

THE UNIVERSITY OF ADELAIDE

**NUMERICAL INVESTIGATION OF
WIND INPUT AND SPECTRAL DISSIPATION
IN EVOLUTION OF WIND WAVES**

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List of Symbols

Symbol	Quantity	SI Unit
a	coefficient of the inherent breaking term	-
a_{et}, a_{ft}	exponents of time-limited growth curves	-
a_{ex}, a_{fx}	exponents of fetch-limited growth curves	-
a_{exp}	experimental value for the coefficient a	-
a_w	wave amplitude	m
A	directional spectral width of wave spectrum	m
A_{et}, A_{ft}	relational coefficients of time-limited growth curves	-
A_{ex}, A_{fx}	relational coefficients of fetch-limited growth curves	-
b	coefficient of the forced breaking term	-
b_{br}	coefficient reflecting the strength of the wave breaking	-
b_{G05}	coefficient of the wave breaking by Gemmrich (2006)	-
b_{MM02}	coefficient of the wave breaking by Melville and Matusov (2002)	-
B	spectral saturation	-
B_n	spectral saturation normalized directionally	-
B_{nT}	normalized threshold spectral saturation	-
c	wave phase speed	m s^{-1}
c_g	group velocity	m s^{-1}
c_p	phase speed of components at the spectral peak	m s^{-1}
d	water depth	m
C_D	atmospheric drag coefficient	-

Symbol	Quantity	SI Unit
C_V	viscous drag coefficient	-
D	integral of dissipation source function	-
$D(f, \theta)$	directional spreading function	$\text{m}^2 \text{s}$
D_n	normalized directional spreading function	$\text{m}^2 \text{s}$
E	total wave energy	m^2
f	frequency 1 / T	s^{-1}
f_{max}, f_{cut}	maximum (cut) frequency of spectral scale	s^{-1}
f_p	frequency of spectral peak	s^{-1}
f_T	transition frequency	s^{-1}
$F(f)$	one-dimensional wave frequency spectrum	$\text{m}^2 \text{s}$
$F(f, \theta)$	directional frequency spectrum	$\text{m}^2 \text{s}$
F_{max}	maximum of wave spectrum	$\text{m}^2 \text{s}$
F_T	threshold spectrum for the dissipation source function	$\text{m}^2 \text{s}$
ΔF_p	residual between the wave spectrum and the threshold spectrum	$\text{m}^2 \text{s}$
$\Phi(k)$	wavenumber spectrum integrated in directional space	$\text{m}^3 \text{rad}^{-1}$
g	gravitational acceleration	ms^{-2}
G	sheltering coefficient	-
G_q	quadruplet nonlinear interaction coupling coefficient	-
H	significant wave height	m
j_{v2k}	transformation Jacobian	-
k	wave number	rad m^{-1}
k_p	wave number of spectral peak	rad m^{-1}
$K(f, \theta)$	directional spreading function	-

Symbol	Quantity	SI Unit
$L(f)$	correction function for wind input source function	-
$M(f)$	wind momentum input spectrum	-
MSS	mean spectral slope	-
n	exponent of the wave spectrum	-
$N(\mathbf{k})$	action density	$\text{m}^2\text{rad}^{-1}$
p	pressure of the air exerted on the water surface	-
$p(f, U_{10} / c_p)$	trough parameter of bimodal directional function	-
$Q(\omega)$	quadrature wave spectrum	-
$Q(s)$	normalization factor for directional spreading	-
R	dissipation rate	-
s	directional spreading exponent	-
s_p	directional spreading exponent at the peak frequency	-
S_{ds}	dissipation source term	m^2
S_{in}	atmospheric input source term	m^2
S_{nl}	quadruplet nonlinear interaction source term	m^2
S_{tot}	total source term	m^2
S_1, S_2, S_{11}, S_{12}	partial integrals of source function	m^2
t	time	s
T	wave period	s
$T(f)$	saturation transformer	-
T_1	inherent breaking term for dissipation function	m^2
T_2	forced breaking term for dissipation function	m^2

Symbol	Quantity	SI Unit
u^*	friction velocity	ms^{-1}
U_{10}	wind speed measured at a height of 10m	ms^{-1}
U_h	wind speed measured at a height h	ms^{-1}
$U_{L/2}$	wind speed measured at a height of $L/2$	ms^{-1}
U_{10} / c_p	inverse wave age	-
x	horizontal length co-ordinate	m
x	fetch length over which the wind blows	m
X	correction factor for the wind input source function	-
V	bimodal directional spreading function	-
W	integral of wind input source function	-
W_1, W_2	partial integrals of wind input source function	-
α	JONSWAP Phillips parameter	-
α_{BY}	scale parameter of wind direction slice of the wavenumber spectrum from Banner and Young (1994)	-
β	spectrum scale parameter by Donelan et al. (1985)	-
γ	growth rate parameter	-
γ_D	peak enhancement factor by Donelan et al. (1985)	-
γ_J	JONSWAP peak enhancement factor	-
δ	wave steepness	-
$\eta(x,t)$	water surface elevation	m
η	correction rate	-
η_o	lower boundary value of the correction rate	-
λ	wave length	m

Symbol	Quantity	SI Unit
$\lambda(k)$	lobe-ratio	-
$A(v)$	the average length of breaking crests per unit area per unit speed interval	m
μ	slope exponent of wind source function	-
v	non-dimensional peak frequency	-
v_c	the crest propagation speed	m/s
ζ	similarity parameter of the spectral energy level	-
π	pi constant	-
ρ_a	density of air	kg/m ³
ρ_w	density of water	kg/m ³
σ	spectral peak width parameter	-
σ^2	variance or total energy of the wave record	m
ε	non-dimensional energy	-
χ	non-dimensional fetch	-
ϕ	wave phase	rad
ω	angular frequency	s ⁻¹
τ	total wind stress	-
τ_t	turbulent stress	-
τ_w	wave-induced stress	-
τ_v	viscous stress	-
ζ	non-dimensional duration	-
θ	wave direction	rad
θ_m	mean wave direction	rad

Symbol	Quantity	SI Unit
θ_{max}	direction of the spectral peak	rad
θ_w	wind direction	rad
θ_H	directional spectral width by Hwang et al. (2000)	rad
$\bar{\theta}$	mean spectral width	rad
$\Delta\theta_f$	total increment of the peak angle along the frequency scale	rad
$\Delta\theta_{ucp}$	total increment during the wave evolution	rad

List of Equations

$$\eta(\mathbf{k}, \omega, \mathbf{x}, t) = a \cdot \sin(\mathbf{kx} - \omega t) \dots\dots\dots(\text{Eq. 2-1}) \dots\dots\dots 10$$

$$\eta(t) = \sum_{i=1}^N a_i \cdot \sin(\omega_i t + \phi_i) \dots\dots\dots(\text{Eq. 2-2}) \dots\dots\dots 11$$

$$F(k) = \frac{cc}{2\pi\omega} g F(f, \theta) \dots\dots\dots(\text{Eq. 2-3}) \dots\dots\dots 12$$

$$F(f, \theta) = \alpha \frac{g^2}{(2\pi)^4} f^{-5} \exp\left[-\frac{5}{4}\left(\frac{f}{f_p}\right)^{-4}\right] \cdot \gamma_J \exp\left[\frac{-(f-f_p)^2}{2\sigma^2 f_p^2}\right] \dots\dots\dots(\text{Eq. 2-4}) \dots\dots\dots 14$$

$$F(f, \theta) = \beta \frac{g^2}{(2\pi)^4} f_p^{-1} f^{-4} \exp\left[-\left(\frac{f}{f_p}\right)^{-4}\right] \cdot \gamma_D \exp\left[\frac{-(f-f_p)^2}{2\sigma^2 f_p^2}\right] \dots\dots\dots(\text{Eq. 2-5}) \dots\dots\dots 15$$

$$\beta = 0.0165v^{0.55} \dots\dots\dots(\text{Eq. 2-6}) \dots\dots\dots 15$$

$$\gamma_D = \begin{cases} 6.489 + 61 \log v & v \geq 0.159 \\ 1.7 & v < 0.159 \end{cases} \dots\dots\dots(\text{Eq. 2-7}) \dots\dots\dots 15$$

$$\sigma = 0.08 + 1.29 \times 10^{-3} v^{-3} \dots\dots\dots(\text{Eq. 2-8}) \dots\dots\dots 15$$

$$f_T \sim \frac{2.5}{\pi} \frac{g}{U_{10}} \dots\dots\dots(\text{Eq. 2-9}) \dots\dots\dots 16$$

$$F(f, \theta) = F(f) \cdot D(f, \theta) \dots\dots\dots(\text{Eq. 2-10}) \dots\dots\dots 17$$

$$\int D(f, \theta) = 1 \dots\dots\dots(\text{Eq. 2-11}) \dots\dots\dots 17$$

$$D(f, \theta) = Q(s) \cos^{2s} \left\{ \frac{\theta - \theta_m(f)}{2} \right\} \dots\dots\dots(\text{Eq. 2-12}) \dots\dots\dots 17$$

$$s = \begin{cases} s_p \left(\frac{f}{f_p} \right)^5, & f < f_p \\ s_p \left(\frac{f}{f_p} \right)^{-2.5}, & f \geq f_p \end{cases} \dots\dots\dots(\text{Eq. 2-13}) \dots\dots\dots 17$$

$$s_p = 11.5 \left(\frac{U_{10}}{c_p} \right)^{-2.5} \dots\dots\dots(\text{Eq. 2-14}) \dots\dots\dots 17$$

$$A(f)^{-1} = \int_{-\pi}^{\pi} D_n(f, \theta) d\theta \dots\dots\dots(\text{Eq. 2-15}) \dots\dots\dots 18$$

$$D_n(f, \theta_{\max}) = 1 \dots\dots\dots(\text{Eq. 2-16}) \dots\dots\dots 18$$

$$MSS = \iint k^4 \Phi(k, \theta) \frac{1}{k} dk d\theta = \int k^4 \Phi(k) d(\ln k) \dots\dots\dots(\text{Eq. 2-17}) \dots\dots\dots 19$$

$$B = \omega^5 E(\omega) / 2g^2 = (2\pi)^4 f^5 F(f) / 2g^2 \dots\dots\dots(\text{Eq. 2-18}) \dots\dots\dots 19$$

$$B_n(\omega) = \frac{\omega^5 E(\omega)}{2g^2} A(\omega) \dots\dots\dots(\text{Eq. 2-19}) \dots\dots\dots 20$$

$$\frac{\partial F}{\partial t} + c_g \times \nabla F = S_{tot} \dots\dots\dots(\text{Eq. 2-20}) \dots\dots\dots 22$$

$$S_{tot} = S_{in} + S_{ds} + S_{nl} \dots\dots\dots(\text{Eq. 2-21}) \dots\dots\dots 23$$

$$\gamma = \left(\frac{1}{\omega E} \right) \frac{\partial E}{\partial t} \dots\dots\dots(\text{Eq. 2-22}) \dots\dots\dots 23$$

$$\frac{\partial E(\omega)}{\partial t} = S_{in}(\omega) = \frac{1}{\rho_w g} \overline{p(x,t) \frac{\partial \eta(x,t)}{\partial t}} \dots\dots\dots(\text{Eq. 2-23}) \dots\dots\dots 24$$

$$S_{in}(f) = 2\pi \rho_a / \rho_w f \gamma(f) F(f) \dots\dots\dots(\text{Eq. 2-24}) \dots\dots\dots 24$$

$$\gamma = G \sqrt{B_n} \cdot \left[\frac{U_{10}}{c} - 1 \right]^2 \dots\dots\dots(\text{Eq. 2-25}) \dots\dots\dots 26$$

$$G = 2.8 - 1.0 \cdot (1 + \tanh(10 \cdot \sqrt{B_n} \cdot \left[\frac{U_{10}}{c} - 1 \right]^2 - 11)) \dots\dots\dots(\text{Eq. 2-26}) \dots\dots\dots 26$$

$$\tau = \tau_t + \tau_w + \tau_v \dots\dots\dots(\text{Eq. 2-27}) \dots\dots\dots 27$$

$$\tau = \rho_a C_d U_{10}^2 \dots\dots\dots(\text{Eq. 2-28}) \dots\dots\dots 28$$

$$S_{ds}(f) = a \cdot \underbrace{f \cdot A(f) \cdot (F(f) - F_T(f))}_{T_1(f)} + b \underbrace{\int_{f_p}^{f_{cut}} A(f) \cdot (F(f) - F_T(f)) df}_{T_2(f)} \dots\dots\dots(\text{Eq. 2-29}) \dots\dots\dots 31$$

$$\begin{cases} \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 + \mathbf{k}_4 \\ \omega_1 + \omega_2 = \omega_3 + \omega_4 \end{cases} \dots\dots\dots(\text{Eq. 2-30}) \dots\dots\dots 32$$

$$\frac{\partial N_1}{\partial t} = \iiint G_q(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4) \delta(\omega_1 + \omega_2 - \omega_3 - \omega_4) \times [N_1 N_3 (N_4 - N_2) + N_2 N_4 (N_3 - N_1)] d\mathbf{k}_2 d\mathbf{k}_3 d\mathbf{k}_4 \dots\dots\dots(\text{Eq. 2-31}) \dots\dots\dots 33$$

$$\varepsilon = \frac{\sigma^2 g^2}{U_h^4} \dots\dots\dots(\text{Eq. 2-32}) \dots\dots\dots 36$$

$$\mathbf{v} = \frac{f_p U_h}{g} \dots\dots\dots(\text{Eq. 2-33}) \dots\dots\dots 36$$

$$\chi = \frac{gx}{U_h^2} \dots\dots\dots(\text{Eq. 2-34}) \dots\dots\dots 36$$

$$\zeta = \frac{gt}{U_h} \dots\dots\dots(\text{Eq. 2-35}) \dots\dots\dots 36$$

$$\begin{aligned} \varepsilon &= A_{et} \cdot \zeta^{a_{\zeta t}} \\ \mathbf{v} &= A_{fv} \cdot \zeta^{a_{\zeta v}} \end{aligned} \dots\dots\dots(\text{Eq. 2-36}) \dots\dots\dots 37$$

$$\begin{aligned} \varepsilon &= A_{ex} \cdot \chi^{a_{\chi}} \\ \mathbf{v} &= A_{fv} \cdot \chi^{a_{\chi v}} \end{aligned} \dots\dots\dots(\text{Eq. 2-37}) \dots\dots\dots 37$$

$$\begin{aligned} \mathbf{v} &= 2.40 \cdot \chi^{-0.275} \\ \varepsilon &= 8.3 \cdot 10^{-6} \cdot \mathbf{v}^{-3.01} \end{aligned} \dots\dots\dots(\text{Eq. 2-38}) \dots\dots\dots 38$$

$$\alpha = \begin{cases} 8.03 \cdot 10^{-2} \cdot \mathbf{v}^{1.24}, & \mathbf{v} \leq 0.23 \\ 13.2 \cdot 10^{-3}, & \mathbf{v} > 0.23 \end{cases} \dots\dots\dots(\text{Eq. 2-39}) \dots\dots\dots 38$$

$$\gamma_J \approx 7.6 \cdot \mathbf{v} \dots\dots\dots(\text{Eq. 2-40}) \dots\dots\dots 38$$

$$\frac{\partial F}{\partial t} = S_{tot} \dots\dots\dots(\text{Eq. 3-1}) \dots\dots\dots 45$$

$$F_{j+1} = F_j + \Delta F = F_j + S_{tot} \cdot \Delta t \dots\dots\dots(\text{Eq. 3-2}) \dots\dots\dots 45$$

$$A_{ft} = A_{fx} \frac{1}{a_{fx} + 1} \left[\frac{R \cdot (a_{fx} + 1)}{2\pi} \right]^{a_{ft}}, \quad a_{ft} = \frac{a_{fx}}{a_{fx} + 1}$$

$$A_{et} = A_{ex} \left[\frac{R \cdot (a_{fx} + 1)}{2\pi A_{fx}} \right]^{a_{et}}, \quad a_{et} = \frac{a_{ex}}{a_{fx} + 1} \dots\dots\dots(\text{Eq. 3-3}) \dots\dots\dots 48$$

$$v = 10.74 \cdot \zeta^{-0.38}$$

$$\varepsilon = 6.54 \cdot 10^{-9} \cdot \zeta^{1.14} \dots\dots\dots(\text{Eq. 3-4}) \dots\dots\dots 49$$

$$\tau_w = \tau - \tau_v \dots\dots\dots(\text{Eq. 4-1}) \dots\dots\dots 68$$

$$\tau_w' = \int M(f) df \dots\dots\dots(\text{Eq. 4-2}) \dots\dots\dots 68$$

$$M(f) = \rho_w g \frac{S_{in}(f)}{c(f)} \dots\dots\dots(\text{Eq. 4-3}) \dots\dots\dots 69$$

$$\tau_w' = \rho_w g \int \frac{S_{in}(f)}{c(f)} df \dots\dots\dots(\text{Eq. 4-4}) \dots\dots\dots 69$$

$$\tau_w' = \tau_w \dots\dots\dots(\text{Eq. 4-5}) \dots\dots\dots 69$$

$$\tau_w = \rho_a U_{10}^2 (C_D - C_V) \dots\dots\dots(\text{Eq. 4-6}) \dots\dots\dots 70$$

$$C_V = -5 \cdot 10^{-5} U_{10} + 1.1 \cdot 10^{-3} \dots\dots\dots(\text{Eq. 4-7}) \dots\dots\dots 70$$

$$C_D = [0.78 + 0.475 \cdot f(\delta) \cdot U_{10}] \times 10^{-3} \dots\dots\dots(\text{Eq. 4-8}) \dots\dots\dots 70$$

$$f(\delta) = 0.85^B A^{1/2} \delta^{-B} \dots\dots\dots(\text{Eq. 4-9}) \dots\dots\dots 70$$

$$\tau_w' = \int_{f_{min}}^{f_0} M(f) df + \int_{f_0}^{f_{max}} M(f) df \dots\dots\dots(\text{Eq. 4-10}) \dots\dots\dots 73$$

$$S_1 = \int_{f_{min}}^{f_0} M(f) df \quad \text{and} \quad S_2 = \int_{f_0}^{f_{max}} M(f) df \dots\dots\dots(\text{Eq. 4-11}) \dots\dots\dots 73$$

$$\begin{cases} \tau_w = S_1 + X \cdot S_2 \\ \tau'_w = S_1 + S_2 \end{cases} \dots\dots\dots(\text{Eq. 4-12}) \dots\dots\dots 73$$

$$X = 1 + \frac{\tau_w - \tau'_w}{S_2} \dots\dots\dots(\text{Eq. 4-13}) \dots\dots\dots 73$$

$$1 + \frac{\tau_w - \tau'_w}{S_2} > 0 \dots\dots\dots(\text{Eq. 4-14}) \dots\dots\dots 76$$

$$S_2 > \tau'_w - \tau_w \dots\dots\dots(\text{Eq. 4-15}) \dots\dots\dots 76$$

$$S_1 < \tau_w \dots\dots\dots(\text{Eq. 4-16}) \dots\dots\dots 76$$

$$L(f) = \exp\left(\frac{f_0 - f}{f} \cdot \eta\right) \dots\dots\dots(\text{Eq. 4-17}) \dots\dots\dots 79$$

$$\int_{f_0}^{f_{\max}} L(f)M(f)df = X \int_{f_0}^{f_{\max}} M(f)df \dots\dots\dots(\text{Eq. 4-18}) \dots\dots\dots 79$$

$$\int_{f_0}^{f_{\max}} L(f)M(f)df = L(f_i) \int_{f_0}^{f_{\max}} M(f)df \dots\dots\dots(\text{Eq. 4-19}) \dots\dots\dots 81$$

$$\begin{cases} L(f_0) = 1 \\ L(f_i) = X \end{cases} \dots\dots\dots(\text{Eq. 4-20}) \dots\dots\dots 81$$

$$\exp\left(\frac{f_0 - f_i}{f_i} \eta\right) = X \dots\dots\dots(\text{Eq. 4-21}) \dots\dots\dots 81$$

$$\eta = \frac{f_i \ln X}{f_0 - f_i} \dots\dots\dots(\text{Eq. 4-22}) \dots\dots\dots 81$$

$$\begin{cases} F(f, \theta) = \beta \frac{g^2}{(2\pi)^4} f_p^{-1} f^{-4} \exp\left[-\left(\frac{f}{f_p}\right)^{-4}\right] \cdot \gamma_D \exp\left[\frac{-(f-f_p)^2}{2\sigma^2 f_p^2}\right] & f \leq f_T \\ F(f, \theta) = \beta \frac{g^2}{(2\pi)^4} f_T f_p^{-1} f^{-5} \exp\left[-\left(\frac{f}{f_p}\right)^{-4}\right] \cdot \gamma_D \exp\left[\frac{-(f-f_p)^2}{2\sigma^2 f_p^2}\right] & f > f_T \end{cases} \dots\dots\dots(\text{Eq. 4-23}) \dots\dots\dots 89$$

$$T(f) = (2\pi)^4 f_p^{n+5} f^{-n} F(f) / 2g^2 \dots\dots\dots(\text{Eq. 4-24}) \dots\dots\dots 90$$

$$T(f) = \left(\frac{f_p}{f}\right)^{n+5} B(f) = \left(\frac{f_p}{f}\right)^{n+5} A^{-1}(f)B_n(f) \quad \text{.....(Eq. 4-25) 90}$$

$$F_T(f) = 2g^2(2\pi)^{-4} f_p^{-(n+5)} f^n T_T(f) \quad \text{.....(Eq. 4-26) 110}$$

$$F_T(f) = 2g^2(2\pi)^{-4} f^{-5} A^{-1}(f)B_{nT}(f) \quad \text{.....(Eq. 4-27) 110}$$

$$\Delta F_p \leq 0 \quad \text{.....(Eq. 4-28) 111}$$

$$R = \frac{\int S_{ds}(f)df}{\int S_{in}(f)df} \quad \text{.....(Eq. 4-29) 116}$$

$$R_{linear} = \begin{cases} -0.12 U_{10}/c_p + 1.52, & 4.5 < U_{10}/c_p \leq 5.8 \\ 0.0031 U_{10}/c_p + 0.96, & 1.5 < U_{10}/c_p \leq 4.5 \\ -0.052 U_{10}/c_p + 1.043, & 0.83 < U_{10}/c_p \leq 1.5 \\ 1, & U_{10}/c_p = 0.83 \end{cases} \quad \text{.....(Eq. 4-30) 116}$$

$$R_{smooth} = \begin{cases} 0.97 - 0.07 \times (1 + \tanh[3(U_{10}/c_p - 5.2)]), & 2 < U_{10}/c_p \leq 5.8 \\ 0.97 + 0.015 \times (1 - \tanh[5(U_{10}/c_p - 1.1)]), & 0.9 < U_{10}/c_p \leq 2 \\ 1, & 0.83 \leq U_{10}/c_p \leq 0.9 \end{cases} \quad \text{(Eq. 4-31) 118}$$

$$\int S_{ds}(f)df = R \int S_{in}(f)df \quad \text{.....(Eq. 4-32) 118}$$

$$W = \int S_{in}(f)df, \quad W_1 = \int_{f_{min}}^{f_p} S_{in}(f)df, \quad W_2 = \int_{f_p}^{f_{cut}} S_{in}(f)df \quad \text{..... 119}$$

$$D = \int S_{ds}(f)df, \quad S_1 = \int T_1(f)df, \quad S_2 = \int T_2(f)df \quad \text{.....(Eq. 4-33) 119}$$

$$S_{11} = \int_{f_{min}}^{f_p} T_1(f)df, \quad S_{12} = \int_{f_p}^{f_{cut}} T_1(f)df \quad \text{..... 119}$$

$$S_{ds}(f) = a_0 \cdot T_1(f) \quad \text{.....(Eq. 4-34) 119}$$

$$\int_{f_{min}}^{f_p} a_0 \cdot T_1(f)df = R \int_{f_{min}}^{f_p} S_{in}(f)df \quad \text{.....(Eq. 4-35) 121}$$

$$a_0 = \frac{RW_1}{S_{11}} = \frac{R \int_{f_{\min}}^{f_p} S_{in}(f) df}{\int_{f_{\min}}^{f_p} f \cdot A(f) \cdot (F(f) - F_T(f)) df} \dots\dots\dots(\text{Eq. 4-36}) \dots\dots\dots 121$$

$$\begin{cases} D = RW \\ D = a_0 S_1 + b_0 S_2 \end{cases} \dots\dots\dots(\text{Eq. 4-37}) \dots\dots\dots 123$$

$$b_0 = \frac{RW - a_0 S_1}{S_2} \dots\dots\dots(\text{Eq. 4-38}) \dots\dots\dots 123$$

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$$a = \begin{cases} a_0, & f \leq f_p \\ a_0 \cdot Z(f), & f > f_p \end{cases} \dots\dots\dots(\text{Eq. 4-45}) \dots\dots\dots 133$$

$$V(\theta, f, U_{10}/c_p) = \begin{cases} V_1(\theta, f, U_{10}/c_p) = A(f) \cdot \exp(-p(\theta + \theta_p)^2), & \theta < 0 \\ V_2(\theta, f, U_{10}/c_p) = A(f) \cdot \exp(-p(\theta - \theta_p)^2), & \theta \geq 0 \end{cases} \dots\dots\dots(\text{Eq. 4-46}) \dots\dots\dots 140$$

$$\begin{cases} \theta_p(f) = \theta_p(f_p) + \Delta\theta_f \frac{(f - f_s)}{(f_{cut} - f_s)} \\ \theta_p(U_{10}/c_p) = \theta_p(f_p) + \Delta\theta_{ucp} \frac{(5.7 - U_{10}/c_p)}{(5.7 - 0.83)} \end{cases} \dots\dots\dots(\text{Eq. 4-47}) \dots\dots\dots 140$$

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$$[b_{br}] = \frac{\left[\frac{m}{s^2} \right] \left[\frac{kg}{m^3} \frac{m}{s^2} m^2 \frac{m}{s \cdot rad} \right]}{\left[\frac{kg}{m^3} \right] \left[\frac{m}{s} \right]^5 \left[\frac{1}{rad} \right]} = [1] \dots\dots\dots(\text{Eq. 4-52}) \dots\dots\dots 155$$

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Abstract

The present study comprised an intensive investigation of the two newly proposed parameterisation forms for the wind input source term S_{in} (Donelan et al., 2006) and the wave dissipation source term S_{ds} (Young and Babanin, 2006) proposed on the basis of the recent experimental findings at Lake George, New South Wales, Australia in 1997-2000. The main objective of this study was to obtain advanced spectral forms for the wind input source function S_{in} and wave spectral dissipation source function S_{ds} , which satisfy important physical constraints.

A new approach was developed to achieve the objectives of this study, within the strong physical framework. This approach resulted in a new balance scheme between the energy source terms in the wave model, mentioned before as the split balance scheme (Badulin, 2006). The wave-induced stress was defined as the main physical constraint for a new wave model including recently suggested source functions for the wind input and wave dissipation source terms. Within this approach, a new methodology was developed for correction of the wind input source function S_{in} . Another important physical constraint was the consistency between the wave dissipation and the wind energy input to the waves. The new parameter, the dissipation rate, R , was introduced in this study, as the ratio of the wave dissipation energy to the wind input energy. The parameterisation form of the dissipation rate is presented as a function of the inverse wave age U_{10} / c_p . Some aspects of wave spectral modelling regarding the shape of the wave spectrum and spectral saturation were revised.

The two-phase behaviour of the spectral dissipation function was investigated in terms of the functional dependency of the coefficients a for the inherent wave breaking term and b for the forced dissipation term. The present study found that the both coefficients have functional dependence on the inverse wave age U_{10} / c_p and the spectral frequency. Based on the experimental data by Young and Babanin (2006), a new directional spreading function of bimodal shape was developed for the wave dissipation source term.

The performance of the new spectral functions of the wind input $S_{in}(f)$ and the wave dissipation $S_{ds}(f)$ source terms was assessed using a new third-generation two-dimensional

research wave model WAVETIME-1. The model incorporating the corrected source functions was able to reproduce the existing experimental data.

Statement of Originality

I hereby state that this thesis contains no material which has been accepted for the award of any other degree or diploma in any University or other tertiary institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except where stated.

I give consent for this thesis to be available for photocopying and loan purposes after depositing in The University of Adelaide library.

The experiments reported in this work were performed by myself and any assistance received from others is acknowledged. To my knowledge, there are no intellectual property issues or conflicts of interest with other persons or organizations with respect to the data presented in this thesis.

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