

Barotropic depth-averaged
and
three-dimensional tidal programs
for
shallow seas

by

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Summary

Barotropic depth-averaged and three-dimensional tidal programs for shallow seas.

This thesis reports the development and testing of three programs for computing tides in shallow seas and their particular application to Spencer Gulf, South Australia.

The first program solves the three-dimensional barotropic non-linear long-wave equations discretised using finite differences. The σ -transformation is used in the vertical with non-uniform depth-level spacing. The overall difference scheme is three-level in time and of leap-frog type for the computation of tidal elevations, and two-level in time for velocity calculations. It uses explicit differencing in the horizontal direction and implicit differencing in the vertical. The second program solves the depth-averaged analogue of these equations. The third program is an adaptation of the depth-averaged program to include the modelling of the covering and uncovering of tidal flats.

All three programs are designed to handle general boundary configurations while using only four distinct element types. Appropriate forms of the discretised equations for a particular element configuration are automatically selected using the element type numbers in algebraic switches.

The coding and accuracy of the differencing used is checked by comparing predicted results with analytical solutions for idealized basins. Wave propagation analyses of the difference formulations for linearized forms of the equations are also carried out.

Depth-averaged and three-dimensional models of Spencer Gulf are developed with the open boundary at the entrance to the Gulf. Height-specified and combined radiation/height-specified open boundary conditions are compared in the case of the depth-averaged model. Three formulations for bottom stress (depth-averaged model) and two forms for vertical eddy viscosity coefficient (three-dimensional model) are examined. The role of the horizontal eddy viscosity terms in introducing artificial friction is discussed. Both models are driven simultaneously by the four major tidal constituents O_1 , K_1 , M_2 and S_2 . Comparisons of results using linear and non-linear equations are made in the depth-averaged case. Results clarify and extend the work from earlier numerical models of Spencer Gulf and predict new features of tidal flow near Wallaroo.

A modification of the method for incorporating the wetting and drying of tidal flats due to Flather and Heaps (1975) is presented and applied in a depth-averaged fine-grid model of Northern Spencer Gulf. It gives stable predictions over a 32 day simulation. Stability and accuracy are enhanced if a de Chèzy coefficient of quadratic friction is used. This suggests that a locally depth-dependent coefficient of quadratic friction is more suited to very shallow tidal modelling than a constant coefficient. An appraisal of the built-in filtering behaviour of some common finite difference formulations and their role in suppressing grid-scale oscillations is presented.

The Northern Spencer Gulf tidal flat model numerically confirms the observation of a residual gyre south of Lowly Point. A complementary gyre is predicted to the north of Lowly Point. The model predicts that Ward Spit, a large sand bar to the east of Port Bonython, is uncovered only at low springs. This prediction is supported by annotation on an old map of the region. A mechanism for the creation and maintenance of Ward Spit is suggested based on a plot of residual vectors for depth-averaged velocity.