

**Calibration of Numerical Models
with Application to Groundwater
Flow in the
Willunga Basin, South Australia**

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Abstract

The process of calibrating a numerical model is examined in this thesis with an application to the flow of groundwater in the Willunga Basin in South Australia. The calibration process involves estimating unknown parameters of the numerical model so that the output obtained from the model is comparable with data that is observed in the field.

Three methods for calibrating numerical models are discussed, these being the steepest descent method, the nonlinear least squares method, and a new method called the response function method. The response function method uses the functional relationship between the model's output and the unknown parameters to determine improved estimates for the unknown parameters. The functional relationships are based on analytic solutions to simplified model problems or from previous experience.

The three calibration methods are compared using a simple function involving one parameter, an idealised steady state model of groundwater flow and an idealised transient model of groundwater flow. The comparison shows that the response function method produces accurate estimates in the least amount of iterations.

A numerical model of groundwater flow in the Willunga Basin in South Australia has been developed and the response function method used to estimate the unknown parameters for this model. The model of the Willunga Basin has been used to examine the sustainable yield of groundwater from the basin. The effect on groundwater levels in the basin using current and estimated extraction rates from the literature for sustainable yield has been examined.

The response function method has also been used to estimate the rate of extraction to return the groundwater levels at a specific location to a desirable level.

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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