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# Finite-Range Regularisation of Chiral Effective Field Theory

Ross D. Young

Supervisors: A/Prof. D. B. Leinweber and Prof. A. W. Thomas

*Special Research Centre for the  
Subatomic Structure of Matter*

and

*Department of Physics,*

*University of Adelaide,*

*Australia*

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## Abstract

Modern calculations of quantum chromodynamics (QCD) in lattice gauge theory are restricted to the simulation of quark masses which are significantly heavier than those observed in nature. In order to compare with experiment, an extrapolation of lattice QCD results is required. A low-energy effective field theory (EFT), chiral perturbation theory ( $\chi$ PT), provides an optimal framework for studying the quark-mass variation of hadron properties. Established as a rigorous expansion about the limit of vanishing quark-masses, convergence of the EFT at the scale of masses explored in lattice QCD is not guaranteed.

Working with various regularisation schemes, a quantitative analysis is performed to investigate the convergence properties of the quark-mass expansion. The traditional formulation of  $\chi$ PT, defined by a minimal subtraction renormalisation scheme, is found to have a limited range of applicability. An alternative scheme, namely finite-range regularisation (FRR), displays significantly enhanced convergence properties. Consequently, the use of FRR permits the reliable extrapolation of modern lattice simulation results to the physical regime.

The study of the quark-mass dependence of nucleon properties in quenched lattice QCD provides valuable insight into the structure of the nucleon. Application of finite-range regularisation reveals a phenomenological link between quenched QCD and the physical theory. This work offers an improved understanding of the dynamical role of the pion field in baryon spectroscopy and electromagnetic structure.

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