Riemannian Non-commutative Geometry

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Abstract

The elements of non-commutative geometry are presented from an operator algebraic viewpoint. Threaded through the presentation is the example of a spectral triple associated to a second countable metrisable locally compact oriented manifold without boundary and without the assumption of spin structure.

Generalisation of the spectral triple associated to such a manifold admits the new notion of a Riemannian representation of a C*-algebra which directly links to the standard theory of von Neumann algebras. The involvement of the standard theory and the reformulation of the axioms of non-commutative geometry in the absence of spin structure are investigated and presented.

The construction of Riemannian representations of C*-algebras is also considered. A new generalisation of a symmetric derivation on a von Neumann algebra R provides the means of constructing Riemannian representations of a C*-subalgebra $A \subset R$ associated to a faithful finite trace on R. The interaction between the standard theory and the generalised symmetric derivation provides new analysis into the structure of K-cycles.

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