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SCALING OF NONPERTURBATIVE RENORMALIZATION OF COMPOSITE OPERATORS WITH OVERLAP FERMIONS

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We compute non-perturbatively the renormalization constants of composite operators for overlap fermions by using the regularization independent scheme. The scaling behavior of the renormalization constants is investigated using the data from three lattices with similar physical volumes and different lattice spacings. The approach of the renormalization constants to the continuum limit is explored.

Keywords: lattice QCD; scaling, non-perturbative renormalization; overlap fermion.

1. Introduction

Following previous papers [1,2] in which we computed non-perturbatively the renormalization constants of composite operators with overlap fermions in quenched QCD, this paper will study the scaling behavior of the renormalization constants.

We adopt the non-perturbative renormalization method which was introduced by Martinelli *et al.* [3]. The method allows a full non-perturbative computation of the matrix elements of composite operators in the Regularization Independent (RI) scheme [3,4] (it is called the RI′ scheme by Chetyrkin [5]). The matching between the RI scheme and $\overline{\mathrm{MS}}$, which is intrinsically perturbative, is computed using continuum perturbation theory, which is well behaved.

The overlap fermion [6,7] was proposed by Narayanan and Neuberger to evade the so called "no-go" theorem [8]. The action in the massless limit preserves a lattice form of chiral symmetry even at finite lattice spacing and volume [7,9]. The use of the overlap action entails many theoretical advantages: it has no additive mass renormalization, there are no order a artifacts, and it has very good scaling [10].

2. Non-perturbative renormalization method

The renormalized operator $O(\mu)$ is related to the bare operator, $O(a)$, calculated on the lattice via

$$O(\mu) = Z_O(\mu a, g(a))\; O(a)\,, \tag{1}$$

Table 1. Lattice parameters.

Action	Size	$N_{\rm Samp}$	β	a (fm)	u_0	Physical Volume (fm^4)
Improved	$16^3 \times 32$	500	4.80	0.093	0.89650	$1.5^3 \times 3.0$
Improved	$12^3 \times 24$	500	4.60	0.123	0.88888	$1.5^3 \times 3.0$
Improved	$8^3 \times 16$	500	4.286	0.190	0.87209	$1.5^3 \times 3.0$

In this work, we will consider the fermion operators

$$O_\Gamma(x) = \bar\psi(x)\Gamma\psi(x)\,, \tag{2}$$

where Γ are the Dirac gamma matrices $\Gamma \in \{1, \gamma_\mu, \gamma_5, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$ and the corresponding notations will be { S, V, P, A, T } respectively.

The renormalization condition is imposed directly on the three-point vertex function $\Gamma_O(pa)$, which is calculated in a fixed gauge, *e.g.*, Landau gauge in our case, at a momentum scale $p^2 = \mu^2$

$$\Gamma_{O,ren}(pa)|_{p^2=\mu^2} = \frac{Z_O(\mu a, g(a))}{Z_\psi(\mu a, g(a))}\Gamma_O(pa)|_{p^2=\mu^2} = 1\,. \tag{3}$$

Here Z_ψ is the field or wave-function renormalization constant, $\Psi_{\rm ren} = Z_\psi^{1/2}\Psi$.

It is apparent that we can only get the ratio of the renormalization constant Z_O for the operator O and the wave-function renormalization constant Z_ψ, from the renormalization condition of Eq. (3). In order to obtain the renormalization constant Z_O for the operator O, one needs to know Z_ψ first. In this work, we will obtain Z_ψ directly from the quark propagator. It can be defined from the Ward Identity (WI) as [3]

$$Z'_\psi = -i\frac{1}{12}\frac{{\rm Tr}\sum_{\mu=1,4}\gamma_\mu(p_\mu a)S(pa)^{-1}}{4\sum_{\mu=1,4}(p_\mu a)^2}\bigg|_{p^2=\mu^2}\,, \tag{4}$$

which, in Landau gauge, differs from Z_ψ by a finite term of order α_s^2. The matching coefficients have been computed using continuum perturbation theory [12].

3. Numerical details

We work on three lattices, each with a different lattice spacing, a, but having similar physical volume. Lattice parameters are summarized in Table 1. The quenched gauge configurations are created using a tadpole improved plaquette plus rectangle (Lüscher-Weisz) gauge action and gauge fixed to the Landau gauge using a Conjugate Gradient Fourier Acceleration algorithm.

The overlap-Dirac operator we use is

$$D(m_q) = \rho + \frac{m_q}{2} + (\rho - \frac{m_q}{2})\gamma_5\epsilon(H). \tag{5}$$

Where ρ is regulator mass and m_q is bare quark mass, and $\epsilon(H)$ is the matrix sign function of an Hermitian operator $H = \gamma_5 D_W$. We use the tadple improved Wilson kernel D_W in the overlap operator, and $\kappa = 0.19163$ is used for the regulator mass ρ for all three lattices. We calculate the overlap quark propagator for 15 bare quark masses on three lattices, they are 53, 59, 71, 83, 94,106, 124, 142, 177, 212, 266, 554, 442, 531, and 620 MeV respectively

Our calculation begins with the evaluation of the inverse of the overlap-Dirac operator. After we calculate the quark propagator in coordinate space for each configuration, we use the Landau gauge fixing transformation matrix to rotate the quark propagator to Landau gauge. Then the discrete Fourier transformation is used to obtain the quark propagator in momentum space. Afterward, we calculate five projected vertex functions $\Gamma_O(pa)$ for each bare quark mass, and extrapolate to the chiral limit. These projected vertex functions $\Gamma_O(pa)$ are in general dependent on $(pa)^2$. The dependence may come from two sources. One is from the usual running of the renormalization constant in the RI scheme. The other is from possible $(pa)^2$ errors and we need to remove this. In order to confront experiment, it is preferable to quote the final results in the $\overline{\mathrm{MS}}$ scheme at a certain scale. One needs to transform the results in the RI scheme to the $\overline{\mathrm{MS}}$ scheme. The detailed analysis can be found in Refs. [1,2].

4. Scaling behaviors

We work on three lattices with similar physical volumes and different spacings a to investigate the the scaling behavior of the renormalization constants. Here we compare the results of renormalization constants Z_ψ, Z_V, Z_S and Z_T on the different lattices in the $\overline{\mathrm{MS}}$ scheme at 2.0 GeV. Fig. 4 shows the four renormalization constants Z_ψ, Z_V, Z_S and Z_T against the square of the lattice spacing a. Because overlap fermions are free of $O(a^2)$ errors, the leading term must be proportional to a^2. We use a simple linear fit $Z_O = c_1 + c_2\, a^2$, and take c_1 as the value of the renormalization constant Z_O in the continuum limit. The numerical values of calculated renormalization constants in the continuum limit in the $\overline{\mathrm{MS}}$ and RI schemes at 2.0 GeV are displayed in Table 2.

In the continuum, the vector current is conserved, so Z_V should be equal to one. For the axial vector current, due to the PCAC relation, it will be conserved at large momenta, where the anomaly has no effect. Our result for Z_V and Z_A in Table 2 compares favorably with 1.

Table 2. Results for Z in the continuum limit.

$Z - factor$	RI scheme at 2 GeV	$\overline{\mathrm{MS}}$ scheme at 2 GeV
Z_ψ	1.059±0.008	1.045±0.008
$Z_V(Z_A)$	0.987±0.011	0.987±0.011
$Z_S(Z_P)$	0.765±0.021	0.898±0.025
Z_T	1.069±0.010	1.047±0.010

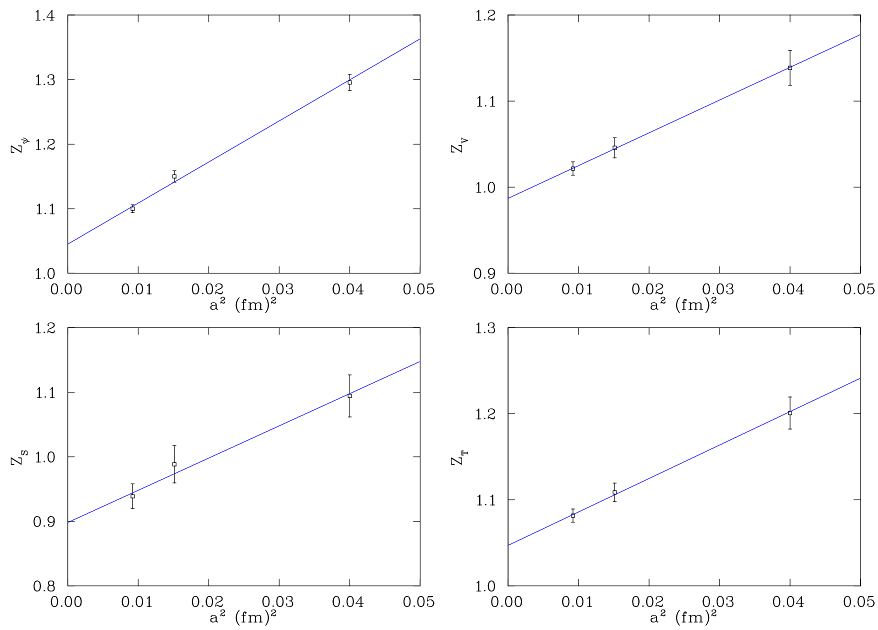

Fig. 1. The renormalization constants Z_ψ, Z_A, Z_S, and Z_T in $\overline{\mathrm{MS}}$ at 2.0 GeV against the square of the lattice spacing a. The straight line is the linear fit $Z_O = c_1 + c_2 a^2$.

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