



The Theory of A_p^q Spaces

H.Kumudini Dharmadasa

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Department of Mathematics
University of Adelaide
South Australia

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Abstract

The aim of the thesis is to extend the notion of A_p^q space from its historical context in the work of Herz and to recognise such spaces as preduals of spaces of intertwining operators of induced representations as suggested by the work of Rieffel. This generalisation of A_p^q spaces involves considering tensor products of a given norm of L_p spaces of Banach space-valued functions (the spaces of induced representations) and constructing a convolution of functions of such spaces. First, the analysis is carried out when the tensor product space is endowed with the greatest cross-norm, and sufficient conditions for the existence of the integral of the convolution are established. Most of this analysis depends upon an identity we derive of Radon-Nikodym derivatives of measures on homogeneous spaces involved.

The elements of the generalised A_p^q space are shown to be cross-sections of a Banach semi-bundle over the double coset space corresponding to the groups from which the representations are induced, and their properties are duly discussed. In particular, the generalised form of the classical result $L_p * L_q \subseteq L_r$, where $1/r = 1/p + 1/q - 1$, is shown to be true in this situation. The result that the A_p^q space is the predual of the space of intertwining operators is then established, under the condition that the intertwining operators can be approximated, in the ultraweak operator topology, by integral operators.

Sufficient conditions under which the above analysis can be carried out, when the tensor product space is endowed with either p-nuclear norm or the Hilbert-Schmidt norm are then given.