. P.** 2004 On the boundary-layer equations for power-law fluids. *Proc. R. Soc. Lond. A* **460**, 3143–3158.

Publications in Refereed Conference Proceedings

- **P.P. Dabrowski** and J.P. Denier, *Blowing-induced boundary-layer separation of shear-thinning fluids*
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Abstracts in Conference Proceedings

- **P P. Dabrowski** and J P. Denier, *Separation of non-Newtonian boundary layer flows*
The 37th Applied Mathematics Conference (ANZIAM 2001), February 3–7, 2001, Barossa Valley, South Australia.

