
**NON-LINEAR INDIVIDUAL AND INTERACTION
PHENOMENA ASSOCIATED WITH FATIGUE CRACK
GROWTH**

By

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ABSTRACT

The fatigue of materials and structures is a subject that has been under investigation for almost 160 years; yet reliable fatigue life predictions are still more of an empirical art than a science. The traditional safe-life approach to fatigue design is based upon the total time to failure of a virtually defect free component. This approach is heavily reliant on the use of safety factors and empirical equations, and therefore much scatter in the fatigue life predictions is normally observed. Furthermore, the safe-life approach is unsuitable for many important applications such as aircraft, pressure vessels, welded structures, and microelectronic devices. In these applications the existence of initial defects is practically unavoidable and the time of propagation from an initial defect to final failure is comparable with the total life of the component.

In the early 1970's, the aircraft industry pioneered a new approach for the analysis of fatigue crack growth, known as damage tolerant design. This approach utilises fracture mechanics principles to consider the propagation of fatigue cracks from an initial crack length until final fracture, or a critical crack length, is reached. Since the first implementation of damage tolerant design, much research and development has been undertaken. In particular, theoretical and experimental fracture mechanics techniques have been utilised for the investigation of a wide variety of fatigue crack growth phenomena. One such example is the retardation and acceleration in crack growth rate caused by spike overloads or underloads. It is generally accepted, however, that the current level of understanding of fatigue crack growth phenomena and the adequacy of fatigue life prediction techniques are still far from satisfactory.

This thesis theoretically investigates various non-linear individual and interaction phenomena associated with fatigue crack growth. Specifically, the effect of plate thickness on crack growth under constant amplitude loading, crack growth retardation due to an overload cycle, and small crack growth from sharp notches are considered. A new semi-analytical method is developed for the investigations, which utilises the distributed dislocation technique and the well-known concept of plasticity-induced crack closure. The effects of plate thickness are included through the use of first-order plate theory and a

fundamental solution for an edge dislocation in plate of arbitrary thickness. Numerical results are obtained via the application of Gauss-Chebyshev quadrature and an iterative procedure. The developed methods are verified against previously published theoretical and experimental data.

The elastic out-of-plane stress and displacement fields are first investigated using the developed method and are found to be in very good agreement with past experimental results and finite element simulations. Crack tip plasticity is then introduced by way of a strip-yield model. The effects of thickness on the crack tip plasticity zone and plasticity-induced crack closure are studied for both small and large-scale yielding conditions. It is shown that, in general, an increase in plate thickness will lead to a reduction in the extent of the plasticity and associated crack closure, and therefore an increase in the crack growth rates. This observation is in agreement with many findings of past experimental and theoretical studies.

An incremental crack growth scheme is implemented into the developed method to allow for the investigation of variable amplitude loading and small fatigue crack growth. The case of a single tensile overload is first investigated for a range of overload ratios and plate thicknesses. This situation is of practical importance as an overload cycle can significantly increase the service life of a cracked component by temporarily retarding the crack growth. Next to be studied is growth of physically small cracks from sharp notches. Fatigue cracks typically initiate from stress concentrations, such as notches, and can grow at rates higher than as predicted for a long established crack. This can lead to non-conservative estimates for the total fatigue life of a structural component. For both the overload and small crack cases, the present theoretical predictions correlate well with past experimental results for a range of materials. Furthermore, trends observed in the experiments match those of the predictions and can be readily explained through use of crack closure arguments.

This thesis is presented in the form of a collection of published or submitted journal articles that are the result of research by the author. These nine articles have been chosen to best demonstrate the development and application of the new theoretical techniques. Additional background information and an introduction into the chosen field of research are provided in order to establish the context and significance of this work.

DECLARATION

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968. The author acknowledges that copyright of published works contained within this thesis (as listed on the following pages) resides with the copyright holder(s) of those works.

John Codrington

Date

THESIS BY PUBLICATION

This thesis is comprised of a combination of peer-reviewed publications and submitted journal articles in accordance with the ‘Academic Program Rules 2008’ of The University of Adelaide. The international journals that these papers have been published in or submitted to are all closely related to the field of the research of this dissertation.

This thesis is based on the following publications:

1. Codrington, J. and Kotousov, A. (2008) Effect of a variation in material properties on the crack tip opening displacement. *International Journal of Pressure Vessels and Piping*, under review.
2. Codrington, J., Kotousov, A. and Ho, S.Y. (2008) Out-of-plane stress and displacement for through-the-thickness cracks in plates of finite thickness. *Journal of Mechanics of Materials and Structures* **3** (2) 261-270.
3. Codrington, J. and Kotousov, A. (2007) Application of the distributed dislocation technique for calculating cyclic crack tip plasticity effects. *Fatigue & Fracture of Engineering Materials & Structures* **30** (12), 1182-1193.
4. Codrington, J. and Kotousov, A. (2007) The distributed dislocation technique for calculating plasticity-induced crack closure in plates of finite thickness. *International Journal of Fracture* **144** (4), 285-295.
5. Codrington, J. (2008) Approximation of the thickness effect on plasticity-induced fatigue crack closure using first-order plate theory. *Theoretical and Applied Fracture Mechanics*, revised version under review.
6. Codrington, J. and Kotousov, A. (2008) A crack closure model of fatigue crack growth in plates of finite thickness under small-scale yielding conditions. *Mechanics of Materials*, doi: 10.1016/j.mechmat.2008.10.002.
7. Codrington, J. and Kotousov, A. (2008) Crack growth retardation following the application of an overload cycle using a strip-yield model. *Engineering Fracture Mechanics*, revised version under review.

8. Codrington, J. (2008) On the effect of specimen thickness on post-overload fatigue crack growth. *International Journal of Fracture (Letters in Fracture and Micromechanics)*, revised version under review.
9. Codrington, J. and Kotousov, A. (2008) Theoretical bounds for the prediction of small fatigue crack growth emanating from sharp notches. *International Journal of Fatigue (Technical Note)*, submitted.

The following conference papers are of close relevance to the present work and are included in the appendices:

- A. Codrington, J., Kotousov, A. and Blazewicz, A. (2007) A computational technique for calculating plasticity-induced crack closure in plates of finite thickness. In: Oñiate, E., Owen, D.R.J. and Suárez, B. (eds), *IX International Conference on Computational Plasticity – Fundamentals and Applications*, COMPLAS IX, Barcelona, September 5-7, pp. 898-901.
- B. ^{1,2}Codrington, J. and Kotousov, A. (2007) Investigation of plasticity-induced fatigue crack closure. In: Veidt, M., Albermani, F., Daniel, B., Griffiths, J., Hargreaves, D., McAree, R., Meehan, P. and Tan, A. (eds), *5th Australian Congress on Applied Mechanics*. ACAM 2007, Brisbane, December 10-12, pp. 127-132.

1. This paper received a 'Postgraduate travel award' given to the best student papers, sponsored by the National Committee on Applied Mechanics.
2. The paper is also accepted for publication in the Australian Journal of Mechanical Engineering:

Codrington, J. and Kotousov, A. (2008) Investigation of plasticity-induced fatigue crack closure. *Australian Journal of Mechanical Engineering*, accepted.

LIST OF PUBLICATIONS

The following journal and conference publications were completed as part of this research candidature, but are not included as part of the thesis.

Journal publications:

1. Codrington, J., Nguyen, P., Ho, S.Y. and Kotousov, A. (2008) Induction heating apparatus for high temperature testing of thermo-mechanical properties. *Applied Thermal Engineering*, revised version under review.
2. Wildy, S.J., Kotousov, A.G. and Codrington, J.D. (2008) A new passive defect detection technique based on the principle of strain compatibility. *Smart Materials & Structures* **17** (4), 045004 (8pp).
3. Wildy, S. Kotousov, A. and Codrington, J. (2008) New passive defect detection technique. *Australian Journal of Mechanical Engineering*. accepted.

Conference publications:

1. Codrington, J., Nguyen, P., Ho, S.Y., Kotousov, A. and Tsukamoto, H. (2006) Experimental apparatus for high temperature testing of them-mechanical properties. In: Hoffman, M. and Price, J. (eds) *International Conference on Structural Integrity and Failure Proceedings*, SIF 2006, Sydney, September 27-29. pp. 148-153.
2. Tsukamoto, H., Kotousov, A. and Codrington, J. (2006) Transformation toughening in Zirconia-enriched multi-phase composites. In: Hoffman, M. and Price, J. (eds) *International Conference on Structural Integrity and Failure Proceedings*, SIF 2006, Sydney, September 27-29. pp. 427-433.
3. Tsukamoto, H., Kotousov, A., Ho, S.Y. and Codrington, J. (2006) Analysis and design of functionally graded thermal coating. In: Hoffman, M. and Price, J. (eds) *International Conference on Structural Integrity and Failure Proceedings*, SIF 2006, Sydney, September 27-29. pp. 25-32.

4. Kotousov, A. and Codrington, J. (2007) Crack under shear loading in a plate of finite thickness. In: Veidt, M., Albermani, F., Daniel, B., Griffiths, J., Hargreaves, D., McAree, R., Meehan, P. and Tan, A. (eds), *5th Australian Congress on Applied Mechanics*. ACAM 2007, Brisbane, December 10-12. pp. 483-488.
5. Wildy, S. Kotousov, A. and Codrington, J. (2007) New passive defect detection technique. In: Veidt, M., Albermani, F., Daniel, B., Griffiths, J., Hargreaves, D., McAree, R., Meehan, P. and Tan, A. (eds), *5th Australian Congress on Applied Mechanics*. ACAM 2007, Brisbane, December 10-12. pp. 221-226.

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TABLE OF CONTENTS

Abstract	i
Declaration	iii
Thesis by Publication	iv
List of Publications	vi
Acknowledgements	viii
Table of Contents	ix
1 Introduction	1
1.1 Overview	3
1.2 Global Objectives	5
1.3 Details of the Publications	6
1.3.1 Preliminary Investigation (Ch. 3)	7
1.3.2 Development of the Distributed Dislocation Technique for Investigating Cracks in Plates of Finite Thickness (Chs. 4-7)	7
1.3.3 Investigation of Various Fatigue Crack Growth Phenomena (Chs. 8-11) ..	9
1.4 References	12
2 Background and Literature Review	15
2.1 Introduction	17
2.2 Fatigue Crack Propagation	19
2.3 Elastic-Plastic Fracture Mechanics	21
2.4 Factors Influencing Fatigue Crack Growth	22
2.4.1 Material Effects	23
2.4.2 Structural Geometry Effects	24
2.4.3 Applied Loading	25
2.4.4 Environmental Conditions	25
2.4.5 Residual Stresses	26
2.5 Theoretical Methods for the Thickness Effect on Crack Closure	26
2.6 The Distributed Dislocation Technique	28
2.7 Summary of Gaps	30
2.8 References	32

3	Effect of a Variation in Material Properties on the Crack Tip Opening	
	Displacement	37
	Statement of Authorship	39
	Abstract	41
	Nomenclature	42
	1 Introduction	43
	2 Characterisation of the Associated Crack Tip Plasticity Effects	44
	3 Direct Plasticity Region, Case (a)	46
	4 Reverse and Direct Plasticity Region, Case (b)	48
	5 Cases (c) and (d)	51
	6 Conclusion	54
	References	55
4	Out-of-Plane Stress and Displacement for Through-the-Thickness Cracks in	
	Plates of Finite Thickness	57
	Statement of Authorship	59
	Abstract	61
	1 Introduction	61
	2 Semiinfinite Crack in a Finite Thickness Plate	62
	3 Results for the Out-of-Plane Stress	65
	4 Results for the Out-of-Plane Displacement	67
	5 Conclusion	69
	References	69
5	Application of the Distributed Dislocation Technique for Calculating Cyclic	
	Crack Tip Plasticity Effects	71
	Statement of Authorship	73
	Abstract	75
	Nomenclature	75
	Introduction	76
	Distributed Dislocation Technique for Semi-Infinite Crack Geometries	77
	Budiansky and Hutchinson Semi-Infinite Crack Model	78
	Finite Crack Results	80
	Results for Semi-Infinite Crack in Finite Thickness Plate	82

Conclusion	84
References	85
6 The Distributed Dislocation Technique for Calculating Plasticity-Induced Crack Closure in Plates of Finite Thickness	87
Statement of Authorship	89
Erratum	90
Abstract	91
1 Introduction	91
2 The Distributed Dislocation Technique for Finite Thickness plates	93
3 Results for Growing Crack with Uniform Thickness Wake	95
4 Results for Growing Crack with Linearly Increasing Wake	96
5 Conclusion	100
References	100
7 Approximation of the Thickness Effect on Plasticity-Induced Fatigue Crack Closure using First-Order Plate Theory	103
Statement of Authorship	105
Abstract	107
1 Introduction	108
2 Through-the-Thickness Cracks in Plates of Finite Thickness	110
3 Formulation of the Distributed Dislocation Technique	112
3.1 Maximum Applied Load	112
3.2 Minimum Applied Load	121
4 Crack Opening Stress	127
5 Conclusions	131
References	133
8 A Crack Closure Model of Fatigue Crack Growth in Plates of Finite Thickness under Small-Scale Yielding Conditions	137
Statement of Authorship	139
Abstract	141
1 Introduction	142
2 Thickness Effect on Crack Closure and Fatigue Crack Growth	144
3 Outline of the Theoretical Approach	146

3.1 Governing Parameters	146
3.2 Description of Model	148
3.3 Mathematical Treatment	149
4 Results for a Fatigue Crack under Small-Scale Yielding Conditions	154
5 Conclusion	160
References	162
9 Crack Growth Retardation Following the Application of an Overload Cycle	
using a Strip-Yield Model	165
Statement of Authorship	167
Abstract	169
Nomenclature	170
1 Introduction	173
2 Steady State Fatigue Cracks	176
3 Fatigue Cracks Growing under Variable Loading	185
4 Results and Discussion	190
5 Comparison with Experimental Data	197
6 Conclusions	201
References	203
10 On the Effect of Specimen Thickness on Post-overload Fatigue Crack Growth ..	207
Statement of Authorship	209
Abstract	211
1 Introduction	212
2 Theoretical Approach	213
3 Crack Opening Load	217
4 Fatigue Crack Growth	218
5 Concluding Remarks	220
References	221
11 Theoretical Bounds for the Prediction of Small Fatigue Crack Growth	
Emanating from Sharp Notches	223
Statement of Authorship	225
Abstract	227
1 Introduction	228

2 Strip-Yield Model for the Crack Opening Stress	231
3 Fatigue Crack Growth from a Sharp Notch	235
4 Summary	237
References	239
12 Summary and Conclusions	241
12.1 Introduction	243
12.2 Preliminary Investigation (Ch. 3)	244
12.3 Development of the Distributed Dislocation Technique for Investigating Cracks in Plates of Finite Thickness (Chs. 4-7)	245
12.4 Investigation of Various Fatigue Crack Growth Phenomena (Chs. 8-11)	246
12.5 Future Work	248
12.6 Conclusion	249
12.7 References	252
 Appendix A. A Computational Technique for Calculating Plasticity-Induced Crack Closure in Plates of Finite Thickness.	 253
Appendix B. Investigation of Plasticity-Induced Fatigue Crack Closure.	259

