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Liquid-gas phase transition in nuclear matter including strangenessP. Wang,¹ D. B. Leinweber,¹ A. W. Thomas,^{1,2} and A. G. Williams¹¹*Special Research Center for the Subatomic Structure of Matter (CSSM) and Department of Physics,
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We apply the chiral SU(3) quark mean field model to study the properties of strange hadronic matter at finite temperature. The liquid-gas phase transition is studied as a function of the strangeness fraction. The pressure of the system cannot remain constant during the phase transition, since there are two independent conserved charges (baryon and strangeness number). In a range of temperatures around 15 MeV (precise values depending on the model used) the equation of state exhibits multiple bifurcates. The difference in the strangeness fraction f_s between the liquid and gas phases is small when they coexist. The critical temperature of strange matter turns out to be a nontrivial function of the strangeness fraction.

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I. INTRODUCTION

The determination of the properties of hadronic matter at finite temperature and density is a fundamental problem in nuclear physics. In particular, the study of the liquid-gas phase transition in medium-energy heavy-ion collisions is of considerable interest. Many intermediate-energy collision experiments have been performed [1] to investigate the unknown features of the highly excited (hot) nuclei formed in such collisions [2,3]. Theoretically, much effort has been devoted to studying the equation of state for nuclear matter and to discussing the critical temperature T_c . Recently, Natowitz *et al.* obtained the limiting temperature by using a number of different experimental measurements [4]. From these observations the authors extracted the critical temperature of infinite nuclear matter, $T_c = 16.6 \pm 0.86$ MeV [5]. We can expect that further experiments may eventually yield the limiting temperature of hypernuclei and the critical temperature for infinite strange hadronic matter. It is therefore interesting to study the liquid-gas phase transition of strange hadronic matter theoretically.

Exploring systems with strangeness, especially with large strangeness fraction, has attracted a lot of interest in recent years. Such a system has many astrophysical and cosmological implications and is indeed interesting by itself. There are many theoretical discussions for both strange hadronic matter [6,7] and strange quark matter [8–11]. However, most discussions are at zero temperature. The properties of strange hadronic matter at finite temperature have not been studied very much yet. Unlike symmetric nuclear matter, for strange hadronic matter, there are two conserved charges, baryon number and strangeness. Glendenning [12] first discussed the phase transition with more than one conserved charge in general and applied it to the possible transition to quark matter in the core of neutron stars. Müller and Serot [13] discussed asymmetric nuclear matter, which has two conserved charges (baryon number and isospin), using the stability conditions on the free energy, the conservation laws, and the Gibbs criterion for the liquid-gas phase transition. The liquid-gas phase transition of asymmetric nuclear matter was also discussed in effective chiral models in Refs. [14,15]. It was

found that the critical temperature decreases with increasing asymmetry parameter α .

For strange hadronic matter, the method is similar to that for asymmetric nuclear matter. In both cases there are two conserved charges. Recently, Yang *et al.* [16] used the extended Furnstahl-Serot-Tang (FST) model [17] to discuss the liquid-gas phase transition of strange hadronic matter. The original FST model satisfies the SU(2) chiral symmetry and was first applied to study nuclear matter [18]. In the extended FST model [16], the authors found a so called critical pressure above which the liquid-gas phase transition cannot exist. As a result no critical strangeness fraction was obtained for a given temperature. This critical pressure exists in finite nuclei together with a limiting temperature T_{lim} . However, in infinite hadronic matter, physically, there should be no such critical pressure, since the pressure of the system can be much higher than the critical pressure. For strange matter with $f_s=2$, i.e., pure Ξ matter, from Fig. 1 of Ref. [16], one sees that the pressure does not increase monotonically with density. This means that at this temperature pure Ξ matter can be in liquid-gas phase coexistence. Therefore, the binodal p - μ diagram at temperature $T=10$ MeV should terminate at $f_s=2$. We will reconsider this problem and show how the critical strangeness fraction can be obtained.

To study the properties of hadronic matter, we need phenomenological models since QCD cannot yet be used directly. The symmetries of QCD can be used to constrain the hadronic interactions and models based on $SU(2)_L \times SU(2)_R$ symmetry and scale invariance have been proposed. These effective models have been widely used to investigate nuclear matter and finite nuclei, both at zero and at finite temperature [18–20]. Papazoglou *et al.* extended the chiral effective models to $SU(3)_L \times SU(3)_R$, including the baryon octets [21,22]. As well as models based on hadronic degrees of freedom, there are some other models based on quark degrees of freedom, such as the quark-meson coupling model [23,24], the cloudy bag model [25], the NJL model [26], and the quark mean field model [27], etc. Recently, we proposed a chiral SU(3) quark mean field model and investigated the properties of hadronic matter as well as quark

matter [28–32]. This model is quite successful in describing the properties of nuclear matter [28], strange matter [30,31], and finite nuclei and hypernuclei [32] at zero temperature. In this paper, we will apply the chiral SU(3) quark mean field model to finite temperature and study the liquid-gas phase transition of strange hadronic matter.

The paper is organized as follows. The model is introduced in Sec. II. In Sec. III we use the model to investigate strange hadronic matter at finite temperature. The numerical results are discussed in Sec. IV, and Sec. V summarizes our findings.

II. THE MODEL

Our considerations are based on the chiral SU(3) quark mean field model (for details see Refs. [30,32]), which contains quarks and mesons as basic degrees of freedom. Quarks are confined in baryons by an effective potential. The quark-meson interaction and meson self-interaction are based on SU(3) chiral symmetry. Through the mechanism of spontaneous chiral symmetry breaking, the resulting constituent quarks and mesons (except for the pseudoscalars) obtain masses. The introduction of an explicit symmetry breaking term in the meson self-interaction generates the masses of the pseudoscalar mesons which satisfy partially conserved axial-vector current (PCAC) relations. The explicit symmetry breaking term of the quark meson interaction leads in turn to reasonable hyperon potentials in hadronic matter. For completeness, we introduce the main concepts of the model in this section.

In the chiral limit, the quark field q can be split into left- and right-handed parts q_L and q_R : $q = q_L + q_R$. Under SU(3)_L × SU(3)_R they transform as

$$q'_L = Lq_L, \quad q'_R = Rq_R. \quad (1)$$

The spin-0 mesons are written in the compact form

$$M(M^+) = \Sigma \pm i\Pi = \frac{1}{\sqrt{2}} \sum_{a=0}^8 (\sigma^a \pm i\pi^a) \lambda^a, \quad (2)$$

where σ^a and π^a are the nonets of scalar and pseudoscalar mesons, respectively, λ^a ($a = 1, \dots, 8$) are the Gell-Mann matrices, and $\lambda^0 = \sqrt{\frac{2}{3}}I$. The alternative plus and minus signs correspond to M and M^+ . Under chiral SU(3) transformations, M and M^+ transform as $M \rightarrow M' = LMR^+$ and $M^+ \rightarrow M'^+ = RM^+L^+$. In a similar way, the spin-1 mesons are introduced through

$$l_\mu(r_\mu) = \frac{1}{2}(V_\mu \pm A_\mu) = \frac{1}{2\sqrt{2}} \sum_{a=0}^8 (v_\mu^a \pm a_\mu^a) \lambda^a \quad (3)$$

with the transformation properties $l_\mu \rightarrow l'_\mu = Ll_\mu L^+$, $r_\mu \rightarrow r'_\mu = Rr_\mu R^+$. The matrices Σ , Π , V_μ , and A_μ can be written in a form where the physical states are explicit. For the scalar and vector nonets, we have the expressions

$$\Sigma = \frac{1}{\sqrt{2}} \sum_{a=0}^8 \sigma^a \lambda^a = \begin{pmatrix} \frac{1}{\sqrt{2}}(\sigma + a_0^0) & a_0^+ & K^{*+} \\ a_0^- & \frac{1}{\sqrt{2}}(\sigma - a_0^0) & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \zeta \end{pmatrix}, \quad (4)$$

$$V_\mu = \frac{1}{\sqrt{2}} \sum_{a=0}^8 v_\mu^a \lambda^a = \begin{pmatrix} \frac{1}{\sqrt{2}}(\omega_\mu + \rho_\mu^0) & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & \frac{1}{\sqrt{2}}(\omega_\mu - \rho_\mu^0) & K_\mu^{*0} \\ K_\mu^{*-} & \bar{K}_\mu^{*0} & \phi_\mu \end{pmatrix}. \quad (5)$$

Pseudoscalar and pseudovector nonet mesons can be written in a similar fashion.

The total effective Lagrangian has the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{q0} + \mathcal{L}_{qM} + \mathcal{L}_{\Sigma\Sigma} + \mathcal{L}_{VV} + \mathcal{L}_{\chi SB} + \mathcal{L}_{\Delta m_s} + \mathcal{L}_h + \mathcal{L}_c, \quad (6)$$

where $\mathcal{L}_{q0} = \bar{q}i\gamma^\mu \partial_\mu q$ is the free part for massless quarks. The quark-meson interaction \mathcal{L}_{qM} can be written in a chiral SU(3) invariant way as

$$\begin{aligned} \mathcal{L}_{qM} = & g_s (\bar{\Psi}_L M \Psi_R + \bar{\Psi}_R M^+ \Psi_L) - g_v (\bar{\Psi}_L \gamma^\mu l_\mu \Psi_L \\ & + \bar{\Psi}_R \gamma^\mu r_\mu \Psi_R) = \frac{g_s}{\sqrt{2}} \bar{\Psi} \left(\sum_{a=0}^8 \sigma_a \lambda_a + i \sum_{a=0}^8 \pi_a \lambda_a \gamma^5 \right) \Psi \\ & - \frac{g_v}{2\sqrt{2}} \bar{\Psi} \left(\sum_{a=0}^8 \gamma^\mu v_\mu^a \lambda_a - \sum_{a=0}^8 \gamma^\mu \gamma^5 a_\mu^a \lambda_a \right) \Psi. \end{aligned} \quad (7)$$

In the mean field approximation, the chiral-invariant scalar meson $\mathcal{L}_{\Sigma\Sigma}$ and vector meson \mathcal{L}_{VV} self-interaction terms are written as [30,32]

$$\begin{aligned} \mathcal{L}_{\Sigma\Sigma} = & -\frac{1}{2} k_0 \chi^2 (\sigma^2 + \zeta^2) + k_1 (\sigma^2 + \zeta^2)^2 + k_2 \left(\frac{\sigma^4}{2} + \zeta^4 \right) \\ & + k_3 \chi \sigma^2 \zeta - k_4 \chi^4 - \frac{1}{4} \chi^4 \ln \frac{\chi^4}{\chi_0^4} + \frac{\delta}{3} \chi^4 \ln \frac{\sigma^2 \zeta}{\sigma_0^2 \zeta_0}, \end{aligned} \quad (8)$$

$$\begin{aligned} \mathcal{L}_{VV} = & \frac{1}{2} \frac{\chi^2}{\chi_0^2} (m_\omega^2 \omega^2 + m_\rho^2 \rho^2 + m_\phi^2 \phi^2) + g_4 (\omega^4 + 6\omega^2 \rho^2 + \rho^4 \\ & + 2\phi^4), \end{aligned} \quad (9)$$

where $\delta = 6/33$; σ_0 , ζ_0 , and χ_0 are the vacuum expectation values of the corresponding mean fields σ , ζ , and χ . The Lagrangian $\mathcal{L}_{\chi SB}$ generates the nonvanishing masses of pseudoscalar mesons

$$\mathcal{L}_{\chi SB} = \frac{\chi^2}{\chi_0^2} \left[m_{\pi}^2 F_{\pi} \sigma + \left(\sqrt{2} m_K^2 F_K - \frac{m_{\pi}^2}{\sqrt{2}} F_{\pi} \right) \zeta \right], \quad (10)$$

leading to a nonvanishing divergence of the axial currents which in turn satisfy the relevant PCAC relations for π and K mesons. The pseudoscalar and scalar mesons as well as the dilaton field χ obtain mass terms by spontaneous breaking of chiral symmetry in the Lagrangian (8). The masses of the u , d , and s quarks are generated by the vacuum expectation values of the two scalar mesons σ and ζ . To obtain the correct constituent mass of the strange quark, an additional mass term has to be added:

$$\mathcal{L}_{\Delta m_s} = -\Delta m_s \bar{q} S q, \quad (11)$$

where $S = \frac{1}{3}(I - \lambda_8 \sqrt{3}) = \text{diag}(0, 0, 1)$ is the strangeness quark matrix. Through these mechanisms, the quark constituent masses are finally given by

$$m_u = m_d = -\frac{g_s}{\sqrt{2}} \sigma_0 \quad \text{and} \quad m_s = -g_s \zeta_0 + \Delta m_s, \quad (12)$$

where g_s and Δm_s are chosen to yield the constituent quark mass in vacuum—in our case, $m_u = m_d = 313$ MeV and $m_s = 490$ MeV. In order to obtain reasonable hyperon potentials in hadronic matter, it has been found necessary to include an additional coupling between strange quarks and the scalar mesons σ and ζ [30]. This term is expressed as

$$\mathcal{L}_h = (h_1 \sigma + h_2 \zeta) \bar{s} s. \quad (13)$$

In the quark mean field model, quarks are confined in baryons by the Lagrangian $\mathcal{L}_c = -\bar{\Psi} \chi_c \Psi$ [with χ_c given in Eq. (14) below]. The Dirac equation for the quark field Ψ_{ij} , under the additional influence of the meson mean fields, is given by

$$[-i \vec{\alpha} \cdot \vec{\nabla} + \chi_c(r) + \beta m_i^*] \Psi_{ij} = e_i^* \Psi_{ij}, \quad (14)$$

where $\vec{\alpha} = \gamma^0 \vec{\gamma}$, $\beta = \gamma^0$, the subscripts i and j denote the quark i ($i = u, d, s$) in a baryon of type j ($j = N, \Lambda, \Sigma, \Xi$), and $\chi_c(r)$ is a confining potential—i.e., a static potential providing confinement of quarks by meson mean field configurations. The quark effective mass m_i^* and energy e_i^* are defined as

$$m_i^* = -g_{\sigma}^i \sigma - g_{\zeta}^i \zeta + m_{i0} \quad (15)$$

and

$$e_i^* = e_i - g_{\omega}^i \omega - g_{\phi}^i \phi, \quad (16)$$

where e_i is the energy of the quark under the influence of the meson mean fields. Here $m_{i0} = 0$ for $i = u, d$ (nonstrange quark) and $m_{i0} = \Delta m_s = 29$ MeV for $i = s$ (strange quark). Using the solution of the Dirac equation (14) for the quark energy e_i^* it has been common to define the effective mass of the baryon j through the ansatz

$$M_j^* = \sqrt{E_j^{*2} - \langle p_{j \text{ c.m.}}^{*2} \rangle}, \quad (17)$$

where $E_j^* = \sum_i n_i e_i^* + E_{j \text{ spin}}$ is the baryon energy and $\langle p_{j \text{ c.m.}}^{*2} \rangle$ is the subtraction of the contribution to the total energy associated with spurious center of mass motion. In the expression

for the baryon energy n_{ij} is the number of quarks with flavor i in a baryon with flavor j , with $j = N\{p, n\}$, $\Sigma\{\Sigma^{\pm}, \Sigma^0\}$, $\Xi\{\Xi^0, \Xi^{-}\}$, Λ , and $E_{j \text{ spin}}$ is the correction to the baryon energy which is determined from a fit to the data for baryon masses. There is an alternative way to remove the spurious c.m. motion and determine the effective baryon masses. In Ref. [33], the removal of the spurious c.m. motion for three quarks moving in a confining, relativistic oscillator potential was studied in some detail. It was found that when an external scalar potential was applied, the effective mass obtained from the interaction Lagrangian could be written as

$$M_j^* = \sum_i n_{ij} e_i^* - E_j^0, \quad (18)$$

where E_j^0 was found to be only very weakly dependent on the external field strength. We therefore use Eq. (18), with E_j^0 a constant, independent of the density, which is adjusted to give a best fit to the free baryon masses.

Using the square root ansatz for the effective baryon mass, Eq. (17), the confining potential χ_c is chosen as a combination of scalar (S) and scalar-vector (SV) potentials as in Ref. [30]:

$$\chi_c(r) = \frac{1}{2} [\chi_c^S(r) + \chi_c^{SV}(r)] \quad (19)$$

with

$$\chi_c^S(r) = \frac{1}{4} k_c r^2 \quad (20)$$

and

$$\chi_c^{SV}(r) = \frac{1}{4} k_c r^2 (1 + \gamma^0). \quad (21)$$

On the other hand, using the linear definition of the effective baryon mass, Eq. (18), the confining potential χ_c is chosen to be the purely scalar potential $\chi_c^S(r)$. The coupling k_c is taken as $k_c = 1$ GeV fm⁻², which yields baryon mean square charge radii (in the absence of a pion cloud [34]) around 0.6 fm.

The properties of infinite nuclear matter and finite nuclei were calculated with these two treatments of effective baryon mass in Ref. [35]. As we explained there, the linear definition of effective baryon mass has been derived using a systematic relativistic approach [33], while to the best of our knowledge no equivalent derivation exists for the square root case. For high baryon density, the predictions of these two treatments are quite different. Many physical quantities change discontinuously at some critical density in the case of the square root ansatz, while the linear definition of baryon mass yields continuous behavior for high density nuclear matter. Both treatments of the spurious c.m. motion fit the saturation properties of nuclear matter and therefore, for densities lower than the saturation density, these two treatments give reasonably similar results. In this paper, we will discuss the liquid-gas phase transition of strange hadronic matter with both treatments. We prefer the linear form because it has been derived. The square root case is reported here because it is

widely used and in fact produces similar results in some regions. However, where they differ we believe that the linear form is the more reliable.

III. STRANGE HADRONIC MATTER AT FINITE TEMPERATURE

Based on the previously defined interaction, the Lagrangian density for strange hadronic matter is written as

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_B(i\gamma^\mu\partial_\mu - M_B^*)\psi_B + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\zeta\partial^\mu\zeta + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi \\ & - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}S_{\mu\nu}S^{\mu\nu} - g_\omega^B\bar{\psi}_B\gamma_\mu\psi_B\omega^\mu - g_\phi^B\bar{\psi}_B\gamma_\mu\psi_B\phi^\mu \\ & + \mathcal{L}_M, \end{aligned} \quad (22)$$

where

$$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \text{ and } S_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu. \quad (23)$$

The term \mathcal{L}_M represents the interaction between mesons which includes the scalar meson self-interaction $\mathcal{L}_{\Sigma\Sigma}$, the vector meson self-interaction \mathcal{L}_{VV} , and the explicit chiral symmetry breaking term $\mathcal{L}_{\chi SB}$, all defined previously. The Lagrangian includes the scalar mesons σ , ζ , and χ , and the vector mesons ω and ϕ . The interactions between quarks and scalar mesons result in the effective baryon masses M_B^* , where subscript B labels the baryon $B=N, \Lambda, \Sigma$, or Ξ . The interactions between quarks and vector mesons generate the baryon-vector meson interaction. The corresponding vector coupling constants g_ω^B and g_ϕ^B are baryon dependent and satisfy the relevant SU(3) relationships. In fact, we find the following relations for the vector coupling constants:

$$g_\omega^\Lambda = g_\omega^\Sigma = 2g_\omega^\Xi = \frac{2}{3}g_\omega^N \text{ and } g_\phi^\Lambda = g_\phi^\Sigma = \frac{1}{2}g_\phi^\Xi = \frac{\sqrt{2}}{3}g_\phi^N. \quad (24)$$

At finite temperature and density, the thermodynamic potential for strange hadronic matter is defined as

$$\begin{aligned} \Omega = & - \sum_{j=N,\Lambda,\Sigma,\Xi} \frac{g_j k_B T}{(2\pi)^3} \int_0^\infty d^3k \{ \ln(1 + e^{-[E_j^*(k) - v_j]/k_B T}) + \ln(1 \\ & + e^{-[E_j^*(k) + v_j]/k_B T}) \} - \mathcal{L}_M, \end{aligned} \quad (25)$$

where $E_j^*(k) = \sqrt{M_j^{*2} + k^2}$ and g_j is the degeneracy of baryon j ($g_{N,\Xi}=2$, $g_\Lambda=1$, and $g_\Sigma=3$). The quantity v_j is related to the usual chemical potential μ_j by $v_j = \mu_j - g_\omega^j \omega - g_\phi^j \phi$. The energy per unit volume and the pressure of the system can be derived as $\varepsilon = \Omega - (1/T)(\partial\Omega/\partial T) + v_j \rho_j$ and $p = -\Omega$, where ρ_j is the baryon density.

The mean field equation for meson ϕ_i is obtained by the formula $\partial\Omega/\partial\phi_i=0$. For example, the equations for σ, ζ are deduced as

$$\begin{aligned} k_0\chi^2\sigma - 4k_1(\sigma^2 + \zeta^2)\sigma - 2k_2\sigma^3 - 2k_3\chi\sigma\zeta - \frac{2\delta}{3\sigma}\chi^4 + \frac{\chi^2}{\chi_0^2}m_\pi^2 F_\pi \\ - \left(\frac{\chi}{\chi_0}\right)^2 m_\omega\omega^2 \frac{\partial m_\omega}{\partial\sigma} + \sum_{j=N,\Lambda,\Sigma,\Xi} \frac{\partial M_j^*}{\partial\sigma} \langle \psi_j \psi_j \rangle = 0, \end{aligned} \quad (26)$$

$$\begin{aligned} k_0\chi^2\zeta - 4k_1(\sigma^2 + \zeta^2)\zeta - 4k_2\zeta^3 - k_3\chi\sigma^2 - \frac{\delta}{3\zeta}\chi^4 + \frac{\chi^2}{\chi_0^2} \left(\sqrt{2}m_k^2 F_k \right. \\ \left. - \frac{1}{\sqrt{2}}m_\pi^2 F_\pi \right) - \left(\frac{\chi}{\chi_0}\right)^2 m_\phi\phi^2 \frac{\partial m_\phi}{\partial\zeta} + \sum_{j=N,\Lambda,\Sigma,\Xi} \frac{\partial M_j^*}{\partial\zeta} \langle \psi_j \psi_j \rangle \\ = 0, \end{aligned} \quad (27)$$

where

$$\langle \bar{\psi}_j \psi_j \rangle = \frac{g_j}{\pi^2} \int_0^\infty dk \frac{k^2 M_j^*}{E_j^*(k)} [n_j(k) + \bar{n}_j(k)]. \quad (28)$$

In the above equation, $n_j(k)$ and $\bar{n}_j(k)$ are the baryon and antibaryon distributions, respectively, expressed as

$$n_j(k) = \frac{1}{\exp\{[E_j^*(k) - v_j]/k_B T\} + 1} \quad (29)$$

and

$$\bar{n}_j(k) = \frac{1}{\exp\{[E_j^*(k) + v_j]/k_B T\} + 1}. \quad (30)$$

The equations for vector mesons ω and ϕ are expressed as

$$\frac{\chi^2}{\chi_0^2} m_\omega^2 \omega + 4g_4 \omega^3 = \sum_{j=N,\Lambda,\Sigma,\Xi} g_\omega^j \rho_j, \quad (31)$$

$$\frac{\chi^2}{\chi_0^2} m_\phi^2 \phi + 8g_4 \phi^3 = \sum_{j=N,\Lambda,\Sigma,\Xi} g_\phi^j \rho_j, \quad (32)$$

where ρ_j is the density of baryons of type j , expressed as

$$\rho_j = \frac{g_j}{\pi^2} \int_0^\infty dk k^2 [n_j(k) - \bar{n}_j(k)]. \quad (33)$$

Let us now discuss the liquid-gas phase transition. For strange hadronic matter, we follow the thermodynamic approach of Refs. [12,13]. The system will be stable against separation into two phases if the free energy of a single phase is lower than the free energy in all two-phase configurations. This requirement can be formulated as [13]

$$F(T, \rho) < (1 - \lambda)F(T, \rho') + \lambda F(T, \rho''), \quad (34)$$

where

$$\rho = (1 - \lambda)\rho' + \lambda\rho'', \quad 0 < \lambda < 1, \quad (35)$$

and F is the Helmholtz free energy per unit volume. The two phases are denoted by a prime and a double prime. If the stability condition is violated, a system with two phases is energetically favorable. The phase coexistence is governed by the Gibbs conditions

$$\mu_j'(T, \rho') = \mu_j''(T, \rho'') \quad (j = N, \Lambda, \Sigma, \Xi), \quad (36)$$

$$p'(T, \rho') = p''(T, \rho''), \quad (37)$$

where the temperature is the same in the two phases. The chemical potentials of the baryons satisfy the following relationship:

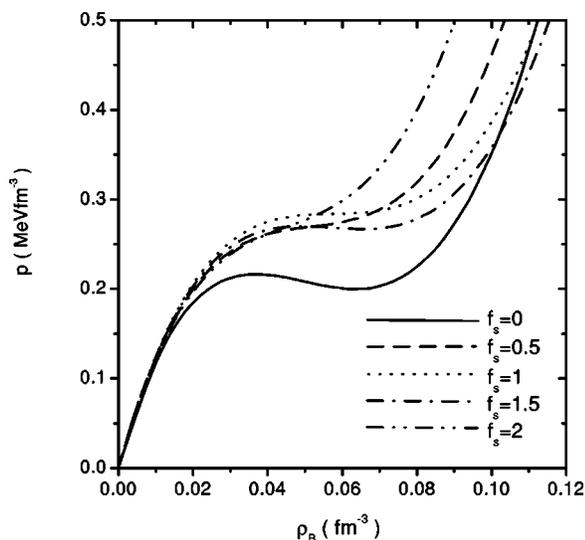


FIG. 1. The pressure of strange hadronic matter p versus baryon density ρ_B with different strangeness fractions f_s at temperature $T = 15$ MeV in the case of the square root ansatz of effective baryon mass.

$$\mu_\Lambda = \mu_\Sigma = (\mu_N + \mu_\Xi)/2. \quad (38)$$

Therefore, there are only two independent chemical potentials for the four kinds of baryons. They are determined by the total baryon density ρ_B and the strangeness fraction f_s , which are defined as $\rho_B = (\rho_N + \rho_\Lambda + \rho_\Sigma + \rho_\Xi)$ and $f_s = (\rho_\Lambda + \rho_\Sigma + 2\rho_\Xi)/\rho_B$.

IV. NUMERICAL RESULTS AND DISCUSSION

The parameters in this model were determined by the meson masses in vacuum and the properties of nuclear matter which were listed in Table I of Ref. [35]. We now discuss the liquid-gas phase transition of strange hadronic matter. In Fig. 1, we plot the pressure of the system versus baryon density for various strangeness fractions f_s at temperature $T = 15$ MeV for the square root ansatz of the effective baryon mass [Eq. (17)]. For nonstrange hadronic matter, the p - ρ_B isotherms exhibit the form of two-phase coexistence with an unphysical region. The nuclear matter can be in a state of liquid-gas coexistence at this temperature. With increasing f_s , the pressure will increase. At a particular value of f_s , the pressure will increase monotonically with increasing density. As we will see later, the strangeness fraction is different in the liquid and gas phases. Therefore, the system can still be in liquid-gas coexistence, even though the pressure increases monotonically with density. The unphysical region appears again in the range of $1.0 < f_s < 1.6$. It is obvious that the behavior of the pressure of strange hadronic matter is not monotonic with f_s . For the linear definition of effective baryon mass [Eq. (18)], the results are plotted in Fig. 2. At small strangeness fraction, say $f_s < 0.4$, there are unphysical regions. In the range $0.4 < f_s < 1.75$, the pressure increases monotonically with increasing density, while for $f_s > 1.75$, the unphysical regions appear again.

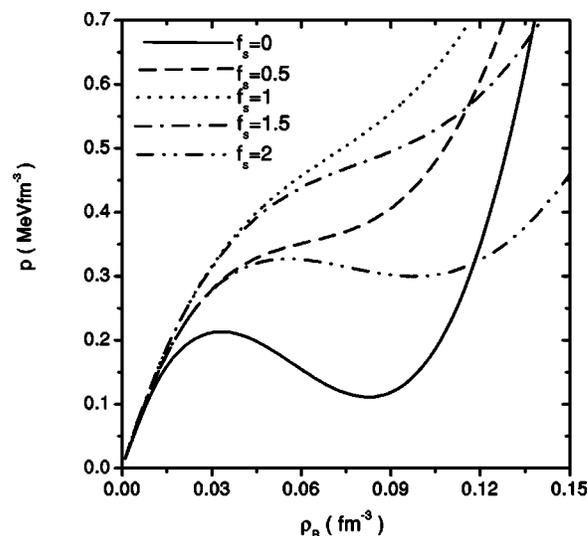


FIG. 2. The pressure of strange hadronic matter p versus baryon density ρ_B with different strangeness fractions f_s at temperature $T = 15$ MeV in the case of the linear definition of effective baryon mass.

As we pointed out earlier, there are two independent chemical potentials for the baryons. We now show how the Gibbs conditions can be satisfied. As an example, we plot the chemical potentials of nucleon and Λ versus f_s at temperature $T = 15$ MeV and pressure $p = 0.23$ MeV fm $^{-3}$ with the square root ansatz for the effective baryon mass in Fig. 3 (for convenience, we use the reduced chemical potential which is defined as $\tilde{\mu}_j = \mu_j - M_j$). The solid and dashed lines are for the nucleon and Λ , respectively. The Gibbs equations (36) and (37) for phase equilibrium demand equal pressure and chemical potentials for two phases with different concentrations. The desired solution can be found by means of the geometrical construction shown in Fig. 3, which guarantees

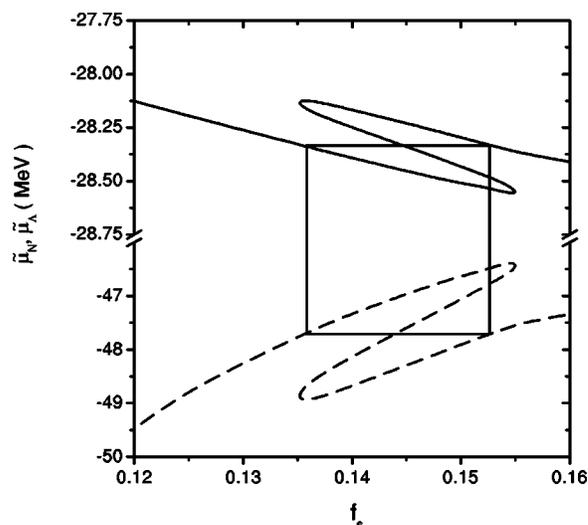


FIG. 3. Geometrical construction used to obtain the chemical potentials and strangeness fractions in the two-phase coexistence at temperature $T = 15$ MeV and $p = 0.23$ MeV fm $^{-3}$. The solid and dashed lines are for the nucleon and Λ , respectively.

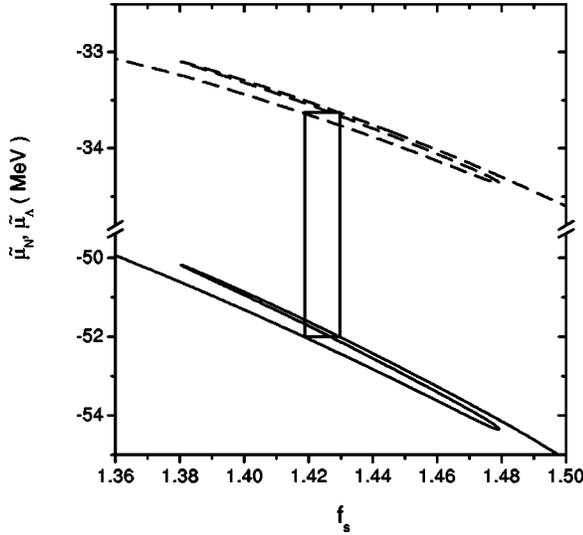


FIG. 4. Geometrical construction used to obtain the chemical potentials and strangeness fractions in the two-phase coexistence at temperature $T=15$ MeV and $p=0.27$ MeV fm $^{-3}$. The solid and dashed lines are for the nucleon and Λ , respectively.

the same pressure and chemical potentials of the nucleon and Λ in the two phases with different f_s . Due to the chemical relationship between the baryons, the chemical potentials of Σ and Ξ are also the same in the two phases.

For asymmetric nuclear matter, there is only one kind of solution which satisfies Eqs. (36) and (37) at the given pressure and temperature. For hadronic matter, there is another solution of the Gibbs equations with higher pressure at the same temperature. We show in Fig. 4 the chemical potentials versus f_s at pressure $p=0.27$ MeV fm $^{-3}$ with the same temperature as in Fig. 3. From the geometrical construction, one can see that the difference of f_s between the two phases is very small. We can discuss in some detail the reason why there is only one solution for asymmetric nuclear matter and two solutions for strange hadronic matter. This is because for asymmetric nuclear matter with $\alpha=(\rho_n-\rho_p)/(\rho_n+\rho_p)$, the α -dependence behavior of the pressure is monotonic—as can be seen clearly in Fig. 3 of Ref. [14]. However, in the case of strange hadronic matter, the dependence on f_s is not monotonic. For example, for temperature $T=15$ MeV, the point where $\partial p/\partial \rho_B=0$ appears in two regions of f_s , i.e., $0 < f_s < 0.2$ and $1.0 < f_s < 1.6$. This means that the system can be in the liquid-gas coexistence phase.

The pairs of solutions shown in Fig. 3 with small strangeness fraction form a binodal curve which is plotted in Fig. 5. There is a critical point A where the pressure is about 0.244 MeV fm $^{-3}$ with the corresponding strangeness fraction $f_s=0.24$. The binodal curve is divided into two branches by the critical point. One branch corresponds to the high density (liquid) phase; the other corresponds to the low density (gas) phase. Assume the system is initially prepared in the low density (gas) phase with $f_s=0.2$. When the pressure increases to some value, the two-phase region is encountered at point a and a liquid phase at b with a low f_s begins to emerge. As the system is compressed, the gas phase evolves from point a to c, while the liquid phase evolves from b to d. If the pressure

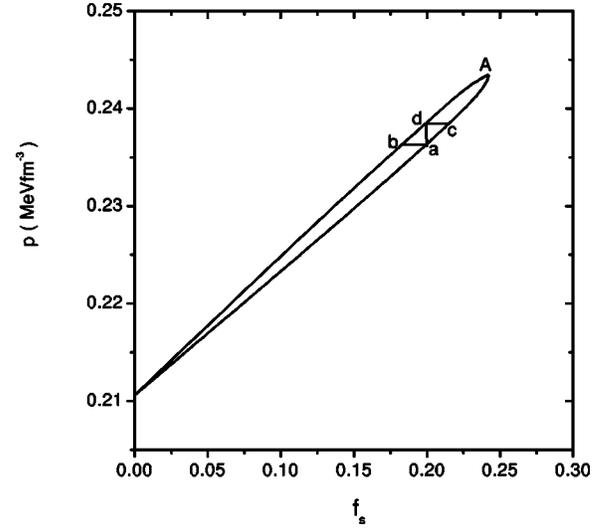


FIG. 5. The first binodal curve with smaller strangeness fraction at temperature $T=15$ MeV. The points a through d denote the liquid-gas phase transition. A is the critical point.

of the system continues to increase, the system will leave the two-phase region at point d. The gas phase disappears and the system is entirely in the liquid phase. This kind of phase transition is different from the normal first order phase transition where the pressure remains constant during the phase transition. If the strangeness is larger than 0.24, there is no phase transition between liquid and gas phases. Therefore, for a given temperature there exists a critical strangeness fraction, above which the system can only be in the gas phase. In other words, for a system with a fixed strangeness fraction f_s there exists a critical temperature, above which the system cannot change completely into the liquid phase however large the pressure.

The solutions shown in Fig. 4 with higher strangeness fraction form another binodal curve and we plot it in Fig. 6. As in Fig. 5, there are two branches divided by the critical

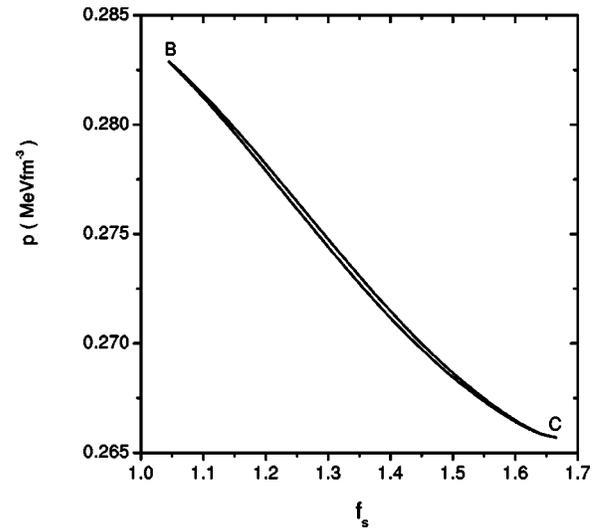


FIG. 6. The second binodal curve with larger strangeness fraction at temperature $T=15$ MeV. B and C are the critical points.

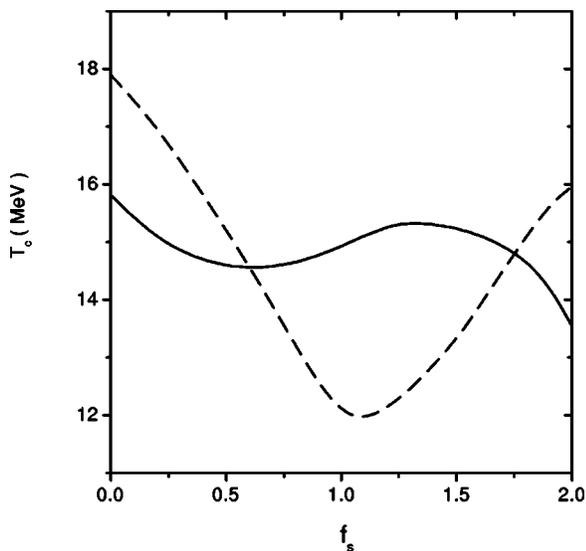


FIG. 7. The critical temperature T_c versus strangeness fraction f_s . The solid and dashed lines correspond to the square root and linear case, respectively.

points B and C . One branch is for the liquid phase; the other is for the gas phase. One can see that the difference of f_s between liquid and gas phase is very small. At the critical points B and C the strangeness fraction is about 1.04 and 1.66, respectively. If f_s is smaller than 1.04 but larger than 0.24, or $f_s > 1.66$, the system can only be in the gas phase. At the critical points B and C , the strangeness fraction of liquid and gas phases is the same. The liquid-gas phase transitions at these two points are the same as in symmetric nuclear matter. The pressure stays constant and f_s does not change during the phase transition.

For the linear case, there are also two kinds of solutions which satisfy the Gibbs equations at $T=15$ MeV. However, the third critical point C disappears because it moves to the point with $f_s=2$. This is because at this temperature the system can be in two-phase coexistence when $f_s > 1.75$. This can be seen clearly from Fig. 2 where the unphysical region exists for very large strangeness fraction f_s .

Therefore, for the case of the square root ansatz, altogether there are three critical strangeness fractions for strange hadronic matter at temperature $T=15$ MeV, while for the linear definition of the effective baryon mass, there are only two critical points. In Fig. 7, we plot the critical temperature versus strangeness fraction. For the square root case, the critical temperature first decreases with increasing f_s . When $0.65 < f_s < 1.3$, T_c increases with f_s . For f_s greater than 1.3, T_c decreases with f_s again. In the range $14.6 < T < 15.3$ MeV, there are three critical strangeness fractions for a given temperature. For example, at $T=15$ MeV, the three critical values of f_s are about 0.24, 1.06, and 1.66. If $0.24 < f_s < 1.04$, the strange hadronic matter can

only be in the gas phase. When $1.04 < f_s < 1.66$, the system can be in a state of liquid-gas coexistence, while for $f_s > 1.66$, the system can once again only be in the gas phase. The critical temperature for strange hadronic matter with $f_s=2$ is about 13.6 MeV. For the case of the linear definition of effective baryon mass, T_c decreases with increasing f_s if f_s is smaller than 1.1. If f_s is larger than 1.1, T_c increases with f_s till $f_s=2.0$. Compared with the square root case, T_c changes more quickly with f_s in this case. The highest and lowest T_c values are 17.9 and 12.0 MeV with the corresponding $f_s=0$ and $f_s=1.1$.

V. SUMMARY

In this paper, we applied the chiral SU(3) quark mean field model to investigate the properties of strange hadronic matter at finite temperature and density. All the parameters have been determined in earlier papers and there is no further parameter to be adjusted. The model works very well at zero temperature. The saturation properties and compression modulus of nuclear matter are reasonable. The hyperon potentials are close to the empirical values for strange hadronic matter. The results of finite nuclei and hypernuclei are also consistent with the experiments. In this paper, the liquid-gas phase transition of strange hadronic matter was studied in this model. For strange hadronic matter, there are two independent conserved charges. The system will be in liquid-gas phase coexistence if the pressure and the chemical potentials of all the baryons are the same in the two phases. During the phase transition, the strangeness fraction f_s of liquid and gas phases is different and changes during the phase transition, though the total f_s is conserved. We found that there are two branches of solutions which satisfy the Gibbs equations at some range of temperature. One corresponds to the phase transition at small f_s , while the other corresponds to the phase transition at large f_s . For the square root ansatz of the effective baryon mass, there are three critical strangeness fractions during $14.6 < T < 15.3$ MeV. For the linear definition of effective mass, there are two critical points when $12.0 < T < 15.9$ MeV since the third one disappears and moves to the point with $f_s=2$. The difference of f_s in the two phases is small, especially in the case of higher strangeness fraction. The critical temperature T_c does not change monotonically with f_s . For the square root case, if $f_s < 0.65$ or $f_s > 1.3$, T_c decreases, while for $0.65 < f_s < 1.3$, T_c increases with increasing f_s . For the linear case, T_c first decreases and then increases with increasing f_s . The minimum critical temperature is about 12.0 MeV with $f_s=1.1$.

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