

# Robust estimation of structure from motion in the uncalibrated case

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# Abstract

A picture of a scene is a 2-dimensional representation of a 3-dimensional world. In the process of projecting the scene onto the 2-dimensional image plane, some of the information about the 3-dimensional scene is inevitably lost. Given a series of images of a scene, typically taken by a video camera, it is sometimes possible to recover some of this lost 3-dimensional information. Within the computer vision literature this process is described as that of recovering structure from motion. If some of the information about the internal geometry of the camera is unknown, then the problem is described as that of recovering structure from motion in the uncalibrated case. It is this uncalibrated version of the problem that is the concern of this thesis.

Optical flow represents the movement of points across the image plane over time. Previous work in the area of structure from motion has given rise to a so-called differential epipolar equation which describes the relationship between optical flow and the motion and internal parameters of the camera. This equation allows the calibration of a camera undergoing unknown motion and having an unknown, and possibly varying, focal length. Obtaining accurate estimates of the camera motion and internal parameters in the presence of noisy optical flow data is critical to the structure recovery process.

We present and compare a variety of methods for estimating the coefficients of the differential epipolar equation. The goal of this process is to derive a tractable total least squares estimator of structure from motion robust to the presence of inaccuracies in the data. Methods are also presented for rectifying optical flow to a particular motion estimate, eliminating outliers from the data, and calculating the relative motion of a camera over an image sequence. The thesis thus explores the application of numerical and statistical techniques for estimation of structure from motion in the uncalibrated case.

# Publications

In carrying out the research that underlies this thesis, a number of papers were published [1, 2, 3, 4, 5, 6, 7]. These papers have largely been co-authored with my supervisors M. J. Brooks and W. Chojnacki. Aspects of the introductory sections of the papers appear in Chapters 1 and 2. The reconstruction formulae upon which Section 3.1 is based appeared originally in [3], as did the exact methods presented in Section 4.1 and the least median of squares scheme from Section 6.2. Aspects of the iteratively reweighted least squares method derived in Section 5.9.1 appeared in [3] and were developed further in [1, 4]. The gradient weighted least squares cost function presented in Section 5.2 was derived in [2], although for the case in which covariance information about the data is available. The error measure based on the smallest angle between the true and estimated motion matrices was used in [2] to measure the performance of different schemes. The rectification procedure for enforcing the cubic constraint on the matrices discussed in Section 2.4.1 appeared in [3] and was used in [1]. The Newton-like method first appeared in [1]. Some of the ideas presented in this thesis have been applied to the case in which covariance information about the data is available [2, 5, 6, 7].

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# Declaration

This thesis contains no material that has been accepted for the award of any other degree or diploma in any university or other tertiary institution. To the best of my knowledge and belief, it contains no material previously published or written by any other person, except where due reference is made in the text.

I give consent for this copy of my thesis, when deposited in the University Library, to be available for loan and photocopying.

Anton van den Hengel  
April, 2000

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