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CONTRIBUTION TO A DISCUSSION OF A PAPER ON INTERVAL ESTIMATION BY M.A. CREASY*

* Creasy, M.A. (1954) Limits for the ratio of means. *Journal of the Royal Statistical Society*, B, 16: 186-194.

The following contribution was received in writing after the meeting.

Professor Sir RONALD FISHER: I did not take part in the discussion in the Research Section on some supposed paradoxes propounded by Miss Creasy, partly because it is not entirely desirable for the President of the Society, as I then was, to take part in controversy, but principally because I could not understand the reasoning from which it was supposed the paradoxes arose.

Though I am still under the second of these disabilities, the importance of eliminating all avoidable confusion in inductive inference is such that I should now like to make a few points.

In Statistical Methods (Ex. 23.1) I illustrate the use of the t test to obtain fiducial limits for the ratio of the potencies of two drugs, supposing this ratio to be the same for each of a number of hospital patients.

Miss Creasy does not refer to this example, but to a number of my papers much more remote from the subject of her discussion. It is, I think, clear from the Example that no hypothesis is introduced as to the distribution of sensitivity among the patients, and no attempt is made to estimate any property of a hypothetical population from which the patients supplying the data might be supposed to have been drawn. Nobody has suggested that they were a random sample even of hospital patients.

In these circumstances the process that seemed appropriate, and of which I have received no criticism, was to find two lines through the origin, such that, for each "Student's" test gave a probability of 2½ per cent. of the observed mean being on the wrong side of the line. No fiducial distribution is mentioned.

Had we had the additional datum "The patients are drawn as a random sample from a population normally distributed in sensitivity" and no more information than this, a more searching test would have been possible, and therefore appropriate. We should have been given a normal bivariate sample, of which the statistics used before are now estimates of the variances and covariance of the errors of the co-ordinates of the estimated centre $(\bar{x}, \bar{y})$, namely

$$P = \frac{1}{N(N-1)} S(x-\bar{x})^2$$

$$Q = \frac{1}{N(N-1)} S(x-\bar{x})(y-\bar{y})$$

$$R = \frac{1}{N(N-1)} S(y-\bar{y})^2;$$

then recognizing, as a simple application of "Student's" test that, for any ratio $\lambda : \mu$,

$$\lambda(\xi-\bar{x}) + \mu(\eta-\bar{y}) > t\sqrt{\lambda^2 P + 2\lambda\mu\, Q + \mu^2 R}$$

with the one-sided probability appropriate to t for $N-1$ degrees of freedom, it would seem that the appropriate fiducial probability of the pair (ξ, η) lying in any elementary region must be

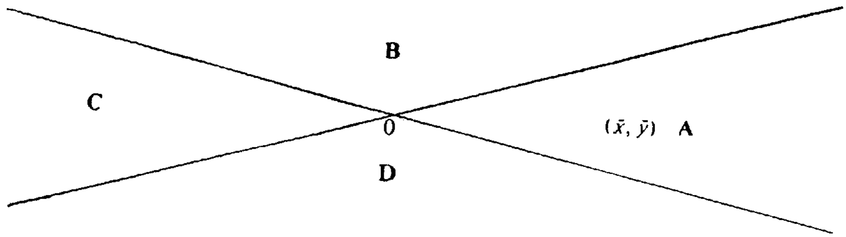
$$\frac{N-1}{2\pi\sqrt{PR-Q^2}} \cdot \frac{d\xi\, d\eta}{(1+r^2)^{\frac{1}{2}(N+1)}},$$

where r is defined by

$$(PR - Q^2)\,r^2 = R(\xi - \bar{x})^2 - 2Q(\xi - \bar{x})(\eta - \bar{y}) + P(\eta - \bar{y})^2,$$

which may be easily generalized to d dimensions, and be integrated over any restricted region to give the fiducial probability to be associated with it.

Were such a method applied to the four sectors made by the lines determined in Ex. 23.1, if the probability content of the sector containing the mean were **A**, and the others **B**, **C**, **D** in order,



the conditions these must fulfil would be

$$\mathbf{A} - \mathbf{C} = 1 - \alpha,$$

$$\mathbf{B} - \mathbf{D} = 0,$$

where α is the probability (e.g. 5 per cent.) measuring the level of significance.

It would be only a matter of somewhat troublesome integration, to satisfy alternative conditions, such as

$$\mathbf{A} = 1 - \alpha$$

or,

$$\mathbf{A} + \mathbf{C} = 1 - \alpha$$

in which the opposite segments are added.

The two approaches distinguished so sharply by Miss Creasy are, in this example, closely concordant. The example is, however, a useful reminder of the truth that in general a change in the data may be expected to lead to a change in the inferences.