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Over-the-Horizon Radar Array Calibration

by

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Contents

Abstract vi

Declaration viii

Acknowledgments ix

List of Figures x

List of Tables xvi

Glossary xvii

Publications xxi

1 Introduction 1

- 1.1 Problem Description 1
- 1.2 Outline of Thesis and Main Contributions 4

2 Background Array Processing and Array Calibration 6

- 2.1 Array Processing Background 6
  - 2.1.1 Maximum Likelihood Method 10
  - 2.1.2 Beamformers 11
  - 2.1.3 Subspace Methods 13
- 2.2 Array Calibration Application Areas 15
  - 2.2.1 Telescope Images 15
  - 2.2.2 General Antenna Arrays 16
  - 2.2.3 Airborne Radar 16
  - 2.2.4 Synthetic Aperture Radar 17
  - 2.2.5 Passive Sonar Towed Arrays 17

2.2.6 Sonar Imaging Systems 18
2.2.7 Synthetic Aperture Sonar Imaging Systems 18
2.2.8 Computer Vision 18
2.2.9 Ultrasound Arrays 19
2.2.10 Microphone Arrays 19
2.3 Previous Array Calibration Methods 19
2.3.1 Methods for Estimating Sensor Positions 20
2.3.2 Methods for Estimating Mutual Coupling 24
2.3.3 Methods for Estimating Sensor Positions and Mutual Coupling 25
2.3.4 Other Methods 26
2.4 Discussion 27

3 Effect of Model Errors on Radar Array Processing 28
3.1 Literature 29
3.2 Description 31
3.3 Performance Measures 32
3.3.1 Signal-to-Noise Ratio and Array Gain 33
3.3.2 Beampointing Error 33
3.3.3 Sidelobe Levels 34
3.4 Non-stationary Interferer Environment 36
3.4.1 Fluctuating Interferer Wavefront Model 38
3.4.2 Fluctuating Interferer DOA Model 38
3.4.3 Sea Clutter Model 40
3.4.4 Results 40
3.5 Conclusion 43

4 Array Calibration using Disjoint Sources 45
4.1 Mutual Coupling Models 46
4.2 Generalised Weiss-Friedlander Method 48
4.3 Performance of Generalised Method 52
4.4 Modified Algorithm for OTH radars 55
4.5 Simulation Example 59
4.6 Monte Carlo Analysis 61
4.7 Alternative Sensor Position Estimator 65

4.8 Performance Criteria 67
4.9 Mutual Coupling Estimation Investigation 70
4.10 Error Surface 72
4.11 Conclusion 79

5 Array Calibration using Disparate Sources **83**
5.1 Signal Model 84
5.2 Time-varying DOA Sources 85
5.3 Time-invariant DOA Sources 86
5.4 Overall Cost Function 87
5.5 An Example 87
5.6 Algorithm 88
    5.6.1 Initialisation 89
    5.6.2 Sensor Position Estimation 91
    5.6.3 Coupling Matrix Estimation 93
    5.6.4 DOA Estimation 93
    5.6.5 Estimation of Complex s's 95
    5.6.6 Assumptions 95
5.7 Simulation Example 96
5.8 Monte Carlo Analysis 99
5.9 Special Case 100
    5.9.1 Simulation Example 103
    5.9.2 Monte Carlo Analysis 104
5.10 Error Surface 109
    5.10.1 4-element Array 110
    5.10.2 Theoretical Analysis of 2-element Array 113
5.11 Performance with Larger Model Errors 118
5.12 Conclusion 119

6 Cramer-Rao Lower Bounds **122**
6.1 CRLB for Disjoint Sources 122
    6.1.1 DOA - DOA Terms 124
    6.1.2 Sensor Position - Sensor Position Terms 124
    6.1.3 Coupling - Coupling Terms 125

6.1.4 Cross Terms 125
6.1.5 CRLB Analysis 126
6.1.6 Algorithm Comparison with CRLB 132
6.1.7 Mutual Coupling Estimation using a Single Source 133
6.2 CRLB for Disparate Sources 142
6.2.1 DOA - DOA Terms 142
6.2.2 Sensor Position - Sensor Position Terms 144
6.2.3 Coupling - Coupling Terms 144
6.2.4 Cross Terms 145
6.2.5 Algorithm Comparison with CRLB 145
6.2.6 CRLB Analysis 146
6.3 Conclusion 147

7 Sources for HF Array Calibration 150
7.1 Jindalee Radar Overview 150
7.2 Meteors 154
7.2.1 Visual Observations 155
7.2.2 Radar Observations 155
7.2.3 Underdense versus Overdense Echoes 156
7.2.4 Shower Meteors and Sporadic Meteors 157
7.2.5 Radio Frequency Dependence 157
7.2.6 Diurnal Variation 160
7.2.7 Range and Altitude Dependence 160
7.2.8 Numerous Disjoint Sources of Opportunity 161
7.2.9 Spatial Stationarity Analysis of Head and Trail Echoes 169
7.2.10 Multimode Echoes 172
7.3 External Noise 173
7.3.1 Suitable Noise Sources 174
7.3.2 HF Noise Statistics 175
7.4 Other Sources 176
7.5 Jindalee Array Calibration 176
7.5.1 Calibration over Receivers 177
7.5.2 Performance Determination 180
7.6 Conclusion 180

8 Conclusion **185**
8.1 Overview and Contributions . . . 185
8.2 Future Work . . . 187

A Sidelobes for 2-D Array with Mutual Coupling **188**

B Mutual Coupling Estimation using a Single Source **190**
B.1 Passive Array Calibration . . . 195
B.2 Active Array Calibration . . . 198
B.3 Interpretation for Non-Identifiable Conditions . . . 200

C Analytic Expressions for Error Surfaces **201**
C.1 Source DOA . . . 202
C.2 Sensor Position Error . . . 203
C.3 Coupling Amplitude . . . 204
C.4 Coupling Phase . . . 205
C.5 Complex Scalar s_n . . . 206

Abstract

Modern over-the-horizon radars are currently being developed which have arrays than can be erected quickly, with minimal site preparation. Due to the rapid deployment of these arrays, antenna/sensor position errors may be present. Further since the antennas have a simple and cost-effective design, mutual coupling may be present. These imperfections, which can degrade radar performance, form the basis for the work conducted in this thesis.

The effect of these model errors on radar performance is first analysed. The degradation in signal-to-noise ratio, array gain, bearing estimation and array sidelobe levels, are determined. The major degradation is observed in the array sidelobe levels, which in turn results in the clutter-to-noise ratio (and hence target detectability) being worsened in the presence of non-stationary interferences. For these reasons, array calibration is required to improve the array sidelobe levels.

New array calibration algorithms are then developed to correct for sensor position errors and mutual coupling, using sources in the radar environment. These algorithms are analysed using simulations, and are found to perform well. The Cramer-Rao lower bound is derived, for the problem scenarios considered, and the algorithms are shown to achieve the bound. Further the Cramer-Rao lower bound is analysed, to obtain useful insight into the array calibration problem and identifiability.

Scattered echoes from meteor trails are shown to be excellent sources of opportunity for over-the-horizon radar array calibration. These echoes are found in general to : have planar wavefronts, be present in large numbers, be sufficiently strong, and be of adequate duration for sufficient snapshots to be obtained for array calibration. It also is shown that meteor head echoes are good sources of opportunity, and their properties along with that of other sources, are determined.

Finally, the receiving array of the Jindalee over-the-horizon radar (located in central Australia) is calibrated using echoes from meteor trails. The results obtained are compared with standard calibration (for this radar), which involves the use of special calibration sources. Array calibration using meteor trail echoes is found to perform as well as the standard array calibration, indicating that echoes from meteor trails can perform good array calibration.

Declaration

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

Signature

Date April, 1998

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List of Figures

1.1 The surface-wave radar environment. 2

1.2 Example of surface-wave radar data. 3

2.1 Planewave signal impinging an array of sensors. 7

3.1 Variation of signal-to-noise ratio with signal-interference separation, for the specified model error - (a) gain errors : σ_{α_m} = 0 (–), 0.2 (– –), 0.4 (–.–), 0.5 (...); (b) phase errors : σ_{ψ_m} = 0° (–), 12° (– –), 24° (–.–), 36° (...); (c) sensor position errors : $\sigma_{x_m} = \sigma_{y_m} = 0$ (–), 0.1d (– –), 0.2d (–.–), 0.3d (...); (d) mutual coupling : element height (h) = 0.25λ (–), 0.4λ (–.–), 0.5λ (– –). 34

3.2 Variation of pointing error with signal-interference separation, for the specified model error - (a) gain errors : σ_{α_m} = 0 (–), 0.2 (– –), 0.4 (–.–), 0.5 (...); (b) phase errors : σ_{ψ_m} = 0° (–), 12° (– –), 24° (–.–), 36° (...); (c) sensor position errors : $\sigma_{x_m} = \sigma_{y_m} = 0$ (–), 0.1d (– –), 0.2d (–.–), 0.3d (...); (d) mutual coupling : element height (h) = 0.25λ (–), 0.4λ (–.–), 0.5λ (– –). . 35

3.3 Variation of average sidelobe level with signal-interference separation, for the specified model error - (a) gain errors : σ_{α_m} = 0 (–), 0.2 (– –), 0.4 (–.–), 0.5 (...); (b) phase errors : σ_{ψ_m} = 0° (–), 12° (– –), 24° (–.–), 36° (...); (c) sensor position errors : $\sigma_{x_m} = \sigma_{y_m} = 0$ (–), 0.1d (– –), 0.2d (–.–), 0.3d (...); (d) mutual coupling : element height (h) = 0.25λ (–), 0.4λ (–.–), 0.5λ (– –). 36

3.4 Variation of peak sidelobe level with signal-interference separation, for the specified model error - (a) gain errors : σ_{α_m} = 0 (–), 0.2 (– –), 0.4 (–.–), 0.5 (...); (b) phase errors : σ_{ψ_m} = 0° (–), 12° (– –), 24° (–.–), 36° (...); (c) sensor position errors : $\sigma_{x_m} = \sigma_{y_m} = 0$ (–), 0.1d (– –), 0.2d (–.–), 0.3d (...); (d) mutual coupling : element height (h) = 0.25λ (–), 0.4λ (–.–), 0.5λ (– –). 37

3.5 Examples illustrating the models used for non-stationary interference analysis. 39

3.6 Example of SCV degradation. The clutter spectra shown are : (a) when no interferer exists and there are no model errors (−); (b) when an interferer exists and there are no model errors (− −); and (c) when an interferer exists and there are model errors (−.−). A target is present at Doppler cell 100. 41

3.7 SCV degradation versus model error standard deviation, for a fluctuating interference DOA model (−) and a fluctuating interference wavefront model (− −). 42

3.8 Beampatterns for the first 128 time samples (sweeps) - (a) position errors are not present; (b) position errors are present. 44

4.1 Generalised Weiss-Friedlander method's performance - Toeplitz coupling matrix. 53

4.2 Average and peak sidelobe levels for parameters estimated in figure 4.1 - nominal parameters (−.−), estimated parameters (− −) and actual parameters (–). 54

4.3 Generalised Weiss-Friedlander method's performance - full measured coupling matrix. 55

4.4 Average and peak sidelobe levels for parameters estimated in figure 4.3 - nominal parameters (−.−), estimated parameters (− −) and actual parameters (–). 56

4.5 Performance of modified method - cost function decreases with iteration number; sharpening of MUSIC; errors in estimated source DOAs; actual coupling matrix and estimated coupling matrix; errors in nominal (o) and estimated (∗) sensor positions. 60

4.6 Average and peak sidelobe levels for parameters estimated in figure 4.5 - nominal parameters (−.−), estimated parameters (− −) and actual parameters (–). 61

4.7 Statistical performance of algorithm. 63

4.8 Algorithm's performance for typical SNRs and number of snapshots obtainable from meteor trail echoes. 64

4.9 Dependence of algorithm on number of sources. 65

4.10 Algorithm's performance with alternative sensor position estimator, for typical SNRs and number of snapshots obtainable from meteor trail echoes. 68

4.11 Performance Criteria - (a) integrated sidelobe level (ISLB) for uncalibrated (−.−), calibrated (−−) and perfectly known (–) cases; (b) measure of performance. 71
4.12 First row of Longreach coupling matrix, at 5.0 MHz. 73
4.13 First row of Longreach coupling matrix for 5-12 MHz. 74
4.14 Average sidelobe levels for Longreach coupling matrices. 75
4.15 Longreach 5.0 MHz coupling matrix estimation. 76
4.16 Longreach 12.0 MHz coupling matrix estimation. 77
4.17 Error surface for third sensor's position. 78
4.18 Error surface for two source DOAs. 80
4.19 Algorithm's performance for larger model errors. Sensor position errors with standard deviation - (a) $0.2d$, (b) $0.3d$, (c) $0.4d$. 81

5.1 Illustration of disjoint clusters : A, B and C are disjoint clusters, each of which may contain a number of non-disjoint sources/signals. 84
5.2 Algorithm Sequence. 89
5.3 Simulation example results : cost function decreases with iteration number; the actual (o) and the estimated (+) signals' DOAs; actual coupling matrix and estimated coupling matrix; errors in nominal (o) and estimated ($*$) sensor positions. 98
5.4 Average and peak sidelobe levels for the nominal parameters (−.−), estimated parameters (−−) and also the actual parameters (–). 99
5.5 Statistical performance of algorithm. 101
5.6 Performance of algorithm for typical number of snapshots and SNRs obtainable from OTH radar sources. 102
5.7 Special case simulation example : cost function decreases with iteration number; errors in estimated signals' DOAs; actual coupling matrix and estimated coupling matrix; errors in nominal (o) and estimated ($*$) sensor positions. . . . 105
5.8 Average and peak sidelobe levels for parameters estimated in figure 5.7 - nominal parameters (−.−), estimated parameters (−−) and also the actual parameters (–) 106
5.9 Statistical performance of special case - asymptotic scenario. 107
5.10 Statistical performance of special case - finite/practical scenario. 108
5.11 Dependence on number of sources, for the special case. 109

5.12 Error surface for third sensor position. 111
5.13 Error surface for DOAs of two sources. 112
5.14 Error Surface for General Case. 114
5.15 Algorithm's performance in the special case, for larger model errors. Sensor position errors with standard deviation - (a) $0.2d$, (b) $0.3d$, (c) $0.4d$. 120
5.16 Algorithm's performance in the general case, for larger model errors. Sensor position errors with standard deviation - (a) $0.2d$, (b) $0.3d$, (c) $0.4d$. 121

6.1 CRLB variation with number of snapshots : third sensor's y-coordinate ($-$) and x-coordinate ($--$), and c_{12}'s amplitude ($-.-$) and phase (...). The standard deviation values for the sensor position are in units of wavelengths, while the coupling value phase is in units of radians. 127
6.2 CRLB variation with SNR : third sensor's y-coordinate ($-$) and x-coordinate ($--$), and c_{12}'s amplitude ($-.-$) and phase (...). The standard deviation values for the sensor position are in units of wavelengths, while the coupling value phase is in units of radians 128
6.3 CRLB variation with number of sources : third sensor's y-coordinate ($-$) and x-coordinate ($--$), and c_{12}'s amplitude ($-.-$) and phase (...). The standard deviation values for the sensor position are in units of wavelengths, while the coupling value phase is in units of radians. 129
6.4 CRLB variation with SNR for several cases. 130
6.5 CRLB variation with range, for the third sensor's y-coordinate, when the elevation angle of meteor trail echoes is considered. 132
6.6 Comparison of algorithm performance with theoretical performance bound - CRLB (solid line), STD (circle) and 99 percent confidence intervals (vertical bar) as a function of SNR, for the position of a sensor and for the amplitude and phase of a coupling parameter. The STD values were obtained from 10 algorithm runs. 133
6.7 Variation of CRLB standard deviations with source DOA, for $c = 0.5$ and $\psi = \pi/2$. 136
6.8 Variation of CRLB standard deviations with source SNR, for $c = 0.5$ and $\psi = \pi/2$. 138
6.9 Variation of CRLB standard deviations with both source DOAs, for $c = 0.5$ and $\psi = \pi/2$. 139

6.10 Variation of CRLB standard deviations with source DOA, for Jindalee 2x2 coupling matrix estimation. 141
6.11 Variation of CRLB standard deviations with both source DOAs, for Jindalee 2x2 coupling matrix estimation. 143
6.12 Algorithm's performance relative to theoretical performance bound : STD (circle) and 99 percent confidence intervals (vertical bar) as a function of SNR, for the position of a sensor and for the amplitude and phase of a coupling parameter. The solid line is the CRLB. Ten algorithm runs were used to obtain each STD value. 146
6.13 CRLB dependence on disjoint/non-disjoint sources - (a) twelve disjoint clusters, each with one signal (-); (b) six disjoint clusters, each with two non-disjoint signals (−−); (c) four disjoint clusters, each with three non-disjoint signals (−.−). 148
6.14 CRLB dependence on signal correlation - (a) uncorrelated signals (-); (b) partially correlated signals (−−); (c) coherent signals (−.−). 149

7.1 Skywave radar propagation environment (Lees et. al.). 151
7.2 Jindalee radar processing of received signals - range processing, digital beamforming and Doppler processing. 152
7.3 Example of Jindalee radar output. The received power is displayed, for each of the 10 finger beams, as a 2-dimensional image; each image comprising of 20 range cells (vertical axis) and 128 Doppler cells (horizontal axis). 153
7.4 Propagation modes for meteor echoes. 154
7.5 Examples of radar meteor echoes (McKinley). 156
7.6 Tracking of Eta Aquarid shower over azimuth and time (Thomas et. al.). . . . 158
7.7 Variation of echo rates with frequency (Thomas et. al.). 159
7.8 Sporadic meteor echo power variation with frequency (Thomas et. al.). Curve A and B are modelled results, while the circles show measured values. 159
7.9 Predicted diurnal variation of sporadic meteor echo rates for different days in the year (Thomas et. al.). 160
7.10 Example data set analysed for investigating wavefront of meteor echoes. . . . 162
7.11 Example illustrating meteor echo wavefront analysis - (a) eigenspectrum for echo; (b) phase values of principal eigenvector ($*$) and straight line fit (−); (c) wavefront phase errors. 164

7.12 Wavefront phase errors for meteor echoes - (a) ionospherically propagated "bad" trail echoes; (b) ionospherically propagated "good" trail echoes; (c) line-of-sight trail echoes; (d) line-of-sight head echoes. 165
7.13 Wavefront amplitude errors for meteor echoes - (a) ionospherically propagated "bad" trail echoes; (b) ionospherically propagated "good" trail echoes; (c) line-of-sight trail echoes; (d) line-of-sight head echoes. 166
7.14 Histogram of number of snapshots obtained from meteor trail echoes - (a) ionospherically propagated echoes; (b) line-of-sight echoes. 168
7.15 Two examples of TSSM plots showing meteor trail echoes. 170
7.16 Meteor head-trail echo pair. 171
7.17 Multimode ionospheric propagation conditions. 172
7.18 Jindalee data showing sources in the HF environment : (a) noise source; (b) beacon, aircraft return and meteor return. 174
7.19 Jindalee calibration weights versus range, for a particular receiver. 178
7.20 Jindalee calibration weights versus receiver number, for a particular range. . . 179
7.21 Actual (o) and estimated (+) calibration weights. 181
7.22 Average and peak sidelobe level difference (+) between the two methods. Positive difference values indicate calibration using meteor echoes is better. . 182
7.23 Comparison between the methods - statistical performance. Positive difference values indicate calibration using meteor echoes is better. 183

List of Tables

4.1 Position Errors for 4-element Array 75
5.1 Effect of near-field sources on ISLB 109
B.1 Ambiguity Conditions 200

Glossary

Abbreviations

ASL	average sidelobe level
C-TLS	constrained total least squares
CRLB	Cramer-Rao lower bound
DOAs	directions-of-arrival
ESPRIT	estimation of signal parameters via rotational invariance techniques
FIM	Fisher information matrix
ISLB	integrated sidelobe level
LS	least squares
ML	maximum likelihood
MUSIC	multiple signal classification
PSL	peak sidelobe level
SCV	sub-clutter visibility
SLB	sidelobe
SNR	signal-to-noise ratio
STD	standard deviation
TLS	total least squares

Symbols

$\mathbf{x}^T$, $\mathbf{X}^T$	transpose
$\mathbf{x}^H$, $\mathbf{X}^H$	conjugate transpose
$\hat{x}$, $\hat{\mathbf{x}}$, $\hat{\mathbf{X}}$	estimates
$trace\{\mathbf{X}\}$	trace of matrix $\mathbf{X}$
$\Re\{x\}$	real part of x
$\Im\{x\}$	imaginary part of x
$conj(x)$, x^*	conjugate of x
$diag\{\mathbf{x}\}$, $\mathbf{D}_\mathbf{x}$	diagonal matrix containing the elements of $\mathbf{x}$ along its main diagonal
$\|\mathbf{x}\|$	vector norm of $\mathbf{x}$
$\|\|\mathbf{X}\|\|_F$	Frobenious norm of matrix $\mathbf{X}$
$\frac{\partial Q}{\partial x}$	partial derivative of Q with respect to x
$\dot{x}_y$	first derivative of x with respect to y
e^x, $\exp(x)$	exponential operator
$\delta(t_1, t_2)$	one when $t_1 = t_2$, and zero otherwise
$(\mathbf{X})^+$	generalised inverse of matrix $\mathbf{X}$
$\odot$	Hadamard product
j	complex operator
$\mathbf{I}_M$	MxM identity matrix
M	number of sensors / receivers
t	time index
T	number of snapshots/samples
N	number of signals
N_C	number of clusters
N_T	total number of signals in all clusters
$N_{\tilde{Z}}$	number of columns in matrix $\tilde{\mathbf{Z}}$
θ, ϕ	bearing (azimuth), direction-of-arrival
Φ	set of bearings not in main beamwidth
(x_m, y_m)	actual position of mth sensor
(x_m^o, y_m^o)	nominal position of mth sensor
$(\Delta x_m, \Delta y_m)$	position error of mth sensor
λ	radar wavelength

f	radar frequency
w	radar angular frequency
v	speed of light
τ_{mn}	time delay for nth signal to arrive at mth sensor
$\mathbf{a}(\theta)$	array steering vector
$\mathbf{a}_o(\theta)$	nominal steering vector
$\mathbf{v}$	actual steering vector (including the effects of mutual coupling)
$\mathbf{A}$	matrix of array steering vectors
$\mathbf{D}(\theta)$, $\mathbf{D}_2(\theta)$	diagonal matrices involved in the derivatives of $\mathbf{a}(\theta)$
$s_n(t)$	baseband waveform of the nth signal at time t
$s_I(t)$	baseband waveform of interference signal at time t
$n_m(t)$	receiver noise at time t
$z_m(t)$	output of mth sensor/receiver at time t
$\mathbf{s}(t)$	vector of baseband waveforms of signals, at time t
$\mathbf{n}(t)$	vector of noise outputs at time t
$\mathbf{n}_T(t)$	vector of total noise contribution (i.e. including interferers) at time t
$\mathbf{z}(t)$	vector of sensor outputs at time t
$\mathbf{z}_C(t)$	clutter signal at time t
$\bar{\mathbf{z}}_m(t)$	output of consecutive receivers starting at mth receiver, at time t
$\tilde{\mathbf{Z}}$	matrix with data from all clusters
$\check{\mathbf{z}}(t)$	vector of sensor outputs for all sources, at time t
$\tilde{z}_\theta(t)$	beamformer output for a beam steered in direction θ, at time t
$\mathbf{w}(\theta)$	beamformer weight vector
$B_\theta(\phi)$	beampattern for steer direction θ
$ASL(\theta)$	average sidelobe level for a beam steered in direction θ
$p(\theta)$	beamformer power output in direction θ
σ_S^2, σ_N^2, σ_I^2	signal, noise, and interferer powers
σ_n^2	power of nth disjoint source
s_n, $\mathbf{s}$, $\mathbf{S}$	complex parameters
$\mathbf{R}$	spatial covariance matrix
$\mathbf{R}_n$	spatial covariance matrix of nth disjoint source/cluster
$\mathbf{P}$	signal covariance matrix
$\mathbf{P}_n$	signal covariance matrix of nth disjoint cluster

$\mathbf{Q}$	noise covariance matrix
$\mathbf{v}_m$, $\mathbf{V}$	mth eigenvector and matrix of eigenvectors
λ_m	mth eigenvalue
$\mathbf{E}$	signal subspace
$\mathbf{U}$	noise subspace
α_m, ψ_m	gain and phase error, for mth element
$\boldsymbol{\Gamma}$	diagonal matrix of receiver gain/phase errors
σ_a	standard deviation of amplitude errors
σ_ψ	standard deviation of phase errors
$\mathbf{S}_C$	scattering matrix
$\mathbf{C}$	coupling matrix
$\mathbf{C}_o$	nominal coupling matrix
$\mathbf{Z}_o$	array impedance matrix
Z_L	scalar load impedance
Q	cost function
$\mathbf{H}$	Hessian matrix
$\mathbf{r}$	gradient vector
$\boldsymbol{\Psi}$	unknowns to be estimated
$\mathbf{J}$	Fisher information matrix
$\mathbf{CRLB}(\boldsymbol{\Psi})$	Cramer-Rao lower bound for estimating $\boldsymbol{\Psi}$

Boldface lower case variables are column vectors.

Boldface upper case variables are matrices.

Publications

The list of publications relating to this thesis are :

- I. S. D. Solomon, Yu. I. Abramovich, D. A. Gray and S. J. Anderson, "OTH radar antenna array calibration analysis", Fourth International Symposium on Signal Processing and its Applications, August 1996, Gold Coast, Australia, pp. 471-474.
- I. S. D. Solomon, D. A. Gray, Yu. I. Abramovich and S. J. Anderson, "Estimating of array mutual coupling and sensor positions for over-the-horizon radar", Digital Signal Processing Applications Conference (TENCON), November 1996, Perth, Australia, pp. 846-851.
- I. S. D. Solomon, D. A. Gray, Yu. I. Abramovich and S. J. Anderson, "Sources for OTH radar array calibration", IEEE Antennas and Propagation Symposium, July 1997, Montreal, Canada, pp. 306-309.
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- I. S. D. Solomon, Yu. I. Abramovich, D. A. Gray and S. J. Anderson, "Performance of OTH radar array calibration", IEEE International Conference on Acoustics, Speech, and

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- I. S. D. Solomon, Yu. I. Abramovich, D. A. Gray and S. J. Anderson, "Mutual coupling estimation", International Radar Symposium, September 1998, Munich, Germany (to appear).
- I. S. D. Solomon, D. A. Gray, Yu. I. Abramovich and S. J. Anderson, "Meteor trail echoes for OTH radar array calibration", *in preparation for* Radio Science.