NON-LINEAR INDIVIDUAL AND INTERACTION PHENOMENA ASSOCIATED WITH FATIGUE CRACK GROWTH

By

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ABSTRACT

The fatigue of materials and structures is a subject that has been under investigation for almost 160 years; yet reliable fatigue life predictions are still more of an empirical art than a science. The traditional safe-life approach to fatigue design is based upon the total time to failure of a virtually defect free component. This approach is heavily reliant on the use of safety factors and empirical equations, and therefore much scatter in the fatigue life predictions is normally observed. Furthermore, the safe-life approach is unsuitable for many important applications such as aircraft, pressure vessels, welded structures, and microelectronic devices. In these applications the existence of initial defects is practically unavoidable and the time of propagation from an initial defect to final failure is comparable with the total life of the component.

In the early 1970’s, the aircraft industry pioneered a new approach for the analysis of fatigue crack growth, known as damage tolerant design. This approach utilises fracture mechanics principles to consider the propagation of fatigue cracks from an initial crack length until final fracture, or a critical crack length, is reached. Since the first implementation of damage tolerant design, much research and development has been undertaken. In particular, theoretical and experimental fracture mechanics techniques have been utilised for the investigation of a wide variety of fatigue crack growth phenomena. One such example is the retardation and acceleration in crack growth rate caused by spike overloads or underloads. It is generally accepted, however, that the current level of understanding of fatigue crack growth phenomena and the adequacy of fatigue life prediction techniques are still far from satisfactory.

This thesis theoretically investigates various non-linear individual and interaction phenomena associated with fatigue crack growth. Specifically, the effect of plate thickness on crack growth under constant amplitude loading, crack growth retardation due to an overload cycle, and small crack growth from sharp notches are considered. A new semi-analytical method is developed for the investigations, which utilises the distributed dislocation technique and the well-known concept of plasticity-induced crack closure. The effects of plate thickness are included through the use of first-order plate theory and a

fundamental solution for an edge dislocation in plate of arbitrary thickness. Numerical results are obtained via the application of Gauss-Chebyshev quadrature and an iterative procedure. The developed methods are verified against previously published theoretical and experimental data.

The elastic out-of-plane stress and displacement fields are first investigated using the developed method and are found to be in very good agreement with past experimental results and finite element simulations. Crack tip plasticity is then introduced by way of a strip-yield model. The effects of thickness on the crack tip plasticity zone and plasticity-induced crack closure are studied for both small and large-scale yielding conditions. It is shown that, in general, an increase in plate thickness will lead to a reduction in the extent of the plasticity and associated crack closure, and therefore an increase in the crack growth rates. This observation is in agreement with many findings of past experimental and theoretical studies.

An incremental crack growth scheme is implemented into the developed method to allow for the investigation of variable amplitude loading and small fatigue crack growth. The case of a single tensile overload is first investigated for a range of overload ratios and plate thicknesses. This situation is of practical importance as an overload cycle can significantly increase the service life of a cracked component by temporarily retarding the crack growth. Next to be studied is growth of physically small cracks from sharp notches. Fatigue cracks typically initiate from stress concentrations, such as notches, and can grow at rates higher than as predicted for a long established crack. This can lead to non-conservative estimates for the total fatigue life of a structural component. For both the overload and small crack cases, the present theoretical predictions correlate well with past experimental results for a range of materials. Furthermore, trends observed in the experiments match those of the predictions and can be readily explained through use of crack closure arguments.

This thesis is presented in the form of a collection of published or submitted journal articles that are the result of research by the author. These nine articles have been chosen to best demonstrate the development and application of the new theoretical techniques. Additional background information and an introduction into the chosen field of research are provided in order to establish the context and significance of this work.

DECLARATION

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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THESIS BY PUBLICATION

This thesis is comprised of a combination of peer-reviewed publications and submitted journal articles in accordance with the 'Academic Program Rules 2008' of The University of Adelaide. The international journals that these papers have been published in or submitted to are all closely related to the field of the research of this dissertation.

This thesis is based on the following publications:

1. Codrington, J. and Kotousov, A. (2008) Effect of a variation in material properties on the crack tip opening displacement. *International Journal of Pressure Vessels and Piping*, under review.
2. Codrington, J., Kotousov, A. and Ho, S.Y. (2008) Out-of-plane stress and displacement for through-the-thickness cracks in plates of finite thickness. *Journal of Mechanics of Materials and Structures* **3** (2) 261-270.
3. Codrington, J. and Kotousov, A. (2007) Application of the distributed dislocation technique for calculating cyclic crack tip plasticity effects. *Fatigue & Fracture of Engineering Materials & Structures* **30** (12), 1182-1193.
4. Codrington, J. and Kotousov, A. (2007) The distributed dislocation technique for calculating plasticity-induced crack closure in plates of finite thickness. *International Journal of Fracture* **144** (4), 285-295.
5. Codrington, J. (2008) Approximation of the thickness effect on plasticity-induced fatigue crack closure using first-order plate theory. *Theoretical and Applied Fracture Mechanics*, revised version under review.
6. Codrington, J. and Kotousov, A. (2008) A crack closure model of fatigue crack growth in plates of finite thickness under small-scale yielding conditions. *Mechanics of Materials*, doi: 10.1016/j.mechmat.2008.10.002.
7. Codrington, J. and Kotousov, A. (2008) Crack growth retardation following the application of an overload cycle using a strip-yield model. *Engineering Fracture Mechanics*, revised version under review.

8. Codrington, J. (2008) On the effect of specimen thickness on post-overload fatigue crack growth. *International Journal of Fracture (Letters in Fracture and Micromechanics)*, revised version under review.
9. Codrington, J. and Kotousov, A. (2008) Theoretical bounds for the prediction of small fatigue crack growth emanating from sharp notches. *International Journal of Fatigue (Technical Note)*, submitted.

The following conference papers are of close relevance to the present work and are included in the appendices:

A. Codrington, J., Kotousov, A. and Blazewicz, A. (2007) A computational technique for calculating plasticity-induced crack closure in plates of finite thickness. In: Oñiate, E., Owen, D.R.J. and Suárez, B. (eds), *IX International Conference on Computational Plasticity – Fundamentals and Applications*, COMPLAS IX, Barcelona, September 5-7, pp. 898-901.

B. [1,2]Codrington, J. and Kotousov, A. (2007) Investigation of plasticity-induced fatigue crack closure. In: Veidt, M., Albermani, F., Daniel, B., Griffiths, J., Hargreaves, D., McAree, R., Meehan, P. and Tan, A. (eds), *5th Australian Congress on Applied Mechanics.* ACAM 2007, Brisbane, December 10-12, pp. 127-132.

1. This paper received a 'Postgraduate travel award' given to the best student papers, sponsored by the National Committee on Applied Mechanics.
2. The paper is also accepted for publication in the Australian Journal of Mechanical Engineering:

   Codrington, J. and Kotousov, A. (2008) Investigation of plasticity-induced fatigue crack closure. *Australian Journal of Mechanical Engineering*, accepted.

LIST OF PUBLICATIONS

The following journal and conference publications were completed as part of this research candidature, but are not included as part of the thesis.

Journal publications:

1. Codrington, J., Nguyen, P., Ho, S.Y. and Kotousov, A. (2008) Induction heating apparatus for high temperature testing of thermo-mechanical properties. *Applied Thermal Engineering*, revised version under review.
2. Wildy, S.J., Kotousov, A.G. and Codrington, J.D. (2008) A new passive defect detection technique based on the principle of strain compatibility. *Smart Materials & Structures* **17** (4), 045004 (8pp).
3. Wildy, S. Kotousov, A. and Codrington, J. (2008) New passive defect detection technique. *Australian Journal of Mechanical Engineering.* accepted.

Conference publications:

1. Codrington, J., Nguyen, P., Ho, S.Y., Kotousov, A. and Tsukamoto, H. (2006) Experimental apparatus for high temperature testing of them-mechanical properties. In: Hoffman, M. and Price, J. (eds) *International Conference on Structural Integrity and Failure Proceedings*, SIF 2006, Sydney, September 27-29. pp. 148-153.
2. Tsukamoto, H., Kotousov, A. and Codrington, J. (2006) Transformation toughening in Zirconia-enriched multi-phase composites. In: Hoffman, M. and Price, J. (eds) *International Conference on Structural Integrity and Failure Proceedings*, SIF 2006, Sydney, September 27-29. pp. 427-433.
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4. Kotousov, A. and Codrington, J. (2007) Crack under shear loading in a plate of finite thickness. In: Veidt, M., Albermani, F., Daniel, B., Griffiths, J., Hargreaves, D., McAree, R., Meehan, P. and Tan, A. (eds), *5th Australian Congress on Applied Mechanics.* ACAM 2007, Brisbane, December 10-12. pp. 483-488.
5. Wildy, S. Kotousov, A. and Codrington, J. (2007) New passive defect detection technique. In: Veidt, M., Albermani, F., Daniel, B., Griffiths, J., Hargreaves, D., McAree, R., Meehan, P. and Tan, A. (eds), *5th Australian Congress on Applied Mechanics.* ACAM 2007, Brisbane, December 10-12. pp. 221-226.

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INTRODUCTION

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INTRODUCTION

1.1 Overview

Innovation and commercial competitiveness sees the need for materials and structures that are lightweight, low cost, can withstand extreme environmental conditions and mechanical loads, and have long service lifetimes. The design of such components requires a detailed understanding of the failure mechanisms involved. Mechanical fatigue failure, in particular, has been found to be one of the most common failure mechanisms of engineering structures (Stephens et al. 2001; Cui 2002; Bhaumik et al. 2008). Between 50 and 90 % of all mechanical failures are reportedly due to fatigue (Stephens et al. 2001). Many examples of the severe consequences of fatigue failure can be seen throughout the literature including ships (Smith 1998), aircraft (Campbell and Lahey 1984; Whitey 1997), railway (Smith and Hillmansen 2004) and bridges (Fisher 1984). Despite considerable advancements being made into the understanding of fatigue over the past decades, several key areas still require further study.

A large number of factors influence the fatigue behaviour of a structural member, including the material properties, component geometry, applied loading, environmental conditions, and residual stresses. The accurate measurement of the influence of these factors in real structures is very difficult, time consuming and expensive. This has lead to the use of laboratory experiments in conjunction with theoretical models to determine crack growth rates, failure criteria and to predict the service life of components. One such approach is damage tolerant design (DTD), which was pioneered by the aircraft industry (Toor 1973; MIL-A-83444 1974; Grandt 2003) and is now widely used in the design of pipelines, pressure vessels and welded structures (BS 7910 2005) such as in the nuclear, civil, and oil and gas industries. In damage tolerant design it is assumed that flaws and defects will always be present and therefore the structure should be able to ‘survive’ a period of time as a fatigue crack grows to a critical length at which failure will occur.

Fatigue life predictions under the damage tolerant design approach are made based on the load history and resulting crack growth from an initial flaw or defect. The flaws are assumed to be located at critically stressed points within the structure and their size estimated from typical manufacturing defects. A fundamental feature of DTD is the continued monitoring and regular inspection of a structure or component using non-destructive techniques, such as fibre optics, vibration based or strain measurement methods (Chang and Liu 2003; Grandt 2003; Wildy et al. 2008). The minimum detectable crack length of the chosen technique may therefore be used as alternative criterion for the initial flaw size. If a crack is detected in the structure, the remaining fatigue life can be evaluated based on predictions of the crack growth. Knowledge of the crack growth obtained from these predictions can also be used to decide the best course of action for the cracked part; for example removal from service, replacement of the part, crack repair (patching, welding, etc) or growth retardation (infiltration, overloading, stop drilling, etc) (Domazet 1996; Shin et al. 1996).

Successful application of damage tolerant design relies on the accurate prediction of fatigue crack growth taking into account such parameters as the load history, specimen geometry, and material properties. The interaction between these parameters must also be well understood. Predictions for the crack growth, and thus fatigue life, require the use of experimental growth rate curves. However, under the classic linear elastic fracture mechanics approach each set of test parameters, for example the applied loading or specimen thickness, will result in a separate growth rate curve. A large number of fatigue tests are therefore required to be undertaken in order to obtain a comprehensive set of fatigue data for a given material. This can be highly expensive and time consuming, and may also be impractical due to limitations on the amount of material available for testing.

The discovery of fatigue crack closure by Elber (1968, 1970, 1971) provided a new insight into fatigue analyses. Elber (1968) attributed the closure to residual plastic deformation remaining on the crack faces as the crack propagates. The crack closure concept, in particular plasticity-induced crack closure, has been used to explain a number of fatigue phenomena, including the effects of load ratio, variable amplitude loading, and specimen thickness (Newman 1998; Skorupa 1999). Furthermore, through the use of plasticity-induced crack closure it is possible to collapse the crack growth rate curves obtained for different applied loadings or plate thicknesses, for a given material, onto a single unique baseline curve (e.g. Elber 1971).

A popular method of accounting for specimen thickness (i.e. stress state) in many simple theoretical models (e.g. Newman 1981) is to use an empirical fitting parameter. However, there are numerous uncertainties with these methods as the choice of the parameter is not always clear and usually requires trial-and-error. The value, or sometimes a simple function, for the fitting parameter is often chosen based on the correlation of constant amplitude fatigue test data (Newman et al. 1999). This greatly limits the ability of the models to accurately predict the interaction between the involved variables, for example the thickness and load history. An alternative means for investigating the thickness effect on crack closure and fatigue crack growth is through the use of finite element methods (Chermahini et al. 1989; Alizadeh et al. 2007). However, finite element methods are limited in application due to their computational requirements, as well as the various numerical issues involved, such as mesh refinement, crack face contact, and the node release scheme. The inclusion of thickness effects into crack closure and fatigue crack growth models is therefore an area that needs further investigation. In particular, there is a clear need for a new theoretical approach which directly takes into account plate thickness, thus allowing the interaction between the involved parameters, such as the material properties and applied load, to be considered.

1.2 Global Objectives

The objective of the current research is to develop new effective methods for investigating non-linear individual and interaction phenomena associated with fatigue crack growth. These methods will include such parameters as the plate thickness, mean stress, load history, and material properties. The specific phenomena to be considered are the effect of plate thickness on crack tip plasticity and plasticity-induced fatigue crack closure, the crack growth retardation following an overload cycle, and small crack growth from sharp notches.

Furthermore, the focus of this study is on through-the-thickness cracks in flat plates with the fatigue crack orientated perpendicular to the maximum in-plane tensile stresses (i.e. mode I). Fatigue cracks typically grow in mode I and therefore this assumption can be applied to many practical situations, including in aircraft, naval, and civil structures.

The developed methods are based on the concept of plasticity-induced crack closure, and utilise the distributed dislocation technique (Hills et al. 1996). The distributed dislocation technique has been chosen as in and out-of-plane geometry can be directly incorporated through the dislocation influence functions. This includes in-plane features like holes and plate width (Hills et al. 1996), as well as the plate thickness (Kotousov and Wang, 2002). In the current study, the thickness effect will be included via first-order plate theory and the solution for an edge dislocation in a finite thickness plate (Kotousov and Wang, 2002). Use of this approach allows for the effects of plate thickness to be taken into account in the analyses, whilst still having an essentially two-dimensional solution procedure. The theoretical techniques developed in this research therefore aim to eliminate the need for empirical correction factors as are often used to account for specimen thickness and the stress state. At the same time, the techniques aim to be simple to use, computationally efficient, and produce repeatable and reliable results, such that they can be readily applied to many practical situations.

1.3 Details of the Publications

This section describes the aims and objectives of each individual publication/submitted article and how they link together with the main objectives of this research (see previous section). The articles included in this thesis have been chosen to best show the progression of the research, which includes the development, validation and application of the theoretical techniques. Brief summaries of the aims and approach for each article are provided below. Here the research is divided into three main stages: *1. Preliminary Investigation* into the effect of a variation in material properties on the crack tip plasticity and crack opening displacement, *2. Development of the distributed dislocation technique for investigating cracks in plates of finite thickness* using both linear elastic and elastic-plastic fracture mechanics approaches, and *3. Investigation of various fatigue crack growth phenomena*.

1.3.1 Preliminary Investigation (Ch. 3)

Ch.3 Effect of a variation in material properties on the crack tip opening displacement.

Elastic-plastic fracture mechanics allows for the investigation of a wide range of non-linear phenomena, which would not be possible to consider under the classic linear elastic approach. One such phenomenon is the effect of a variation in material properties on the crack tip plasticity and crack tip opening displacement. This will in turn have implications on fatigue crack growth due to cyclic loading, and may itself also be a source of crack growth. In this study, analytical equations are developed based on the strip-yield hypothesis (Dugdale 1960) and the complex potential method of Budiansky and Hutchinson (1978).

The specific aim of this investigation is to first investigate the effect that a variation in material properties has on the crack tip plasticity zone, using the complex potential method. This situation may occur due to a change in the operating temperature, phase transformations, or a deterioration of the material properties over time.

1.3.2 Development of the Distributed Dislocation Technique for Investigating Cracks in Plates of Finite Thickness (Chs. 4-7)

The following articles are concerned with the development of a new approach to directly account for plate thickness effects in fracture and fatigue analysis. The theoretical methods are based on the distributed dislocation technique and the solution for an edge dislocation in a plate of finite thickness (Kotousov and Wang 2002). Several practically important generalisations of the crack geometry are considered: namely the embedded crack, or finite length crack, and the semi-infinite crack. Numerical results are obtained through the use of Gauss-Chebyshev quadrature and an iterative procedure. The case of linear elastic material properties is first studied, and the results obtained for the stress and displacement fields are validated against finite element and experimental data. Next, the elastic-plastic case is investigated by utilising a modified strip-yield model. Both stationary cracks, or cracks that have had limited growth, and fatigue cracks growing under constant amplitude loading are considered. Small and large-scale yielding conditions are also investigated. Results are again compared with previous finite element values and found to be in good agreement.

The aims of each article in chapters 4-7 are detailed below.

Ch. 4 Out-of-plane stress and displacement for through-the-thickness cracks in plates of finite thickness.

Aims:

- to develop a new method of directly accounting for the effects of plate thickness based on the distributed dislocation technique,
- to study the resultant linear elastic stress and strain fields through the out-of plane constraint factor and the out-of-plane displacement, and
- to compare the obtained results with previous numerical and experimental data and thus validate the developed procedures.

Ch. 5 Application of the distributed dislocation technique for calculating cyclic crack tip plasticity effects.

Aims:

- to develop a new method for investigating cyclic crack tip plasticity effects using the distributed dislocation technique,
- to validate the developed method against a previous analytical model by Budiansky & Hutchinson (1978), and
- to study the effects of plate thickness on the crack tip plasticity zone of a fatigue crack under constant amplitude loading and small-scale yielding conditions.

Ch. 6 The distributed dislocation technique for calculating plasticity-induced crack closure in plates of finite thickness.

Aims:

- to extend the developed methods to consider large-scale yielding and plasticity-induced crack closure in plates of finite thickness,
- to investigate the effects of plate thickness on the crack opening displacement and plastic stretch for several wake distributions (parallel and linear), and
- to compare the obtained results for the crack opening stress (linear wake case) for both plane stress conditions and finite thickness plate.

*Ch. 7 Approximation of the thickness effect on plasticity-induced fatigue crack closure using first-order plate theory***.**

Aims:

- to investigate the crack tip opening displacement and crack opening stress (linear wake distribution) as a function of the governing non-dimensional parameters,
- to further refine the already developed methods, and, in particular, to remove the error associated with placing zone edges at integration points,
- to compare the obtained results for the crack opening stress with finite element results as a function thickness, and
- to use the theoretical results to correlate experimental fatigue crack growth data for plates of several thicknesses.

1.3.3 Investigation of Various Fatigue Crack Growth Phenomena (Chs. 8-11)

The next stage in the research is the application of the developed methods to investigate various non-linear fatigue crack growth phenomena. First, a unified model is presented for determining the crack opening load and thus the normalised load ratio parameter under small-scale yielding conditions. The normalised load ratio parameter is frequently utilised in the correlation of fatigue crack growth data and can minimise the scatter obtained for specimens of different thicknesses.

An incremental crack growth scheme is then introduced into the theoretical procedure to allow for the investigation of variable amplitude loading. In particular, the crack growth retardation due to an overload cycle is studied for plane stress conditions and for the case of a finite thickness plate. This situation is of considerable importance in many engineering applications where variable or random cyclic loading takes place. In addition, overload cycles can be used as a means of temporarily slowing crack propagation and thus extending the service life of a cracked component.

Lastly, the developed methods are utilised to examine the behaviour of small fatigue cracks emanating from sharp notches. It is well known that small fatigue cracks grow at much faster rates than long established cracks (Newman 1998). Fatigue life predictions made based on traditional long crack data will therefore overestimate the total life of a component or structure. The small crack phenomenon has been attributed to the plasticity-induced crack closure mechanism and the developed methods will therefore be used to investigate this.

The aims of the journal articles in chapters 8-11 will now be outlined.

Ch. 8 A crack closure model of fatigue crack growth in plates of finite thickness under small-scale yielding conditions.

Aims:

- to present a unified model for determining the normalised load ratio parameter as a function of the applied load, plate thickness and material properties,
- to make the model applicable to a wide range of crack and plate geometries by using the small-scale plasticity generalisation, and
- to use the obtained results to reduce the scatter usually found in fatigue crack growth data for specimens of different thickness.

Ch. 9 Crack growth retardation following the application of an overload cycle using a strip-yield model.

Aims:

- to further extend the theoretical methods developed in the previous chapters to include an incremental crack growth scheme such that variable loading can be considered,
- to investigate the crack growth retardation following a tensile overload cycle under plane stress conditions,
- to use the results for the stress and displacement to describe the various stages of post-overload crack growth in relation to plasticity-induced crack closure, and
- to predict the fatigue crack growth for several materials and loading conditions and to compare the predictions with previous experimental results.

Ch. 10 On the effect of specimen thickness on post-overload fatigue crack growth.

Aims:

- to investigate the effect of specimen thickness on crack growth retardation following an overload cycle as a function of overload ratio, material properties and applied loading.
- to compare theoretical predictions for the fatigue crack growth with past experimental data for specimens of varying thickness and loading conditions.

Ch. 11 Theoretical bounds for the prediction of small fatigue crack growth emanating from sharp notches.

Aims:

- to extend the developed crack closure models of the previous chapters for the investigation of small fatigue crack growth emanating from sharp notches, and
- to compare predictions made using the theory with past experimental results.

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CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

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BACKGROUND AND LITERATURE REVIEW

2.1 Introduction

The purpose of this chapter is to provide a general overview into issues relating to non-linear fracture mechanics and fatigue crack growth. More detailed examinations of specific areas of research are contained in the introduction sections of each of the journal publications and submitted articles that make up the subsequent chapters.

Mechanical fatigue is defined as a process of the cycle-by-cycle accumulation of damage in a component due to fluctuating stresses and strains (Almar-Naess 1985). In addition to mechanical fatigue, other forms of fatigue include thermomechanical, corrosion, rolling contact and fretting fatigue (Suresh, 1991). The focus of the current research is on mechanical fatigue and from hereafter is referred to simply as fatigue. A major feature of fatigue is that damage can occur at loads much less than those required for immediate failure. Fatigue damage may be in the form of diffuse damage, such as distributions of micro-cracks, as well as in the form of a dominant fatigue crack. The latter form of damage is the focus of this work. The total fatigue lifetime can be divided into five main stages (Cui 2002): (1) crack nucleation, (2) microstructurally small crack growth (crack lengths in the order of 0.1 μm to 10 μm), (3) physically small crack growth (10 μm to around 1 mm), (4) long crack growth (greater than 1 mm) and (5) final fracture of the component once the remaining uncracked section is unable to support the applied load. The crack lengths indicated in the parenthesis are of course only typical values and will vary for different materials and situations. Further, depending on whether or not the fatigue crack is growing from an initial flaw, and the flaw size, the early stages may not be present.

There are two main approaches to investigating and predicting fatigue, these are: cumulative damage theories and crack propagation theories. Cumulative damage theories involve characterising the total fatigue life to failure based on the cyclic stress range or the cyclic strain range (plastic or total). The presence of any initial flaws or defects is ignored and thus the predicted fatigue life includes the number of cycles required to initiate a crack as well as the various growth stages. Not surprisingly, these methods are usually referred to as a total-life or safe-life design approach. A range of techniques have been developed over the years to incorporate the effects of mean stress, variable loading, stress concentrators, and environmental conditions (see for example Fatemi and Yang 1998). However, large scatter is often observed with the predicted lifetimes ranging between 0.02 to 50 times the actual lifetimes (Berkovits et al. 1998). This variation can be primarily attributed to the inconsistent nature of crack initiation, which is highly dependent on the material microstructure and manufacturing procedures employed. Life predictions made using cumulative damage theories can therefore only be used with limited confidence.

The second approach to fatigue analysis is based on the use of fracture mechanics principles to study the crack propagation from an initial flaw or defect. These fatigue crack propagation methods form the basis of damage tolerant design. Empirical crack growth rate laws are derived from experimental observations and are used in conjunction with theoretical fracture and fatigue models. These models are then used to predict the growth of a crack taking into account such factors as: component geometry, including thickness, crack shape, and stress concentrators; load history, which includes the mean stress, and variable or spectrum loading; and also environmental conditions, such as tests in air or seawater.

Cumulative damage and fatigue crack propagation theories each have their own benefits and disadvantages, though a detailed review is beyond the scope of this work. Past reviews on both cumulative damage and crack propagation theories are provided by Suresh (1991), Newman (1998) and Cui (2002), among others. One significant advantage of the crack propagation approach is that it provides knowledge of the fatigue crack growth as a function of the involved parameters, in particular the component geometry and applied cyclic loading. This information can therefore be used to help take action to prolong the service life as mentioned previously. In addition, by ignoring the crack initiation period the scatter between predictions is drastically reduced. The remaining sections of this thesis will consequently focus on the use of fracture mechanics principles to investigate fatigue crack propagation for a range of specimen geometry and loading conditions.

2.2 Fatigue Crack Propagation

The growth of established, 'long' fatigue cracks is generally described by a log-log plot of the crack growth per cycle, da/dN, versus the cyclic elastic stress intensity factor range, ΔK, at the tip of the crack. The stress intensity factor range ΔK is a measure of the magnitude of the singularity at the crack tip and is a function of applied load and the crack/specimen geometry. A sigmoidel growth rate curve typical of many engineering metals is shown in Fig. 1.1 for the case of constant amplitude loading.

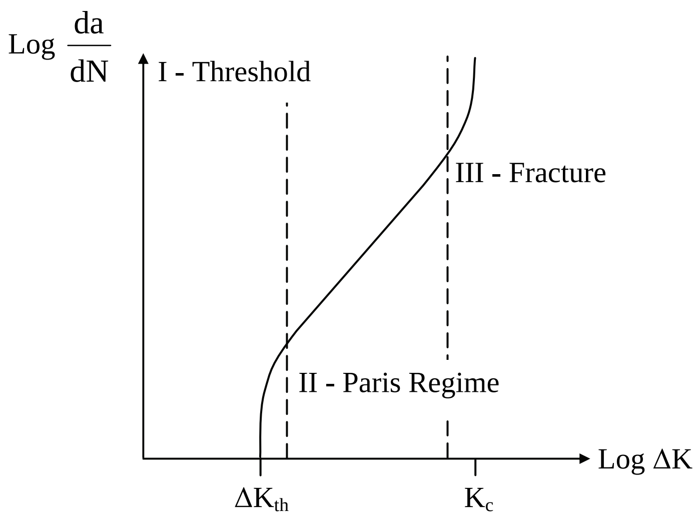


Figure 1.1. Typical sigmoidel log-log fatigue crack growth rate curve.

Region I is the near threshold regime where minimal crack growth occurs with a threshold value, ΔK_{th}, below which there is no detectable crack growth. This value is determined experimentally by decreasing the applied ΔK until the crack growth becomes negligible. In practice, the threshold value is usually taken as the ΔK at a growth of 10^{-10} m/cycle (ASTM E 647). The next section of the curve, Region II, is where the main stable macroscopic crack growth occurs. Within this region the log-log crack growth rate is practically linear with respect to changes in the stress intensity factor range. This region was first introduced by Paris et al. (1961, 1963), who used linear elastic fracture mechanics (LEFM) to describe the crack growth rate by the following relationship:

$$\frac{da}{dN} = C(\Delta K)^m , \tag{1.1}$$

where C and m are constants dependent on the specimen geometry, material properties and load history. As a result of the work by Paris et al. (1961, 1963), Region II is often referred to as the Paris regime. Through integration of equation (1.1), for a given crack increment and applied loading, the number of cycles required for failure to occur can be determined.

The last section of the curve is Region III or the fracture regime. In this region crack growth rates are very high as the maximum stress intensity factor approaches the material's fracture toughness, K_c, where final failure occurs. Since the work by Paris et al. many other empirical relationships have been developed to describe the crack growth behaviour in Regions I, II and III, including those by Forman et al. (1967), Donahue et al. (1972), McEvily and Groeger (1977), and Kujawski (2001), as only a small example.

As technology has continued to develop, so has the ability to detect smaller and smaller cracks. This has lead to the discovery that small cracks can grow at different rates than predicted by long crack theory and at ΔK below the long crack threshold (e.g. Shin and Smith 1985; Newman 1998). In addition, small cracks are greatly influenced by material discontinuities and manufacturing defects such as inclusions, grain boundaries, notch plasticity and service damage. Figure 1.2 shows typical log-log growth rate curves for various small cracks in comparison to traditional long crack theory. Small cracks are a major concern for structures working close to yield conditions, such as in welded joints where there can be large residual tensile stresses (Berkovits et al. 1998), as long crack techniques are not conservative.

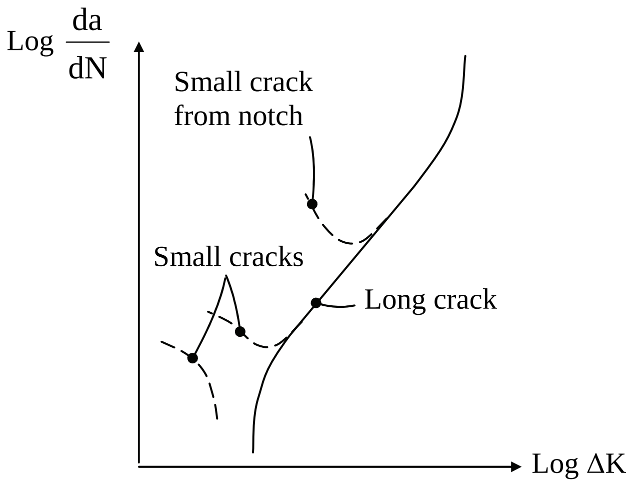
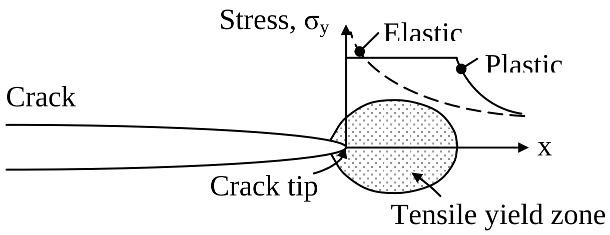


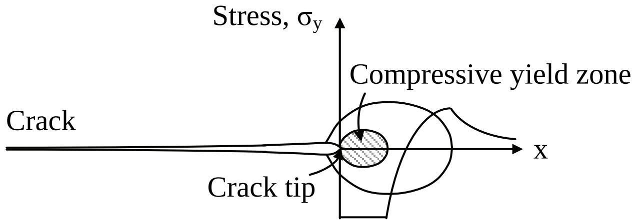
Figure 1.2. Long and small fatigue crack growth.

2.3 Elastic-Plastic Fracture Mechanics

One of the major phenomena associated with fracture and fatigue crack growth is the presence of crack tip plasticity, which occurs to some extent in almost all engineering materials (Hills et al. 1996). Before describing the implications of elastic-plastic fracture mechanics (EPFM) it is first necessary to outline the classic linear elastic fracture mechanics approach. Under this notion the presence of any plasticity is ignored and the resulting stress field in the vicinity of the crack tip is singular. This gives rise to infinite stresses at the crack tip, which has little physical meaning. The application of LEFM is based on the principle that away from the crack tip the true stress and strain fields tend towards the LEFM field, and the infinite behaviour at the tip can be ignored. When using the EPFM approach, however, the infinite tensile stresses are alleviated by allowing plastic deformation to occur. This results in the formation of small zones of tensile plasticity ahead of each crack tip (Fig. 1.3a). An equivalent situation arises during the unloading or compressive part of the load cycle, whereby reverse plastic deformation takes place at the crack tip once the compressive yield stress is reached (Fig 1.3b). The residual deformation due to the tensile loading however, is not fully reversed upon unloading and props the crack open at the tip. The extent and shape of the plastic deformation is dependent largely on the applied load, crack geometry and specimen thickness.



a) Maximum applied load



b) Minimum applied load

Figure 1.3. Schematic illustration of crack tip plasticity and the associated stress field at a) maximum applied load and b) minimum applied load.

2.4 Factors Influencing Fatigue Crack Growth

A large number of factors can affect the fatigue mechanism and thus the crack growth rate and fatigue life of a component or structure. It is vital that these parameters be considered in any fatigue analysis in order to provide greater applicability of the results to practical engineering situations. The various factors influencing fatigue crack growth can be divided into five main groups, as summarised in the following sections. These are *material effects, geometry effects, applied loading, environmental conditions,* and *residual stresses.* There is of course some interrelation between different items from each group.

2.4.1 Material Effects

Material effects include:

i. Material properties/characteristics – basic properties such as Young's modulus, Poisson's ratio, yield strength, ultimate strength, strain hardening exponent, fracture toughness, Bauschinger effect, etc.

ii. Material type and preparation – this includes whether the material is ductile or brittle, the use of cold/hot forming, or any heat treatments, the resulting surface finish and grain structure, etc.

iii. Fatigue limit – the fatigue limit is traditionally associated with cumulative damage theories and is a loading limit under which significant cracks will not form (Cui 2002).

iv. Fatigue crack propagation threshold – used in crack propagation theories and defined as the load limit below which a crack will not significantly grow.

v. Crack closure – it has been found that the faces of a fatigue crack can remain closed, or partially closed, even if the applied load is tensile (Elber 1970, 1971). This closure has been attributed to a variety of sources including residual plasticity, oxide formation, particle debris, crack surface roughness, phase transformation, and viscous fluid closure (Suresh 1991; Anderson 1995). Several of the main mechanisms are depicted in Fig. 1.4.

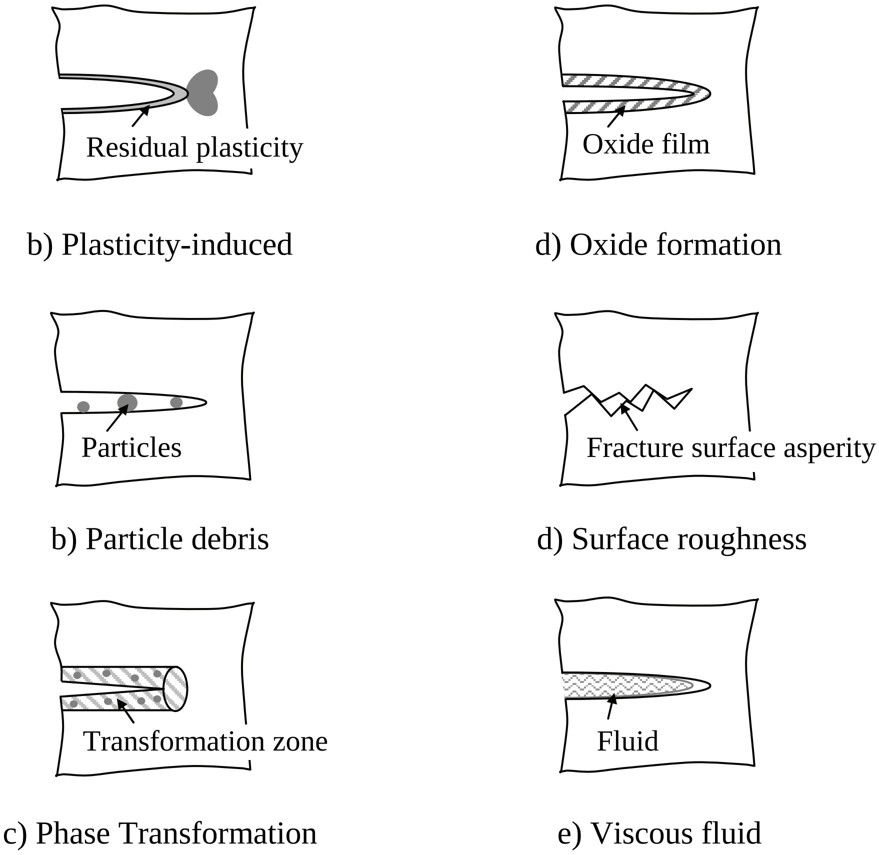


Figure 1.4. Mechanisms of crack closure.

2.4.2 Structural Geometry Effects

Structural Geometry effects include:

i. Component geometry – the size and shape of the component has a significant effect on the stress levels experienced by a fatigue crack at a particular location. Important parameters include the material thickness and the presence of any stress concentrators such as bolt holes or sharp corners.

ii. Manufacturing defects – welding defects, notches or surface scratches, etc create stress concentrations and locations for cracks to propagate from.

2.4.3 Applied Loading

Parameters related to the applied loading are:

i. Loading direction and type – loading may be axial, flexural, or torsional; also may be uni-axial or multi-axial loading, normal, in-plane shear or out-of-plane shear loading, etc.

ii. Mean stress – the mean stress is defined as the average of the maximum and minimum applied stress for a given load cycle. Numerous experiments have shown that for a fixed ΔK an increase in crack growth rate occurs with an increase in mean stress (see for example Unangst et al. 1977; Costa and Ferreira 1998).

iii. Load amplitude – crack growth under constant amplitude loading will generally produce linear growth rates in the log-log plane (see Fig. 1.2). However, under variable amplitude loading (for example overloads, underloads, spectrum loading) load interaction phenomena (Skorupa 1998, 1999) can lead to both acceleration and retardation of the crack growth. The extent of acceleration/retardation is greatly dependent on the load history as well as other factors such as the material properties and specimen geometry.

iv. Frequency – experiments have shown that when heating and corrosion effects are negligible, the loading frequency has only a small influence on the total fatigue life for frequencies less than 1 Hz to around 200 Hz (Stephens et al. 2001).

2.4.4 Environmental Conditions

The most considerable environmental effects on fatigue crack growth are due to the operating temperature and the presence of any corrosive substances.

i. Temperature – a variation in temperature will cause a change in the basic material properties and can also lead to phase transformations and thus changes in the microstructure. At higher temperatures there is an increase in the amount of plastic deformation, due to a reduction in the yield stress, and creep can become quite significant. On the other hand, at low temperatures the fracture toughness can be dramatically reduced.

ii. Corrosion fatigue - fatigue crack growth in corrosive environments is known as corrosion fatigue and involves complex interactions between the corrosive substance and the cyclic loading. The combined effect of corrosion and mechanical fatigue is greater than either of the individual processes acting separately (Cui 2002).

2.4.5 Residual stresses

The main causes of residual stresses are the manufacturing and fabrication processes employed in the production of the material and/or component. In particular, welding can result in residual tensile and compressive stresses, which have a profound effect on the fatigue life (McClung 2007). Residual stresses may also be introduced intentionally to improve the fatigue life through processes such as shot peening, hammer peening, or overloading and underloading. Inclusion of residual stresses into life prediction models is difficult though, as stress relaxation can occur due to the externally applied cyclic loading.

2.5 Theoretical Methods for the Thickness Effect on Crack Closure

There are two main approaches of accounting for plate thickness effects (i.e. stress state) in plasticity-induced crack closure analyses. The first method is to use a two-dimensional crack closure model with a plastic constraint factor to simulate the thickness effect. The second method involves modelling the three-dimensional geometry with a finite element mesh. Specific and detailed reviews of the various techniques used to model the thickness effect in both linear elastic plates and crack closure analyses, are given in the introduction sections of the journal papers that make up the main body of this thesis (particularly chapters 4-8). However, for completeness this section provides a brief summary of the key issues relating to the different approaches.

The most popular method of accounting for the effects of plate thickness is through the use of an empirical correction factor, which was first introduced by Newman (1981). In his model (Newman 1981); the tensile yield stress was multiplied by a scale factor, the so-called plastic constraint factor, to increase the value at which yielding occurs. The physical meaning behind the constraint factor is that it increases the material’s yield stress in order to simulate the out-of-plane constraint caused by the plate thickness. A constraint factor equal to 1 represents plane stress conditions, while a value of 3 is often assumed for plane strain conditions. Stress states in between these limits are determined by linear interpolation, trial-and-error, or by curve fitting finite element or experimental data (Newman et al. 1995, 1999). This creates much ambiguity as the degree of out-of-plane constraint varies within the plastic zone, and also with the cracked geometry and applied loading (Kelly and Nowell 2000; Codrington et al. 2008). Despite this, the constraint factor approach does have the benefits of simplicity, in terms of mathematical implementation, which is why it has become so widely used (e.g. de Koning and Liefting 1988; Wang and Blom 1991; Daniewicz et al. 1994; Newman et al. 1999).

Newman’s (1981) plasticity-induced crack closure model was based on the Dugdale (1960) strip-yield hypothesis, which was modified to leave a wake of plasticity as the crack propagates. The numerical procedure utilised boundary elements to represent the crack opening displacement and plastic deformation along the length of the crack and in the crack tip plastic zone. A large number of strip-yield crack closure models have been developed over the years using a range of analytical and numerical techniques including the method of complex potentials (Budiansky and Hutchinson 1978; Wang and Rose 1999), weight functions (Wang and Blom 1991; Daniewicz et al. 1994; Kim and Lee 2000), and dislocation boundary elements (Nowell 1998). The weight function approach, in particular, has been utilised to model the effects of in-plane plate geometry on crack closure such as in centre-cracked tension and bend specimens as well as compact tension specimens (Wang and Blom 1991; Kim and Lee 2000).

An alternative approach to the use of the constraint factor for modelling plate thickness effects is to use finite element methods (e.g. Chermahini and Blom 1991; Roychowdhury and Dodds 2003; Alizadeh et al. 2007). Finite element methods allow for virtually any two or three-dimensional cracked geometry to be modelled and investigated. The variation in the extent of plastic deformation and thus crack closure across the plate thickness can also be examined (Chermahini and Blom 1991; Roychowdhury and Dodds 2003). However, numerical issues such as mesh refinement, solution convergence, crack face contact, crack advancement and node release scheme, etc (Solanki et al. 2004) can limit the practical application of finite element methods. In addition, finite element calculations often require extensive computational power and time, which makes them unsuitable for systematic studies of a large number of input parameters or for fatigue life prediction in industry.

2.6 The Distributed Dislocation Technique

The distributed dislocation technique (DDT) is a powerful analytical method that is frequently employed in fracture mechanics analyses. Based on the pioneering work of Eshelby (1957, 1959) and Bilby et al. (1963, 1968), this technique involves the representation of a crack by an unknown distribution of ‘strain nuclei’. Use is then made of the principle of superposition for the stress field that would be present in an uncracked body subject to external forces, together with the stresses produced by the distribution of strain nuclei. The unknown distribution of nuclei can thus be determined by enforcing the requirement that the crack faces remain traction free.

One particular type of strain nuclei that is of interest to the present study is the edge dislocation, as depicted in Fig. 1.5a. The creation of an edge dislocation may be thought of as making a cut along the negative x-axis, then inserting a thin strip of material before rejoining (Hills et al. 1996). By adding more thin strips distributed along the x-axis (Fig 1.5b) and by taking others away (fig 1.5c), the crack geometry can be recovered (Fig 1.5d). The insertion of thin strips of material inside the crack is used here as a mathematical tool to simulate the crack face separation and to provide a means of generating the resultant stress-field within the cracked body. The actual crack is, of course, empty. Furthermore, the edge dislocation is usually associated with lattice distortions due to the insertion of an extra half-plane of atoms within a lattice structure. When modelling a crack via the DDT, however, it should be noted that the presence of any lattice defects is not implied.

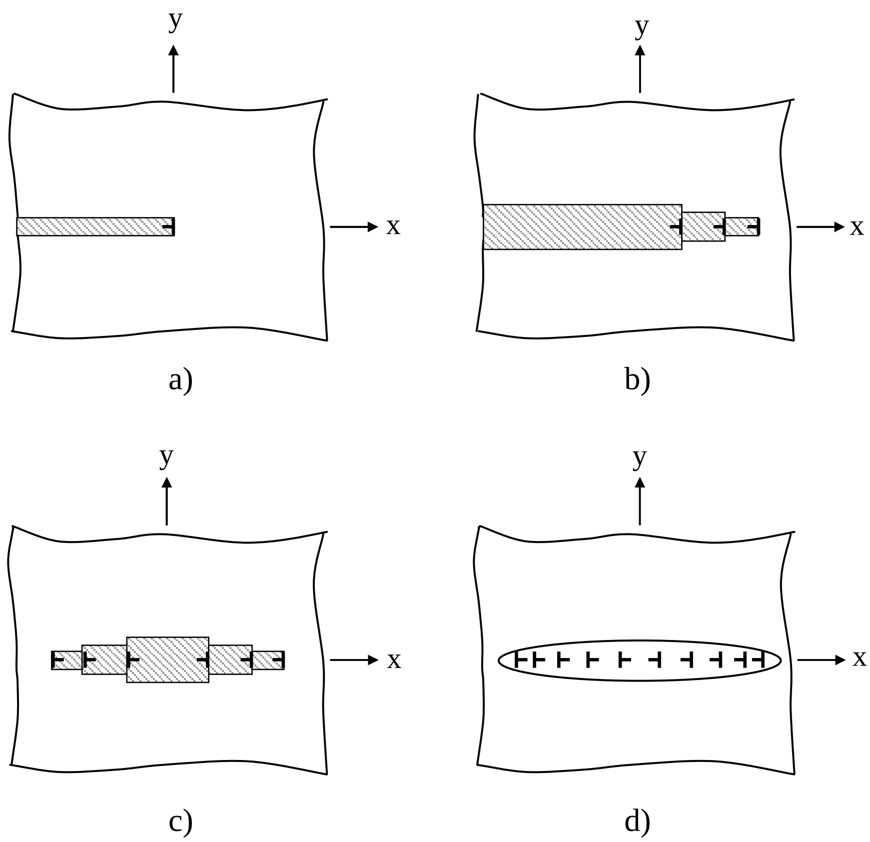


Figure 1.5. Formation of a centre crack using edge dislocations: a) single dislocation, b) addition of more dislocations, c) removal of dislocations and d) final crack geometry.

The resultant stress and strain fields generated by the continuous distribution of dislocations can be found by integrating the solution for a single edge dislocation over the length of the crack (Hills et al. 1996). This produces a singular integral equation with the well known Cauchy kernel, which can be readily solved via standard techniques. One of the most widely used and easily implemented methods is the Gauss-Chebyshev quadrature developed by Erdogan et al. (1972, 1973).

The distributed dislocation technique has been utilised for the analysis of a wide variety of elastic crack problems, which have included geometrical features such as circular and elliptical inclusions (Dundurs and Mura 1964; Warren 1983), kinked cracks (Li and Hills 1990), multilayered composites (Erdogan and Gupta 1971), finite boundaries (Keer, Lee and Mura 1983) and surface breaking cracks (Nowell and Hills 1987). However, these investigations have only considered plane stress or plane strain stress states. Kotousov and Wang (2002) derived the solution for a through-the-thickness edge dislocation in a plate of finite thickness. This solution is based on first-order plate theory and provides an approximation to the three-dimensional stress and strain fields using a two-dimensional approach (Codrington et al. 2008). In a few cases full three-dimensional geometry has been considered using dislocation loops to model, for example, penny-shaped, elliptical and square cracks in infinite bodies (Hills et al. 1996). However, the numerical treatment involved is highly complex and requires the use of two-dimensional discretisation schemes to solve the singular integral equations. A thorough review of the DDT and the treatment of various crack problems are provided by Hills et al. (1996).

Various researchers have also made use of the DDT for the analysis of non-linear crack problems. The first was Bilby et al. (1963) who modelled plastic yielding at the tip of a crack under modes I, II and III loading. Keer and Mura (1966) also modelled the direct plasticity zone for mode III cracks and then determined the crack tip yield zone for penny shaped cracks under uniform tensile loading. More recent examples include the work by Nowell (1998) who employed dislocation boundary elements rather than a continuous distribution, and the current research by the thesis author (e.g. Codrington and Kotousov 2007a, 2007b). In all of the aforementioned analyses plastic yielding was assumed to be confined along the plane of the crack. Riedel (1976) and Atkinson and Kanninen (1977) have investigated the plane strain problem by using dislocations to model the plasticity zones located along ‘wings’ extending either side of the crack front, i.e. planes of shear.

2.7 Summary of Gaps

This section provides a summary of several keys areas of research, in relation to non-linear fatigue crack growth, that have been identified as needing further investigation. Firstly, plasticity-induced crack closure, and the crack closure concept in general, has been widely embraced by a large number of researchers for correlating and predicting fatigue crack growth. The plasticity-induced closure mechanism has provided a crucial understanding into a number of individual fatigue phenomena including variable amplitude loading, load ratio effects, plate thickness effects, etc, as well as the interaction between these phenomena

(Skorupa 1999). However, current theoretical techniques for modelling plasticity-induced crack closure are based largely on the use of empirical correction factors or finite element (FE) methods (or both).

The determination of the geometry correction factors, such as those used to incorporate plate thickness effects, can be very difficult as limited data is available for the values of these factors. Consequently, curve fitting of experimental fatigue crack growth data and trial-and-error are often used (Newman et al. 1999). This greatly reduces any original physical meanings behind these parameters. Furthermore, correction factors are usually assumed to be constant, or of several constant values, throughout the analysis. The interaction between various parameters, for example the specimen thickness and the loading sequence, is therefore not accurately considered.

Finite element methods have provided an alternative to the use of geometry correction factors. It has been found, however, that these techniques suffer from many issues due to mesh refinement, crack advancement scheme, and so forth (see for example Solanki et al. 2004). The use of FE methods requires extensive calculations for each different geometry and load configuration under investigation. The significant time and computational effort involved therefore make finite element methods impractical for many engineering applications. There is therefore a need for new methods that eliminate the use of correction factors, and are computationally efficient so that they can be used in practical situations.

The distributed dislocation technique (DDT) has been shown to be a very powerful tool in fracture mechanics. This technique can allow for a range of geometric features to be considered in the analysis including holes, free surfaces and plate thickness. The convergence and accuracy of the results obtained with the DDT is usually very high and can be easily controlled by varying the number of integration points (Hills et al. 1996). Methods developed with the DDT would offer a viable alternative to the use of empirical correction factors and also to FE calculations, which are often time consuming, have problems with convergence and can be very sensitive to the various parameters involved.

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CHAPTER 3

EFFECT OF A VARIATION IN MATERIAL PROPERTIES ON THE CRACK TIP OPENING DISPLACEMENT

STATEMENT OF AUTHORSHIP

Effect of a Variation in Material Properties on the Crack Tip Opening Displacement

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Codrington, J. (Candidate)

Created theoretical models, performed all analyses, interpreted data, and wrote manuscript.

Signed _______________ Date ________________

Kotousov, A. (Supervisor)

Supervised development of work, participated in discussions of work, and manuscript evaluation.

Signed _______________ Date ________________

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Effect of a Variation in Material Properties on the Crack Tip Opening Displacement

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Abstract:

This paper theoretically investigates the effect of a variation in material properties on the crack tip opening displacement when the crack is opened by a steady-state load. This situation is typical when a structure is subjected to a slow temperature fluctuation or the material properties undergo some changes due to a phase transformation, deterioration over time, etc. A theoretical study is made of the implications of a strip-yield model for the analysis of the plasticity effects associated with such variations. Results of calculations over a wide range of material properties are presented.

Keywords: Crack tip opening displacement, Crack tip plasticity, Effect of material properties, Small-scale yielding.

Nomenclature

(0)	Superscript for before change in material Properties
(1)	Superscript for before change in material Properties
+,-	Subscripts referring to the limit of $\lambda \to 0$ ($\lambda > 0$) such that $f_{\pm} = f(x \pm i\lambda)$
a, c	Sizes of the yield zones
A	Constant term as part of solution procedure
i	Standard imaginary unit
E	Young's modulus
g(x)	Function for the crack opening displacement and plastic stretch curves
K	Remotely applied stress intensity factor
δ	Crack opening displacement and plastic stretch
δ_t	Crack tip opening displacement
$\Delta\delta_t$	Change in crack tip opening displacement
η	Non-dimensional Yield stress parameter
ρ	Non-dimensional Young's modulus parameter
σ_y	Y-stresses along x-axis
σ_Y	Yield stress
ϕ	Muskhelishvili complex potential
Φ, ϕ'	First derivative of the Muskhelishvili complex potential
ω	Tensile plastic zone size
$\chi(z), \hat{\chi}(x)$	Auxiliary functions

1 Introduction

The investigations of crack tip plasticity phenomena are of significant interest in relation to failure and lifetime assessments. The plasticity effects, which occur near the stationary or growing crack have been vastly investigated experimentally, numerically and analytically, see [1-12] to point a few. It has been shown that many factors including the applied stress and stress ratio, plate thickness and cracked geometry, transverse and residual stresses, crack surface condition, crack closure and material properties affect the plastic deformations in the vicinity of the crack tip and crack growth controlling parameters.

One of the most frequently utilised theoretical methods for investigating crack tip plasticity effects is the Dugdale strip-yield model [1]. This simple model assumes that all plastic deformation is confined to an infinitesimal strip along the line of the crack. In 1978, Budiansky and Hutchinson [9] developed an analytical model for plasticity-induced fatigue crack closure using the Dugdale hypothesis and the theory of complex potentials. The developed model lent support to the existence of crack closure and provided some justification for the adoption of an effective stress-intensity factor range based on closure effects. Since then several researchers have extended the Budiansky and Hutchinson [9] model to incorporate various factors as mentioned above, including refs [10-12]. The current work will also make use of the Budiansky and Hutchinson [9] methodologies for investigating the effects of a variation in material properties.

Over the years, finite element methods have been extensively used to simulate crack tip plasticity effects [3-6]. Complex 3D geometry and load conditions can be realised relatively easily via these methods. The process usually involves creating a mesh with an initial crack, and then loading the mesh by applied tractions leading to the formation of the plastic zone. Despite the simplicity of the modelling concept; there are several issues, which apply significant limitations to numerical procedures, influence their accuracy and are partially responsible for existing controversy in the numerical results obtained by different researchers or by using different methods. Such issues include mesh refinement, finite boundaries, and in the case of a growing crack the crack surface contact, crack advancement scheme, etc [6].

The majority of the previous investigations on crack-tip plasticity phenomena have been carried out assuming constant material properties. However, these investigations do not address many practical situations where significant variations in material properties can occur during the life of the cracked component. Even at a steady-state mechanical loading, for example during relatively slow heating or cooling of the structural component (when the thermal stresses are negligible), the variation of mechanical properties alone could significantly influence the crack tip plasticity behaviour. This will consequently affect the failure conditions and lifetime of the component. In order to investigate this situation, a theoretical model is developed and results are presented for a wide range of the material parameters.

2 Characterisation of the Associated Crack Tip Plasticity Effects

The theoretical analysis below adopts the methods presented by Budiansky and Hutchinson [9]. The material is characterised by two mechanical properties, namely, Young's modulus, E, and the yield strength, σ_Y. To describe the variations in the material properties we introduce the following dimensionless variables:

$$\rho = \frac{E^{(1)}}{E^{(0)}}, \qquad (1a)$$

$$\eta = \frac{\sigma_Y^{(1)}}{\sigma_Y^{(0)}}, \qquad (1b)$$

where the superscripts (0) and (1) refer to material properties before and after the variation, respectively.

Figure 1 illustrates the four distinct cases of the crack tip plasticity effects associated with the values of the introduced non-dimensional variables (1). The crack tip deformation effects corresponding to these four cases is illustrated schematically Fig. 2. A fifth trivial region also exists along the $\rho = 1$, $\eta \geq 1$ boundary line where no changes in the crack-tip opening displacement (CTOD) occur after the variation of the mechanical properties.

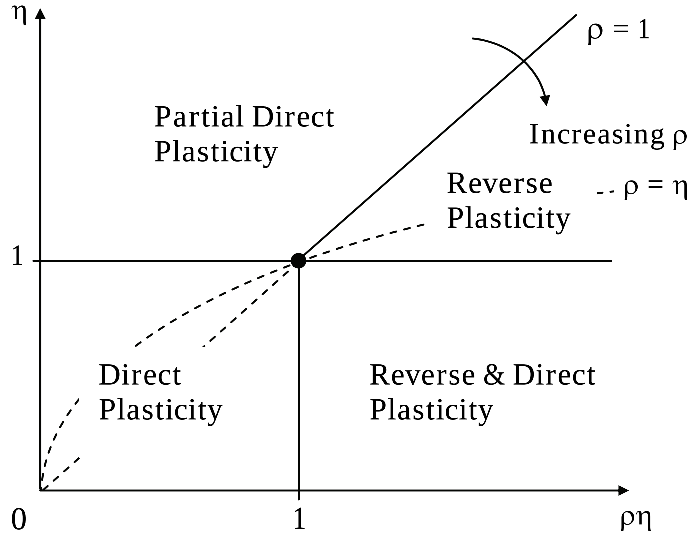
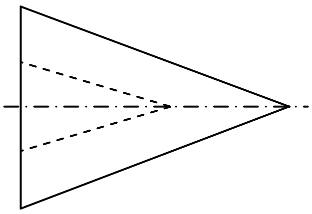
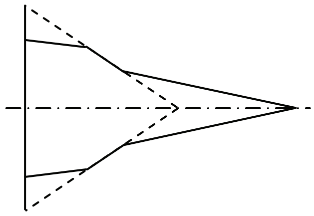


Fig.1. Characterization of crack tip plasticity effects.



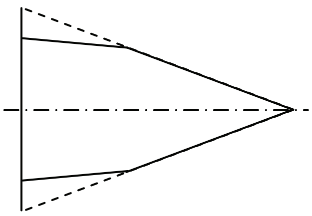
(a) $\eta \le 1$, $\rho\eta \le 1$

(Direct plasticity)



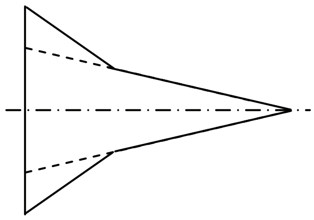
(b) $\eta \le 1$, $\rho\eta \ge 1$

(Reverse and direct plasticity)



(c) $\eta \ge 1$, $\rho \ge 1$

(Reverse plasticity)



(d) $\eta \ge 1$, $\rho \le 1$

(Partial-direct plasticity)

- - - - - - - Plastic zone before variation

———— Plastic zone after variation

Fig.2. Schematic illustration of the crack-tip plasticity effects for the four cases of (a) direct plasticity, (b) reverse and direct plasticity, (c) reverse plasticity, and (d) partial-direct plasticity.

For example, an increase in the temperature of a structure is normally associated with a reduction in the values of the elastic modulus and yield stress ($\rho < 1$ and $\eta < 1$), which corresponds to the first case (a). Alternatively, a decrease of the operating temperature ($\rho > 1$ and $\eta > 1$) will result in reverse plastic deformations as shown schematically in Fig.2c. The deterioration of material properties over time is a fairly complicated phenomenon, which includes many competing mechanisms, and in a general case can lead to any of the crack-tip plasticity effects illustrated in Fig.2. Each of these cases will be analysed further in the subsequent sections. The following assumptions have been made in order to develop the theoretical model: (a) the concept of small-scale plasticity is adapted, which means that the plastic zone is confined to a small region in the vicinity of the crack tip; (b) the crack is assumed to be fully opened by a steady state load at all times; and (c) there is no interaction with the residual stretch, which can exist behind the crack tip due to previous fatigue crack growth.

3 Direct Plasticity Region, Case (a)

This is the simplest case and the change in the plastic zone size follows straight from the Dugdale solution [1] for the plastic stretch ahead of a semi-infinite crack. The associated Dugdale type boundary conditions can be written as:

$$\begin{aligned} &\sigma_y = 0, && -\infty < x < 0, \\ &\sigma_y = \sigma_Y^{(1)}, && 0 < x < \omega^{(1)}, \end{aligned} \tag{2}$$

where σ_y is the y- stresses along the x-axis, and $\omega^{(1)}$ is the size of the tensile plastic zone after the change in material properties.

Fig. 3 shows a schematic diagram of these conditions as well as the resulting plastic stretch and crack opening displacement. The final size of the plastic zone is given by [7,9]:

$$\omega^{(1)} = \frac{\pi}{8}\left(\frac{K}{\eta\sigma_Y^{(0)}}\right)^2, \tag{3}$$

and the plane stress CTOD is:

$$\delta_t^{(1)} = \frac{K^2}{\rho E^{(0)} \eta \sigma_Y^{(0)}} . \tag{4}$$

The plastic stretch in the interval (0, $\omega^{(1)}$) and the crack opening displacement for $x < 0$ is given by [7,9]:

$$\frac{\delta^{(1)}(x/\omega^{(1)})}{\delta_t^{(1)}} = g(x/\omega^{(1)}) , \tag{5}$$

where:

$$g(\xi) = \sqrt{1-\xi} - \frac{\xi}{2} \log\left|\frac{1+\sqrt{1-\xi}}{1-\sqrt{1-\xi}}\right| . \tag{6}$$

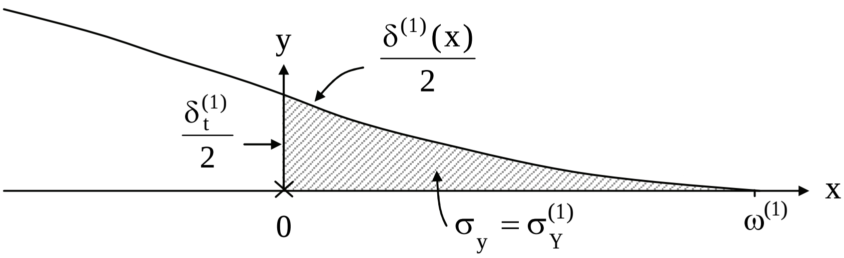


Fig.3. Dugdale solution.

Results for the change of the crack-tip opening displacement, normalised by the initial CTOD, are given in Fig. 4 as a function of the normalised material properties ρ and η.

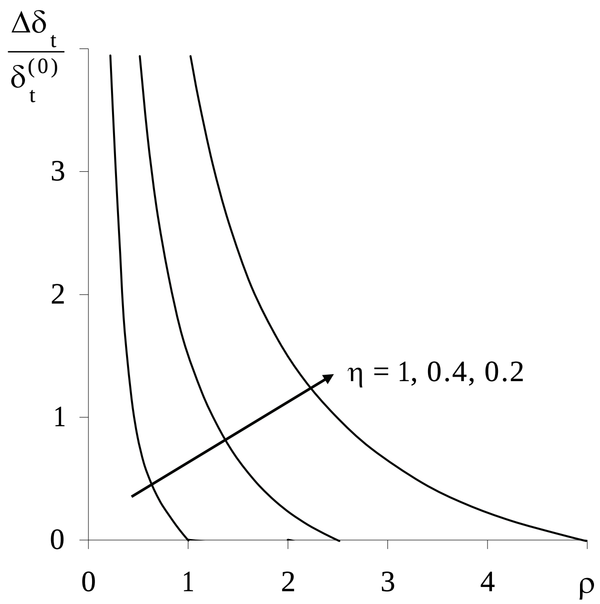


Fig.4. Change in crack-tip opening displacement as a function of material properties ($\eta \le 1$, $\rho\eta \le 1$).

4 Reverse and Direct Plasticity Region, Case (b)

For the remaining plasticity regions we follow a similar procedure to that of Budiansky and Hutchinson [9]. As discussed in Section 2, we will assume that the residual stretch due to the fatigue crack growth does not affect the boundary conditions, which for case (b) are shown in Fig. 5. In this figure, a is the size of the reverse yielding zone, c is the start of the direct yielding zone and $\omega^{(1)}$ is the final overall plastic zone size. The initial plastic zone size $\omega^{(0)}$, crack-tip opening displacement $\delta_t^{(0)}$, and plastic stretch $\delta^{(0)}$, are given by (3) to (6) with $\rho = 1$, $\eta = 1$. The boundary conditions are then realised by applying complex Muskhelishvili potentials [13].

Along the x-axis this provides the y-stress equation:

$$\sigma_y = \Phi_+ + \Phi_- . \tag{7}$$

The jump in the y-direction displacement gradient across the x-axis, in the case of plane stress, is given by:

$$\left(\frac{\partial\delta}{\partial x}\right)_+ - \left(\frac{\partial\delta}{\partial x}\right)_- = \frac{4}{iE}\left[\Phi_+ - \Phi_-\right], \tag{8}$$

where $\Phi = \phi'$ is the first derivative of the Muskhelishvili potential ϕ, the $\pm$ subscripts correspond to $f_\pm = f(x \pm i\lambda)$ for $\lambda > 0$, $\lambda \to 0$ and i is the standard imaginary unit.

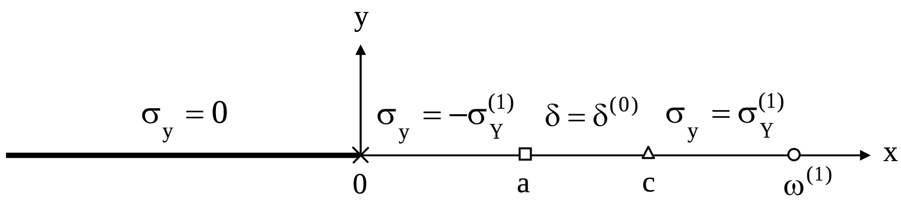


Fig.5. Boundary conditions for case (b).

The solution to the problem can be found by asserting the boundary conditions of Fig. 5 via (7) and (8). Before this we must introduce an auxiliary function $\chi(z) = \sqrt{z - \omega^{(1)}}\sqrt{z-c}\sqrt{z-a}$ (branch cut on the interval (-∞, a) and (c, $\omega^{(1)}$) with $\chi(z > \omega^{(1)}) = +\sqrt{z - \omega^{(1)}}\sqrt{z-c}\sqrt{z-a}$). This leads to the following conditions on $\chi\Phi$ along the x-axis:

$$\left.\begin{aligned}
(\chi\Phi)_+ - (\chi\Phi)_- &= 0 && \text{for} \quad x < 0 \\
&= -\eta\sigma_Y\chi_+ && 0 < x < a \\
&= \frac{i\rho E\chi}{4}\frac{d\delta^{(0)}}{dx} && a < x < c \\
&= \eta\sigma_Y\chi_+ && c < x < \omega^{(1)} \\
&= 0 && x > \omega^{(1)}
\end{aligned}\right\} \tag{9}$$

The general solution for $\chi\Phi$ is determined via the application of Plemelj integrals [9] such that:

$$\chi(z)\Phi(z) = -\frac{\eta\sigma_Y^{(0)}}{2\pi}\int_0^a \frac{\hat{\chi}(x)}{x-z}dx + \frac{\rho E^{(0)}}{8\pi}\int_a^c \frac{\chi(x)}{x-z}\cdot\frac{d\delta^{(0)}}{dx}dx$$
$$+\frac{\eta\sigma_Y^{(0)}}{2\pi}\int_c^{\omega^{(1)}} \frac{\hat{\chi}(x)}{x-z}dx + \frac{K}{2\sqrt{2\pi}}z + A\,, \tag{10}$$

where:

$$\hat{\chi}(x) = \sqrt{\omega^{(1)} - x}\sqrt{x-c}\sqrt{x-a}\,, \tag{11}$$

and use has been made of the potential associated with the far-field elastic stresses [9] and A is a constant to be determined. The boundedness of the stresses at x = a, c and $\omega^{(1)}$ can be enforced by making the Cauchy principal value of the right-hand side of (10) vanish. This provides three relations among the quantities $\omega^{(1)}$, a, c and A. The fourth condition necessary to determine these quantities is the continuity of the plastic stretch at point c, or:

$$\delta^{(1)}(c/\omega^{(1)}) = \delta^{(0)}(c/\omega^{(0)})\,. \tag{12}$$

For various assumed values of η ($\eta \le 1$) and ρ ($\rho\eta \ge 1$), it was straightforward to solve these four equations (we omit details) for $\omega^{(1)}$, a, c and A.

Figure 6 shows the results for the normalised change in the crack-tip opening displacement as a function of the two governing parameters ρ and η. The results were obtained via equation (8) by substituting the calculated values of $\omega^{(1)}$, a, c and A into the complex potential Φ.

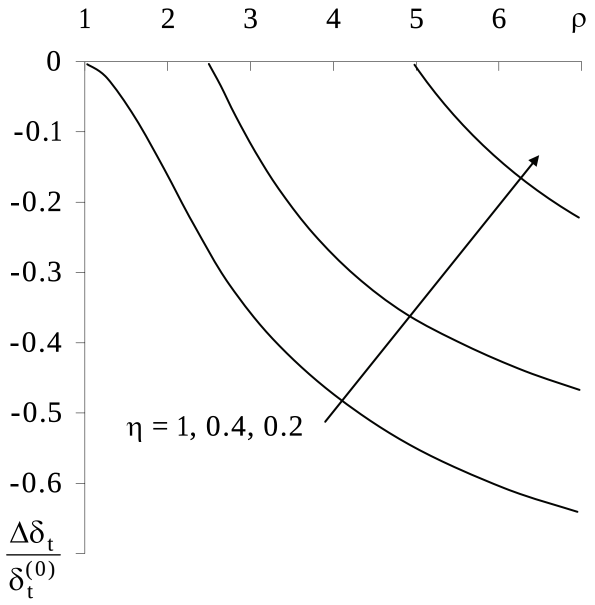


Fig.6. Change in crack-tip opening displacement as a function material of properties ($\eta \le 1$, $\rho\eta \ge 1$).

5 Cases (c) and (d)

The final two cases can be obtained as special cases of the considered above case (b). For case (c) $\omega^{(1)} = c$ and is in fact equal to the initial plastic zone size, $\omega^{(0)}$. Substituting these conditions into (10) provides the following general solution for $\chi\Phi$ along the x-axis:

$$\chi(z)\Phi(z) = -\frac{\eta\sigma_Y^{(0)}}{2\pi}\int_0^a \frac{\hat{\chi}(x)}{x-z}dx + \frac{\rho E^{(0)}}{8\pi}\int_a^{\omega^{(0)}} \frac{\chi(x)}{x-z}\cdot\frac{d\delta^{(0)}}{dx}dx + \frac{K}{2\sqrt{2\pi}}z + A\,, \tag{13}$$

where $\hat{\chi}(x)$ and $\chi(x)$ are as described previously. The boundedness of the stresses at x = a and $\omega^{(0)}$ produces a system of two equations with two unknowns, a and A, which can be easily solved to give the size of the reverse plasticity zone as:

$$a = \omega^{(1)} \cdot \left(\frac{\rho - 1}{\rho + \eta}\right)^2 . \tag{14}$$

Quite similarly for case (d), the general solution for $\chi\Phi$ along the x-axis is found to be:

$$\chi(z)\Phi(z) = \frac{\eta\sigma_Y^{(0)}}{2\pi}\int_0^a \frac{\hat{\chi}(x)}{x-z}dx + \frac{\rho E^{(0)}}{8\pi}\int_a^{\omega^{(0)}} \frac{\chi(x)}{x-z}\cdot\frac{d\delta^{(0)}}{dx}dx + \frac{K}{2\sqrt{2\pi}}z + A\,. \tag{15}$$

Again, the boundedness of the stresses at x = a and $\omega^{(0)}$ produces a system of two equations with two unknowns, a and A. The size of the direct plasticity zone is thus determined to be:

$$a = \omega^{(1)}\cdot\left(\frac{\rho-1}{\rho-\eta}\right)^2. \tag{16}$$

The dependence of the ratio of the CTOD before and after the variation of the material properties calculated using Eq. (8) for cases (c) and (d) are shown in Figs. 7 and 8, respectively.

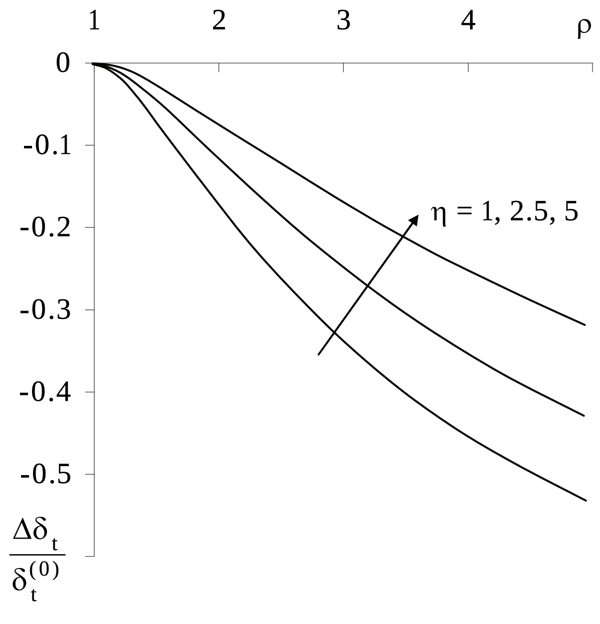


Fig.7. Change in crack-tip opening displacement as a function of material properties ($\eta \geq 1$, $\rho \geq 1$).

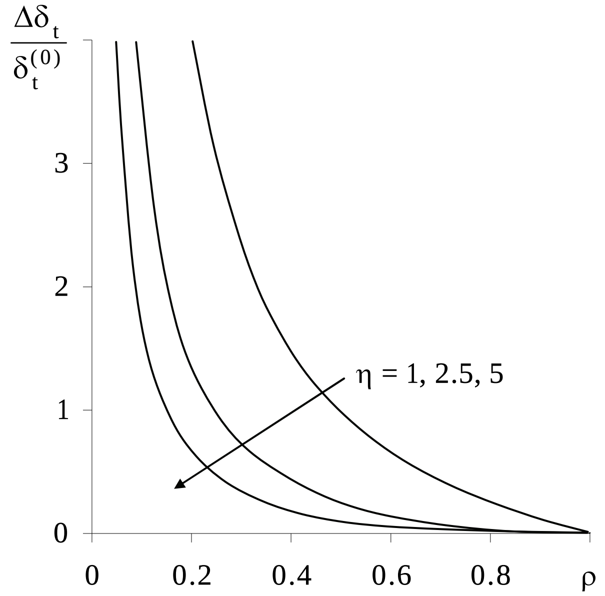


Fig.8. Change in crack-tip opening displacement as a function of material properties ($\eta \geq 1$, $\rho \leq 1$).

As one can see from Fig. 4 and 6-8, a typical variation in the mechanical properties, say $\pm 50\%$, has a significant effect of on the crack-tip opening displacement for all four cases considered. It seems the most significant effect is for cases (a) direct plasticity and (d) partial direct plasticity. Comparably less change occurs at the same level of material properties variation for cases (b) and (c), which are connected with the reverse plasticity. In the two last cases a possible interaction with the residual stretch due to fatigue crack growth can also be important. Although, such interaction was beyond the scope of the current paper.

The considered mechanism could itself be a source for fatigue crack growth or assist the fatigue crack growth caused, for example, by applied cyclic tractions. However, despite the significant effect that a variation in the material properties has on the crack tip opening displacement, it seems, there is no experimental studies available in the literature against which the obtained theoretical results can be compared and validated. One of the possible reasons is that most fatigue crack growth investigations have adopted the linear fracture mechanics concept. Within this concept the problem under investigation vanishes, as the stress intensity factor in most cases is a function of the geometry and applied loading only and not the material properties. Thus, linear fracture mechanics, which is also the framework for many fatigue failure assessment codes and standards, cannot provide a guidance for the assessment of the effects associated with a variation of material properties on the crack growth rate and failure conditions.

6 Conclusion

The effects of a change in material properties were investigated by extending the Budiansky and Hutchinson [9] model. Four cases of crack tip plasticity effects were identified for the different combinations of parameters controlling the change of material properties. A general integral equation was obtained for all these cases. Enforcing the boundedness of the stresses and using an additional condition of continuity, the integral equation was reduced to a system of algebraic equations from which all sizes characterising the extent of the direct and reverse plastic zones were obtained. Results for the change in crack-tip opening displacement were presented for all these cases. However, extensive experimental work is required to properly understand the obtained results and the physical implications of a variation in the material properties on crack tip plasticity zone.

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CHAPTER 4

OUT-OF-PLANE STRESS AND DISPLACEMENT FOR THROUGH-THE-THICKNESS CRACKS IN PLATES OF FINITE THICKNESS

STATEMENT OF AUTHORSHIP

Out-of-Plane Stress and Displacement for Through-the-Thickness Cracks in Plates of Finite Thickness.

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Codrington, J. (Candidate)
Created theoretical models, performed all analyses, interpreted data, and wrote manuscript.

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Kotousov, A. (Supervisor)
Supervised development of work, participated in discussions of work, and manuscript evaluation.

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Ho, S.Y. (External supervisor)
Supervised development of work, and manuscript review.

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OUT-OF-PLANE STRESS AND DISPLACEMENT FOR THROUGH-THE-THICKNESS CRACKS IN PLATES OF FINITE THICKNESS

JOHN CODRINGTON, ANDREI KOTOUSOV AND SOOK YING HO

The out-of-plane stress and displacement fields are investigated analytically for mode I through-the-thickness cracks in an infinite plate of finite thickness within the first-order plate theory. The developed method is based on the distributed dislocation approach and an earlier derived three-dimensional solution for an edge dislocation. Numerical results are obtained through application of Gauss–Chebyshev quadrature for both finite length and semiinfinite crack cases. The calculated stress and displacement fields are found to be in good agreement with already published experimental and finite element studies. Further results for the averaged through-the-thickness stress intensity factor are given and again found to be in good agreement with previous finite element values. The developed solutions can therefore be used in experimental techniques for the assessment of the stress intensity factor using the out-of-plane displacement measurements, for example by the interferometry method.

Codrington, J., Kotousov, A. & Ho, S.Y. (2008) Out-of-plane stress and displacement for through-the-thickness cracks in plates of finite thickness.
Journal of Mechanics of Materials and Structures, v. 3 (2), pp. 261-270

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This publication is included on pages 61-70 in the print copy of the thesis held in the University of Adelaide Library.

Keywords: distributed dislocation technique, edge dislocation, out-of-plane constraint factor, out-of-plane displacement, plate thickness effect, through-the-thickness crack.

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CHAPTER 5

APPLICATION OF THE DISTRIBUTED DISLOCATION TECHNIQUE FOR CALCULATING CYCLIC CRACK TIP PLASTICITY EFFECTS

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Application of the Distributed Dislocation Technique for Calculating Cyclic Crack Tip Plasticity Effects.

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Application of the distributed dislocation technique for calculating cyclic crack tip plasticity effects

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ABSTRACT This paper describes a method for modelling cyclic crack tip plasticity effects based on the distributed dislocation technique (DDT). A strip-yield model is utilised to allow for the determination of the crack opening displacement, size of the plastic zones and in the case of a fatigue crack, the wake of plasticity. The DDT can be easily implemented for a wide range of cracked geometries with reliable control over the accuracy and convergence. Thickness effects can also be incorporated through a recently obtained solution for an edge dislocation in an infinite plate of finite thickness. Results for finite length cracks that have had limited growth, such that there is no plastic wake, are presented for a range of applied loads and R-ratios. Further results are provided for a steady-state fatigue crack in a plate of finite thickness. The present results are compared with analytical solutions and they show an excellent agreement.

Keywords: crack closure; crack tip plasticity; distributed dislocation technique; Gauss–Chebyshev quadrature; semi-infinite crack.

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CHAPTER 6

THE DISTRIBUTED DISLOCATION TECHNIQUE FOR CALCULATING PLASTICITY-INDUCED CRACK CLOSURE IN PLATES OF FINITE THICKNESS

STATEMENT OF AUTHORSHIP

The Distributed Dislocation Technique for Calculating Plasticity-Induced Crack Closure in Plates of Finite Thickness.

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Created theoretical models, performed all analyses, interpreted data, and wrote manuscript.

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Kotousov, A. (Supervisor)

Supervised development of work, participated in discussions of work, and manuscript evaluation.

Signed ________________ Date ________________

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ERRATUM

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Equations (16) and (28) in the above paper were printed containing minor errors. The correct version of (16) is:

$$\bar{\sigma}(t_k) - \bar{\sigma}_{yy}^{\infty}(t_k) = \frac{(a + r_p)}{2N+1} \sum_{i=1}^{N} \bar{\phi}(s_i)(s_i + 1)\left[\bar{G}(t_k, s_i) - \bar{G}(t_k, -s_i)\right] \quad k = 1\ldots N, \tag{16}$$

and (28) is:

$$\frac{dg(x)}{dx} = \frac{\delta_R}{a - a_0} \qquad \beta \le |x| < a, \tag{28}$$

The authors apologise for any inconvenience and would like to assure the readers that these are only typing errors and in no way affect the presented results.

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ORIGINAL PAPER

The distributed dislocation technique for calculating plasticity-induced crack closure in plates of finite thickness

John Codrington · Andrei Kotousov

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Abstract An analytical method for calculating plasticity-induced fatigue crack closure in plates of finite thickness is presented. The developed method utilizes the distributed dislocation technique (DDT) and Gauss-Chebyshev quadrature. Crack tip plasticity is incorporated by adopting a Dugdale type strip yield model. The finite plate thickness effects are taken into account by using a recently obtained three-dimensional solution for an edge dislocation in an infinite plate. Numerical results for the ratio of the size of the crack tip plasticity zones are presented for the cases of uniform thickness wake and linearly increasing wake for a range of plate thickness to crack length ratios and applied load ratios. The results show a very good agreement with previous analytical solutions in the limiting cases of very thick and very thin plates. Further results for the opening stress to maximum stress ratio are also provided and are compared with known three-dimensional finite element (FE) solutions. A good agreement is observed. The developed method is shown to be an effective and very powerful tool in modeling the crack closure phenomenon.

Keywords Crack closure · Wake of plasticity · Plate thickness effect · Through-the-thickness crack · Distributed dislocation technique · Edge dislocation

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CHAPTER 7

APPROXIMATION OF THE THICKNESS EFFECT ON PLASTICITY-INDUCED FATIGUE CRACK CLOSURE USING FIRST-ORDER PLATE THEORY

STATEMENT OF AUTHORSHIP

Approximation of the Thickness Effect on Plasticity-Induced Fatigue Crack Closure using First-Order Plate Theory.

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Created theoretical models, performed all analyses, interpreted data, and wrote manuscript.

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Approximation of the thickness effect on plasticity-induced fatigue crack closure using first order plate theory

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Abstract:

This paper describes a semi-analytical approach for approximating the effects of plate thickness on plasticity-induced fatigue crack closure. The developed approach is based on application of the distributed dislocation technique with thickness effects being included into the analysis through the use of first-order plate theory. The regions of plastic deformation ahead of the crack tip and the plastic wake are represented by a modified strip-yield model. Numerical results are obtained via Gauss-Chebyshev quadrature and are presented for the size of the tensile plastic zone. Additional results are provided for the cyclic crack tip opening displacement and crack opening stress, which are important parameters frequently used in the correlation and prediction of fatigue crack growth. A very good agreement is observed between the present results for the crack opening stress ratio and through-the-thickness average values from past three-dimensional finite element simulations. The developed approach therefore offers a suitable compromise between simplified plane stress analyses and complex finite element models.

Keywords: Crack opening stress, Distributed dislocation technique, First-order plate theory, Plasticity-induced crack closure, Plate thickness effect, Through-the-thickness crack.

1 Introduction

The fatigue of materials has been widely investigated for many years and has been found to be one of the most common failure mechanisms of engineering structures [1]. There are many factors that influence the fatigue behaviour of a structural member including the nature and magnitude of the applied load, the local geometry, the material properties, residual stresses, the environmental operating conditions, etc. The classic linear elastic approach says that fatigue crack growth rates can be described as a function of the linear elastic stress intensity factor range, $\Delta K = K_{max} - K_{min}$. However, this method is unable to account for many crack growth phenomena, which stem from the above mentioned factors and are present in almost all practical situations. Such examples include the effects of the loading history, plate thickness, crack size and the presence of any stress concentrators.

Considered by many as a landmark event in the study of fatigue crack growth was the discovery of crack closure by Elber [2,3] in the early 1970's. Elber observed that the surfaces of fatigue cracks close together during the unloading process while the remotely applied load is still tensile and can remain closed for a significant part of the load cycle. The closure was attributed [2,3] to crack tip plasticity and the formation of a plastic wake along the faces of the crack as it propagates. A variety of sources for crack closure have since been identified, such as surface roughness, with comprehensive summaries provided in refs [4,5]. The focus of this study, however, is on the investigation of the plasticity-induced crack closure concept. It was proposed by Elber [2,3] that fatigue damage is minimal when the crack is closed and that significant crack growth will only occur when the crack is fully open. He then suggested the use of an effective stress intensity factor range:

$$\Delta K_{eff} = K_{max} - K_{open}, \tag{1}$$

where K_{open} refers to the stress intensity factor at the point when the crack tip just re-opens. It was found that unlike the elastic stress intensity factor range, the effective range, ΔK_{eff}, is able to account for R ratio effects when correlating fatigue crack growth data.

A number of experimental investigations have been undertaken to try and determine the extent of crack closure. The main techniques that have been employed can be divided into two groups, those that measure closure on the specimen surface and those that measure an average value across the thickness. Surface techniques include compliance methods, which utilise surface strain gauges located at various positions along the crack and/or ahead of the crack [3,6,7], and optical techniques like Moiré interferometry [8,9]. On the other hand, the average through-thickness crack closure has been measured by such techniques as the crack mouth opening displacement or back face strain gauge compliance methods [6,7,10], the potential drop method [11] and by acoustic emission [7]. It is well understood, however, that crack closure varies throughout the specimen thickness [12]. In an attempt to investigate this effect, Fleck and Smith [10] used a pushrod compliance technique to provide crack closure measurements at specific locations in the centre of a wide specimen. Overall, experimental studies have shown the importance of accounting for crack closure and that there are many contributing parameters that still need further investigation.

Theoretical analysis of plasticity-induced crack closure by means of analytical or numerical techniques can provide vital insight into the effect of each of the different variables involved. This includes specimen geometry, load ratio effects and material properties to name just a few. A large number of authors [13-19] have made use of the Dugdale [20] strip-yield model in their investigations. In all of these analyses, plastic yielding is assumed to be confined to a thin strip along the line of the crack. This assumption is most applicable when plane stress conditions prevail at the crack tip. Kanninen and Atkinson [21] presented a method for predicting plane strain fatigue crack growth by utilising an inclined strip yield model where the plasticity zones were located along ‘wings’ extending either side of the crack front, i.e. planes of shear.

True crack tip stress fields are three-dimensional (3D) and consequently no two-dimensional, that is plane stress, analysis will be able to fully describe crack closure under general stress states. Finite element methods (FE) have been most widely used to model complex two- and three-dimensional geometry [12,22-25], however, there are many difficulties such as mesh refinement, crack face contact, required computational effort, etc. This has lead to the use of a so-called plastic constraint factor, α, by many researchers to account for plate thickness effects in their simplified models [14-17]. However, linear interpolation or trial and error are usually necessary to determine a value for the constraint factor when considering general stress states other than plane stress. This makes use of the constraint factor rather difficult, as in addition it requires the availability of numerical or experimental data for similar materials, specimen geometry, and load conditions.

The purpose of this study is to present a simple method for the approximation of plate thickness effects on plasticity-induced crack closure. The semi-analytical approach is based on the distributed dislocation technique and a modified strip-yield model. This involves replacing the crack and plastic deformation zones by a continuous distribution of edge dislocations, which are chosen in order to satisfy the same stress and displacement boundary conditions that would be generated by the presence of the actual crack. Thickness effects are included through use of the solution for a through-the-thickness edge dislocation [26,27]. This solution is derived from first-order plate theory and is therefore an approximation to the true 3D case (this is discussed further in the next sections). By directly taking into account thickness effects through the dislocation solution, the need for any empirical thickness correction factors, such as the constraint factor, is eliminated. The developed approach has the simplicity of a two-dimensional plane stress model, while providing an improved prediction of the effects of plate thickness on crack closure.

Detailed first is the procedure for determining the crack opening displacement and plastic stretch curves, for both maximum and minimum applied loads under constant amplitude loading. Numerical results are obtained via Gauss-Chebyshev quadrature and are presented for the tensile plastic zone size, cyclic crack tip opening displacement and the crack tip stress field. In the subsequent section a method for determining the crack opening stress is outlined and results are given for the crack opening stress ratio for a wide range of applied loads and plate thickness to crack length ratios. Results for the crack opening stress ratio as a function of the plate thickness are compared with those from a finite element study. A very good agreement between through-the-thickness average FE values and the present results is found.

2 Through-the-Thickness Cracks in Plates of Finite Thickness

The situation of a through-the-thickness crack in a plate of finite thickness has been under close investigation for many years [24-35]. It is well known that the crack tip stress and strain fields are three-dimensional in nature and vary not only with the applied load and crack length, but also across the plate thickness. Differences in the level of constraint through the plate thickness can lead to curvature of the crack front as it propagates. Furthermore, the intersection of the crack front with the free surface of the plate will introduce an additional geometric discontinuity. All of these factors can create many issues when trying to accurately model 3D through-the-thickness cracks, and can raise questions as to how best include the effects of plate thickness into the analysis.

Past studies into the three-dimensional crack front include the works by Hartranft and Sih [28,29], Bažant and Estenssoro [30], Nakamura and Parks [31], Su and Sun [32], and de Matos and Nowell [24], as only a very small example. Early on it was identified that for surface breaking cracks the corner, or vertex, point where the crack front meets the free surface will create a deviation in the regular square-root singular behaviour [33]. This corner singularity will dominate the stress and strain fields in the region near the free surface while the traditional square-root singularity prevails on the interior of the plate. It has also been suggested (see for example ref [34]) that for surface breaking cracks, in addition to the corner singularity, it is necessary to correctly account for the surface-volume interaction at the plate surface. A FE study of static load increasing crack growth by Sih and Chen [35], accommodated for this interaction by varying the effective yield strength across the plate thickness and by implementing a strain energy density criterion for the crack extension.

Several recent finite element investigations have considered the effect of the corner singularity and crack front curvature on plasticity-induced crack closure [24,25]. It was shown by de Matos and Nowell [24] that the shape of the near surface crack front has only a small influence on the calculated values for the crack opening stress. Although, in their analysis [24] the shape of the crack front was held constant as the crack propagates. Branco et al. [25], however, calculated values for the stress intensity factor across the plate thickness to determine the variation in the crack front curvature each time the crack is extended. Based on this approach, it was found that the crack curvature has greatest effect on the opening stress values calculated on the plate interior and has negligible influence on the surface values.

The practical application of any crack closure model requires the developed techniques to be easy to use, computationally efficient, and readily adaptable to a variety of load and geometry conditions. For this reason, many popular life prediction codes are based on simplified plane stress analyses with the aid of a constraint factor to account for 3D effects (for example STRIPY [15] as used in the NASGRO software, and FASTRAN [36], among others). In the current work it has therefore been chosen to employ the first-order plate theory of Kane and Mindlin [37] to approximate thickness effects on plasticity-induced crack closure. This theory assumes that generalised plane strain conditions exist whereby the out-of-plane displacement is linear in the thickness direction. It is also assumed that the crack front is straight and perpendicular to the crack length, and that the singularity is square-root across the whole crack front. These assumptions are, of course, not physically correct, but will provide for a suitable compromise between modelling simplicity and the actual 3D case. Previous investigations have shown that results from the first-order plate theory are in close agreement with the average through-the-thickness values from FE simulations [38].

3 Formulation of the Distributed Dislocation Technique

3.1 Maximum Applied Load

The approach developed in this study for calculating the effects of plasticity-induced crack closure is an extension of earlier work by the author [18,19,38]. For completeness a brief review is presented here along with a description of the improvements made to the technique. The main modification includes separation of the governing integral equation into individual integrals for each boundary zone. This provides for greater control over the placement of the integration points and zone edges, such as the plastic and closure zones.

We first consider a through-the-thickness crack of length 2a which lies in an infinite plate of thickness 2h and is subjected to the remotely applied mode I stress, $\sigma_{yy}^{\infty}(x)$ (Fig. 1). The size of the tensile, or direct, plastic yield zone produced ahead of the crack tip is given by r_p and the crack tip stretch by δ. In this study a rigid perfect-plastic strip-yield model is employed such that the plastic deformation is confined to an infinitesimal strip along the x-axis. Furthermore, as a first-order estimate it is assumed that the stress components and plastic deformation are constant across the thickness of the plate. It is understood that the strip-yield simplification is most applicable to the case of a plane stress analysis. However, previous studies (including refs [14-18,39]) have already proven the capability of this method for determining the crack opening stresses or crack tip opening displacement which are frequently used in the correlation and prediction of fatigue crack growth in three-dimensional cracked geometries. The techniques developed in this study aim to eliminate the empiricalism involved with determining the constraint factor, as used to account for plate thickness effects [23]. In addition, the developed methods will offer a powerful alternative to FE analyses, which suffer from many issues.

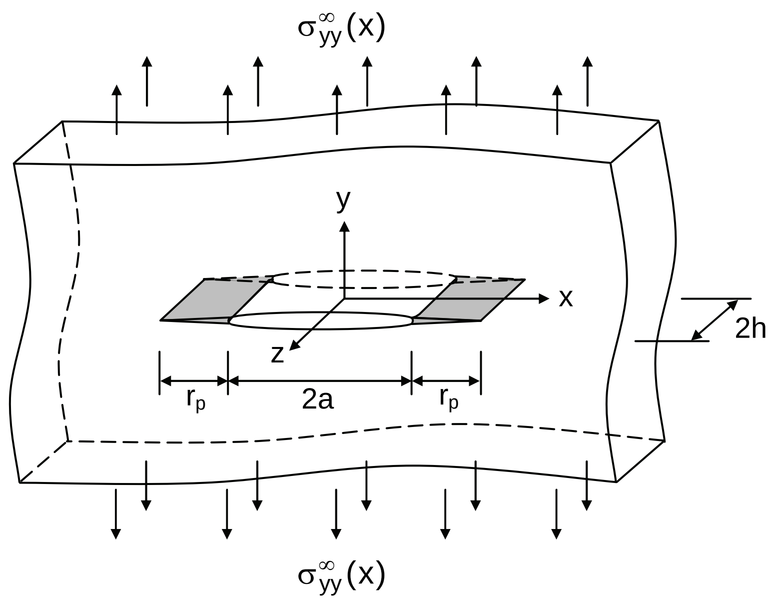


Figure. 1. Through-the-thickness crack under tensile loading.

By replacing the crack and plastic yield zones with a continuous distribution of edge dislocations, and through application of the superposition principle, the following singular integral equation can be produced for the stress field along the x-axis:

$$\sigma(x) = \frac{1}{\pi} \int_{-a-r_p}^{a+r_p} B_y(\xi)\, G(x,\xi)\, d\xi + \sigma_{yy}^{\infty}(x)\,, \tag{2}$$

where $B_y(\xi)$ is the unknown dislocation density function and $G(x, \xi)$, often referred to as the dislocation influence function, is the singular kernel of the system. The influence function,

$G(x, \xi)$, represents the non-dimensional stress at a point x due to a dislocation at the point ξ with a unit Burgers vector in the y-direction. The separation of the crack faces, $g(x)$, is related to the dislocation density function such that:

$$B_y(\xi) = -\frac{dg(\xi)}{d\xi}. \tag{3}$$

The influence functions for a through-the-thickness edge dislocation in a finite thickness plate are given by Kotousov and Wang [26] as:

$$G_{xx}(x,\xi) = \frac{E}{4(1-\nu^2)}\frac{1}{\rho}\left[\frac{4\nu^2}{(\lambda\rho)^2} + (1-\nu^2) - 2\nu^2 K_0(\lambda|\rho|) - \frac{4\nu^2 K_1(\lambda|\rho|)}{\lambda|\rho|}\right] \tag{4}$$

for the x-direction:

$$G_{yy}(x,\xi) = -\frac{E}{4(1-\nu^2)}\frac{1}{\rho}\left[\frac{4\nu^2}{(\lambda\rho)^2} - (1-\nu^2) - 2\nu^2 K_0(\lambda|\rho|) - \frac{2(2+\lambda^2\rho^2)\nu^2 K_1(\lambda|\rho|)}{\lambda|\rho|}\right] \tag{5}$$

for the y-direction and:

$$G_{zz}(x,\xi) = \frac{E}{2(1-\nu^2)}\lambda\nu K_1(\lambda|\rho|)\frac{\rho}{|\rho|} \tag{6}$$

for the z-direction. In the previous equations $\rho = x - \xi$, E is Young's modulus, ν is Poisson's ratio, K_0 and K_1 are modified Bessel functions of the second kind, and the thickness parameter is given by:

$$\lambda = \frac{1}{h}\sqrt{\frac{6}{1-\nu}}. \tag{7}$$

In the limiting cases of plane stress and plane strain the x and y kernels recover the known solutions [40]:

$$G_{xx}(x,\xi) = G_{yy}(x,\xi) = \frac{2\mu}{\kappa+1}\frac{1}{\rho}, \tag{8}$$

where μ is the shear modulus and κ is Kolosov's constant, equal to $(3 - \nu)/(1 + \nu)$ in plane stress and $3 - 4\nu$ in plane strain.

If a Tresca yield criterion is used then, assuming that $\sigma_{yy} \geq \sigma_{xx} \geq \sigma_{zz}$, the stresses in the plastic zone can be written as $\left|\sigma_{yy} - \sigma_{zz}\right| = \sigma_o$, where σ_o is the material's flow stress. The boundary conditions for the governing integral (2) at maximum applied load, $\sigma_{yy}^{\infty}(x) = \sigma_{max}^{\infty}$, therefore become:

$$\left.\begin{aligned} G(x,\xi) &= G_{yy}(x,\xi) \\ \sigma(x) &= \sigma_{yy}(x) = 0 \end{aligned}\right\} \qquad |x| < a, \tag{9}$$

and:

$$\left.\begin{aligned} G(x,\xi) &= G_{yy}(x,\xi) - G_{zz}(x,\xi) \\ \sigma(x) &= \sigma_{yy}(x) - \sigma_{zz}(x) = \sigma_o \end{aligned}\right\} \qquad a \leq |x| \leq a + r_p. \tag{10}$$

When $|x| \geq a + r_p$ the displacement condition $g(x) = 0$ applies and this has already been taken into account in the integral (2).

Symmetry of the crack problem can be utilised to reduce the range of integration to only half of the total 'crack' length, which provides:

$$\sigma(x) = \frac{1}{\pi}\int_0^{a+r_p} B_y(\xi)\left[G(x,\xi) - G(x,-\xi)\right]d\xi + \sigma_{yy}^{\infty}(x). \tag{11}$$

The resulting singular integral equation will be solved numerically via application of Gauss-Chebyshev quadrature. Therefore (11) is split into two separate integrals in order to provide control of the number of integration points that are placed in each of the respective zones along the crack length. The first integral is taken over the half-length of the actual crack and the second over the length of the direct plastic zone, such that:

$$\sigma(x) = \sigma_1(x) + \sigma_2(x) + \sigma_{yy}^{\infty}(x), \tag{12}$$

where:

$$\sigma_1(x) = \frac{1}{\pi}\int_0^a B_y(\xi)\left[G(x,\xi) - G(x,-\xi)\right]d\xi, \tag{13}$$

and:

$$\sigma_2(x) = \frac{1}{\pi}\int_a^{a+r_p} B_y(\xi)\left[G(x,\xi) - G(x,-\xi)\right]d\xi. \tag{14}$$

Separating the integral equations as above allows for exact placement of the edge of the plastic zone, rather then being limited by the location of the integration points [18,19]. This in turn reduces the number of points required to reach convergence, hence improving the efficiency of the technique.

Equations (13) and (14) are then transformed to the range -1 to 1, which gives:

$$\sigma_1(x) = \frac{1}{\pi}\int_{-1}^{1} \overline{B}_y(s_1)\left[\overline{G}(x,s_1) - \overline{G}(x,-s_1)\right]\frac{a}{2}ds_1, \tag{15}$$

where:

$$\xi = \frac{a}{2}(s_1 + 1), \tag{16}$$

and:

$$\sigma_2(x) = \frac{1}{\pi}\int_{-1}^{1} \overline{B}_y(s_2)\left[\overline{G}(x,s_2) - \overline{G}(x,-s_2)\right]\frac{r_p}{2}ds_2, \tag{17}$$

where:

$$\xi = \frac{r_p}{2}(s_2 + 1) + a \,. \tag{18}$$

We now introduce the function $\overline{\phi}(s)$ such that:

$$\overline{B}_y(s) = \overline{\phi}(s)(1+s)^{+1/2}(1-s)^{-1/2}, \tag{19}$$

and through application of Gauss-Chebyshev quadrature the integrals (15) and (17) can each be reduced to a linear series summation in n unknowns, $\overline{\phi}(s_i)$ for i = 1… n. This leads to the following equations:

$$\sigma_1(x) = \frac{a}{2n_1+1}\sum_{i=1}^{n_1}\overline{\phi}_1(s_{1,i})(s_{1,i}+1)\left[\overline{G}(x, s_{1,i}) - \overline{G}(x, -s_{1,i})\right], \tag{20}$$

and:

$$\sigma_2(x) = \frac{r_p}{2n_2+1}\sum_{i=1}^{n_2}\overline{\phi}_2(s_{2,i})(s_{2,i}+1)\left[\overline{G}(x, s_{2,i}) - \overline{G}(x, -s_{2,i})\right], \tag{21}$$

where the integration points are given in both cases as:

$$s_{j,i} = \cos\left(\pi\frac{2i-1}{2n_j+1}\right), \qquad i = 1\ldots n_j, \qquad j = 1, 2. \tag{22}$$

Within the crack and plastic zones the stress functions (20) and (21), respectively, are only valid at the collocation points, which are defined by:

$$t_{j,k} = \cos\left(\pi\frac{2k}{2n_j+1}\right), \qquad k = 1\ldots n_j, \qquad j = 1, 2. \tag{23}$$

making use of the transformations:

$$x_{1,k} = \frac{a}{2}(t_{1,k} + 1), \qquad k = 1 \ldots n_1, \qquad \text{for } 0 < x < a, \tag{24}$$

and:

$$x_{2,k} = \frac{r_p}{2}(t_{2,k} + 1) + a, \qquad k = 1 \ldots n_2, \qquad \text{for } a < x < a + r_p. \tag{25}$$

Outside of these intervals however, that is $x > a + r_p$, (20) and (21) may be evaluated at any point. Analogous conditions apply for $x < 0$.

The unknown functions $\overline{\phi}_j(s_{j,i})$, for $i = 1\ldots n_j$ and $j = 1, 2$, can now be determined by substituting (20) and (21) back into (12) and by enforcing the stress boundary conditions over the length of the crack and direct plastic zone. This gives a total system of $n_1 + n_2$ linear equations in the $n_1 + n_2$ unknowns:

$$\sigma(x_{1,k}) = \sigma_1(x_{1,k}) + \sigma_2(x_{1,k}) + \sigma_{yy}^{\infty}(x_{1,k}), \qquad k = 1\ldots n_1, \qquad \text{for } 0 < x < a, \tag{26}$$

and:

$$\sigma(x_{2,k}) = \sigma_1(x_{2,k}) + \sigma_2(x_{2,k}) + \sigma_{yy}^{\infty}(x_{2,k}), \qquad k = 1\ldots n_2, \qquad \text{for } a < x < a + r_p, \tag{27}$$

where $\sigma(x)$ and $\sigma_{yy}^{\infty}(x)$ are given by the boundary conditions (9) and (10).

The size of the direct plastic zone, r_p, is determined through iteration by first making an initial guess, for example an average of the plane stress and plane strain values. Use is then made of the requirement that the stress at the tip of the plastic zone must be finite, which means that $K_I(a + r_p) = 0$. Here K_I refers to the mode I stress intensity factor and is found from an asymptotic analysis of stress field ahead of the crack tip as:

$$K_I = \sqrt{2\pi r_p}\,\frac{E}{4(1-\nu^2)}\,\bar{\phi}_2(s_{2,1})\ . \tag{28}$$

Figure 2 displays the results from calculations for the normalised direct plastic zone size as a function of the maximum applied stress to flow stress ratio. Curves for various plate thickness to crack length ratios are provided along with the plane stress and plane strain limits. A total of 200 integration points were used to ensure a high level of convergence in the solution over the range of applied loads considered. It can be seen that as $\sigma^{\infty}_{max}/\sigma_o \to 0$ and $\sigma^{\infty}_{max}/\sigma_o \to 1$ plane strain and plane stress conditions prevail, respectively. Further, as $h/a \to 0$, that is as the plate thickness decreases or the crack length increases, the plane stress solution is recovered. Results for the triaxial stress field along the line of the crack are given in Fig. 3 for a crack under remote tensile loading with h/a = 1 and $\sigma^{\infty}_{max}/\sigma_o = 0.6$. The y-stress component is determined via (12), (20) and (21) by setting $G(x, \xi) = G_{yy}(x, \xi)$ for all x. The x- and z- components are obtained in a similar manner by replacing the y-direction kernel, with either (4) or (6), and removing the $\sigma^{\infty}_{yy}(x)$ term. The results show that the assumption of $\sigma_{yy} \geq \sigma_{xx} \geq \sigma_{zz}$ is indeed correct and how the stress components vary over the length of the plastic zone.

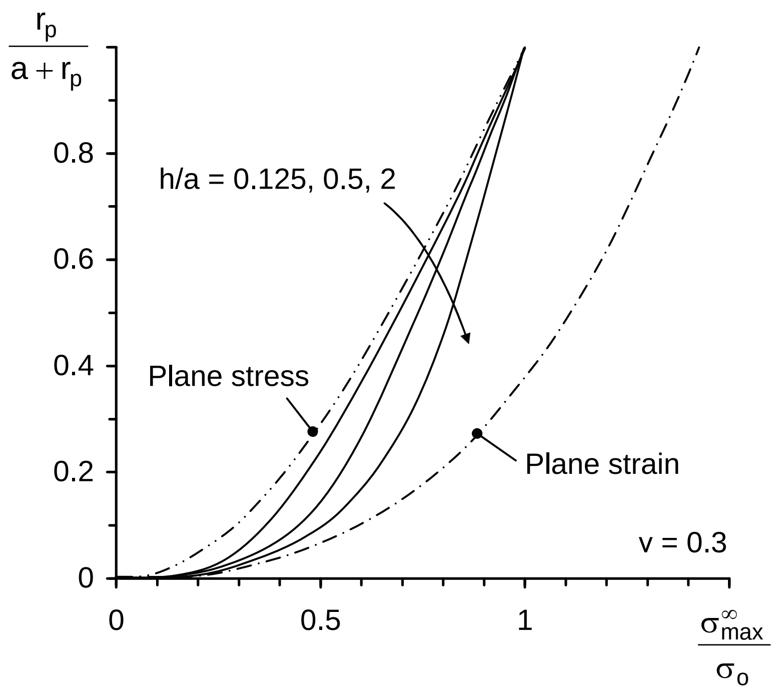


Figure 2. Normalised direct plastic zone size as a function of the maximum applied stress to flow stress ratio.

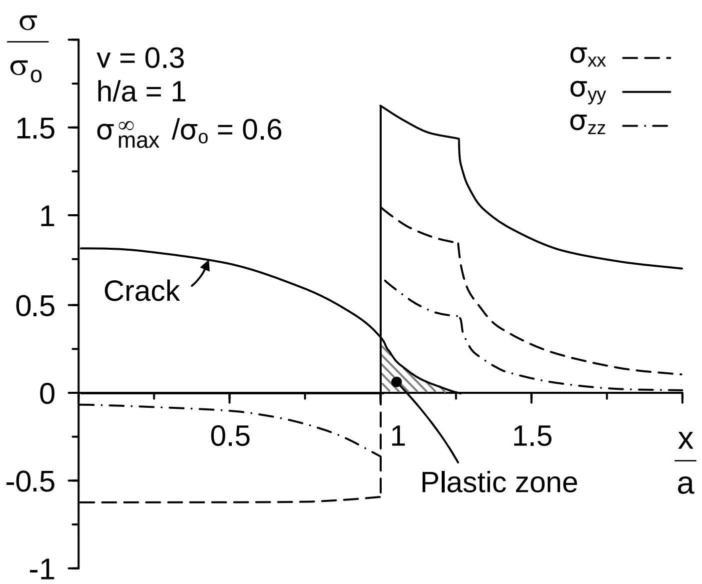
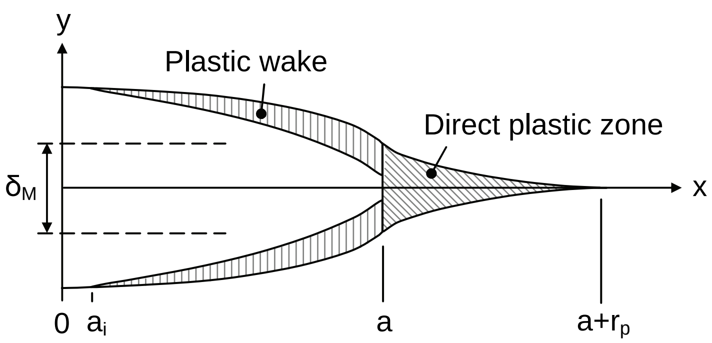


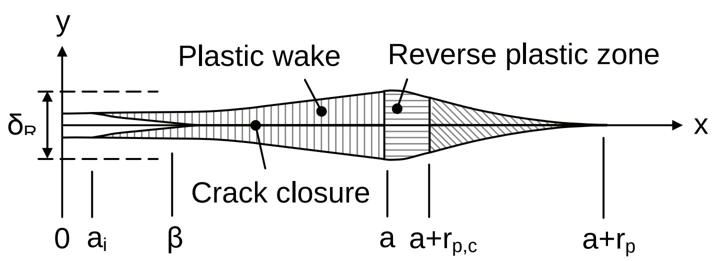
Figure 3. Triaxial stress field along the x-axis for a crack under tensile loading.

3.2 Minimum Applied Load

To investigate the case of minimum applied load it is assumed that the crack has been growing under constant amplitude cyclic loading from an initial crack size of $2a_i$. In order to eliminate the need to manually grow the crack with each load cycle, it is assumed that the thickness of the plastic wake can be described by a linear function of the crack half length, a [19]. This approximation becomes most appropriate once the crack has grown several times its original length, for example $a > 3a_i$. Strictly speaking the linear wake assumption is only applicable to plane stress analyses where the maximum crack tip stretch and the minimum crack tip stretch, with no plastic wake present, are both directly proportional to the crack half length. However, the linear wake idealisation will still give a reasonable estimate of the actual plastic wake in the case of a finite thickness plate and thus provide a means of investigating thickness effects on crack closure. A schematic of a fatigue crack growing with a linear wake at maximum and minimum applied loads is shown in Figs. 4a and 4b, respectively. Here $r_{p,c}$ is the size of the reverse, or cyclic, plastic zone, β is the half-length of the portion of the crack that remains open at minimum load, δ_M is the maximum crack tip stretch and δ_R is the residual crack tip stretch.



a) Maximum applied load, σ^{∞}_{max}



b) Minimum applied load, σ^{∞}_{min}

Figure. 4. Schematic of growing crack at a) maximum and b) minimum applied load.

A similar process to the maximum load case is followed at minimum load, with the singular integral equation (12) now being divided into four intervals along the x-axis, namely 0 to β, β to a, a to a + $r_{p,c}$ and a + $r_{p,c}$ to a + r_p. The y-stresses along the x-axis at minimum applied load can therefore be written as:

$$\sigma(x) = \sigma_3(x) + \sigma_4(x) + \sigma_5(x) + \sigma_6(x) + \sigma_{yy}^{\infty}(x)\,. \tag{29}$$

Through application of Gauss-Chebyshev quadrature to each of the integrals the stress functions therefore become:

$$\sigma_j(x) = \frac{A_j}{2n_j + 1}\sum_{i=1}^{n_j}\bar{\phi}_j(s_{j,i})(s_{j,i}+1)\left[\bar{G}(x, s_{j,i}) - \bar{G}(x, -s_{j,i})\right], \qquad j = 3, 4, 5, 6, \tag{30}$$

for:

$$A_3 = \beta,\; A_4 = a - \beta,\; A_5 = r_{p,c},\; \text{and}\; A_6 = r_p - r_{p,c}\,. \tag{31}$$

The ξ-s coordinate transformations for each of the intervals are given by:

$$\xi = \frac{\beta}{2}(s_3 + 1)\,, \tag{32}$$

$$\xi = \frac{a-\beta}{2}s_4 + \frac{a+\beta}{2}\,, \tag{33}$$

$$\xi = \frac{r_{p,c}}{2}(s_5 + 1) + a\,, \tag{34}$$

and:

$$\xi = \frac{r_p - r_{p,c}}{2}s_6 + \frac{r_p + r_{p,c}}{2} + a\,, \tag{35}$$

where the integration points s_j are given by (22) using the appropriate values for n. As before, when $0 < x < a + r_p$ the stress functions (30) are only valid at the collocation points within each of the various regions. These can be determined by use of (23) and the equivalent x-t transformations based on (32) to (35).

The boundary conditions for minimum applied load can now be derived in the same manner as for the maximum load case. In the present study it is assumed that the tensile flow stress, $\sigma_{o,T}$, and the compressive flow stress, $\sigma_{o,C}$, are of equal magnitude, σ_o. For simplicity any cyclic hardening effects have been ignored. Furthermore, the out-of-plane constraint is removed during compressive yielding and is assumed to occur when $|\sigma_{yy}| = \sigma_o$. This assumption was first suggested by Newman [14] and has been utilised in various subsequent investigations (e.g. refs [15-17]). The physical reasons were not clearly stated in these studies, but described as a “loss of constraint under compression” [36].

In addition to the Bauschinger effect, which leads to non-symmetry of the direct and reverse yielding; another factor contributing to the reduction of the apparent compressive yield stress is the residual deformations left from the positive cycle. This is due to the difference in the Poisson’s ratio of the material subjected to plastic deformation and the surrounding elastic material. The reduction in the apparent yield stress at reverse loading can also be observed in past experimental work [41] and numerical simulations of the problem [22], which show a decrease in the crack opening stress occurs with an increase in plate thickness. This corresponds to an increase in the relative size of the yield zone produced under compressive loading compared to that of the maximum tensile yield zone. That is, there is an increase in the out-of-plane constraint during tensile yielding without the same level of increase in constraint during compressive yielding. The reverse yielding phenomenon is very complicated and affected by many factors. The assumptions made in the current work, regarding the reverse yielding, are therefore considered to be appropriate based on the numerical results as well as the previous attempts to model the cyclic wake of plasticity using the Dugdale model.

The boundary conditions at minimum load, $\sigma^{\infty}_{yy}(x) = \sigma^{\infty}_{min}$, are:

$$\left.\begin{aligned} &G(x,\xi) = G_{yy}(x,\xi), \\ &\sigma(x) = 0, \end{aligned}\right\} \quad |x| < \beta, \tag{36}$$

$$\frac{dg(x)}{dx} = \frac{\delta_R}{a - a_i}, \qquad \beta \le |x| < a, \tag{37}$$

$$\left.\begin{aligned} &G(x,\xi) = G_{yy}(x,\xi), \\ &\sigma(x) = -\sigma_o, \end{aligned}\right\} \quad a \le |x| < a + r_{p,c}, \tag{38}$$

and:

$$\frac{dg(x)}{dx} = \frac{dg_{max}(x)}{dx}, \qquad a + r_{p,c} \le |x| \le a + r_p. \tag{39}$$

where the subscript max refers to the initial, or maximum load, configuration. Once again $g(x) = 0$ for $|x| \ge a + r_p$. The displacement gradient conditions (37) and (39) can be applied via the relations (3) and (19). It should be noted that even for $R < 0$ the conditions (36) and (37) still remain true, when $a_i = 0$, in order to prevent the discontinuity in the slope of the wake, and hence singular stresses, that would occur at $x = 0$ if (36) was removed (i.e. $\beta = 0$). Similarly, when $a_i > 0$ and for significantly large magnitudes of R, $R < 0$, we need to introduce an extra boundary condition along $|x| < \gamma$, where γ is less then a_i, with the requirement $g(x) = 0$. The condition (36) would still apply over the range $\gamma < |x| < \beta$, therefore preventing any discontinuity at $|x| = a_i$. Full details on the analysis, however, are not included here.

Solution to the problem follows by substituting the set of equations (30) into (29), and making use of the boundary conditions (36) through (39), to provide a system of $n_3 + n_4 + n_5 + n_6$ linear equations in the $n_3 + n_4 + n_5 + n_6$ unknowns. The size of the reverse plastic zone, $r_{p,c}$, and the region of non-closure, β, are determined through an iterative procedure using the requirement that the slope of the deflection/plastic stretch curve must be continuous between each of the integration ranges. This can be achieved simply by minimising and maximising the residual crack tip stretch, δ_R, to give $r_{p,c}$ and β, respectively. As before an initial guess is necessary for each of the unknowns including δ_R, which then converges very rapidly through back substitution into the boundary condition (37). The crack opening displacement and plastic stretch curves, g(x), at maximum and minimum applied loads, and therefore δ_M and δ_R, can be determined by numerical integration of (3) making use of (19). The simple trapezium rule is usually sufficient.

An important parameter often utilised in the correlation of fatigue crack growth rates is the cyclic crack tip opening displacement [39] defined by:

$$\Delta\delta = \delta_M - \delta_R \,. \tag{40}$$

Simulations were undertaken for a range of applied loads and R ratios as well as for various plate thickness to crack length ratios. In these calculation the initial crack length was taken as $a_i = 0$. Figs. 5 and 6 give the results for the normalised cyclic crack tip opening displacement as a function of the maximum applied stress to flow stress ratio and the load ratio, respectively. From the results in Figs. 5 and 6 it can be seen that the plate thickness to crack length ratio has decreasing effect as the maximum applied load ratio decreases or the R ratio increases. For medium to large values of $\sigma^{\infty}_{max}/\sigma_o$ or for negative R ratios, however, the effect of plate thickness is quite significant. Fig. 7 provides results for the triaxial stress field along the line of the crack for the case of minimum applied load. For these calculations the values h/a = 1, $\sigma^{\infty}_{max}/\sigma_o$ = 0.6 and R = -0.2 where used.

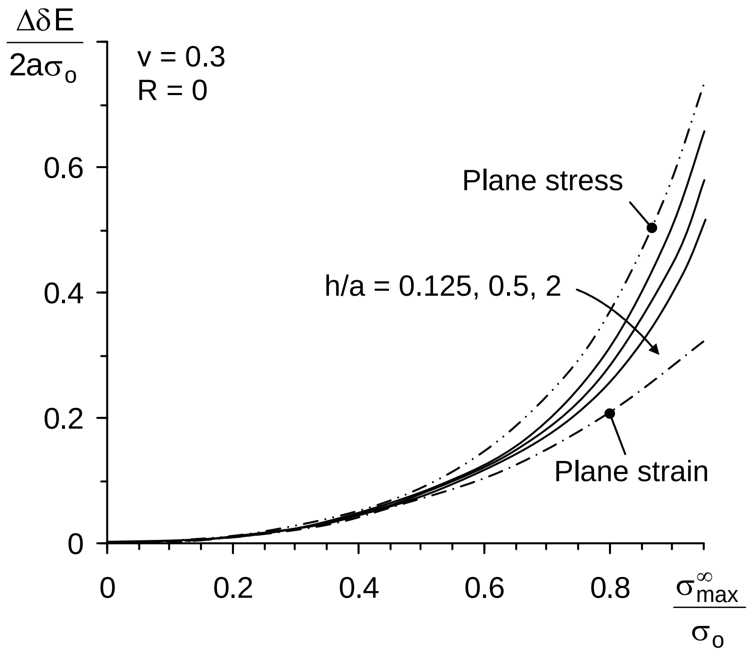


Figure 5. Normalised cyclic crack tip opening displacement as a function of the maximum applied stress to flow stress ratio.

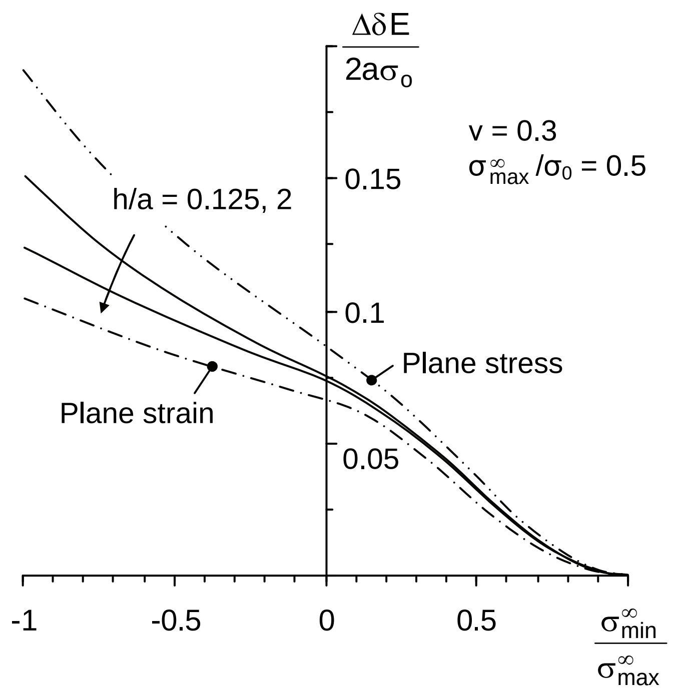


Figure 6. Normalised cyclic crack tip opening displacement as a function of the load ratio.

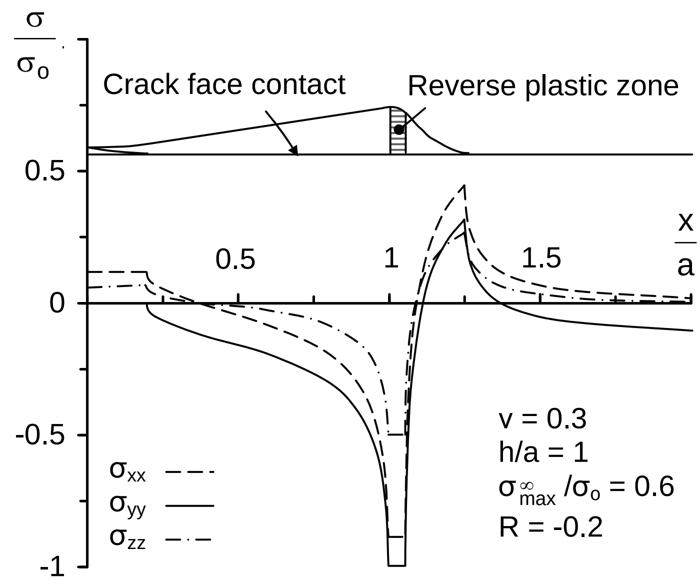


Figure 7. Triaxial stress field along the x-axis at minimum applied load.

4 Crack Opening Stress

The effective stress intensity factor range has become a very important concept in the analysis fatigue crack growth. Under this approach it is assumed that the fatigue crack growth rate is a function of the effective stress intensity range rather then the traditional linear elastic stress intensity range. Many researchers have embraced this theory and used it in the correlation and prediction of fatigue crack growth (see for example ref [14]). The effective stress intensity factor range, as given by (2), can be written in terms of the effective stress range such that:

$$\Delta K_{eff} = \Delta\sigma_{eff} Y \sqrt{\pi a}, \qquad (41)$$

where:

$$\Delta\sigma_{eff} = \sigma^{\infty}_{max} - \sigma^{\infty}_{open}, \qquad (42)$$

and Y is a function specific to the cracked geometry of the component and σ^{∞}_{open} is the remotely applied stress at the point when the crack just re-opens. The effective stress range may also be defined in terms of the closure stress, σ^{∞}_{close}, although only the former definition is considered here.

The distributed dislocation techniques developed in section 2 can now be applied to determine the crack opening stress. The boundary conditions for the singular integral (11) at the point of crack opening, $\sigma^{\infty}_{yy}(x) = \sigma^{\infty}_{open}$, therefore become:

$$\left.\begin{aligned} G(x,\xi) &= G_{yy}(x,\xi), \\ \sigma(x) &= 0, \end{aligned}\right\} \qquad |x| < a, \tag{43}$$

and:

$$\frac{dg(x)}{dx} = \frac{dg_{min}(x)}{dx}, \qquad a \leq |x| \leq a + r_p. \tag{44}$$

Results for the ratio of the crack opening stress to maximum stress are given in Figs. 8 and 9 as a function of the normalised direct plastic zone size and load ratio, respectively. The plane stress and plane strain limits are provided along with curves for several plate thickness to crack length ratios. For these calculations the initial crack length was again $a_i = 0$. The graphs show that as $\sigma^{\infty}_{max}/\sigma_o \to 1$ the plane stress solution is recovered. It can be seen that for a given crack length, the crack opening stresses decrease with an increase in the plate thickness. This is in agreement with the general trends observed in crack closure experiments [41]. Conversely, for a constant plate thickness the crack opening stresses will increase with an increase in crack length. As $\sigma^{\infty}_{max}/\sigma_o \to 0$ and $h/a \to 0$ the crack opening stress ratio does recover the plane stress solution, though this is not visible for the range of h/a shown in Fig. 8. Overall the opening stress ratio is quite high, indicating to the significance of accounting for plasticity-induced crack closure in fatigue analyses. As the load ratio decreases, however, the effects of plasticity-induced closure become less prevalent (Fig. 9).

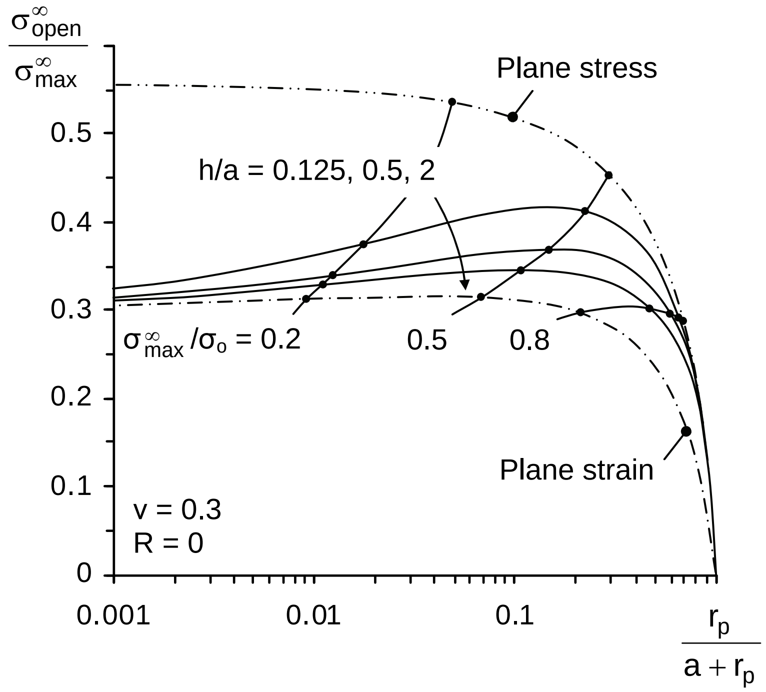


Figure 8. Opening stress ratio as a function of the normalised direct plastic zone size.

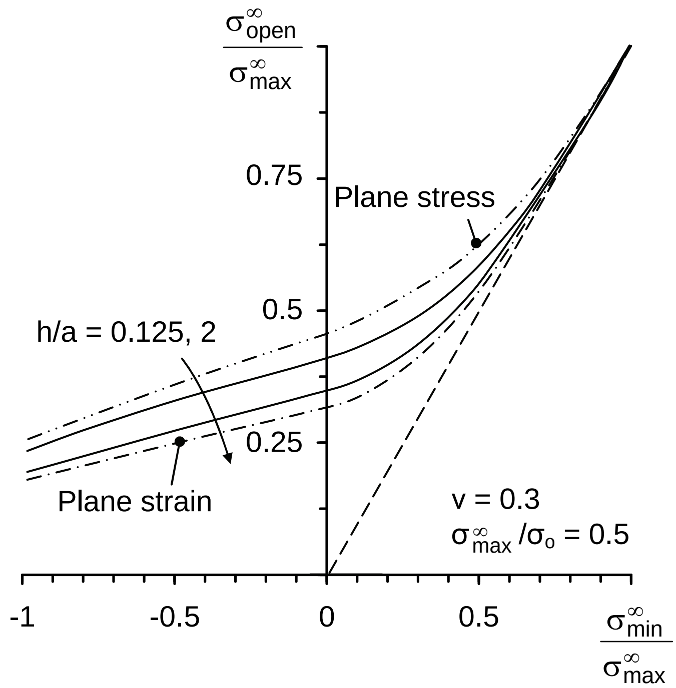


Figure 9. Opening stress ratio as a function of the load ratio.

In Fig. 10 results for the crack opening stress ratio as a function of plate thickness are compared to those from a three-dimensional FE investigation by Chermahini et al. [22]. Here the plate thickness has been normalised by maximum plate thickness, H = 101.6 mm. To provide for a better comparison the crack length and plate thickness as used in the current analysis, a crack in an infinite plate, were adjusted in order to keep the crack tip stress intensity factor and h/a ratio the same as that for the FE model, a centre cracked tension specimen. In addition, the final crack half length has been implied from ref [22] and set as a = 20.6 mm. A very good agreement can be seen between the present results and the average through-the-thickness FE values. The variation between the results can be explained by the different modelling assumptions made in the present analysis compared to the FE investigation.

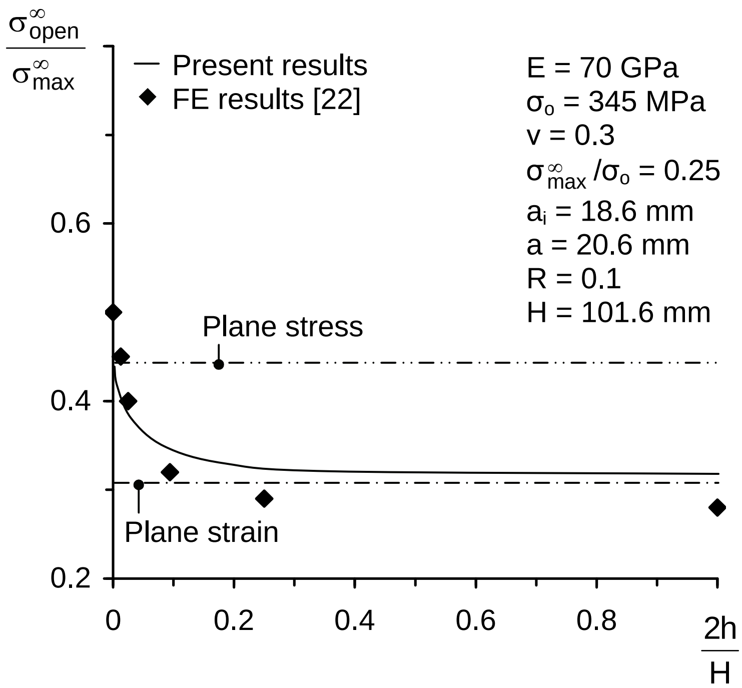


Figure 10. Crack opening stress ratio as a function of the normalised plate thickness.

As a final demonstration of the developed procedures, the crack opening stress results obtained from the current work are used to correlate experimental fatigue crack growth data. Fig. 11 displays best-fit lines to the results from constant amplitude fatigue tests of aluminium alloy (LY12-CZ) compact tension specimens for two different plate thicknesses [39]. In Fig. 11a the results are plotted against the elastic stress intensity factor range, while in Fig. 11b the effective stress intensity range is used. Figure 11a shows that the crack growth rates are higher for the thicker specimen, which agrees with the observation that the effects of crack closure decrease with increasing plate thickness. When plotted against the effective stress intensity range the crack growth data for the two specimens, collapse towards a single unique curve. This provides support for the use of plasticity-induced crack closure and baseline crack growth rate curves in predicting fatigue crack growth. Further, it shows that thickness effects can be considered without the need for empirical curve fitting via the plastic constraint factor.

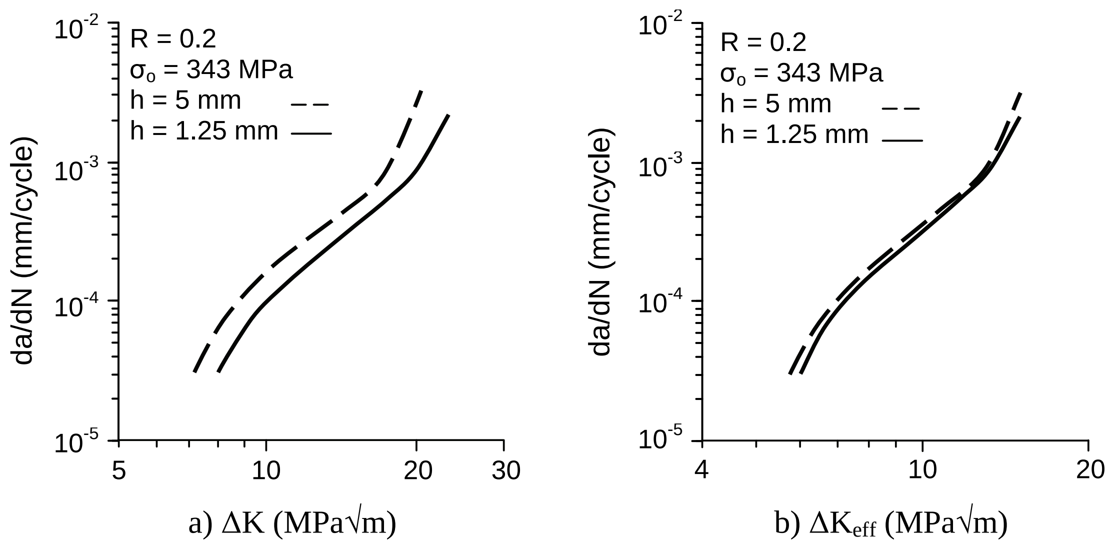


Figure 11. Fatigue crack growth data for LY12-CZ aluminium alloy compact tension specimens [39] plotted against a) the elastic stress intensity range and b) the effective stress intensity range.

5 Conclusions

This paper describes the development of a semi-analytical approach for approximating the effects of plasticity-induced crack closure in plates of finite thickness. The distributed dislocation technique has been utilised to model the crack and plastic yield zones via application of the strip-yield assumption. The developed approach was used to investigate crack closure in stable growing fatigue cracks under constant amplitude loading. Boundary conditions for the governing singular integral were described for the cases of maximum and minimum loads in the load cycle as well as for the crack opening load. To improve the efficiency of accuracy of the numerical solution, the governing integral equation was separated into several intervals over the length of the crack according to the prescribed boundary conditions. Numerical results were then obtained via application of Gauss-Chebyshev quadrature.

Results for the size of the direct plastic zone were presented for a range of plate thickness to crack length ratios. It was shown that as $\sigma_{max}^{\infty}/\sigma_o \to 0$ and $\sigma_{max}^{\infty}/\sigma_o \to 1$ plane strain and plane stress conditions prevail, respectively. In addition, as expected, the plate thickness to crack length ratio increases the solution tends towards plane strain and as the ratio decreases the solution tends towards plane stress. Further results were provided for the cyclic crack tip opening displacement and the crack opening stress ratio for a wide range of load conditions. Similar trends to that of the direct plastic zone were noted in regards to the effect of plate thickness. A comparison of the crack opening stress ratio as a function of plate thickness with finite element values displayed a very good agreement. It was also shown that the crack opening stresses decrease with an increase in plate thickness, which corresponds with previous experimental observations. Results from the developed methods were then used to collapse fatigue crack growth rate data for different plate thicknesses onto a single baseline curve.

The cyclic crack tip opening displacement and crack opening stress ratio are important parameters regularly used in the correlation and prediction of fatigue crack growth. The main techniques that are currently employed to account for plate thickness effects on crack closure are based on finite element analysis or the use of a so-called plastic constraint factor, or both. These methods, however, have many limitations as discussed in detail throughout this paper. In particular FE methods require a great deal of computational time and effort, which makes them unsuitable for most practically important situations. The developed techniques therefore offer a powerful alternative method for determining the various parameters used in fatigue analyses, such as the effective stress intensity factor or cyclic crack tip opening displacement.

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CHAPTER 8

A CRACK CLOSURE MODEL OF FATIGUE CRACK GROWTH IN PLATES OF FINITE THICKNESS UNDER SMALL-SCALE YIELDING CONDITIONS

STATEMENT OF AUTHORSHIP

A Crack Closure Model of Fatigue Crack Growth in Plates of Finite Thickness under Small-Scale Yielding Conditions.

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Codrington, J. (Candidate)
Created theoretical models, performed all analyses, interpreted data, and co-wrote manuscript.

Signed ________________ Date _________________

Kotousov, A. (Supervisor)
Supervised development of work, participated in discussions of work, assisted in data interpretation, and co-wrote manuscript.

Signed ________________ Date _________________

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A crack closure model of fatigue crack growth in plates of finite thickness under small-scale yielding conditions

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Abstract:

Crack growth rates are significantly affected by the thickness of the specimen when all other parameters are kept constant. A quantitative estimation of the thickness effect is thus necessary to make predictions of crack growth rates more accurate and reliable. For this purpose a theoretical model was developed based on the strip-yield assumption and first-order plate theory. No empirical or fitting parameters were used in this work unlike some previous studies. The theoretical values obtained for the normalised load ratio parameter, U, were employed to describe experimental data, obtained under small-scale yielding conditions, at various load ratios and plate thicknesses. Such a representation considerably narrowed the scatter in the crack growth rates versus the effective stress intensity factor range, ΔK_{eff}, demonstrating the potential of the theoretical model.

Keywords**:** Crack opening stress, Fatigue crack growth rate, Plasticity-induced crack closure, Small-scale yielding, Thickness effect.

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Mechanics of Materials, v. 41 (2), pp. 165-173

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This publication is included on pages 141-164 in the print copy of the thesis held in the University of Adelaide Library.

It is also available online to authorised users at:

http://dx.doi.org/10.1016/j.mechmat.2008.10.002

CHAPTER 9

CRACK GROWTH RETARDATION FOLLOWING THE APPLICATION OF AN OVERLOAD CYCLE USING A STRIP-YIELD MODEL

STATEMENT OF AUTHORSHIP

Crack Growth Retardation following the Application of an Overload Cycle using a Strip-Yield Model.

Submitted to *Engineering Fracture Mechanics*.

Codrington, J. (Candidate)
Created theoretical models, performed all analyses, interpreted data, and wrote manuscript.

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Kotousov, A. (Supervisor)
Supervised development of work, participated in discussions of work, and manuscript evaluation.

Signed ________________ Date ________________

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Crack growth retardation following the application of an overload cycle using a strip-yield model

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Abstract:

Experiments have shown that the application of an overload cycle can act to retard crack growth and even potentially lead to crack arrest. This paper describes a new method for investigating crack growth after the application of an overload cycle under plane stress conditions. The developed method is based on the concept of plasticity-induced crack closure and utilises the distributed dislocation technique and a modified strip-yield model. The present results are compared to previous experimental data for several materials. A good agreement is found, with the predictions showing the same trends in the various stages of post-overload crack growth.

Keywords: Crack growth retardation, Distributed dislocation technique, Fatigue crack closure, Overload effect, Plasticity-induced crack closure.

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CHAPTER 10

ON THE EFFECT OF SPECIMEN THICKNESS ON POST-OVERLOAD FATIGUE CRACK GROWTH

STATEMENT OF AUTHORSHIP

On the effect of Specimen Thickness on Post-Overload Fatigue Crack Growth.

Submitted to *International Journal of Fracture (Letters in Fracture and Micromechanics).*

Codrington, J. (Candidate)

Created theoretical models, performed all analyses, interpreted data, and wrote manuscript.

Signed ________________ Date _________________

NOTE:
Statements of authorship appear in the print copy of the thesis held in the University of Adelaide Library.

On the effect of specimen thickness on post-overload fatigue crack growth

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Abstract:

Specimen thickness can have a profound effect on the fatigue crack growth following the application of an overload cycle. This short paper presents a modified strip-yield model for determining the effects of specimen thickness based on the concept of plasticity-induced crack closure. Comparisons are made with experimental data for the case of a single tensile overload cycle applied under otherwise constant ΔK loading. Theoretical results for the crack growth and growth rates as well as the calculated number of delay cycles, are found to be in good agreement with the measured values.

***Keywords*:** Fatigue crack growth, Overload retardation, Plasticity-induced crack closure, Specimen thickness.

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CHAPTER 11

THEORETICAL BOUNDS FOR THE PREDICTION OF SMALL FATIGUE CRACK GROWTH EMANATING FROM SHARP NOTCHES

STATEMENT OF AUTHORSHIP

Theoretical Bounds for the Prediction of Small Fatigue Crack Growth Emanating from Sharp Notches.

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Codrington, J. (Candidate)
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Theoretical bounds for the prediction of small fatigue crack growth emanating from sharp notches

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Abstract:

In this paper, plane stress and plane strain prediction bounds are presented for the fatigue crack growth of small cracks emanating from sharp notches. The theoretical methods are based on a modified strip-yield model and the distributed dislocation technique. Results for the fatigue crack growth are compared with past experimental data for a range of materials under constant amplitude loading. The trends observed in the predictions match those of the experiments. In all of the cases considered the plane stress and plane strain curves provide upper and lower bounds for the experimental values.

Keywords: Plasticity-induced crack closure, Fatigue crack growth, Sharp notches, Small cracks, Strip-yield model.

1 Introduction

The initiation of fatigue cracks in structural components usually occurs at stress concentrators like holes or notches. Such geometrical features may be part of the component design (bolt holes, key ways, weld toes, etc) or could be defects introduced during manufacturing (welding defects, surface scratches, etc). These stress concentrators have a significant influence on the early stages of fatigue crack growth and therefore need to be incorporated into predictions made for the service-life of a component. Numerous experiments [1-9] have found that small cracks emanating from holes and notches tend to propagate at much faster rates in comparison to long established cracks under the same applied loading. In addition, small cracks can grow at an applied elastic stress intensity factor range, ΔK, well below that of the long crack threshold, ΔK_{th}. Typical small crack growth from a notch starts at a growth rate higher than would be predicated by long crack theory. The growth rate then decreases with increasing ΔK to a minimum value, which is still normally above the long crack growth rates. With further propagation the crack growth rate increases from the minimum towards the traditional long crack values.

Research into the behaviour of small cracks has provided a range of possible explanations and governing mechanisms. In general, these studies can be divided into two groups; those relating to metallurgical or microstructural effects and those relating to plasticity effects. During the initial stages of growth, when the crack size is comparable to the microstructure, factors such as the local grain orientation, small particles and inclusions will have a great influence [10,11]. As the crack increases in size, however, these effects will become less significant and the growth rate will be an average over several grain lengths. Use can be made of this to provide a further description of the different stages of small crack growth, namely *microstructurally small* (< ~10μm) and *physically small* (~10μm – 1mm). The exact ranges will of course vary with the material and microstructure under consideration. Other descriptions have also been suggested including mechanically and chemically small cracks [12]. The remaining discussions in this paper will focus on physically small cracks where the crack length is greater than any significant microstructural features.

For fatigue cracks growing from notches, one of the contributing factors to the differences observed in growth rates between small and long cracks is due to the notch plasticity. In the uncracked body a field of plastic deformation will surround a small area ahead of the notch root. Initially, the crack will be entirely within the notch plastic zone and the classic linear elastic approach to fatigue becomes invalid. Linear elastic fracture mechanics relies on the length scale of any plastic deformation being far less than that of the propagating crack. This approach is generally suited to long cracks where the crack tip will be far from any influence of the notch stress/strain field and plastic zone. An intermediate stage therefore exists when the crack tip has left the notch plastic zone, but is still within the notch elastic field.

The majority of the models presented in the literature for describing small crack behaviour are based on plasticity effects in one way or another, though the chosen parameter may vary. For example, Hammouda et al. [14] suggested that the crack growth rate was a function of the total (elastic and plastic) shear deformation at the crack tip On the other hand, Smith and Miller [14] related the variations in crack growth to the relative contribution of the crack tip and notch plasticity. More recently, Hammouda et al. [15] used finite element simulations to derive a parameter based on the combined extents of the monotonic and cyclic plastic zones as well as an equivalent length term. An alternative approach, which has generated some interest stems from the notion that within the notch plastic zone the crack should be under strain control. This has lead to the use of strain based intensity factors [1,4] as well as the cyclic J-integral, ΔJ [16].

All of these different methods have shown varying degrees of success in predicting and correlating small crack growth data with long crack results. There are also some discrepancies between the models as to whether the accelerated growth rates of small cracks, compared to long crack data, still occur once the crack has left the notch plastic zone. Experiments [4] have shown that the transition from the notch affected growth to normal long crack growth occurs when the length of the crack has reached the end of the original notch plastic zone. Though many other experiments (e.g. refs [3,6,9]) have found that accelerated crack growth still occurs outside the notch plastic zone.

The most noteworthy mechanism for small crack behaviour involves the use of the plasticity-induced crack closure concept [17,18]. This approach employs a modified linear elastic stress intensity factor range, ΔK_{eff}, which only considers the part of the load cycle when the crack tip is fully open. Plasticity-induced crack closure is based on the idea that any residual plastically deformed material ahead of the crack tip will remain on the crack faces. This will alter the amount of time for which the crack tip is fully open. A newly initiated crack will have a limited growth history and thus a significant plastic wake will not yet be developed. Small cracks will therefore grow at faster rates than long cracks where the plastic wake is fully developed. The use of crack closure can explain a number of small crack phenomena including the accelerated growth rates outside of the notch plastic zone [3], crack arrest [2], as well as the higher growth rates observed in small cracks that have initiated in the absence of a notch [11]. A number of theoretical crack closure models have since been produced, which allow for the prediction and correlation of small fatigue crack growth [19-21].

Plastic yielding at a notch root or crack tip is highly dependent on the local stress state, which in turn is a function of the specimen or component thickness. For example, a very thin plate or shell component will be largely under a state of plane stress. As the thickness increases the stress state will transition towards more plane strain conditions. In addition, for thicker components, the stress state will vary across the thickness with plane stress conditions at the free surfaces and plane strain on the interior. This has a considerable effect on the growth of fatigue cracks, be they small or long, as fatigue damage is closely associated with the extent of cyclic plastic deformation at the crack tip. The influence of plate thickness on fatigue crack growth rates can be easily rationalised through use of the plasticity-induced crack closure concept [22,23].

The purpose of this paper is to present theoretical bounds for the prediction of small fatigue crack growth from sharp notches. Past work by the authors [23] has shown that the experimental scatter observed due to specimen thickness lies within the plane stress and plane strain prediction bounds. This work [24] involved use of a finite thickness strip-yield model to study the effects of plate thickness on plasticity-induced crack closure. An extension of the developed methods for variable amplitude [24] loading will be used here to investigate small crack behaviour. Results for the crack opening stress ratio are presented as a function of the crack growth. The developed methods are then used to predict the fatigue crack growth from a sharp notch and the results are compared with the experimental data of Shin and Smith [5,6].

2 Strip-yield Model for the Crack Opening Stress

To investigate the small crack phenomenon we will utilise the well established concept of plasticity-induced fatigue crack closure. This approach requires the determination of the effective stress intensity factor range, ΔK_{eff}, as a function of the crack propagation. The effective stress intensity range is given by:

$$\Delta K_{eff} = K_{max} - K_{open} = (\sigma_{max} - \sigma_{open}) Y \sqrt{\pi a}\,, \tag{1}$$

where K_{max} is the maximum applied stress intensity factor, K_{open} is the stress intensity factor at which the crack tip just starts to re-open, σ_{max} and σ_{open} are the analogous stress values, respectively, Y is a geometry correction factor and a is the crack length. In this study, the crack opening stresses will be determined by employing an earlier developed plasticity-induced crack closure model [23,24]. These methods are based on a modified strip-yield model and the distributed dislocation technique for central cracks under both small and large-scale yielding conditions.

In the case of a small crack emanating from a long and narrow notch in a large plate, the notch geometry can be well approximated by an edge crack of the same length (as depicted in Figs. 1a and 1b). Here, the notch depth is given by c, the crack length by a, $r_{p,N}$ is the length of the original notch plastic zone, and σ_{app} is the remotely applied cyclic stresses. Through use of the distributed dislocation technique the notch, crack and plastic zone are replaced with an array of dislocations (Fig. 1b) chosen to satisfy the necessary stress and displacement boundary conditions. Only the case of a straight mode I crack is considered. Furthermore, no initial plastic wake is assumed to exist along the notch length as this would contradict the small crack hypothesis. It is also possible to create the exact notch and crack geometry by using dislocations to ‘cut’ any desired shapes [25], though for simplicity this is not considered here.

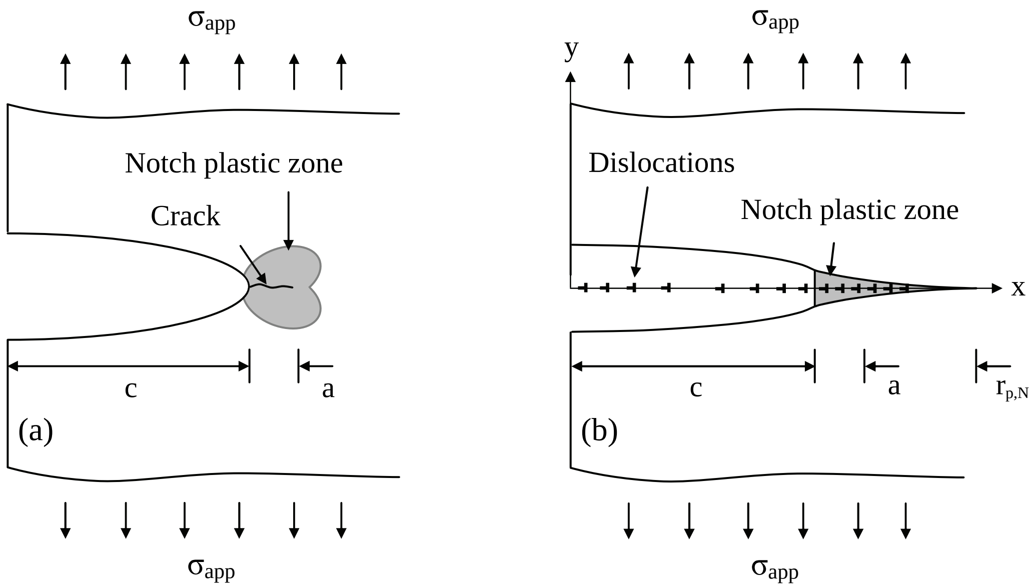

Fig. 1. Schematic of a) small fatigue crack growth from notch and b) simplified dislocation representation.

Full details of the developed techniques have already been outlined elsewhere [23,24] and only the modifications for small crack growth will be mentioned here. By replacing the notch and crack geometry with edge dislocations, the resultant stress along the y-axis can be obtained via:

$$\sigma_{yy}(x) = \frac{E}{4\pi D}\int_0^A B_y(\xi)\, K(x;\xi)\, d\xi + \sigma_{app}(x), \tag{2}$$

where E is Young's modulus, D = 1 for plane stress and D = 1 – ν^2 for plane strain, ν is Poisson's ratio, $\sigma_{yy}^{\infty}(x)$ is the remotely applied mode I stresses, and $B_y(\xi)$ is the dislocation density function. For the case of an edge crack, the plane stress/strain influence function K(x; ξ) is given by [26]:

$$K(x;\xi) = \frac{1}{x-\xi} - \frac{1}{x+\xi} - \frac{2\xi}{(x+\xi)^2} + \frac{4\xi^2}{(x+\xi)^3}. \tag{3}$$

Initially, in equation (2) the ‘imaginary’ crack length, A, is given by the notch length plus notch plastic zone, that is $A = c + r_{p,N}$. As the crack propagates the imaginary crack length will become the sum of the notch length, small crack length and crack tip plastic zone, r_p (i.e. $A = c + a + r_p$).

The unknown function $B_y(\xi)$ is obtained through application of Gauss-Chebyshev quadrature as well as through direct placement of the edge dislocations (see ref [24]). For each applied load, e.g maximum, minimum and opening load, appropriate stress or displacement boundary conditions are assigned depending on whether or not the crack will be open/closed, and whether there will be tensile/compressive yielding. A Tresca yield criterion and strip-yield model are used to determine the distribution of plastic deformation in the notch and crack tip plastic zones. The variation in opening stress with crack length is found by incrementally growing the crack based on the applied loading history. In this current investigation we only consider the case of constant amplitude loading such that:

$$\Delta\sigma = \sigma_{max} - \sigma_{min} = \sigma_{max}(1 - R), \tag{4}$$

where σ_{min} is the minimum applied stress in the load cycle and R is the load ratio.

Results for the variation in the opening stress ratio, $\sigma_{open}/\sigma_{max}$, as the crack propagates are shown in Fig. 2. These values are for plane stress conditions and are given as a function of the maximum applied stress to flow stress ratio, σ_{max}/σ_o, (Fig. 2a) and as a function of the load ratio, R (Fig. 2b). The crack length, a, has been normalised by the initial notch plastic zone size, $r_{p,N}$. It can be seen that as the crack length increases the crack opening stress also increases towards a steady state value. This behaviour is as expected since there will be a gradual build up of residual plastic material on the newly formed crack faces. The results for the transient behaviour under plane strain conditions show the same trends as for the plane stress state. Thus detailed results are omitted.

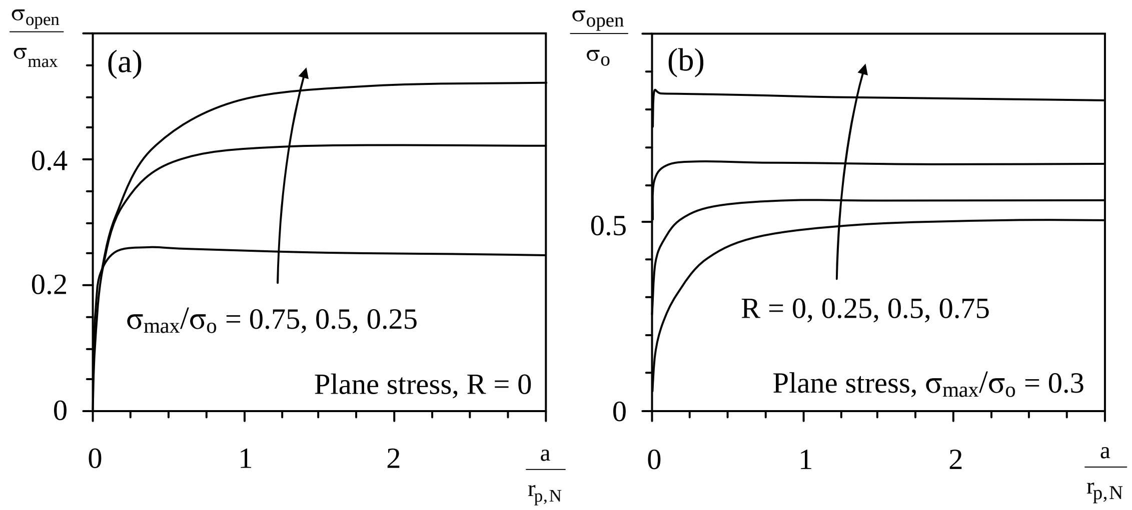

Fig. 2. Variation in the plane stress crack opening stress ratio as a function of a) the maximum applied stress to flow stress ratio and b) the load ratio.

A summary of the steady state values for plane stress and plane strain, and hence the bounds for the finite thickness case, are provided in Figs. 3a and 3b. These results show that for low to medium values of σ_{max}/σ_o fatigue cracks will grow faster in thicker specimens where the stress state is nearer to plane strain. This is in agreement with various experimental studies [22]. Figure 3a also implies that there is a reversal of this behaviour at $\sigma_{max}/\sigma_o \sim 0.7$, although this needs further investigation.

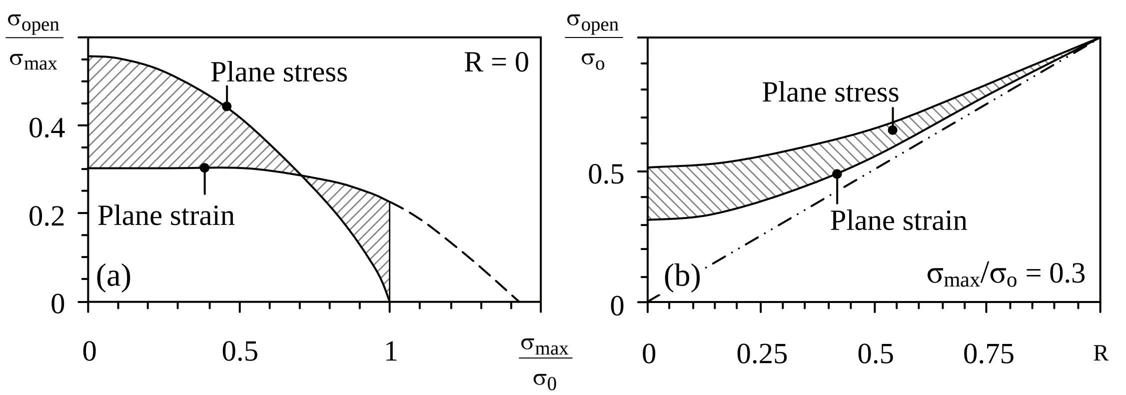

Fig. 3. Bounds for the steady state crack opening stress ratio as a function of a) the maximum applied load to flow stress ratio and b) the load ratio.

3 Fatigue Crack Growth from a Sharp Notch

We will now consider the experimental results of Shin and Smith [5,6] for the fatigue crack growth from sharp notches. The first comparisons will be made with data from double edge notched AISI 316 stainless steel specimens, which are 2.6 mm in thickness. Elliptical notches of a length of 35 mm were spark-machined into the specimens for several notch root radii. The results compared to in this study are for the notch root radius of 0.4 mm. The material properties for the stainless steel are a yield stress of 256 MPa and an ultimate strength of 574 MPa. Tests were conducted under constant amplitude loading with a load ratio of $R = 0.05$.

The theoretical predictions are made using the Paris law and the effective stress intensity factor range as given by (1). The crack opening stress values were determined by matching the theoretical and experimental elastic crack tip stress intensity factor as the crack propagates. In these calculations the flow stress was taken as being equal to the yield stress. Results for the plane stress and plane strain predictions of the crack growth rate for two load cases are given in Fig. 4 along with the experimental values. The hatched area in Fig. 4a indicates the prediction range between the plane stress and plane strain bounds. It can be seen that the plane stress prediction provides an upper bound while the plane strain provides a lower bound for both of these cases. Initially the experimental values are nearer the plane strain predictions, but as the crack length increases they tend towards the plane stress values.

Further results for single edge notched specimens made from BS 4360 grade 50B steel and BS 1470 SiC aluminium alloy are presented in Figs. 5 and 6, respectively. The 50B steel has a yield stress of 352 MPa and an ultimate strength of 519 MPa, while the aluminium alloy as a yield stress of 115 MPa and an ultimate strength of 125 MPa. The experiments were again conducted under constant amplitude loading with $R = 0.05$ and the notch root radius of the chosen results is 0.4 mm. As before, the plane stress and plane strain prediction provide upper and lower bounds for the experimental results. In all of the predictions, the theoretical trends match those of the experimental ones, whereby the initial high crack growth rate decreases towards a minimum value and then approaches the long crack data.

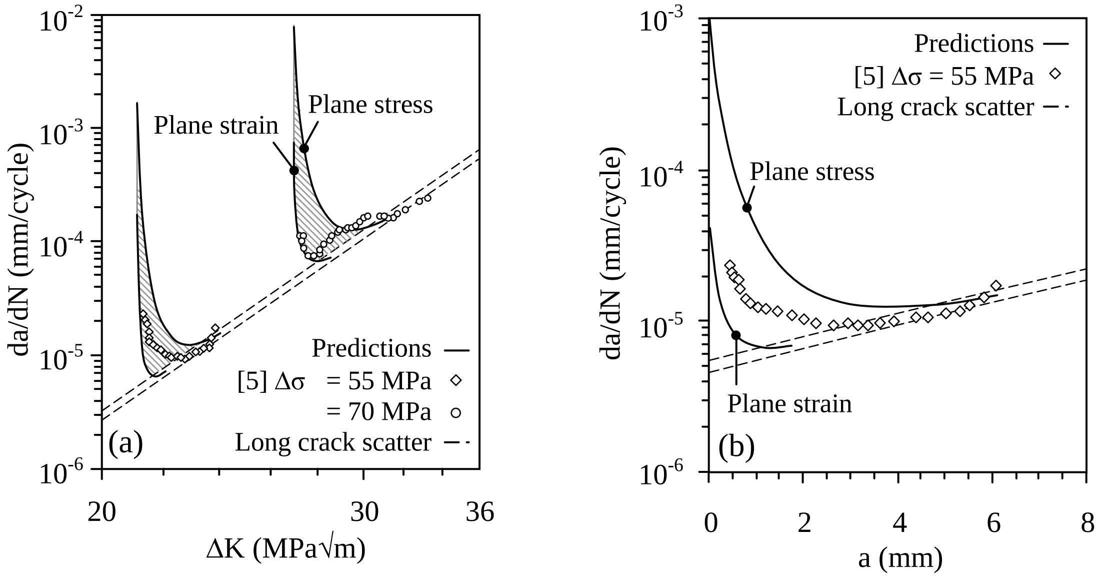

Fig. 4. Prediction bounds for the fatigue crack growth rate of small cracks in stainless steel [5] as a function of a) the stress intensity factor range and b) the crack length (R = 0.05).

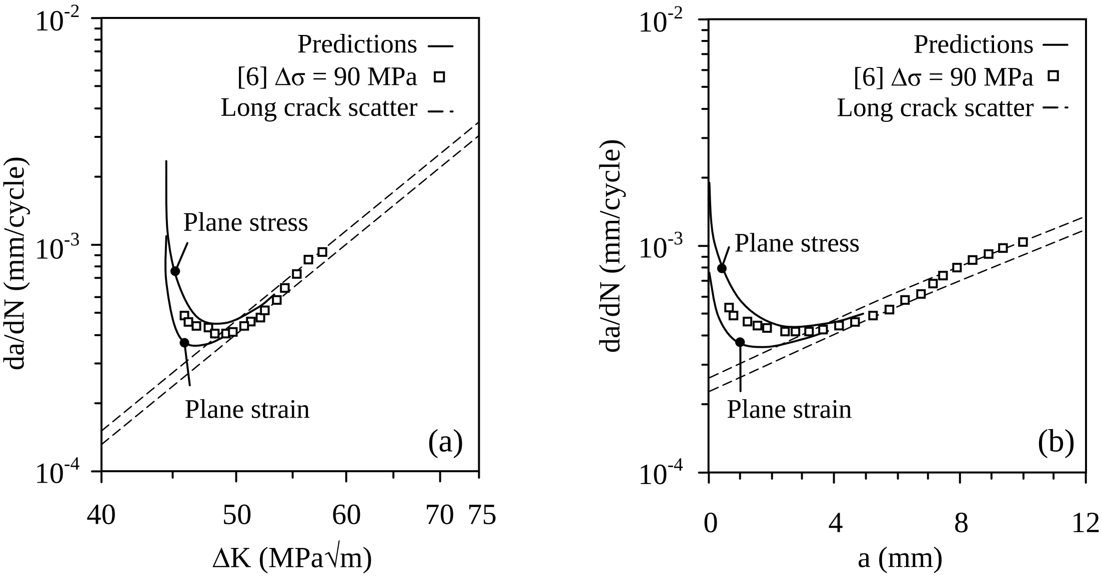

Fig. 5. Predictions of the fatigue crack growth rate in 50B steel [6] as a function of a) the stress intensity factor range and b) the crack length (R = 0.05).

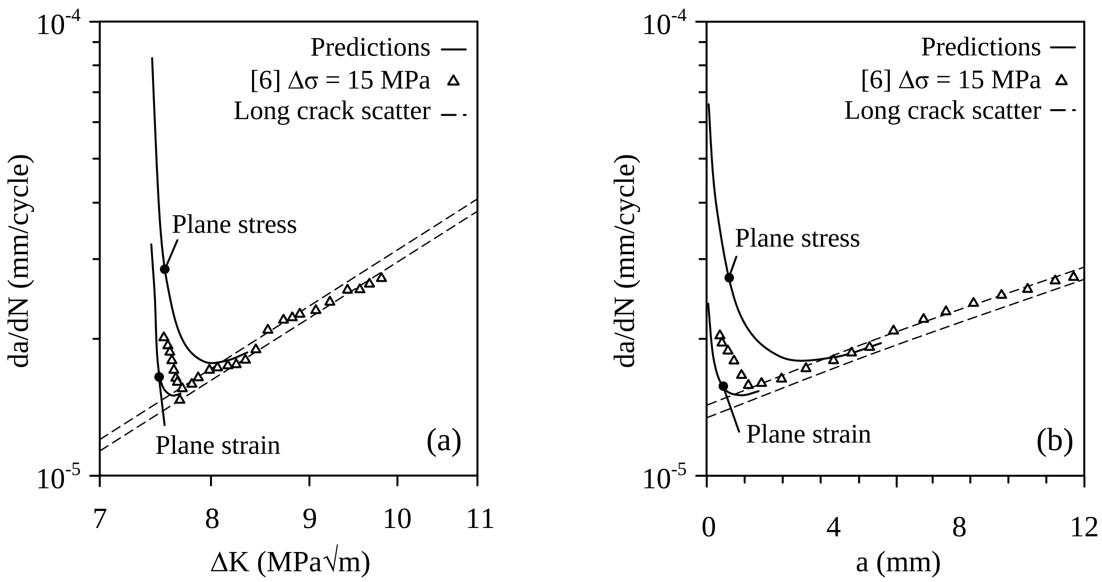

Fig. 6. Predictions of the fatigue crack growth rate in aluminium alloy [6] as a function of a) the stress intensity factor range and b) the crack length (R = 0.05).

4 Summary

Modifications to a strip-yield plasticity-induced crack closure model [24] were presented for considering the small crack fatigue growth from sharp notches. The developed approach is based on the distributed dislocation technique and allows for the determination of the crack opening stress as a function of the crack propagation from an initial sharp edged notch. Results for the crack opening stress for plane stress and plane strain were provided for a range of load ratios for constant amplitude loading. In the past it has been shown that the plane stress and strain values provide suitable upper and lower bounds for fatigue crack growth in plates of different thicknesses [23].

Theoretical predictions made using the developed methods were compared with the experimental data of Shin and Smith [5,6] for a range of materials and load conditions. In all cases the theoretical predictions provided bounds for the experimental results. In addition, the trends observed in the predictions matched those of the experiments. This shows that by using the plasticity-induced crack closure mechanism it is possible to account for the small crack phenomenon.

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CHAPTER 12

SUMMARY AND CONCLUSIONS

CHAPTER 12

SUMMARY AND CONCLUSIONS

12.1 Introduction

Investigations into fatigue crack growth are of great importance as fatigue has been identified as one of the main causes of the failure in engineering structures (Stephens et al. 2001). A careful literature review found that further research is needed into the understanding and modelling of the interaction behaviour between the various parameters involved, such as cracked geometry (i.e. plate thickness), material properties, and the load history. The current research aimed to develop a new theoretical approach based on the distributed dislocation technique, first-order plate theory and the concept of plasticity-induced fatigue crack closure. It was desired that the developed approach eliminates the need for the use of correction factors, which are often employed in an attempt to account for thickness effects and thus the crack tip stress state (Newman 1981). A further requirement was that the developed methods be computationally efficient and repeatable, and therefore suitable for use in practical engineering applications.

This study was divided into three main stages: *1. Preliminary investigation*, *2. Development of the distributed dislocation technique for investigating cracks in plates of finite thickness*, and *3. Investigation of various fatigue crack growth phenomena*. Major discussions of the specific investigations undertaken in this research can be found in each of the journal publications that make up Chapters 3-11. The purpose of the following sections is to provide a brief discussion of the major outcomes of each of three main stages, including details of some of the difficulties that were encountered. Recommendations for future work are also outlined.

12.2 Preliminary Investigation (Ch. 3)

The first stage of this research involved a preliminary investigation into the effect that a variation in material properties can have on the crack tip plasticity zone and crack tip opening displacement. This situation can occur due to a change in operating temperature, phase transformations, or deterioration over time of the material properties. The crack tip opening displacement is closely related to the propagation of a fatigue crack and has been used in the past for the correlation of fatigue crack growth data (see for example Guo et al. 1999). However, in these past studies the material properties are assumed as being constant throughout the analysis.

The theory developed in this investigation utilised the strip-yield hypothesis and the analytical complex potential method of Budiansky and Hutchinson (1978). It was first shown that a variation in the material properties can have a significant effect on the crack tip opening displacement, either causing an increase or decrease depending on the change in properties. This mechanism therefore has the potential to assist, or be a source of, fatigue crack growth. In addition it may also act to impair fatigue crack growth. To the best of the author’s knowledge there is no suitable experimental or finite element data available to compare the obtained results with. This is therefore an area for potential future experimental and theoretical work.

By undertaking the preliminary investigation, the suitability of the complex potential method for fatigue crack growth modelling was determined. In general, the use of complex potentials is fairly simple to implement and without the need for numerical discretisation. However, it is unsuitable for more complicated analyses, such as when the crack is incrementally grown under variable loading, where the crack opening displacement and plastic stretch distributions are highly varied along the crack length. This means that the specific boundary conditions at each location along the crack length may not necessarily be known ahead of time and need to be found by iteration. The complex potential equations would therefore have to be reformulated for each guess of the boundary condition for each boundary zone along the crack length.

12.3 Development of the Distributed Dislocation Technique for Investigating Cracks in Plates of Finite Thickness (Chs. 4-7)

The most popular methods for investigating fatigue crack growth in plates of finite thickness presented in the literature are based on the use of empirical correction factors or finite element simulations. The major outcome of this stage of research was the development of new methods for investigating the effects of plate thickness based on the use of the distributed dislocation technique. These methods directly take into account three-dimensional thickness effects, while still having the simplicity of a two-dimensional approach. This allows for interaction between the involved parameters, such as the material properties, applied loading and plate thickness, to be investigated. First to be considered was the linear elastic out-of-plane stress and displacement fields surrounding the tip of a crack (Ch. 4). A good agreement with past experimental results was found. This demonstrates how the theoretical models can be used in conjunction with experimental results for the determination of the crack tip stress intensity factor. Furthermore, it also provides a validation of the developed techniques.

The elastic-plastic case was next considered by modelling the crack tip plasticity and plasticity-induced crack closure for stationary and growing cracks (Chs. 5-7). For the growing cracks, the assumptions of a parallel or linear wake distribution were used to avoid the need to incrementally grow the crack from its initial to final length. Both small and large scale yielding were also considered. Results for the crack opening stress ratio showed a good agreement with previous finite element values for both plane stress and finite thickness analyses. By not using an incremental crack growth scheme in these models, however, the effects of variable load sequences are unable to be investigated. Furthermore, the use of a linear wake distribution has limited applicability for large-scale yielding in a finite thickness plate, as the wake distribution is actually non-linear.

Several difficulties were found in regards to the placement of the zone edges of the various boundary conditions along the crack length (e.g. plastic zones or closure zones). The use of Gauss-Chebyshev quadrature means that the placement of the numerical integration and collocation points is fixed by the quadrature scheme. Two ways of overcoming this problem were proposed. The first approach was to use linear interpolation between the results from the zone edge, say the tensile plastic zone, being placed at the integration point either side of the ‘exact’ solution (Ch. 5). The second method involved separating the governing integral equation into several intervals coinciding with each of the zones along the crack length. This allowed the integration intervals to be chosen to coincide exactly with the zones edges and thus removed the error involved with point placement (Ch. 7). These difficulties could also be simply overcome by increasing the number of integration points, but the approach developed here made no need for this.

Lastly, in regards to compressive yield zone at the crack tip, the assumption was made that yielding occurs under a plane stress Tresca criterion. This assumption was used for the plane stress and plane strain analyses as well as the finite thickness plate models. For the tensile yield zone, a three-dimensional Tresca criterion was utilised for the finite thickness plate analysis. Similar assumptions are made in plane stress strip-yield models, which utilise the constraint factor for plate thickness effects (Newman 1981), though no real reasons are given. The reverse yielding process is highly complex and material factors such as the Bauschinger effect and the Poisson effect can alter the apparent yield stress. The assumption of a loss of constraint on reverse yielding is supported by finite element and experimental observations and was considered to be appropriate for the current work. However, this is an area where further investigation is still needed.

12.4 Investigation of Various Fatigue Crack Growth Phenomena (Chs. 8-11)

One of the desired outcomes of this research was to develop theoretical models which are simple to implement and that the results are appropriate to a wide range of practical situations. Therefore, the first journal article (Ch. 8) in this stage of work presented a unified model for plasticity-induced fatigue crack closure in plates of finite thickness. In order to generalise the obtained results small-scale yielding conditions were utilised. Within the small-scale-yielding regime the effects of in-plane plate geometry are minimal. An equation for the normalised load ratio parameter, U, was provided as a function of the load ratio, R, and the non-dimensional parameter, $\eta = K_{max}/(\sigma_f\sqrt{h})$. In this parameter, K_{max} is the maximum applied stress intensity factor, σ_f is the flow stress, and 2h is the plate thickness. The normalised load ratio parameter, U, is used to determine the effective stress intensity factor range, ΔK_{eff}, which in turn is used to reduce the scatter in fatigue crack growth rate data due to specimen thickness and load ratio. This investigation, however, was limited to constant amplitude loading.

To investigate the effects of variable loading, and the interaction between other parameters such as plate thickness, it was necessary to include an incremental growth scheme into the closure models. Of particular interest was the crack growth retardation that occurs due to the application of a tensile overload cycle. Two studies were undertaken into this phenomenon, one using plane stress conditions (Ch. 9) and one for the finite thickness plate (Ch. 10). Results from both studies for the crack opening stress and crack growth rate displayed the same trends as are observed in many past experimental investigations. These trends can be directly related to the plasticity-induced crack closure mechanism. An initial acceleration in growth rate occurs due to the increased crack tip opening displacement and thus a reduction in the crack opening level. This is followed by a decrease in the growth rate towards a minimum value, which corresponds to the crack tip propagating through the plastic ‘hump’ caused by the overload. After this point the crack opening stress gradually returns to the steady state value and therefore so does the crack growth rate.

It was noted in both the plane stress and finite thickness investigations that the predicated crack growth rates generally returned to the steady state values more quickly than in the experiments. All of the experiments showed significant retardation well outside the overload plastic zone (up to 5 or 6 times the plastic zone length). The predictions did display continued retardation outside of the plastic zone, though it was at a much lower level. This effect has been partly attributed to strain hardening (Pommier and de Freitas 2002), which was not considered in the present work and is an area for future investigation. In addition, the averaging nature of the first-order plate theory employed in the finite thickness calculations, will remove the effect of the varying stress state across the specimen thickness. That is, near the specimen surfaces plane stress conditions prevail, while in the centre plane strain conditions are dominant. This will cause a variation the level of crack closure across the specimen thickness and therefore will alter the post-overload crack growth behaviour. Despite these issues, predictions made using the plane stress and finite thickness closure models were in good agreement with the experimental values.

The use of an incremental growth scheme also allowed for the investigation of small crack behaviour (Ch. 11). Here the crack was grown from an initial sharp notch to its final length. Small crack behaviour can be readily explained by the plasticity-induced crack closure mechanism. As the crack propagates, a plastic wake will begin to develop and the initially high crack growth rate will be reduced down towards the long crack value. In Chapter 8, the plane stress and plane strain limits were shown to provide upper and lower bounds for the variation in growth rates due to specimen thickness. Therefore in this study (Ch. 11), the developed dislocation models were used to present plane stress and plane strain prediction bounds for small fatigue crack growth from sharp notches. Additionally, the use of prediction bounds reduces the ambiguity in the results due to any variation in the chosen values for the Paris constants. The experimental results for three different materials were examined and in each case the theoretical predictions provided upper and lower bounds for the data.

12.5 Future Work

Several general recommendations for areas of further research are:

- Under large-scale yielding conditions the in-plane plate geometry will have a greater significance than under small-scale yielding conditions. Further investigation could be made into the effect of in-plane specimen geometry such as plate width, specimen type (e.g. compact tension, double edge notched), stress concentrators (e.g. holes, inclusions, blunt notches), etc. This can be achieved for plane stress/strain conditions using already derived dislocation solutions (Hills et al. 1996). For the plane stress/strain cases, it is also possible to ‘cut’ arbitrary specimen geometry using dislocations (Hills et al. 1996; Dai 2002) and this method could be extended to the finite thickness case.
- As briefly discussed in earlier sections, further investigation is needed into the assumption made for the compressive yielding. This may involve examination of the plastic yield zones through experimental or finite element techniques.
- The methods developed in the present study utilised a strip-yield model with a perfect-plastic material behaviour in the yield zone. Further analysis could

incorporate strain hardening and the Bauschinger effect, as well as inclined yield zones, or an alternative yield criterion.

- Experimental work could include the examination of the plastic zone and plastic wake, as well as the associated fatigue crack growth, for different materials, specimen geometry, loading sequences, etc,
- Further research may also involve a continued investigation of the effect of a variation in material properties on the crack tip opening displacement. This could be achieved through experiments (i.e. apply a gradual temperature change, provided thermal stresses are minimum and no phase transitions occur in the given range) or finite element simulations.

12.6 Conclusion

The primary objective of this research was to develop a new effective method for investigating the individual and interaction effects of various non-linear fatigue crack growth phenomena. This was achieved by utilising the distributed dislocation technique and the plasticity-induced crack closure mechanism. Theoretical models were developed for through-the-thickness cracks under both small and large-scale yielding conditions. Results from these models were validated against past numerical and experimental data. The effects of plate thickness were directly taken into account through first-order plate theory, which eliminated the need for any empirical correction factors. This allowed for the investigation of the effect of the plate thickness on fatigue crack closure and growth, as well as the effect of the interaction between the other involved parameters, namely: the material properties and applied loading. An incremental growth scheme was then implemented into the developed methods to allow for the investigation of fatigue crack growth from an initial crack length to a final crack length. By incrementally growing the crack it was also possible to examine variable amplitude load sequences.

A systematic study into the effect of plate thickness, load ratio and material properties for constant amplitude loading, provided much insight into the interaction between these parameters. It was shown that, in general, the extent of plasticity-induced crack closure decreases with an increase in plate thickness. Similarly, as the applied load is increased, all other parameters held constant, the crack opening stress increases and eventually reaches the plane stress solution. On the other hand, if the applied load is decreased the crack opening stress decreases and the plane strain value is reached. The increase in crack opening stress with an increase in applied load or decrease in plate thickness is due to the greater amount of tensile plastic yielding at the crack tip produced under these conditions. This increase in tensile yielding is the result of a reduction in the out-of-plane constraint. A lowering of the yield stress has an equivalent effect. One of the implications of the thickness effect on crack closure and thus fatigue crack growth is that for the same applied loading and material properties, a fatigue crack will grow faster in a thick plate compared to a thin plate. This theoretical finding agrees with the observations of many experimental studies.

It is well known that an overload cycle can temporarily retard crack growth and even lead to crack arrest. For this reason the case of a single tensile overload was next investigated. Predictions for the crack opening stress ratio where made using the developed methods for a range of applied loadings, material properties and plate thicknesses. The various stages of post-overload crack growth observed in past experimental studies could be readily explained via these results and thus the plasticity-induced crack closure mechanism. After an initial acceleration in crack growth rate, corresponding to a drop in the crack opening stress, the crack propagates into the large tensile plastic zone produced by the overload cycle. This leads to a steady increase in the crack opening stress level above the pre-overload value, which in turn causes a reduction in the crack growth rate. As the crack tip starts to leave the overload plastic zone, the crack growth rate decreases back towards the pre-overload rate.

A similar behaviour was found with the investigation into small fatigue crack growth from sharp notches. Initially there is no plastic wake, since there has been no prior crack growth, and therefore the crack opening stress is equal to zero, or the minimum load if $R > 0$. This means that to start with, the short crack growth rate will be greater than the rate predicted by the traditional long crack data. As the crack propagates into the notch plastic zone a wake of plastic deformation develops. This results in the crack opening stress increasing and the crack growth rate decreasing. With continued growth a steady-state value is reached and the crack growth rate recovers the long crack value. It was also shown that the plane stress and plane strain limits provide upper and lower bounds, respectively, for the actual crack growth in a finite thickness plate.

Fatigue crack growth predictions for both the overload retardation and small crack growth were compared with previous experimental studies for several materials and a range of load conditions and plate thicknesses. A good correlation was seen between the theoretical and experimental results. In all cases, the trends in the various stages of crack growth predicted by the theoretical methods matched those of the experiments. Further improvements in the predictions could be obtained by incorporating such effects as strain hardening or the variation in stress state across the plate thickness.

The collection of journal papers presented in this thesis have shown the development and investigation of a new theoretical approach for predicting plasticity-induced crack closure and fatigue crack growth in plates of finite thickness. The application of the distributed dislocation technique and first-order plate theory removed the need for the use of any empirical constraint factors to account for thickness effects. In addition, the developed methods are, in most cases, more computationally efficient than finite element methods for the study of fatigue crack closure. Overall, the theoretical methods developed by this research have provided a means for better investigating the effects of plate thickness on crack closure and fatigue crack growth. This in turn, has allowed for an improved understanding of a range of individual non-linear phenomena associated with fatigue crack growth as well as the interaction between these phenomena.

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APPENDIX A

A COMPUTATIONAL TECHNIQUE FOR CALCULATING PLASTICITY-INDUCED CRACK CLOSURE IN PLATES OF FINITE THICKNESS

Codrington, J., Kotousov, A. and Blazewicz, A. (2007) A computational technique for calculating plasticity-induced crack closure in plates of finite thickness. In: Oñiate, E., Owen, D.R.J. and Suárez, B. (eds), *IX International Conference on Computational Plasticity – Fundamentals and Applications*, COMPLAS IX, Barcelona, September 5-7. pp. 898-901.

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A COMPUTATIONAL TECHNIQUE FOR CALCULATING PLASTICITY-INDUCED CRACK CLOSURE IN PLATES OF FINITE THICKNESS

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Key words: Crack Closure, Plate Thickness Effect, Through-the-Thickness Crack, Wake of Plasticity, Distributed Dislocation Technique, Edge Dislocation.

Summary. In this paper a new method will be presented for calculating the effects of plasticity induced fatigue crack closure for through-the-thickness cracks in plates of arbitrary thickness.

1 INTRODUCTION

The traditional linear elastic approach to fatigue is unable to account for all the important parameters. During his experimental studies in the early 1970's Elber[1,2] discovered that fatigue cracks often remain closed for a significant part of the load cycle, which has a great effect on the crack driving force. Elber attributed this closure to the formation of plasticity zones ahead of the crack tips during tensile loading and proposed the use of an effective stress intensity factor range for describing fatigue crack growth.

Most attempts to model crack closure usually involve greatly simplifying the analysis to that of a plane stress or plane strain problem. One of the first and most well known is the analytical model for plasticity induced closure in fatigue cracks[3]. Based on the strip-yield hypothesis[4] and the theory of complex potentials, the plane stress model demonstrated the concept of plasticity induced crack closure under small-scale yielding conditions. The inclusion of plate thickness and local geometry effects into crack closure analyses are usually based on empirical correction factors or finite element (FE) results[5]. However, data is only available for a very small range of material, load conditions and specimen geometry.

The presented method is based on the distributed dislocation technique (DDT) and the fundamental solution for an edge dislocation[6]. Theoretical models for determining the direct/reverse plastic zone size, plastic wake and crack opening stress are presented for stable growing cracks under constant amplitude loading. Numerical results are given for a wide range of plate thickness to crack length ratios and load conditions, and are also compared with current literature data.

Codrington, J., Kotousov, A. & Blazewicz, A. (2007) A computational technique for calculating plasticity-induced crack closure in plates of finite thickness.
In: Oñiate, E., Owen, D.R.J. and Suárez, B. (eds), *IX International Conference on Computational Plasticity - Fundamentals and Applications, COMPLAS IX, Barcelona, September 5- 7, pp. 898-901.*

NOTE:
This publication is included on pages 255-258 in the print copy of the thesis held in the University of Adelaide Library.

APPENDIX B

INVESTIGATION OF PLASTICITY-INDUCED FATIGUE CRACK CLOSURE

Codrington, J. and Kotousov, A. (2007) Investigation of plasticity-induced fatigue crack closure. In: Veidt, M., Albermani, F., Daniel, B., Griffiths, J., Hargreaves, D., McAree, R., Meehan, P. and Tan, A. (eds), *5th Australian Congress on Applied Mechanics.* ACAM 2007, Brisbane, December 10-12. pp. 127-132.

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Investigation of plasticity-induced fatigue crack closure

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Abstract: Plasticity induced crack closure and constraint effects due to finite plate thickness are both fundamental aspects in the mechanics of fatigue cracks. Moreover, plasticity induced crack closure provides an effective first-order correction to the crack driving force, as used in the correlation and prediction of fatigue crack growth. The approach developed in this study utilises the distributed dislocation technique to model fatigue cracks growing under constant amplitude loading in finite thickness plates. Numerical results are obtained through the application of Gauss-Chebyshev quadrature and are presented for the crack opening stress ratio. An excellent agreement is observed with previous three-dimensional finite element studies.

Keywords: crack closure, crack opening stress, crack tip plasticity, distributed dislocation technique, edge dislocation, plate thickness effect, through-the-thickness crack.

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In: Veidt, M., Albermani, F., Daniel, B., Griffiths, J., Hargreaves, D., McAree, R., Meehan, P. & Tan, A. (eds), *5th Australian Congress on Applied Mechanics. ACAM 2007, Brisbane, December 10-12, pp. 127-132.*