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NON-LINEAR FREE-SURFACE FLOWS
ABOUT BLUNT BODIES

by

Lawrence K. Forbes

B.Sc.(Hons.), University of Adelaide, South Australia.

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SUMMARY

In this thesis, three problems of inviscid free-surface hydrodynamics are investigated. The behaviour of the fluid at the free surface is governed by non-linear equations, and the free surface of the stream is disturbed by an immersed body. The body may be regarded as blunt, in the sense that its forward face is somewhere perpendicular to the direction of the flow of the oncoming stream. There is thus a stagnation point on the forward face of the body, so that approximate theories which involve assumptions of body slenderness may not be applied directly.

The first of these problems is discussed in chapter one. Here, a blunt-nosed body, such as a bridge pier, is considered to be standing in a fast-flowing shallow stream. The non-linear shallow water equations are used, and the bow wave upstream of the body is regarded as a shock front (hydraulic jump), across which suitable jump conditions are prescribed. The problem is then solved inversely, by assuming a known shape for the upstream bow wave, and seeking to determine the position and shape of the body. The flow variables are expressed as Taylor series expansions about the bow wave, and the computer is used to obtain numerical values for the coefficients. Several singularities in the flow field are discovered and discussed.

The second problem, discussed in chapter two, concerns two-dimensional flow of an ideal fluid in a horizontal stream, attached to the bottom of which is a semi-circular obstruction.

Infinitely far upstream, the fluid flows uniformly, with a known Froude number F. A new linearized theory is presented, which is valid for semi-circular obstructions of small radius, and accounts for the behaviour of the fluid at the stagnation points on the bottom. This theory predicts a train of downstream waves whenever the flow is subcritical $(F < 1)$, and a symmetric wave-free surface profile whenever the flow is supercritical $(F > 1)$. The exact non-linear equations are then solved numerically at the free surface using a boundary-integral technique and a Newton-Raphson procedure. Non-linear solutions possessing a train of downstream waves are obtained for $F < 1$, and solutions free of waves for $F > 1$. The non-linear results suggest that the validity of the wave-like solution may extend into $F > 1$, overlapping with the domain of validity of the wave-free solutions.

In chapter three, the semi-circular obstruction of the previous chapter is generalized to include the case of a semi-elliptical body attached to the bottom. Attention is confined to the subcritical region $F < 1$, where it is shown that, for ellipses of certain special lengths, the non-linear downstream wave amplitude may be made to vanish, resulting in zero wave resistance.

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