Regularized Equivariant Euler Classes and Gamma Functions

Rongmin Lu

Thesis submitted for the degree of
Doctor of Philosophy
in
Pure Mathematics
at
The University of Adelaide

Discipline of Pure Mathematics

December 2008

For my parents,
Mr Looi Chee Sin *and* Mdm Lee Lai Chan
on the occasion of their 30th anniversary.

til rose skjenke

Contents

Abstract

We consider the regularization of some equivariant Euler classes of certain infinite-dimensional vector bundles over a finite-dimensional manifold M using the framework of zeta-regularized products [35, 53, 59]. An example of such a regularization is the Atiyah–Witten regularization of the $\mathbb{T}$-equivariant Euler class of the normal bundle $\nu(TM)$ of M in the free loop space LM [2].

In this thesis, we propose a new regularization procedure — W-regularization — which can be shown to reduce to the Atiyah–Witten regularization when applied to the case of $\nu(TM)$. This new regularization yields a new multiplicative genus (in the sense of Hirzebruch [26]) — the $\hat{\Gamma}$-genus — when applied to the more general case of a complex spin vector bundle of complex rank ≥ 2 over M, as opposed to the case of the complexification of TM for the Atiyah–Witten regularization. Some of its properties are investigated and some tantalizing connections to other areas of mathematics are also discussed.

We also consider the application of W-regularization to the regularization of $\mathbb{T}^2$-equivariant Euler classes associated to the case of the double free loop space LLM. We find that the theory of zeta-regularized products, as set out by Jorgenson–Lang [35], Quine et al [53] and Voros [59], amongst others, provides a good framework for comparing the regularizations that have been considered so far. In particular, it reveals relations between some of the genera that appeared in elliptic cohomology, allowing us to clarify and prove an assertion of Liu [44] on the $\hat{\Theta}$-genus, as well as to recover the Witten genus. The $\hat{\Gamma}_2$-genus, a new genus generated by a function based on Barnes' double gamma function [5, 6], is also derived in a similar way to the $\hat{\Gamma}$-genus.

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

I acknowledge that the copyright of published works contained within this thesis (as listed below) resides with the copyright holder(s) of those works.

1. Rongmin Lu, *The* $\hat{\Gamma}$*-genus and a regularization of an* S^1*-equivariant Euler class*, J. Phys. A: Math. Theor. **41** (2008), 425204.

SIGNED: ..... DATE: .....

Acknowledgements

It is a pleasure to thank my supervisors, Professor Varghese Mathai and Associate Professor Nicholas Buchdahl, for their patience and guidance. As principal supervisor, Professor Mathai has guided the development of this work, from his suggestion of the initial problem, through to its many twists and turns. Associate Professor Buchdahl has been an encouraging co-supervisor, and his probing questions have stimulated further inquiries that have enriched this work. I would also like to record my thanks to Professor Michael K. Murray and Associate Professor Finnur Lárusson for useful conversations I had with them.

Part of this work has been presented at various conferences. The first presentation was at the 2007 ICE-EM Graduate School at the University of Queensland, where I benefited from conversations with Professor Nolan Wallach, of the University of Califonia at San Diego. Thanks are due to the organizers of the School, whose generous hospitality ensured a wonderful learning experience. Subsequent presentations were made at the 51st Annual Meeting of the AustMS at La Trobe University, and at the Workshop on Geometry and Integrability at the University of Melbourne. I wish to thank the organizers of these two conferences for the opportunity to present my work.

Postgraduate life at Adelaide would not have been complete without my fellow postgraduates. The Strings Journal Club is a stimulating venue to learn from experts and fellow postgraduates alike, the latter of whom included, at some point, David Baraglia, Richard "Ric" Green, Jens Kroeske, Tyson Ritter, David M. Roberts, Raymond Vozzo and James Wallbridge. They have been wonderful audiences of the talks that I have given there. I have also benefited from useful conversations with Richard and David. David, in particular, has suggested the line of analysis that I have only very briefly sketched in §8.3.

I have also been helped in many other ways. In the course of my work, I have had to read papers in French and German. Tony Scoleri checked through my English translation of a crucial French paper and reassured me that I had an adequate understanding of French. In matters Teutonic, Jens also reassured me, in his own way, that I was capable of reading German papers. Thanks also to Alice Bednarz, David Butler, Alys Clark, Sam Cohen, Glenis Crane, Nick Crouch, Aiden Fisher, Ariella Helfgott, Jessica Kasza, Eder Kikianty, Kevin Mark, Daniel Marshall, Ross McNaughton, Allison Plant, Kate Simms, Jono Tuke, Roger Brian Webby and Jason Whyte, for being there to help, listen, or share their knowledge and wisdom.

Outside of campus, I have benefited from the hospitality of Lincoln College during the last two years of my candidature. My stay in Lincoln was also a transitional period for the college, which welcomed its first female principal, Bec Pannell. She and her predecessor, Ken Webb, have made the college a vibrant place for its students — whether undergraduate or postgraduate, domestic or international — to live in, and I greatly treasure the friendships I have made here.

Finally, but most importantly, I would like to thank my parents, Mr Looi Chee Sin and Mdm Lee Lai Chan. Without their invaluable and tireless support, none of this would have been possible.