Optimisation techniques for horn loaded loudspeakers

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Abstract

Horn loaded loudspeakers increase the efficiency and control the spatial distribution of the sound radiated from the horn mouth. They are often used as components in cinema sound systems where the sound can be broadcast evenly onto the audience at all frequencies, improving the listening experience. The sound distribution, or beamwidth, is related to the shape of the horn and is not predicted adequately by existing analytical horn models. The aim of the work described in this thesis is to develop a method to optimise the shape of the horn to give a specified beamwidth, which is ideally frequency independent, thus giving a high quality listening experience.

This thesis begins with a thorough review of the literature relevant to modelling and optimising horn loaded loudspeakers. It gives an introduction to horn loaded loudspeakers, and describes traditional modelling approaches and their limitations. The applications of alternative modelling techniques to horn loaded loudspeakers, which have been found in the literature, are critiqued as are horn optimisation techniques.

To examine the validity of the plane wave radiation assumption made by a number of horn models, experiments were undertaken to measure the sound field at the mouth of two small horns. These horns are representative of the size and design required for cinema loudspeaker systems, but are axisymmetric. The sound field was measured by an automated microphone traverse with almost 3500 measurements made across the face of each horn, providing a high spatial resolution.

The results of the measurements showed that at low frequencies the sound fields from both the conical and exponential horns were similar and that above a certain frequency the sound field became more complex. An analysis of the data, using a modal decomposition with cylindrical duct modes of the same diameter as the mouth of the horns, revealed that almost all of the energy in the system existed in modes with no circumferential variation, and that above a certain limiting frequency, plane waves ceased to exist at the mouth of each horn. This work showed that any numerical model developed must be capable of efficiently modelling variations in the sound field across the mouth of the horn, and that models based on plane wave approximations should not be used for modelling these experimental horns, at least above a certain critical frequency.

Numerical models able to accurately and quickly calculate the far field pressure from arbitrary shapes are also investigated. Calculations of the beamwidth from the analytical solutions for a 45° vibrating spherical cap, mounted on the surface of a unit sphere, were compared with those obtained from an implementation of the direct Boundary Element Method (BEM) and a source superposition technique. The investigation found excellent agreement between these results for mesh densities of 6 elements per wavelength, the generally recommended minimum mesh density for BEM simulations. The source superposition technique was significantly faster than the direct BEM for comparable accuracy in the far field.

There was also excellent agreement between analytical calculations and all of the numerical methods for a mesh density of 3 elements per wavelength. This is a significant finding as it allows a reduction in mesh density, and hence matrix size and solution time, for a given accuracy of far field solution. Alternatively, higher frequencies can be reached for a given mesh density. It was also found that the source superposition technique produced matrices that are highly diagonally dominant, and well suited to fast iterative solution techniques.

The validation of the numerical methods for modelling the beamwidth of horn loaded loudspeakers was undertaken by comparing the source superposition technique to experiment, as well as with an alternative numerical method, the direct BEM. It was shown that such models are capable of modelling the sound field generated by a horn loaded loudspeaker from a specification of the horn geometry. This accuracy of the model is adequate for design purposes within the given frequency range. Both the direct BEM and the source superposition technique are capable of modelling the experimental beamwidth; however, the source superposition technique is considerably faster and hence more suitable for use in optimisation techniques.

During the literature review, a type of sonar transducer called a Constant Beamwidth Transducer (CBT) was found that was able to produce an easily specified frequency independent beamwidth. These are desirable characteristics for the design of a horn. The concept used in the development of these transducers, a specified velocity profile over the surface of a sphere, is explored in this thesis in relation to horn design.

A semi-analytical technique, using solutions to the Helmholtz equation in spherical coordinates and numerical integration of Legendre functions, was developed to efficiently calculate the beamwidth for an arbitrary velocity profile over the surface of a sphere. It was used to calculate the beamwidth for four different velocity profiles: a spherical cap mounted on the surface of a sphere; a CBT profile; and two smooth tailed CBT velocity profiles. The results showed that the smooth tailed CBT velocity profiles produce the smoothest beamwidth, possibly at the expense of low frequency performance. It was also found that the performance of each velocity profile is consistent with CBT theory, with the best performing profile having the highest rate of energy decay in the spherical Legendre modes.

CBT theory also suggests that the performance of the CBT transducers is unaffected by the removal of the inactive part of the sphere, i.e. that part over which the velocity profile is zero. This was confirmed numerically by simulations using the source superposition technique.

The numerical model developed to investigate the CBT was used to test robust optimisation techniques suitable for optimising horn loaded loudspeakers. Two different objective functions were considered, one that uses least squares to drive the velocity profile to a minimum, and the other that uses a constrained optimisation of a smoothness parameter. It was found that constrained optimisation was able to robustly find an optimal solution in an acceptable number of objective function evaluations. As the cost of evaluating the objective function for horn loaded loudspeakers is high, the potential of surrogate modelling techniques, designed to reduce the overall number of objective function evaluations, was investigated. Optimal solutions were found for two different parameterisations of the velocity profile. One parameterisation was similar to the smooth tailed CBT velocity profiles and the other, which allowed a more variable velocity profile, was defined by Bézier curves.

The idea of CBT theory, that is, defining an optimal velocity profile over a spherical cap, was applied to the optimisation of horn loaded loudspeakers. A number of different horn geometry parameterisations were developed, with the aim of producing an optimal velocity profile over the mouth of the horn. The robust optimisation techniques developed previously were applied, and an optimal horn geometry calculated. It was found that a very simple geometry parameterisation could produce near constant beamwidth performance while keeping the desired design (or nominal) beamwidth, and that a more complicated parameterisation (using splines) could not keep the nominal beamwidth but provided superior constant beamwidth performance. A series of optimisations using the spline parameterisation were undertaken to map the design space, with the result being a design chart for constant beamwidth horns. The desired performance characteristics of a constant beamwidth horn such as length, mouth to throat ratio or nominal beamwidth can be specified, and the horn performance and specifications easily read from a chart.

The overall aim of this thesis was to develop fast and reliable optimisation techniques for horn loaded loudspeakers to achieve a robust horn design method for cinema loud-

speakers. This thesis achieved this aim for axisymmetric horn geometries by: developing fast and well validated numerical methods for calculating the beamwidth of horn loaded loudspeakers; by examining how optimal beamwidth control is achieved in CBTs, and how this can be achieved in horn loaded loudspeakers; by developing robust objective functions and optimisation techniques capable of finding an optimal beamwidth from a parameterised geometry; and by developing a design chart for constant beamwidth horns.

Statement of originality

To the best of my knowledge, except where otherwise referenced and cited, everything that is presented in this thesis is my own original work and has not been presented previously for the award of any other degree or diploma in any University. If accepted for the award of the degree of Ph.D. in Mechanical Engineering, I consent that this thesis be made available for loan and photocopying.

Richard C. Morgans

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