PHYSICAL REVIEW D **71,** 034010 (2005)

Neutral pion decay in dense Skyrmion matter

Alexander C. Kalloniatis[1,*] and Byung-Yoon Park[1,2,†]

[1]*CSSM, University of Adelaide, Adelaide 5005, Australia*

[2]*Department of Physics, Chungnam National University, Daejon 305-764, Korea*

(Received 21 October 2004; published 8 February 2005)

We study the density dependence of the decay $\pi^0 \rightarrow \gamma\gamma$ using the Skyrme Lagrangian to describe simultaneously both the matter background and mesonic fluctuations. The classical ground state configuration has different chiral properties depending on the Skyrmion density, which is reflected in the physical properties of pion fluctuating on top of the classical background. This leads to large suppression at high density of both photo-production from the neutral pion and the reverse process. The effective charges of $\pi^\pm$ are also discussed in the same framework.

DOI: 10.1103/PhysRevD.71.034010 PACS numbers: 12.39.Dc, 12.39.Fe, 13.20.Cz, 21.65.+f

I. INTRODUCTION

At high temperature and/or density, the properties of hadrons are expected to change dramatically and understanding these changes under extreme conditions is important not only in nuclear and particle physics but also in many other related fields such as astrophysics. Data from high-energy heavy ion colliders, astronomical observations on compact stars and some theoretical considerations suggest that the phase diagram of hadronic matter is far richer than the simple confinement/deconfinement picture seen in finite temperature lattice QCD simulations and include interesting QCD phases such as color superconductivity. Moreover effective theories can be derived for these extreme conditions, using macroscopic degrees of freedom, by matching them to QCD at a scale close to the chiral scale $\Lambda_\chi \sim 4\pi f_\pi \sim 1$ GeV.

Chiral symmetry, which under normal conditions is spontaneously broken, is believed to be restored under such extreme conditions by virtue of its seeming to go hand-in-hand with confinement. The value of the quark condensate $\langle \bar{q}q \rangle$ of QCD is an order parameter of this symmetry and is expected to decrease as the temperature and/or density of hadronic matter are increased. Since the pion is the Goldstone boson associated with spontaneously broken chiral symmetry, the various patterns in which the symmetry is realized in QCD will be directly reflected in the in-medium properties of the pion, such as its mass m_π and decay constant f_π.

These quantities have been the subject of previous studies [1–4] in the formalism adopted here. In those works, the Skyrme picture is used to describe both pions and dense baryonic matter. The basic strategy of the approach begins with the Skyrme conjecture that a soliton (Skyrmion) of the meson Lagrangian can be taken as a baryon, so that dense baryonic matter can be approximated as a system of infinitely many Skyrmions. The pion is then incorporated as a fluctuation over this dense Skyrmion matter. The chiral properties of the classical background solution are directly reflected in the pion fluctuation properties, which may be interpreted as in-medium modifications. If we accept that the Skyrme model can be applied up to some density, its unique feature of a unified meson-baryon description provides an interesting framework to investigate nonperturbatively the meson properties in dense baryonic matter. That is, we do not have to assume any density dependence of the in-medium parameters. We work with a single model Lagrangian whose parameters are fixed for mesons in free space. Only the classical ground state describing the dense Skyrmion matter becomes highly density dependent and thus naturally in turn so do the fluctuating mesons on top of this dense background. There are however a few drawbacks in the approach. Firstly, the lowest energy configurations for Skyrmionic matter are available thus far only for a crystal structure [5]; we cannot yet describe the liquid structure of normal nuclear matter nor its behavior at high temperature. Next, there are a few undetermined parameters in the model. They limit us to a qualitative understanding of the related physics. Leaving these drawbacks to further improvements, in this paper we apply the approach to the anomalous decay of the neutral pion into two photons, $\pi^0 \rightarrow \gamma\gamma$ at finite density. Though this electromagnetic process may not be regarded as being so important in the fireball phase of relativistic heavy ion collisions, it is potentially relevant, for example, through $\gamma\gamma \rightarrow \pi^0 \rightarrow \nu\bar{\nu}$, to astrophysical phenomena such as core collapse supernovae and neutron stars [6,7].

The dependence of the process $\pi^0 \rightarrow \gamma\gamma$ on temperature has been investigated by various authors [8–11]. It is well understood that though the anomaly, which drives the decay, is temperature independent, nonetheless the amplitude does depend on T through the phase space or through the modification of the thermal quark propagators in evaluating explicitly the triangle diagram. In particular, [10,11] consider temperature effects in the Wess-Zumino-Witten (WZW) effective Lagrangian which incorporates

*Electronic address: akalloni@physics.adelaide.edu.au

†Electronic address: bypark@cnu.ac.kr

the anomaly. The coefficient of the anomalous term corresponding to $\pi^0 \to \gamma\gamma$ can pick up temperature dependence from loops [9], which is close in spirit to what will emerge in our study of density dependence below though for us the coefficient modifications come not from loops but at tree-level in the background field approach.

Several works have studied also the density dependence of the process, with quite different results emerging [12–15]. In [12] the decay is computed from the triangle diagram but keeping pion quantities such as mass and coupling fixed at their *in vacuo* values with the result that the decay width increases with density. In [13] a diagrammatic approach in the three-flavor NJL model is used with the result that the decay width decreases with density despite a corresponding increase in the pion mass. In [14] the neutral pion decay width was computed by assuming its free-space form from the anomaly and inputting two, quite different, model (such as Nambu–Jona-Lasinio) scenarios for the behavior of the pion mass and decay constant at the phase transition point in temperature or density with correspondingly different results, increasing and decreasing, for the decay width dependence on density.

In distinction to this, by using the Skyrme Lagrangian we obtain all in-medium quantities such as pion mass and decay constant from the same Lagrangian from which we compute the decay width. We stress that though our approach uses some of the machinery of the Skyrme approach, the most important aspect of the calculation is that the density dependence is obtained from the *same effective chiral Lagrangian* as that for free space using the fluctuation formalism. Other details of the Skyrme model do not play a significant role.

In [15], a similar process $\tilde{\pi}^0 \to \tilde{\gamma}\,\tilde{\gamma}$ was analyzed for the generalized pion fluctuations and the generalized photons in the color-flavor-locked (CFL) phase using the corresponding Wess-Zumino-Witten term [16] for that phase. It was shown that the decay of the generalized pion is constrained by geometry and vanishes at large density. The color-flavor-locked phase can be taken as the high density limit of our approach, in the region where a hadronic description is no longer valid.

The paper is structured as follows: the next section reviews the Skyrme Lagrangian and the properties of the dense Skyrmion matter. In Sec. III we consider meson fluctuations on this matter background and extract the in-medium modifications to the pion observables. There is a brief statement of conclusions in Sec. IV.

II. MODEL LAGRANGIAN AND DENSE SKYRMION MATTER

We begin with a modified Skyrme model Lagrangian [17], which also incorporates the scale anomaly of QCD in terms of a scalar dilaton field:

$$\mathcal{L} = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^2 \mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \mathcal{L}_{\rm sk} + \frac{f_\pi^2 m_\pi^2}{4} \times \left(\frac{\chi}{f_\chi}\right)^3 \mathrm{Tr}(U^\dagger + U - 2) + \mathcal{L}_{\rm WZW} + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{m_\chi^2 f_\chi^2}{4}\left((\chi/f_\chi)^4\left[\ln(\chi/f_\chi) - \frac{1}{4}\right] + \frac{1}{4}\right) \tag{1}$$

where $U = \exp(i\vec{\tau}\cdot\vec{\pi}/f_\pi) \in \mathrm{SU}(2)$ and χ is the scalar dilaton field. The parameters in Eq. (1) correspond to physical properties of the corresponding mesons: m_π and f_π (m_χ and f_χ) are the pion (respectively, dilaton) mass and decay constant in free space. The Skyrme term $\mathcal{L}_{\rm sk}$ is the higher derivative term introduced into the Lagrangian to stabilize the soliton solution of the Lagrangian, namely

$$\mathcal{L}_{\rm sk} = \frac{1}{32e^2}\mathrm{Tr}([L_\mu, L_\nu]^2), \tag{2}$$

where $L_\mu \equiv (\partial_\mu U)U^\dagger$ $(R_\mu \equiv U^\dagger(\partial_\mu U))$. Finally, the Wess-Zumino-Witten term $\mathcal{L}_{\rm WZW}$ is necessary to break the symmetry of Eq. (1) under $U \to U^\dagger$ which is not a genuine symmetry of QCD. The corresponding action can be written locally as [18]

$$S_{\rm WZW} = -\frac{iN_c}{240\pi^2}\int \varepsilon^{\mu\nu\lambda\rho\sigma} d^5x \mathrm{Tr}(L_\mu \cdots L_\sigma) \tag{3}$$

in a five-dimensional space whose boundary is ordinary space and time. For $U \in \mathrm{SU}(2)$, namely, for two flavors, $S_{\rm WZW}$ trivially vanishes (for three flavors this gives the hypothesized process $KK \to \pi\pi\pi$). This will change when we couple to photons, as discussed below.

The model Lagrangian has a few parameters which are fixed by the meson dynamics in baryon-free space. For the pions, we can fix the associated parameters to the empirical values as $f_\pi = 93$ MeV and $m_\pi = 138$ MeV. On the other hand, for the dilaton field, there are no available well-established empirical values for the mass and decay constant (or equivalently the vacuum expectation value). There have been a number of theoretical discussions on the field itself [19]. We use the values phenomenologically determined in nuclear matter studies [20]. There is another unknown parameter which has not been determined by any directly associated experiment, the Skyrme parameter e, for which we use the conventionally used value [21,22]. Because of the ambiguities in these model parameters our study should be taken as having only qualitative relevance.

The Lagrangian is invariant under global charge rotations, $U \to U + i\varepsilon[Q, U]$, where ε is a constant and $Q = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3})$ is the charge matrix for light quarks. The coupling of the pions and the photons can be incorporated by promoting this to a local symmetry, $\delta U \to \varepsilon(x)[Q, U]$. This can be done by replacing the derivatives acting on the pion fields (not the neutral dilaton field) by covariant derivatives, $\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$ where the photon field $A_\mu(x)$ transforms as $A_\mu \to A_\mu - (1/e)\partial_\mu\varepsilon$ and e is the charge of the proton. For example, the current algebra term for the pions is rewritten via

$$\frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^2\mathrm{Tr}(\partial_\mu U^\dagger\partial^\mu U)\Rightarrow\frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^2\mathrm{Tr}(D_\mu U^\dagger D^\mu U). \quad (4)$$

On the other hand, as is well known, minimal substitution does not work in gauging the Wess-Zumino action. The so-called trial and error Noether method [18] gives the gauged Wess-Zumino-Witten action

$$\begin{aligned}\tilde{\Gamma}_{\mathrm{WZW}}(U, A_\mu) = \Gamma_{\mathrm{WZW}}(U) - e\int d^4x A^\mu J_\mu^{\mathrm{an}} + \frac{ie^2}{24\pi^2}\\ \times\int d^4x\varepsilon^{\mu\nu\alpha\beta}(\partial_\mu A_\nu)A_\alpha\times\mathrm{Tr}[Q^2L_\beta\\ + Q^2R_\beta + QUQU^\dagger L_\beta],\end{aligned} \quad (5)$$

where the anomalous part of the electromagnetic current of pions is defined as

$$J_\mu^{\mathrm{an}} = \frac{1}{48\pi^2}\varepsilon_{\mu\nu\alpha\beta}\int d^3x\mathrm{Tr}[Q(L_\nu L_\alpha L_\beta + R_\nu R_\alpha R_\beta)]. \quad (6)$$

The normal part of the current coming from the current algebra term (4) is

$$J_\mu^{\mathrm{n}} = i\frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^2\mathrm{Tr}[Q(R_\mu - L_\mu)] \quad (7)$$

and is conserved by the classical equations of motion of the fields.

For meson dynamics in baryon-free space, the vacuum solutions for the pions and scalar field are

$$U_{\mathrm{vac}} = 1, \qquad \chi_{\mathrm{vac}} = f_\chi. \quad (8)$$

The fluctuations on top of this free-space vacuum are then incorporated via

$$U = U_\pi = \exp(i\vec{\tau}\cdot\vec{\pi}/f_\pi), \quad \text{and} \quad \chi = f_\chi + \tilde{\chi}. \quad (9)$$

To lowest nontrivial order we obtain

$$\begin{aligned}\mathcal{L} = \frac{1}{2}\partial_\mu\pi_a\partial^\mu\pi_a - \frac{1}{2}m_\pi^2\pi_a\pi_a + \frac{1}{2}\partial_\mu\tilde{\chi}\partial^\mu\tilde{\chi} - \frac{1}{2}m_\chi^2\tilde{\chi}^2\\ + \frac{i}{2}eA^\mu(\partial_\mu\pi^+\pi^- - \partial_\mu\pi^-\pi^+)\\ + \frac{N_c e^2}{96\pi^2 f_\pi}\pi^0\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\\ - \frac{ieN_c}{12\pi^2 f_\pi^3}\varepsilon^{\mu\nu\alpha\beta}A_\mu\partial_\nu\pi^+\partial_\alpha\pi^-\partial_\beta\pi^0 + \cdots.\end{aligned} \quad (10)$$

Thus we generate the usual pion-photon interactions. Note that the normal electromagnetic current of pions leads to the isovector current associated with two charged pions. Correspondingly, the anomalous current gives the isoscalar current for the neutral and charged pions. The two terms with N_c in Eq. (10) come from the gauged Wess-Zumino-Witten term and describe the anomalous $\pi^0\to\gamma\gamma$ decays and $\gamma\to\pi^+\pi^-\pi^0$. At tree-level, it leads to the standard $\pi^0\to\gamma\gamma$ decay width *in vacuo*

$$\Gamma_{\pi^0\gamma\gamma} = \frac{m_\pi^3}{64\pi}\left(\frac{N_c e^2}{12\pi^2 f_\pi}\right)^2. \quad (11)$$

On the other hand, the nonlinearity of the Lagrangian supports soliton solutions (Skyrmions) carrying nontrivial topological winding numbers. With Skyrme's conjecture interpreting the winding number as baryon number we may simulate dense baryonic matter by using the meson Lagrangian as a system made of many Skyrmions. Let the lowest energy configuration for a given baryon number density be $U_0(\vec{r})\equiv n_0 + i\vec{\tau}\cdot\vec{n}$ and $\chi_0(\vec{r})$. The Classical lowest energy state of the multi-Skyrmion system is a crystal and there has been intensive work in the late 1980's on a model containing only pions [5]. Of course, such a crystal structure is unrealistic for a multibaryon system and a randomizing of the multi-Skyrmion arrangement is an open problem. There have been a few pioneering works attempting to incorporate statistical fluctuations (for example, see [23]), which can be adopted in our approach in principle. However, none of the properties we consider below will depend explicitly on the specific periodic properties of the crystal structure.

Two well-separated Skyrmions are in a lowest energy configuration when relatively rotated in isospin space about an axis perpendicular to the line joining their centers. We thus consider as the lowest energy state of Skyrmionic matter at relatively low density that for a face centered cubic (FCC) crystal where well localized single Skyrmions are arranged on each lattice site such that the 12 nearest Skyrmions are each oriented corresponding to the above lowest energy configuration for a Skyrmion pair. At higher density, Skyrmion tails start overlapping each other and the system undergoes a phase transition to a more symmetric configuration, the so-called the "half-Skyrmion" cubic crystal. There, one half of the baryon number carried by the single Skyrmion is concentrated at an original FCC site where $U_0 = -1$ while the other is concentrated on the links where $U_0 = +1$. Such a system has an additional symmetry with respect to $U_0\to -U_0$, which results in the vanishing of $\langle U_0\rangle$. This is often interpreted as a restoration of the chiral symmetry in the literature [5]. More precisely though, it is only the average value of U over space that vanishes while the chiral circle still has a fixed radius f_π. We call this a "pseudogap" phase to distinguish it from the genuine chiral symmetry restored phase for which the chiral circle shrinks to a point at $U = 0$.

The dilaton field, when introduced in the Lagrangian Eq. (1) to restore scale symmetry, effectively plays the role of a "radial" field for U and the restriction on the chiral radius is then relaxed. The pseudogap phase still remains as a transient process unless the dilaton mass is sufficiently small [3]. Shown in Fig. 1 is a typical numerical result for

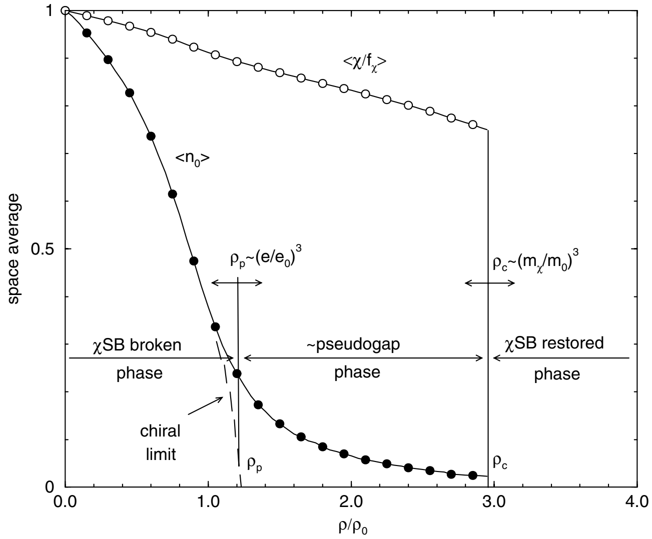

FIG. 1. Average values of σ and χ/f_χ of the lowest energy crystal configuration at a given baryon number density.

the average value of n_0 and χ_0/f_χ in the lowest energy field configuration for a given baryon number density. In the figure, the baryon number density is given in units of normal nuclear matter density ρ_0. It should be taken only as a rough guide since density dependence scales strongly with the model parameters, especially on the "Skyrme parameter" e and the scalar mass m_χ as $\rho_p \propto e^3$ and $\rho_c \propto m_\chi^3$. One can see, nevertheless, that as the density increases the average value $\langle n_0\rangle$ drops quickly and almost reaches zero around the density ρ_p, where the system enters the pseudogap phase in the half-Skyrmion configuration. Compared to the chiral limit $m_\pi = 0$ where $\langle n_0\rangle$ vanishes exactly at $\rho = \rho_p$ (as illustrated in Fig. 1 by the dashed line), the transition happens rather smoothly and thus can be called an approximate or *quasipseudogap* phase. As we increase the density further, the system remains in this approximate pseudogap phase for some range of ρ but the average value of χ/f_χ continues slowly to decrease. At density ρ_c, the $\langle\chi/f_\chi\rangle \neq 0$ phase and the $\langle\chi/f_\chi\rangle = 0$ phase have the same energy. Then, at density higher than ρ_c, the latter comes to have lower energy and finally chiral symmetry is restored. To conclude this discussion, the dilaton in this approach is important for setting up a consistent mechanism of chiral symmetry restoration within the Skyrme approach. In the mean field approach we will use in the following the dilaton will not play any further role beyond this enabling a correct framework for change of symmetry properties approaching the phase transition.

III. FLUCTUATIONS ON TOP OF THE DENSE SKYRMION MATTER

We now introduce mesonic fluctuations on the dense baryonic medium just described via

$$U = \sqrt{U_\pi}U_0\sqrt{U_\pi}, \qquad \chi = \chi_0 + \tilde{\chi} \tag{12}$$

in Eq. (1). Expanding in U_π we thus obtain the Lagrangian for the *in-medium* fields

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}G_{ab}(\vec{r})\partial_\mu\pi_a\partial^\mu\pi_b - \frac{1}{2}M_\pi^2(\vec{r})\pi^2 + \frac{1}{2}\partial_\mu\tilde{\chi}\partial^\mu\tilde{\chi} \\ &- \frac{1}{2}M_\chi^2(\vec{r})\tilde{\chi}^2 + \frac{i}{2}eC(\vec{r})A^\mu(\partial_\mu\pi^+\pi^- - \partial_\mu\pi^-\pi^+) \\ &+ \frac{N_c e^2}{96\pi^2 f_\pi}D(\vec{r})\pi^0\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \\ &- \frac{ieN_c}{12\pi^2 f_\pi^3}F(\vec{r})\varepsilon^{\mu\nu\alpha\beta}A_\mu\partial_\nu\pi^+\partial_\alpha\pi^-\partial_\beta\pi^0 + \cdots.\end{aligned} \tag{13}$$

where we have written only the terms corresponding to those in Eq. (10) to emphasize the in-medium modifications of the various contributions. Note that this does not alter the anomaly structure of the Lagrangian, which is fully respected by the Lagrangian equation (1). Only the coefficients of the terms corresponding to each process receive *local effective* corrections from the background matter through the potentials:

$$\begin{aligned}&G_{ab}(\vec{r}) = (\chi_0/f_\chi)^2(n_0^2\delta_{ab} + n_a n_b),\\ &M_\pi^2(\vec{r}) = m_\pi^2(\chi_0/f_\chi)^3 n_0,\\ &M_\chi^2(\vec{r}) = m_\chi^2(\chi_0/f_\chi)^2[3\ln(\chi_0/f_\chi) + 1] + (f_\pi/f_\chi)^2\\ &\qquad \times[(\partial_i n_\alpha)^2 + 6m_\pi^2(1-n_0)(\chi_0/f_\chi)],\\ &C(\vec{r}) = (\chi_0/f_\chi)^2 n_0^2, \qquad D(\vec{r}) = n_0^2 + n_3^2, \qquad F(\vec{r}) = n_0^2.\end{aligned}$$

In a mean field treatment of these potentials, we may take their spatial averages so that they reduce to constants, namely $\langle G_{ab}(\vec{r})\rangle \equiv G\delta_{ab}$, $\langle M_{\pi,\chi}^2(\vec{r})\rangle \equiv M_{\pi,\chi}^2$, $\langle C(\vec{r}), D(\vec{r}), F(\vec{r})\rangle \equiv C, D, F$. Then the Lagrangian can be rewritten simply as

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}\partial_\mu\pi_a^*\partial^\mu\pi_a^* - \frac{1}{2}m_\pi^{*2}\pi_a^*\pi_a^* + \frac{1}{2}\partial_\mu\tilde{\chi}\partial^\mu\tilde{\chi} \\ &- \frac{1}{2}m_\chi^{*2}\tilde{\chi}^2 + \frac{i}{2}e_\pi^* A^\mu(\partial_\mu\pi^{*+}\pi^{*-} - \partial_\mu\pi^{*-}\pi^{*+}) \\ &+ \frac{(N_c e^2)^*_{\pi\gamma^2}}{96\pi^2 f_\pi^*}\pi^{*0}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \\ &- \frac{i(eN_c)^*_{\gamma\pi^3}}{12\pi^2 f_\pi^{*3}}\varepsilon^{\mu\nu\alpha\beta}A_\mu\partial_\nu\pi^{*+}\partial_\alpha\pi^{*-}\partial_\beta\pi^{*0}\cdots\end{aligned} \tag{14}$$

where we have carried out a wave function renormalization $\pi_a^* = \sqrt{\langle G\rangle}\pi_a$ for the pion fields. *All* the physical parameters are evidently modified by the medium. Their density dependence is given by the following relations:

$$f_\pi^*/f_\pi = \sqrt{G},$$

$$m_\pi^*/m_\pi = \sqrt{M_\pi^2/G},$$

$$e_\pi^*/e = (C/G),$$

$$(eN_c)^*_{\gamma\pi^3}/(eN_c) = F,$$

$$(e^2N_c)^*_{\pi\gamma^2}/(e^2N_c) = D.$$

We now see that three different kinds of effective electric charge of pions appear depending on the different electromagnetic processes involved: that of charged pions (denoted by e_π^*), that appearing in the vertices for $\pi^0 \to \gamma\gamma$ and thirdly that for the process $\gamma \to \pi^+\pi^0\pi^-$. All three charges were simply a unit of electric charge in the Lagrangian equation (10) for the pions in baryon-free space. However, as illustrated in Fig. 2, all three charges originate from the electric charge of the quarks and thus how each term gets the charge factor e in its effective vertex is based on completely different detailed dynamics of the quarks. The baryonic matter influences each process in a different way so that each effective charge develops its own density dependence.

Above all, the ''effective'' electric charge of the charged pions illustrated as Fig. 2(a) comes from the effect of the surrounding medium just as in the case of electrons in condensed matter. As for the other processes, we also have to take into account not only the modification of the electric charge of the quarks but also the changes in the number of colors N_c^* involved in the triangular or square anomaly diagram of Figs. 2(b) and 2(c). The latter is expected to take place through the modifications of the quark propagator and the quark vacuum. As explicitly shown in [16], for example, the evaluation of the anomalous triangle diagram in the color-flavor-locked phase does not pick up the N_c factor from the quark loop.

Note that in Eqs. (14) $F/D = \langle n_0^2\rangle/\langle n_0^2 + n_3^2\rangle < 1$ so that $(eN_c)^*_{\gamma\pi^3}/(eN_c)_{\gamma\pi^3} < (e^2N_c)^*_{\pi\gamma^2}/(eN_c)_{\pi\gamma^2}$, which implies that fewer colors would be effectively involved in the box diagram of process (c) than in the triangle diagram of process (b).

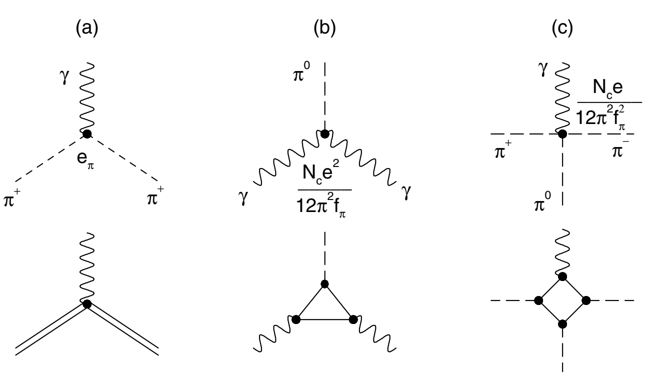

FIG. 2. The pion vertices for the processes and the corresponding quark processes.

Shown in Fig. 3 is the density dependence of the parameters appearing in Eqs. (14). Quantities here are normalized with respect to their values in free space and all are reduced by the effect of the baryonic medium. Only the pion mass appears stable at low density corresponding to the symmetry broken phase. The other quantities show quite strong dependence on the medium density even at low density. On the other hand, the pion mass drops quickly to zero in the pseudogap phase, while the other parameters exhibit plateau behavior. In the chiral symmetry restored phase, all the parameters rapidly go to zero after ρ_c.

As a consequence of changes in the in-medium parameters, the $\pi^0 \to \gamma\gamma$ decay width is modified, with the result

$$\Gamma^*_{\pi^0\gamma\gamma} = \frac{m_\pi^{*3}}{64\pi}\left(\frac{(N_c e^2)^*_{\pi\gamma^2}}{12\pi^2 f_\pi^*}\right)^2. \quad (15)$$

That this result is identical in structure to Eq. (11) up to the replacements $m_\pi \to m_\pi^*$, $f_\pi \to f_\pi^*$ and so on, is a consequence of the preservation of the chiral anomaly in the presence of dense matter. Thus, for example, the pion mass coming from the phase factor of the process is replaced by the effective pion mass. In obtaining Eq. (15), we have used a very naive mean field approximation. The higher order contributions in the medium-generated potentials of Eqs. (14) can be incorporated systematically. However, as shown in the previous works on the other properties of pion [1,3], after the resummation they lead only a minor corrections compared to the zeroth order values that are nothing but those obtained in the mean field approximation.

Temperature effects could have been revealed, for example, in [11] from the pion loops, because the pion propagator at finite temperature differs from that at zero temperature by discrete Matsubara frequencies. One may extract a density dependence by evaluating similar pion loops by using the modified pion propagator with effective pion mass. We expect, however, that the result is less important than the dominant nonperturbative one through direct in-medium modification of the effective parameters.

Since the dilaton field is decoupled from the anomalous WZW term of the Lagrangian and is neutral, this particle cannot be involved directly in electromagnetic processes such as $\pi^0 \to \gamma\gamma$. The presence of background medium could generate a π − matter−χ coupling, which can be absorbed into the resummation process like other terms higher order in the potentials Eqs. (14) and leads to small corrections on top of the mean field result we have pursued here. However, as discussed in [2], the dilaton plays its most important role in the chiral symmetry restoration of the background matter. Thus, the dilaton field is indirectly relevant to the process $\pi^0 \to \gamma\gamma$, as mentioned above, by providing for a consistent context of chiral symmetry

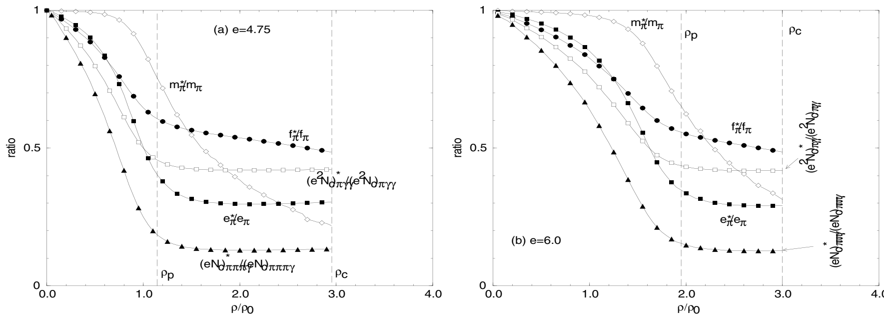

FIG. 3. Ratios of the in-medium quantities to the free ones. In order to illustrate the strong parameter dependence of ρ_p, we present two results obtained with (a) $e = 4.75$ and (b) $e = 6.0$. The other parameters are fixed as $f_\pi = 93$ MeV, $m_\pi = 138$ MeV, $f_\chi = 240$ MeV, and $m_\chi = 720$ MeV.

restoration within which the anomalous process can be studied. We shall return to this again below.

Numerical results for Eq. (15) are presented in Fig. 4. One can see that the neutral pion decay process is strongly suppressed as the matter density increases. This comes from the reduction in the pion mass (solid line in the inset graph) and from the reduction in the strength of the corresponding vertex (dashed line), where the former plays more a dominant role in the pseudogap phase and the latter in the symmetry broken phase.

We now compare our results to other work specifically focusing on qualitative behavior, whether the decay width is enhanced or suppressed. To this end, it is useful to state the three broad approaches within which temperature/density dependence is reflected in pion properties and chiral symmetry restoration at the critical point.

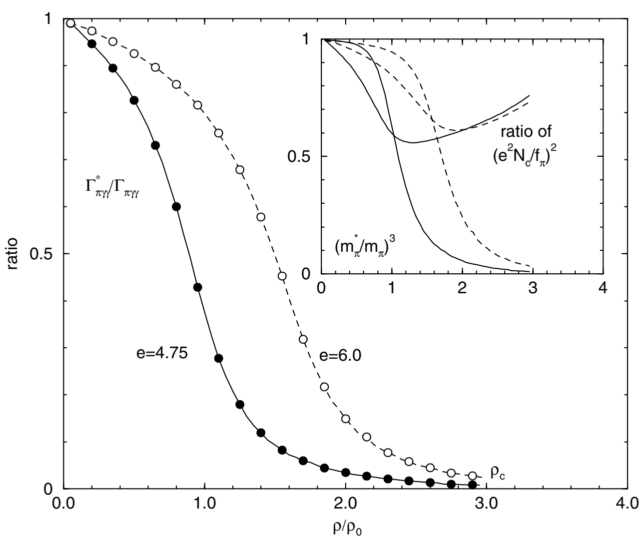

FIG. 4. Ratios of the decay width of $\pi^0 \to \gamma\gamma$ to the free-space value. The curves of the inset graph are the ratios $(m_\pi^*/m_\pi)^3$ and $((eN_c/f_\pi)^*/(eN_c/f_\pi))^2$. Again the results obtained with two different values of the Skyrme parameter: $e = 4.75$ (solid lines) and $e = 6.0$ (dashed lines).

Phenomenologically these different approaches center around different ways of realizing that the ratio of $m_\pi f_\pi$ in-medium vs *in vacuo* must be equal to the corresponding ratio for $m_q\langle\bar{q}q\rangle$ [Gell-Mann–Oakes–Renner (GMOR) relation], which must in turn vanish if chiral symmetry is restored at the phase transition. Also relevant is that upon restoration of the symmetry, the pion and its chiral partner the σ, here represented by the dilaton, must become degenerate. Thus we can label as approach A the case of the pion mass increasing to become degenerate with its chiral partner, while f_π to approaches zero to fulfill the ratio of GMOR relations. Approach B would have both the pion and its chiral partner decreasing in mass to become eventually degenerate, precisely the case for our work with the pion and dilaton becoming degenerate. No experimental evidence yet invalidates one of these against the other. In comparing our results with the other approaches mentioned in the introduction the difference in the final result for the decay width will ultimately depend on which approach (A or B) has been realized for chiral symmetry restoration. Furthermore, to compare with works that only include finite temperature effects we will follow the general consensus and take our density dependence as qualitatively indicative also of the behavior as a function of temperature.

First, our decay width suppression disagrees with [12] which, however, does not incorporate medium (temperature or density) dependences in quantities such as the pion mass and the decay constant. To that extent [12] corresponds neither to approach A or B above and in fact can only be relevant for low temperature or density far away from any transition to chiral symmetry restoration. Our result agrees with [13] in the decay width, though in our calculation the pion mass decreases with density while [13] have chiral symmetry restoration realized via approach A: the pion mass increases to that of the σ meson to achieve symmetry restoration. Our approach is closest to [14] in that the temperature/density dependence of the pion mass and the decay width are incorporated. In particular, a vanishing pion mass at the phase transition point corresponds to case II of [14] and approach B delineated above.

However, in [14] the mass is almost constant in temperature and density until the phase transition and then either suddenly increasing (case I) or decreasing (II). Consequently, the temperature/density dependence of the pion decay constant plays the most important role, which makes the decay width enhanced over some range in both cases but near the phase transition dropping suddenly to zero in case II.

In our approach the pion mass is almost stable in the chiral symmetry broken phase, then decreases steadily through the pseudogap phase to vanish at the transition to the symmetry restored phase, thereby realizing restoration via approach B, as above. On the other hand, both the effective pion decay constant f_π^* and the effective values for $(N_c e^2)^*$ decrease monotonically in the chiral symmetry broken phase but become stable in the pseudogap phase. If we do not take into account the change in $(N_c e^2)$, the decreasing pion decay constant alone would lead to an enhancement of the decay width (as in case II of [14]) in the low density regime where the chiral symmetry is still broken. Since $(N_c e^2)^2$ decreases faster than f_π^*, the decay width becomes decreasing in that region. This in-medium modification of $(N_c e^2)$ then is the key result distinguishing our result for the $\pi^0 \rightarrow \gamma\gamma$ decay width from others while realizing chiral symmetry restoration via decreasing pion mass. Figures 3 and 4 illustrate the degree of sensitivity of our results on the Skyrme parameters. Either way, we see in the two curves of Fig. 4 that qualitatively the decay width suppression is robust against such variations. This result smoothly matches to the results of [15] where the analogous process of generalized pion decay $\tilde{\pi}^0 \rightarrow \tilde{\gamma}\,\tilde{\gamma}$ is also suppressed in the color-flavor-locking phase which would take place at higher density above ρ_c.

IV. CONCLUSIONS

The main result of this work is given in Fig. 4 which shows suppression of the process $\pi^0 \rightarrow \gamma\gamma$ computed in a unified framework of the Skyrme model for pion fluctuations in a dense baryonic matter background. The in-medium dependences of the pion mass and decay constant arise in the same effective chiral theory as the anomaly which drives this process. The key new result emerging from this Skyrme approach is that $(N_c e^2)^*$ can also be medium dependent in the *hadronic phase* just as it does in the CFL phase [15].

We repeat that this study of $\pi^0 \rightarrow \gamma\gamma$ decay at finite density using Skyrmions is qualitative, due to the naivety in the crystal structure of the background baryons, the absence of a treatment of the Fermi statistics of the nuclear matter and the absence of loop contributions.

Our result would appear to be not so relevant to relativistic heavy ion collisions, for example, the observation of suppression of neutral pions created in central vs peripheral collisions as seen at PHENIX [24]. However, the scenario of low temperature and high density matter is particularly relevant to the context of compact stars. Applications of these results to this problem are underway.

ACKNOWLEDGMENTS

B-Y. P. is grateful for the hospitality of the Special Research Centre for the Subatomic Structure of Matter at the University of Adelaide, where part of this work has been done. A. C. K. is supported by the Australian Research Council. The authors are grateful to Professor B. Hong for discussions on heavy ion collisions.

[1] H.-J. Lee, B.-Y. Park, D.-P. Min, M. Rho, and V. Vento, Nucl. Phys. **A723**, 427 (2003).
[2] H.-J. Lee, B.-Y. Park, M. Rho, and V. Vento, Nucl. Phys. **A726**, 69 (2003).
[3] H.-J. Lee, B.-Y. Park, M. Rho, and V. Vento, Nucl. Phys. **A741**, 161 (2004).
[4] B.-Y. Park, M. Rho, and V. Vento, Nucl. Phys. **A736**, 129 (2004).
[5] I. Klebanov, Nucl. Phys. **B262**, 133 (1985); L. Castillejo, P. S. J. Jones, A. D. Jackson, J. J. M. Verbaarschot, and A. Jackson, Nucl. Phys. **A501**, 801 (1989); M. Kugler and S. Shtrikman, Phys. Lett. B **208**, 491 (1988); Phys. Rev. D **40**, 3421 (1989); A. D. Jackson, A. Wirzba, and L. Castillejo, Nucl. Phys. **A486**, 634 (1988); Phys. Lett. B **198**, 315 (1987); H. Forkel, A. D. Jackson, M. Rho, and C. Weiss, Nucl. Phys. **A504**, 818 (1989); D. Diakonov and A. D. Mirlin, Sov. J. Nucl. Phys. **47**, 21 (1988).
[6] For a review, see M. Prakash, J. M. Lattimer, R. F. Sawyer, and R. R. Volkas, Annu. Rev. Nucl. Part. Sci. **51**, 295 (2001).
[7] F. Arretche, A. A. Natale, and D. N. Voskresensky, Phys. Rev. C **68**, 035807 (2003).
[8] H. Itoyama and A. H. Muller, Nucl. Phys. **B218**, 349 (1983); T. Hashimoto, K. Hirose, T. Kanki, and O. Miyamura, Phys. Rev. D **37**, 3331 (1988); C. Contreras and M. Loewe, Z. Phys. C **40**, 253 (1988); A. A. Rawlinson, D. Jackson, and R. J. Crewther, Z. Phys. C **56**, 679 (1992); P. Z. Bi and J. Rafelski, Mod. Phys. Lett. A **7**, 2493 (1992); **7**, 3439(E) (1992).
[9] R. D. Pisarski, Phys. Rev. Lett. **76**, 3084 (1996).
[10] R. D. Pisarski, T. L. Trueman, and M. H. G. Tytgat, Phys. Rev. D **56**, 7077 (1997).
[11] R. F. Alvarez-Estrada, A. Dobado, and A. Gómez Nicola, Phys. Lett. B **324**, 345 (1994).

[12] A. Gómez Nicola and R. F. Alvarez-Estrada, Z. Phys. C **60**, 711 (1993).
[13] P. Costa, M. C. Ruivo, Yu. L. Kalinovsky, Phys. Lett. B **577**, 129 (2003); **581**, 274 (2004).
[14] H. Caldas, Phys. Rev. C **69**, 035204 (2004).
[15] M. A. Nowak, M. Rho, A. Wirzba, and I. Zahed, Phys. Lett. B **497**, 85 (2001).
[16] D. K. Hong, M. Rho, and I. Zahed, Phys. Lett. B **468**, 111 (1999).
[17] J. Ellis and J. Lanik, Phys. Lett. B **150**, 289 (1985); V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B165**, 67 (1980); H. Gomm, P. Jain, R. Johnson, and J. Schechter, Phys. Rev. D **33**, 3476 (1986); P. Jain, R. Johnson, and J. Schechter, Phys. Rev. D **35**, 2230 (1987); G. E. Brown and M. Rho, Phys. Rev. Lett. **66**, 2720 (1991).
[18] E. Witten, Nucl. Phys. **B223**, 422 (1983).
[19] See, for example, V. Vento, hep-ph/0401218.
[20] R. J. Furnstahl, H.-B. Tang, and S. D. Serot, Phys. Rev. C **52**, 1368 (1995); S. R. Beane and U. van Kolck, Phys. Lett. B **328**, 137 (1994); C. Song, G. E. Brown, D.-P. Min, and M. Rho, Phys. Rev. C **56**, 2244 (1997).
[21] A. D. Jackson and M. Rho, Phys. Rev. Lett. **51**, 751 (1983).
[22] G. Adkins, C. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).
[23] Oliver Schwindt and Niels R. Walet, hep-ph/0201203
[24] PHENIX Collaboration, K. Adcox *et al.*, Phys. Rev. Lett. **88**, 022301 (2002); **88**, 242301 (2002).