INTERACTING TACHYONS IN CLASSICAL AND QUANTUM PHYSICS

by

C. M. Ey, B.Sc.(Hons)

A thesis submitted in accordance with the requirements of the degree of Doctor of Philosophy.

Department of Mathematical Physics

The University of Adelaide

South Australia

February 1979

Awarded August 1979

CONTENTS

SUMMARY

This thesis examines classical and quantum-mechanical aspects of the hypothetical faster-than-light particles called tachyons. Chapter 1 demonstrates that tachyons do not violate the principles of relativity, and that, with the aid of a reinterpretation principle to eliminate negative energies, tachyons can be characterized as particles of real, spacelike 4-momentum. We find that neither special relativity nor requirements of macroscopic causality can eliminate as legitimate objects of experimental and theoretical enquiry tachyons that cannot be used for faster-than-light signal transmission. Lacking any experimental observations of tachyons, we propose that a theoretical study of tachyons within current physical theories be made which, as far as possible, keeps the structure of those theories intact. Theories which attempt to extend special relativity to include faster-than-light reference frames are not in accord with this program, and, as explained in chapter 2, these approaches seem unsatisfactory.

In chapter 3 we treat the classical, charged tachyon within conventional electromagnetic theory, and in an explicitly Lorentz-invariant way. We prove that a charged tachyon would not emit electromagnetic radiation in a vacuum regardless of its state of motion. This is done by showing that the Lorentz-invariant equation of motion which can be derived for a tachyon cannot have radiation reaction terms, so that by energy-momentum conservation, the emission of radiation is impossible. The only possible radiation reaction terms are noncovariant. The method of deriving the equation may be applied to slower-than-light charged particles, and a new derivation

of the Lorentz-Dirac equation results.

We consider the application of quantum field theory to tachyons in chapter 4. A theory based on the real-energy solutions of the Klein-Gordon equation with imaginary mass is shown to provide the best opportunity for describing spinless tachyons in quantum field theory. We require that the theory be Lorentz-invariant, incorporate the reinterpretation principle in some way to remove negative energies, and be as close as possible to conventional quantum theory. After an examination of various quantization procedures, we adopt the proposal of Arons and Sudarshan as best fulfilling these requirements. We show that reinterpretation must necessarily be applied only to the matrix elements of the theory, and cannot be formulated as an operation on the states.

Difficulties in describing the interactions of tachyons in quantum theory are discussed in chapter 5. We consider a perturbation-type expansion for the S-matrix and show that if the first order term is any polynomial in the tachyon field and its conjugate, then the reinterpreted S-matrix will violate unitarity. A derivative-type coupling and the usual coupling to the electromagnetic field are similarly ruled out. The indications are that if a consistent interacting theory of tachyons were possible, then it would be nonlocal. This conclusion is supported by two simple model calculations, based on the fixed-source and pair models, in which the source function for the interactions must, for consistency, be nonlocal, so that the tachyons only interact over an extended spatial region.

STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university, and to the best of my knowledge and belief, contains no material previously published or written by another person, except when due reference is made in the text.

Christopher M. Ey.

ACKNOWLEDGMENTS

I would like to thank my supervisor, Professor C.A. Hurst, for his constant assistance and encouragement throughout the course of this work, and also Dr. A.L. Carey for most helpful discussions.

I am grateful for the receipt of a Commonwealth Postgraduate Research Award (1974-1978) and a University Research Grant (Feb.-April 1978).

CHAPTER I

INTRODUCTION

The aim of this thesis is to examine, both within the special theory of relativity and in quantum field theory, an hypothesis concerned with the existence of particles travelling faster than light in a vacuum. It was only comparatively recently that the generally-held belief that faster-than-light particles were forbidden by special relativity was convincingly challenged. In this first chapter we shall present the arguments which show that faster-than-light particles, called tachyons, are not incompatible with the principles of special relativity, and that a concept of a tachyon can be arrived at which is wholly consistent with that theory. Later chapters will use this concept to treat tachyons in the framework of classical electromagnetism and quantum field theory, for we believe that the success or otherwise of incorporating tachyons into such theories is an indication of their possible existence, at least to the extent which these theories reflect the true workings of nature.

1.1 The Possibility of Faster-than-Light Particles

Before the advent of the special theory of relativity in 1905, and against the background of Newtonian concepts of space and time, particle velocities in excess of that of light were considered unproblematic, if somewhat remarkable. Sommerfeld [1,2,3] and others [4,5] considered the electromagnetic fields associated with charged superluminal (faster-than-light) particles. It was concluded that such particles would emit electromagnetic radiation even if they were in a vacuum. The modern descendents of the particles considered by Sommerfeld, now called tachyons [6], are also under the suspicion of spontaneously radiating in a vacuum if they are electrically charged. This problem will form the subject of our chapter 3.

For a long time after the arrival of special relativity, faster-than-light particles were considered impossible for the reason that they seemed to be forbidden by that theory. The proof consisted of taking the relativistic formulae for the energy and momentum of a particle of rest mass m_0 and velocity $\underline{v}$. These are (see, for example, ref. [7])

$$E = \frac{m_0}{(1-v^2)^{\frac{1}{2}}} \tag{1.1}$$

and

$$|\underline{p}| = \frac{m_0 v}{(1-v^2)^{\frac{1}{2}}} \quad , \tag{1.2}$$

where $v \equiv |\underline{v}| < c$, and we use units in which the speed of light c is equal to 1. From (1.1) and (1.2) it is seen that as v approaches the speed of light from below, both the energy and momentum of the particle become infinite. Further, were the velocity to be greater than 1, both the energy and momentum would become imaginary. Then, if more evidence against superluminal particles was required, there was Tolman's paradox [8], which seemed to show that if superluminal particles existed, then communication with the past would be possible.

The apparent strength of these arguments virtually eliminated any consideration of superluminal particles or effects, but some exceptions were the efforts of Schmidt [9], Arzeliès [10], Tanaka [11], Terletskii [12,13], and Bilaniuk, Deshpande, and Sudarshan [14], which will be considered in due course. It was the last mentioned paper which gave rise to a great increase in the interest shown in the possibility of superluminal particles, for it seemed to provide answers to all the objections arising out of special relativity. Let us explain how the above objections were overcome by Bilaniuk, Deshpande, and Sudarshan, thus allowing a coherent description of a tachyon which does not violate relativistic principles. We postpone a treatment of the causality arguments such as Tolman's until section 1.3 because, as we shall show, those arguments are not based solely on the special theory of relativity.

Firstly, the superluminal particles we shall be treating are to be regarded as a new class of particles quite distinct from the usual slower-than-light particles (hereafter collectively called bradyons [15,16,17]) and the luxons (particles like photons and neutrinos which always travel at the speed of light). There is no question of accelerating bradyons to superluminal velocities. In fact it is only this process which Eqs. (1.1) and (1.2) disallow. This is clearly the sense in which we must take Einstein's statement concerning the impossibility of velocities greater than light [18], for it was made after an examination of the energy formula (1.1). Tachyons, then, are supposed to be created with velocities greater than light. They do not have to be accelerated to such speeds just as photons and neutrinos do not have to be accelerated up to their speed which is always equal to c.

Now we must require that tachyons have real energy and momentum because these are measurable quantities which are exchanged with other particles during interaction. But from (1.1) and (1.2) we see that this can only be the case if the tachyon has an imaginary rest mass, so that if

$$m_0 = im, \tag{1.3}$$

then the energy and momentum of a tachyon will be real and given by

$$E = \frac{m}{(v^2-1)^{\frac{1}{2}}} \tag{1.4}$$

and

$$|\underline{p}| = \frac{mv}{(v^2-1)^{\frac{1}{2}}} , \tag{1.5}$$

where v is greater than 1. We see that the velocity of a tachyon is given by the usual expression

$$v = \frac{|\underline{p}|}{E} \tag{1.6}$$

and that the energy-momentum 4-vector

$$p^{\mu} = (E,\underline{p}) \tag{1.7}$$

is spacelike since

$$[p]^2 = E^2 - |\underline{p}|^2 = -m^2 \tag{1.8}$$

is negative. (See appendix A for the notational conventions which we shall use throughout).

That we attribute an imaginary restmass to a tachyon is no cause for concern because this is not a measurable quantity if tachyons can never be brought to rest. We see from (1.4) and (1.5) that the measurable quantities of energy and momentum will be real so long as the velocity v of the tachyon remains in excess of $c = 1$. As the velocity decreases towards 1, both the energy and momentum tend to infinity so that for tachyons the speed of light is just as unobtainable as it is for bradyons. For tachyons, however, it is a minimum velocity and they slow down as they gain energy. Such behaviour is indeed strange but no inconsistency is involved. When we refer to the mass of a tachyon we shall mean the real quantity m appearing in the equations above. There is no real need to speak of imaginary quantities like $m_0 = im$ at all.

At infinite velocity the energy of a tachyon is zero and the momentum $|\underline{p}|$ takes its smallest value m. In a reference frame in which the tachyon has infinite velocity it appears to be instantaneously everywhere on a straight line in space. Such tachyons will be called <u>transcendent</u>.

Having a spacelike 4-momentum, tachyons are open to another objection: by an ordinary (proper, orthochronous) Lorentz transformation we may change the sign of the zero-component of a spacelike 4-vector so the energy of a tachyon can change sign under some Lorentz transformations. This may be seen from an examination of the surfaces in the four dimensional energy-momentum space that are defined by the points $(E,\underline{p})$ satisfying the condition for a fixed mass m, which is given by (1.8) for tachyons. The surface obtained in this way is an hyperboloid of revolution about the energy axis and is illustrated in Fig. 1.1. Notice that it is single-sheeted, and that proper

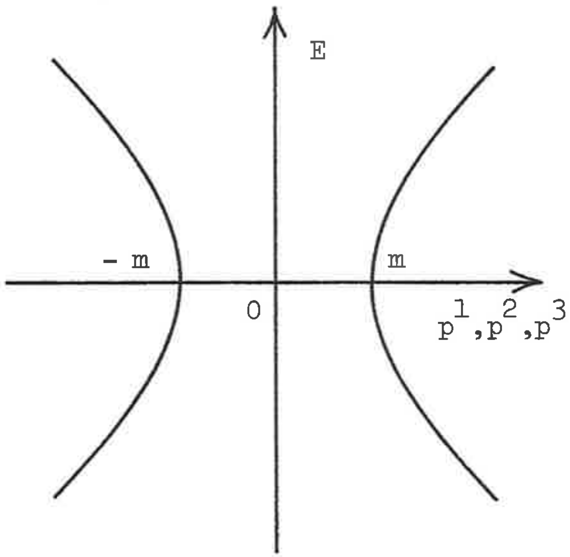

FIGURE 1.1. The tachyon energy-momentum surface defined by $E^2 - |\underline{p}|^2 = -m^2$.

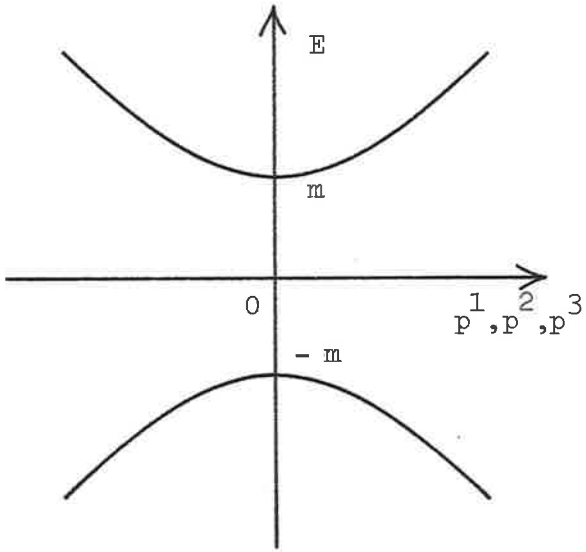

FIGURE 1.2. The bradyon energy-momentum surface defined by $E^2 - |\underline{p}|^2 = m^2$.

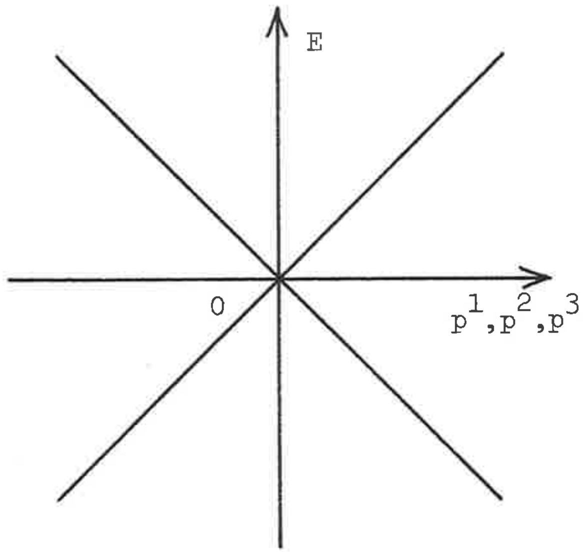

FIGURE 1.3. The luxon energy-momentum surface defined by $E^2 - |\underline{p}|^2 = 0$.

orthochronous Lorentz transformations simply move the point $p^\mu = (E,\underline{p})$ around on the surface, so that by a continuous transformation we can change the energy from positive to negative. This is a most important difference from the case for bradyons where the corresponding surface is defined by

$$[p]^2 = m^2$$

with m real and positive. This surface, illustrated in Fig. 1.2, consists of two separated sheets, one with points for any positive energy greater than m and the other containing all negative energies less than -m. Proper Lorentz transformations will not take points from the upper to the lower sheet or vice versa. The sign of the energy is also an invariant for luxons which satisfy

$$[p]^2 = 0.$$

The energy-momentum surface so defined consists of two cones joined at their apexes (Fig. 1.3).

So if the 4-momentum of a positive-energy tachyon according to a frame S is

$$p^\mu = (E,\underline{p}) \ , \qquad E > 0 \ ,$$

then there are frames S', moving with a velocity $\underline{w}$ with respect to S, in which the energy is negative. The Lorentz transformation of the energy reads [7]

$$\begin{aligned} E' &= (1 - w^2)^{-\frac{1}{2}} (E - \underline{w}.\underline{p}) \\ &= (1 - w^2)^{-\frac{1}{2}} E(1 - \underline{w}.\underline{v}) \ , \end{aligned} \qquad (1.9)$$

where $\underline{v}$ is the velocity of the tachyon in the frame S. Now since the velocity of the tachyon is greater than light, we can find a frame S' with velocity $\underline{w}$ such that

$$\underline{w}.\underline{v} > 1 .$$

If for simplicity we take $\underline{w}$ parallel to $\underline{v}$, then we may choose

$$w > \frac{1}{v} , \qquad (1.10)$$

and this can be fulfilled since the right-hand side of (1.10) is less than 1. The energy E' in the frame S' will then be negative. Now negative energies cannot be tolerated because they would allow events to occur which are clearly unphysical. For example, the vacuum would not be stable because groups of positive- and negative-energy particles could be produced whilst conserving energy and momentum in the process. The problem of negative energies for tachyons can be overcome in a way which can also be used to solve the related problem of inverted temporal ordering, so we shall first explain this latter problem.

The worldline of a tachyon is a spacelike path because the tachyon always travels faster than light. That is, if

$$dx^{\mu} = (dt, dx, dy, dz)$$

is the 4-vector which connects two nearby points on the worldline of the tachyon, then the invariant quantity

$$ds^2 \equiv dt^2 - dx^2 - dy^2 - dz^2 = dt^2 - d\underline{x}^2$$

is negative, so that the invariant Minkowski interval ds and the proper time

$$d\tau = \frac{ds}{c} ,$$

are imaginary. As for the rest mass, and other "proper" quantities like length, the fact that the proper time is imaginary is not a problem because it is not measurable. We may simply use the invariant $-ds^2$ and define the invariant parameter along the worldline of the tachyon to be

$$d\lambda \equiv \sqrt{-ds^2} = dt(v^2 - 1)^{\frac{1}{2}} , \tag{1.11}$$

where

$$\underline{v} = \frac{d\underline{x}}{dt} ,$$

so that the 4-velocity of the tachyon is

$$v^\mu = \frac{dx^\mu}{d\lambda} = (v^2 - 1)^{-\frac{1}{2}} (1, \underline{v}) , \tag{1.12}$$

and is a spacelike 4-vector with

$$[v]^2 = -1 . \tag{1.13}$$

Multiplying the 4-velocity by the real mass parameter we get the 4-momentum

$$p^\mu = mv^\mu \tag{1.14}$$

for which we have

$$[p]^2 = -m^2 \quad , \qquad (1.15)$$

which is just Eq. (1.8).

As the 4-vector displacement dx^μ along a small segment of the worldline of a tachyon is spacelike, an ordinary Lorentz transformation can change the sign of the time separation. So, in the frame S, if A is the event of the emission at the spacetime point x^μ of a tachyon of velocity v, and B is the absorption at a later time at the spacetime point $x^\mu + dx^\mu$, with $dx^0 > 0$, then there are other Lorentz frames S' for which dx'^0 is negative. That is, in such a frame S', the absorption of the tachyon occurs before it was created. From the Lorentz transformation for the time,

$$dx'^0 = (1 - w^2)^{-\frac{1}{2}} (dx^0 - \underline{w}.d\underline{x})$$

$$= (1 - w^2)^{-\frac{1}{2}} dx^0 (1 - \underline{w}.\underline{v}) \quad ,$$

we see that this will appear to be the case in all Lorentz frames which move at a velocity $\underline{w}$ with respect to S such that

$$\underline{w}.\underline{v} > 1 \quad .$$

But, from (1.9), this is precisely the situation in which the energy changes sign. So with the 4-momentum given by (1.14), we find that in these frames we have a negative-energy tachyon travelling backwards in time from its absorption to its creation. The recognition that the problem of negative energies and this problem of inverted temporal ordering of events are essentially the same led Bilaniuk, Deshpande, and Sudarshan [14] to formulate an interesting solution which they called the "reinterpretation principle" [19].

The reinterpretation principle

According to this principle, the observer S' will not see a negative-energy tachyon of 4-momentum $(E',\underline{p}')$ propagating backwards in time from its absorption at B to its emission at A. Instead S' will observe a positive-energy tachyon with the 4-momentum $(-E',-\underline{p}')$ emitted at the event B at the time $x'^0 + dx'^0$, which then travels forwards in time until it is absorbed at the event A at the time x'^0. The absorption at A is later than the emission at B because dx'^0 is negative. The observer S' will therefore see the whole segment of the worldline between the events in the opposite order to which S sees them, and will see as emission what S describes as absorption. This is a consistent interpretation of the events in the frame S' because the absorption of an amount of negative energy at B, for example, is equivalent to the emission of an amount of positive energy. We notice that a similar inversion happens to all variables like charge during reinterpretation. If the absorption of a quantity of charge is observed from a frame in which the time ordering is reversed, then, according to the reinterpretation principle, it will look like the emission of the negative of that charge. We appear to have a change between particle and antiparticle upon reinterpretation of tachyons. In fact the reinterpretation principle is similar to the Stückelburg-Feynman [20,21] interpretation of positrons as negative-energy electrons travelling backwards in time. The negative-energy electrons would be associated with the lower sheet of the mass hyperboloid in Fig. 1.2.

By the use of this principle, all the objections to tachyons which arise from special relativity can be removed. All observers use the same physical laws and will see the same events but give different interpretations to them.

1.2 Properties of Tachyons

Having obtained a consistent description of tachyons within special relativity, we are now able to examine some of the properties which relativity together with the reinterpretation principle attributes to these particles. Firstly we note that the velocity addition law remains valid. Let S' be a frame moving with a velocity w along the positive x-axis of a frame S in which a tachyon has velocity $\underline{v}$. Then, by transforming the 4-velocity (1.12) for example, the velocity of the tachyon in the frame S' will be

$$\underline{v}' = \left(\frac{v^1-w}{(1-wv^1)}\, ,\, \frac{v^2}{\gamma_w(1-wv^1)}\, ,\, \frac{v^3}{\gamma_w(1-wv^1)}\right) \qquad (1.16)$$

where

$$\gamma_w = (1 - w^2)^{-\frac{1}{2}} \quad .$$

From (1.16) we may obtain

$$1 - |\underline{v}'|^2 = \frac{(1-|\underline{v}|^2)(1-|\underline{w}|^2)}{(1-wv^1)^2} \qquad (1.17)$$

from which we see that if v is greater 1, then v' will always be greater than 1 because the relative velocity w of the frames S and S' must be less than 1. That is, tachyons travel faster than light according to all observers.

By using the conservation of 4-momentum we may determine the types of processes which tachyons might engage in. Some of these are quite different from processes allowed to bradyons and luxons, and they have suggested ways in which experiments might detect tachyons.

Below we shall mention some allowed processes. We need only consider tachyon 4-momenta having positive energy because we know from the reinterpretation principle that a process in which a negative-energy tachyon of 4-momentum p^μ appears in the final state, say, is actually observed as a process in which a positive-energy tachyon of 4-momentum $-p^\mu$ was present among the initial particles.

One unusual process which is allowed by energy-momentum conservation is the decay of a tachyon of mass m into two or more tachyons, all having the same mass m. In fact two tachyons of mass m may combine to form a bradyon, a luxon, or another tachyon. This may be demonstrated by adding two tachyon 4-momenta, p^μ and q^μ, which may be parametrized as

$$p^\mu = m(\sinh\alpha, \cosh\alpha\, \hat{\underline{p}})$$

$$q^\mu = m(\sinh\beta, \cosh\beta\, \hat{\underline{q}}) \quad , \qquad \alpha,\beta \geqslant 0,$$

where $\hat{\underline{p}}$ and $\hat{\underline{q}}$ are unit 3-vectors in the direction of the respective 3-momenta, and α and β must be positive in order that p^μ and q^μ have positive energy. The tachyons then have mass m because

$$[p]^2 = [q]^2 = -m^2 \quad ,$$

and they may combine to form a particle, the square of the mass of which is

$$[p+q]^2 = 2m^2(\sinh\alpha\sinh\beta - \cosh\alpha\cosh\beta\cos\theta - 1), \qquad (1.18)$$

where θ is the angle between $\hat{\underline{p}}$ and $\hat{\underline{q}}$. Clearly α,β, and θ may be chosen so that (1.18) is positive, negative, or zero, so that the composite particle may be a bradyon of any mass, a tachyon of any mass, or a luxon. Therefore, it is energetically possible for bradyons and luxons to decay into any number of tachyons, so if tachyons exist their interactions with ordinary particles must be such that the stable particles which we observe are protected from these decays.

An ordinary particle of mass M may also emit a tachyon while preserving its identity. However it can only do so if it is in motion. If p is the initial 4-momentum of the bradyon and p' its final 4-momentum after emitting a tachyon with 4-momentum q, when we must have

$$p - q = p' \quad .$$

Squaring, we obtain

$$-\frac{m^2}{2} = p.q = p^0 q^0 - \underline{p}.\underline{q} \quad , \tag{1.19}$$

where m is the mass of the tachyon. This cannot be satisfied for an initially stationary bradyon, for which

$$p^{\mu} = (M,0,0,0) \quad ,$$

unless the energy of the tachyon, q^0, is negative. But, by the reinterpretation principle, if q^0 were negative this process would appear as the absorption of a tachyon by a stationary bradyon, and not as emission.

Energy and momentum can also be conserved in the process of a tachyon of mass m emitting a photon without changing its mass. A tachyon of any energy may perform this emission which for charged tachyons has been called vacuum Cherenkov radiation. If p and q are the initial and final tachyon 4-momenta and k is the photon 4-momentum, then conservation requires that

$$\cos\theta = \frac{\sqrt{\underline{q}^2 - m^2}}{|\underline{q}|} = \frac{1}{v} \quad , \tag{1.20}$$

where θ is the angle between the initial tachyon 3-momentum and the 3-momentum of the emitted photon, and v is the initial tachyon velocity. As $|\underline{q}|$ tends to m, that is, as the initial tachyon approaches the zero-energy transcendent state, the angle θ tends towards 90°.

If the initial tachyon emitted more energy than was required to reduce it to the transcendent state, then the final tachyon would have negative energy. By reinterpretation, this would appear as a process in which two tachyons were present initially and only the photon occurred in the final state. This would be described as pair annihilation. For tachyons then, Cherenkov radiation and pair annihilation are essentially the same process in that one can be converted into the other by a Lorentz transformation [6].

Other processes involving tachyons may easily be checked for energy-momentum conservation restrictions in a similar way. Those mentioned above will be of some later use.

1.3 Causality

It has been shown that by the use of the reinterpretation principle, an idea which is by no means unfamiliar in physics, one is able to obtain a satisfactory description of tachyons and discuss processes which are allowed by energy-momentum conservation. In fact the most stringent theoretical restrictions on the existence of tachyons come not from special relativity alone, but from the extra requirements of macroscopic causality. If faster-than-light effects are allowed unrestricted admission into special relativity, then there are numerous arguments to show that causal paradoxes will arise out of the lack of an invariant time ordering along spacelike worldlines. The prototype of such arguments was given by Tolman [8], and we shall consider it now.

Tolman's paradox

Suppose that an observer A at the origin of a reference frame S uses tachyons with a speed $v > 1$ to send a signal to another observer B who is at rest in a frame S' which is moving with a speed $w(< 1)$ along the positive x-axis of the frame S. It is supposed moreover that

$$w > \frac{1}{v} \quad . \tag{1.21}$$

This condition is required for the paradox to work [6]. The situation is most clearly seen from a spacetime diagram. In Fig. 1.4, A_1 is the event of the emission of the signal by A and B_1 is its reception by the observer B. The observer B replies to A with another tachyon signal which is sent at a speed $u > 1$ relative to the frame S'. The observer A receives the reply in the event A_2. But it is clear that the speed of the reply signal, u, can be chosen so that the event A_2 in the frame S occurs before the event A_1 as shown in Fig. 1.4. So although the events A_1 and B_1 are in a causal relationship (A_1 caused B_1) and B_1 in turn caused A_2, it appears that the effect A_2 is in a position to interfere with its original cause which was the event A_1. This is a violation of

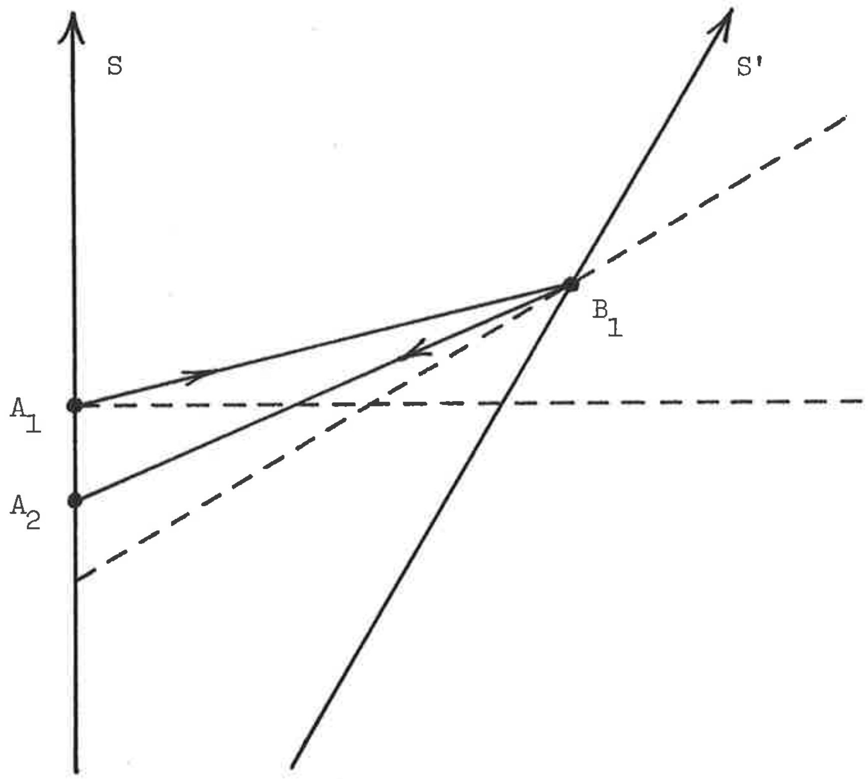

FIGURE 1.4. Tolman's Paradox.

The observer A in the frame S sends a tachyon signal to B who is in the moving frame S' ($A_1 \rightarrow B_1$). The observer in S' replies ($B_1 \rightarrow A_2$), but the event A_2 occurs before A_1. The causal chain of events closes upon itself ($A_1 \rightarrow B_1 \rightarrow A_2 \rightarrow A_1$), which is a logical contradiction. The dashed lines through A_1 and B_1 are lines of simultaneity for A and B respectively.

the principle of causality because effects precede their causes, and the observer A seems capable of changing his own past history. This type of logical contradiction, called a closed causal cycle, can, of course, not be tolerated. From this and similar paradoxes, many drew the conclusion that faster-than-light particles are an impossibility.

Reinterpretation

However, the introduction of the reinterpretation principle at first seemed to answer all causality objections [14,19,22,23]. With this principle in mind, it is clear that the observer A will not see the event A_2 as absorption or detection of tachyons, but will see it as emission. It was assumed, therefore, that according to A both events A_1 and A_2 are causes which have their effects in the event B_1. Similarly, the observer B sees both signals as emissions from his own apparatus, and sees A_1 and A_2 as effects having their causes in B_1.

In Tolman's paradox, and later causal paradoxes, there must always occur what amounts to the sending of signals between two observers in relative motion who do not agree upon the time ordering of the two ends of the worldline of one or more of the signals. This is the reason for the requirement (1.21) in Tolman's paradox. So for each observer, the reinterpretation principle is always able to break the closed causal cycles by reinterpreting those parts of the cycle in which the signal is being transmitted into the past because, in order to close the cycle and obtain the logical contradiction, there must always be such signals somewhere in the cycle. In this view, the assignment of events as causes or effects is not regarded as invariant from one frame to another, and the designation is altered by the reinterpretation principle so that every observer sees causes preceding their effects.

Feinberg [6] did not assume this total reversal of cause and effect. He believed that the observer A in the situation above would describe events at A_2 as an uncorrelated spontaneous emission from his apparatus. The paradox was interpreted as proving that reliable signalling using tachyons was impossible, and that there could be no control over either production or absorption of tachyons. We shall return to this view later.

More paradoxes

So, whereas some deduced from Tolman's paradox that faster-than-light particles were impossible, others, who used the reinterpretation principle, seemed to be saying that it carried no weight whatsoever. A number of more complicated paradoxes were then put forward to challenge the solution provided by the reinterpretation principle. The paradox originally described by Bohm [24] sought to remove the ambiguity about sender and receiver in the Tolman paradox by introducing two more reference frames. The event B_1 is supposed to coincide with the origin of a frame S" which is stationary with respect to the frame S. The observer A then sends his signal to the observer in S" who will receive it in the event B_1. Both observers agree that A sent the signal because they are stationary with respect to each other and there can be no inversion of temporal ordering for them. The observer in S" may then communicate the signal from A to the observer B in S', who is passing by at the time. This may be done by conventional means (a light signal for example) so that both agree that the signal had its origin with the observer in S". Similarly another frame S"' having its origin at the event A_2 and stationary with respect S' is introduced to remove the ambiguity about the direction of the signal from B_1 to A_2. The observer in S"' then communicates the message to the observer A in S and the causal cycle is reestablished.

But, if the solution to the Tolman paradox is to be believed, we can solve this paradox by noting that A will see the message as transmitted from A_2 and absorbed at B_1 even though the observers in S' and S"' behave as though the cause-effect relation between these events is in the reverse [23]. Thus A will not regard what the observer in S"' has to tell him at A_2 as information from his own future. It is supposed that by looking at the events according to each observer separately in this way, we find that each observer will describe a consistent and causal sequence of events.

Pirani [25] invented a paradox which involved four observers, A, B, C, and D, all in a prescribed relative motion. He showed that, by using tachyon signals, a message initially sent by A could be sent from one observer to another until the message arrived back at A at a time which is prior to the transmission of the original signal from A. It was claimed that for each observer only the detection and emission of positive-energy signals was required, and that there was no opportunity for applying the reinterpretation principle to resolve the paradox. However it was shown by Parmentola and Yee [26] that this is not the case, for although each observer agrees that he receives the message and then transmits it to the next observer, he will not agree that the other observers in the cycle acted in this way. So again there is the opportunity for reinterpretation to break the causal cycle by reversing the cause-effect relation in the remaining signal worldlines.

We shall return to Tolman's paradox described earlier because it is now clear that it contains all the essential features of the more complicated paradoxes. Let us suppose that, instead of merely sending a message from event B_1 to event A_2, the observer B in the frame S' decided to send a very intense tachyon beam which destroyed the apparatus belonging to A. If A's apparatus had been destroyed at A_2, then how could he possibly have sent the signal at A_1 which was supposed to initiate the whole sequence of events? Scenarios such as this were invented to try to obtain a clear statement of which event was the cause and which the effect. But this was avoided by explaining the situation as follows. In the frame S', B emits both signals. One destroys A's apparatus at A_2 and the other, arriving later, cannot be intercepted by A who no longer has any apparatus so the signal by-passes A at A_1 and proceeds towards its eventual absorption at some remote place in the universe.

Now in the frame S, A sees his apparatus self-destruct at A_2 whilst emitting a very strong tachyon signal to B. Later, at A_1, A realizes that he has been passed by a tachyon beam from remote sources. This beam, surprisingly, lands on B's detector at B_1. Reinterpretation therefore restores to each observer a logically consistent sequence of events, but we now see that any description of tachyon signals must take remote sources into account, as was pointed out by Root and Trefil [27]. In some frames it looks as though some very remote tachyon sources have obligingly conspired together to produce the very signals required to eliminate any causality paradox.

Signalling and irreversibility

It is now becoming clear that there is something most unsatisfactory about the solutions which have been offered for the paradoxes so far.

Observers in all frames certainly see logically consistent events and causes always precede effects, but there occur some wildly improbable events. This is precisely the matter which has been overlooked in dismissing the paradoxes by merely performing a thorough going reinterpretation at every possible opportunity. In fact Terletskii [12,13], who did not use the reinterpretation principle, drew essentially the correct conclusions concerning the existence of faster-than-light particles from the causal paradoxes. The important point which has so far been ignored is that for a recognizable signal to be transmitted there must occur macroscopic, irreversible processes in both the transmitter and the receiver. To have a cause-effect relationship established by a signal transmission, there must be a transfer of information which is to be defined in terms of entropy [13]. With this definition of cause and effect, a Lorentz transformation may reverse the time ordering of two events if they are connected by a tachyon signal, but it cannot interchange cause and effect. Therefore the assumption made previously that the reinterpretation principle acts on cause and effect in the same way it acts on emission and absorption is wrong.

Here it is useful to keep in mind the definition of cause and effect used by Newton [28]. Suppose that an occurrence e_1 can be produced at will and that it is made to occur at time intervals given by random number tables, or by some other procedure which we can be sure is random. Then another occurrence e_2 is said to be an effect of the cause e_1 if there is a statistical correlation measured between the events. This definition of the causal direction is independent of the time ordering, and the assignment of some events as causes and others as effects will be invariant. Conceivably the events

e_2 could precede the e_1 and yet e_1 is still recognized as the cause on the basis of this definition using statistical correlations. But if this were the case, we could use the paradoxes above to arrive at a logical contradiction [29]. The reinterpretation principle cannot help here because it is not able to interchange cause and effect, so that we must admit Terletskii's [13] conclusion that faster-than-light signal transmission, or information transfer, of any kind is impossible. This is similar to Feinberg's [6] conclusion quoted earlier.

By correctly taking the unidirectionality of information flow into account, later authors [29,30,31] also concluded that tachyonic information transfer would lead to the logical contradictions described in the paradoxes. We now see that by ignoring this point and interchanging cause and effect along with emission and absorption upon reinterpretation, we were eliminating contradictions at the price of necessarily having everything completely predetermined. All the worldlines in spacetime were just regarded as "already" being there so that contradictions were not possible. The description of events according to any observer was then found by simply letting the hyperplane of simultaneity for that observer sweep over the worldlines which were frozen into spacetime [32]. We reject this view for the reason that it is rather unappealing and is simply not open to disproof or to any further useful discussion. One may also wonder whether doing experiments would have any meaning under such a scheme.

Therefore, on the basis of the quite general requirements of macroscopic causality, we must reject the possibility of superluminal information transfer. But it is equally important to realize that such arguments do not apply to the microscopic world, or to chance observation of tachyons, so the conclusion that tachyons could not interact with ordinary matter in any way [33] is unwarranted. Terletskii [13] points out that there is no way to assign causal directions for the microscopic world of elementary particles, and that the concept of causality which is implicit in the paradoxes described above, and which is of importance in scientific discussions, has a purely macroscopic origin.

So all that may be concluded from an examination of the causal paradoxes is that tachyons can only exist if they cannot be used to carry information or if certain experiments cannot be performed [31]. As we have seen, the experiments referred to here involve signalling at a speed $v > 1$ such that $vw > 1$ where w is the relative speed of the observers who are signalling to each other. It has been pointed out by Bilaniuk and Sudarshan [34] that this restriction could be understood in terms of cosmological boundary conditions. Assume that in a certain frame S_0 the background flux of tachyons from the rest of the universe is finite. Let this flux be zero for simplicity. Now observers in the frame S_0 can emit any number of tachyons with any velocity $v > 1$. But an observer in a frame S' moving at a velocity w with respect to S_0 would not be able to emit tachyons which had a velocity greater than $1/w$ because if they were emitted to distant regions, then in the frame S_0 they would appear as an incoming tachyon background. This contradicts the assumption that the tachyon background is zero in S_0. The paradoxes are then solved by referring to the standard frame S_0 to see which signals could not be sent. One should remember that the frame S_0 is singled out by the boundary conditions, and not by the theory itself, so that the relativistic postulate of the equivalence of all inertial frames is not violated here.

Another alternative is mentioned by Csonka [35] and Vilela Mendes [36]. In our thought experiments we have been treating tachyons as though certain classical concepts still applied. We essentially treated them as localized particles and supposed that we could freely specify any initial conditions we pleased in setting up the experiments. This may not be valid and when speaking of tachyons one should perhaps give some thought to the extent which one is using concepts built up from experience with ordinary particles. Csonka [35] and Vilela Mendes [36] point out that the dynamics of tachyons may be such that the boundary conditions required to set up the paradoxes are self-contradictory.

For the present it does not matter which proposal, if any, will assist us best in the treatment of tachyons. The point which has been made in this section is that special relativity and the principle of causality together cannot provide a proof that tachyons do not exist.

1.4 Experimental Searches

The previous sections have shown that the theoretical argument against the existence of tachyons is unconvincing so that there is ample reason for attempting experimental searches. A number of experiments have been performed but none has found conclusive evidence for tachyons. We point out, however, that some of the experiments which have been performed were based on assumptions about the properties of tachyons which are either not in accord with the description arrived at above, or else have subsequently been shown to be incorrect.

Alväger and Kreisler [37] carried out the first experiment to detect tachyons. They searched for charged tachyons (of about 1 to 2 electronic charges) which might be produced by 800keV gamma rays incident on lead by looking for any Cherenkov radiation that they might produce in a vacuum. Alväger and Kreisler calculated that charged tachyons would quickly lose their energy in this way so they set up an electrostatic field to impart some energy to the tachyons which would then radiate it again. The Cherenkov radiation was looked for in a direction about 90° to the presumed line of flight because, from Eq. (1.20), it is in this direction that low-energy tachyons should predominately radiate. No tachyons were found in this way. Davis, Kreisler, and Alväger [38] later used a modified apparatus, but one which operated on the same detection principle, to detect charged tachyons produced by 1MeV gamma rays on lead. Again none were found, indicating that the limit for tachyon photo-production in this way was over eight orders of magnitude smaller than the cross section for the production of electron-positron pairs.

Bartlett and Lahana [39], using photoproduction in lead and water, tried to detect tachyons carrying a magnetic charge (tachyon monopoles). Their experimental setup was similar to that used by Davis et al except that a magnetic field replaced the electrostatic field. It was hoped to detect vacuum Cherenkov radiation from the superluminal magnetic charge, but none was found.

There are now strong theoretical grounds for believing that experiments like these which try to detect vacuum Cherenkov radiation from charged tachyons will not succeed. In Chapter 3 we shall explain this further.

Experiments which look for a negative "missing mass squared" should detect tachyons if they are produced in the reaction being studied because this method of detection involves very few assumptions about the properties of tachyons. It only relies on tachyons having a spacelike 4-momentum. Baltay et al [40] searched for neutral tachyons by examining reactions of the type

$$K^- p \rightarrow \Lambda^0 X$$

and $\bar{p}\ p \rightarrow \pi^+ \pi^- X$,

where X refers to neutral reaction products which carry away 4-momentum from the reaction. The 4-momenta of the initial particles and that of the final particles, apart from that carried by the X, was measured. By energy-momentum conservation the 4-momentum of X could be calculated. If X consisted of a single tachyon then this missing 4-momentum would be spacelike. If X contained more than one tachyon, then a plot of the missing mass squared would show negative as well as positive values because, as we saw in section 1.2, spacelike 4-momenta can combine to produce either a spacelike or timelike total 4-momentum. No evidence for tachyons was found in this way, indicating that the ratio of tachyon production to that of ordinary particles was less than 10^{-3} for the energies used in these reactions. Later, an experiment by Danburg et al [41] found no evidence for charged tachyon pair production in the K^-p interaction. They used the missing mass technique and assumed that charged tachyons would produce tracks in a bubble chamber.

From Eq. (1.19) we know that a stationary bradyon can only emit a tachyon if that tachyon has negative energy. The bradyon then takes on positive energy. This process for protons was looked for by Danburg and Kalbfleisch [42] by examining bubble chamber photographs for evidence of protons spontaneously acquiring energy. Ramana Murthy [43] searched for such "elastic decays" of the protons and electrons contained in the gas of a Geiger-Müller counter by looking for ionization when no particles were incident. Both experiments returned negative results, as did a similar experiment by Ljubičić et al [44] which looked for elastic electron decays. In this experiment the lower limit to the lifetime of the electron was placed at 4.6×10^{13} years. It was realized that according to the reinterpretation principle negative energy tachyons do not exist, so that the process being looked for would in fact appear as absorption of an incoming tachyon by the stationary bradyon. Therefore the negative results cannot strictly be attributed to a large lifetime for the bradyon. They should be seen as either implying a weak coupling between the bradyon and tachyon, or indicating a low incident flux of tachyons, or both. The problem of a possibly low tachyon flux would be avoided by looking for decays in flight of bradyons.

Ramana Murthy [45] surmised that tachyons would be produced by cosmic rays in the upper atmosphere. If so, the tachyons would arrive at the ground before the cosmic ray air shower which travels at almost the speed of light. He therefore looked at the time period after the detection of a potential tachyon by a photomultiplier-scintillator arrangement for the onset of an air shower. If a statistical correlation existed between these initial detections and the arrival of air showers, it would constitute evidence for faster-than-light particles. However no such correlation was found. An improved method of storing cosmic ray data was used by Clay and Crouch [46]. Their apparatus was able to store $128\ \mu$ sec of data and continually record new data while discarding the earliest data. If an air shower was detected, the recording could be stopped and the data for the previous $128\ \mu$ sec would be available for a search for prearrivals. More energetic showers than those detected by Ramana Murthy were considered.

But again tachyons did not reveal themselves [47].

All the experiments above looked for stable tachyons, but Gleeson et al [48] suggested a search for evidence of unstable tachyon resonances. They report some preliminary success in interpreting peaks in particle differential cross sections at negative values of the mass squared in terms of a tachyon resonance. Although inconclusive at present, further analysis along these lines might yield a clearer picture.

1.5 Outlook

From the initial proposal for considering faster-than-light particles put forward by Bilaniuk, Deshpande, and Sudarshan [14], there has arisen the concept of a tachyon which has been explained in this chapter. Although tachyons have not been found as yet, their discovery would challenge physicists to give a theoretical account of their behaviour. Even the nonexistence of tachyons would not remove the necessity for a theoretical consideration of faster-than-light effects because we have found that neither the special theory of relativity nor the requirements of macroscopic causality has provided a prohibition of a type of tachyon which cannot be used for macroscopic signalling. If the conclusion that tachyons do not exist is correct, then we should not wish to arrive at the right answer for the wrong reasons. It is suggested, therefore, that further theoretical consideration be given to tachyons.

Present physical theories have been tested against a vast body of experimental experience, and it would be foolish to suggest their wholesale dismantling for the sake of being able to include tachyons in a theory. The theoretical program should instead seek to determine whether present prejudices against tachyons are unfounded, and whether the prohibition of tachyons in present theory is as superficial as it has been shown to be in the case of special relativity. If such be our aim then we ought to be willing to make only minimal theoretical changes to accomodate tachyons. Some deviations from this procedure are considered in the next chapter, but thereafter we shall follow this policy when we consider a problem in the electrodynamics of the charged classical tachyon, and the problems associated with treating tachyons in quantum field theory.

CHAPTER 2

SUPERLUMINAL LORENTZ TRANSFORMATIONS AND EXTENDED RELATIVITY

In this chapter we shall examine a number of attempts which have been made to extend the special theory of relativity to include faster-than-light inertial frames because some such extended theory forms the basis of a great deal of the literature on tachyons. The superluminal frames are meant to play the role of rest frames for tachyons, but there is no suggestion that a boost from a subluminal to a superluminal frame could be physically performed. Rather, the use to which the superluminal Lorentz transformations are put is the application to a familiar process involving bradyons in order to arrive at a description of a process which is supposed to occur for tachyons. This is believed to be legitimate because some extended principle of relativity, which declares the equivalence of all inertial frames, is usually assumed to hold. That is, in their rest frames, or in frames moving at subluminal speeds with respect to these frames, tachyons are supposed to behave like ordinary bradyons.

In general, we disagree with all approaches of this kind. Our reasons for doing so, and the approach which we shall favour, are explained.

2.1 Transformations in One Dimension

Superluminal Lorentz transformations in one spatial dimension have been arrived at independently by Gilson [49], Mariwalla [50], Parker [51], and Antippa [52]. It is hoped that a satisfactory extended theory of relativity can be obtained by adding these new transformations to those of the proper Lorentz group in one spatial dimension, which we write here in the form

$$t' = \frac{1}{(1-v^2)^{\frac{1}{2}}}(t+vx) \quad , \quad x' = \frac{1}{(1-v^2)^{\frac{1}{2}}}(x+vt) \quad , \qquad (2.1)$$

where $|v| < c = 1$. The ordinary Lorentz transformations (2.1) preserve the square of the Minkowski interval:

$$t'^2 - x'^2 = t^2 - x^2 \quad . \qquad (2.2)$$

It is well known that the Lorentz transformations can be derived from the assumptions that the coordinate transformation between two inertial frames S and S' is linear, that space is isotropic, that spacetime is homogeneous, and that the velocity of light is the same for all frames. (See, for example, Rindler [7]). Parker [51] made these same assumptions to derive his superluminal transformations which are supposed to give the coordinate transformation between frames S and S' that move with a velocity $v > 1$ with respect to each other. Recall that during the derivation of the Lorentz transformations, one arrives at the requirement

$$t'^2 - x'^2 = \pm(t^2-x^2) \quad . \qquad (2.3)$$

For relative velocities v less than light, the plus sign in (2.3) must be taken because we must be able to obtain the identity transformation in the limit of zero relative velocity. We then obtain the transformations (2.1) satisfying (2.2). But for superluminal relative velocities, Parker takes the minus sign in (2.3) to avoid the introduction of imaginary quantities in the transformation. Then, apart from space and time inversions, the superluminal transformation has the unique form

$$t' = \frac{1}{(1-v^2)^{\frac{1}{2}}} \frac{v}{|v|} (t+vx) \quad , \quad x' = \frac{1}{(1-v^2)^{\frac{1}{2}}} \frac{v}{|v|} (x+vt) \quad , \quad (2.4)$$

where $|v| > 1$. We note that Parker's transformations (2.4) are misquoted in references [53] and [54].

The transformations (2.4) taken together with the ordinary Lorentz transformations (2.1) form a group which Parker calls the extended Lorentz group in one spatial dimension. This is the real group of transformations which preserve the form t^2-x^2, not exactly, but only up to a sign as in (2.3). Alternatively, we may view it as the group generated by the transformations of the two dimensional Lorentz group and the transformation of interchanging the time and space axes:

$$t' = x \quad , \quad x' = t \quad . \qquad (2.5)$$

We see that (2.5) is the superluminal transformation corresponding to infinite relative velocity, so that a bradyon at rest at the origin in the frame S, for example, is a transcendent tachyon according to the frame S'.

Given a particular frame S, subluminal for example, we can define the set of subluminal frames as all those frames which are obtained from S by an ordinary Lorentz transformation, and the set of superluminal frames as those obtained from S by applying all the superluminal transformations. The same sets are obtained no matter which subluminal frame S is chosen. Moreover, any two superluminal frames are related by an ordinary Lorentz transformation. There is complete symmetry between the two sets of reference frames. Because of this "duality" as he calls it, Parker is led to postulate the so-called extended principle of relativity which holds that the laws of physics have the same form relative to the superluminal frames as they do relative to the subluminal frames. Therefore, superluminal particles in their rest frames (the superluminal frames) behave like subluminal particles in subluminal frames.

The quantities E and p transform like t and x respectively under ordinary homogeneous Lorentz transformations, and it is assumed that this applies to superluminal transformations as well. This is what is meant by calling (E,p) an extended Lorentz vector [51]. Then it is found that a particle of mass m_0 which is at rest in the frame S, so that

$$E = m_0 \quad \text{and} \quad p = 0 \quad ,$$

has the usual properties ascribed to a tachyon according to a frame S' moving superluminally with respect to S. For if S and S' have relative velocity v $(v > 1)$ then we have, from (2.4), that

$$E' = \frac{v}{|v|} \frac{m_0}{(v^2-1)^{\frac{1}{2}}} \quad \text{and} \quad p' = \frac{m_0 \, |v|}{(v^2-1)^{\frac{1}{2}}} \quad .$$

But according to the reinterpretation principle, when the energy E is negative, the tachyon is observed to have the energy-momentum $(-E,-p)$ so we find that

$$E' = \frac{m_0}{(v^2-1)^{\frac{1}{2}}} \quad \text{and} \quad p' = \frac{m_0 v}{(v^2-1)^{\frac{1}{2}}} \qquad (2.6)$$

in the frame S', and these are just the usual expressions, (1.4) and (1.5), for the energy and momentum of a tachyon of velocity $v > 1$. We see that the mass m_0 of the tachyon appearing in equations (2.6) is real. Moreover, m_0 here is the "rest-mass" of the tachyon in the sense that a frame exists in extended relativity in which the tachyon appears as a stationary bradyon of (real) mass m_0. The speed of light is invariant for the whole extended Lorentz group, but a superluminal transformation can change the sign of the energy of a photon so the reinterpretation principle is applied to photons as well in this case.

As the speed of light is the same for both types of frame, the extended principle of relativity, or duality principle, implies that photons, and other luxons in general, appear the same to both types of frame. According to this argument, we can determine the electromagnetic interactions of tachyons by knowing the electromagnetic interactions of bradyons and the superluminal transformations. It is on this basis that Parker considers the possible transformation properties of the electromagnetic field. He notes that such considerations are only heuristic since his extended theory of relativity is purely one-dimensional. Under the superluminal transformation (2.4) for infinite velocity, (t,x) is taken into (x,t) as in (2.5). Such a transformation does not change the observed energy and momentum of a photon (that is, after the reinterpretation principle is applied if necessary), so the transformation must leave $\underline{E} \times \underline{B}$ and $\underline{E}^2 + \underline{B}^2$ unchanged if $\underline{E}$ and $\underline{B}$ are the electromagnetic fields for a photon moving on the x-axis. With the identity transformation of the fields already corresponding to the ordinary Lorentz transformation (2.1) with zero velocity, the transformation of the fields for this superluminal transformation is taken to be

$$\underline{E}' = -\underline{B} \quad \text{and} \quad \underline{B}' = \underline{E} \ , \tag{2.7}$$

at least to within a rotation about the x-axis. Parker thus makes the interesting suggestion that a charged superluminal particle at rest in a superluminal frame S would appear in the subluminal frame moving at infinite speed relative to S as an infinite velocity magnetic monopole. A good deal is made of this result in one of the attempts at a three-dimensional extended relativity described below.

As experimental verification, Parker suggests the examination of photon-photon scattering because, from duality, it is clear that the process appears the same from both subluminal and superluminal frames. Only having a one-dimensional theory, he says that the predictions only apply to backward scattering from head-on collisions. But, if no interference effects occur, the probability for this process should be twice as great as that predicted by just considering virtual electron-positron pair contributions since, by duality, tachyon-antitachyon pairs should appear on the same footing. He notes that a negative result may show that charged tachyons do not exist, but clearly this would invalidate the strict form of duality which requires the existence of tachyonic electrons, positrons, and so forth.

From the foregoing discussion it seems possible to develop a reasonable extension in one dimension of the special theory of relativity. The extension is essentially unique and preserves the basic principles of relativity. Looking at Lorentz transformations on a spacetime diagram as rotations of the axes, we see that the superluminal transformations (2.4) continue the rotation until the time and space axes exchange places. The obvious difficulty in extending this procedure to three dimensions is that a spatial direction would have to be singled out in order to specify the exchange of axes and such a preferred direction would violate relativity principles. The usual questions of causality still arise in the extended theory above and the considerations of section 1.3 would apply here.

It has been pointed out by Antippa and Everett [55] that for one spatial dimension causal cycles can be avoided in a simple way. They note that in one spatial dimension the positive-energy bradyons together with the set of positive-momentum tachyons are not sufficient for the construction of causal cycles. The latter set is invariant under the ordinary Lorentz transformations in one dimension simply because we do not have the spatial rotations in this case. It is only in this sense that the theory is consistent with special relativity. Taking the velocity v of a tachyon to be given by the ratio of the momentum to the energy, $\frac{p}{E}$, where (E,p) forms a Lorentz vector, Antippa and Everett show that in one dimension the four regions of the energy-momentum diagram consisting of the positive- and negative-energy bradyons, and the positive- and negative-momentum tachyons, are separately left invariant by ordinary Lorentz transformations. Further, using particles from two adjacent regions, such as the positive-energy bradyons and the positive-momentum tachyons, causal cycles cannot be constructed because the energy-momentum vector (and the space-time displacement vector), always have positive projections on the line $E = p$ (or $t = x$) for these particles. So using these two sets of particles, we cannot transmit information back to its starting point in spacetime. Their main reason for only considering positive momentum tachyons seems to rest on consideration of Parker's superluminal transformations which were derived by Antippa independently [52]. Writing the superluminal transformations (2.4) in the form

$$t' = x \cosh\alpha + t \sinh\alpha$$
$$x' = x \sinh\alpha + t \cosh\alpha \qquad (2.8)$$

where $\tanh\alpha = \frac{1}{v}$,

we see that the momentum p' of a tachyon according to a subluminal frame S' is

$$p' = p \sinh\alpha + E \cosh\alpha \quad , \qquad (2.9)$$

where in the superluminal frame S, the tachyon appears as an ordinary bradyon which must have positive energy. Therefore, from (2.9), it must have positive momentum according to the observer S'.

Now a positive-momentum tachyon may yet have negative energy and, by the discussion of the reinterpretation principle, be travelling backward in time so that it will appear as a positive-energy tachyon travelling forward in time in the negative spatial direction. An observer will therefore see tachyons travelling in both directions along the x-axis. Nevertheless, Antippa and Everett maintain [56] that the "causal direction", that is, the unique direction in which tachyons may transmit information, is the forward spatial direction. Here they are using the cause-effect distinction used by Newton [28]. The idea being used here is that the time and space axes are interchanged for tachyons compared with bradyons so that the transmission of information by a tachyon must be only in the positive x-direction because, in the rest frame of the tachyon, this is the direction of the future along the tachyon time axis.

Parker's superluminal transformations, as interpreted by Antippa and Everett, have been derived by Antippa [57] from a number of postulates. However we do not consider this seriously because the postulates are clearly engineered to reproduce the above construction placed upon the Parker transformations.

The inverse velocity parameter ξ, defined by

$$\xi v = 1 \quad , \tag{2.10}$$

can be used to achieve some symmetry in the appearance of bradyon and tachyon laws. Antippa writes the transformations (2.4) in the form [57]

$$\begin{aligned} t' &= \frac{x + \xi t}{\sqrt{1-\xi^2}} \\ x' &= \frac{t + \xi x}{\sqrt{1-\xi^2}} \qquad |\xi| < 1. \end{aligned} \tag{2.11}$$

A comparison between Eq. (2.11) for a superluminal transformation and Eq. (2.1) for a subluminal transformation leads Antippa to speak of a "reciprocity principle" according to which it is proposed that equations describing tachyons be derived from equations describing bradyons by the replacement of t,x, and v by x,t, and ξ respectively.

We distinguish here another approach to tachyons which uses the generalized Lorentz transformations of Gilson [49]. Naranan [58] points out that these transformations, if they have any physical significance imply that tachyons would never be observed as particles with a velocity greater than light in any inertial frame. The transformations are arrived at by first forming the space and time variables into a complex number

$$z = x + i\,t \quad .$$

Because of its use of this complex variable, Naranan rightly questions the physical significance of this scheme, but we shall see in section 2.5 that the appearance of complex quantities has not deterred a number of authors from considering three-dimensional forms of extended Lorentz transformations. In terms of the variable z, the ordinary Lorentz transformations (2.1) are the real and imaginary parts of the equation

$$z' = \gamma(z + ivz^*) \qquad (2.12)$$

where

$$\gamma = (1 - v^2)^{-\frac{1}{2}} \qquad (2.13)$$

and $|v| < 1$. The ordinary Lorentz transformations are extended by merely assuming that (2.12) is also valid for superluminal velocities v, in which case γ becomes imaginary:

$$\gamma = \pm\, i(v^2 - 1)^{-\frac{1}{2}} \quad , \qquad |v| > 1 \quad . \qquad (2.14)$$

To find the superluminal transformations in terms of x and t, the real and imaginary parts of (2.12), in which γ is given by (2.14), is taken. The resulting transformations may be written as

$$\pm\, t' = \Gamma\,(t + Vx) \quad ,$$

$$\mp x' = \Gamma\,(x + Vt) \quad , \tag{2.15}$$

and the above, where $V = \frac{1}{v}$ and $\Gamma = (1 - V^2)^{-\frac{1}{2}}$.

Eqs.(2.15) have the same form as the ordinary Lorentz transformations (2.1) because for $|v|$ greater than 1, $|V|$ will be less than 1. Supposing that S is the rest frame of a tachyon, then in a frame S' with relative velocity v greater than light, the tachyon should appear to have velocity v. But the transformations (2.15) are interpreted to mean that an observer in the frame S' will not see a tachyon of velocity $v > 1$, but an ordinary particle with a subluminal velocity $V = \frac{1}{v}$ which has undergone either a space or a time reflection.

Certainly the derivation of the "superluminal transformations" (2.15) does not constitute a compelling physical argument. In fact the manipulations above have just obtained, apart from either a space or a time reflection, a reparametrization of the ordinary Lorentz transformations (2.1) by using $V = \frac{1}{v}$. Eqs. (2.15) should not then be considered as satisfactory superluminal transformations. The same criticism may be made of the transformations used by Mariwalla [50], Rachman and Dutheil [59], and Arzeliès [60] because by various manipulations these authors arrive at transformations which are essentially the same as (2.15), apart from factors of a sign.

We conclude that Parker's group of transformations forms the most satisfactory extension of special relativity in one spatial dimension to include superluminal inertial frames, in that the transformations are real, form a group, and are consistent with properties which space is believed to have, such as isotropy. The symmetry between the superluminal and subluminal frames gives rise to the ideas of an extended principle of relativity and duality. However, unless a similar success is to be had in deriving extended transformations in three spatial dimensions, the idea of an extended relativity theory will not be of any physical or theoretical importance to the consideration of tachyons.

2.2 Extensions to Three Dimensions

It is known that the success in extending Lorentz transformations in one dimension cannot be repeated in the case of three. This was shown by Gorini [61] who considered (n+1)-dimensional spacetimes and the types of linear kinematical groups of transformations which could exist on the spacetimes subject to the requirements of the isotropy of space. He wrote the coordinate transformation between two frames S and S' as

$$x'_\mu = \sum_{\nu=0}^{n} \mathsf{L}_{\mu\nu} \, x_\nu \quad ,$$

where L is a real $(n+1) \times (n+1)$ nonsingular matrix, and $x, x' \in R^{n+1}$ are the spacetime coordinates of an event according to the frames S and S' respectively, with the zero component being the time. Let L be the group of the matrix transformations L and suppose that the subgroup of L which consists of transformations between frames at rest with respect to each other consists of transformations of the form

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & & & & \\ 0 & & X & & \\ \vdots & & & & \\ 0 & & & & \end{bmatrix} ,$$

where X is an $n \times n$ real orthogonal matrix with unit determinant. This assumption just means that certain conventions have been chosen for the coordinate systems. The homogeneity of spacetime and the isotropy of space are involved here because it has been assumed that the axes of the coordinate systems can be made to coincide and that they use the same standards of length and time. Gorini then proved, subject to this assumption, that for $n \geqslant 3$ the only possible groups L are the proper orthochronous Lorentz and Galilei groups. The theorem does not hold for $n = 1$ and a counter example is provided by the Parker group.

Gorini's theorem shows that in three spatial dimensions there can be no extended theory of special relativity in which the transformations form a real linear group and space is isotropic. But a number of authors have shown themselves willing to dispense with some of these conditions and produce an extended theory of relativity, the nature of which is highly dependent upon which of these conditions are discarded.

2.3 Extensions Lacking Spatial Isotropy

Antippa and Everett [56,62] arrive at a (3+1)-dimensional extended theory of relativity by using a rather heavy-handed extension of their interpretation of the Parker transformations which destroys the isotropy of space. In three dimensional space, a preferred spatial direction, called the "tachyon corridor", is chosen. The set of reference frames with relative velocities along that direction form a set of preferred coordinate systems in that any two coordinate systems of this set are connected by the Parker transformation group, given by (2.1) and (2.4), to which is appended

$$y' = y \quad \text{and} \quad z' = z \quad , \tag{2.16}$$

where the x-direction is taken along the tachyon corridor. A prescription is then made for transforming between two arbitrary nonpreferred frames. Calling the sets of subluminal and superluminal reference frames the frames of class I and II respectively, it is seen that ordinary Lorentz transformations connect frames of the same class. To transform from one class to another, an ordinary Lorentz transformation is first of all made to a preferred frame of the same class. An extended Lorentz transformation is then made to one of the preferred frames of the other class. Finally, an ordinary Lorentz transformation is made from the resulting preferred frame to the non-preferred frame at which we wanted to arrive.

The simple manner in which causality was preserved in the one-dimensional theory in section 2.1 is repeated in the three dimensional theory by stating that the interval of a tachyon worldline taken in the "causal direction" always has a positive projection along the tachyon corridor. This feature has been preserved in the three dimensional theory at the expense of rotational and full Lorentz invariance. Using a superluminal transformation along the preferred direction, we see from (2.4) and (2.16) that the Minkowski spacetime interval is not preserved. Instead we find

$$-t'^2 + x'^2 - y'^2 - z'^2 = t^2 - x^2 - y^2 - z^2 \quad , \qquad (2.17)$$

showing that a light signal in a class I coordinate system will not have speed 1 in class II systems unless it is moving along the x-direction which is the tachyon corridor. Antippa and Everett therefore suppose that what are electromagnetic fields to one class of observers are not electromagnetic fields for observers of the other class. Antippa and Everett realize that according to this theory, rotational invariance would be violated even for processes involving only bradyons because of virtual tachyon effects. They are then concerned with limiting the magnitude of the coupling between their tachyons and bradyons to values small enough not to conflict with experimental checks on rotational invariance [56]. We believe that the three-dimensional version of the Antippa-Everett theory is not a satisfactory extension of special relativity for the reason that it preserves for tachyons very little of the theory it is supposed to extend. It may be of some relevance to the explicitly non-Lorentz invariant theory of Everett [63], but we have determined not to follow such approaches in the present work.

Lemke [64] also uses the Antippa-Everett transformations and bases his version of tachyon electrodynamics upon them, but his interpretation of the preferred direction is slightly different. He tries to lessen the damage done to spatial isotropy by assuming that the preferred direction along which the superluminal boosts may be applied is not a fixed direction in space, but the direction of motion of the superluminal source which is producing the fields. But again the speed of light is not invariant due to the transformation (2.17) of the spacetime interval, so that the behaviour of light is dependent upon the motion of its source if its source is superluminal. Another feature which we believe is unsatisfactory is that there is some ambiguity about how the electromagnetic fields transform in this theory, although Lemke [64] tries to justify his particular choice. In section 2.6 we shall observe a similar uncertainty concerning the electromagnetic field in another form of extended relativity.

2.4 Transformations not forming a Group

Goldoni [65] has tried a rather unusual way of introducing superluminal frames. He says that the Minkowski metric

$$d = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

does not have universal applicability, and that the appropriate metric depends on the class of phenomena being studied. This is a way of avoiding the noninvariance in the form of the spacetime interval which occurs, as in (2.17), when the Parker transformations are combined with the identity transformation (2.16) on the remaining two spatial coordinates. Taking a particular inertial frame, there are supposed to exist three classes of phenomena which are superluminal with respect to this frame, and they have the metrics

$$d_x = \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}, \quad d_y = \begin{bmatrix} -1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix}, \text{ and } \quad d_z = \begin{bmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{bmatrix}.$$

The transformations between the frames connected with these different types of phenomena do not form a group, and the separate classes of particles are described in different metric spaces. For this reason there is no interaction between the classes of superluminal particles and bradyons which involves any exchange of 4-vector or 4-tensor properties or spin.

Another extension of relativity which gives up the group property of the set of transformations has been given by Gonzalez-Gascon [66]. He uses real transformation formulae, which we shall not describe here, and instead of having only one singular speed, that of light, he considers a sequence $c_1 = c, c_2, c_3, \ldots,$ of singular speeds, each being the speed of propagation of some long-range field of force. The isotropy of space and the homogeneity of spacetime are retained in this suggestion. As partial justification for introducing a sequence of singular speeds, Gonzalez-Gascon quotes the opinions of some physicists who believe that the velocity of propagation of gravitational signals or of neutrinos is not that of light. The scheme is acknowledged as being highly speculative but he considers his formulae a first attempt at broadening relativity. We mention it here as one of a number of attempts to generalize relativity while avoiding the consequences of Gorini's theorem. In this and the previous case it is achieved by not having the set of transformations forming a group.

2.5 Complex Transformations

The extended Lorentz transformations of Recami and Mignani have provoked by far the greatest amount of discussion. These authors, unlike many others, require of their extended relativity that the speed of light be invariant, that space be isotropic, and that all inertial frames be equivalent. Thus their theory tries to embody most of the aspects of ordinary special relativity. However their extended relativity group is a group of complex transformations, as indeed it must be by Gorini's theorem, given these requirements.

Their generalized Lorentz transformations were first introduced in reference [67], but a revised version of them appears in reference [68], and it is the latter generalized Lorentz transformations which form the basis of an extended theory of relativity which is extensively discussed in reference [53]. For the case of motion along the x-axis, the transformations may be written in the form [53]

$$\begin{aligned} t' &= \eta\gamma(t - x\tan\phi) \;, \\ x' &= \eta\gamma(x - t\tan\phi) \;, \qquad -\frac{\pi}{4} \leqslant \phi \leqslant \frac{7\pi}{4} \qquad (2.18) \\ y' &= \eta\delta y \;, \quad \text{and} \quad z' = \eta\delta z \end{aligned}$$

where

$$v = \tan\phi \quad , \quad \gamma = |1 - \tan^2\phi|^{-\frac{1}{2}} \quad , \quad \eta = \frac{\cos\phi}{|\cos\phi|}\,\delta^2 \quad ,$$

and $\delta = \left[\dfrac{1 - \tan^2\phi}{|1 - \tan^2\phi|}\right]^{\frac{1}{2}}$.

The whole group, G, of the extended theory of Recami and Mignani is the complex group of transformations which preserves the square of the Minkowski interval up to a sign:

$$t'^2 - x'^2 - y'^2 - z'^2 = \pm\,(t^2 - x^2 - y^2 - z^2) \quad , \tag{2.19}$$

where the plus sign holds for subluminal transformations and the minus sign holds for superluminal transformations. If ϕ is in the range $(\frac{\pi}{4}\,,\,\frac{3\pi}{4})$ in the transformations (2.18), we obtain superluminal transformations which may be written as

$$\begin{aligned} t' &= \frac{v}{|v|}\,\gamma(t + vx) \quad , \\ x' &= \frac{v}{|v|}\,\gamma(x + vt) \quad , \\ y' &= i\,\frac{v}{|v|}\,y \quad , \\ \text{and} \quad z' &= i\,\frac{v}{|v|}\,z \quad , \end{aligned} \tag{2.20}$$

where $\gamma = (v^2 - 1)^{-\frac{1}{2}}$ and $|v| > 1$. We see that, as far as the variables x and t are concerned, Eqs. (2.20) are the superluminal transformations of Parker, Eqs. (2.4). Most importantly, we notice that condition (2.19) is satisfied by the introduction of imaginary quantities in the transformation of 4-vector components which are perpendicular to the direction of relative motion. A more general transformation than (2.20) would involve complex quantities.

Because of the appearance of complex quantities, the physical meaning of this extended theory of relativity is unclear and it remains so despite numerous attempts to clarify the matter. Pavšič and Recami [69] recall that the conformal group may be viewed as a group of transformations which preserves not the value of the square of the Minkowski interval ds^2 exactly but only the relation

$$ds^2 = 0 \quad .$$

That is, the light cone is transformed into the light cone. This group contains, besides the Poincaré transformations, the extra transformations consisting of dilatations and the special conformal transformations. But the ordinary conformal group cannot interchange timelike and spacelike vectors which is what is required for superluminal transformations for which (2.19) with the minus sign holds. So, in order to consider the extended Lorentz group G as a subgroup of the conformal group, Pavšič and Recami resort to considering imaginary dilatations. Again there arises the problem of understanding what imaginary physical quantities mean and Pavšič and Recami [69] concede that a complete physical interpretation of the imaginary quantities is still lacking.

Recami and Mignani acknowledge that the extended group of transformations must be complex, and yet maintain that they do not have a complex spacetime [67], and that all observers, both subluminal and superluminal, measure only real quantities. Taking the real parts of complex quantities at appropriate points, as in reference [70], is evidently not intended [67]. Lee and Kalotas [54] suggest that the real quantities measured by observers should perhaps be combined into complex quantities in some way prior to the application of the extended theory.

Yaccarini [71] does not believe that the superluminal transformations like (2.20) perform their intended function which is the interchange of spacelike and timelike vectors and the interchange of tachyonic and bradyonic phenomena in general. He would say that the superluminal transformation between frames S and S', which satisfies

$$t'^2 - x'^2 - y'^2 - z'^2 = -(t^2 - x^2 - y^2 - z^2) , \quad (2.21)$$

only amounts to a change of metric from diag (+1,-1,-1,-1) to diag (-1,+1,+1,+1) for the reason that an inversion which changes the set of spacelike vectors into the set of timelike vectors and vice versa is only possible in a two dimensional spacetime, unless a complex spacetime is used. Mignani and Recami deny that their theory involves either a change of metric [72] or a complex spacetime. Supposing that the subluminal frame S and the superluminal frame S' are connected by a superluminal transformation, then the only interpretation offered for the imaginary quantities seems to involve the idea that "if an observer S wants to look at spacetime not directly - and not through observations by a subluminal observer - but through observations by an observer S', then he will have to multiply them by the imaginary unit, i, before manipulating them as if they were observations from a subluminal inertial frame" [72]. This does not seem to us to be particularly illuminating.

Recami and Mignani state that the problem of formally and rigorously interpreting the imaginary quantities remains unresolved [73], while believing that any objections to their theory can be overcome by an appeal to some physical reasoning. We would prefer to see a proper interpretation of the theory before engaging in such reasoning. One attempt to provide the interpretation was the consideration of a six-dimensional spacetime which has three space and three time dimensions [73]. This followed a suggestion of Demers [74] who tried to achieve complete symmetry between space and time in special relativity by assuming that an event P was represented by a vector

$$P = (x,y,z,\ it_x,\ it_y,it_z) \quad ,$$

and that a quadratic form on the space was defined by

$$P^2 = x^2 + y^2 + z^2 - (t_x{}^2 + t_y{}^2 + t_z{}^2) = \underline{x}^2 - \underline{t}^2 \quad .$$

It is assumed that only the modulus

$$t = |\underline{t}| = (t_x{}^2 + t_y{}^2 + t_z{}^2)^{\frac{1}{2}}$$

has direct physical meaning. Cole [75] has written down both subluminal and superluminal transformations in this spacetime but notes that linear transformations on the six variables $(\underline{x},\ i\underline{t})$ would not correspond to linear transformations among the space variables $\underline{x}$ and the physical time $|\underline{t}|$. Moreover, we now have the task of interpreting physically two extra time dimensions, so that this proposal has not yet brought us any closer to understanding superluminal transformations.

In Corben's view [76,77], the fact that a quantity which results from making a superluminal transformation is imaginary indicates that

that quantity is unmeasurable in that frame, an idea which he illustrates with a thought experiment. Suppose that S and S' move with a superluminal relative speed and that S holds a rod of length L at right-angles to the direction of relative motion. The observer in S could calculate this length from the time taken for a light ray to travel from one end of the rod to the other. But the observer in the frame S' cannot measure the length of the rod in this way because a light signal emitted from one end of the rod will never catch up with the other end. This, says Corben, is the significance of the imaginary quantities in the transverse components in the superluminal transformation (2.20) - these components cannot be measured by S'. The whole formalism seems to have collapsed back into one dimension. However, although these distances cannot be measured by S' using light rays, Lee and Kalotas [54] point out that other methods could be imagined, such as the use of a grid of synchronized clocks by the observer in S', in which case the problem of determining what the imaginary quantities mean still remains.

Corben takes the presence of the imaginary quantities seriously, and by taking them into account he seeks to rule out the possibility of vacuum Cherenkov radiation from a tachyon. By transforming with a superluminal Lorentz transformation to the rest frame of the Cherenkov-radiating tachyon, we see that the process appears to the tachyon as the emission of a photon which has some imaginary momentum components. These are the imaginary quantities introduced by superluminal transformations into the transverse components. Corben says that this shows that vacuum Cherenkov radiation cannot occur, for he asserts that only real processes (those not involving imaginary quantities) are allowed to objects which appear as bradyons to the observer [77,78].

It would appear that Corben has decided against the possibility of interaction between tachyons and bradyons, because he believes that in any interaction or exchange of 4-momentum in any way, bradyons would acquire imaginary momentum components [78].

2.6 Electromagnetism and Complex Transformations

The behaviour of the electromagnetic field in the complex transformation theory will be mentioned here because it illustrates the lack of unambiguous results to be had from the extended relativity of Recami and Mignani.

From Parker's extended principle of relativity, to which Recami and Mignani also subscribe, tachyons in their rest frames should behave just like bradyons. Therefore, one would expect that a consistent extended theory which preserved the speed of light should, given the very satisfactory relativistic treatment of electromagnetism for bradyons, yield some useful, unambiguous information about the interaction of tachyons with light and with electromagnetic fields in general.

This does not appear to be the case for the theory of Recami and Mignani. They obtain the transformations for the electromagnetic fields $\underline{E}$ and $\underline{H}$ [79,80,53] by first observing that, under an ordinary Lorentz transformation, the transformation of E_2 and H_3,

$$E'_2 = \gamma(E_2 + vH_3)$$

$$H'_3 = \gamma(H_3 + vE_2) \quad ,$$

looks like the transformation (2.1) for x and t. The components E_2 and H_3 are then chosen to behave like x and t under superluminal transformations as well, and E_1 and H_1 are elected to pick up the imaginary factors associated with the transverse components in a transformation like (2.20). So according to Recami and Mignani [81], objects which are vectors and tensors in ordinary relativity, like the electromagnetic field tensor $F_{\mu\nu}$, are not necessarily vectors and tensors under the transformations of the whole extended group G.

With the above type of transformation for the fields, the theory of Recami and Mignani predicts that superluminal charges appear as magnetic monopoles. It should be noted that the imaginary quantities arising in a transformation do not cause a problem because Maxwell's equations are written in terms of the self-dual tensor [53]

$$T_{\mu\nu} = F_{\mu\nu} + \tfrac{1}{2}\varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \quad ,$$

which already has complex components simply due to the choice of using vectors

$$x^{\mu} = (x^0, x^1, x^2, x^3) = (t, ix, iy, iz) \quad ,$$

as pointed out by Corben and Honig [82]. But the origin of the conventional factors i here is quite different from that of the imaginary quantities involved in superluminal transformations.

Corben and Honig [76,82] choose the field tensor $F_{\mu\nu}$ to be a tensor under all of the extended transformations. This is, perhaps, more reasonable, and under this assumption superluminal charges no longer appear as magnetic monopoles. This alternative was mentioned and rejected by Recami and Mignani [53].

Therefore, in extended relativity, whether charged tachyons behave as magnetic monopoles or not is purely a matter of which transformations are chosen for the electromagnetic field.

2.7 Discussion

Of course experimental evidence may eventually decide in favour of one of the theories mentioned in this chapter. But as such evidence is lacking at present, those who wish to approach tachyons by using an extended relativity allowing superluminal reference frames are faced with a wide choice of possible theories, all involving different assumptions and departures from conventional relativity. In fact the result due to Gorini shows that there is no simple, natural extension of relativity in four-dimensional spacetime, so that extra assumptions and additions to the concept of a tachyon are an unavoidable feature of such extended theories. Aside from the nonuniqueness of the extension, many of the theories proposed suffer from problems of interpretation.

We reiterate that our approach is to take seriously the proposal put forward by Bilaniuk, Deshpande, and Sudarshan which promises to allow faster-than-light particles to be treated within conventional physical theories like special relativity. The program suggested is to attempt consistently to do just this, and not to introduce immediately a whole range of further assumptions.

The extended theories may eventually lead to useful results but we feel that at present they do not seem to be giving tachyons the full consideration which they deserve. The introduction of rest frames for tachyons appears to be motivated by a desire to remove the unfamiliar aspects of tachyons and be able to regard them as ordinary particles in those frames. In fact ideas like duality, in their strictest form, say that all the types of particles with which we are familiar have their counterparts in the parallel universe of tachyons. This enables us to study ordinary particles, and later make a superluminal transformation to see how tachyons should behave. There may even be problems with doing this consistently, given the ambiguous results obtained for electromagnetism in one of the extended theories.

CHAPTER 3

THE CLASSICAL, CHARGED TACHYON

Following the discussion in the last chapter, we do not propose to alter the special theory of relativity in any essential way in order to treat tachyons, and we shall not speculate further on what forms a successor to this theory might take. We return now to the classical tachyon described in the Introduction, and we shall retain explicit Lorentz invariance in future discussions to ensure that our results are compatible with special relativity. In this chapter we shall apply classical electromagnetic theory to the charged tachyon. We stress that our electromagnetism is entirely conventional in that the fields obey Maxwell's equations. The only unusual feature is that the field set up by the tachyon is due to a spacelike 4-current. We prove that a charged tachyon will not emit electromagnetic radiation in a vacuum. This conclusion is arrived at using only the theoretical apparatus mentioned here, together with the appropriate treatment of divergences which arise because distributions must be multiplied together. The need for performing such improper operations on distributions is present even in the theory of charged bradyons. A new derivation of the Lorentz-Dirac equation for bradyons highlights this fact which has been obscured in previous derivations.

3.1 The Problem of Vacuum Cherenkov Radiation

As mentioned in the Introduction, determining the behaviour of charged superluminal particles is an old problem. With Sommerfeld's results in mind, the first writers on tachyons suggested that they would most probably emit Cherenkov radiation in a vacuum [14,19,6]. This type of radiation is observed when charged particles travel in a material medium at a velocity greater than light in that medium. The radiation is emitted in a cone, the axis of which is the line of flight of the particle, and the particle itself is at the apex of the cone. The electromagnetic fields due to the particle are confined to the interior and surface of the cone. A classical electromagnetic treatment of the Cherenkov effect for bradyons produces results which are in close accord with experimental observations [83,84], and the calculations are virtually identical to those performed by Sommerfeld. Hence it was expected that a charged tachyon would emit Cherenkov radiation in a vacuum. But, in the case of bradyons, it should be remembered that from a fundamental point of view, the Cherenkov radiation actually comes from the medium, the particles of which are excited by the electromagnetic fields of the moving charge [85]. However the calculations using the macroscopic theory may ignore these microscopic effects as was explained by Tamm [84]. So it is by no means certain that we should expect vacuum Cherenkov radiation from a charged tachyon.

It is important to resolve this question so that we may better interpret the results of the experiments described in section 1.4 which attempted to detect charged tachyons by means of their vacuum Cherenkov radiation. In these experiments it is assumed that the formula for the energy radiated by a charged bradyon per unit path length is also valid for tachyons. The formula is given by

$$\frac{dE}{d\ell} = e^2 \int_{v n>1} \left(1 - \frac{1}{v^2 n^2(\omega)}\right) \omega \, d\,\omega \qquad (3.1)$$

where v is the velocity of the particle, e its charge, and $n(\omega)$ is the refractive index of the medium as a function of the frequency ω

of the radiation. In a vacuum all electromagnetic radiation travels at the same speed regardless of the frequency so that we must put n equal to unity if we are to apply (3.1) to tachyons in a vacuum. The range of integration in (3.1) then becomes infinite because v is always greater than one, so the energy radiated threatens to be infinite. Correspondingly the fields on the surface of the Cherenkov cone become singular. To obtain a finite result from (3.1) a frequency cutoff was made [37,38,39]. It was assumed that a tachyon of energy E could not radiate more than this energy so that the highest frequency possible for the emitted radiation should be

$$\omega_{max} = E/\hbar \quad . \tag{3.2}$$

Finite emission rates were obtained and lifetimes for decay to the zero-energy transcendent state were calculated. Values obtained in this way were used in the experimental design [37,38,39].

But it can be seen that the frequency cutoff (3.2) for the formula (3.1) is wrong on the grounds of Lorentz invariance [86,87,88]. A charged tachyon in a vacuum cannot settle down into the transcendent state as seen by any particular frame because in other frames the tachyon will have a non-zero energy and should continue to radiate if Lorentz invariance is not to be violated. Of course for a bradyon emitting Cherenkov radiation, the medium singles out a preferred frame and the particle gradually loses energy in this way until it is travelling at a speed less than light in the medium. But for the tachyon in a vacuum there can be no such preferred frame if we strictly adhere to special relativity as we have decided to do. This state of affairs has been given the status of a paradox by Barrowes [89] and, together with the causality paradoxes, it induces him to break with Lorentz invariance and introduce a preferred frame. However the lack of invariance of the transcendent state may be interpreted as indicating that the tachyon will never achieve a constant-velocity radiationless state, but continue to radiate even after its energy becomes negative. This, by reinterpretation, would just appear to be the annihilation of a tachyon-antitachyon pair [34,86]. The worldline of the tachyon would never become straight but curve back into the past, and for different observers different events on the worldline would correspond to the annihilation.

Sommerfeld used an extended particle to obtain a finite rate of energy loss. The calculation was done in pre-relativistic times but this type of argument convinced Wimmel [86] that charged tachyons are always subject to Cherenkov energy loss. It is further noted by Wimmel that a tachyon which is accelerating due to Cherenkov energy loss and which is travelling in a straight line in 3-space in one frame, will, in other frames move on some curved path. Apparently the acceleration depends not only on the velocity but also on other variables [86,88]. But Jones [90] realized that this situation is not unusual in that even for normal particles the equation of motion which takes the radiation reaction force into account involves higher derivatives of the position than the second. By the use of an extended charge distribution, Jones [90] derived an equation of motion for a Cherenkov-emitting tachyon which produced the type of tachyon worldline described above. Jones claimed that his formulation is Lorentz invariant, but this is not correct as we shall see in section 3.5.

The problem of vacuum Cherenkov radiation may be trivially avoided in the theories of extended relativity. Mignani and Recami [91] show that it is not predicted by their theory by the following argument. Given a uniformly moving tachyon, we may transform to a superluminal frame in which the tachyon is at rest and is supposed to appear simply as an ordinary bradyon. A charged bradyon at rest does not radiate so that neither will a constant-velocity tachyon, as emission of radiation is a frame-independent occurrence. Of course such a proof is not available in ordinary relativity which we choose to keep.

We will not treat tachyons in general relativity, but we note here that even if tachyons are uncharged there will arise the very similar problem of gravitational Cherenkov radiation since tachyons should interact gravitationally because of their real energy-momentum [34]. It has been shown that a Cherenkov-type cone upon which the fields are singular occurs here also and that gravitational radiation should be expected [92,93,94,86,87]. In fact Lapedes and Jacobs [94] pointed out a close similarity to electromagnetic Cherenkov radiation by using the linearized field equations of general relativity and a point tachyon.

Evidently a solution to the problem of vacuum Cherenkov radiation for tachyons is required. In particular, there is a need for a treatment of the problem using unextended relativity and orthodox classical electrodynamic theory.

3.2 The Classical Theory of Charged Particles

As we are dealing with classical theories, we can only sensibly speak of point particles. Extended-particle models require that assertions be made about the shape and internal charge distribution of the particle. Moreover, cohesive forces must be introduced in some way to ensure the stability of the finite charge distribution.

All of these matters are outside the scope of classical theory. The main reason that extended particles instead of point particles were considered is that the self-energy for the latter diverges, but in more recent times we have come to accept methods for treating those divergences which troubled the physicists at the turn of the century.

Clearly, a consistent particle electrodynamics must include the dynamical consequences of the particle's energy loss due to the emission of radiation, for otherwise energy-momentum cannot be conserved. The loss of energy may be accounted for as a "radiation damping force" or a force of "radiation reaction" experienced by the particle. Within the limits of classical theory the Lorentz-Dirac equation is generally accepted as giving the correct account of radiation reaction, provided that unphysical, runaway solutions are excluded in some way, such as by the addition of asymptotic conditions (see, for example, Rohrlich [95]). This equation has existed since the time of the Abraham-Lorentz theory, but an important advance in classical point particle theory was made in 1938 when Dirac [96] derived it for a strictly point particle from fundamental classical electrodynamics alone, that is, from Maxwell's equations and the Lorentz force law.

Dirac derived the equation of motion from energy-momentum conservation. He took an arbitrary length of the worldline of the particle and considered it to be surrounded by a thin tube of radius ε. If the worldline of the particle is described as a function $z^\mu(\tau)$ of the propertime, then the points x^μ on the surface of the tube are given by

$$x^\mu = z^\mu(\tau) + \gamma^\mu \quad , \qquad (3.3)$$

where γ^μ is a small spacelike 4-vector satisfying

$$[\gamma]^2 = -\varepsilon^2$$

and $\gamma^\mu v_\mu = 0$,

where v^μ is the 4-velocity of the particle. This means that in the frame in which the particle is at rest, the tube is a sphere of radius ε. The flux of energy-momentum through the surface of this tube is then calculated and it is found that it can be expressed as an integral along the tube in the form

$$\int_{\tau_1}^{\tau_2} \left(\frac{e^2}{2\varepsilon} \dot{v}_\mu + e F^{Ext}_{\mu\nu} v^\nu - \frac{2}{3} e^2 \ddot{v}_\mu - \frac{2}{3} e^2 [\dot{v}]^2 v_\mu \right) d\tau \quad , (3.4)$$

where $F^{Ext}_{\mu\nu}$ is any external field present and dots denote differentiation with respect to the proper time. Terms that vanish with ε are ignored. Conservation of energy-momentum is then used by requiring that this flux must be balanced by the net flow in at the ends of the tube. Since the flux (3.4) only depends on the conditions at the ends of the tube, the integrand must be a perfect differential, say $\dot{B}_\mu$. Dirac chose the simplest form allowed for B_μ, namely

$$B_\mu = kv_\mu \quad ,$$

where k is a constant. If k is now chosen to have the form

$$k = \frac{e^2}{2\varepsilon} - m \quad . \qquad (3.5)$$

then the limit as ε tends to zero can be taken and there results the equation of motion

$$m\dot{v}_\mu - \frac{2}{3}e^2(\ddot{v}_\mu + [\dot{v}]^2 v_\mu) = -e\, F^{Ext}_{\mu\nu}\, v^\nu \quad . \tag{3.6}$$

This is the Lorentz-Dirac equation. An interesting feature of Dirac's derivation is the introduction of the observed mass m by way of the mass renormalization (3.5) which removes the terms that diverge when ε vanishes.

Advanced fields were actually incorporated into the calculation by Dirac when he defined the radiation field at a point as the difference of the retarded and advanced fields there. But Havas [97] pointed out that this was only done so that a field which was finite on the particle worldline could be obtained. The divergences are incorporated into the mass term. Thus no essential use is made of the advanced fields and the vital steps of the calculation, like the calculation of the flux, use only the familiar retarded fields. Fleury, Lopes, and Oberlechner [98] obtained a finite energy loss from a charged tachyon by writing down a type of Dirac radiation field. It is not derived in any way and, in any case, the solutions of the field equations at any point in space due to a charged tachyon are both retarded (or both advanced) solutions because the backward (or forward) light cone from any point will either make two or no intersections with a tachyon worldline. So to define a finite radiation field one must take the difference of either two advanced or two retarded solutions. Such a definition appears somewhat arbitrary.

Later derivations of the Lorentz-Dirac equation (refs. [99,100], for example) have also shown that only retarded fields are required. Teitelboim [99] derived the equation for a point particle by including the retarded field of the particle in the Lorentz force law and calculating the self-force. The retarded field is singular at the position of the particle so Teitelboim redefined the field at a point on the worldline of the particle to be the field obtained by averaging over a surface enclosing the point. Then the surface was shrunk down onto the point and in the process a divergent term appears. This term is absorbed into the mass in the Lorentz force law. Thus radiation reaction can be explained as a purely retarded-field effect if that field is modified a little. Another derivation of the Lorentz-Dirac equation has been given by Barut [100] who also used retarded fields and a different limiting procedure, the physical meaning of which is harder to picture.

That the self-force must be included in point-particle electrodynamics is explained by Teitelboim [99]. It is a consistency requirement and also necessary from the field point of view which says that energy and momentum are localized in the electromagnetic field. Certainly a self-force calculation is the most direct path to radiation reaction and the Lorentz-Dirac equation in point-particle electrodynamics, where the Maxwell-Lorentz equations are the only ones we possess. We will now give a derivation of the Lorentz-Dirac equation for bradyons by way of the self-force. The method suggests an extension to tachyons which has not been possible for the previous derivations of the equations of motion of a point particle.

3.3 The Lorentz-Dirac Equation for Bradyons

Let the worldline of the bradyon be described by a position 4-vector $z^\mu(\tau)$, where τ is the proper time. Differentiating with respect to the proper time produces the 4-velocity

$$v^\mu(\tau) = \dot{z}^\mu(\tau)$$

which, for a bradyon, is timelike with

$$v^\mu v_\mu \equiv [v]^2 = +1 .$$

Such a particle with charge e creates at the spacetime point x^μ a 4-current given by

$$j^\mu(x) = e \int_{-\infty}^{\infty} d\tau \; \delta^4(x - z(\tau)) \; v^\mu(\tau) \quad , \tag{3.7}$$

where the point charge distribution is described by a δ-function.

Maxwell's equations give the electromagnetic field produced by this current. In Gaussian units with $c = 1$, we can write Maxwell's equations in Lorentz gauge as

$$\Box A^\mu(x) = 4\pi \; j^\mu(x) \quad , \tag{3.8}$$

where $A^\mu(x)$ is the field's 4-vector potential and

$$F_{\mu\nu}(x) = A_{\mu,\nu}(x) - A_{\nu,\mu}(x) \tag{3.9}$$

is the electromagnetic field tensor. The solution of Maxwell's equations for the retarded 4-potential may be written as the convolution

$$A^\mu(x) = 4\pi \int d^4y \; D_R(x-y) \; j^\mu(y) \quad , \tag{3.10}$$

where D_R is the retarded Green's function which satisfies

$$\Box D_R(x) = \delta^4(x) \quad , \tag{3.11}$$

and may be represented either as

$$D_R(x) = \frac{1}{2\pi} \delta([x]^2) \; \theta(x) \quad , \tag{3.12}$$

where the step function θ ensures that we have a retarded solution of (3.11), or as

$$D_R(x) = \frac{-1}{(2\pi)^4} \int_{C_R} d^4k \, \frac{\exp(-ik.x)}{[k]^2} \quad . \qquad (3.13)$$

The contour C_R in (3.13) is drawn in the imaginary k_0-plane to pass above the poles at $k^0 = \pm|\underline{k}|$ in the direction of increasing k^0. The distribution D_R has support on the future light cone. By using (3.13) we get for the field tensor

$$F_{\mu\nu}(x) = \frac{ie}{4\pi^3} \int d^4y \int_{C_R} d^4k \int d\tau \, [v_\mu(\tau)k_\nu - v_\nu(\tau)k_\mu]\delta^4(y-z(\tau))$$

$$.\exp\frac{(-ik.(x-y))}{[k]^2} \quad . \qquad (3.14)$$

The self-force at the point $z^\mu(s)$ on the worldline is given by the Lorentz force law as

$$K_\mu(s) \equiv K_\mu(z(s)) = -e\, F_{\mu\nu}(z(s))v^\nu(s) \quad . \qquad (3.15)$$

This expression is singular since it involves evaluating the field, which is a distribution strictly speaking, at a point on the worldline where it diverges. The limiting procedures used by Dirac [96] and Teitelboim [99] and in all the other derivations of the Lorentz-Dirac equation from the Lorentz-force law may be seen as schemes for handling this improper operation on a distribution. Using Eq. (3.14) for the field, we get

$$K_\mu(s) = \frac{-ie^2}{4\pi^3} \int_{C_R} d^4k \int d\tau \, \exp[-ik.(z(s)-z(\tau)] \, [v_\mu(\tau)k_\nu - v_\nu(\tau)k_\mu]$$

$$.v^\nu(s)\, \frac{1}{[k]^2} \qquad (3.16)$$

as the self-force on the bradyon.

We begin to evaluate (3.16) by first making the simple change of variables

$$z'^{\mu} = z^{\mu}(\tau) - z^{\mu}(s) \tag{3.17}$$

and then choosing a Lorentz transformation $L^{\mu}{}_{\nu}$ such that

$$z'^{\mu} = L^{\mu}{}_{\nu}\ \xi^{\nu} \quad ,$$

where

$$\xi^{\mu} = ((\tau-s)\zeta,0,0,0) \quad .$$

Such a transformation can be found because the bradyon worldline is timelike so that z'^{μ} is a timelike 4-vector. The transformation $L^{\mu}{}_{\nu}$ may be written explicitly as

$$L^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \frac{z'^{\mu}z'_{\nu}+z'^{\mu}\xi_{\nu}+\xi^{\mu}z'_{\nu}+\xi^{\mu}\xi_{\nu}}{[z']^2 + \xi.z'} + \frac{2z'^{\mu}\xi_{\nu}}{[z']^2} \quad , \tag{3.18}$$

provided that

$$[z']^2 = [\xi]^2 = (\tau-s)^2\zeta^2 \quad .$$

By expanding z'^{μ} in a power series in $\tau-s$, this condition is seen to require that

$$\zeta(\tau) = 1 - \frac{(\tau-s)^2}{24}\,[\dot{v}(s)]^2 + 0(3) \quad . \tag{3.19}$$

Making the Lorentz transformation in Eq. (3.16) for the self-force yields

$$K_\mu(s) = -\frac{ie^2}{4\pi^3}\int_{C_R} d^4k \int d\tau\, \exp[ik_0\zeta.(\tau-s)][v_\mu(\tau)L_\nu{}^\rho - v_\nu(\tau)L_\mu{}^\rho]$$

$$.k_\rho v^\nu(s)\,\frac{1}{[k]^2} \qquad (3.20)$$

$$= -\frac{ie^2}{4\pi^3}\int_{C_R} d^4k \int d\tau\, \exp(ik_0\tau)\; G_\mu{}^0(\tau)k_0\,\frac{1}{[k]^2} \quad , \qquad (3.21)$$

where

$$G_\mu{}^\rho(\tau-s) = [v_\mu(\tau)L_\nu{}^\rho - v_\nu(\tau)L_\mu{}^\rho]v^\nu(s) \quad , \qquad (3.22)$$

and a change of variables from $\tau-s$ to τ has been made. Moreover, ζ has been dropped from the exponential, since the final result should only depend on the value of ζ near $\tau = s$ and $\zeta(s) = 1$ from Eq. (3.19). Only the $\rho = 0$ term in the summation in (3.20) survives because other values of ρ make the integrand an odd function in k_ρ.

Now we do the contour integration but we note that, if τ is positive, the contour must be closed in the upper half k_0 plane giving zero. Hence the step function θ in the following:

$$K_\mu(s) = \frac{-e^2}{\pi}\int_0^\infty k^2 dk \int_{-\infty}^\infty d\tau\; G_\mu{}^0(\tau)(\exp(ik\tau) + \exp(-ik\tau))\;\theta(-\tau)$$

$$= \frac{-e^2}{\pi}\int_{-\infty}^\infty dk \int_{-\infty}^\infty d\tau\; G_\mu{}^0(\tau)\; k^2\, \exp(ik\tau)\;\theta(-\tau) \quad ,$$

where we use polar coordinates with $k = |\underline{k}|$. The integration over k gives that

$$K_\mu(s) = 2e^2 \int_{-\infty}^{\infty} d\tau\, G_\mu{}^0(\tau)\, \theta(-\tau)\, \delta''(-\tau) \quad ,$$

which becomes, after integrating twice by parts,

$$K_\mu(s) = 2e^2 \left[G_\mu{}^{0''}(0)\, \theta(0) - 2G_\mu{}^{0'}(0)\, \delta(0) + G_\mu{}^0(0)\, \delta'(0) \right] \quad (3.23)$$

By substituting the form (3.18) of the Lorentz transformation into Eq. (3.22) for $G_\mu{}^\rho$ and expanding all functions of τ in power series in $\tau - s$, we get

$$G_\mu{}^0(\tau) = \frac{\tau}{2}\, \dot{v}_\mu(s) + \frac{\tau^2}{3}\, (\ddot{v}_\mu(s) + [\dot{v}(s)]^2\, v_\mu(s)) + O(3) \quad ,$$

so that the self-force (3.23) becomes

$$K_\mu(s) = \frac{4e^2}{3}\, (\ddot{v}_\mu + [\dot{v}]^2\, v_\mu)\, \theta(0) - 2e^2\, \dot{v}_\mu\, \delta(0) \quad . \quad (3.24)$$

From Newtonian mechanics, $M_{mech}\, \dot{v}_\mu$, where M_{mech} is the "mechanical" mass, is equal to the total force on the particle in which we must now include the self-force (3.24). Making this equality we find that

$$(M_{mech} + 2e^2\, \delta(0))\, \dot{v}_\mu - \frac{4}{3}\, e^2(\ddot{v}_\mu + [\dot{v}]^2\, v_\mu)\, \theta(0) = -eF^{ext}_{\mu\nu}\, v^\nu \quad , (3.25)$$

where $F^{ext}_{\mu\nu}$ is any external field. Now the infinite term $2e^2\, \delta(0)$ is combined with the mechanical mass to give the observed mass m, and $\theta(0)$ is taken to be one-half. Under these assumptions, (3.25) is just the Lorentz-Dirac equation (3.6). The necessity of these or equivalent assumptions in the calculation arises because an improper operation must be performed with certain distributions. In section 3.5 we shall exhibit the distributions which are involved, and look more closely at the way in which they are being handled.

3.4 The Equation of Motion of a Charged Tachyon

We parametrize the worldline $z^\mu(\lambda)$ of the tachyon by the invariant λ defined by Eq. (1.11). The 4-velocity of the tachyon is then defined by (1.12) and is a spacelike 4-vector:

$$[v]^2 = -1 .$$

As for the bradyon, the 4-current produced by a point tachyon with charge e is taken to be

$$j^\mu(x) = e \int_{-\infty}^{\infty} d\lambda \ \delta^4(x - z(\lambda)) \ v^\mu(\lambda) \quad ,$$

and, as was also done for the bradyon, we may solve Maxwell's equations and obtain the retarded field which may then be used to calculate the self-force at the point $z^\mu(s)$ on the worldline. We obtain

$$K_\mu(s) = -\frac{ie^2}{4\pi^3} \int_{C_R} d^4k \int d\lambda \ \exp\left[-ik.(z(s)-z(\lambda))\right].\left[v_\mu(\lambda)k_\nu - v_\nu(\lambda)k_\mu\right] .v^\nu(s) \frac{1}{[k]^2} \quad . \tag{3.26}$$

Because

$$z'^\mu \equiv z^\mu(\lambda) - z^\mu(s)$$

is a spacelike vector, a Lorentz transformation $L^\mu{}_\nu$ may be found such that

$$z'^\mu = L^\mu{}_\nu \ \xi^\nu \quad ,$$

where

$$\xi^\mu = (0,0,0,(\lambda-s)\eta) \quad .$$

The Lorentz transformation is given explicitly by (3.18).

The justification for many steps in this calculation is the same as that for equivalent steps appearing in the bradyon derivation, as we will pass over these details here. Thus we find that

$$\eta(\lambda) = 1 + \frac{(\lambda-s)^2}{24}[\dot{v}(s)]^2 + O(3) \quad .$$

Making the Lorentz transformation in the integrand of (3.26), we get

$$K_\mu(s) = -\frac{ie^2}{4\pi^3}\int_{C_R} d^4k \int d\lambda \, \exp(ik_3\eta.(\lambda-s))\left[v_\mu(\lambda)L_\nu{}^\rho - v_\nu(\lambda)L_\mu{}^\rho\right]$$

$$.k_\rho \; v^\nu(s) \frac{1}{[k]^2} \qquad (3.27)$$

$$= -\frac{ie^2}{4\pi^3}\int_{C_R} d^4k \int d\lambda \, \exp(ik_3\lambda)\left[M_\mu{}^0(\lambda)\, k_0 + M_\mu{}^3(\lambda)\, k_3\right]\frac{1}{[k]^2} \quad , (3.28)$$

since only the $\rho=0$ and $\rho=3$ terms contribute. We have written

$$M_\mu{}^\rho(\lambda-s) = (v_\mu(\lambda)L_\nu{}^\rho - v_\nu(\lambda)L_\mu{}^\rho)\, v^\nu(s) \quad .$$

The k_0 contour integration is seen to eliminate the term in (3.28) containing $M_\mu{}^3(\lambda)$ immediately, but such is not the case for the term with $M_\mu{}^0(\lambda)$. In fact the remaining k integration would give a divergent result. A subtraction procedure is evidently required and we choose the method of regulators, which replaces the propagator $1/[k]^2$ by the regular form

$$\text{Reg}\,\frac{1}{[k]^2} = \frac{1}{[k]^2} - \frac{1}{[k]^2-M^2} \quad ,$$

where M is an auxiliary mass which is allowed to tend to infinity in the last step of a calculation. With this expression replacing the propagator in (3.28), we arrive at

$$K_\mu(s) = -\frac{ie^2}{4\pi^3}\int_{C_R} d^4k \int d\lambda \exp(ik_3\lambda)\, M_\mu{}^0(\lambda)\, k_0$$

$$\cdot\left[\frac{1}{2|\underline{k}|}\left(\frac{1}{k_0-|\underline{k}|} - \frac{1}{k_0+|\underline{k}|}\right) - \frac{1}{2\sqrt{|\underline{k}|^2+M^2}}\left(\frac{1}{k_0-\sqrt{|\underline{k}|^2+M^2}} - \frac{1}{k_0+\sqrt{|\underline{k}|^2+M^2}}\right)\right]$$

which we find is zero upon doing the contour integration. Hence the self-force vanishes, giving an equation of motion for the tachyon which is entirely lacking radiation reaction terms:

$$m\dot{v}_\mu = -e\,F^{ext}_{\mu\nu}\,v^\nu \quad . \qquad (3.29)$$

Note that in (3.29), v^μ is the real spacelike 4-velocity of the tachyon, which is given by (1.12), and that m is the real mass parameter appearing in (1.15), and not the imaginary rest mass which is equal to im.

The Eq. (3.29), derived for an arbitrary tachyon motion, does not allow radiation from the tachyon in any form as there are no radiation reaction terms. In particular, vacuum Cherenkov radiation is not permitted. The absence of radiation may be demonstrated by calculating the energy-momentum flux from some region containing a segment of the tachyon's worldline. By Gauss' theorem, this flux is equal to the divergence of the electromagnetic energy-momentum tensor integrated over that region. Therefore the flux is

$$\int d^4x\, \phi(x)\, T_\mu{}^\rho{}_{,\rho}(x) \quad ,$$

where $\phi(x)$ is the characteristic function of the region, and $T_\mu{}^\rho$ is the electromagnetic energy-momentum tensor which is given by

$$T_{\mu\rho} = \frac{1}{4\pi}\,(F_{\mu\nu}\, F^\nu{}_\rho + \tfrac{1}{4}\, g_{\mu\rho}\, F_{\alpha\beta}\, F^{\alpha\beta})$$

in terms of the field tensor. By a well-known manipulation

$$\begin{aligned} -T_\mu{}^\rho{}_{,\rho}(x) &= -\frac{1}{4\pi}\, F_{\mu\nu}(x)\, F^{\nu\rho}{}_{,\rho}(x) \\ &= -\, F_{\mu\nu}(x)\, j^\nu(x) \\ &= K_\mu(x) \quad , \end{aligned}$$

which is the Lorentz force at the general point x^μ of spacetime (whereas (3.15) gives the Lorentz force on a particle).

Thus the flux of $T_{\mu\rho}$ from a spacetime region whose characteristic function is $\phi(x)$ is equal to

$$\int d^4x\, \phi(x)\, T_\mu{}^\rho{}_{,\rho}(x) = \int d^4x\, \phi(x)\, F_{\mu\nu}(x)\, j^\nu(x)$$

$$= \frac{ie^2}{4\pi^3} \int d^4x \int d^4y \int_{C_R} d^4k \int d\tau \int d\lambda\, \phi(x)[\, v_\mu(\tau)\, k_\nu \,-\, v_\nu(\tau)\, k_\mu]$$

$$.\,\delta^4(y-z(\tau))\, \frac{\exp[-ik.(x-y)]}{[k]^2}\, \delta^4(x-z(\lambda))\, v^\nu(\lambda) \quad ,$$

where we have used Eq. (3.7) for the current and Eq. (3.14) for the field. By doing the x and y integration we find that this flux is equal to

$$\frac{ie^2}{4\pi^3}\int_{C_R} d^4k \int d\tau \int d\lambda\ \phi(z(\lambda))\ [v_\mu(\tau)\ k_\nu - v_\nu(\tau)\ k_\mu]\ v^\nu(\lambda)$$

$$\cdot \frac{\exp[-ik.(z(\lambda)-z(\tau))]}{[k]^2} \quad .$$

Repeating the evaluation of (3.26), we find that the flux is zero so that the tachyon does not radiate.

3.5 Multiplication of Distributions

Let us determine the types of distributions which we have been implicitly dealing with in the previous sections. If we take the retarded Green's function in the alternative form (3.12), then the field tensor (3.9) for the bradyon's field may, using (3.10), be written as

$$F_{\mu\nu}(x) = 2e \int d\tau\ (v_\mu(\tau)\partial_\nu - v_\nu(\tau)\ \partial_\mu)\ \delta([x-z(\tau)]^2)\ \theta(x-z(\tau)) \ . \quad (3.30)$$

To find the self-force (3.15) we need to evaluate the field at a point $z^\mu(s)$ on the worldline and although the field diverges here we nevertheless write

$$F_{\mu\nu}(z(s)) = \int d^4x\ \delta^4(x-z(s))\ F_{\mu\nu}(x) \quad . \qquad (3.31)$$

This expression would be well-defined if $F_{\mu\nu}$ were continuous at $x^\mu = z^\mu(s)$, but of course it is not, and the right hand side of (3.31) is a product of distributions. Using (3.30) and (3.31) we can calculate the self-force and arrive at Eq. (3.20) written in the form

$$K_\mu(s) = -2e^2 \int d^4x \int d\tau\ \delta^4(x)\ G_\mu{}^\rho(\tau-s)\partial_\rho \Big[\delta((x^0-(\tau-s)\zeta)^2-(x^1)^2$$

$$-(x^2)^2-(x^3)^2)\ \theta(x^0-(\tau-s)\zeta)\Big] \ , \quad (3.32)$$

where $G_\mu{}^\rho$ is defined by (3.22). As happened with equation (3.20), only the $\rho=0$ term of (3.32) survives, this time because the differentiation for $\rho=1,2,$ and 3 gives the derivative of the δ-function multiplied by $-2x^\rho$ and this then vanishes as another δ-function occurs in the integrand and we know that

$$y\,\delta(y) = 0 \quad .$$

So in place of (3.21) we have

$$K_\mu(s) = -2e^2 \int dx^0 \int d\tau\; G_\mu{}^0(\tau-s)\; \delta(x^0)\; \partial_0 \Big[\delta((x^0-(\tau-s)\;\zeta)^2)\; \theta(x^0-(\tau-s)\;\zeta)\Big] \qquad (3.33)$$

after doing the spatial x integration.

We notice that (3.33) involves the distributional product

$$\delta(x^2)\;\theta(x) \quad , \qquad (3.34)$$

where x is a single variable. The limiting procedures, such as that used by Dirac [96], must be seen as methods of interpreting the expression (3.34) which occurs implicitly in the calculation. By using the thin tube surrounding the worldline, Dirac does not evaluate the field at the point $z^\mu(s)$ on the worldline directly, as in (3.31), but actually evaluates it at the point

$$x^\mu = z^\mu(s) + \gamma^\mu \quad ,$$

where γ^μ is the small spacelike vector described in equation (3.3) and following. In this way one is not dealing with $[z(s) - z(\tau)]^2$ as the argument of the δ-function in the field tensor (3.30) (in which τ would have to equal s) but with the expression

$$[z(s) - z(\tau) + \gamma]^2$$

in which $(\tau-s)$ is nonzero as long as the length ε of γ^μ is nonzero. The method of using the thin tube around the worldline is therefore equivalent to interpreting the distribution (3.34) as

$$\lim_{\varepsilon\to 0+} \delta(x^2-\varepsilon^2)\,\theta(x) = \lim_{\varepsilon\to 0+} \frac{\delta(x+\varepsilon)+\delta(x-\varepsilon)}{2\varepsilon}\,\theta(x)$$

$$= \lim_{\varepsilon\to 0+} \frac{\delta(x-\varepsilon)}{2\varepsilon}$$

$$= \lim_{\varepsilon\to 0+} \left[\frac{\delta(x)}{2\varepsilon} + \frac{\delta(x-\varepsilon)-\delta(x)}{2\varepsilon}\right]$$

$$= \lim_{\varepsilon\to 0+} \frac{\delta(x)}{2\varepsilon} - \tfrac{1}{2}\,\delta'(x) \quad . \qquad (3.35)$$

With this definition of the expression $\delta(x^2)\,\theta(x)$, equation (3.33) will then yield the Lorentz-Dirac equation if the first term in (3.35) is eliminated in a mass renormalization.

Other definitions of $\delta(x^2)\,\theta(x)$ yield slightly different results. For example, in Teitelboim's derivation [99], the first term in (3.35) has a factor $\frac{2}{3}$ instead of $\frac{1}{2}$. This may not seem particularly important because this term is absorbed into the observed mass eventually, but it partially illustrates the fact that it is not possible to write $\delta(x^2)\,\theta(x)$ in terms of proper distributions and at the same time avoid the introduction of arbitrary constants. In fact nonuniqueness must be accepted if we wish to multiply distributions together, especially distributions with coinciding singularities. For example Bogolubov, Logunov, and Todorov [101](p.100) consider the product $\delta(x)\,\theta(x)$ (which is just the $\theta(0)$ appearing in equation

(3.25) and show that it can only be sensibly defined to within an arbitrary constant C:

$$\delta(x)\ \theta(x) = C\ \delta(x) \quad , \tag{3.36}$$

so that $\theta(0)$ is not uniquely one half but can in fact be any constant. The method of Bogolubov et al for defining products of distributions is very similar to a method used by Güttinger [102] which we shall now explain and use to evaluate the expression $\delta(x^2)\ \theta(x)$.

In general, many of the familiar features of products must be discarded, such as uniqueness, associativity, and sometimes commutativity, but the method attempts to retain a maximum number of the usual properties. In fact the rule for the differentiation of a product can still be made to hold, and applying the method to a test function and any distribution gives the normal result of distributional analysis. These and other properties make the method quite satisfactory for applications. Briefly, the product of two distributions is determined by first finding a subspace U of the space of test functions D on which the product may be defined unambiguously, and then extending the definition to a linear functional defined on the whole space D. This extension is non-unique, bringing in a number of completely arbitrary, but finite, constants.

Returning to the product $\delta(x^2)\ \theta(x)$ we see that it may be defined on a subspace U containing all C^{∞} functions ϕ for which

$$\phi(0) = \phi'(0) = 0 \quad ,$$

as then we may write

$$\phi(x) = \frac{x^2}{2}\,\phi''(0) + \frac{x^3}{6}\,\phi'''(0) + \ldots \quad ,$$

and the method gives, for all ϕ in U,

$$\langle \delta(x^2)\;\theta(x)\;,\;\phi(x)\rangle \;=\; \langle \delta(x^2)\;,\;\frac{x^2}{2}\,\phi''(0) + \ldots\rangle \;, \quad \text{for} \quad x \geqslant 0$$

$$= \langle \delta(y)\;,\;\frac{y}{2}\,\phi''(0) + \ldots\rangle$$

$$= 0 \quad . \qquad (3.37)$$

To perform the extension to the whole of D, we note that any function ψ in D may be written as

$$\psi(x) = \psi(0)\;\psi_0(x) + \psi'(0)\;\psi_1(x) + \phi(x) \quad ,$$

where ϕ belongs to U and ψ_0 and ψ_1 are arbitrary functions of D apart from the restrictions

$$\psi_0(0) = \psi_1'(0) = 1$$

and

$$\psi_0'(0) = \psi_1(0) = 0 \quad .$$

Now if S is the distribution, defined on the whole of D, that corresponds to the product $\delta(x^2)\;\theta(x)$, then we have

$$\langle S,\,\psi(x)\rangle \;=\; \psi(0)\,\langle S,\,\psi_0(x)\rangle \;+\; \psi'(0)\,\langle S,\,\psi_1(x)\rangle \;+\; \langle S,\,\phi(x)\rangle$$

$$= A\;\psi(0) - B\;\psi'(0) \quad ,$$

where A and B are undetermined constants (since ψ_0 and ψ_1 are arbitrary) and, from (3.37), S gives zero when applied to ϕ. Therefore

$$\langle S, \psi(x) \rangle = A \langle \delta(x), \psi(x) \rangle + B \langle \delta'(x), \psi(x) \rangle ,$$

so that

$$\delta(x^2)\, \theta(x) = A\, \delta(x) + B\, \delta'(x) , \qquad (3.38)$$

where A and B are arbitrary constants. We may handle $\delta(x)\, \theta(x)$ in a similar way and obtain the formula (3.36) again.

Using the result (3.38) we may evaluate the self-force (3.33) for the bradyon and derive the equation of motion

$$m\dot{v}_{\mu} - \frac{4}{3} e^2 B(\ddot{v}_{\mu} + [\dot{v}]^2\, v_{\mu}) = -e\, F^{ext}_{\mu\nu}\, v^{\nu} , \qquad (3.39)$$

which would be the Lorentz-Dirac equation if B were chosen to be ½. The arbitrary constant A in (3.38), has been absorbed into the mass term in Eq. (3.39). We may say that classical point-particle electrodynamics gives Eq. (3.39) as the equation of motion of a charged bradyon, and that this equation is only the Lorentz-Dirac equation if a particular choice is made when interpreting a product of distributions. In this connection it is interesting to note that Eliezer [103], by exploiting the arbitrariness in the definition of the Dirac radiation field, is led to consider equations of the form (3.39) in which B is equal to (k+½). Here k is an arbitrary constant which is introduced by defining the radiation field to be the difference of the retarded and advanced fields multiplied by k.

Likewise for tachyons we may use the form (3.12) for the retarded Green's function in the expression for the field tensor and derive, in place of (3.27), the following expression for the self-force:

$$K_\mu(s) = -2e^2 \int d^4x \int d\lambda\ \delta^4(x)\ M_\mu{}^\rho(\lambda-s)\ \partial_\rho \Big[\delta((x^0)^2-(x^1)^2-(x^2)^2$$

$$- (x^3-(\lambda-s)\ \eta)^2)\ \theta(x^0)\Big] \ . \qquad (3.40)$$

Again only the $\rho=0$ and $\rho=3$ terms contribute, and these give

$$K_\mu(s) = -2e^2 \int dx^0 \int d\lambda\ M_\mu{}^0(\lambda-s)\ \delta(x^0)\ \partial_0 \Big[\delta((x^0)^2-(\lambda-s)^2\eta^2)\ \theta(x^0)\Big]$$

$$-2e^2 \int dx^0 \int dx^3 \int d\lambda\ M_\mu{}^3(\lambda-s)\ \delta(x^3)\ \partial_1 \Big[\delta((x^0)^2-(x^3-(\lambda-s)\eta)^2)$$

$$\theta(x^0)\Big] \ . \qquad (3.41)$$

To evaluate (3.41) we need the formula

$$\delta'(x)\ \theta(x) = C\ \delta'(x) + D\ \delta(x) \qquad (3.42)$$

in the first term, and the formulae

$$\delta(x)\ \theta(x) = C\ \delta(x) \qquad (3.36)$$

and $$\delta(x^2) = K\ \delta(x) \qquad (3.43)$$

to treat the second term. The formulae (3.42) and (3.43) (see [104]) may be derived by the same method used to obtain (3.38). In the above, C, D, and K are aribtrary constants, but the constant C appearing in (3.42) is the same as that in (3.36) since in Güttinger's method for multiplying distributions the product rule of differentiation is still valid [102]. Using these formulae in (3.41), a calculation similar to those performed previously will yield the equation of motion

$$(M_{mech} - e^2CK)\dot{v}_\mu - 2e^2DK \left[(v^0 v_\mu + \delta_\mu{}^0) - \frac{v^0}{(v^3+1)}(v^3 v_\mu + \delta_\mu{}^3)\right]$$

$$= - e F^{ext}_{\mu\nu} v^{\nu} \quad . \qquad (3.44)$$

In (3.44) the combination CK may be combined with the "bare" mechanical mass to give the observed mass of the tachyon. But we notice that the second term in (3.44) is non-covariant and even predicts a non-zero self-force when the tachyon is moving with constant velocity. This is the effect on the tachyon of would-be Cherenkov radiation, but Lorentz-invariance requires that the arbitrary constant D be set equal to zero to remove this term, and we get (3.29) as the equation of motion once again. The derivation of section 3.4 used regulators to treat the divergences, and no non-covariant terms arose in that derivation because the method of regulators retains Lorentz invariance at all times. So that while the formula for the distributional product $\delta'(x)\,\theta(x)$ is given by (3.42) in general, we demand Lorentz invariance in our application of it and this extra requirement calls for D to be put equal to zero.

We are now in a position to see the origin of the finite energy loss due to Cherenkov radiation obtained by Jones [90]. He calculates that the rate of change of energy with proper length for an extended tachyon charge distribution is proportional to

$$\frac{v}{(v^2-1)^{\frac{1}{2}}} \qquad (3.45)$$

in a derivation which is not manifestly covariant. For a comparison with the results above, consider a tachyon of constant velocity v along the z-axis so that its 4-velocity is

$$v^{\mu} = \frac{1}{(v^2-1)^{\frac{1}{2}}}\,(1,0,0,v) \quad . \qquad (3.46)$$

We then substitute this 4-velocity into the non-covariant term of equation (3.44) and consider the $\mu=0$ component of this term since we are interested in the energy balance. The result is just (3.45). Jones' non-invariant calculation has therefore uncovered the $\mu=0$ component of the non-covariant expression in Eq. (3.44). In fact his calculation proceeds from this point by merely filling in the other three components to make a 4-vector equation.

3.6 Discussion

The self-energy problem for the classical point tachyon is certainly more difficult than that for ordinary charged bradyons, as Cawley [17] has noted, but we have seen that it is nevertheless possible to determine the behaviour of charged tachyons from classical electrodynamics. Indeed, a comparison of the treatment of bradyons in section 3.3 with that for the tachyon shows that the solutions are in principle the same. For both we must handle divergences resulting from the multiplication of distributions which in turn arise because we are treating point particles in a classical field theory. For bradyons there exist a number of derivations of the equation of motion, and these all effectively choose a particular definition for the distributional product which occurs. The divergences encountered when treating tachyons are more severe, but methods exist for removing them.

Our treatment of charged tachyons in classical electrodynamics has led to the conclusion that vacuum Cherenkov radiation is disallowed because, from energy-momentum conservation, there would have to be a velocity-dependent radiation reaction term in the equation of motion, and we have seen that such a term is not present. In fact no radiation reaction terms of any kind are present so that a charged tachyon would not emit electromagnetic radiation under any circumstances, even if it were undergoing an arbitrary accelerated motion.

We stress that it is our retention of Lorentz invariance which has eliminated the possibility of a tachyon emitting electromagnetic radiation. The only way to secure such radiation from a tachyon is to break with Lorentz invariance by, for example, introducing a preferred frame.

Therefore, assuming the validity of classical electrodynamics for tachyons, we may say that tachyons are rather elusive objects, and that their detection could be very difficult.

CHAPTER 4

TACHYONS AND QUANTUM FIELD THEORY

We have found that while the tachyon is not forbidden existence as a classical object, it is, nevertheless, subject to some quite restrictive conditions. The general requirement of macroscopic causality does not allow tachyons to transmit information at superluminal speed. In addition, it has been shown that Lorentz invariance forbids a charged tachyon emitting electromagnetic radiation even when it is accelerating, so that the opportunity for this form of classical interaction is severely limited. Yet this does not necessarily mean that tachyons could only exist as mere phantoms which could never be detected. Tachyons might, for instance, interact with ordinary elementary particles to the extent that observable consequences might result. Classical theories would not be applicable in such a situation, and we would require an extension of quantum theory that covered tachyons. Therefore, we now attempt to place the tachyon in a quantum theory. To be satisfactory, a quantum theory of tachyons should include the essential features of the classical tachyon which emerged from the discussion in the Introduction, but at the same time we should want it to be as close as possible to conventional quantum theory.

4.1 The Scalar Tachyon

Throughout our discussion of the classical tachyon, we strictly adhered to Lorentz invariance, and we shall now take it as a most important requirement for a satisfactory quantum theory. We recall that Lorentz invariance is incorporated into ordinary quantum theory at quite a fundamental level. The set of states of an elementary particle or system provide a representation space for a unitary irreducible representation of the inhomogeneous Lorentz group (Poincaré group), and from this representation there arises the 4-momentum operator and the physical quantities of mass and spin. Wigner [105] described all the unitary, irreducible representations of the Poincaré group, and amongst them there are spacelike representations, or representations corresponding to an elementary particle with a spacelike 4-momentum. These were always discarded as being unphysical, but if tachyons are able to be treated in quantum theory, these representations may finally be associated with physical particles.

The solutions ϕ of the Klein-Gordon equation with imaginary mass,

$$(\Box - m^2)\ \phi(x) = 0 \quad , \tag{4.1}$$

provide a representation space for the one-dimensional, or scalar, representation corresponding to spacelike 4-momentum. This is the simplest of such unitary, irreducible representations as all the others are infinite-dimensional. So for simplicity we shall examine quantization procedures for the scalar tachyon field $\phi(x)$ which satisfies (4.1), and whose Fourier transform has its support on the mass hyperboloid given by

$$[p]^2 = -m^2 \quad . \tag{4.2}$$

In fact all the difficulties associated with describing tachyons in quantum field theory are present in the case of the scalar tachyon, for they arise from the unusual nature of the tachyon mass hyperboloid (4.2) which was described in the Introduction and is illustrated in Fig. 1.1.

We shall mention some of these problems now.

Since the mass hyperboloid is single-sheeted, the first major problem to be overcome in developing a quantum field theory of tachyons is in reconciling the apparent conflict between Lorentz invariance and the need to have only positive energies capable of being observed according to the theory. That is, the reinterpretation principle must be implemented in the theory without destroying Lorentz invariance. In section 4.2 we shall look at some methods of quantization. We find that we must be very careful in the way we use the reinterpretation principle in quantum field theory because Lorentz invariance can be lost in a rather subtle way.

Another problem, the inability to obtain a strictly localized solution of (4.1), was pointed out by Feinberg [6]. This arises because the 3-momentum of a tachyon of mass m has a minimum possible length of $|\underline{p}| = m$. If the Fourier transform $\tilde{\phi}(p)$ of a function $\phi(x)$ vanishes for values of $|\underline{p}|$ less than m, then $\phi(x)$ cannot vanish outside of any compact region, although we can arrange that $\phi(x)$ fall off like any power of $|\underline{x}|^{-1}$ at large spatial distances $|\underline{x}|$ [6]. Thus we cannot construct a wave-packet solution of (4.1) which has only a finite spatial extent if the 3-momentum satisfies

$$|\underline{p}| \geqslant m \quad . \tag{4.3}$$

By maintaining this restriction on the momenta, we are not worried by doubts which have been raised about whether Eq. (4.1) is able to describe superluminal propagation. We consider this matter after examining the invariant functions associated with Eq. (4.1).

Invariant functions

In future discussions it will be of use to know the Lorentz-invariant solutions, and also the Green's functions, of Eq. (4.1). For example, the commutator function for the free, quantized, scalar tachyon field should be a Lorentz-invariant solution to the homogeneous equation (4.1). Such functions were described first by Schmidt [9], and were also treated by other authors [11,106-110].

One invariant solution of Eq. (4.1) is given by

$$G^1(x) = \frac{i}{(2\pi)^4}\int_{C_1} d^4p\, \frac{e^{-ip.x}}{[p]^2+m^2} \quad , \tag{4.4}$$

where the closed contour C_1 in the complex p^0 plane is given in Fig. 4.1. The contour C_1 gives the prescription for avoiding the singularities of the integrand in (4.4) which are located at the following values of p^0:

$$\pm\,\omega_{\mathbf{p}} = \pm\sqrt{|\underline{p}|^2-m^2} \quad , \qquad \text{where} \quad |\underline{p}| \geqslant m, \tag{4.5}$$

and

$$\pm\, i\chi_{\mathbf{p}} = \pm\, i\sqrt{m^2-|\underline{p}|^2} \quad , \qquad \text{where} \quad |\underline{p}| < m. \tag{4.6}$$

We find the function G^1 may be written in the form

$$G^1(x) = \frac{1}{(2\pi)^3}\int_{|\underline{p}|\geqslant m} \frac{d^3p}{\omega_{\mathbf{p}}}\; e^{i\,\underline{p}.\underline{x}}\cos(\omega_{\mathbf{p}}x^0) \quad , \tag{4.7}$$

from which we can see that the invariant function G^1 is even,

$$G^1(x) = G^1(-x) \quad , \tag{4.8}$$

and that it does not vanish for spacelike argument since

$$G^1(x)\Big|_{x^0=0} = \frac{1}{(2\pi)^3}\int_{|\underline{p}|\geqslant m} \frac{d^3p}{\omega_{\mathbf{p}}}\; e^{i\,\underline{p}.\underline{x}} \neq 0 \quad . \tag{4.9}$$

We also find that

$$\left(\frac{\partial\, G^1(x)}{\partial x^0}\right)_{x^0=0} = 0 \quad . \tag{4.10}$$

Another invariant solution of (4.1) is the function

$$G(x) = \frac{-1}{(2\pi)^4}\int_C d^4p\, \frac{e^{-ip.x}}{[p]^2+m^2} \quad , \tag{4.11}$$

where the contour C is given in Fig. 4.2. We may write this function in the form

$$G(x) = \frac{1}{(2\pi)^3}\int_{|\underline{p}|\geq m} \frac{d^3\underline{p}}{\omega_p}\, e^{i\underline{p}.\underline{x}}\, \sin(\omega_p x^0)$$
$$+ \frac{1}{(2\pi)^3}\int_{|\underline{p}|<m} \frac{d^3\underline{p}}{\chi_p}\, e^{i\underline{p}.\underline{x}}\, \sinh(\chi_p x^0) \quad , \tag{4.12}$$

from which we find the properties

$$G(-x) = -G(x) \quad , \tag{4.13}$$

$$G(x)\Big|_{x^0=0} = 0 \quad , \tag{4.14}$$

and

$$\left(\frac{\partial\, G(x)}{\partial x^0}\right)_{x^0=0} = \delta^3(\underline{x}) \quad . \tag{4.15}$$

From (4.12) it can be seen that the imaginary energies $\pm\, i\chi_p$ arise in the momentum representation of the invariant function G, but the separation of the two terms in (4.12) does not yield Lorentz-invariant functions. So, for example, although the function G^2 defined by

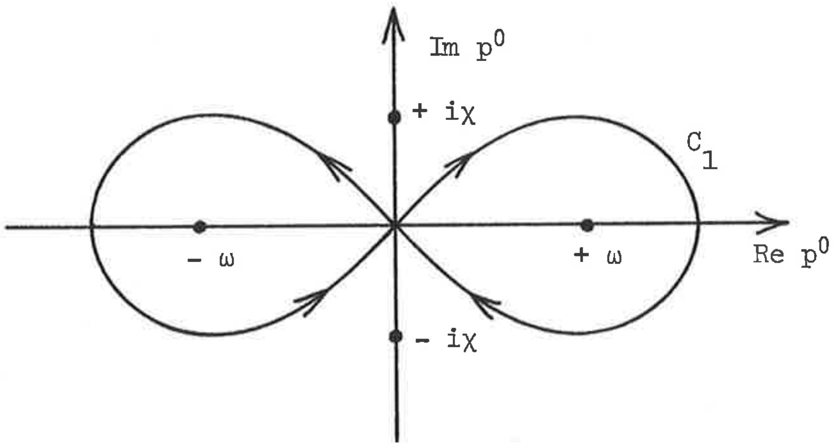

FIGURE 4.1. The Contour C_1.

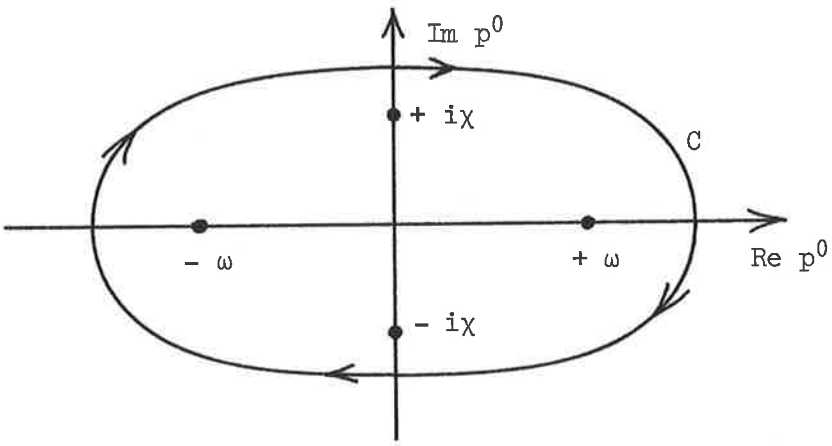

FIGURE 4.2. The Contour C.

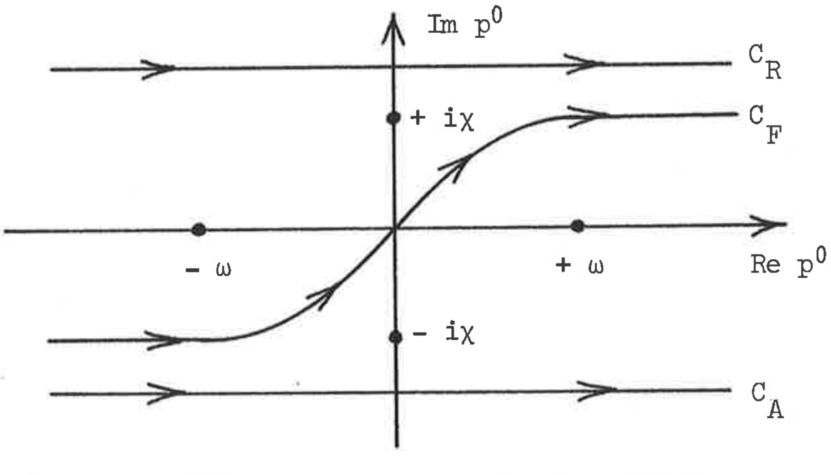

FIGURE 4.3. The Contours C_R, C_A, and C_F.

$$G^2(x) = \frac{1}{(2\pi)^3} \int_{|\underline{p}| \geqslant m} \frac{d^3p}{\omega_p} \; e^{i\underline{p}.\underline{x}} \sin(\omega_p x^0) \tag{4.16}$$

satisfies Eq. (4.1), it is not an invariant function.

Because of the properties (4.14) and (4.15), the function G can be used to solve the initial-value problem of the imaginary-mass Klein-Gordon equation. In fact by putting imaginary mass into the function $\Delta(x)$ for the ordinary, real-mass Klein-Gordon equation (see Schweber [111]), we obtain the function $-G(x)$.

We may also find the Green's functions, or the invariant solutions to the inhomogeneous equation

$$(\Box - m^2)\,\phi(x) = \delta^4(x) \quad . \tag{4.17}$$

Three such functions are

$$G^R(x) = \frac{-1}{(2\pi)^4} \int_{C_R} d^4p \, \frac{e^{-ip.x}}{[p]^2+m^2} = \theta(x)\, G(x) \quad , \tag{4.18}$$

$$G^A(x) = \frac{-1}{(2\pi)^4} \int_{C_A} d^4p \, \frac{e^{-ip.x}}{[p]^2+m^2} = -\theta(-x)\, G(x) \quad , \tag{4.19}$$

and

$$G^F(x) = \frac{2i}{(2\pi)^4} \int_{C_F} d^4p \, \frac{e^{-ip.x}}{[p]^2+m^2} \quad , \tag{4.20}$$

where the contours C_R, C_A, and C_F are given in Fig. 4.3. As suggested by the notation, G^R and G^A are the retarded and advanced solutions of (4.17), respectively.

<u>Propagation of the scalar tachyon field</u>

Feinberg [6] considered the particular initial-value problem for the scalar tachyon field in which the initial conditions $\phi(0,\underline{x})$ and

$(\partial\phi/\partial x^0)(0,\underline{x})$ are restricted to be functions whose Fourier transforms $\tilde{\phi}(p)$ vanish when $|\underline{p}| < m$. Then no imaginary energies are involved and the solution is given just in terms of the noninvariant part $G^2(x)$ of the function $G(x)$ by the usual type of expression. We have that

$$\phi(x^0,\underline{x}) = \int d^3y\ [G^2(x^0,\underline{x}-\underline{y}) \frac{\partial\phi}{\partial x^0}(0,y) - \frac{\partial G^2}{\partial x^0}(x^0,\underline{x}-\underline{y})\ \phi(0,\underline{y})] \ .$$

Note that we are not describing the propagation of a strictly localized disturbance in this way because we have seen that under the above conditions on the Fourier transform, the function $\phi(0,\underline{x})$ cannot vanish at large values of $|\underline{x}|$.

Some authors [112,113,114], recalling that the characteristic surfaces of a differential equation like (4.1) are determined by the terms containing the highest derivatives and not, for example, by the mass term in (4.1), conclude that the sign of the mass term is irrelevant to the propagation character of the field, and that, just as for the real-mass Klein-Gordon equation, only subluminal propagation would occur. This matter was actually explained by Tanaka [11] some time ago. The point is that the speed of light is a maximum speed for the propagation of sharply defined features of the wave, like a pulse which has vanishing amplitude outside of a small spatial region. But such highly localized disturbances contain Fourier components which are non-vanishing for $|\underline{p}| < m$. Thus imaginary energies are involved, and the propagation is described by the whole function $G(x)$, and is subluminal due to the properties of this function.

But if only real energies are permitted, then we must require the Fourier transform of the solutions of (4.1) to vanish for $|\underline{p}| < m$. Then the speed of light is a minimum speed of propagation [115,116], and we have true superluminal behaviour in that the initial conditions at a spacetime point can influence the solution of the equation at a spacetime point from which it has a spacelike separation.

The imaginary energies have been included in some quantization procedures for tachyons. Tanaka [11] was the first to attempt this, but his quantization is not Lorentz-invariant [107,117]. Schroer [118] and Streit and Klauder [119] also included imaginary energies and obtained causal commutation relations, in the sense that the commutator function is $G(x)$ which vanishes for spacelike argument. But in these quantizations one may be troubled by the lack of a ground state or the need for an indefinite metric.

The inclusion of imaginary energies during quantization might be a reasonable response from those who see the problem just as one of "quantizing" the equation (4.1). But this is not our view. We are interested in the real-energy solutions of (4.1) for the representation of the Poincaré group which they carry, and we want to use the classical field satisfying (4.1) as a base for the construction of a quantum field theory of the scalar tachyon. We therefore require superluminal propagation, real energies, and, as far as possible, all the things that characterize the classical tachyon. So we shall not consider the imaginary-energy quantizations further.

Excluding imaginary energy, we are left with the function $G^1(x)$ as the only nontrivial candidate for the commutator function of the quantized scalar tachyon field. This function does not vanish for spacelike argument so that it seems we must violate "microscopic causality" in a quantum theory of tachyons. We have therefore found that two of the assumptions made in ordinary quantum field theory are under threat when we treat tachyons. Firstly, having a spacelike

4-momentum, the semidefiniteness of the energy operator is in question, although we hope some form of reinterpretation will yield positive physical energies. Secondly, local commutativity or microscopic causality is in jeopardy. This axiom reflects the idea that measurements of the field at one point in spacetime do not affect field quantities at spacetime points which are separated from it by a spacelike interval. So for the tachyons the violation of this principle should not have been unexpected. We note that if these two axioms do not strictly hold for tachyons, then the proof by Schwartz [120] that imaginary mass free fields are impossible no longer applies.

We enquire now to what extent our classical tachyon can be treated in a conventional quantum field theory. Following the remarks above, we do this by trying to base such a theory on the solutions of (4.1) subject to the requirement that the energy be real so that the 3-momentum satisfies (4.3).

4.2 Tachyons and Quantization

Feinberg's quantization [6] was the first attempt to incorporate the reinterpretation principle in a quantum field theory for tachyons. It tried to remove negative energies from the outset by its particular description of the field operator in terms of annihilation and creation operators. Arons and Sudarshan [117] pointed out that this method is not Lorentz invariant by showing that the unitary operators representing Lorentz transformations do not, strictly speaking, exist in the Fock space of Feinberg's theory. They mentioned that this is connected with Segal's comment concerning the lack of an invariant separation between the positive- and negative-frequency parts of a solution of the Klein-Gordon equation with imaginary mass. Ecker [106] also stresses the importance of properly specifying the Hilbert space of states in tachyon field theories.

These comments may be illustrated by trying to apply the algebraic quantization formalism set up by Segal [121]. A great advantage of this approach is that the essential features of a quantum theory, like the commutation relations, are used in the construction of an abstract C^*-algebra, and that we are not committed to a particular representation of the commutation relations in a specific Fock space.

Briefly, the algebraic formulation of a quantized theory based on Eq. (4.1) would proceed by taking the space M of real solutions of Eq. (4.1). The space M may be thought of as the classical phase space. We may write the elements of M as

$$\phi(x) = (2\pi)^{-3/2} \int_{p^0=\pm\omega_p} \frac{d^3p}{2|p^0|} e^{-ip.x} \tilde{\phi}(p) \quad , \qquad (4.21)$$

where the integral is taken over the whole mass hyperboloid (4.2) so that the energy can be either positive or negative:

$$p^0 = \pm \omega_p \equiv \pm\sqrt{|\underline{p}|^2-m^2} \quad , \qquad (4.22)$$

and we shall only ever be considering 3-momenta satisfying (4.3). Because of the reality of the solutions $\phi(x)$, we have

$$\tilde{\phi}(p)^* = \tilde{\phi}(-p) \qquad (4.23)$$

with * denoting complex conjugation. With an inner product given by

$$(\phi,\psi) = \int_{p^0=\pm\omega_p} \frac{d^3p}{2|p^0|} \tilde{\phi}(p) \tilde{\psi}(p)^* \quad , \qquad (4.24)$$

M is a real Hilbert space.

Quantization using commutators requires the existence of a nondegenerate, bilinear, antisymmetric form (called a symplectic form) on the space M. Just as for the ordinary Klein-Gordon equation, such a form is given by

$$B(\phi,\psi) = \int d^3x\ (\phi(x)\ \dot{\psi}(x) - \dot{\phi}(x)\ \psi(x)).$$

Writing this in momentum space we get

$$B(\phi,\psi) = i \int_{p^0>0} \frac{d^3p}{2|p^0|}\ (\tilde{\phi}(p)\ \tilde{\psi}(p)^* - \tilde{\phi}(p)^*\ \tilde{\psi}(p)) \quad , \quad (4.25)$$

where the integral is only over positive energies, p^0.

A Weyl system over (M,B) is now defined to be a set of unitary operators $W(\phi)$ in some Hilbert space H, which satisfy a certain continuity condition [121] and for which the Weyl relations,

$$W(\phi)\ W(\psi) = e^{\frac{i}{2}B(\phi,\psi)}\ W(\phi+\psi) \quad , \qquad (4.26)$$

are satisfied. In the language of ordinary field theory in a Hilbert space, the self-adjoint generators of the operators W can be regarded as being smoothed field operators, and the form B as being derived from the commutator function. Now the important physical entity is the abstract Weyl algebra of which a Weyl system over (M,B) is regarded as a representation in a specific Hilbert space. In the case of the free bradyon Klein-Gordon field, certain physical requirements concerning the vacuum and the particle number single out a particular representation, namely, the Fock representation. A Weyl system is known to exist [121] if there is a real linear operator, J, on M satisfying

$$J^2 = -1 \quad ,$$

$$B(J\phi, J\psi) = B(\phi,\psi) \quad , \qquad (4.27)$$

and $B(\phi, J\phi) > 0$ iff $\phi \neq 0$.

The space M becomes the complex Hilbert space H with inner product

$$(\phi,\psi)_J = B(\phi, J\psi) - i\, B(\phi,\psi) \quad . \qquad (4.28)$$

For both bradyons and tachyons the properties (4.27) are possessed by the operator

$$J : \tilde{\phi}(p) \to i\, \varepsilon(p)\, \tilde{\phi}(p) \quad . \qquad (4.29)$$

The complex structure provided by J in (4.29) is directly related to the separation of the field into positive- and negative-energy parts and the definition of creation and annihilation operators in conventional field theory.

For tachyons, the trouble lies with Lorentz invariance. Physical transformations, which should be unitary transformations in the representation space H, arise from real linear symplectic transformations in M; that is, transformations preserving the form B. Now the elements of M, being scalars, behave in the following way under a Poincaré transformation (Λ, a):

$$\phi(x) \to \phi(\Lambda^{-1}(x - a)) \qquad (4.30)$$

$$\tilde{\phi}(p) \to e^{ip.a}\, \tilde{\phi}(\Lambda^{-1}p) \quad . \qquad (4.31)$$

Thus, from Eq. (4.25) we see that B is not Lorentz invariant because a Lorentz transformation can change the sign of the energy. We cannot obtain a Lorentz invariant form by just extending the integration in (4.25) to cover negative energies as well because the resulting expression will vanish due to the reality condition (4.23). There is no invariant symplectic form so that Lorentz transformations are not unitary with respect to (4.28) as Segal noted [121]. Recalling the relation between B and the commutator function, this can be seen as a restatement of the fact that no odd invariant solution of (4.1) involving only real energies in the momentum space representation exists. (We saw in the last section that the odd invariant function G(x) contains imaginary energies). This is also equivalent to Feinberg's observation that quantization with commutators is not invariant for tachyons. He then proposes the use of anticommutators.

The algebraic quantization with anticommutators uses the space M above and a non-degenerate, symmetric bilinear form S, and considers the Clifford algebra over (M,S) [122]. We take

$$S(\phi,\psi) = \int_{p^0=\pm\omega_p} \frac{d^3p}{2|p^0|} \, \tilde{\phi}(p) \, \tilde{\psi}(p)^* \, , \tag{4.32}$$

and, with transformations which preserve S taking the place of the symplectic transformations of the boson case, we see that we have in this case a definite abstract C*-algebra in which Poincaré transformations generate *-automorphisms (see [122]). But the construction of a representation of this abstract Clifford algebra in a conventional Fock space would, similarly to the boson case, require a complex structure, J, which made M a complex Hilbert space H with inner product

$$(\phi,\psi)_J = S(\phi,\psi) - i\, S(J\phi,\psi) \; .$$

With S as in (4.32) and J given by (4.29), we find

$$(\phi,\psi)_J = \int_{p^0=+\omega_p} \frac{d^3p}{|p^0|} \tilde{\phi}(p)\, \tilde{\psi}(p)^* \quad , \tag{4.33}$$

and this is the Feinberg quantization.

From (4.33) it can be seen that rotations and translations commute with J and preserve S, and will therefore be represented as unitary transformations in the Fock space H of Feinberg's theory. But it is evident that J does not commute with Lorentz transformations, which will therefore not be unitary transformations with respect to (4.33). That is, for the Feinberg theory, there is no fixed Fock representation space upon which Lorentz transformations act as unitary operators. Assuming that such operators do exist in the representation space produces results like that obtained by Arons and Sudarshan [117], who showed that the Lorentz-transformed vacuum of Feinberg's theory contains infinitely-many particles.

It may be countered that a non-invariant vacuum is not especially serious. In fact one arises during the algebraic formulation of the quantization of the electromagnetic field by the Fermi method [123]. Nevertheless, a Hilbert space displaying a representation of the abstract Weyl algebra and a unitary representation of the Poincaré group can be constructed [123]. This Hilbert space is the direct integral of Hilbert spaces labelled by a momentum-like vector q. To attempt a similar construction here, we would consider Hilbert spaces H_q which are constructed from (M,S) by using the complex structure

$$J(q) = \Lambda(q)\; J\; \Lambda(q)^{-1} \quad , \tag{4.34}$$

where J is given by (4.29) and $\Lambda(q)$ is the Lorentz transformation that takes the standard vector $\hat{q} = (1,0,0,0)$ to the vector q. That is, $q = \Lambda(q)\hat{q}$, and we are labelling Lorentz transformations by the "momentum" vector q. Now

$$J(q)\ \phi(x) \equiv \Lambda(q)\ J\ \Lambda(q)^{-1}\ \phi(x)$$

$$= i\ (2\pi)^{-3/2}\int_{(\Lambda(q)^{-1}p)^0>0} \frac{d^3p}{2|p^0|}\ (e^{-ip.x}\tilde{\phi}(p) - e^{ip.x}\ \tilde{\phi}(-p)), \qquad (4.35)$$

so that H_q is the Hilbert space with inner product

$$(\phi,\psi)_q = S(\phi,\psi) - i\ S(J(q)\phi,\psi)$$

$$= \int_{(\Lambda(q)^{-1}p)^0>0} \frac{d^3p}{2|p^0|}\ \tilde{\phi}(p)\ \tilde{\psi}(p)^* \ , \qquad (4.36)$$

and we get a Fock representation on each H_q by using the complex structure $J(q)$.

Unfortunately, taking the direct integral of the spaces H_q is not now going to solve our problems because we are worried by negative energies again. This can be seen by noting that although time translations have a positive generator in $H = H_{\hat{q}}$, they do not have a positive generator in H_q where the inner product is (4.36) because p^0 in (4.36) can be negative even though $(\Lambda(q)^{-1}p)^0$ is always positive.

We conclude that there is no satisfactory way of taking the reinterpretation principle into account during the basic quantization in a theory which is conventional to the extent that Lorentz-invariant fields, invariant (anti-) commutation relations, and an ordinary Fock space are retained. The unusual properties of Lorentz transformations in Feinberg's theory [6,124] may be seen as a reflection of the fact that such transformations do not exist as unitary operators in a single Fock space, rather than an expression of some property of tachyons.

The method of quantization used by Ecker [106] is a formal version of that described by Korff and Fried [125], and in this method the field operator no longer transforms as a Lorentz scalar. In addition, the reinterpretation principle does not operate in quite the way described in chapter 1, for a Lorentz transformation applied to a state can convert a tachyon into an antitachyon, for example, but emission and absorption are not interchanged. This scheme appears to involve too great a departure from the idea of the classical tachyon which has been developed.

The results of this section suggest that the only way to treat tachyons in a conventional quantum theory is to allow the negative energies into the initial stages of quantization and attempt to eliminate them at some later stage, for otherwise Lorentz invariance will be violated. Such is the approach of the Arons-Sudarshan quantization.

4.3 The Arons-Sudarshan Quantization

This method of quantization is also illustrated by using it on the scalar tachyon field, but it is clearly applicable to fields which transform under other unitary, spacelike representations of the Poincaré group. The configuration space wavefunction for a tachyon with a fixed mass m is given by

$$\phi(x) = (2\pi)^{-3/2} \int_{\pm} (dp)\, e^{-ip.x}\, \tilde{\phi}(p) \quad , \qquad (4.37)$$

where we have the same invariant integration as in (4.21) so that

$$\int_{\pm} (dp) \equiv \int d^4p\ \delta([p]^2+m^2)\ \theta(|\underline{p}|-m) \quad , \quad \text{and} \quad (dp) = \frac{d^3p}{2\omega_p} \ . \qquad (4.38)$$

In (4.37) the momentum-space function, $\tilde{\phi}(p)$, could be taken to be a Poincaré-covariant wavefunction [126]. Note that a single representation of the Lorentz group would be carried by the function $\tilde{\phi}(p)$ defined on the single-sheeted hyperboloid $[p]^2 = -m^2$. This is to be contrasted with the situation for bradyons where the positive- and negative-energy branches of the mass hyperboloid carry different representations and both are required to obtain a local configuration-space wavefunction [126]. This should make one wary of attempting any sort of separation of positive and negative energies during quantization for tachyons whilst still requiring Lorentz invariance.

The Arons-Sudarshan [117] quantization avoids this difficulty by expanding the field operator, which we also write as $\phi(x)$, solely in annihilation operators, $a(p)$, so that we have

$$\begin{aligned} \phi(x) &= (2\pi)^{-3/2} \int_{\pm} (dp)\ e^{-ip.x}\, a(p) \\ &= (2\pi)^{-3/2} \int (dp) \left(e^{-i\omega_p p^0 + i\underline{p}.\underline{x}}\, a_+(\underline{p}) + e^{i\omega_p p^0 + i\underline{p}.\underline{x}}\, a_-(\underline{p}) \right) \qquad (4.39) \end{aligned}$$

where

$$a_{\pm}(\underline{p}) \equiv a(\pm\omega_p, \underline{p}) \quad ,$$

and the field is nonhermitian. Either commutation or anticommutation relations may be used, and we choose the former for the scalar field:

$$[a_\varepsilon(\underline{p})\ ,\ a^\dagger_{\varepsilon'}(\underline{p}')] = 2\omega_p\ \delta^3(\underline{p} - \underline{p}')\ \delta_{\varepsilon\varepsilon'} \quad , \qquad (4.40)$$

where ε is the sign of the energy. For the field (4.39) these give

$$[\phi(x), \phi^\dagger(x')] = (2\pi)^{-3} \int_{|\underline{p}|\geqslant m} \frac{d^3p}{\omega_p} e^{i\underline{p}.(\underline{x}-\underline{x}')} \cos \omega_p (x^0-x'^0)$$

$$= G^1(x - x') \quad . \qquad (4.41)$$

That the commutator function in (4.41) is the invariant function $G^1(x-x')$ can be seen from (4.7). As was noted in section 4.1, G^1 does not vanish for spacelike argument. We have also noted that local commutativity is expected to fail for tachyons, but it makes for a serious difference from bradyon field theory in that it severely hampers attempts to describe interactions for tachyons.

A conventional Fock space, H, is now constructed on an invariant vacuum $|0\rangle$. We have

$$a_\pm(\underline{p})\ |0\rangle = 0 \ , \quad \text{for all} \ |\underline{p}| \geqslant m \ , \qquad (4.42)$$

and the many-particle states are the

$$\left| \begin{matrix} p_1, p_2, \ldots, p_m \\ q_1, q_2, \ldots, q_n \end{matrix} \right\rangle = \frac{1}{\sqrt{m!n!}}\ a_+^\dagger(\underline{p}_1)a_+^\dagger(\underline{p}_2)\ldots a_+^\dagger(\underline{p}_m)a_-^\dagger(q_1)a_-^\dagger(q_2)\ldots$$

$$\ldots a_-^\dagger(\underline{q}_n)\ |0\rangle \ , \qquad (4.43)$$

where, on the left-hand side, a momentum vector p in the upper (lower) position designates a tachyon of 3-momentum $\underline{p}$ and energy ω_p $(-\omega_p)$. The states (4.43) satisfy the orthonormality relation

$$\left\langle \begin{matrix} p_1, \ldots, p_m \\ q_1, \ldots, q_n \end{matrix} \middle| \begin{matrix} r_1, \ldots, r_k \\ s_1, \ldots, s_\ell \end{matrix} \right\rangle = \delta_{mk}\ \delta_{n\ell}\ \frac{1}{m!n!} \sum_{p(\mu)} \sum_{p(\nu)} \prod_{i=1}^{m} \prod_{j=1}^{n}$$

$$\cdot 2|p_i^0|\,\delta^3(\underline{p}_i - \underline{r}_{\mu(i)})2|q_j^0|\,\delta^3(\underline{q}_j - \underline{S}_{\nu(j)}) \,, \qquad (4.44)$$

where the sums are taken over all permutations of the sets $\{\mu(1),\ldots,\mu(m)\}$ and $\{\nu(1),\ldots,\nu(n)\}$. The completeness relation for these states in the space H then has the form

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\int(dp_1)\ldots\int(dp_m)\int(dq_1)\ldots\int(dq_n)$$

$$\cdot\left|\begin{matrix}p_1,p_2,\ldots,p_m\\ q_1,q_2,\ldots,q_n\end{matrix}\right\rangle\left\langle\begin{matrix}p_1,p_2,\ldots,p_m\\ q_1,q_2,\ldots,q_n\end{matrix}\right| = I \qquad (4.45)$$

Under a Poincaré transformation (Λ,a), the field transforms as a scalar (see (4.30)) and the operators $a(p)$ transform as follows:

$$a(p) \rightarrow e^{ip.a}\, a(\Lambda^{-1}p) \quad . \qquad (4.46)$$

A Lorentz transformation can therefore change the sign of the energy and, acting on the states (4.43) of H, mix the upper and lower sets of momenta. The generators of spacetime translations are the operators

$$P_\mu = \int_{\pm}(dp)\; p_\mu\; a^\dagger(p)\; a(p) \qquad (4.47)$$

since

$$[\phi(x), P_\mu] = i\,\partial_\mu\,\phi(x) \quad . \qquad (4.48)$$

We see again from (4.47) that the space H contains positive- and negative-energy states which, of course, are required if we are to have a unitary representation of the Poincaré group in H.

The reinterpretation principle

An application of the reinterpretation principle is eventually needed to remove any mention of negative energies, but Arons and Sudarshan want to avoid destroying the Lorentz invariance of the theory in the process, so the implementation of the reinterpretation principle is postponed for as long as possible. They correctly observe that the scattering, or transition, amplitudes are the only point of contact between a theory and physical observation, so the quantum theory may keep negative- energy tachyons right up to the point of calculating amplitudes for processes. The interactions are imagined to have been calculated, and we suppose that we know the (Lorentz invariant) S-matrix which, using the interaction representation [111], connects in- and out-states chosen from H.

It is now postulated that the physical amplitudes are obtained by replacing negative-energy tachyons of momentum p^μ in the initial (final) state of a transition amplitude by positive-energy antitachyons of momentum $-p^\mu$ in the final (initial) state. This form of the reinterpretation principle may be expressed as follows:

$$\left\langle \begin{matrix} p_1, p_2, \ldots, p_m \\ q_1, q_2, \ldots, q_n \end{matrix} \right| S \left| \begin{matrix} r_1, r_2, \ldots, r_k \\ s_1, s_2, \ldots, s_\ell \end{matrix} \right\rangle$$

$$= \left\langle \begin{matrix} p_1, p_2, \ldots, p_m \\ -s_1, -s_2, \ldots, -s_\ell \end{matrix} \right| \tilde{S} \left| \begin{matrix} r_1, r_2, \ldots, r_k \\ -q_1, -q_2, \ldots, -q_n \end{matrix} \right\rangle \qquad (4.49)$$

where S is the invariant S-matrix in the space H and the right-hand side is a matrix element of the physical, or reinterpreted, S-matrix $\tilde{S}$ calculated between initial and final physical states in which the momenta of the tachyons (antitachyons) are written in the upper (lower) position and all energies are positive. In using antitachyons as the reinterpreted particles, we are using a slight modification of the reinterpretation principle which was proposed by Kamoi and Kamefuchi [107], and which allows Eq. (4.49) to be applied to the whole S-matrix.

Due to the Lorentz invariance of S, its matrix elements between states of H are left unchanged by Lorentz transformations upon those states, so that from (4.49), the process described in terms of physical states has a value for the matrix element which is the same as that for the Lorentz-transformed process. The preservation of the value of matrix elements under Lorentz transformations is the minimal requirement for the Lorentz invariance of a theory, and it is therefore satisfied for the Arons-Sudarshan theory. But we see that Lorentz transformations which change the sign of the energy of tachyons in a matrix element of S effect, from (4.49), a shift of the tachyon from the initial to the final state, or vice versa, in the physical matrix element. This means that processes involving tachyons may appear quite different when viewed from another Lorentz frame, with different numbers of particles in the initial and final states, and yet the amplitude for the process is the same. So that although the basic physical requirement of the equality of matrix elements is respected as it is for bradyons (because of (4.49)), tachyons do not satisfy the additional property of the preservation of the "in" and "out" spaces which bradyons do.

The physical states

We should like to investigate further the physical states which make their first appearance in the relation (4.49). A greater similarity with conventional field theory, and therefore a better chance of future progress, might be had if we were able to work with the physical states from the outset. But clearly there is no space of reinterpreted physical states, $\tilde{H}$, which is left invariant under Lorentz transformations, since we have seen that Lorentz transformations can shift tachyons between the in and out states of a physical matrix element.

So we are not permitted to think of positive-energy tachyons in the initial state, for example, in isolation, and independent from, positive-energy antitachyons in the final state. This might suggest that we took as a "generalized state" in the formal approach mentioned in section 4.2, an element of the direct sum

$$\mathcal{M} = M^2 \oplus M^1 \quad , \tag{4.50}$$

where $M^1(M^2)$ is the space M of section 4.2 for the in (out) space. We write such a generalized state as

$$\Phi = \{\phi^2,\phi^1\} \quad , \tag{4.51}$$

where $\phi^1 \in M^1$ and $\phi^2 \in M^2$. We could take as inner product the expression

$$(\Phi,\Psi)_{\mathcal{M}} = \int_{\pm} (dp)\ (\tilde{\phi}^2(p)\ \tilde{\psi}^2(p)^* + \tilde{\phi}^1(p)\ \tilde{\psi}^1(p)^*) \tag{4.52}$$

where $\Phi = \{\phi^2,\phi^1\}$ and $\Psi = \{\psi^2,\psi^1\}$ are elements of $\mathcal{M}$. The inner product (4.52) makes $\mathcal{M}$ a real Hilbert space.

We would then want to define the action of Poincaré transformations on the space $\mathcal{M}$ in such a way that Lorentz transformations would be able to effect a shift between the in and out spaces in the manner described earlier. To begin with, we consider an element ψ^1 of M^1 containing only positive energies, so that we may write

$$\psi^1(x) = \int_+ (dp)\, e^{-ip.x}\, \psi^1(p) \quad . \tag{4.53}$$

Since ψ^1 is a scalar function, we see from (4.30) that it would normally transform as follows under a Poincaré transformation (Λ,a):

$$\begin{aligned}\psi^1(x) \to \psi^1(\Lambda^{-1}(x - a)) &= \int_+ (dp)\, e^{-ip.\Lambda^{-1}(x-a)}\, \tilde{\psi}^1(p) \\ &= \int_{p^0>0;\,(\Lambda^{-1}p)^0>0} (dp)\, e^{-ip.x+ip.a}\, \tilde{\psi}^1(\Lambda^{-1}p) \\ &\quad + \int_{p^0<0;\,(\Lambda^{-1}p)^0>0} (dp)\, e^{-ip.x+ip.a}\, \tilde{\psi}^1(\Lambda^{-1}p) \\ &\equiv \psi^{1\prime}_+(x) + \psi^{1\prime}_-(x) \quad ,\end{aligned} \tag{4.54}$$

where negative energies are now present in the term $\psi^{1\prime}_-(x)$. The foregoing description of Lorentz transformations on the physical states in the Arons-Sudarshan quantization suggests that if ψ^1 was contained in the incoming part of the state Ψ, then the part $\psi^{1\prime}_-(x)$ in (4.54) which results from a change in the sign of the energy under the Lorentz transformation Λ, should be transferred to the outgoing part (or M^2 side) of the generalized state. Furthermore, the discussion of the classical reinterpretation principle shows that energies and momenta should be reversed as well, so we also apply a complex conjugation to $\psi^{1\prime}_-(x)$. Applying this technique whenever the energy changes sign under a Lorentz transformation, we are led to make the following definition of the action of a Poincaré transformation (Λ,a) on the generalized state $\Phi = \{\phi^2,\phi^1\}$:

$$(\Lambda,a)\Phi = \Big\{\int_+ (dp)\, e^{-ip.x+ip.a}\left[\theta(\Lambda^{-1}p)\tilde{\phi}^2(\Lambda^{-1}p)+\theta(-\Lambda^{-1}p)\tilde{\phi}^1(-\Lambda^{-1}p)^*\right]$$

$$+ \int_- (dp)\, e^{-ip.x+ip.a}\left[\theta(-\Lambda^{-1}p)\tilde{\phi}^2(\Lambda^{-1}p)+\theta(\Lambda^{-1}p)\tilde{\phi}^1(-\Lambda^{-1}p)^*\right],$$

$$\int_+ (dp)\, e^{-ip.x+ip.a}\left[\theta(\Lambda^{-1}p)\tilde{\phi}^1(\Lambda^{-1}p)+\theta(-\Lambda^{-1}p)\tilde{\phi}^2(-\Lambda^{-1}p)^*\right]$$

$$+ \int_- (dp)\, e^{-ip.x+ip.a}\left[\theta(-\Lambda^{-1}p)\tilde{\phi}^1(\Lambda^{-1}p)+\theta(\Lambda^{-1}p)\tilde{\phi}^2(-\Lambda^{-1}p)^*\right]\Big\} . \tag{4.55}$$

This definition of Poincaré transformations on M preserves the inner product (4.52), and also preserves a symplectic form on M given by

$$B(\Phi,\Psi) = i \int_+ (dp)(\tilde{\phi}^2(p)\ \tilde{\psi}^2(p)^* - \tilde{\phi}^2(p)^*\ \tilde{\psi}^2(p)$$

$$- \tilde{\phi}^1(p)\ \tilde{\psi}^1(p)^* + \tilde{\phi}^1(p)^*\ \tilde{\phi}^1(p)) \quad , \tag{4.56}$$

where $\Phi = \{\phi^2,\phi^1\}$ and $\Psi = \{\psi^2,\psi^1\}$ are elements of M.

For the complex structure on M we have the operation J which consists of applying the J of (4.29) to the component ϕ^2 of $\Phi = \{\phi^2,\phi^1\}$ and $-J$ to the component ϕ^1. This operator J satisfies the conditions (4.27) if the symplectic form is taken as in (4.56), and it also commutes with the Lorentz transformations of (4.55) so that we will have an invariant vacuum.

The inner product defined by

$$(\Phi,\Psi)_{\mathcal{J}} = B(\Phi, \mathcal{J}\Psi) - i\, B(\Phi,\Psi) \tag{4.57}$$

makes the space M a complex Hilbert space H, and Lorentz transformations will be unitary transformations on H. We have then a conventional, invariant quantization by commutators, but now we find that the energy is not positive definite. If T_t is the unitary operator in H for a translation in time by an amount t, then we find that

$$-\mathcal{J} \frac{d}{dt} (T_t \Phi) \Big|_{t=0} = -\mathcal{J} \left\{ \int_{\pm} (dp)\, e^{-ip.x} (ip^0)\, \tilde{\phi}^2(p),\ \int_{\pm} (dp)\, e^{-ip.x} (ip^0) \tilde{\phi}^1(p) \right\}$$

$$= \left\{ \int_{\pm} (dp)\, e^{-ip.x}\, (\pm p^0) \tilde{\phi}^2(p)\ ,\ \int_{\pm} (dp)\, e^{-ip.x}\, (\mp p^0) \tilde{\phi}^1(p) \right\} . \tag{4.58}$$

From the sign of p^0 in the M^1-component in (4.58), we can see that the group of time translations does not have a positive generator with respect to the complex Hilbert space structure of H. These troubles cannot be remedied, for it is found that when we try other complex structures and other definitions for Lorentz transformations on M, either the negative energies still appear or, what is worse, we have to contend with noninvariance of the symplectic form.

In view of this, we may wish to abandon the interpretation of the space M as a sum of the incoming and outgoing spaces, and accept the presence of negative energies. In that case, however, the quantization on M described here cannot be taken to be the Arons-Sudarshan quantization prior to the application of the reinterpretation principle, for the field $\Phi(x)$ clearly does not transform as a Lorentz scalar. So, after considering various methods of quantization for tachyons, the Arons-Sudarshan scheme appears to be the most acceptable even though negative energies appear in the initial stages, and the equal-time commutation relations are not in any sense "canonical".

This attempt at quantizing the tachyon field using a direct sum of in- and out-spaces was made because it seemed that such a scheme most faithfully depicted the reinterpretation principle as it is described by (4.49) while at the same time using reinterpreted, physical states from the outset. From the failure of this attempt we conclude that it is essential to allow negative-energy states into the theory initially, and later to apply the reinterpretation only to matrix elements and not to states. That is, we are left with expressions like (4.49) as the full statement of what can be known about the physical states in the Arons-Sudarshan theory, and we are not able to construct a Hilbert space of "physical states" or describe reinterpretation in terms of states alone.

4.4 Discussion

Obviously, many difficulties are encountered when one tries to develop a quantum field theory of tachyons. We have examined the nature of these difficulties and considered the suitability of some candidates for such a theory. Using the criteria that a satisfactory quantum field theory of tachyons should, firstly, embody our main ideas about the classical tachyon, and, secondly, be as close as possible to conventional quantum theory in form, we judge that the Arons-Sudarshan method of quantization is the most successful. In this scheme the reinterpretation principle operates in much the same way as the classical reinterpretation principle. Also, Lorentz invariance of the quantum theory is instituted in the usual manner by having the field transform under a spacelike unitary, irreducible representation of the Poincaré group.

Tachyons and Spin

Actually, as we have already mentioned in the last section, other representations besides the scalar one could conceivably be used in the Arons-Sudarshan quantization. This leads us to speculate that we might be able to describe a tachyon with non-zero spin. We could reasonably assume that the property of spin for the tachyon is determined in the usual way by the representation carried by the field. The spacelike unitary, irreducible representations of the Poincaré group are specified by giving the mass, m, and the irreducible, unitary representation of the tachyon little group which is $SU(1,1)$. The spin would then be connected with the value of the Casimir invariant of the little group. But we observe from the nature of the unitary representations of the tachyon little group (see, for example, Shirokov [127]) that the spin of the tachyon could be either continuous or discrete. There is no a priori reason for restricting ourselves to either of these alternatives. Now all the unitary representations of the little group of the tachyon, apart from the trivial representation, are infinite dimensional, so that if we wish to consider tachyons with non-zero spin, we must work with infinite-component fields.

If we attempt to describe a tachyon by a finite-component field, say by using the solutions to the Dirac equation with imaginary mass [128], we would be working with a non-unitary representation of the little group [127]. Describing "spin ½" tachyons by such an adaption of the Dirac equation, Bandukwala and Shay [128] found that there is no conserved particle number and that neither a scalar probability nor a momentum 4-vector could be constructed. Shay [129] uses other finite-dimensional non-unitary representations in describing tachyons of other spins. This is done by using a similar adaption of the Weinberg method for describing higher-spin bradyon fields [130]. But we are inclined to think that the most convincing account of spin for tachyons would, by analogy with spin for bradyons, be based on the unitary representations of the little group, and therefore involve infinite-component fields. We will not pursue this matter here because problems still remain with the theory of the scalar tachyon, and we make this our first priority.

With the mention here of infinite-component fields, we recall that virtually all infinite-component wave equations have solutions with spacelike 4-momentum which cannot simply be ignored because they must be included to obtain a complete set of solutions [131,132,133]. The first infinite-component wave equation to be studied was the Majorana equation [134], the infinite-component spinor solutions of which transform under the simplest infinite-dimensional unitary representations of the Lorentz group. To find the masses and spins of the particles described by the Majorana field, one determines which unitary, irreducible representations of the Poincaré group the solutions of the equation generate. It is found [135,136] that the tachyon solutions, those with spacelike 4-momentum, have the continuous "spin" mentioned above. Also, there is a spectrum of masses with the mass proportional to the inverse of a spin-like parameter which varies continuously from zero to infinity. Quantizations of the Majorana field [135,136] have expanded the field operator in annihilation operators only, as done in the Arons-Sudarshan quantization of the scalar tachyon field.

Thus the Arons-Sudarshan method of quantization gives a theory of the free scalar tachyon field which resembles conventional quantum theory in many respects. But the differences from the usual theory have been sufficient to prevent any consistent theory of interactions being developed. This is the most serious shortcoming of the Arons-Sudarshan theory, and we shall investigate the difficulties involved in obtaining an interacting theory of tachyons in the next chapter.

CHAPTER 5

TOWARDS INTERACTION

We have seen that if one wishes to treat tachyons in quantum field theory, then the Arons-Sudarshan method is the most suitable way of quantizing free tachyon fields, given our desire to arrive at the most conventional theory which still embodies our ideas about the classical tachyon. But it is clear from the manner in which the reinterpretation principle is implemented in the Arons-Sudarshan quantization, that to examine the viability of this theory further, one should ideally be able to describe interactions and derive the S-matrix. That this is very difficult, and to date has not been accomplished for tachyons, is due largely to the lack of spacelike commutativity, as we explain in section 5.1. Later we consider a class of unreinterpreted S-matrices, which are obtained heuristically, and which are the result of what might be called local polynomial-type interaction terms. The proof that they cannot yield unitary physical, or reinterpreted, S-matrices suggests that if tachyons interact at all, they must do so nonlocally. This is supported by the simple model calculations of section 5.3 in which the "form factor" for the interaction must in principle be nonlocal, and cannot be viewed as approximating an actual point interaction.

5.1 Difficulties in Describing the Interaction of Tachyons

Lacking spacelike commutativity for the tachyon fields, we lose much of the apparatus for describing interactions which is present in the usual bradyon theory. The Lagrangian approach is not applicable, for although we may consider that the field equation (4.1) is derivable from a Lagrangian, the equal-time commutation relations obtained from (4.41) are not canonical, as may be seen from the property (4.9) of the commutator function G^1.

Furthermore, without spacelike commutativity, the usual derivations of the S-matrix are not valid. In bradyon field theory, the S-matrix in the interaction picture is given by the infinite time limit $U(\infty,-\infty)$ of the time displacement operator $U(t,t_0)$ which satisfies

$$i \partial_t U(t,t_0) = H_I(t) U(t,t_0) \quad , \qquad (5.1)$$

where $H_I(t)$ is the interaction part of the Hamiltonian operator. The S-matrix may be written as

$$S = T \exp\left[-i \int_{-\infty}^{\infty} d^4x \, \mathcal{H}_I(x)\right] \quad , \qquad (5.2)$$

where $\mathcal{H}_I(x)$ is the interaction Hamiltonian density, and T is the time-ordering operation [111]. It is essential for the derivation of (5.2) that the interaction Hamiltonian density commutes at spacelike separations, so Eq. (5.2) cannot be derived in the case of tachyon interactions. In any case, we can see that Eq. (5.2) cannot be correct for tachyons because we know that the time-ordering operator T is a noninvariant operation for tachyon fields, and the S-matrix must be Lorentz-invariant. We could, however, take for the S-matrix involving tachyon interactions the expression

$$S = \exp\left[-i \int_{-\infty}^{\infty} d^4x \, \mathcal{H}_I(x)\right] \quad , \qquad (5.3)$$

for an arbitrarily chosen but Lorentz-invariant $\mathcal{H}_I$, considering this to be a reasonable generalization from the bradyon situation. Kamoi and Kamefuchi [107] take this approach, and we shall expand upon it in the next section. Nevertheless, we must remember that without the T-operator, the S of (5.3) is not even formally the limit of the solution of (5.1) for large times.

In order to point out some difficulties involved with the restriction (4.3) on the 3-momenta of tachyon fields, Kamoi and Kamefuchi [107] consider the definition of the S-matrix in terms of Heisenberg fields. The interacting field may be taken to be the solution of

$$(\Box - m^2)\, \phi(x) = g\, J(x) \quad , \tag{5.4}$$

where g is a coupling constant, and the current $J(x)$ may be a polynomial in the tachyon fields and other fields, all evaluated at the point x. We may write solutions of (5.4), satisfying certain boundary conditions, in terms of Green's functions and the Heisenberg in- and out-operators, ϕ_{in} and $\phi_{out}(x)$, which satisfy the free-field equation (4.1) and have the free-field commutation relations (4.41). For example, the solution

$$\phi(x) = \phi_{in}(x) + g \int_{-\infty}^{\infty} d^4y\, G^R(x-y)\, J(y) \quad , \tag{5.5}$$

where G^R is the retarded function (4.18), is equal to ϕ_{in} in the remote past, and the solution

$$\phi(x) = \phi_{out}(x) + g \int_{-\infty}^{\infty} d^4y\, G^A(x-y)\, J(y) \quad , \tag{5.6}$$

where G^A is the advanced function (4.19), is equal to ϕ_{out} in the remote future.

Now the S-matrix in the space H (the unreinterpreted S-matrix) relates the in- and out-fields:

$$\phi_{out}(x) = S^{-1}\,\phi_{in}(x)\,S \quad . \tag{5.7}$$

But from (5.5) and (5.6) we obtain

$$\phi_{out}(x) = \phi_{in}(x) + g\int_{-\infty}^{\infty} d^4y\; G(x-y)\;J(y) \quad , \tag{5.8}$$

where G is the function defined by (4.11) since, from Eqs. (4.18) and (4.19),

$$G^R(x) - G^A(x) = G(x) \quad .$$

So, if the S-matrix has the expansion

$$S = I - ig\int_{-\infty}^{\infty} d^4y\; H(y) + O(g^2) \quad ,$$

then, using (5.7) and (5.8), H must satisfy

$$\int d^4y \left[\phi_{in}(x) \; , H(y)\right] = i\int d^4y\; G(x-y)\;J(y) \quad , \tag{5.9}$$

where only terms of first order in g have been retained. Kamoi and Kamefuchi [107] then note that if the Fourier transform of (5.9) is taken with respect to x, then the left-hand side will not have Fourier components for $|\underline{p}| < m$ because of the free-field nature of $\phi_{in}(x)$. But such is not the case in general for the right-hand side because, from Eq. (4.12), G has Fourier components for $|\underline{p}| < m$, and so will J if it is a product of field operators. For consistency then, we must require that J not have such Fourier components, and this would entail the use of non-local form factors if the current J is defined in terms of the field operators. Kamoi and Kamefuchi [107] doubt whether such non-local form-factors can be constructed for all orders in g, and turn to a model S-matrix based on (5.3) as a description of a possible tachyon interaction. We do likewise in the next section, and in section 5.3 we turn to a model interaction obtained by using a classical current J in the above approach which uses the Heisenberg fields.

5.2 Unitarity

As we do not have a procedure for describing general interactions for tachyons, we go to the S-matrix directly, and consider what restrictions are placed upon it by physical requirements. The most important requirement is Lorentz invariance, and we note that this is satisfied by (5.3) as long as $H_I(x)$ is a Lorentz invariant operator. It could, for example, be a polynomial in the tachyon field operator, (4.39), and its conjugate.

The conservation of energy-momentum is also required, but, with the lack of either spacelike commutativity or positive-definite energy, little else of a general nature can be said about the tachyon S-matrix in the space H. The analyticity of S-matrix elements, which seems to arise from microcausality [137], cannot be invoked for tachyons. In further contrast to bradyon theories, the tachyon S-matrix is not expected to exhibit thresholds [137] with respect to the energy because, as shown in section 1.2, it is energetically possible for a positive-energy tachyon to decay into two, or more, positive-energy tachyons of the same mass.

There is, however, an important property which the physical S-matrix, $\tilde{S}$, defined by (4.49), should possess. This is the unitarity of $\tilde{S}$ for it expresses probability conservation. Without the unitarity of the physical S-matrix, we would be forced into a major revision of the probability interpretation of the matrix elements in our quantum theory, and it is not obvious how this might be done.

The unitarity condition for the physical S-matrix, $\tilde{S}$, may be written as

$$\tilde{S}^{\dagger}\tilde{S} = \tilde{I} = \tilde{S}\tilde{S}^{\dagger} ,$$

or, writing the first equation in terms of matrix elements,

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\int_{+}(dp_1)\ldots\int_{+}(dp_m)\int_{+}(dq_1)\ldots\int_{+}(dq_n) \, .$$

$$\cdot\left\langle \begin{matrix} p_1,\ldots,p_m \\ q_1,\ldots,q_n \end{matrix} \middle| \tilde{S} \middle| \begin{matrix} r_1,\ldots,r_k \\ s_1,\ldots,s_l \end{matrix} \right\rangle^{*} \left\langle \begin{matrix} p_1,\ldots,p_m \\ q_1,\ldots,q_n \end{matrix} \middle| \tilde{S} \middle| \begin{matrix} u_1,\ldots,u_i \\ v_1,\ldots,v_j \end{matrix} \right\rangle$$

$$= \left\langle \begin{matrix} r_1,\ldots,r_k \\ s_1,\ldots,s_l \end{matrix} \middle| \tilde{I} \middle| \begin{matrix} u_1,\ldots,u_i \\ v_1,\ldots,v_j \end{matrix} \right\rangle , \qquad (5.10)$$

where * denotes the complex conjugate, and the right-hand side is defined by (4.49) for S replaced by I, the identity in the space H. The summation and integration is over all possible (positive-energy) states containing tachyons and antitachyons. Using (4.49) we may rewrite (5.10) as a requirement on the matrix elements of S.

This gives

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \int_{+}(dp_1)\ldots\int_{+}(dp_m)\int_{+}(dq_1)\ldots\int_{+}(dq_n) \,.$$

$$\cdot \left\langle \begin{matrix} p_1,\ldots,p_m \\ -s_1,\ldots,-s_l \end{matrix} \right| S \left| \begin{matrix} r_1,\ldots,r_k \\ -q_1,\ldots,-q_n \end{matrix} \right\rangle^{*} \left\langle \begin{matrix} p_1,\ldots,p_m \\ -v_1,\ldots,-v_j \end{matrix} \right| S \left| \begin{matrix} u_1,\ldots,u_i \\ -q_1,\ldots,-q_n \end{matrix} \right\rangle$$

$$= \left\langle \begin{matrix} r_1,\ldots,r_k \\ -v_1,\ldots,-v_j \end{matrix} \middle| \begin{matrix} u_1,\ldots,u_i \\ -s_1,\ldots,-s_l \end{matrix} \right\rangle \qquad (5.11)$$

as the rather awkward expression which the S-operator must satisfy in the space H. We want to know if such S-matrices can exist. Note that under Lorentz transformations, particles may be shifted between the initial and final states of the matrix elements of $\tilde{S}$, so that Eq. (5.10) does not retain its form under a Lorentz transformation. In the new frame, the transformed equation no longer expresses the unitarity of $\tilde{S}$, which has to be specified anew in that frame.

To investigate the possibility of obtaining unitary physical S-matrices for tachyons, Kamoi and Kamefuchi [107] took one particular example for S. They took S to have the form (5.3) with

$$H_I(x) = g\, \phi^{\dagger}(x)^2\, \phi(x)^2 \qquad (5.12)$$

so that

$$S = \exp[-ig\, U] = I - ig\, U - \tfrac{1}{2} g^2 U^2 + \ldots \,, \qquad (5.13)$$

where

$$U = (2\pi)^{-2} \int_{\pm}(dp)\int_{\pm}(dq)\int_{\pm}(dr)\int_{\pm}(ds)\ \delta^4(p+q-r-s)a^{\dagger}(p)a^{\dagger}(q)a(r)a(s)\ . \tag{5.14}$$

The expansion (5.13) for S produces a similar expansion for the physical S-matrix so that

$$\tilde{S} = \tilde{I} - ig\,\tilde{U}_1 - \tfrac{1}{2}g^2\,\tilde{U}_2 + \dots\ , \tag{5.15}$$

where $\tilde{U}_n$ is the result of applying the reinterpretation principle (4.49) to the operator U^n. From (5.15) we see that U must be hermitian if $\tilde{S}$ is to satisfy the unitarity relation to first order in the coupling constant g. Eq. (5.14) shows that U is hermitian. In fact all the reinterpreted matrices $\tilde{U}_n$ in (5.15) are hermitian too, because all the U^n are. The hermiticity of U means that S is actually unitary in H, but we note that there is no physical requirement that this be so. Unitarity to second order in g requires that

$$\tilde{U}_1\,\tilde{U}_1 = \tilde{U}_2\ , \tag{5.16}$$

since $\tilde{U}_2$ is hermitian from above. Kamoi and Kamefuchi then show that there is a nonvanishing matrix element of the form

$$\left\langle \begin{matrix} p \\ 0 \end{matrix} \middle| \tilde{U}_1\,\tilde{U}_1 \middle| \begin{matrix} p \\ 0 \end{matrix} \right\rangle \equiv \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \int_{+}(du_1)\dots\int_{+}(du_m)\int_{+}(dv_1)\dots\int_{+}(dv_n)\ .$$

$$\cdot \left\langle \begin{matrix} p \\ 0 \end{matrix} \middle| \tilde{U}_1 \middle| \begin{matrix} u_1,\dots,u_m \\ v_1,\dots,v_n \end{matrix} \right\rangle \left\langle \begin{matrix} u_1,\dots,u_m \\ v_1,\dots,v_n \end{matrix} \middle| \tilde{U}_1 \middle| \begin{matrix} p \\ 0 \end{matrix} \right\rangle ,$$

but it can be seen from (5.14) that $\tilde{U}_2$ does not have such a matrix element. The unitarity of the physical S-matrix is therefore violated, for the particular example chosen by Kamoi and Kamefuchi, for the reason that a certain matrix element vanishes when unitarity (Eq. (5.14)) requires it to be non-zero.

Now if we choose S to have the form (5.13) and take U to be a sum of terms of the form

$$U(m,n) = (2\pi)^{4-\frac{3}{2}(m+n)} \int_{\pm}(dp_1)\ldots\int_{\pm}(dp_m)\int_{\pm}(dq_1)\ldots\int_{\pm}(dq_n)\,\delta^4\Big(\sum_{i=1}^{m} p_i - \sum_{j=1}^{n} q_j\Big)$$

$$.a^{\dagger}(p_1)\ldots a^{\dagger}(p_m)a(q_1)\ldots a(q_n), \qquad (5.17)$$

then, by simply requiring certain matrix elements to exist for one side of Eq. (5.16) when they exist on the other, we can see that U must contain particular U(m,n). For example, we find that U must contain elements of the form U(1,n) and U(n,1), for some n. This rules out the expression (5.14) as a possible form for U. We shall not pursue the argument based on the existence of matrix elements in Eq. (5.16) here, for there is a more serious consideration which eliminates a large class of possible S-matrices due to their violation of unitarity, and it is to this that we now turn.

Since there is only a requirement that the reinterpreted S-matrix, $\tilde{S}$, be unitary, but none that S itself be unitary, we need not necessarily choose for S the form (5.13) in which U is hermitian. But let us assume that there is an expansion for the unreinterpreted S-matrix in powers of a coupling constant g, so that

$$S = I + ig\,S_1 + g^2\,S_2 + \ldots \quad , \qquad (5.18)$$

and therefore $\tilde{S}$ has a similar expansion:

$$\tilde{S} = \tilde{I} + ig\,\tilde{S}_1 + g^2\,\tilde{S}_2 + \dots \quad , \tag{5.19}$$

where $\tilde{S}_n$ is obtained from S_n by the reinterpretation principle (4.49). From the properties of S discussed above, we know that S, and each of the S_n, must be Lorentz invariant operators.

To first order in g, the unitarity equation for the $\tilde{S}$ of (5.19) requires that $\tilde{S}_1$, and therefore S_1, be hermitian. In second order we require

$$\tilde{S}_1^{\dagger}\tilde{S}_1 (= \tilde{S}_1\,\tilde{S}_1) = -\,\tilde{S}_2^{\dagger} - \tilde{S}_2 \quad , \tag{5.20}$$

from which we see that the matrix elements of $\tilde{S}_1\,\tilde{S}_1$ between "physical states" must be Lorentz invariant expressions because we know that the right-hand side of (5.20) is obtained from the Lorentz invariant operator S_2.

Without assuming any particular form for S_2, let us take for S_1 the example mentioned above. That is,

$$S_1 = \int d^4x\ \phi^{\dagger}(x)^2\ \phi(x)^2$$

$$= (2\pi)^{-2} \int_{\pm}(dp)\int_{\pm}(dq)\int_{\pm}(dr)\int_{\pm}(ds)\,\delta^4(p+q-r-s)$$

$$.a^{\dagger}(p)a^{\dagger}(q)a(r)a(s) \quad . \tag{5.21}$$

The only "connected parts" (see [137]) of the matrix elements of this S_1, and the corresponding matrix elements of $\tilde{S}_1$, are

$$\left\langle \begin{matrix} p,q \\ 0 \end{matrix} \right| S_1 \left| \begin{matrix} r,s \\ 0 \end{matrix} \right\rangle = \frac{1}{2\pi^2}\,\delta^4(p+q-r-s) = \left\langle \begin{matrix} p,q \\ 0 \end{matrix} \right| \tilde{S}_1 \left| \begin{matrix} r,s \\ 0 \end{matrix} \right\rangle ,$$

$$\left\langle \begin{matrix} p,q \\ 0 \end{matrix} \right| S_1 \left| \begin{matrix} r \\ -s \end{matrix} \right\rangle = \frac{1}{\sqrt{2}\pi^2}\,\delta^4(p+q-r+s) = \left\langle \begin{matrix} p,q \\ s \end{matrix} \right| \tilde{S}_1 \left| \begin{matrix} r \\ 0 \end{matrix} \right\rangle ,$$

$$\left\langle \begin{matrix} p \\ -q \end{matrix} \right| S_1 \left| \begin{matrix} r,s \\ 0 \end{matrix} \right\rangle = \frac{1}{\sqrt{2}\pi^2}\,\delta^4(p-q-r-s) = \left\langle \begin{matrix} p \\ 0 \end{matrix} \right| \tilde{S}_1 \left| \begin{matrix} r,s \\ q \end{matrix} \right\rangle ,$$

$$\left\langle \begin{matrix} p \\ -q \end{matrix} \right| S_1 \left| \begin{matrix} r \\ -s \end{matrix} \right\rangle = \frac{1}{\pi^2}\,\delta^4(p-q-r+s) = \left\langle \begin{matrix} p \\ s \end{matrix} \right| \tilde{S}_1 \left| \begin{matrix} r \\ q \end{matrix} \right\rangle , \qquad (5.22)$$

$$\left\langle \begin{matrix} p \\ -q \end{matrix} \right| S_1 \left| \begin{matrix} 0 \\ -r,-s \end{matrix} \right\rangle = \frac{1}{\sqrt{2}\pi^2}\,\delta^4(p-q+r+s) = \left\langle \begin{matrix} p \\ r,s \end{matrix} \right| \tilde{S}_1 \left| \begin{matrix} 0 \\ q \end{matrix} \right\rangle ,$$

$$\left\langle \begin{matrix} 0 \\ -p,-q \end{matrix} \right| S_1 \left| \begin{matrix} r \\ -s \end{matrix} \right\rangle = \frac{1}{\sqrt{2}\pi^2}\,\delta^4(-p-q-r+s) = \left\langle \begin{matrix} 0 \\ s \end{matrix} \right| \tilde{S}_1 \left| \begin{matrix} r \\ p,q \end{matrix} \right\rangle ,$$

and

$$\left\langle \begin{matrix} 0 \\ -p,-q \end{matrix} \right| S_1 \left| \begin{matrix} 0 \\ -r,-s \end{matrix} \right\rangle = \frac{1}{2\pi^2}\,\delta^4(-p-q+r+s) = \left\langle \begin{matrix} 0 \\ r,s \end{matrix} \right| \tilde{S}_1 \left| \begin{matrix} 0 \\ p,q \end{matrix} \right\rangle ,$$

where the 4-momenta $p,q,r,$ and s are taken to have positive energy. The states of H referred to here are normalized as in (4.43). Using the expressions in (5.22) we can calculate the matrix elements of $\tilde{S}_1\,\tilde{S}_1$, and we shall consider the matrix element

$$\left\langle \begin{matrix} p \\ 0 \end{matrix} \right| \tilde{S}_1\,\tilde{S}_1 \left| \begin{matrix} q,r \\ s \end{matrix} \right\rangle \qquad (5.23)$$

which, using (5.22), is equal to

$$\int_+ (dk_1) \int_+ (dk_2) \int_+ (dk_3) \left\langle \begin{matrix} p \\ 0 \end{matrix} \right| \tilde{S}_1 \left| \begin{matrix} k_1,k_2 \\ k_3 \end{matrix} \right\rangle \left\langle \begin{matrix} k_1,k_2 \\ k_3 \end{matrix} \right| \tilde{S}_1 \left| \begin{matrix} q,r \\ s \end{matrix} \right\rangle$$

$$= \frac{1}{2\sqrt{2}\pi^4}\,\delta^4(p-q-r-s)\int_+(dk_1)\int_+(dk_2)\,\Big[\delta^4(p-s-k_1-k_2)$$

$$+ 2\delta^4(p-q-k_1-k_2) + 2\delta^4(p-r-k_1-k_2)\Big] \quad (5.24)$$

This, by (5.20), should be equal to the same matrix element of $-\tilde{S}^{\dagger}_2 - \tilde{S}_2$, which we know is Lorentz invariant.

The expression (5.24) will be evaluated for a convenient choice of the four tachyon momenta $p,q,r,$ and s. We are free to choose these momenta provided that they all have positive energy and satisfy energy-momentum conservation and the mass shell condition. Thus

$$p = q + r + s \quad \text{and} \quad [p]^2 = [q]^2 = [r]^2 = [s]^2 = -m^2 \quad . \quad (5.25)$$

If we now choose the momenta such that

$$[p-r]^2 = [p-s]^2 = -m^2 \quad , \quad (5.26)$$

then the relations (5.25) will require that $[p-q]^2 = -2m^2$. The matrix element (5.24) is then a sum of integrals of the type which are evaluated in appendix B. Equation (B11) applies and we obtain

$$\left\langle \begin{matrix} p \\ 0 \end{matrix} \middle| \tilde{S}_1 \tilde{S}_1 \middle| \begin{matrix} q,r \\ s \end{matrix} \right\rangle = \frac{1}{4\sqrt{2}\pi^3}\,\delta^4(p-q-r-s)\left[\frac{p^0-s^0}{|\underline{p}-\underline{s}|} + 2\frac{p^0-q^0}{|\underline{p}-\underline{q}|} + 2\frac{p^0-r^0}{|\underline{p}-\underline{r}|}\right], (5.27)$$

which involves a sum of terms of the type $x^0/|\underline{x}|$, where x is a 4-vector. Now, unless x is a null 4-vector, which is not the case in (5.27) for the particular choice of momenta made in (5.26), $x^0/|\underline{x}|$ is not Lorentz-invariant. Thus the matrix element (5.27), which is a sum of such terms, cannot be Lorentz-invariant*, and therefore cannot equal the matrix element

$$\left\langle \begin{matrix} p \\ 0 \end{matrix} \right| -\tilde{S}^{\dagger}_{2} - \tilde{S}_{2} \left| \begin{matrix} q,r \\ s \end{matrix} \right\rangle ,$$

since $\tilde{S}_2$ must be derived from some Lorentz-invariant operator S_2. This shows that the reinterpreted S-matrix $\tilde{S}$ cannot satisfy unitarity to second order (Eq. (5.20)) if S has an expansion of the form (5.18) in which S_1 is given by (5.21).

---

* This is most easily seen in the case when the sum contains a term $x^0/|\underline{x}|$ in which x is a timelike 4-vector. We may choose the tachyon 4-momenta p,q,r, and s such that (5.24) has a finite value and such that $x = p - s$, say, is timelike with the integral in (5.24) containing this difference of the tachyon 4-momenta requiring Eq. (B10) of appendix B for its evaluation. The matrix element (5.23) then contains a term proportional to $x^0/|\underline{x}|$ for this choice of momenta, and such a term becomes infinite when we transform to the rest frame of the timelike vector x. This shows directly that the value of the matrix element is not invariant under Lorentz transformations.

More generally, we may consider expansions (5.18) for S in which S_1 is a linear combination of a finite number of operators of the form

$$S(m,n) = \int d^4x \; \phi^\dagger(x)^m \; \phi(x)^n \quad , \qquad (5.28)$$

where m and n are any positive integers. In terms of creation and annihilation operators, $S(m,n)$ takes the form (5.17). That is, we are considering operators S_1 of the form

$$S_1 = \int d^4x \; P(\phi^\dagger(x),\phi(x)), \qquad (5.29)$$

where P is a normal-ordered polynomial in the tachyon field and its conjugate , and P has finite order so that for the $S(m,n)$ appearing in (5.29), m and n are finite. We have seen that for $\tilde{S}$ to be unitary, S_1 must be hermitian so if S_1 contains $S(m,n)$, it will also contain the hermitian adjoint, $S(n,m)$.

Considering these more general forms (5.29) for S_1, we find that there will always be matrix elements of $\tilde{S}_1.\tilde{S}_1$ which, for sufficiently large numbers of tachyons and antitachyons in the initial and final states, are non-zero and have connected components which involve only two integrations over intermediate momenta, as we found in (5.24). It is sufficient to consider only the connected components since the Lorentz invariance of the remainder is required separately by considering matrix elements of $\tilde{S}_1.\tilde{S}_1$ with fewer incoming and outgoing momenta.

We shall illustrate these remarks by using the form (5.29) for S_1 and evaluating a specific matrix element of $\tilde{S}_1.\tilde{S}_1$. The discussion below may also be done in terms of bubble diagrams [137]. The polynomial P is of finite order so we find the term $S(M,N)$ in S_1 which has the largest values for $M + N$ and M. We have already considered an S_1 of the form (5.21), so we now take M and N such that

$$M + N \geqslant 4.$$

Consider the matrix element

$$\left\langle \begin{matrix} p \\ 0 \end{matrix} \middle| \tilde{S}_1 . \tilde{S}_1 \middle| \begin{matrix} s_1, \ldots, s_{2N-2} \\ t_1, \ldots, t_{2M-3} \end{matrix} \right\rangle = \left\langle \begin{matrix} (1) \\ 0 \end{matrix} \middle| \tilde{S}_1 . \tilde{S}_1 \middle| \begin{matrix} (2N-2) \\ (2M-3) \end{matrix} \right\rangle \quad (5.30)$$

of $\tilde{S}_1 . \tilde{S}_1$, where on the right-hand side we use the abbreviation of only indicating the number of tachyons and antitachyons in the initial and final states, instead of writing in all the momenta as on the left-hand side. By definition, the matrix element (5.30) is equal to

$$\sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \int_+ (dq_1) \ldots \int_+ (dq_k) \int_+ (dr_1) \ldots \int_+ (dr_l)$$

$$\cdot \left\langle \begin{matrix} (1) \\ 0 \end{matrix} \middle| \tilde{S}_1 \middle| \begin{matrix} q_1, \ldots, q_k \\ r_1, \ldots, r_l \end{matrix} \right\rangle \left\langle \begin{matrix} q_1, \ldots, q_k \\ r_1, \ldots, r_l \end{matrix} \middle| \tilde{S}_1 \middle| \begin{matrix} (2N-2) \\ (2M-3) \end{matrix} \right\rangle . \quad (5.31)$$

In the evaluation of (5.31), as in the evaluation of (5.23), each term requires at most two nontrivial integrations under the above assumption about $M + N$.* We now indicate how this may be seen.

---

*If $N=0$, this situation holds for the following matrix element which we may consider in place of (5.30): $\left\langle \begin{matrix} (1) \\ 0 \end{matrix} \middle| \tilde{S}_1 . \tilde{S}_1 \middle| \begin{matrix} (M-2) \\ (M-3) \end{matrix} \right\rangle$.

In order to evaluate (5.31), we need to know the nonvanishing matrix elements of $\tilde{S}_1$, and, as previously, these are found from the known matrix elements of S_1 by using the reinterpretation principle (4.49). For example, in the sum of terms (5.31), the matrix elements of $\tilde{S}_1$ appearing as the second factors in each term are given by

$$\left\langle \begin{matrix} q_1,\ldots,q_k \\ r_1,\ldots,r_l \end{matrix} \middle| \tilde{S}_1 \middle| \begin{matrix} s_1,\ldots,s_{2N-2} \\ t_1,\ldots,t_{2M-3} \end{matrix} \right\rangle = \left\langle \begin{matrix} q_1,\ldots,q_k \\ -t_1,\ldots,-t_{2M-3} \end{matrix} \middle| S_1 \middle| \begin{matrix} s_1,\ldots,s_{2N-2} \\ -r_1,\ldots,-r_l \end{matrix} \right\rangle \tag{5.32}$$

in terms of the matrix elements of S_1. Now S_1 is a linear combination of terms $S(m,n)$, and the contribution of a certain $S(m,n)$ to the expression (5.32) is given by

$$\langle 0 | a_+(q_1)\ldots a_+(q_k) \frac{1}{\sqrt{k!}} a_-(-t_1)\ldots a_-(-t_{2M-3}) \frac{1}{\sqrt{(2M-3)!}} U(m,n)$$

$$\cdot \frac{1}{\sqrt{(2N-2)!}} a_+^\dagger(s_1)\ldots a_+^\dagger(s_{2N-2}) \frac{1}{\sqrt{l!}} a_-^\dagger(-r_1)\ldots a_-^\dagger(-r_l)|0\rangle, \tag{5.33}$$

where $U(m,n)$ is given by (5.17), and the incoming and outgoing states of the matrix element are normalized as in (4.43). Now $U(m,n)$ is a product of m creation operators and n annihilation operators, and if

$$k + 2M-3 = m \quad \text{and} \quad 2N-2 + l = n \tag{5.34}$$

in (5.33), then we may evaluate (5.33) by making all possible contractions between the operators, so we find that the contribution of $S(m,n)$ to the matrix element (5.32) of S_1 is proportional to

$$\delta^4\left(\sum_{i=1}^{k} q_i - \sum_{i=1}^{2M-3} t_i - \sum_{i=1}^{2N-2} s_i + \sum_{i=1}^{l} r_i\right) \quad . \tag{5.35}$$

Due to the condition (5.34), there are $m+n$ momenta in the argument of this δ-function. Such a matrix element we call connected.

If k and l are less than is required for (5.34) to hold, then after making all the available contractions between the operators a and $a^\dagger$, there will be a normal-ordered residue which will make the expression (5.33) vanish.

But for larger k and l, that is, an excess of operators in the incoming and outgoing status of (5.33) over the number $m'+n'$ contained in the expression for a particular $U(m',n')$, the matrix element (5.33) will, if not zero,be unconnected. By this we mean that when we evaluate (5.33) by making all the possible contractions, we find that in each case $m'+n'$ of the momenta q_i ,t_i ,s_i and r_i go into a factor of the form (5.35), and that multiplying it are factors which contain the remaining momenta, and these factors are of the form

$$2|q^0_{i'}|\,\delta^3(\underline{q}_{i'} - \underline{s}_{j'}) \quad , \tag{5.36}$$

where, for instance, one of the q_i and one of the s_i remained after having contracted with all the operators residing in the $U(m',n')$. So that a matrix element of the form (5.33) which is unconnected is equal to a sum of terms, in each of which there are $m'+n'$ external momenta being combined into a factor like (5.35), and the remainder are unconnected and appear in factors like (5.36).

Now in the evaluation of one term of (5.31) (a term with a particular k and l) the presence of unconnected momenta in the second factor would mean that the integration over these momenta is immediate. But such is not the case for the integration over momenta which are connected, or appear in factors like (5.35). We want to find the maximum number of integrations over connected momenta which are involved in the evaluation of (5.31). Consider a term of the sum (5.31) in which

$$k + l < M + N - 1 \quad , \tag{5.37}$$

and consider the second factor

$$\left\langle \begin{matrix}(k)\\(l)\end{matrix} \middle| \tilde{S}_1 \middle| \begin{matrix}(2N-2)\\(2M-3)\end{matrix} \right\rangle = \left\langle \begin{matrix}(k)\\(2M-3)\end{matrix} \middle| S_1 \middle| \begin{matrix}(2N-2)\\(l)\end{matrix} \right\rangle . \tag{5.38}$$

Suppose that in the evaluation of (5.38), at most i of the k positive-energy tachyon momenta, and j of the l negative-energy momenta, can be connected. Then, with the remaining momenta in factors like (5.36), there will be a factor like (5.35), arising from a certain S(m,n), for which

$$m + n = 2M + 2N - 5 - k - l + 2(i+j) \quad . \tag{5.39}$$

Now, using (5.37), we find that

$$m + n > M + N - 4 + 2(i+j) \quad , \tag{5.40}$$

showing that the number $i+j$ of integrations over connected momenta cannot be two or greater, for otherwise $m+n$ would be greater than $M+N$, contradicting our assumption that there are no $S(m,n)$ in S_1 with $m+n$ greater than $M+N$.

A similar argument, this time using the first factor of a term in (5.31), may be constructed for the case in which $k+l$ is greater than $M+N-1$. We find that in (5.31), for the maximum number of integrations over connected momenta, $k+l$ must be equal to $M+N-1$. In this case there is an integration over two connected momenta and, with M being the largest number m for which there is an $S(m,n)$ in S_1, only $S(M,N)$ contributes to the two factors of each term.

So, to evaluate (5.31), we need to consider the term with $k=N$ and $l=M-1$. The first factor of this term is connected and proportional to

$$\delta^4(p - q_1 - \ldots - q_N - r_1 - \ldots - r_{M-1}) \quad .$$

The second factor is unconnected and is equal to (5.33) with

$$k = N, \; l = M - 1, \; m = M, \quad \text{and} \quad n = N \quad .$$

Evaluating this factor, only two momenta of the set $\{q_i, r_j\}$, say k_1 and k_2, can be connected, the remaining $M+N-3$ momenta being unconnected, adding up to the momentum y, say, and joined by δ-functions like (5.36) to $M+N-3$ of the momenta $\{s_i, t_j\}$. Let z be the sum of the remaining $M+N-2$ momenta of $\{s_i, t_j\}$ that are not joined to the momenta in y by δ-functions. The momenta in z are then connected, so that the second factor is proportional to $\delta^4(k_1+k_2-z)$ multiplied by the δ-functions involving the momenta in y. Thus the integration over the momenta of $\{q_i, r_j\}$ which add up to y can be done immediately, so that the integral of the product of the two factors is proportional to

$$\sum \int_+ (dk_1) \int_+ (dk_2) \delta^4(p - y - k_1 - k_2) \delta^4(k_1 + k_2 - z)$$

$$= \sum \delta^4(p - y - z) \int_+ (dk_1) \int_+ (dk_2) \delta^4((p - y) - k_1 - k_2) \quad , \quad (5.41)$$

where the sum is taken over all ways of choosing k_1, k_2, and the momenta in z from the available sets.

The matrix element (5.31) thus contains a term of the form (5.41), which is composed of a sum of integrals of the type which have been evaluated in appendix B and found to be generally noninvariant. As indicated previously, the sum (5.41) of such integrals, and therefore also the matrix element (5.30), will be a noninvariant function of the external momenta $\{p, s_i, t_j\}$. We see again that unitarity to second order (Eq. (5.20)) must be violated.

We conclude that all physical tachyon S-matrices, $\tilde{S}$, which are derived by the reinterpretation principle (4.49) from Lorentz-invariant operators S of the form (5.18) in which S_1 is as given by (5.29) (a local-polynomial-type interaction term), violate unitarity.

In view of this result one may wish to include derivatives of the field in the form for S_1. For example, we may take

$$S_1 = \int d^4x \, \partial^\mu \phi^\dagger(x) \, \partial_\mu \phi^\dagger(x) \, \phi(x)^2 \quad ,$$

and calculate the matrix element (5.23). In doing so, we find that we need to evaluate integrals similar to those treated in appendix B, but which contain scalar products in the integrand. These give rise to a great many more noninvariant terms and unitarity cannot hold for the reasons already explained above.

The "electromagnetic" interaction

Using the Arons-Sudarshan quantization, we have seen that physical S-matrices which arise from "local polynomial" interactions involving only the tachyon fields are not unitary. By forming Lorentz-invariant combinations of the tachyon field with other fields, we may similarly obtain an S-matrix describing a possible interaction between tachyons and these other fields. We shall mention here what might be called an electromagnetic interaction, but we find, as above, that the physical S-matrix cannot be unitary.

The tachyon field in the Arons-Sudarshan quantization, (4.39), is nonhermitian, and from it we may construct a current 4-vector J^μ which is defined by

$$J^\mu(x) = i\, e(\phi^\dagger(x)\, \partial^\mu\, \phi(x) - \partial^\mu\, \phi^\dagger(x)\, \phi(x)) \quad , \qquad (5.42)$$

and which can be written in the form

$$J^\mu(x) = \frac{e}{(2\pi)^3} \int_\pm (dp) \int_\pm (dq)\; e^{i(p-q).x}\, (p^\mu + q^\mu)\, a^\dagger(p)\, a(q) \qquad (5.43)$$

by using (4.39). We find that this current satisfies the equation

$$\partial^\mu\, J_\mu(x) = 0 \qquad (5.44)$$

because the field satisfies the Klein-Gordon equation for a tachyon, Eq. (4.1).

With the current $J^\mu(x)$ and a 4-vector field such as the 4-vector potential $A^\mu(x)$ of the electromagnetic field, we may form the Lorentz scalar $J^\mu(x)A_\mu(x)$, and assume, in the spirit of the earlier work of this section, that the unreinterpreted S-matrix is given by the expansion

$$S = I + e\, S_1 + e^2\, S_2 + \ldots \, , \qquad (5.45)$$

where

$$e\, S_1 = i \int d^4x\, J^\mu(x)\, A_\mu(x) \quad . \qquad (5.46)$$

For the quantization of the electromagnetic field we expand the 4-potential as [111]

$$A_\mu(x) = (2\pi)^{-3/2} \int_+ (dq)(e^{-iq.x}\, a_\mu(\underline{q}) + e^{iq.x}\, a_\mu^\dagger(\underline{q}))\, , \qquad (5.47)$$

where $q^0 = |\underline{q}|$, and where we have the commutation relations

$$[a_\mu(\underline{p}),\, a_\nu^\dagger(\underline{q})] = -g_{\mu\nu}\, 2p^0 \delta^3(\underline{p} - \underline{q}) \quad . \qquad (5.48)$$

If $\varepsilon_\mu^{(\lambda)}(\underline{p})$, for $\lambda = 0,1,2,$ or 3, are four linearly independent, orthonormal polarization vectors, then the annihilation operators $a^{(\lambda)}(\underline{q})$ for a photon of a specific polarization are given by

$$a_\mu(\underline{q}) = \sum_{\lambda=0}^{3} \varepsilon_\mu^{(\lambda)}(\underline{q})\, a^{(\lambda)}(\underline{q}) \quad .$$

The physical one-photon states $|q,\lambda\rangle$ of 4-momentum q and polarization $\varepsilon_\mu^{(\lambda)}(\underline{q})$ are those for which

$$\varepsilon^{(\lambda)}(\underline{q}) \, . \, q = 0 \quad .$$

Using the expressions (5.43) and (5.47) for the current and the 4-vector potential, we can evaluate (5.46), the operator S to first order in e. As previously, we may then determine the nonvanishing matrix elements of S_1, and, by applying the reinterpretation principle (4.49), those of $\tilde{S}_1$. We now find that any physical S-matrix $\tilde{S}$ derived from the unreinterpreted S of (5.45) in which S_1 is given by (5.46), cannot be unitary, for again we find that (5.20), which expresses unitarity to second order in e, fails. To show this, the matrix element

$$\left\langle \begin{matrix} p \\ 0 \end{matrix} \middle| \tilde{S}_1^\dagger \tilde{S}_1 \middle| \begin{matrix} s \\ 0 \end{matrix} \right\rangle = \frac{2}{\pi} \int_+ (dr) \sum_\lambda \int_+ (dq) p.\varepsilon^{(\lambda)}(\underline{q}) \delta^4(p-q-r)$$

$$.s.\varepsilon^{(\lambda)}(\underline{q}) \delta^4(q+r-s) \qquad (5.49)$$

of $\tilde{S}_1^\dagger \tilde{S}_1$ may be evaluated by the method of appendix B, giving

$$-\delta^4(p - s) \, m^2 \, \frac{p^0}{|\underline{p}|} \quad .$$

This is not a Lorentz invariant function, and, as explained earlier, unitarity must therefore fail.

The noninvariance which we have been observing throughout this section, and which has prevented the so-called local polynomial interactions from yielding unitary physical S-matrices, evidently arises from only performing integrations over positive energies, but this is all we are allowed to do because the reinterpreted particles, the physical tachyons and antitachyons, can only have positive energy.

Doubts have been raised [107] as to whether the Arons-Sudarshan theory admits any nontrivial, Lorentz-invariant, unitary, physical S-matrices. Our demonstration here that what would be called local polynomial interactions must yield a nonunitary physical S-matrix, removes from consideration a large class of possible interactions which, in conventional field theory, at least provide S-matrices which satisfy the conditions above. It has also been shown that a derivative-type coupling and the usual coupling to the electromagnetic field fail for the same reason.

These results suggest that if tachyons have a role to play in quantum field theory, then they will enter the theory in a way more unfamiliar than has been considered here, and it appears that non-local interactions may be required. That the interactions in a consistent quantum theory of tachyons must be non-local is also suggested by the following model calculations.

5.3 Simple Soluble Interactions

Dhar and Sudarshan [138] were the first to attempt an interacting theory of scalar tachyons. Using a quantization of the Arons-Sudarshan type, they expanded the tachyon field $\phi(x)$ in annihilation operators only, giving a nonhermitian field. Creation operators only appear in the conjugate field $\phi^{\dagger}(x)$, and in order to have a field which contained both creation and annihilation operators, Dhar and

Sudarshan introduced the hermitian field

$$\chi(x) = \frac{1}{\sqrt{2}}[\phi(x) + \phi^{\dagger}(x)] \quad . \qquad (5.50)$$

But their theory is not Lorentz-invariant, for they choose noninvariant commutation relations for the field $\phi(x)$ [107], which result in the field $\chi(x)$ having the commutation relations

$$[\chi(x), \chi(x')] = -i\ G^2(x - x') \quad ,$$

where the function G^2 is given by (4.16). As we have seen, the function G^2 is not Lorentz invariant. Furthermore, Boulware [139] has shown that the Dhar-Sudarshan theory is not unitary.

Jue [140] has applied a soluble field theoretic model to tachyons. He uses an hermitian tachyon field of the form (5.50) but uses the invariant commutation relations (4.41) for the field ϕ. The commutator of the field χ is then found to vanish identically:

$$[\chi(x), \chi(x')] = 0 \quad .$$

This is in accordance with his belief that tachyons only exist as virtual particles, and he does not include terms involving tachyon operators in the free Hamiltonian. It is supposed that tachyons are not observable as physical particles, so that the reinterpretation principle is not needed, even though the tachyons are allowed to have both positive and negative energies. Jue then looks at Schweber's scalar field model [111] in which the neutral scalar bradyon field has been replaced by the hermitian field $\chi(x)$, and he obtains the unitary "dressing" transformation. The dressing of the bare particles is the full extent of the interaction.

Kamoi and Kamefuchi [110] have considered the propagation of the interacting tachyon field that satisfies (5.4). As pointed out in section 5.1, the solution (5.5) suffers from the problem that, for a general current J, it contains Fourier components violating the condition (4.3). Furthermore, from (4.18) and (4.12), we see that the presence of these components produces a field which increases exponentially in future timelike directions. Kamoi and Kamefuchi therefore propose the use of the Green's function G^F in place of the retarded function G^R in (5.5), for the reason that this function gives a field in which the Fourier components violating (4.3) are exponentially damped for large times, both past and future. But there are problems with this approach because the physical interpretation of such a solution is unclear as we are not solving a conventional initial-value problem [110].

We should like to point out here that a simple interaction between the usual Arons-Sudarshan tachyon field ϕ and a classical current that satisfies a certain nonlocality condition can be described using the ordinary retarded Green's function. Although we are not able to offer a scheme for describing more general or realistic interactions, we shall discuss this simple interaction now because it uses the ordinary tachyon field ϕ, rather than χ, and it is based on a reasonably familiar form of initial-value problem.

<u>Interaction with a fixed source</u>

Let us consider the interaction of the non-hermitian Arons-Sudarshan tachyon field $\phi(x)$ with a classical current which is described by a complex scalar source function $J(x)$, so that the equation of motion for the tachyon field is

$$(\square - m^2)\ \phi(x) = J(x) \quad . \tag{5.51}$$

This type of model is well-known for a scalar bradyon under the name of the van Hove model, and has been discussed by many authors, among whom we mention Wentzel [141], Friedrichs [142], Henley and Thirring [143], Segal [121], and Emch [144].

We solve (5.51) with the aid of the Green's functions as indicated in section 5.1. We want to find the operator S which gives the relation between the incoming and outgoing fields, and which satisfies

$$S^{-1}\phi_{in}(x)\ S = \phi_{in}(x) + \int d^4y\ G(x-y)\ J(y) \quad , \tag{5.52}$$

where G is defined by (4.11).

In this model the tachyon field ϕ is quantized but the current J is some prescribed classical (unquantized) source function. Thus the fields ϕ_{in} and ϕ_{out}, which satisfy free-field equations, are taken to be free Arons-Sudarshan fields for which we have the decomposition

$$\phi_{in}(x) = (2\pi)^{-3/2}\int \frac{d^3p}{2\omega_p}\ e^{i\underline{p}\cdot\underline{x}}\ (e^{-i\omega x^0} a_{in+}(\underline{p}) + e^{i\omega x^0} a_{in-}(\underline{p})), \tag{5.53}$$

for example, and for the unquantized complex function J we consider the Fourier transform

$$J(x) = (2\pi)^{-4}\int d^4p\, e^{-ip.x}\, j(p) \quad . \tag{5.54}$$

Substituting (5.53), (5.54), and the form (4.12) of the function G into the equation (5.52), we find that S is the solution of

$$\int (dp)\, e^{i\underline{p}.\underline{x}}(e^{-i\omega x^0}\, S^{-1}a_{in+}(\underline{p})S + e^{i\omega x^0}\, S^{-1}a_{in-}(\underline{p})S)$$

$$= \int (dp)\, e^{i\underline{p}.\underline{x}}\, (e^{-i\omega x^0} a_{in+}(\underline{p}) + e^{i\omega x^0} a_{in-}(\underline{p}))$$

$$+ (2\pi)^{-3/2}\Big[\int_{|\underline{p}|\geqslant m} \frac{d^3p}{2\omega}\, e^{i\underline{p}.\underline{x}}\, (e^{-i\omega x^0} ij(\omega,\underline{p}) - e^{i\omega x^0}\, ij(-\omega,\underline{p}))$$

$$+ \int_{|\underline{p}|<m} \frac{d^3p}{2\chi}\, e^{i\underline{p}.\underline{x}}\, (e^{\chi x^0}\, j(i\chi,\underline{p}) - e^{-\chi x^0}\, j(-i\chi,\underline{p}))\Big] \quad , \tag{5.55}$$

where ω and χ are given by (4.5) and (4.6) respectively.

In (5.55) we notice the introduction of field components with $|\underline{p}|<m$ and imaginary energies $\pm i\chi_p$. But the outgoing field behaves as a free field and such a field, in the Arons-Sudarshan quantization, has only real energy and only components with momentum $|\underline{p}|\geqslant m$. For consistency, then, the last term in (5.55) must vanish and this can only occur if the source function $J(x)$ is such that the Fourier components appearing in this term vanish. We must therefore require that the Fourier transform of the source function satisfy the condition

$$j(p) = 0 \quad \text{for} \quad |\underline{p}| < m. \tag{5.56}$$

We know that such a condition on a function means that it is spatially extended and cannot vanish outside a finite closed region of 3-space.

Using a source complying with (5.56), we see from (5.55) that the operator S which describes the interaction is the solution of

$$S^{-1}a_{in+}(\underline{p})\,S = a_{in+}(\underline{p}) + i(2\pi)^{-3/2}\,j(\omega,\underline{p})$$
$$S^{-1}a_{in-}(\underline{p})\,S = a_{in-}(\underline{p}) - i(2\pi)^{-3/2}\,j(-\omega,\underline{p}) \quad . \tag{5.57}$$

We may find the explicit solution of the equations (5.57), and it is the unitary operator

$$S = \exp\Big[i(2\pi)^{-3/2}\int(dp)(a^{\dagger}_{in+}(\underline{p})j(\omega,\underline{p}) - a^{\dagger}_{in-}(\underline{p})j(-\omega,\underline{p}))$$
$$+ i(2\pi)^{-3/2}\int(dp)(a_{in+}(\underline{p})j^{*}(\omega,\underline{p}) - a_{in-}(\underline{p})j^{*}(-\omega,\underline{p}))\Big] \quad . \tag{5.58}$$

This operator S solves the problem and, as we shall see below, predicts the emission of tachyons if the source possesses the Fourier components which appear in (5.58). For a static (time-independent) source, however, there is no emission of particles and the scattering matrix is the identity. This may be seen [121,144] by noting that the Hamiltonian describing the interaction with a static source is

$$H = \tfrac{1}{2}\int d^3p\Big[a^{\dagger}_{+}(\underline{p})a_{+}(\underline{p}) - a^{\dagger}_{-}(\underline{p})a_{-}(\underline{p}) - i(2\pi)^{-3/2}j^{*}(\underline{p})(a_{+}(\underline{p})+a_{-}(\underline{p}))$$
$$+ i(2\pi)^{-3/2}j(\underline{p})(a^{\dagger}_{+}(\underline{p}) + a^{\dagger}_{-}(\underline{p}))\Big] \quad , \tag{5.59}$$

instead of the free Hamiltonian which is given by P^0 of (4.47). But, for sufficiently smooth source functions, (5.59) is unitarily equivalent to the free Hamiltonian. That is, there is a unitary transformation like (5.58) which diagonalizes the interacting Hamiltonian, and in such a case there is no interaction between the source and the tachyon field apart from the dressing of the source.

We shall want to find the matrix elements of S between states with a specific number of tachyons, so that we need to normal-order the expression (5.58). This can be done by using the formula

$$e^{A+B} = e^{-\frac{1}{2}[A,B]} e^{A} e^{B} \quad , \tag{5.60}$$

which is valid for operators A and B if the commutator [A,B] is a c-number. With the aid of (5.60), we find that the probability amplitude for r positive-energy, and s negative-energy tachyons of specified momenta to be emitted if there were m positive-energy and n negative-energy tachyons present initially is given by

$$\left\langle \begin{matrix} k_1,\ldots,k_r \\ l_1,\ldots,l_s \end{matrix} \text{ out} \middle| \begin{matrix} p_1,\ldots,p_m \\ q_1,\ldots,q_n \end{matrix} \text{ in} \right\rangle = \left\langle \begin{matrix} k_1,\ldots,k_r \\ l_1,\ldots,l_s \end{matrix} \text{ in} \middle| S \middle| \begin{matrix} p_1,\ldots,p_m \\ q_1,\ldots,q_n \end{matrix} \text{ in} \right\rangle$$

$$= \exp\left[-\tfrac{1}{2}(2\pi)^{-3} \int (dp)(|j_+(\underline{p})|^2 + |j_-(\underline{p})|^2)\right].(2\pi)^{-\frac{3}{2}(r+s+m+n)}$$

$$\cdot \frac{i^r}{\sqrt{r!}} j_+(k_1)\ldots j_+(k_r) \frac{(-i)^s}{\sqrt{s!}} j_-(l_1)\ldots j_-(l_s)$$

$$\cdot \frac{i^m}{\sqrt{m!}} j_+^*(p_1)\ldots j_+^*(p_m) \frac{(-i)^n}{\sqrt{n!}} j_-^*(q_1)\ldots j_-^*(q_n) \quad , \tag{5.61}$$

where, for convenience, we have written $j_\pm(p)$ in place of $j(\pm\omega_p, \underline{p})$. To determine what physical process this corresponds to, we have to apply the reinterpretation principle as given by (4.49).

We then find that (5.61) is equal to the probability amplitude for r tachyons and n antitachyons to be emitted if the initial state contained m tachyons and s antitachyons.

Consider the case that no tachyons or antitachyons are present initially, so that any particles which are present in the final state will have been emitted by the source. In particular, let us determine the probability, $P(N;0)$, that N particles of either type are emitted if originally none were present. That is, for simplicity, we shall not distinguish between tachyons and antitachyons, so that the probability $P(N;0)$ is obtained by summing the probabilities for all those processes which, upon reinterpretation, appear as the emission of N physical tachyons, regardless of whether they be tachyons or antitachyons, with no particles of either type present initially. As we are only interested in the number of particles present, we also integrate over the momenta. Using (5.61), we find that the probability $P(N;0)$ is given by

$$P(N;0) = \sum_{\substack{n=0 \\ n+r=N}}^{\infty} \sum_{r=0}^{\infty} \int (dk_1) \ldots \int (dk_r) \int (dq_1) \ldots \int (dq_n)$$

$$\cdot \left| \left\langle \begin{matrix} k_1, \ldots, k_n \\ 0 \end{matrix} \right| S \left| \begin{matrix} 0 \\ q_1, \ldots, q_n \end{matrix} \right\rangle \right|^2$$

$$= \frac{\bar{N}^N}{N!} \, e^{-\bar{N}} \, , \qquad (5.62)$$

where

$$\bar{N} = \frac{1}{(2\pi)^3} \int \frac{d^3p}{2\omega_p} \left(|j(\omega,\underline{p})|^2 + |j(-\omega,\underline{p})|^2 \right) \, . \qquad (5.63)$$

From (5.62) we see that the number of tachyons emitted by the source obeys a Poisson distribution in which the mean number emitted is given by (5.63), and that adding the probabilities for all possible N gives unity.

Our model thus describes the random emission of tachyons from an extended classical source. Note that the source is not quantized in this model and that the source function is a predetermined complex scalar function, so that the source is entirely unaffected by the emission or absorption of the tachyons.

Reinterpretation and antitachyons

In the above, we did not wish to distinguish between tachyons and antitachyons merely for reasons of convenience, and in finding the probability that N particles be emitted, we summed over the probabilities corresponding to all partitions of those N particles into tachyons and antitachyons. But we mention here that there may be grounds for regarding what we have called tachyons and antitachyons as inherently indistinguishable.

It has generally been assumed that the physical particles obtained from reinterpreting negative-energy tachyons bear an antiparticle-particle relationship to those particles obtained from reinterpreting positive-energy tachyons because discrete quantities like charge would change sign during the reinterpretation process in the former case but not in the latter. But we see that the charge associated with the 4-current defined by (5.42) is given by

$$Q = \int d^3x \; J^0(x)$$

$$= e\int (dp)(a_+^\dagger(\underline{p})a_+(\underline{p}) - a_-^\dagger(\underline{p})a_-(\underline{p})) \quad ,$$

so that positive- and negative-energy tachyons have opposite charges. This should hold generally for such quantities, for if the 4-current associated with them is spacelike, then by Lorentz invariance the sign of the zero-component of the 4-current cannot be invariant. Actually, the conserved charge associated with the Majorana field [135,136] has the same sign for both the positive- and negative-energy spacelike solutions. But here the 4-current appears to be timelike, even for the spacelike solutions [136], suggesting that the charge is somehow not being transported superluminally by that part of the field with spacelike 4-momentum. If such is the case, then reinterpretation should not affect the sign of the charge, and the physical tachyons may all have charges of the same sign. But one may speculate that all quantities like charge which are transported superluminally by tachyons are connected with a spacelike 4-current, so that they take opposite signs for positive-and negative-energy tachyons. Therefore, upon reinterpretation, all the tachyons will have charges of the same sign, so that there may in fact be no distinction between particle and antiparticle.

In addition, we recall the description of antiparticles which comes from the usual bradyon theory [101,126]. The bradyon field operator carries distinct representations of the Poincaré group belonging to the two separate branches of the bradyon mass hyperboloid, and charge conjugation consists of a mapping from one representation of the Poincaré group to the complex conjugate representation. But as we noted in section 4.3, the field operator of the scalar tachyon carries a single representation of the Poincaré group that belongs to the single-sheeted tachyon mass hyperboloid, so that perhaps we should consider that only one type of particle is ultimately described by the tachyon field operator.

So if we still wish to call some of the reinterpreted tachyons antitachyons, these comments caution us that they may not be antiparticles in the usual sense.

The pair model

We can also apply another soluble model, the Wentzel pair model, to tachyons. This model has been extensively studied for bradyons [145,146,147,142,143], and here we mainly follow Klein and McCormick [146]. For this model the interaction term is more complicated and we have

$$(\Box - m^2)\ \phi(x) = -\lambda \int d^4y\ Q(x,y)\ \phi(y) \quad , \qquad (5.64)$$

for which we consider the case of a time-independent separable interaction,

$$Q(x,y) = \delta(x^0 - y^0)\ U(\underline{x})\ U^*(\underline{y}) \quad , \qquad (5.65)$$

as it can be solved exactly. We shall only consider this model briefly. There is no production of real particles and only one-channel scattering occurs.

With the interaction given by (5.65), the equation of motion becomes

$$(\Box - m^2)\ \phi(t,\underline{x}) = -\lambda\ U(\underline{x}) \int d^3y\ U^*(\underline{y})\ \phi(t,\underline{y}) \quad , \qquad (5.66)$$

the solution of which satisfies

$$\phi(t,\underline{x}) = \phi_{in,\,out}(t,\underline{x}) - \lambda\int d^4y\int d^3z\ G^{R,\,A}(t-y^0,\underline{x}-\underline{y})U(\underline{y})U^*(\underline{z})$$

$$.\phi(y^0,\underline{z}) \quad . \qquad (5.67)$$

The expression (5.67), and similar expressions below, refer to pairs of equations like (5.5) and (5.6). In one equation of the pair, the labels "in" and R appear, and in the other equation we have the labels "out" and A. Defining the Fourier transforms

$$\psi(p^0,\underline{x}) = \frac{1}{2\pi}\int dt\ e^{ip^0t}\ \psi(t,\underline{x}) \quad ,$$

where ψ is ϕ, ϕ_{in}, or ϕ_{out}, and

$$G^{R,\,A}(p^0,\underline{x}) = \int dt\ e^{ip^0t}\ G^{R,\,A}(t,\underline{x}) \quad ,$$

equation (5.67) becomes

$$\phi(p^0,\underline{x}) = \phi_{in,\,out}(p^0,\underline{x}) - \lambda\int d^3y\int d^3z\ G^{R,A}(p^0,\underline{x}-\underline{y})U(\underline{y})U^*(\underline{z})\phi(p^0,\underline{z}) \ . \qquad (5.68)$$

The solution of this integral equation is

$$\phi(p^0,\underline{x}) = \phi_{in,\,out}(p^0,\underline{x}) - \lambda\int d^3y\ G^{R,\,A}(p^0,\underline{x}-\underline{y})U(\underline{y})\int d^3z\ U^*(\underline{z})$$

$$.\phi_{in,\,out}(p^0,\underline{z})\left[1 + \lambda\int d^3y\int d^3z\ U^*(\underline{y})G^{R,\,A}(p^0,\underline{y}-\underline{z})U(\underline{z})\right]^{-1} \ . \qquad (5.69)$$

Because of the free-field nature of ϕ_{in} and ϕ_{out}, they may be Fourier-decomposed in the manner of (5.53), which we write as

$$\phi^{(+)}_{in,\,out}(p^0,\underline{p}) = (2\pi)^{-3/2}\,\frac{1}{2\omega_p}\;a_{in,\,out+}(p^0,\underline{p})\,\delta(p^0-\omega_p)$$

and (5.70)

$$\phi^{(-)}_{in,\,out}(p^0,\underline{p}) = (2\pi)^{-3/2}\,\frac{1}{2\omega_p}\;a_{in,\,out-}(p^0,\underline{p})\,\delta(p^0+\omega_p)$$

for the fields ϕ_{in} and ϕ_{out}. Using (5.70), we can write the Fourier transform of Eq. (5.69) as

$$\phi^{(+)}(p^0,\underline{p}) = (2\pi)^{-3/2}\int\frac{d^3r}{2\omega_r}\left[\delta^3(\underline{p}-\underline{r}) + \frac{\lambda}{(2\pi)^3}\,\frac{\tilde{U}(\underline{p})\tilde{U}^*(\underline{r})}{\underline{r}^2-\underline{p}^2\pm i\eta}\cdot\frac{1}{D_\pm(\underline{r}^2)}\right]$$

$$\cdot\, a_{in,\,out}(p^0,\underline{r})\,\delta(p^0-\omega_r) \qquad (5.71)$$

and

$$\phi^{(-)}(p^0,\underline{p}) = (2\pi)^{-3/2}\int\frac{d^3r}{2\omega_r}\left[\delta^3(\underline{p}-\underline{r}) + \frac{\lambda}{(2\pi)^3}\,\frac{\tilde{U}(\underline{p})\tilde{U}^*(\underline{r})}{\underline{r}^2-\underline{p}^2\mp i\eta}\cdot\frac{1}{D_\mp(\underline{r}^2)}\right]$$

$$\cdot\, a_{in,\,out}(p^0,\underline{r})\,\delta(p^0+\omega_r)\quad , \qquad (5.72)$$

where we have defined

$$D_\pm(\underline{r}^2) = 1 + \frac{\lambda}{(2\pi)^3}\;d^3q\;\frac{|\tilde{U}(\underline{q})|^2}{\underline{q}^2-(\underline{r}^2\pm i\eta)}\quad , \qquad (5.73)$$

and we have used the following representations of the retarded and advanced Green's functions:

$$G^{R,\,A}(x) = \frac{-1}{(2\pi)^4}\int d^4p\;\frac{e^{-ip.x}}{(p^0\pm i\eta)^2-\underline{p}^2+m^2}\quad , \qquad (5.74)$$

where $\eta > m$. Of the two signs in front of η in Eqs. (5.71) to (5.74), the upper (lower) sign replaces the designation retarded (advanced), and is to be paired with the label in (out) in expressions

(5.71) and (5.72).

It is important to note that to be able to derive (5.71) and (5.72) for the tachyon field, it is necessary that the Fourier transform $\tilde{U}(\underline{p})$ of the source function $U(\underline{x})$ vanish for $|\underline{p}| < m$. We have to make this assumption here for the same reasons that the condition (5.56) had to be applied in the previous model. If we take a source function which satisfies this condition, then we can write the Heisenberg field ϕ as

$$\phi(x) = (2\pi)^{-\frac{3}{2}}\int d^3p\int d^3r\ \frac{e^{i\underline{p}.\underline{x}}}{2\omega_r}\Big[\Omega_{\pm}(\underline{p},\underline{r})\ e^{-i\omega_r t}\ a_{in,out+}(\underline{r})$$

$$+\ \Omega_{\mp}(\underline{p},\underline{r})\ e^{i\omega_r t}\ a_{in,out-}(\underline{r})\Big]\ , \qquad (5.75)$$

where we define

$$\Omega_{\pm}(\underline{p},\underline{r}) = \delta^3(\underline{p}-\underline{r}) + \frac{\lambda}{(2\pi)^3}\ \frac{\tilde{U}(\underline{p})\tilde{U}^*(\underline{r})}{\underline{r}^2-\underline{p}^2\pm i\eta}\cdot\frac{1}{D_{\pm}(\underline{r}^2)}\ , \qquad (5.76)$$

and the field (5.75) is the solution of the problem.

The tachyon fields $\phi_{in,out}(x)$ are free Arons-Sudarshan fields so that they satisfy the commutation relations

$$[\phi_{in,out}(x)\ ,\ \phi^{\dagger}_{in,out}(x')] = G^1(x - x')\ ,$$

which, as we have seen, do not yield canonical equal-time commutation relations. We can calculate the equal-time commutation relations of the field $\phi(x)$ given by (5.75), and we find that the interacting field $\phi(x)$ will satisfy the same equal-time commutation relations as the free fields, ϕ_{in} and ϕ_{out}, if Ω satisfies a form of unitarity requirement. For a spherically symmetric source function $U(\underline{x})$, this condition may be proved by the same procedure which is used in the bradyon case [146].

The S-matrix gives the relation between the operators a_{in} and a_{out},

$$a_{out} = S^{-1} a_{in} S \quad ,$$

as in Eq. (5.7), and we can determine the action of S on the a_{in} by using the two equations contained in the expression (5.75). As in the case for the model applied to bradyons [146], we find that

$$S^{-1} a_{in+}(\underline{p}) S = \exp(2i\delta(\underline{p}))\, a_{in+}(\underline{p})$$

$$S^{-1} a_{in-}(\underline{p}) S = \exp(-2i\delta(\underline{p}))\, a_{in-}(\underline{p}) \quad , \tag{5.77}$$

where

$$\tan\delta(\underline{p}) = -\frac{\lambda}{4\pi}\, |\underline{p}|\, |\tilde{U}(\underline{p})|^2 \left[1 + \frac{\lambda}{(2\pi)^3} \int d^3q\, \frac{|\tilde{U}(\underline{q})|^2}{\underline{q}^2 - \underline{p}^2}\right]^{-1} \quad . \tag{5.78}$$

Thus, from the form of the S-matrix, we see that there is no particle production, and that outgoing states can only differ from incoming states by a possible phase shift (5.78).

Again we see that a simple interaction between tachyons and a source can be treated in a fairly conventional way. It is to be noted, however, that in this model the tachyons must interact with an extended scatterer, specified by $U(\underline{x})$, just as in the previous model the tachyons were produced and absorbed by an extended classical current.

5.4 Discussion

Although we would like to be able to describe the possible interactions of tachyons, in doing so we should not demolish the quantization tachnique which was developed in the last chapter. We have seen that this constraint has severely impeded the progress towards a general interacting theory, because the quantization of the free tachyon field has two features which prevent the straightforward application of bradyon theories; the lack of spacelike commutativity of the fields, and a non-zero minimum possible value to the 3-momentum of the tachyon.

Our examination in section 5.2 of a class of S-matrices which a priori seemed satisfactory for describing tachyons, and moreover, were not too dissimilar from S-matrices which involve only bradyon interactions, showed us that an attempt to describe a local polynomial-type interaction for tachyons would produce nonunitary S-matrices. This result indicates that if an interacting theory of tachyons were ever to be formulated, it would be a nonlocal theory.

Because of the difficulties encountered in developing an interacting theory of tachyons, Kamoi and Kamefuchi [107] at one point surmise that the whole formalism of conventional quantum field theory is inapplicable to tachyons. But the model calculations of section 5.3 may be indicating that the situation is not quite as bad as all that. In both models the tachyon field is quantized by the Arons-Sudarshan method, and we neither introduce any other type of tachyon field, nor alter the quantization procedure from that found to be best for quantizing the free field. Also, the quantum field used in the models is derived from a classical field whose progress is described using conventional retarded and advanced functions, so that for the classical field one can specify a reasonably familiar sort of initial-value problem. To be able to apply the models consistently, the tachyon field had to interact in a nonlocal way, and this supports our earlier suspicion about the nonlocality of tachyon interactions, and shows the probable path which further investigation of tachyon interactions will take. We must admit, however, that the models of section 5.3 used very simple interactions with unquantized sources, and that they do not immediately suggest how more general interactions might be tackled.

The models allow for the presence of free physical tachyons, but because of the simplicity of the models we cannot, strictly speaking, use them to construct causality violations. The first model allowed only for the random emission and absorption of tachyons, and we have seen that causality violations using tachyons rest upon being able to transmit information with tachyons. This would require the modulation of a tachyon beam, for example, by the use of a shutter in front of the source, and would also require the reliable detection of tachyons. Such requirements obviously demand more complicated interactions to occur than those envisaged by our simple models.

Nonlocality may provide tachyons in quantum theory with a mechanism, unavailable to the classical point tachyon, whereby the objections raised against tachyons by macroscopic causality can be avoided. In section 4.1 we saw that we could not construct a well-defined pulse from classical tachyon fields, and that although diffuse features which moved faster than light were described by the retarded solutions, the future propagation behaviour depended on initial values on the entire spacelike initial-value surface. Later, in the quantum theory, we saw that tachyons necessarily interact over an extended region. In a full theory of interaction, these factors may serve to rule out superluminal signal propagation by making impossible the arrangement of those initial conditions that an observer considered necessary for giving him the capability of sending a reliable signal.

CHAPTER 6

CONCLUSIONS

Interest in tachyons began when it was demonstrated that the existence of faster-than-light particles would not necessarily invalidate special relativity, and that an idea of a classical tachyon could be developed which was in no way inconsistent with relativistic principles. We have discussed this and subsequent work on tachyons in the preceding chapters, and have studied tachyons in classical electrodynamics and quantum theory. Let us now make a brief assessment of the present theoretical status of tachyons, and speculate on their future prospects.

In our discussions of tachyons we have always tried to secure for tachyons as much as possible of established physical theories, and we believe that further work on tachyons should endeavour to do likewise. We have observed that such a program is difficult to execute, but surely it would yield the most reliable results because we must believe that present theories encapsulate a great deal of knowledge about the physical world. Thus, if tachyons can be shown to be consistent with the bulk of present theories, we should tend to think their existence in nature more likely. Of course one does not expect to be able to apply directly to tachyons theories which have been developed with only bradyons in mind. Rather, we should find out whether the obstructions preventing us from doing this are in principle as superficial as those arising from special relativity. We recall here that often the requirement of causality is taken to be equivalent to the prohibition of faster-than-light particles, but we have seen that this is not the case.

On the other hand, if theoretical studies showed that the introduction of any concept of a tachyon so conflicted with current theories that it would entail their virtual abandonment, then we would be inclined to decide against the possibility of tachyons existing. It is with such ideas in mind that the theoretical studies of tachyons should be assessed.

On this basis we are led to conclude that the existence of tachyons as classical point particles is most unlikely. We found that tachyons, if they exist, must not be able to be used for superluminal information transfer, and it is difficult to imagine a classical mechanism to achieve this. If tachyons undergo any detectable interaction with ordinary matter, then their emission and absorption must be completely random, for any regularity whatsoever could be exploited to transmit information. Such is not the behaviour of a classical object.

Moreover, we have shown that, in further contrast to classical particles, a classical, charged, point tachyon will not emit electromagnetic radiation, even when undergoing an arbitrary accelerated motion. Hence if such tachyons existed, they would be very hard to detect. The study of an extended classical tachyon might be informative. Here Lorentz-invariance would have to be maintained and problems arise in specifying the extension of the tachyon in an invariant way.

However, we have found that quantum theory is a much less hostile environment for tachyons. Arguments about macroscopic causality cannot be used against short-lived and virtual particles, and tachyons may well exist in such forms and be detectable. Also, the spacelike unitary irreducible representations of the Poincaré group await use, and we have found that the scalar field provides a way of taking the classical idea of a tachyon across into quantum theory. It produces a reasonable quantum theory of tachyons, although some features of conventional quantum theory had to be given up. It was necessary to violate spacelike commutativity, and, although we have Lorentz invariance, there is a more liberal account of the preservation of matrix elements under Lorentz transformations due to the manner in which the reinterpretation principle acts.

It is promising to have an acceptable quantization of the free tachyon field, but a real test of the theory would be the description of interactions. Further work on the question of the interactions of tachyons in quantum field theory is evidently needed, and we have shown that this should involve nonlocal interactions. This circumstance suggests that progress may well not be easy, but at the same time it might be just what is required to allow tachyons a place in quantum field theory.

APPENDIX A

NOTATION AND TERMINOLOGY

Throughout the text we use Lorentz 4-vector and 4-tensor notation (see, for example, Rindler [7]), with the metric tensor

$$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1) \quad .$$

The inner product of two 4-vectors x and y with contravariant components x^{μ} and y^{μ} is written as

$$x.y = g_{\mu\nu} \, x^{\mu} \, y^{\nu} = x_{\mu} \, y^{\mu} = x^{0} \, y^{0} - \underline{x}.\underline{y} \quad ,$$

where $\underline{x}$ and $\underline{y}$ are the usual 3-vectors. The square of the Minkowski length of a 4-vector x is denoted

$$[x]^{2} \equiv x_{\mu} \, x^{\mu} \quad ,$$

and the d'Alembertian operator is

$$\Box \equiv \partial^{\mu} \, \partial_{\mu} \equiv \frac{\partial^{2}}{\partial t^{2}} - \underline{\nabla}^{2} \quad .$$

We always use units in which the speed of light is unity:

$$c = 1 \quad .$$

If x is a 4-vector, then $\delta^{4}(x)$ is the 4-dimensional Dirac δ-function

$$\delta^4(x) = \delta(x^0)\delta(x^1)\delta(x^2)\delta(x^3) \quad ,$$

$\delta^3(\underline{x})$ is the corresponding 3-dimensional δ-function, and $\theta(x)$ is the step function

$$\theta(x) \equiv \theta(x^0) = \begin{cases} 1 & \text{if } x^0 > 0 \\ 0 & \text{if } x^0 < 0 \end{cases} .$$

The Lorentz-invariant measure on the mass-hyperboloids

$$[p]^2 = \text{const.} \gtrless 0$$

is denoted by (dp) so that

$$(dp) = \begin{cases} d^3p/2\sqrt{|\underline{p}|^2-m^2} & \text{if } [p]^2 = -m^2 \\ d^3p/2\sqrt{|\underline{p}|^2+m^2} & \text{if } [p]^2 = +m^2 \\ d^3p/2|\underline{p}| & \text{if } [p]^2 = 0 \end{cases} ,$$

where m is real and positive.

The name _tachyon_ for a faster-than-light particle was coined by Feinberg [6]. It is useful to have a generic name for the normal slower-than-light particles and we call them _bradyons_ [15,16,17]. _Luxon_ is the name given to a particle like a photon or neutrino that always travels at the speed of light. The _reinterpretation principle_ is largely responsible for making faster-than-light particles conceivable within special relativity and it is explained in section 1.1. We call a tachyon _transcendent_ if it has infinite velocity and therefore zero energy (see section 1.1).

APPENDIX B

THE EVALUATION OF AN INTEGRAL

We want to evaluate the integral

$$I(x) = \int_{+}(dk_1)\int_{+}(dk_2)\delta^4(x - k_1 - k_2) \quad , \tag{B1}$$

where x is a positive-energy 4-momentum for which the invariant $[x]^2$ may take an arbitrary positive or negative value so that x may be timelike, spacelike, or null. In (B1), k_1 and k_2 are positive-energy tachyon 4-momenta of mass m.

Recalling (4.38), we do the integration over $\underline{k}_2$ in (B1) to obtain

$$I(x) = \int_{+}(dk)\,\frac{\delta(x^0-k^0-\sqrt{(\underline{x}-\underline{k})^2 - m^2})}{2\sqrt{(\underline{x}-\underline{k})^2 - m^2}} \quad , \tag{B2}$$

in which the sign of the square-root in the integrand is only as shown since from (B1) we see that k_2 (and k) are restricted to positive energies. By introducing the polar coordinates

$$\begin{aligned} \underline{k} &= m \cosh\alpha\,(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta) \\ k^0 &= m \sinh\alpha \ , \end{aligned} \tag{B3}$$

in which θ is the angle between the vectors $\underline{k}$ and $\underline{x}$, Eq. (B2) becomes

$$I(x) = \frac{m^2}{4}\int_0^\infty d\alpha\,\cosh^2\alpha\int_{-1}^{1}dz\int_0^{2\pi}d\phi$$

$$\frac{\delta(x^0-m\sinh\alpha-(|\underline{x}|^2+m^2\sinh^2\alpha-2m|\underline{x}|\cosh\alpha.z)^{\frac{1}{2}})}{(|\underline{x}|^2+m^2\sinh^2\alpha-2m|\underline{x}|\cosh\alpha.z)^{\frac{1}{2}}} \quad , \tag{B4}$$

where $z = \cos\theta$. If we consider the argument of the δ-function in the integrand of (B4) to be a function f of z, then we may use

$$\delta(f(z)) = \frac{\delta(z-z_0)}{|f'(z_0)|} \quad , \tag{B5}$$

where z_0 satisfies $f(z_0) = 0$, and express z in terms of α. Eq. (B4) then becomes

$$I(x) = \frac{m}{4}\int_0^{\alpha'} d\alpha \cosh\alpha \int_{-1}^{1} dz \int_0^{2\pi} d\phi \frac{\delta(z-z_0)}{|\underline{x}|} \quad , \tag{B6}$$

where

$$z_0 = \frac{(2mx^0\sinh\alpha-[x]^2)}{2m|\underline{x}|\cosh\alpha} \quad , \tag{B7}$$

and α must be less than $\alpha' = \sinh^{-1}\frac{x^0}{m}$ because of the sign of the square-root in the δ-function in Eq. (B4). (The equations following (B4) can strictly only be used for $|\underline{x}|$ nonzero, but by treating the case $[x]^2 = a^2 > 0$ with $|\underline{x}| = 0$ separately we obtain the expression (B9) below).

Now z is only integrated over the range [-1,1] in (B6), so the integration over α is further restricted to those values for which z_0 in (B7) lies in this range. The nature of these extra restrictions varies according to whether x is timelike, spacelike, or null, and the restrictions, together with the resulting value of the integral (B6), are given in the following:

1) For $[x]^2 = 0$, the range of α integration is $[0,\sinh^{-1}\frac{x^0}{m}]$ and (B6) can be evaluated to give

$$I(x) = \frac{\pi}{2} \tag{B8}$$

2) For $[x]^2 = a^2 > 0$, and $v \equiv \tanh^{-1} \frac{|\underline{x}|}{x^0} < \sinh^{-1} \frac{a}{2m}$, the range of α is $[\sinh^{-1} \frac{a}{2m} - v , \sinh^{-1} \frac{a}{2m} + v]$ (where $a > 0$), which yields

$$I(x) = \frac{\pi m}{a} \cosh (\sinh^{-1} \frac{a}{2m}) \quad . \tag{B9}$$

If v defined above is greater than $\sinh^{-1} \frac{a}{2m}$, then the range of α is $[0, \sinh^{-1}(\frac{a}{m} \cosh v)]$, giving

$$I(x) = \frac{\pi}{2} \frac{x^0}{|\underline{x}|} \quad . \tag{B10}$$

3) For $[x]^2 = -\mu^2 < 0$, and $\mu < 2m$, the range of α is $[0, \sinh^{-1} \frac{x^0}{m}]$ so that

$$I(x) = \frac{\pi}{2} \frac{x^0}{|\underline{x}|} \quad . \tag{B11}$$

If $\mu \geqslant 2m$, and $u \equiv \tanh^{-1} \frac{x^0}{|\underline{x}|} < \cosh^{-1} \frac{\mu}{2m}$, then there is no suitable range of α, so

$$I(x) = 0 \quad . \tag{B12}$$

If $\mu \geqslant 2m$, and $u \equiv \tanh^{-1} \frac{x^0}{|\underline{x}|} > \cosh^{-1} \frac{\mu}{2m}$, then there are two regions of α integration: $[0,u-\cosh^{-1}\frac{\mu}{2m}]$ and $[u+\cosh^{-1}\frac{\mu}{2m}, \sinh^{-1} \frac{x^0}{m}]$. The integral (B6) then becomes

$$I(x) = \frac{\pi}{2} \left[\frac{x^0}{|\underline{x}|} - \frac{m}{\mu} \sinh(\cosh^{-1} \frac{\mu}{2m})\right] \quad . \tag{B13}$$

We note that the expressions (B10), (B11), and (B13) are not Lorentz-invariant, and it is this observation which is used in section 5.2.

REFERENCES

[1] A. Sommerfeld, Amsterdam Academy Proc. Nov. 26 (1904), quoted in [3]

[2] A.Sommerfeld, Göttinger Nachrichten, 201 (1905), quoted in [3].

[3] A.Sommerfeld, "Optics", Academic Press Inc., New York, N.Y. (1954).

[4] O. Heaviside, "Electrical Papers", Vol. 2, p. 496,Macmillan and Co., London (1892).

[5] J.J. Thomson, Phil. Mag. 28, 13 (1889).

[6] G. Feinberg, Phys. Rev., 159, 1089 (1967).

[7] W. Rindler, "Special Relativity", Oliver and Boyd, Edinburgh (1966).

[8] R.C. Tolman, "The Theory of Relativity of Motion", Univ. of California Press, Berkeley (1917).

[9] H. Schmidt, Z. Phys. 151, 365, 408(1958).

[10] H. Arzeliès, C.R. Acad. Sci. 244, 2698 (1957).

[11] S. Tanaka, Progr. Theor. Phys. 24, 177 (1960).

[12] Ya. P. Terletskii, Sov. Phys. - Dokl. 5, 782 (1961).

[13] Ya. P. Terletskii, "Paradoxes in the Theory of Relativity", Plenum Press, N.Y. (1968).

[14] O.M.P. Bilaniuk, V.K. Deshpande, and E.C.G. Sudarshan, Am.J. Phys. 30, 718 (1962).

[15] M. Baldo, G. Fonte, and E. Recami, Lett. Nuovo Cimento 4, 241 (1970).

[16] R. Goldoni, Nuovo Cimento 14A, 501 (1973).

[17] R.G. Cawley, Phys. Rev. D 2, 276 (1970).

[18] A. Einstein, Ann. Phys. (Leipzig) 17, 891 (1905).

[19] O.-M. Bilaniuk and E.C.G. Sudarshan, Phys. Today 22, no. 5, 43 (1969).

[20] R.P. Feynman, Phys. Rev. 76, 749 (1949).

[21] E.C.G. Stückelburg, Helv. Phys. Acta. 15, 23 (1942).

[22] O.-M. Bilaniuk and E.C.G. Sudarshan, Nature (London)223, 386 (1969).

[23] E.C.G. Sudarshan, Symposia on Theoretical Physics and Mathematics 10, 129, ed. A. Ramakrishnan, Plenum Press, N.Y. (1970).

[24] D. Bohm, "The Special Theory of Relativity", W.A. Benjamin, N.Y. (1965).

[25] F.A.E. Pirani, Phys. Rev. D 1, 3224 (1970).

[26] J.A. Parmentola and D.D.H. Yee, Phys. Rev. D 4, 1912 (1971).

[27] R.G. Root and J.S. Trefil, Lett. Nuovo Cimento 3, 412 (1970).

[28] R.G. Newton, Phys. Rev. 162, 1274 (1967).

[29] R.G. Newton, Science 167, 1569 (1970).

[30] G.A. Benford, D.L. Book, and W.A. Newcomb, Phys. Rev. D 2, 263 (1970).

[31] W.B. Rolnick, Phys. Rev. D 6, 2300 (1972).

[32] L.S. Schulman, Am. J. Phys. 39, 481 (1971).

[33] W.B. Rolnick, Phys. Rev. 183, 1105 (1969).

[34] O.-M. Bilaniuk, S.L. Brown, B. DeWitt, W.A. Newcomb, M. Sachs, E.C.G. Sudarshan, and S. Yoshikawa, Phys. Today 22, no. 12, 47 (1969).

[35] P.L. Csonka, Nucl. Phys. B 21, 436 (1970).

[36] R. Vilela Mendes, Phys. Rev. D 14, 600 (1976).

[37] T. Alväger and M.N. Kreisler, Phys. Rev. 171, 1357 (1968).

[38] M.B. Davis, M.N. Kreisler, and T. Alväger, Phys. Rev. 183, 1132 (1969).

[39] D.F. Bartlett and M.D. Lahana, Phys. Rev. D 6, 1817 (1972).

[40] C. Baltay, G. Feinberg, N. Yeh, and R. Linsker, Phys. Rev. D 1, 759 (1970).

[41] J.S. Danburg, G.R. Kalbfleisch, S.R. Borenstein, R.C. Strand, V. VanderBurg, J.W. Chapman, and J. Lys, Phys. Rev. D 4, 53 (1971).

[42] J.S. Danburg and G.R. Kalbfleisch, Phys. Rev. D 5, 1575 (1972).

[43] P.V. Ramana Murthy, Phys. Rev. D 7, 2252 (1973).

[44] A. Ljubičić, Ž. Pavlović, K. Pisk, and B.A. Logan, Phys. Rev. D 11, 696 (1975).

[45] P.V. Ramana Murthy, Lett. Nuovo Cimento 1, 908 (1971).

[46] R.W. Clay and P.C. Crouch, Nature (London) 248, 28 (1974).

[47] J.R. Prescott, J. Phys. G 2, 261 (1976).

[48] A.M. Gleeson, M.G. Grundzik, E.C.G. Sudarshan, and A. Pagnamenta, Phys. Rev. D 6, 807 (1972).

[49] J.G. Gilson, Math. Gazette 52, 162 (1968).

[50] K.H. Mariwalla, Am. J. Phys. 37, 1281 (1969).

[51] L. Parker, Phys. Rev. 188, 2287 (1969).

[52] A.F. Antippa (1970) Unpublished report. Quoted in Phys. Rev. D 4, 2198 (1971).

[53] E. Recami and R. Mignani, Riv. Nuovo Cimento 4, 209 (1974).

[54] A.R. Lee and T.M. Kalotas, Nuovo Cimento 41B, 365 (1977).

[55] A.F. Antippa and A.E. Everett, Phys. Rev. D 4, 2198 (1971).

[56] A.F. Antippa and A.E. Everett, Phys. Rev. D 8, 2352 (1973).

[57] A.F. Antippa, Nuovo Cimento 10A, 389 (1972).

[58] S. Naranan, Lett. Nuovo Cimento 3, 623 (1972).

[59] A. Rachman and R. Dutheil, Lett. Nuovo Cimento 8, 611 (1973).

[60] H. Arzeliès, C.R. Acad. Sci., série A 280, 1653 (1975).

[61] V. Gorini, Commun. Math. Phys. 21, 150 (1971).

[62] A.F. Antippa, Phys. Rev. D 11, 724 (1975).

[63] A.E. Everett, Phys. Rev. D 13, 785, 795 (1976).

[64] H. Lemke, Nuovo Cimento 27A, 141 (1975).

[65] R. Goldoni, Lett. Nuovo Cimento 5, 495 (1972).

[66] F. González Gascón, Nuovo Cimento 41B, 317 (1977).

[67] E. Recami and R. Mignani, Lett. Nuovo Cimento 4, 144 (1972).

[68] R. Mignani and E. Recami, Nuovo Cimento 14A, 169 (1973).

[69] M. Pavšič and E. Recami, Lett. Nuovo Cimento 19, 273 (1977).

[70] G. Ramachandran, S.G. Tagare, and A.S. Kolaskar, Lett. Nuovo Cimento 4, 141 (1972).

[71] A. Yaccarini, Lett. Nuovo Cimento 9, 354 (1974).

[72] R. Mignani and E. Recami, Lett. Nuovo Cimento 9, 357 (1974).

[73] R. Mignani and E. Recami, Lett. Nuovo Cimento 16, 449 (1976).

[74] P. Demers, Can. J. Phys. 53, 1687 (1975).

[75] E.A.B. Cole, Nuovo Cimento 40A, 171 (1977).

[76] H.C. Corben, Lett. Nuovo Cimento 11, 533 (1974).

[77] H.C. Corben, Int. J. Theor. Phys. 15, 703 (1976).

[78] H.C. Corben, Nuovo Cimento 29A, 415 (1975).

[79] R. Mignani and E. Recami, Lett. Nuovo Cimento 9, 367 (1974).

[80] E. Recami and R. Mignani, Lett. Nuovo Cimento 9, 479 (1974).

[81] R. Mignani and E. Recami, Lett. Nuovo Cimento 13, 589 (1975).

[82] H.C. Corben and E. Honig, Lett. Nuovo Cimento 13, 586 (1975).

[83] I. Frank and Ig. Tamm, C.R. (Dokl.) Acad. Sci. URSS 14, no. 3, 109 (1937).

[84] Ig. Tamm, J. Phys. (Moscow) 1, 439 (1939).

[85] L.D. Landau and E.M. Lifshitz, "Electrodynamics of Continuous Media", Vol. 8 of "Course of Theoretical Physics," Pergamon Press, Oxford, England (1960).

[86] H.K. Wimmel, Lett. Nuovo Cimento 1, 645 (1971).

[87] M. Glück, Nuovo Cimento 1A, 467 (1971).

[88] H.K. Wimmel, Nature (London), Phys. Sci. 236, 79 (1972).

[89] S.C. Barrowes, Foundations Phys. 7, 617 (1977).

[90] F.C. Jones, Phys. Rev. D 6, 2727 (1972).

[91] R. Mignani and E. Recami, Lett. Nuovo Cimento 7, 388 (1973).

[92] A. Peres, Phys. Lett. A 31, 361 (1970).

[93] L.S. Schulman, Nuovo Cimento 2B, 38 (1971).

[94] A.S. Lapedes and K.C. Jacobs, Nature (London), Phys. Sci. 235, 6 (1972).

[95] F. Rohrlich, "Classical Charged Particles. Foundations of Their Theory". Addison-Wesley Publishing Company, Inc., Reading, Mass. (1965).

[96] P.A.M. Dirac, Proc. Roy. Soc. London 167A, 148 (1938).

[97] P. Havas, Phys. Rev. 74, 456 (1948).

[98] N. Fleury, J.L. Lopes, and G. Oberlechner, Acta Phys. Austriaca 38, 113 (1973).

[99] C. Teitelboim, Phys. Rev. D 4, 345 (1971).

[100] A.O. Barut, Phys. Rev. D 10, 3335 (1974).

[101] N.N. Bogolubov, A.A. Logunov, and I.T. Todorov, "Introduction to Axiomatic Quantum Field Theory", W.A. Benjamin, Inc., Reading, Mass. (1975).

[102] W. Güttinger, Progr. Theor. Phys. 13, 612 (1955).

[103] C.J. Eliezer, Rev. Mod. Phys. 19, 147 (1947).

[104] W. Güttinger, Fortschr. Phys. 14, 483 (1966).

[105] E.P. Wigner, Annals of Mathematics 40, 149 (1939).

[106] G. Ecker, Ann. Phys. (N.Y.) 58, 303 (1970).

[107] K. Kamoi and S. Kamefuchi, Progr. Theor. Phys. 45, 1646 (1971).

[108] J.E. Murphy, Phys. Rev. D 6, 426, 1808 (1972).

[109] J.E. Murphy, Phys. Rev. D 7, 1260 (1973).

[110] K. Kamoi and S. Kamefuchi, Lett. Nuovo Cimento 19, 413 (1977).

[111] S.S. Schweber, "An Introduction to Relativistic Quantum Field Theory", Harper and Row, N.Y. (1961).

[112] Y. Aharonov, A. Komar, and L. Susskind, Phys. Rev. 182, 1400 (1969).

[113] R. Fox, C.G.Kuper, and S.G. Lipson, Proc. Roy. Soc. Lond. A 316, 515 (1970).

[114] A. Bers, R. Fox, C.G. Kuper, and S.G. Lipson, in "Relativity and Gravitation", p. 41. Ed. C.G. Kuper and A. Peres, Gordon and Breach, Science Publishers, Inc., N.Y. (1971).

[115] E.P. Wigner, in "Theoretical Physics", p. 59, International Atomic Energy Agency, Vienna (1963).

[116] P.M. Mathews and M. Seetharaman, Phys. Rev. D 8, 1815 (1973).

[117] M.E. Arons and E.C.G. Sudarshan, Phys. Rev. 173, 1622 (1968).

[118] B. Schroer, Phys. Rev. D 3, 1764 (1971).

[119] L. Streit and J.R. Klauder, in "Lectures in Theoretical Physics" 14B, 539, ed. W.E. Britten, Colorado Associated University Press, Boulder, Colorado (1973).

[120] J. Schwartz, J. Math. Phys. 2, 271 (1961).

[121] I.E. Segal, "Mathematical Problems of Relativistic Physics", Am. Math. Soc., Providence, R.I. (1963).

[122] P.J.M. Bongaarts, in "Mathematics of Contemporary Physics", ed. R.F. Streater, Academic Press, N.Y. (1972).

[123] A.L. Carey, J.M. Gaffney, and C.A. Hurst, J. Math. Phys. 18, 629 (1977).

[124] G. Feinberg, Phys. Rev. D 17, 1651 (1978).

[125] D. Korff and Z. Fried, Nuovo Cimento 52A, 173 (1967).

[126] U.H. Niederer and L. O'Raifeartaigh, Fortschr. Phys. 22, 111, 131 (1974).

[127] Iu. M. Shirokov. Sov. Phys. - JETP 6, 664, 919, 929 (1958).

[128] J. Bandukwala and D. Shay, Phys. Rev. D 9, 889 (1974).

[129] D. Shay, J. Math. Phys. 16, 1934 (1975).

[130] S. Weinberg, Phys. Rev. 133, B 1318 (1964).

[131] S.-J. Chang and L. O'Raifeartaigh, Phys. Rev. 170, 1316 (1968).

[132] I.T. Grodsky and R.F. Streater, Phys. Rev. Letters 20, 695 (1968).

[133] W. Rühl, Commun. Math. Phys. 6, 312 (1967).

[134] E. Majorana, Nuovo Cimento 9, 335 (1932).

[135] D. Tz. Stoyanov and I.T. Todorov, J. Math. Phys. 9, 2146 (1968).

[136] E.C.G. Sudarshan and N. Mukunda, Phys. Rev. D 1, 571 (1970).

[137] R.J. Eden, P.V. Landshoff, D.I. Olive, and J.C. Polkinghorne, "The Analytic S-Matrix", C.U.P. (1966).

[138] J. Dhar and E.C.G. Sudarshan, Phys. Rev. 174, 1808 (1968).

[139] D.G. Boulware, Phys. Rev. D 1, 2426 (1970).

[140] C. Jue, Phys. Rev. D 8, 1757 (1973).

[141] G. Wentzel, "Quantum Theory of Fields", Interscience Publishers, Inc., N.Y. (1949).

[142] K.O. Friedrichs, "Mathematical Aspects of the Quantum Theory of Fields", Interscience Publishers, Inc., N.Y. (1953).

[143] E.M. Henley and W. Thirring, "Elementary Quantum Field Theory", McGraw-Hill Book Company, Inc., N.Y. (1962).

[144] G.G. Emch, "Algebraic Methods in Statistical Mechanics and Quantum Field Theory", Wiley-Interscience, N.Y. (1972).

[145] G. Wentzel, Helv. Phys. Acta 15, 111 (1942).

[146] A. Klein and B.H. McCormick, Phys. Rev. 98, 1428 (1955).

[147] A. Chevalier and G. Rideau, Nuovo Cimento 10, 228 (1958).