The Influence of Textbooks on Mathematical Readiness for University Calculus —A Material Requirements Planning (MRP) Approach

Richard K. Burton

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Table of Contents

List of Tables

List of Figures

Abstract

There is a troubling trend regarding the competency of first-year university students in the subject of mathematics. Literature reflects a perspicuous disappointment in secondary school graduates' ability to perform simple mathematical tasks for employers or to succeed in upper-level mathematics courses in colleges/universities. The profoundness of inability is revealed, in part, by the proliferation of remedial mathematical courses at colleges/universities and the increase in elementary training programs implemented by employers so that employees can function at a minimum requirements level.

This research investigates, via survey data collection, descriptive statistics, initial factor models (using SPSS v20.0), and confirmatory factor analysis and Structured Equation Modelling (via Mplus v7.1) the interaction of possible contributing causes to ascertain whether the absence of mathematical maturity in secondary school graduates can be linked to the trend of mathematical textbooks having a content and presentation inconsistent with the entrance requirements of the college/university sectors. Specifically, this work focuses on the preparation of year-12 precalculus students to succeed in their first-year university calculus course by analysis of precalculus textbook content, trust, and use.

This study uses a mixed methods approach to integrate and optimize the affirming value of qualitative and quantitative data analysis. It uses a Likert scale questionnaire and demographic survey questions that included identification of the textbooks in use so they could be acquired and qualitatively analyzed through a rigor measuring tool developed as part of this project.

This study suggests the applicability of Material Requirements Planning (MRP) to the academic environment as a tool for restoring tertiary institutions as the proper drivers for secondary mathematical exit requirements and for guiding development of secondary

mathematics curricula, syllabi, and textbook content. MRP has at its core a focus on customer requirements for a particular product; when applied to the mathematical education of inbound tertiary mathematics students, MRP can identify tertiary institutions' minimum requirements of mathematical maturity as the customer requirements that secondary schools can then prioritize in their mathematics programs.

This research demonstrates the solid connection between mathematical maturity and textbook content in that textbook completion influenced year-12 student and teacher confidence in student readiness for year-13 calculus. This finding prompted creation of a unique tool for measuring the rigor of precalculus textbooks designed to prepare secondary students for their university elementary calculus course. *Rigor* is defined as the extent to which a textbook facilitates student mastery of relevant mathematical concepts while simultaneously preparing them to master ever more complex concepts. Accordingly, the rigor tool described in this study assesses the extent to which a precalculus textbook presents core calculus prerequisite topics recursively, employing a cohesive continuum of topic introduction, topic relevance, topic theory, and topic practice while maintaining a maturing connection to previous topics. Using the rigor tool, year-12 precalculus textbook rigor was qualitatively analyzed into measurable quantitative data from which a rigor score was derived for selected textbooks. The level of misalignment between the year-13 calculus prerequisites and the year-12 targeted outcomes was made apparent through this process.

Declaration Page

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint award of this degree.

I give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library Search and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

I acknowledge the support I have received for my research through the provision of an Australian Government Research Training Program Scholarship.

Signed:

Date: February 27, 2022

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Finally, to the Lord God of Heaven and Earth, in whom are all the treasures of wisdom and knowledge, my thanks, my praise, and my pledge to serve Him all the days of my life.

Chapter 1. Introduction

1.1. Project Overview: Aims and Purpose

There is a troubling trend regarding the competency of year-12[1] graduates in the subject of mathematics. Both the scholarly literature and popular media reflect disappointment in the preparedness of secondary school graduates to perform simple mathematical tasks for employers (Comrey & Lee, 1992, as cited in Flanagan, 2006, p. 203; Greene, 2000; Lester, 2008) or to succeed in upper-level mathematics courses in colleges/universities (Higgins et al., 2010; Van Rooij & Jansen, 2018). The lack of mathematical competency of incoming students has forced universities to provide remedial math courses to prepare students for upper-level courses; some have even begun offering credit toward baccalaureate completion for courses that used to be taught at the secondary school level (Bettinger & Long, 2009; Snead et al., 2021; Zachry Rutschow, 2019). The purpose of this study was to explore the instructional centrality of the year-12 precalculus textbook on year-12 student readiness for year-13 calculus and on student and teacher perception of student readiness, and to determine the extent to which the content of year-12 precalculus textbooks aligned with year-13 calculus requirements. The study describes the overarching architecture for data collection and processing that yields results that demonstrate a need for textbook reformation and the restoration of tertiary institutions as the proper drivers for secondary mathematical exit requirements. It introduces a unique tool based on Material Requirements Planning (MRP) principles for secondary school mathematics departments and teachers to use to assess the rigor of precalculus textbooks to better align their precalculus courses with year-13 calculus course requirements.

[1] Throughout this study, the terms year-12 and year-13 will be used to designate the last year of secondary school and first year of university respectively. Alternatively, the terms secondary and tertiary will be used.

1.2. Problem Scope, Cause, and Ramifications

There is widespread agreement in the literature among scholars and laypersons that year-13 college students in the United States and Australia are often ill-prepared for success in upper-level mathematics courses and that there is an academic misalignment between secondary exit standards and tertiary entrance standards in mathematics (Dos Reis et al., 2019; Gewertz, 2018a; Mather & Tadros, 2014; Thomson, 2016; VOA Student Union, 2018; Wesley, 2008). While there are a number of reasons why this is the case, this project focuses on investigating the mathematical maturity of secondary school graduates and looked for influences connected to mathematical textbooks.

1.2.1. Mathematical Maturity and Textbook Rigor

Research indicates that the primacy of rigor in the mathematics textbook facilitates not only the mastery of mathematics, but also the learning of thinking (Schmidt et al., 2004). For the purposes of this study, *rigor* is defined as the extent to which a textbook facilitates student mastery of relevant mathematical concepts while simultaneously preparing them to master ever more complex concepts (Raubenheimer et al., 2010). Rigor is multi-faceted—addressing conceptual, methodological, and analytic factors (Houston, 2019). The conceptual, methodological, and analytic facets, applied to mathematical rigor, address the inclusion of proofs for elementary and mature concepts. Their omission could be considered a degrading of rigor in the mathematics textbook (Smoryński, 2017). A rigorous precalculus textbook is one that presents core calculus prerequisite topics recursively, employing a cohesive continuum of topic introduction, topic relevance, topic theory, and topic practice while maintaining a maturing connection to previous topics. A rigorous textbook will teach students to think mathematically (Mun & Hertzog, 2018) and mature them mathematically for subsequent coursework (Faulkner et al., 2019). The uninterrupted, sequential format of the rigorous textbook, along with a comprehensive example, question, and review question bank, provide the necessary practice for the student in the area of mathematical sophistication and intellectual maturity (Heyneman, 2002; Kindle & Gentimis, 2018; Shield & Dole, 2013; Valverde, 2002). In the absence of this maturity, the student will fail to practice rational thinking that is foundational for any advanced study, particularly in the mathematical sciences (Rezat, 2009; Valverde, 2002).

The primary function of a mathematics textbook is to facilitate student mastery of mathematical concepts that has been suggested can only be grasped through a gradual, systematic process (O'Keeffe & O'Donoghue, 2010; Sherman et al., 2020). Knowledge acquisition is “a complex, gradual process requiring both time and effort” (Wood & Kardash, 2002, p. 250); it stands in stark contrast to technological innovation which is characterized by rapid change. The “Technology Progress” index (TPI) shows a doubling rate of 1.5 years (Roser & Ritchie, 2013). The speed of change of technology has posed a conflict for textbook writers/publishers who find themselves trying to simultaneously cement foundational mathematical concepts that never change while introducing students to the latest technological advances such as calculators, computers, and computer algebra systems (Fitriati, 2019; Howard, 2013). Educators and publishers understand that the exponential growth of information availability (Alhumaid, 2019; Hull, 2003) and information itself in the rapidly changing digital culture has established the need for the educated individual to be conceptually savvy with the underpinnings of the technology that now thread through most facets of both academic and non-academic environments (Hilbert, 2015). They also understand, from the degradation of mathematics and reading skills that has accompanied the infusion of technology, that students cannot acquire and master knowledge as rapidly as technology changes (Alhumaid, 2019; Zheng, 1998).

There are hundreds of studies in the pedagogy of the mathematics curriculum and the mathematics textbook and there are competing ideologies contained in these studies; however, by and large educators have continued to insert technology into their mathematics curricula and publishers have followed suit by incorporating it into textbooks. In so doing, they have precipitously dropped topics from the mathematical textbook (Gu, 2010). Additionally, while publishers have felt constrained to update textbooks to accommodate the various key stroke nuances of the latest electronic calculator versions, they have also felt compelled to appeal to the entertainment-oriented *zeitgeist* that prevails today. Consequently, mathematical textbooks:

- have increased in size (Durkin et al., 2021; Gordon & Gordon, 2018)
- have omitted core topics (Duffrin, 2005)
- have computational emphasis with machine-specific calculator or computer instructions and exercises (Brown et al., 2007; Frank & Thompson, 2021)
- have made cultural accommodation with visual effects (i.e., have included cartoons and colorful images) (Baker & Gilbey, 2016)

The result of these changes is that many modern mathematical textbooks lack topic rigor (Duffrin, 2005) and are regarded by some as shallow—containing cursory topic coverage and minimal conceptual reinforcement. (Brown & LaVine Brown, 2007; Duffrin, 2005; Gu, 2010; Schmidt et al., 1997). Chapter 7 will describe the concept of textbook “rigor” in detail.

1.2.2. Universities Forced to Address Declining Mathematical Maturity

Ever since “new math” was introduced in the United States and other countries in the 1960s, there has been an explicit recognition of the declining mathematical proficiency of secondary students among academicians (Chen, 2016; CUPM Panel, 1987; Ngo & Kwon, 2015; Zachry Rutschow, 2019). An increase in remedial education provided at universities is the least ambiguous indicator that the secondary student is less prepared than required. (See Chapter 2, Section 2.2. and Chapter 4, Section 4.3.2 for details.) Thus, universities must make efforts (through remedial course work) to bring the student to a level of maturity and sophistication such that the student will be able to complete the academic program.

1.3. Using a Data-Driven Approach to Address the Cause of the Problem

A proliferation of experimentation and publication by academicians eager to understand and solve the vexing problem of student mathematical readiness deficiency has yielded little agreement as to cause or solutions (Islam & Rouse, 2021; Melguizo & Ngo, 2020; Van den Broeck et al., 2020). See Chapter 2, sections 2.3 and 2.4 for details. A key factor underlying the plethora of causes and solutions is that the academic community has yet to develop and implement solutions based on a truly *data-driven* model of probable causality. A data-driven model holds out the very plausible outcome of minimum bias due to the questioning strategy invoked in the data-driven model scenarios (Anfara & Donhost, 2010). Chapter 3 provides details on the data-driven approach used to conduct this study.

1.4. Putting Universities Back in the (Requirements) Driver's Seat

As the host for year-13 courses, it is imperative that universities drive strategic requirements for university student preparedness. The overarching aim of this research was not to assign blame but rather to identify strongly correlated (possibly causal) components that can be addressed, diagnosed, and repaired. The constructs identified by this research suggest a path to align tertiary entrance requirements with secondary exit standards. Additionally, this research led to the development of a rigor measurement tool that secondary schools can use to assess alignment between entrance and exit requirements of precalculus and calculus courses to help them select the optimal textbooks for preparing secondary school students for their tertiary mathematics courses. Selective procurement of rigorous texts could, in turn, drive the requirement for textbook writers to tailor textbooks to incorporate a consistent minimum standard of rigor that satisfies both secondary school exit standards and university entrance standards.

1.4.1. Universities as Customers—Material Requirements Planning (MRP) as an Alignment Tool

The manufacturing sector offers a relevant model for aligning secondary school exit standards with tertiary school entrance standards. In the business world, customers establish requirements for products, and manufacturers strive to develop, produce, and deliver those products consumer-ready, at the agreed time, and with the advertised quantity and quality. The MRP process includes:

- Creating a <u>Bill of Materials (BOM)</u>. The BOM is a list of the raw materials, sub-assemblies, intermediate assemblies, sub-components, parts, and the quantities of each needed to manufacture an end product.
- Developing a <u>Work Breakdown Structure (WBS)</u>. The WBS is a deliverable-oriented, hierarchical decomposition of the work to be executed by the project team to accomplish the project objectives and create the required deliverables. The WBS organizes and defines the total scope of the project. Each descending level

represents an increasingly detailed definition of the project work. The deliverable orientation of the hierarchy includes both internal and external deliverables.[2]

- Determining a Critical Path by identifying the longest stretch of dependent activities and measuring the time required to complete them from start to finish. So, the critical path is the, timewise, incompressible schedule.
- Establishing a Production Schedule by using Backward Scheduling[3]—working backwards from the due date (or time) to the start date (or time) in order to compute the materials and time required at every operation or stage of production.
- Compiling a Master Schedule (summary-level project schedule) that identifies the major deliverables and work breakdown structure components and key schedule milestones[4].
- Calculating Total Float. The amount of time that a scheduled activity can be delayed (less than the critical path) or extended without delaying the scheduled finish date (Wong, 1964; Tang, 2003). When the total float is exceeded, the schedule encounters a crash and producers must implement remedial contingencies (such as omitting processes) so that the delivery date is not compromised. Alternatively, the producer can slip the delivery date (Brooks, 1975) or arrange for requirements (such as software updates) to be incorporated into products after delivery.

Applying the MRP process to the alignment of secondary school mathematical standards and university mathematical requirements provides a novel framework for measuring secondary

[2] Work Breakdown Structure (WBS) as defined in Georgia Technology Authority, Glossary of Terms and Definitions Supporting Policies, Standards and Guidelines for Information Technology and Information Security.
[3] This definition of backward scheduling was retrieved from: http://www.businessdictionary.com/definition/backwards-scheduling.html.
[4] Master Schedule as defined in Georgia Technology Authority, Glossary of Terms and Definitions Supporting Policies, Standards and Guidelines for Information Technology and Information Security. https://gta-psg.georgia.gov/glossary-terms.

school effectiveness and determining corrective actions. In this construct, the university is the customer, and the required product is a student who is optimally prepared for tertiary level mathematics courses. Success in year-13 mathematical studies presupposes adequate preparation in year 12 which presupposes adequate preparation in year 11, and so on. The university provides high level requirements that can form the basis for creating a BOM that identifies prerequisite knowledge and pre-prerequisite courses that must occur in a chronologically succinct order—i.e., the Master Schedule. (For the purposes of this study, the general content of year-13 calculus textbooks became the driver for identifying the prerequisite preparation of year-12 precalculus students. This will be described in further detail in Chapter 7.) As the "customer," the university dictates the orderly deliverable requirements by implementing the backward scheduling practice for timely deliverable product minimal requirements—that is; the incoming year-13 student trained, matured, and ready for tertiary work. Although the university is focused on the incoming (year-13) student and, thus, on student accomplishment in year 12, its hardened requirements will necessarily also drive the downstream secondary prerequisite content of year 10 and 11 in the secondary environment. Thus, not only will the university need its own backward schedule for timely and high-quality baccalaureate completion, but the secondary school will also need the comparable backward scheduling strategy for the timely and high-quality secondary diploma. The university, as the customer, dictates the deliverable requirements and that is the backward scheduling practice when it places time and requirements limitations on the program both for entry requirements (the secondary school graduate) and exit requirements (the bachelor's degree).

In the MRP process, allowances need to be made for corrective actions; however, this presupposes that there is time in the schedule to do so. Thus, producers must calculate allowable slippage into the Master Schedule. Once all the allowable slippage has been calculated and subtracted from the schedule, what remains is the Critical Path—any further slippage directly affects deliverable timing and quality. Time that is built in for necessary slippage (holidays, sick-days, unplanned events etc.) constitute the Total Float in the backward schedule, so float is the maximum allowable schedule slippage. With time and requirements limitations, the backward scheduling model is forced to degrade the end product when the float is consumed. This degradation is a significant downward step in the quality of the degree and the quality of the end product (as evidenced in the literature).

Figure 1.1 illustrates the application of the MRP process to the production of optimally prepared year-12 graduates for tertiary mathematical studies. It depicts how the secondary school can calculate float in the master schedule and can calculate the critical path (from Start to Drop Dead Delivery) with a recognition that if float is over-consumed, a crash will occur, and topics will have to be omitted. In other words, the corrective action is facilitated by the MRP strategy.

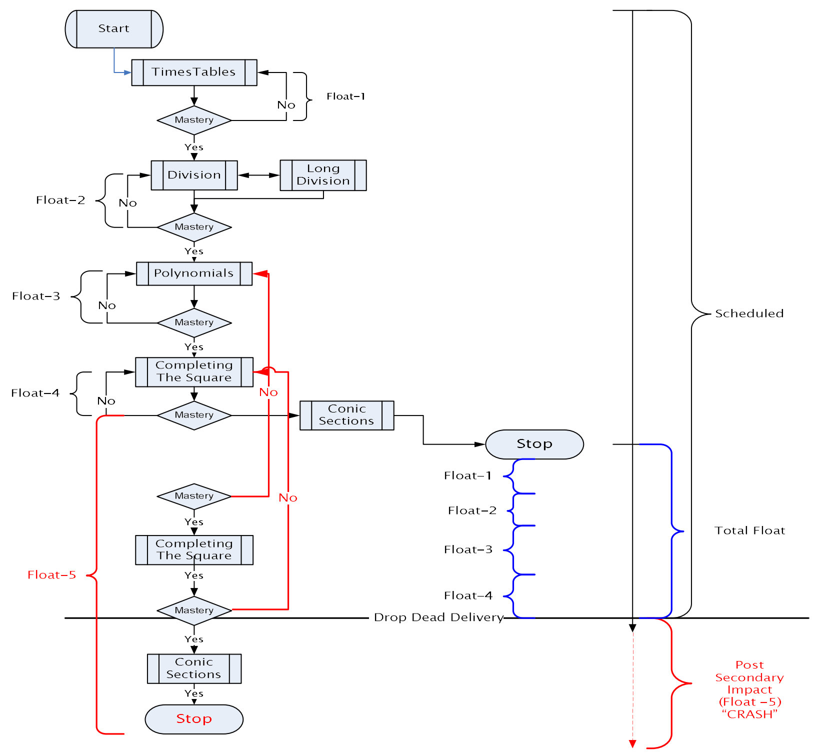

Figure 1.1 Application of MRP Process to Preparation of Year-12 Mathematics Students

With correlation and causal constructs confirming a positive relationship between secondary mathematical textbook rigor and tertiary mathematical success, the university can re-acquire control of the incoming cohort of students by responsibly and analytically providing guidance and requirements to the secondary schools and testing agencies. Additionally, the MRP approach can provide tools to sensibly merge and manage tertiary entrance requirements with secondary exit standards. As an example, Chapters 7 and 8 describe a rigor algorithm based on MRP principles that could be used to assess the rigor of precalculus textbooks being used to prepare students for their year-13 calculus courses.

1.5. Facilitating Mathematical Maturity

This research project led to the development of a rigor algorithm for math educators to use to select the most desirable text which will, in turn, drive the requirements for textbook writers to address the findings and better tailor the textbooks to both the secondary and the tertiary requirements. Merisotis and Phipps (2000) identified four overarching areas in programs that are underway to reduce the need for remediation in higher education. The first area they identify is the requirement for alignment between secondary exit requirements and tertiary entrance competencies. The gap in the alignment between secondary exit standards and tertiary entrance requirements is student maturity—both topic and intellectual. This misalignment is an indication that a rigorous, sequential flow of information and experience that constantly links the current topic with mastered prior experiences to facilitate the appropriation of more complex ideas is missing from many current mathematics textbooks. Maturity is the ability to continually synthesize and create new ideas by the effective use of old or mastered ones. For instance, the ability to grasp simple abstract concepts leads to the ability to master more complex abstractions. For the purposes of this study, the recursive approach to maturing mastery is the Core Cohesive Continuum (CCC).

1.5.1. Core Cohesive Continuum (CCC) Described

As mentioned earlier, as a naturally recursive discipline, mathematics is a good example of truly recursive concept acquisition; this can be illustrated via a concept map. Building on the work of Ausubel (1968) and Toulmin (1972), Novak and Gowin (1984), defined "concept" as "perceived regularity or patterns in events or objects, or records of events or objects, designated by a label" (Cañas & Novak, 2009, para. 2) and developed concept maps to graphically depict knowledge acquisition. In the diagram sequence that follows based on their work, the concept of "Bird" is never disenfranchised from the maturing understanding of the student studying the bird object. See Figures 1.2-1.5.

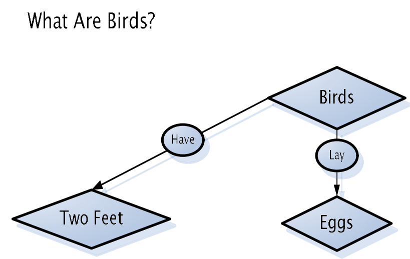

Figure 1.2 Elementary Identification of Bird Characteristics

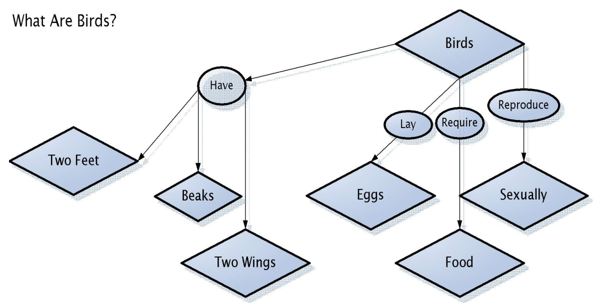

Figure 1.3 Primary Identification of Bird Characteristics

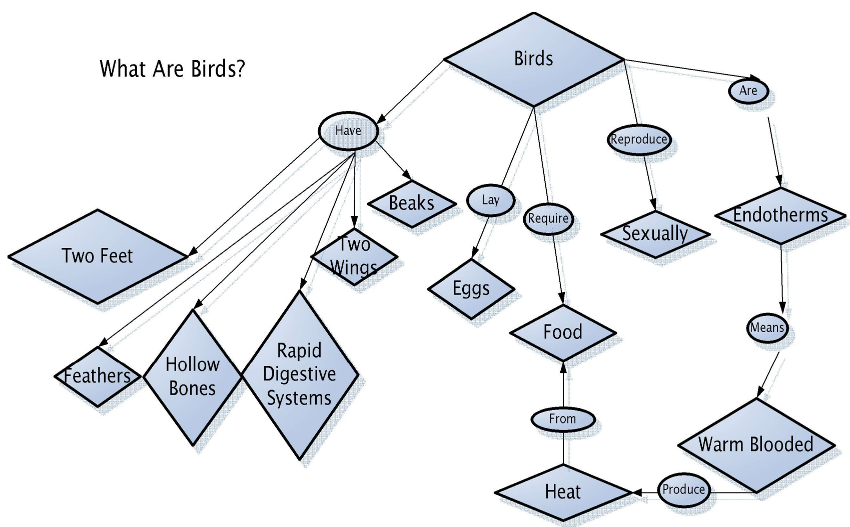

Figure 1.4 Secondary Identification of Bird Characteristics

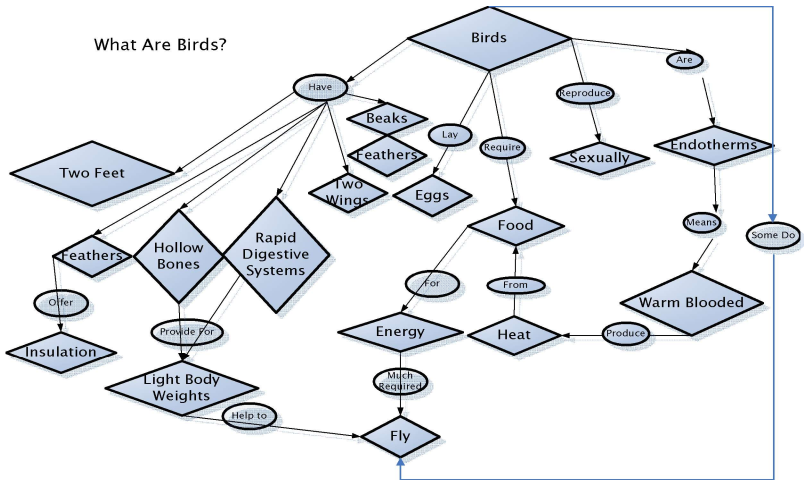

Figure 1.5 Conceptually Mature (Tertiary) Identification of Bird Characteristics

Note that, as subject maturity develops, the proof of the concept is hardened, and the acquisition of the concept leads to intellectual corollaries that are all logically interconnected.

Flavell (Flavell et al., 2002) defines a concept as a “mental grouping of different entities into a single category on the basis of some underlying similarity”—some way in which all the entities are alike, some common core that makes them all, in some sense, the same thing. The label for most concepts is a single word, although sometimes symbols such as + or % are used, and sometimes more than one word is used.

Concepts, like mathematics are critically recursive in that, no matter on what branch of the map a person is located, the clarity of the original notion remains perspicuous, and its maturing understanding is always connected to the root; thus, the textbook should incorporate root concepts and previous topics into the next topic (Ausubel, 2000; Shin & Bryant, 2015).

This dependent inter-connectedness of concepts closely tracks with the mathematics textbook as it defines, hardens, rehearses, builds, defines and so on (Valverde & Houang, 2002). Not only interconnected operationally but interconnected with two-way (more maturity or less maturity) relevance. Mathematical relevance is critical. It must be maintained by a continuum of more mature concepts built on (not supplanting destructively) a less mature (but simple and solid) foundation (Balmaceda, 2020; Nathan & Walkington, 2017; Piesch et al., 2020). Note that current maturity sits on a foundation that is a recursive aggregation of earlier maturities. Thus, it is critical that a continuum be maintained as a concept matures so that the cement of the foundation is properly set.

1.6. Research Schema

1.6.1. Origin

In 2012-2013, while relief teaching grade 12 Specialist Mathematics (secondary school calculus) at a college in South Australia, this researcher was presenting the Definite Integral to approximately 15 students and was surprised to learn that they had never been exposed to polynomial long division—a building block for the concept of factoring (reduction of operational complexity) and something they should have learned in grade 10 or grade 11. Thinking that, perhaps, they had simply forgotten this topic, follow up was made with teachers and a search in grade 10/11 textbooks was conducted which revealed that the topic was given minimal attention. This incident brought to mind a previous experience serving as a year 12 Specialist Mathematics teacher at another South Australian college in 2010 where the textbook contained trivial, rather than rigorous, treatment of several topics, including limits and quadratic iterations. These two encounters helped shape this study to determine the reasons behind degraded mathematical maturity and readiness for year-13 calculus.

1.6.2. Research Questions

The general research question addressed in this thesis is

Is there a rigor level change in year-12 precalculus textbooks' concept presentation that is negatively impacting students' mathematical maturity and preparedness for their year-13 calculus course?

In order to answer this question, the centrality of the mathematics textbook in the delivery of the year-12 curriculum needed to be measured as did the content of precalculus textbooks to include instruction format, appropriate topics, presentation order, reinforcement of topics (commitment to interleaving), and attention to a cohesive flow of maturing mathematical concepts consistent with year-13 calculus prerequisites. The specific research questions were:

1. Is the textbook central to instruction and what is the relationship between factors such as year-12 student use, trust, and like of their year-12 textbook and year-13 student like and confidence in their year-12 textbook and like and use of their year-13 calculus textbook?
2. What is the relationship of the textbook to factors relating to secondary student and teacher perception of student readiness and to students' actual mathematical maturity?
3. Can MRP-derived year-13 calculus prerequisites measure rigor adequacy of year-12 precalculus textbooks?

Question 1 was addressed through survey questions targeting:

- Textbook Centrality (trust, like, and use) to:
  - Year-12 students
  - Year-12 teachers
  - Year-13 students (relating to their year-12 textbook)

Question 2 was addressed through survey questions targeting:

- Student Readiness

- Year-12 student perceived readiness
- Year-12 teacher perception of student readiness for year-13 calculus
- Actual year-13 student readiness from perspective of university professors

Question 3 was addressed through development of an MRP-derived rigor algorithm targeting:

- Textbook content (measuring the textbook rigor and adequacy)
  - Textbook content alignment with year-13 calculus prerequisites
  - Maintenance of a cohesive continuum
  - Recursive concept hardening

An underlying hypothesis was that if the secondary mathematics textbook is central, trustworthy, liked, and useful to the secondary student and teacher, then the textbook (as the curriculum delivery device) could become the measure for successful completion of the curriculum and the basis for secondary student and teacher confidence in the student's mathematical preparation for year-13 university calculus.

1.6.3. Theoretical Significance

If the research concludes that there is a positive connection between the precalculus textbook rigor and student mathematical maturity and readiness, this should elevate awareness of the importance of textbook rigor over other competing theories about poor student readiness and should drive efforts to address the content and sequencing of content within precalculus textbooks.

1.6.4. Practical Significance

If the research concludes that there is a positive connection between the precalculus textbook rigor and student mathematical maturity and readiness, the rigor algorithm proposed in Chapter 7 will provide a method that administrators, teachers, and publishers can apply to determine the rigor of textbooks and whether additional resources need to be used to augment or modify the textbook presentation.

1.6.5. Methodological Significance

Finally, if the research concludes that there is a positive connection between the precalculus textbook rigor and student mathematical maturity and readiness, the rigor algorithm proposed will demonstrate the novel approach of applying Material Requirements Planning (MRP) techniques to solve the problem. College/university mathematics departments will be recognized as one of the "customers" of secondary mathematics departments who are seeking a "product" designed as they want it (in this case mathematically mature and prepared students), and who are prioritized as the appropriate drivers of standards for minimum content and proper sequencing of prerequisites in the secondary mathematics curriculum/textbooks.

1.7. Research Strategy

Student and professor/teacher surveys, detailed in Chapter 3, sought to investigate the centrality of the textbook in the year-12 precalculus experience. Centrality was measured in several different ways through questions asked of students and their teachers/professors. SPSS v20.0 was implemented for descriptive statistics and exploratory factor analysis (EFA) and Mplus v7.1 was used for confirmatory factor analysis (CFA) and the secondary and tertiary student Structured Equation models (SEM). Descriptive statistics were integrated with the models to provide further clarity of the year-12 textbook centrality detailed in Chapters 5 and 6. When textbook centrality was affirmed, alignment of year-12 textbook outcomes and year-13 calculus prerequisites were measured with an MRP-based strategy detailed in Chapters 7 and 8.

1.8. Origin of the Rigor Algorithm

After measuring and testing the centrality, acceptance, and trust of the year-12 precalculus textbook to affirm its centrality to student readiness for year-13 calculus, a rigor algorithm inspired by MRP principles of backward scheduling was developed by compiling the preponderance of topics contained in commonly used college/university calculus texts and synthesizing those into an empirical tool that can be applied to any precalculus textbook to determine whether minimum calculus prerequisites are effectively met.

1.9. Project Overview

The thesis is sectioned into nine chapters. Chapter 1 identifies the problem, potential likely causes, and the research questions and methodology used to address the problem. Chapter 2 provides an overview of relevant scholarly literature on the problem and exposes gaps in that scholarship that this research addresses with an MRP construct. Chapter 3, Research Methodology and Survey Instrument Design, describes the research methodology and the development of the survey instruments used to collect the data that enabled analysis of the relationship between the rigor of secondary precalculus textbooks and the preparedness of year-12 graduates for their year-13 calculus course. This chapter explains how data-driven approaches and MRP backward scheduling can be employed to restore universities to their proper role as drivers of entrance requirements and, thus of mathematical textbook content. Chapter 4, Sampling Justification, explains the methodology used to validate that the experimental samples used for this study are representative of the universal population. Chapter 5 details the process used to do data screening and cleansing of year-12 and year-13 student survey responses in preparation for exploratory and confirmatory factor analyses that, in turn, prepared the data for structured equation modeling (SEM). Chapter 6 details the process used to derive and create a SEM that demonstrated a correlation between textbook rigor and student mathematical preparedness for course work. It also shows how the qualitative data and quantitative data were integrated to further reinforce this correlation. Chapter 7 describes the development of a unique textbook rigor tool that publishers and schools can use to determine the extent to which a textbook will or will not contribute to student mathematical readiness and demonstrates its use. Chapter 8 describes the application of the rigor tool to a sample set of 19 U.S. and Australian precalculus textbooks published over a span of six decades (1965 to 2012). Chapter 9 summarizes the results of the research.

Chapter 2. Literature Review

2.1. Introduction

This chapter will provide an overview of literature relating to the central problem addressed in this project and its proposed solution. Section 2.2 summarizes the literature addressing the question: why are year-13 university students in the United States and Australia often ill-prepared for success in upper-level mathematics courses? Section 2.3 focuses on scholarship that presents a continuum of historic events describing how and why secondary mathematics curricula has undergone so many changes since the early 1900s. Section 2.4 highlights the literature addressing the degradation of mathematics textbooks over time—the key focus of this study. Section 2.5 summarizes scholarship relating to MRP and explains how it inspired a novel strategy to assess secondary mathematical textbook content.

2.2. Ill-prepared High School Graduates

2.2.1. U.S. Graduates Ill-Prepared for Higher Education

Since the function of the secondary school is to prepare graduates for tertiary education or for employment, evaluation of incoming students to each of these sectors should reveal whether educational strategies have been successful. It is clear from the literature that serious shortcomings have been evident for some time. Universities and employers have observed a lack of mathematical readiness in the typical secondary school graduate (Cogan et al., 2001; Corbishley & Truxaw, 2010; Greene, 2000). Recognition of academic competency shortcomings is available empirically from tertiary institutions in the form of test scores, first-year university attrition data and, notably, the proliferation of remedial university course offerings for first-year students (Bettinger & Long, 2009; Cipra, 1988; Froese, 2019; Hieb et al., 2015). Almost seventy years ago, Williams (1954) reported that 17% of U.S. universities offered remedial mathematics courses. Bettinger and Long (2009) found that 98% of community (two-year colleges) and 80% of public four-year colleges were providing remedial courses in 2006, many of which were integrated into curricula whereas in years past, remedial course work had not be awarded credits. Boser and Burd (2009) noted that 99% of public two-year colleges were offering remedial courses and cited an alarming study that found that only 34% of graduating students actually met minimum qualifications for admission to a four-year postsecondary institution. A more recent study reported that almost 70% of incoming students at two-year colleges and 40% of incoming students at four-year colleges were enrolled in remedial classes. Of those in two-year colleges, 59% were taking mathematics remediation (including some multi-year courses) while 33% of the students in remedial programs at four-year universities were enrolled in remedial math courses (Zachry Rutschow, 2019). While the tenfold increase in student enrollment in higher education between 1954 and 2019 (National Center for Education Statistics, 2020a, 2020b) explains some of this increase in remedial course offerings, it does not explain all of it. The irony of the statistics about remedial course offerings is the high level of confidence secondary school graduates have in their readiness for university studies. In one study, more than 80% of students enrolled in remedial courses in universities were surprised by their lack of preparation (Boser & Burd, 2009; Zachry Rutschow, 2019). Secondary schools had prepared students in accordance with secondary curriculum and standards but some of the skills and mastery required to succeed in university courses, such as good cognitive strategies, discipline, thinking skills, foundational concept understanding, and topic mastery, “are not an explicit part of the high school curriculum” (Van Rooij & Jansen, 2018).

2.2.2. Australian Graduates Ill-Prepared for Higher Education

Scholars have noted a similar problem with lack of readiness of secondary students for higher education in Australia. In 2003, Cuthbert and MacGillvray identified a decades-long decline in foundational mathematical skills in Australia that they attributed to “a widespread lack of understanding of the pivotal and underpinning roles of specific and generic mathematical skills, the time necessary for their development, the need to provide nurturing across the full spectrum of mathematical capabilities, and the interdependence of mathematics and technology” (Cuthbert & MacGillivray, 2003, p. 360). Higgins et al., (2010) identified “a significant drop in the abilities of students entering first year mathematics at James Cook University” over the previous 20 years (pp. C641-C642). Galligan and Hobohm (2015) cited the “maths crisis” identified by the Australian Mathematical Sciences Institute in a 2012 report. They described lack of required skills, few graduates, and fewer incoming students with adequate preparation such that the universities were faced with two choices: remedial programs or lowering of the standards. Snead et al., (2021) reported that, in order to keep students enrolled, some mathematics programs in Australia have been redesigned to make the first-year mathematics classes easier to pass—that is, the standard has been lowered. Scholars agree on the worldwide nature of the problem of ill-prepared students for tertiary mathematics (Croft et al., 2009; Higgins et al.; Snead et al.; Van Rooij & Jansen). There is a well-documented mismatch between what is being taught in secondary curricula versus what is needed for success in tertiary mathematics courses (Clark & Lovric, 2009; CUPM Panel, 1987; Hourigan & O'Donoghue, 2007; Nortvedt & Siqveland, 2019).

2.2.3. U.S. and Australian Graduates Ill-Prepared for Employment

The business sector has also consistently highlighted competency shortcomings in the apprentice/entry level work-force market, further reflecting that secondary exit skills fall woefully short of requirements. Flanagan (2006) states that "More than 60 percent of employers report that high school graduates have poor math skills" (p. 12). The costs incurred by businesses and institutions of higher learning to address deficient academic skills was estimated at $601 million per year for Michigan alone in 2000 and at approximately $16.6 billion per year for the United States as a whole (Greene, 2000). Greene added that "in addition to these monetary costs, the human costs are incalculable" (p. 1). Bertonneau (2000) explained this human cost as the "human tragedy" of students' lack of knowledge of grammar and their poor vocabulary skills. He blamed the "unchallenging textbooks" of their high school curriculum, noting that textbooks train students in subjective responses; that is, they do not reinforce the practice of using intellect to solve problems (p. 21). When confronted with societal problems, students' lack of intellectual training results in emotional responses only. They have a general inability for rational thought. Bertonneau concluded that educational failure had "cheated" thousands of students and was scandalous (p. 27).

The U.S. government took notice of the downward trend in high school student readiness in mathematics and science, appointing a special commission headed by Senator John Glenn in 1999 to study the problem. In its report, titled "Before It's Too Late," the commission noted that "it is abundantly clear from the evidence already at hand that we are not doing the job that we should do—or can do—in teaching our children to understand and use ideas from these fields. Our children are falling behind; they are simply not 'world-class learners' when it comes to mathematics and science" (National Commission on Mathematics and Science

Teaching for the 21st Century, 2000, p. 4). The commission reported that the Third International Mathematics and Science Study (TIMSS) had tested the students of 41 nations and that, while U.S. children were "among the leaders in the fourth-grade assessment," by the time they graduated from high school "they were almost last" (NCMST, 2000, p. 4). Commission members recognized that a century earlier the nation's schools had risen to the challenge of educating students to meet the demands of an industrializing economy but cited a "scary" Midwest think tank statistic that "60% of all new jobs in the early 21st century will require skills that are possessed by only 20% of the current workforce" (NCMST, 2000, p. 6 & 13).

The negative trends identified by Glenn's commission continued. In 2005, assessments of students across the world indicated that U.S. students still lagged their peers in other industrialized nations in mathematics scores (Gropp, 2005). Five years later, a report from the Corporate Voices for Working Families highlighted the continuing disconnect between employers' need for qualified workers and the deficient skills of high school graduates. The report noted that "employers in need of better prepared workers and educational systems that fail to produce an adequate supply of skilled graduates have been on a collision course, creating a growing skills gap in the marketplace" (CVWF, 2010, p. 3). As recently as 2018, Gewertz (2018b) noted that employers are saying that high school graduates have not been adequately trained to read, write, or have conversational skills.

In Australia, Thomas et al., (2009) linked the shortage of mathematics teachers in the country to a sharp decline in enrollment in mathematical studies and had observed that the lack of mathematicians was impacting industry such that Eastern European and Asian mathematicians were being invited to immigrate to Australia to fill the gap. Ten years later, Wilkie and Tan

(2019) described continued declining enrollment in year-11 and year-12 advanced mathematics courses leading to mathematically-intensive higher level studies (university mathematics) that was depleting the employment sector of qualified Science, Technology, Engineering, and Mathematics (STEM) graduates and also impacting the quantity and quality of students seeking to become mathematics teachers.

2.2.4. Defining Readiness

There is no "industry standard" definition for university readiness. McCormick and Lucas (2011) note that there are several perspectives on how to measure and define it including, but not limited to: "high school courses completed, high school grade point average, mathematics content and procedural standards, scores on national tests and college placement exams, and success in first year college courses" (p. 5). Van Rooij and Jansen (2018) point out that the definition of readiness often differs between secondary school teachers and university professors with the former believing that graduating from secondary school implies readiness for college whereas the university professor defines readiness as students who have mastered the content taught in high school and who "possess sufficient learning skills, such as an ability to deal with large amounts of content" (p. 10). Conley's statement that "college readiness can be defined operationally as the level of preparation a student needs in order to enroll and succeed—without remediation—in a credit-bearing general education course at a postsecondary institution that offers a baccalaureate degree or transfer to a baccalaureate program" offers a helpful summary of perspectives (Conley, 2007, p. 5). Conley's definition, in the presence of data identifying inadequacies in readiness, has led the educational community to infer that secondary rigor and/or testing is suspect, and/or that the university standards are mismatched with the incoming cohort skill level, and/or that university entrance qualifications fall short of predicting the level of mastery necessary for student university success. This coupled with universities lowering mathematical standards for engineering and science studies is evidence that university preparedness in mathematics is degrading with solutions still elusive (Prince, 2016; Zietara, 2016).

While definitions of readiness have differed, scholarly literature about the proliferation of remedial courses indicates that there is a serious disconnect between secondary school teachers and university professors on what constitutes readiness. Much weighting of university readiness is judged by the scores on institutional placement exams or, in the case of the U.S., on nationally recognized college entrance exams like the American College Test (ACT) (Kaye et al., 2006) . Recognizing the deficiencies in student performance, many U.S. states sought to align their high school exit requirements with university prerequisites (Boser & Burd, 2009). Yet the 2015 ACT Key Findings Report showed that just 31% of students who took the test met any of the four key benchmarks (English, Reading, Mathematics, Science). It reported a downward trend for mathematics achievement between 2011 and 2015, with only 42% of students meeting the mathematics benchmark standard (ACT, 2015). More concerning was that even in the rarified classroom climate of STEM students, only 53% met the ACT mathematical benchmark. Just 3 years later, *Education Week* reported that ACT scores had dipped to a 20-year low where only 4 of 10 scored well enough to meet the mathematics benchmark (Gewertz, 2018a).

2.3. Fighting Over Solutions—The "Math Wars"

The problem of ill-prepared secondary school graduates is an old one and there has been no consensus as to the cause. Many secondary mathematics curriculum changes have been implemented over the decades and battles to decide "how much" and "when" were gathering inertia as early as 1915 (Klein, 2003). It was during these initial controversies that the idea that lasting and valuable knowledge is supported by self-discovery was pressed into the development of the mathematics curriculum by the educational community. Osborne and Crosswhite reported that, by 1920, battle lines were drawn between the mathematical community and educational elites over control of secondary mathematical requirements. These battle lines pitted mathematicians promoting traditional teaching practices against mainstream progressive educators promoting student-centric, discovery forms of learning (Klein, 2003). When Professor William Heard Kilpatrick of the Teachers College at Columbia University was asked in 1915 to chair the National Education Association's commission to study problems associated with teaching mathematics in the secondary schools, there were no mathematicians on his committee. Kilpatrick considered mathematics education "harmful" to thinking normally in society (Kilpatrick, quoted in, Klein, 2003, p. 7). His committee's report, titled *The Problem of Mathematics in Secondary Education*, recommended that nothing in the mathematics curriculum be included in schools unless a "probable value" could be attached and, even then, only for selected students. This report met with resistance from the mathematical community. In 1916, the Mathematical Association of America (MAA) appointed its own National Committee on Mathematical Readiness as a counter to the NEA's commission. The committee's report, completed in 1921, included vigorous opposition to the Kilpatrick report (Bureau of Education, 1922; National Committee on Mathematical Requirements, 1923).

The battle for leadership of steering secondary mathematics curriculums that Klein labelled, "Math Wars," was lost by the mathematicians when, in 1999, amid the protests of 200 university mathematicians, Fields Medal winners, Nobel Laureates and math department chairs, the U.S. Department of Education recommended to 15,000 school districts a list of mathematics textbook to be considered for adoption in the secondary schools that had, in the opinion of the mathematicians, “radically diminished content and a dearth of basic skills” (Klein, p. 3).

Despite pushback from mathematicians, the educational community continued to gain ground in controlling the mathematics curriculum. Educational theorists, fueled in large measure by Jean Piaget’s “stages of learning” and Lev Semenovich Vygotsky’s “Zone of Proximal Development (ZPD),” increasingly became the authorized providers of the content of the mathematics curriculum. Piaget was a psychological constructivist; in his view, learning proceeded by the interplay of assimilation (adjusting new experiences to fit prior concepts) and accommodation (adjusting concepts to fit new experiences) now labelled “discovery” or “experiential” learning (Thompson, 2019, p. 9). Vygotsky developed ZPD to argue against the use of academic, knowledge-based tests as a means to gauge students’ intelligence (Wass & Golding, 2014). Piaget’s and Vygotsky’s educational theories provided the arguments that widened the gap between the textbook development done by the mathematical community and that done by the secondary educational community (Wass & Golding, 2014). Notably, ZPD conceptually is a procedural approach of achieving topic mastery in any subject, but perhaps more in a subject that represents step by step concept maturation (Silalahi, 2019). That is, the teaching of mathematics that is intended to develop stepwise mastery might be well suited to ZPD and the scaffolding approach, provided the mathematics teacher’s content knowledge is rigorous and comprehensive (Kim et al., 2011). The student in the classroom of such a teacher is likely to be assigned in-class and out-of-class assignments to fully acquire the concept presented so that, in addition to the resident master in the form of the fully-competent teacher, the textbook must support the rigorous lesson once the student has left the classroom (Hu, 1993; Valverde, 2002). The requirement for content relevance, rigor, and topic perspicuity in textbooks inspired this research into textbook rigor.

2.4. Degraded Mathematical Maturity—The Source

Intellectual maturity is the ability to continually synthesize and create new ideas by the effective use of old or mastered ones. It is not achieved through the conventional strategy of "thinking outside the box," but rather by increasing the dimensions of the "box" to encompass and include more processes and a maturing understanding (Cowan, 2007). It is through this expanding the box process that mastery is approached and the confidence garnered in mastery allows for the maturing of the student (Insurance Newslink, 2013; Pickett et al., 2006). When mathematical concepts are ordered logically in a textbook, presented clearly, and reinforced systematically, the "box" Cowan addresses will expand inclusively for students and, "as collective intelligence emerges, collective wisdom becomes possible" (Adams & Anderson, 2015, p. 62). Mastery, which can be measured with any number of investigations, becomes the measure of maturity. So, the target for maturing the student is mastery in the subject (Bardach et al., 2019). Mathematically mature students have the self-confidence to continue to move forward in their studies and the ownership (mastery) of each new topic promotes self-confidence and the perception of success which in turn provides the enjoyment that compels students to keep moving along in their coursework (Tapia, 1996).

Student maturity is promoted by rigor in academic coursework but since mathematics standards have measurably dropped over the years the inclination has been to blame the curriculum and academic standards. Taylor (2018) places the blame on the fact that the "secondary school mathematics curriculum is narrow in scope and technical in character; this is quite different from the nature of the discipline itself" (p. 1). In 2004, the American Diploma Project (2004) found that "nearly half the states require students to pass exit exams to graduate, but these exams are generally pegged to 8th and 9th grade material, rather than reflecting the knowledge and skills students must acquire by the time they complete high school" (p. 7). McCormick and Lucas (2011), citing a 2006 report prepared by the National Math Panel of the ACT, noted "an actual loss of momentum in students' progress toward college readiness during their high school years" and attributed it to "a lack of direction on the part of the states in establishing and defining specific course standards and expectations for mathematics achievement in high schools" (p. 12). They further noted: "While more than two thirds of high school teachers surveyed believe they are meeting state standards for preparing students for college-level mathematics, approximately the same ratio of post-secondary educators believe students are coming to college unprepared" (p. 12).

Maturity can be impacted by "teaching to the test" (Zakharov & Carnoy, 2021) and often these tests are "not well aligned with postsecondary learning" (Conley, 2007, p. 9). Teachers at the secondary level tend to assume that they need to cover a broad range of topics in advanced mathematics courses while teachers at the post-secondary level are more concerned that high school students have a proper grounding in math fundamentals (Chait & Venezia, 2009).

2.4.1. Taking Student Maturity Into Account in Survey Design

Formal mathematical structures and concepts presented in the proper rigorous context (usually in a textbook) help develop, foster, and enhance a student's intellectual maturity (Hadar, 2017; Pirie & Schwarzenberger, 1988; Wijaya et al., 2015), and the proper rigorous context can be fashioned into survey question criteria or steps (Khoukhi, 2013) that are listed and defined in the maturity steps described below:

2.4.1.1. Maturity Step 0: Pre-Year-12 and Campus Influences

If the secondary student believes that the teacher is trustworthy and believable, the student then becomes attentive to the curriculum (i.e., matures in the subject area) and its delivery via the textbook (Jukic Matic, 2019; Shield & Dole, 2013). As a result of the textbook and teacher engagement, the textbook is afforded a value to the student which becomes the inertia needed for the student to mature in the concepts being taught (Hughes, 2011; Vincent & Stacey, 2008; Walker, 2008).

2.4.1.2. Maturity Step 1: Socio-Mental Maturity as a Catalyst for Academic Maturity

Thomas Arena notes: “Mental maturity is probably the best single predictor of academic achievement available…” (Arena, 1970, p. 21). Arena did not connect mental maturity to social maturity but the meta-analysis by Ma indicates a connection between social and mental maturity (Ma, 1999). Additionally, Steinberg demonstrated that the connections between formal expectations of the parent, teacher, and delivery method of mathematics (the textbook for one) are significant in the social and mental maturing of the student (Steinberg et al., 1989). Maturity (as opposed to gender or culture or promotion of self-esteem) blossoms as a predictor of academic achievement—notably in Mathematics (Steinberg et al.; Zenkl, 2021).

2.4.1.3. Maturity Step 2: Transitional Maturity

Transitional maturity recursively develops the student from mental maturity to intellectual maturity. The existence of the transitional maturity becomes stark when one sees students who are highly intelligent and intellectually mature and others who are highly intelligent yet intellectually immature (Eckert, 1934; Klafter, 2020). Students must face the problems which are brought about by environmental factors like school atmosphere, family atmosphere, peer group relationship, and gang influence, etc. The unhealthy atmospheres of family, school, and poor peer group relationships cast a bad influence upon the social behavior of adolescents (Klafter). Naturally, the converse is true as well (Lawrence & Jesudoss, 2011). This likely factored into one of the many reasons for the differentiation noted by Eckert in her "Traits Listed" section that included "sense of values," organizational skills, and ability to apply principles to social settings as characteristics of intellectually mature students (Eckert, p. 479).

2.4.1.4. Maturity Step 3: From Social to Intellectual Maturity

"Man is basically a social animal. His existence without social structure can hardly be imagined. He is born in a society, develops in a society, and works and progresses in a society" (Lawrence, p. 244). As individuals age, society expects that their behaviors will change too (Auyang, 2009; Kaur, 2020). Under normal conditions, "social" maturity increases over time, in the form of selflessness and other expected norms of behavior commensurate with age. Individuals learn to be in a group, share, care for others, and respect the norms and values of the society (Nadaf & Patil, 2020). Intellectual maturity develops according to normal, predictable situations that the student encounters (Talebi BahmanBeigloo & Khosravi, 2021; Yani et al., 2019). Lawrence adds, "As adolescence is the age for an individual to express mature behavior, education should inculcate noble human values through various activities along with the normal curriculum" (Lawrence & Jesudoss, 2011, p. 244). There is an expectation that intellectual maturity grows recursively through continued immersion in social and academic challenges that exercise acquired maturity. A formal approach in training, instruction, remediation, encouragement, admonition, and critical thinking will foster the overall process of social and intellectual maturity and the rehearsing of these will then lead to the balanced and mature approach to mathematics and thinking in general. Thus, with a formal and rigorous mathematics textbook, the student will have the advantage of rehearsing these maturing disciplines in the form of a structured approach to learning mathematics (Hurlock, 1967; Kelly & Kotthoff, 2016; Ma, 1999; Steinberg et al., 1989).

2.4.2. Convergence Toward the Target of a Mathematically Mature Student

The following progression results in a mathematically mature student:

- Successful preparation permits a healthy self-awareness (Jones et al., 2016; Schoenfeld, 2016) that promotes teacher acceptance and environmental agnosticism so that teacher acceptance promotes interest in studying the text.
- When the text is rigorous it challenges the students in a healthy way (Gupta & Elby, 2011; Redish & Smith, 2008) to think conceptually (Ferguson, 2012; Ganter & Barker, 2004).
- A rigorous textbook promotes systematic progress and maturity (Niss & Højgaard, 2011) that enhances the student's capability to better utilize the rigorous textbook (Jones et al., 2016) so that maturing instructor interaction promotes trust in the textbook (Gainsburg, 2012) such that the student is prepared for the tertiary experience (European Society for Engineering Education (SEFI), 2013).

Figure 2.1 depicts the chronological flow of the secondary student's mathematical exposure and experience. It highlights the fact that mathematical maturity is multifaceted, linked to the student, environment, and teacher and influenced by the rigor of a sequentially well-presented year-12 mathematical textbook that has, as its focus, the successful acquisition of competency to be measured in this research by success in the year-13 calculus examinations.

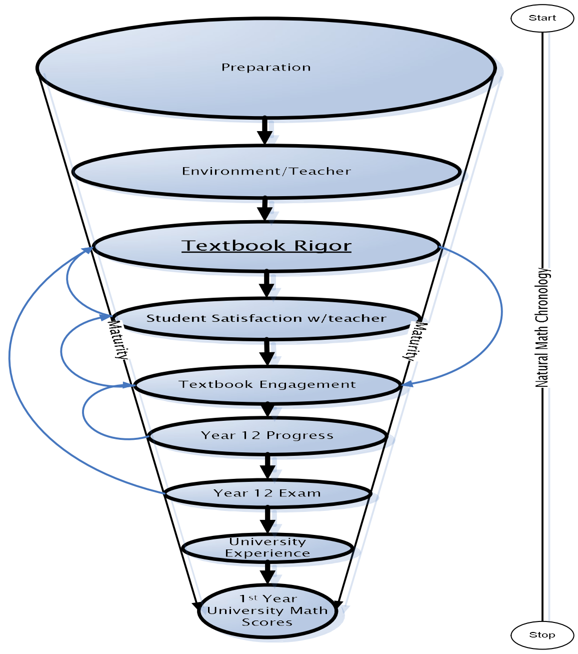

Figure 2.1 Maturity Flow and Mathematical Mastery Progression by Categories

The looping enhancement of skill and maturity has at its center a rigorous textbook (Hadar, 2017; Wijaya et al., 2015). The rigorous textbook is the central resource that competently presents cohesive topics with varying maturity levels of examples and problems, thus allowing the student to continuously review and move toward topic mastery (Rohrer et al., 2020; Shield & Dole, 2013) . In fact, the rigorous textbook gives the student the confidence of having acquired mastery which itself enhances mathematical maturity leading to the development of new ideas (Prabawanto, 2018; Steen, 1983). Accordingly, it was determined that the survey instrument design had to solicit answers to survey questions that would reveal whether the textbook experience had promoted maturity and demanded mastery (Redish & Smith, 2008).

2.5. Drivers of Readiness

While the core requirements of the student, the environment, and the teacher are all valuable considerations for developing mathematically mature students (Shield & Dole, 2013), they meet, in a sense, in the textbook. That is, the intersection of the three core components are not detached from the textbook as seen in the Rezat tetrahedron model, Figure 2.2 below (Rezat, 2009, p. 1261).

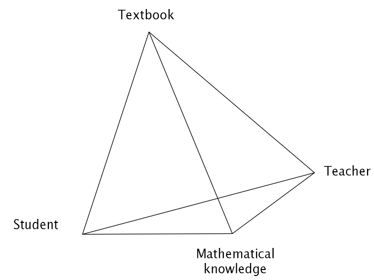

Figure 2.2 Rezat's Tetrahedron Model

With this intersection, the textbook may act as the resource that provides for the student's need for guided, maturing tasks (Glasnovic Gracin, 2018; Rezat, 2009); it can also provide the direction of lessons and order for the teacher (Usiskin, 2013; Wijaya et al., 2015) and the associated cosmos (environment) which may be expressed as a contributor to the acquisition of mathematical knowledge (Hadar, 2017; Wijaya et al.). Thus, the mathematics textbook's centrality emerges as a key driver enhancing or degrading the student, the environment, and the teacher effectivity (Lin, 1983).

2.6. Textbooks as Key Driver of Readiness

Scholars have noted that a rigorous textbook can be a teacher by itself—serving a highly influential role in the mathematical education of the secondary student. In fact, instructional materials are found repeatedly to influence between 75 and 90 percent of instruction decisions made in the classroom (Shield & Dole, 2013; Squire, 1985). Rezat (2009) describes the intrinsic value of the textbook, and in particular the mathematics textbook, as one of the most important tools in the teaching of mathematics while Shield and Dole (2013) observe that the textbook and its environment contribute measurably to providing "rich and connected mathematics knowledge" to students (p. 195). Glasnovic Gracin (2018) emphasizes the important role textbooks play in mathematics education and promoting mathematical maturity in students, citing studies by Howson (2013) and Usiskin (2013) that show that teachers use the textbook as the tool of lesson preparation even more so than curriculum outlines. Pellerin (2005) states that high schools play a role as "socializing agents" (p. 283) and, in this capacity are responsible for setting the direction and commitment to academic standards and emphasis. Selecting appropriate textbooks would be one of those commitments and, in fact, at least one scholar has characterized the adoption of inadequate textbooks as "professional misconduct" in the education community (Heyneman, 2002, p. 1664).

2.6.1. How Students Use Textbooks

Valverde (2002) argues that the structure of mathematics textbooks advances a distinct pedagogical model and is likely to have an impact on actual classroom instruction. Rezat (2009) emphasizes that the mathematics textbook is foundational to the teaching of mathematics and adds that, in order to utilize the textbook content (building mathematical skill and maturity) the textbook must be structured so that a cohesive continuum of topics order, topic rehearsal, and topic utilization be clearly in place throughout the material presented. Rezat's study concluded that students do not only use the mathematics textbook when they are told to by the teacher; they also use the textbook in a self-directed manner. He observed that mathematics students practice complexity reduction and problem solving, deliberately pursue increasing their mathematical knowledge base, and even research interesting mathematical activities as part of their textbook use.

2.6.2. Optimal Topic Order

A rigorous approach to learning is one that is distinctly different from an experiential (ZPD) approach (Dean & Kuhn, 2007; Kirschner et al., 2006; Mayer, 2004). Mathematical topic rigor in the textbook requires directed (or guided) emphasis on the theoretical constructs that demand student mastery and that reinforce theory and practice by diligently and systematically exercising previously presented material while addressing the minimum mastery prerequisites for the sequential, next course which, in the case of this study, is year-13 calculus. The rigorous approach deemphasizes the experiential or discovery approach on the grounds that the precalculus student is beyond the concrete (non-abstract) stage of knowledge acquisition; but, even in the concrete stage, directed (or guided) instruction is measurably more effective and should be emphasized in the mathematics curriculum and textbook (Ausubel, 1964; Colliver, 2000; Klahr & Nigam, 2004). Directed (or guided) and formal theoretical discussions should be sequential for each topic and the order of topics presented should be consistent with the intent of the coursework (Goonatilake & Chappa, 2010; Kindle & Gentimis, 2018).

The rubric for the order of topics in any mathematics textbook would be, first, a brief review of what the student is expected to already know, followed by a structured, precept upon precept, topic presentation (Kindle & Gentimis, 2018). Therefore, to analytically examine rigor (a textbook implementation of directed or guided topic presentation), the textbook will need to be assessed on compliance with the maintenance of a recognized and approved order of topics presented, an uninterrupted and relevant continuum within the overarching topic order, and a commitment to recursively acknowledging previous topics by relevant integration into the current topic (now labelled as interleaving). As an example, there is common agreement among mathematicians and mathematics teachers that polynomials and advanced factoring be presented as the necessary launching to the topic of the reduction of operational complexity and of foundational value in calculus (Wagner et al., 2015; Weiss, 2016) followed by the ability to understand core attributes of analyzing functions. Additionally, the integration of function specifications, definitions and operations with the study of exponents and logarithms are considered to be critically important in calculus readiness and, as a result of the need for function inverse for the successful study of exponents and logarithms, a chapter order begins to come into view (Carlson et al., 2015).

Topic order, or instructional sequencing, has been characterized as "vital" to successful learning progression (Fonger et al., 2018); so vital, in fact, that when a curriculum demanded an order of topics that was not sequenced according to the needed mathematical topic maturity, Choppin (2011) reported negative student effects were measurable in student concept acquisition. In a sampling of students by Carlson, deficiencies were identified in trigonometry course work by test scores reflecting that the textbook did not adequately explain the foundational concept that trigonometric functions are circular functions. Test results also indicated that the necessary preparation in polynomial and rational functions had not been addressed, indicating a probable failure in topic order maintenance (Walsh et al., 2017). Chapter 7 describes development of an algorithm designed to facilitate the assessment of rigor in year-12 precalculus textbooks.

2.6.3. Textbook Content Shifting While Prerequisites Remain Static

Topic order in the precalculus textbook is important, and following the order suggested in the scholarly literature is likely to be instrumental in the mathematical maturing of the secondary student. Yet, precalculus textbooks are constantly changing with little regard for maintaining content order.

Precalculus textbook content change has been dynamic as evidenced by constant course redesign (Gruber et al., 2021; Jones & Lanaghan, 2021). For example, STEM is influencing course content toward a graphing and algebraic logic calculator basis and emphasis (Usiskin, 2013). Electronic calculators have systematically replaced some laborious mathematical operations, such as looking up information in assorted mathematical tables (Bates, 2021; Cipra, 1988; Hieb et al., 2015), and have even displaced some core operations, such as addition and subtraction, in the lower grades (Fischer et al., 2019). Reliance on calculators has impacted student awareness of the correctness of final solutions (LaCour et al., 2019). The integration of calculators into the textbook, and the deemphasis of mathematical tables and their associated mathematics, has necessarily transformed the textbook into a concept-deprived, yet calculator-rich, rapidly changing curriculum delivery system such that the exclusion of needed arithmetic skills deprives the student of a key factor positively influencing mathematical achievement (Kaeley, 1993).

Imagery inclusion is being used for concept reinforcement and problem solving in mathematics classes (Baker & Gilbey, 2016) and is implemented in many precalculus book chapters. The damaging effect of trivializing concepts by user-friendly presentations (Clark & Lovric, 2008; Ghedamsi & Lecorre, 2021) has adversely affected the students, the classroom, and the community (Geisler & Rolka, 2020) and thereby negatively impacted student satisfaction that is linked to mathematical skill and success (Rezat, 2009; Valverde, 2002). Henningsen and Stein (1997) measured conceptual acquisition degradation in the presence of irrelevant discussions and distractions in the form of activities, stories, and illustrations that were not concept specific and concept building. Careless illustrations effectively collapsed the acquisition of the concept being presented (Henningsen & Stein, 1997). Durkin et al., (2021) note that an increased emphasis on group work (discussion forums) in mathematics classrooms has resulted in an increase in the number of pages devoted to examples to promote and support these discussions.

While nations like Singapore are purposely limiting the number of topics introduced to students in mathematics courses (Close-up Media, 2011), scholars have commented on the detrimental addition of new topics into the mathematics curricula and textbooks in Australia and the United States (Hurst, 2015; Sherman et al., 2020). For example, due to massive enrollment in undergraduate statistics classes, one or more chapters dealing with various statistical topics have been incorporated into many precalculus textbooks (Gordon & Gordon, 2018). Often, topics are presented as a smorgasbord of delicacies for students and instructors to choose from based on whatever the current prevailing trend happens to be (Usiskin, 2013). It is no surprise that some scholars caption modern mathematical curricula and textbooks as "a mile wide and an inch deep" (Brown & LaVine Brown, 2007; Duffrin, 2005; Gu, 2010; Schmidt et al., 1997). Gu (2010) noted that the U.S. mathematics textbooks are 800-900 pages in length as compared to 200-300 pages in China and Singapore where, he notes, the students are more proficient in mathematics. The additional pages contain non-conceptual ideas such as pictures, calculator exercises, and computer games which interfere with the textbook as the trainer for rational thinking (Gu, 2010).

The problem with constantly changing precalculus textbooks is that the essential content and style of year-13 calculus courses/textbooks have not change markedly for decades (Sofronas et al., 2015; Tucker, 2013). Some have even expressed concern that the calculus texts are not keeping up with trends and technology (Sevimli, 2016). Because precalculus textbooks are constantly changing and presenting more topics at a superficial level with technology and imagery and group discussions while calculus texts have remained somewhat static (by professor preference according to Sevimli), a mismatch in necessary knowledge and skills is occurring between the standard secondary school precalculus course and the standard university calculus course (Schoen & Hirsch, 2003; Sevimli, 2016). Scholars have documented the difficulty secondary students are having transitioning from the informal presentations of concepts in secondary precalculus textbooks to the formal presentation of concepts via axiom, definition, theorem, and corollary sequences common to university calculus textbooks (Frank & Thompson, 2021; Ghedamsi & Lecorre, 2021).

2.7. Realigning Secondary and Tertiary Mathematics Requirements Through MRP

Because prerequisites for elementary calculus have remained relatively static, alterations to precalculus curricula and textbooks have caused a misalignment between secondary exit standards and tertiary entrance requirements. This project asserts that by considering the calculus-ready graduate of secondary school as the "product" required by university calculus professors, one can employ the backward scheduling approach integral to the business world's MRP approach as a plausible strategy to examine, diagnose, prescribe, and, if needed, remediate the readiness of incoming students to year-13 calculus courses.

2.7.1. History of MRP

MRP as a concept for inventory management emerged as early as 1913 as a procedure discovered and implemented by Ford Whitman Harris to structure inventory such that a factory would store just enough components needed for projected factory operations so as not to incur storage costs (Essex, 2020). In the 1960s and 1970s, with the increased availability of main-frame and personal computing, MRP evolved into an automated process whereby efficiency measures could be implemented, monitored, and optimized for use in the manufacturing sector (Bogataj & Bogataj, 2019; Lambert et al., 2017; McCue, 2020). The advent of the mainframe implementation of MRP paved the way to advance from managing very simple production needs to managing complex component- and resource-laden products.

Miller and Sprague (1975) emphasized how the MRP process, by providing visibility to the deliverable (i.e., the end product or finished goods), necessarily drove the demand for the ordered acquisition of component parts and their assembling into those finished goods.

By the 1980's, the MRP process was firmly entrenched in the manufacturing sector and hundreds of commercial and home-grown versions of MRP automated systems were emerging, modeled on the computer program introduced in 1964 by IBM engineer, Joseph Orlicky, who incorporated the efficiency scheme from the Toyota Production System (Essex).

2.7.2. Evolution of MRP

The efficiency levels attained through an automated MRP system sparked interest as to how such a system could be applied when the end item, or finished good, was something other than devices or material goods. That is, since MRP was providing an automated capability to efficiently process and schedule the manufacture of components through bills of material (BOM) and critical path method (CPM) to manage order and timing of assembly, could it also promote efficient management when the BOM were not simply components but, instead, resources or even services? In 1983, an extension of MRP called MRP II (manufacturing resource planning) enlarged MRP beyond the BOM-centric system to include resources, services, and even capacity planning. By 1990, MRP II had further developed into enterprise resource planning (ERP) to manage things like accounting, human resources, supply chains, and higher educational settings (Essex, 2020; Kumar et al., 2021; Zhao & Tu, 2021).

For those industries that remained component and product assembly centric, the move to the ERP to manage other business processes did not eliminate the core MRP logic. That is, the ERP system had in no wise divorced MRP and, although MRP is a predecessor of ERP, it remains a necessary and important part in the ERP implementations (Salimi et al., 2006). For those industries that do not support a manufacturing environment, ERP implementation, with its concomitant MRP, contains, by virtue of BOM logic, the logic construct called the critical path. CPM is simply the identification of the longest time stretch of dependent tasks that are inseparably involved in the producing of the finished goods (Kim, 2009; Kotani et al., 1987; Tawanda, 2018).

2.7.3. Applying MRP Processes to Higher Education

The CPM component of MRP has also been applied globally in the higher education environment for myriad purposes to include scheduling classes, forecasting and managing supporting resources like classrooms, laboratories, equipment, and professors (Cox & Jesse Jr, 1981; Habeeb & Alsabaj, 2011; Noaman & Ahmed, 2015), and even for streamlining the collection of data used to document accreditation standards (Budiman et al., 2021). Abugabah and Sanzogni (2010) report that: "ERP systems have become a standard feature of most Australian higher education institutions" (p. 396).

2.7.4. Applying MRP Processes to Calculus Students

While MRP and ERP are employed on higher education campuses worldwide, absent from scholarly literature is any indication that they have been used to drive requirements planning for student course readiness (preparedness). Accordingly, this thesis will theorize a strategic implementation of an MRP solution that facilitates designing of a set of course prerequisites to drive needed mathematical material mastery and mapping of those prerequisites into secondary school curriculum that is implemented via the textbook (Hadar, 2017).

The precalculus textbook is the target of this experiment and the minimum requirements (end product or deliverables) will be determined in a BOM and CPM analysis of the mastery expectation identified via a survey of year-13 calculus textbooks. The precalculus textbook target is eminently suitable because, as scholars have noted, textbooks provide order and structure (Hadar, 2017; Palló, 2006), drive curriculum (Usiskin, 2013), and provide classroom focus as teachers move through the textbook in a particular sequence of topic presentation (Fan et al., 2013; Hadar, 2017). The sequencing and content of topics can be evaluated against calculus prerequisites using MRP processes. MRP's critical path strategy applies aptly to the entire secondary mathematics curriculum, but when the calculus prerequisites are defined as the BOM for incoming calculus students, the CPM succinctly defines the individual skills within the topics that must be present and ordered to ensure timely student success.

MRP applied to mathematical competency for university mathematics appears to be absent in the literature as a strategy for course preparation and this paper will suggest that MRP is a useful and straight forward way to address prerequisites in any setting. This thesis thus fills a gap in existing literature (and practice) as it suggests the use of MRP as the optimal way to drive calculus prerequisites from the university down to the secondary school so that university mathematics departments, as the "customers" of secondary schools, can become the rightful drivers of secondary mathematics curricula.

2.8. Summary

Secondary mathematical student testing from several countries demonstrates inadequate preparation for tertiary mathematics. Additionally, literary evidence supports the existence of mathematical inadequacies for students moving from their last year of secondary mathematics into their first year of tertiary mathematics. Educational research is prolific with affirmations of this problem and there are many proposed, implemented, and documented solutions. However, very few look to the tertiary mathematics prerequisites as the driver for secondary mathematical outcomes. The misalignment of year-12 mathematical exit requirements and year-13 mathematical prerequisites, calls for a deliberate focus on tooling designed to enable alignment and subsequent on-time delivery of the finished product—the adequately prepared year-12 student for year-13 calculus.

Chapter 3. Research Methodology and Survey Instrument Design

3.1. Chapter Overview

The design and delivery of the survey instruments described in this chapter sought to uncover correlated and potentially causal components for student mathematical readiness deficiencies as described in Chapters 1 and 2, particularly as they relate to mathematics textbooks. This chapter describes the research methodology and the development of the pilot and final surveys used to collect data to enable analysis of the relationship between the rigor of secondary mathematical textbooks and the preparedness of year-12 graduates for their year-13 mathematics courses by a determination of the academic centrality of the year-12 precalculus textbook; with this in mind, the use and confidence in the secondary precalculus textbook was the objective in the survey questions.

3.2. Research Methodology

The data collection design for this study was based on a mixed methods approach (mixing of qualitative and quantitative data) because the study combined quantitative and qualitative research into a systematic view (Hong et al., 2020). Data collected at both secondary and tertiary institutions included student and teacher demographics, student and teacher textbook experiences, teacher appraisal of student readiness, and a list of textbooks in use by the surveyed students and teachers at the time. While survey data (demographic and Likert) were entirely quantitative, mixed methods was brought to bear when architecting the methodology for textbook rigor assessment with the belief that this approach to building a rigor algorithm helps develop rich insights into various phenomena of interest that could not have been fully understood using only a quantitative or a qualitative method (Venkatesh et al., 2013). Saldaña supports this:

> Mixed methods research has been present for several decades, but only recently has the genre emerged as an approach that brings the once-separated quantitative and qualitative paradigms together to form a new epistemological, theoretical, and methodological way of working, when appropriate for the research purpose and questions (Saldaña, 2011, p. 11).

The mixed method approach applied a Convergent Triangulation Design as depicted in Figure 3.1 (Creswell et al., 2003, p. 226) in that Calculus textbooks were gathered and qualitatively analyzed for common content across several decades so that a prerequisite model could be defined. This model was implemented as a pattern to discern quantitatively whether the year-12 precalculus textbook content and presentation aligned with the year-13 calculus prerequisites which led to a quantitative rating of conformity. This strategy allowed for corroboration of quantitative results and qualitative findings as developed in the structured models that followed the Factor Analyses in chapters 5 and 6 and the building and implementation of the rigor algorithm described in Chapters 7 and 8.

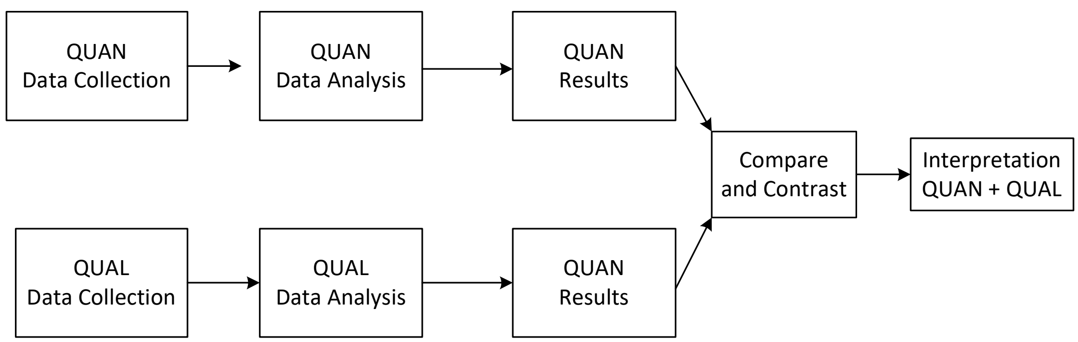

Figure 3.1 Mixed Methods Concurrent/Convergent Triangulation Design

Qualitative research, a generic term for investigative methodologies (Mishra, 2016) must complement information sources and provide data-driven conclusions that minimize and control both researcher and participant bias. When information sources drive the questions that are asked, the questions will yield answers already indicated by other data sources, enabling the researcher to deductively formulate highly probably causal components with minimal researcher and participant bias. In so doing they can provide theory that is holistic and consistent with the information collected. This study uses an integrated approach to maintain a data-driven paradigm, to mix qualitative and quantitative data elements—commonly called *convergent* (parallel) design, to provide greater data reliability (Klassen et al., 2012).

3.2.1. Data Collection Focus

Student readiness for tertiary study is a product of more than one factor. Researchers have routinely cited three overarching factors. Ranked in order of influence, they are the student, the environment (campus and non-campus), and the teacher (Elliott & Healy, 2001; Guffey & Slater, 2020; Hartman & Schmidt, 1995; Hill et al., 2005; Nye et al., 2001; Yin et al., 2020). The research methodology had to account for the broad nature of these factors and the multiple common threads connecting them. In this case, it was determined that survey instruments targeting the three most cited overarching factors was the optimal approach for addressing the problem of ill-prepared secondary school graduates. Thus, survey design sought to intersect the mathematics textbook with the student, the environment, and the teacher.

Additionally, the survey design stressed the importance of constructing, connecting, and comparing categories of maturity, textbook rigor, and mathematical skill with these three factors as shown in Figure 3.2 and to apply emerging theories from the data collected to enable identification of the relationships between categories (Glaser, 1992). R denotes the rigor of year-12 mathematics textbooks, an independent (experimental) variable. T denotes test success in year-13 mathematics, a dependent variable. Independent variable E represents uncontrolled/error or spurious variables, and independent variable C represents those variables that are controlled and can be made constant or eliminated by virtue of design.

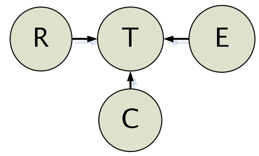

Figure 3.2 Depiction of Relationship Between Variables

The surveys also sought to measure and test the centrality, acceptance, and trust of the year-12 precalculus textbook with the understanding that, if the textbook was demonstrated to be central, then its adequacy would also need to be examined. This was done via the MRP-derived alignment analysis of the year-12 precalculus textbook with the year-13 university calculus requirements detailed in Chapters 7 and 8.

3.3. Data Collection—Survey Instruments

Four surveys were developed to collect research data. The first survey targeted students in their final year of secondary school who were enrolled in precalculus courses designed to prepare them for college/university calculus courses. The second survey targeted these students' teachers. The third survey targeted year-13 calculus students and the fourth survey targeted their professors.

The secondary school survey instrument was fashioned so that question emphasis was on the year-12 student's view of the value and usability of the mathematics textbook and confidence in their mathematical knowledge. The secondary teacher survey targeted the year-12 teacher's view of the textbook's value and its usability, the year-12 teacher's environmental views, to include the year-12 teacher's appraisal of the level of student mathematical knowledge implied by the satisfactory completion of the textbook-prescribed curriculum requirements. The year-13 student surveys intentionally targeted students' self-perception of their mathematical readiness at the commencement of the school year and their evaluation their year-12 precalculus textbook, while the professor surveys targeted professors' appraisal of actual student readiness for the course.

3.3.1. Establishing the Scope

To narrow the scope of the research geographically, the secondary school survey instruments targeted precalculus students and teachers at selected schools in South Australia and South Dakota. The tertiary surveys collected data from students and their professors at the University of Adelaide in Adelaide, South Australia and at the South Dakota School of Mines and Technology in Rapid City, South Dakota. The overall intent of the survey instruments was to answer the overarching research question concerning whether rigor level changes in year-12 precalculus textbooks are negatively impacting students' mathematical maturity and preparedness for their year-13 calculus course at the two universities in Australia and the United States. Both universities are respected institutions of higher learning ("South Dakota Mines again receives top rankings in state and nation," 2020; Times Higher Education, 2021) and both had thousands of students enrolled in programs that require calculus for first-year students, thus providing a large pool of students to take the year-13 student survey. There were, on average, 10,475 students enrolled annually in mathematics and sciences programs at UA between 2014-2016 and 2,800 students enrolled at SDSM&T (South Dakota Board of Regents, 2017; The University of Adelaide, 2017).

To narrow the scope of research to a manageable subset of secondary and tertiary students, it was decided to target the surveys at year-12 and year-13 mathematics students who were already predisposed as mathematically adept by selecting those who were studying precalculus and calculus because their academic path required them to do so. Thus, the pilot surveys and final surveys were delivered in year-12 precalculus classrooms and year-13 engineering mathematics classrooms.

3.4. Pilot Survey Creation, Delivery, and Modification

The flow of main tasks and outputs of this phase of the research project are represented in Figure 3.3, which is explained in more detail below.

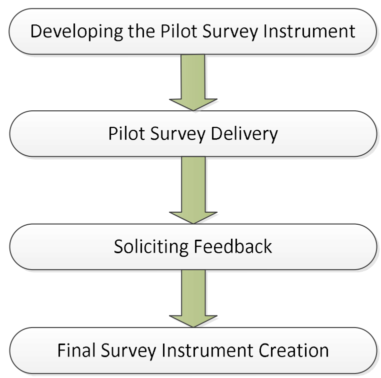

Figure 3.3 Flow of Main Tasks and Output

3.4.1. Selecting a Survey Model

The pilot survey utilized the five major rationales for mixed-method research validation identified by Hayvaert, et al. and Greene, et al. (Greene et al., 1989; Heyvaert et al., 2018) to help determine the containers and the questions within the categories:

i. <u>Triangulation</u> - Seeking convergence and corroboration of results from different methods and designs studying the same phenomenon.

ii. <u>Complementarity</u> - Seeking elaboration, enhancement, illustration, and clarification of the results from one method with results from the other method.

iii. <u>Initiation</u> - Discovering paradoxes and contradictions that lead to a reframing of the research question.

iv. Development - Using the findings from one method to help inform the other method.

v. Expansion - Seeking to expand the breadth and range of research by using different methods for different inquiry components.

Each of these rationales were needed to attempt to determine and reduce influential causality that was not textbook specific. Initiation, the negative approach of Triangulation, affirming controversy rather than congruence, and Triangulation were implemented by virtue of the use of a comparing/contrasting strategy in a two-pronged approach: First, the surveys were designed in such a way that questions were asked that were repetitive in meaning but distinct in wording so that each response could be compared against other responses to similar questions asked in a different way. For example, secondary student survey questions number 5 (“The year 12 textbook is too complicated”), number 16 (“There is enough detail in the textbook to master the topics”), and number 19 (“Without the teacher, the textbook would be useless”). A student who answered “Disagree” to question 16 would most likely answer “Agree” to questions 5 and 19. Conversely, a student who answered “Agree” to question 16 would most likely answer “Disagree” to questions 5 and 19. This technique of survey construction helped identify students who were providing random answers on the survey. Appendix A illustrates how questions were grouped to help facilitate this data-driven survey reliability process. Second, the year-13 survey questions were designed to compare and contrast the year-12 student and teacher perceptions of mathematical readiness from the context of textbook use (Greene et al., 1989; Heyvaert et al., 2018). Based on data-driven, mixed method strategy, twelve groups were developed and molded into survey containers with appropriate validation overlap using questions based on items i, ii, and iii above.

3.4.2. Framing Survey Questions

To guide framing of survey questions relating to principles of textbook rigor and student mathematical maturity, they were grouped into containers (Dolnicar, 2013). The container specifications were guided by evidence and information from the literature and provided verbally from students and teachers in interviews and pilot survey discussions.

The first eight containers reflect the proposition that textbook rigor is connected to tertiary mathematical success. The last four containers deal with the proposition that textbook rigor promotes maturity both academically and socially.

3.4.2.1. Textbook Rigor Question Containers

The following questions related to textbook rigor guided survey instrument design:

1. What are the relationships between prior math influences and textbook rigor and between prior math influences and environmental/teacher textbook engagement?
2. What relationships exist between year-12 preparedness from prior math influences, and year-12 math preparedness with regard to teacher and student textbook engagement?
3. What relationship exists between math textbook rigor and the student use of the math textbook?
4. What are the connections between the student textbook engagement and the math teacher textbook engagement and between the student and environmental/teacher textbook engagement to the student progress?
5. What is the relationship between the math textbook engagement and the outcome of the year-12 math terminal examinations?

6. What relationships exist between year-12 preparation and secondary and terminal progress/success?
7. What relationship exists between year-12 examination outcomes and year-13 examination outcomes?
8. What relationship exists between textbook rigor and secondary progress and textbook rigor and terminal outcome?

3.4.2.2. Student Maturity Question Containers

The following questions related to student maturity guided survey instrument design:

9. What relationship exists between student satisfaction with the secondary math teacher and the student textbook engagement?
10. What relationship exists between student satisfaction with the secondary math teacher and the teacher textbook engagement?
11. What relationships exist between the year-12 math textbook rigor and the year-13 math professor and university math textbook?
12. What relationship exists between the university professor, the university textbook, and the year-13 examination outcomes?

3.4.2.2.1. Framework for Maturity Question Crafting

As documented in the Literature review sections 2.4.1.1 - 2.4.1.4, the four steps of student maturity (Maturity Step 0 – Maturity Step 4) provided the framework for the questions that needed to be answered and placed onto the Pilot Survey. The survey instruments examine the Maturity Steps by assessing:

Maturity Step 0 Pre-Year-12 and Campus Influences

- Prior Preparation: Questions address general readiness and cross examines to determine whether preparation is adequate.
- Previous Staff Exposure: Questions address the effect of positive and negative experience with the year-12 math teacher.

Maturity Step 1 Socio-Mental Maturity as a Catalyst for Academic Maturity

- Interaction with Teachers: Questions address student/teacher interaction (i.e., whether the student requests assistance from teacher, whether they see the teacher as a necessary addition to the textbook, etc.).
- Homework: Questions address how much homework is assigned, how much homework is done, and how much extra homework is done.

Maturity Step 2 - Transitional Maturity

- Student Self-opinion: Questions address students' general opinion of themselves and their perception of how they are doing academically.
- Actual Status: Questions ascertain how students are *actually* doing.
- Work Habits: Questions address students' changes in homework habits and habits related to extra work.

Maturity Step 3 - From Social to Intellectual Maturity

- Student Self-opinion: Questions address students' general opinion of themselves and their perception of how they are doing academically.
- Actual Status: Questions ascertain how students are *actually* doing.

- Work Habits: Questions address students' changes in homework habits and habits related to extra work.

- Teacher and Professor Opinions: Questions address convergence and divergence of opinion of year-12 teachers and year-13 professors regarding students' readiness for year-13 calculus.

3.4.3. Crafting Questions for the Pilot Survey

The crafting of specific questions on the pilot survey was supported by the author's experience as a licensed mathematics teacher and considered these four factors: (1) how to define what is being measured, (2) how many questions to ask, (3) how to ask each question, and (4) how to collect the responses to each question (Dolnicar, 2013). The survey questions related to textbook use and student maturity solicited answers based on a Likert[5] scale to determine both student and instructor sentiments and concerns about the textbook in use. A Likert scale enables the researcher to "answer questions that have been raised, to solve problems that have been posed or observed, to assess needs and set goals, to determine whether or not specific objectives have been met, to establish baselines against which future comparisons can be made, to analyze trends across time, and generally, to describe what exists, in what amount, and in what context" (Isaac & Michael, 1997, p. 136). The remaining questions (those related to demographics) collected non-Likert responses. Figures B-1, B-3, B-5, and B-7 in Appendix B show the secondary and tertiary pilot surveys.

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[5] A Likert scale is an ordered scale from which respondents choose one option that best aligns with their view. It is often used to measure respondents' attitudes by asking the extent to which they agree or disagree with a particular question or statement. A typical scale might be "Strongly disagree, Disagree, Neutral, Agree, Strongly agree." https://www.cdc.gov/dhdsp/pubs/docs/cb_february_14_2012.pdf>

3.5. Pilot Survey Instrument Delivery

To examine the face validity of the pilot surveys, they were given at one secondary school in Australia and one tertiary institution in the United States. The secondary student and teacher pilot surveys were given to 54 students and 7 teachers at Tatachilla College in Australia. The tertiary student and professor pilot surveys were proctored in the United States at the South Dakota School of Mines and Technology (SDSM&T) for 375 students by 6 professors.

3.6. Soliciting Feedback on the Pilot Survey

Since the purpose of the pilot survey was to help refine the survey instrument into its final form, each pilot survey provided space for participants to provide feedback on format (survey length) and on the perceived effectiveness of questions to potentially validate the research thesis. To solicit additional feedback, discussion groups comprised of five teachers at Tatachilla and eight professors at SDSM&T were held. Though informal, the discussion groups were set up to resemble the Nominal Group Technique (Claxton et al., 1980) for the purpose of developing and refining the survey questions associated with year-12 mathematics textbook rigor. This technique allowed the review participants to discuss their own classroom experiences and issues rather than defining the problem based on extant literature. This approach was the catalyst to understanding how a rigorous text and curriculum promote (or degrade when absent) the mathematical and social maturity of students. The interviews at SDSM&T revealed a perception among professors that the incoming engineering students were poorly prepared for their elementary calculus courses.

Discussion group interviews and this author’s experience as a teacher indicated that the omission from the modern textbook of fundamental mathematical concepts and the lack of practice options in the text (the topic examples) were primary factors in the lack of mathematical maturity of year-12 graduates. One important and unexpected finding was that many university professors have had to create their own remedial teaching materials to address topics the students had not learned in their precalculus course. This information pointed to the need to identify the textbooks used by students so that these textbooks could be acquired and analyzed.

3.7. Building the Final Survey Instrument

Questions in the final survey were framed to incorporate feedback from survey participants and interviews with teachers and professors. These interviews revealed unambiguous intersections of concerns and comments when the questions were specific to mathematical maturity. Feedback from the discussion groups and participants validated the selected survey containers but highlighted the need for some changes to final survey instrument design. The following modifications were incorporated into the final survey instruments as shown in Appendix B, Figures B-2, B-4, B-6, and B-8:

- Reduced length of each survey to 1 page so that they could be taken in approximately 10 minutes. This was accomplished by modifying spacing and by removing feedback sections and administrative comments.
- Deleted certain questions (such as name of country and school) since these could be recorded by the proctor.
- Reduced number of questions targeting how teachers/professors use textbooks.
- Changed verbiage of some questions to enhance universal understanding.

In crafting the final survey instrument, steps were taken to address the possibility that students might simply check boxes at random or check boxes with a pattern of their choice. Questions were crafted so that student responses were affirmed via like questions with different wording as described in section 3.3. That is, the survey incorporated multiple strategies to address the multi-faceted aspect of students' sociological maturity. Additionally, survey questions were crafted so that diversity (or lack thereof) of individual responses would reveal the mathematical maturity of students and provide insights into whether their responses to questions about their textbook were valid. The instructor surveys provided validation of student responses.

3.8. Data-Driven Conceptual Model

With maturity data and textbook data, the multiple items or questions to measure a construct gave rise to a conceptual dependency model. The dependency framework depicted in Figure 3.4 (with inferred causation) was the basis for survey question verbiage, order, and repetition for secondary and tertiary student and teacher survey instruments. The framework was constructed with the concurrent/convergent triangulation from Greene (1989) and Creswell (Creswell & Plano Clark, 2011). In Figure 3.4 the connections inferred, and the data-driven categories discovered in the research questions, are linked to provide a visual conceptual model for the survey content and are constructed using triangulation and then complementarity strategies. The diagram shows the progression of influences from prior-to-year-12, year-12, and year-13 connected events. These connections hypothesize the cause-and-effect hierarchies and the latent variable constructs that the survey instrument has been designed to collect and that the subsequent CFA in Chapter 6 will help to confirm. The numerals on the diagram lines refer to the twelve questions listed in sections 3.4.2.1 and 3.4.2.2.

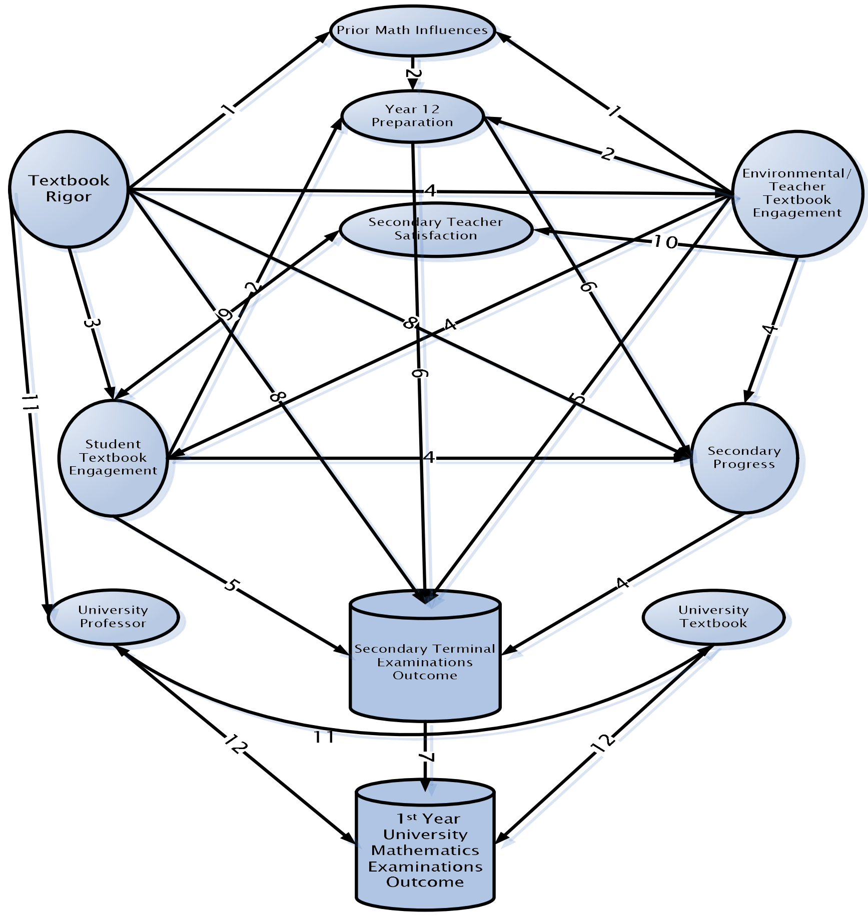


Figure 3.4 Data-Driven Conceptual Model

Note that in this model, the numbered arrows relate to the survey containers addressing textbook rigor and student maturity detailed in sections 3.4.2.1 and 3.4.2.2.

3.8.1. Determining Causality

The selected survey approach recognized that designs contain implicit assumptions about causal links and causal processes (Oppenheim, 1966). While good use can be made of proven techniques like replicability or data disaggregation, there must be an analytic process that avoids a derivation of causal relationships. The analytic relation survey is set up specifically to explore the associations between particular variables (Oppenheim) as illustrated in Figure 3.2 above and integrated in the causal flow in Figure 3.5 below.

The surveys for this project needed to answer the following: Given independent (experimental) variable R [rigor of 12th grade mathematics textbook], dependent variable T [test success in year-13 calculus], independent variable E [uncontrolled/error or spurious variables], and independent variable C [controlled and can be made constant or eliminated by virtue of design], how are T, E and C to be solely influenced by R? Figure 3.2 illustrates the logic just mentioned with the understanding that numerous variables contribute to math test scores.

The survey design had to address T, E and C in such a way that R was the predominant contributor to T. This design was necessarily iterative (including questions that would validate other questions) and would, when complete, resemble the impression diagrammed in Figure 3.5 where the size of the circle is proportional to the influence of the variables:

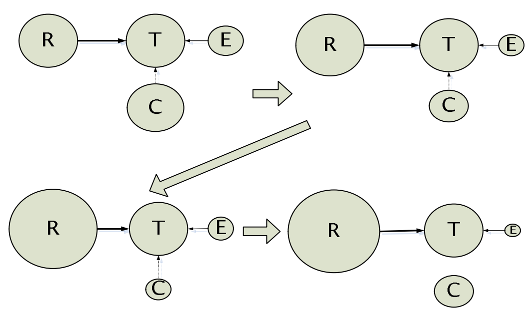


Figure 3.5 Depiction of Systematic Reduction of Influential Causality from E and Effective Removal of C (Made a Constant)

3.8.2. Triangulation as Validation

The data-driven conceptual model in Figure 3.4 uses triangulation as validation. Triangulation has been used as a validation enhancer in mixed methods data analysis (Caudle, 1994; Guba & Lincoln, 1994). Additionally, according to Caudle: “Qualitative evaluation measurements generally are very personal and reflect the evaluator’s perceptions, values, and professional training” (1994, p. 85). Thus, she recommends the use of triangulation as a means of improving the credibility of qualitative research. She defines triangulation even more broadly, as “the combining of methods, data sources, and other factors in examining what is under study” (1994, p. 89). Caudle further notes that congruence and/or complementarity of results from each method is the goal of triangulation. She defines congruence as similarity, consistency, or convergence of results, and explains that complementarity refers to one set of results expanding upon, clarifying, or illustrating the other. If done properly, Caudle asserts, triangulation should rely on independent assessments with offsetting kinds of bias and measurement errors (1994, pp. 89-90; Rhineberger et al., 2005). In the case of this research study, the triangulation from the teacher/professor data and the year-12/year-13 student data, provide a rich analysis of survey responses that effectively mix survey answers across surveyed groups to provide clarity for the potential influential factors that are exploited in chapters 5, 6 and 7 as Likert and demographic responses converged with textbook analysis techniques (Smith, 2018).

Thus, triangulation across survey responses, mixed with textbook appraisal decisions and the literature, has provided the mixed method approach criterion that will enhance data interpretation, sensitivity to rigor considerations, and centrality of the mathematics textbook in the teaching of year-12 precalculus. The data collected from the year-12 surveys is that which indicates the student’s and teacher’s appraisal of the value and use of the year-12 textbook for year-13 elementary calculus preparedness, while the data from the year-13 surveys is that which reflects the student’s and professor’s appraisal of mathematical preparedness for elementary calculus. Additional data was collected through demographic questions relating to student and teacher age, teaching experience, whether the year-13 student was placed into the course by a selection process, and which textbook students had used in the year-12 precalculus class. It should also be noted that additional quantitative findings are derived or embedded in the combining of the qualitative and quantitative data under triangulation (Dzekoe, 2013; Haidar et al., 2020). These embedded connections provided insight for refining the subsequent factor analyses (Dzekoe, 2013; Haidar et al., 2020). So, the embedded quantitative data that is indicative of a well-defined and well-documented actuality (mathematical preparedness), is understood quantitatively from the merging and analyzing of the qualitative data (Tunarosa & Glynn, 2017) such that the analysis of the university calculus textbooks described in Chapter 7 provided the qualitative foundation to quantitatively measure secondary precalculus textbook rigor, and the triangulation component mentioned above is implemented in the survey design and the rigor algorithm.

3.8.3. Parameter Summary for Factor Analytic Possibilities: Theoretical Influence Model

The Data-Driven Conceptual Model (Figure 3.4) showing the relationships between containers #1 through #12, coupled with the Maturity Flow and Mathematical Mastery Progression Model (Figure 2.1) showing the recursive value of the rigorous textbook, leads to the Theoretical Influence Model depicted in Figure 3.6. This summary model will provide an initial basis for factor analytic statistics in chapters 5 and 6. In this model, the following influence model specifications are identified and operationalized:

- Prior Math Influences and Year-12 Preparation as *Mathematical Readiness*. Reflects students' perceptions of readiness for year-12 precalculus based on confidence in their year-11 preparation. Also reflects confidence in a good exam outcome as affirmed by mid-term exam results.
- Environmental and Teacher Satisfaction as *Mathematical Enjoyment*. Reflects year-12 student opinion that the textbook presents topics in a non-complicated way

and that textbook explanations align with teacher explanations. Students prefer their textbook over handouts and believe that the textbook could stand alone if the teacher was absent. Note: As the experiment progressed, this variable was replaced by *Student Textbook Dislike* (Section 5.4.1.1) which was reversed to become *Student Textbook Like* (Section 6.3.1). This variable combined with *Student Textbook Trust* to become *Student Textbook Comfort* (Section 6.3.1).

- Teacher/Textbook Engagement as *Textbook Use*. Reflects year-12 student practice of taking the textbook home regularly and their opinion that if the book had more work problems, they would spend more time using the textbook.
- Textbook Rigor and Textbook Engagement as *Textbook Trust*. Reflects year-12 students' observation that the teacher uses and refers to the textbook in class, that the textbook examples help them understand the topics, that textbook chapters follow a logical sequence, that the teacher values the textbook, and that the textbook contains sufficient detail to help students master topics. Note: This variable later combines with *Student Textbook Like* to present a second-level latent variable called *Student Textbook Comfort* (Section 6.3.1).
- Teacher/Textbook Engagement, Secondary Progress and Textbook Rigor as *Textbook Enjoyment*. Like, *Mathematical Enjoyment*, this variable reflects year-12 student preference for textbooks over handouts and their opinion that textbook explanations align with teacher explanations. As the experiment progressed, this variable, like *Mathematical Enjoyment,* was later absorbed into the more descriptive latent variable, *Student Textbook Comfort* (Section 6.3.1).

- Secondary Terminal Examinations as *Student Maturity*. Reflects year-12 students' opinion that they did not need help with their homework and that their textbook could stand alone without the teacher such that they did not need to ask their teacher for help.

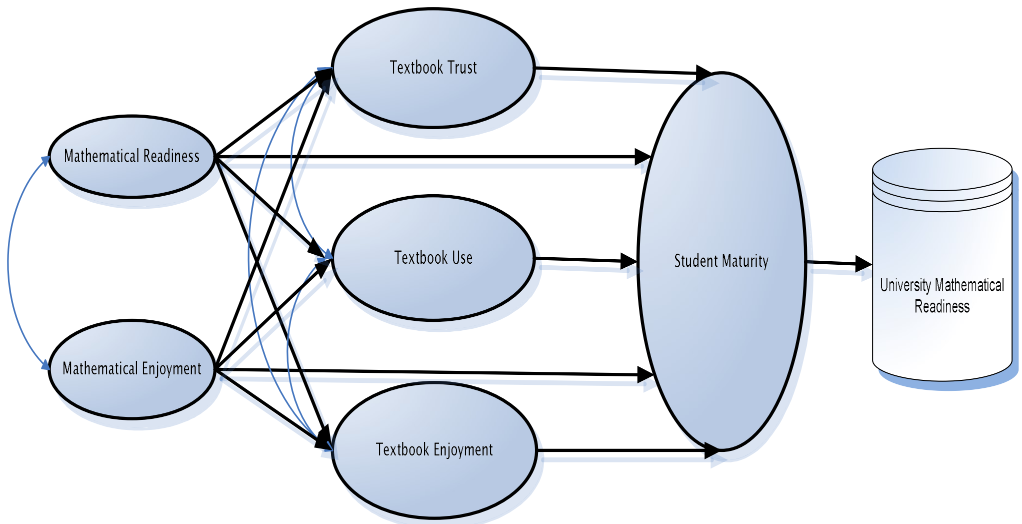


Figure 3.6 Theoretical Influence Model

The influences listed above cannot speak to the use of the university textbook or the university professor although we theorize that a successful exposure to a rigorous mathematical textbook will likely enhance successful university textbook use and allow the student to have a mature connection with the university professor.

3.9. Final Survey Instrument Delivery

3.9.1. Delivery Strategy

Surveys were implemented early in the midterm of the first semester of the Fall (called Autumn in Australia) so that maturity questions and midterm outcome questions were timely. The Fall (or first semester) term occurs in the February to May timeframe in the southern hemisphere and August to November in the northern hemisphere.

3.9.2. Total Surveys Delivered

The final survey instrument was delivered in 2014, 2015, and 2016 to 2,836 students, teachers, principals, and professors. Forty-four percent of the student surveys were proctored by a teacher or professor, 13 percent were proctored by a surveyor (the researcher or his late wife), and the remaining 43 percent of surveys were delivered and completed via the internet. Chapter 4 provides demographics on secondary and tertiary student and teacher/professor survey respondents.

3.10. Conclusion

This chapter has presented the research methodology and survey strategy used to collect data to enable analysis of the relationship between the rigor of secondary mathematical textbooks and the preparedness of year-12 graduates for their year-13 calculus course. It has demonstrated that, just as focus interviews about student readiness and textbook use (qualitative), allowed for question and survey layout input strategy, the mixed methodology approach permitted the qualitative data to integrate with the quantitative data so that survey answer reliability could be improved while also providing essential insights into textbook rigor measurement procedures.

The data-driven survey instrument described in this chapter framed the response context by:

- Student gender, age, and how they valued the text
- Teacher validation
- Student response consistency within the individual survey
- Student maturity
- Teacher experience
- Textbook value to teacher and student

The next chapter will provide demographic details on survey respondents and statistically validate that survey respondents are representative of the universal population.

Chapter 4. Sampling Justification (Preamble for Data Analysis)

4.1. Introduction

This chapter will describe the demographics of survey respondents (secondary and tertiary students and teachers) and will demonstrate the methodology used to validate that the data elements acquired for this study are representative of the universal population. Ideally, the universal population of tertiary students would be all the students in Australian and U.S. universities who studied calculus in their first year of university immediately following advanced mathematics coursework in their secondary schools. Since it is infeasible to survey this population, it was necessary to select a representative sample so that variance, without a "true" population, could be calculated. This process is described below.

4.2. Survey Respondent Demographics

The survey instrument was designed to collect data (demographic and Likert scale) that could be used to assemble a model, or models, that would demonstrate the centrality of mathematics textbook rigor as an essential component in the readiness of year-12 students to successfully complete their year-13 elementary calculus course. This section provides demographic information about survey respondents.[6]

4.2.1. Secondary Student Demographics

The secondary student survey was given to 566 year-12 students in the United States and Australia. The students who took this survey are representative students in the sense that they attended a variety of different types of schools, were the age of typical year-12 students, and comprised a gender ratio consistent with worldwide averages. The U.S. and Australian secondary schools were all co-educational and a mix of private, public, religious, and secular

[6] The total of 2,836 survey respondents included nine secondary school principals who took the survey—six in Australia and three in the United States. The demographic data below does not address these respondents.

schools. The U.S. schools ranged in size from 214 students to 800 students and catered to grades 8-12 and 10-12. The Australian schools were all private schools and were a mixture of religious and secular. The student population ranged from 700 to nearly 1,500 and catered to a mix of K-12 and 8-12.

The mean age of secondary students who completed the survey was 17.5. This is consistent with the fact that secondary schools have grade level age consistency. Of the 544 students who answered the question about gender, 307 (56%) were male and 237 (44%) were female. By way of comparison, females comprised 45.19% of secondary school students in Australia in 2016 and 48.79% of students in the United States in 2016 (Global Change Data Lab, 2021). The small difference in gender ratio between the representative survey sample and the overall secondary student population in the U.S. and Australia is explained by a fewer number of female students enrolled in advanced secondary mathematics (Lubienski et al., 2021; Mejía-Roíguez et al., 2020; Perez Mejias et al., 2021).

Table 4.1 shows the number of students who completed the survey by year.

Table 4.1 Secondary Student Year of Survey Completion

Year of Survey Completion	U.S. Students	Aust. Students	Total # of Students
2014	60	111	171
2015	59	177	236
2016	73	86	159
TOTALS	192	374	566

4.2.2. Tertiary Student Demographics

The tertiary student survey was completed by 2,195 year-13 students in the United States and Australia. The tertiary students in the United States were attending the South Dakota School of

Mines & Technology. The tertiary students in Australia were attending the University of Adelaide. Table 4.2 shows the number of students who completed the survey by year.

Table 4.2 Tertiary Student Year of Survey Completion

Year of Survey Completion	U.S. Students	Aust. Students	Total # of Students
2014	867	352	1,219
2015	457	194	651
2016	75	250	325
TOTALS	1,399	796	2,195

The mean age of tertiary students who completed the survey was 19.5. The oldest student was 72 years old and the youngest students (dual enrolled for college credit while still in high school) were 16 and 17 years old. Table 4.3 shows the year of graduation from secondary school for the 2,154 tertiary students for whom year of graduation was successfully collected from the survey answers. Seventy-two percent (n=1,581) of the students who completed the survey were in their first year of college. The proximity of the dates when tertiary students completed the survey and when they had graduated from high school affirms useful data consistency regarding year-12 and year-13 transitional considerations.

Table 4.3 Tertiary Student Year of Secondary School Graduation

Year of Graduation	# of Students	% of Total
1960s	1	0.05
1970s	0	0.00
1980s	10	0.46
1990s	23	1.07
2000s	105	4.87
2010	24	1.11
2011	42	1.95
2012	126	5.85
2013	565	26.23
2014	625	29.01
2015	509	23.63
2016	81	3.76
2017*	2	0.10
2018*	2	0.10
Missing	41	1.81
Total	2,154	100.00

* Dual Enrollment Students

Note that Table 4.3 also indicates an 83% drop in the number of students completing the survey in 2016. This can be explained by the fact that, in 2016, the survey that had previously been administered via a written questionnaire was converted to an online survey containing the same questions. U.S. professors had predicted that there would be a decrease in survey participation at their school due to the number on online surveys their students were subjected to each year. Their prediction proved to be correct.

Of the 2,147 tertiary students who answered the question about gender, 1,602 were male and 545 were female. The proportion of females (25.38%) is consistent with the U.S. Census report of 2011 showing that females comprise 26% of engineering majors in the United States (Sax et al., 2016) and with the Australian Bureau of Statistics showing 19% for female engineering majors in 2010-2011 (Broadley, 2015). The percentage of tertiary students who entered their year-13 mathematics class by placement test was 44.7% (n=982 students). Table 4.4 shows the courses into which they were placed.

Table 4.4 Tertiary Student Placement Into Mathematical Courses

Course	# of Students	% of Total
Calculus I	358	36.4
Trigonometry[R]	509	51.8
Algebra[R]	115	11.8

[R] Remedial

The remaining 55.3% of students (n=1,213) entered their year-13 mathematics class via valid university transfers or College-Level Examination Program (CLEP) results that met or exceeded entrance requirements.

4.2.3. Secondary Teacher Demographics

The secondary school teacher survey was completed by 41 teachers in the United States and Australia. Thirty-two percent (n=13) of the teachers took the survey in the United States and

68% (n=28) took it in Australia. The average age of the teachers who responded to the survey instrument was 41.2 years. Their average teaching time was 18.2 years. Their education level was as follows: 80.5% (n=33) had a bachelor's degree, 14.6% (n=6) had a master's degree and 2.4% (n=1) had a doctoral degree. (See Tables 4.5 through 4.7).

Table 4.5 Secondary Teacher Age in Years at Year End

Age	Frequency	Percent	Cumulative Percent
27	2	4.9	4.9
30	1	2.4	7.3
35	11	26.8	34.1
40	1	2.4	36.6
42	2	4.9	41.5
44	1	2.4	43.9
45	22	53.7	97.6
61	1	2.4	100.0
Total	41	100.0	

Average Age = 41.17

The number of 45-year-old teachers was double the number of 35-year-olds. This spike coincides with a national initiative in the 1990s to put credentialed mathematics teachers into classrooms (National Commission on Excellence in Education, 1983). Teachers who earned credentials during that initiative would have been in their 40s when they took the secondary teacher survey.

Table 4.6 Secondary Teacher Years Teaching Experience

Years Exp.	Frequency	Percent	Cumulative Percent
1	2	4.9	4.9
5	1	2.4	7.3
6	1	2.4	9.8
7	2	4.9	14.6
8	4	9.8	24.4
11	1	2.4	26.8
13	1	2.4	29.3
14	5	12.2	41.5
15	2	4.9	46.3
20	1	2.4	48.8

Table 4.6 (continued)

Years Exp.	Frequency	Percent	Cumulative Percent
21	2	4.9	53.7
22	1	2.4	56.1
24	6	14.6	70.7
25	9	22.0	92.7
32	1	2.4	95.1
38	1	2.4	97.6
40	1	2.4	100.0
Total	41	100.0	

Average Teaching Experience = 18.2 years

The same spike is evident in Table 4.6 where the largest number of teachers had 25 years of teaching experience.

Table 4.7 Secondary Teacher Educational Level

Education Level*	Frequency	Percent	Cumulative Percent
1	1	2.4	2.4
2	33	80.5	82.9
3	6	14.6	97.6
4	1	2.4	100.0
Total	41	100.0	

* 1=Diploma, 2=Bachelor, 3=Master, 4=Doctorate)
Average educational level = 2.17 (Bachelor and above)

The push for teachers with a college degree in mathematics rather than just diplomas during the 1990s likely explains the large percentage of teachers with a least a bachelor's degree who took the survey.

4.2.4. Tertiary Teacher Demographics

The tertiary teacher survey was completed by 17 teachers in the United States and Australia. Seventy-one percent (n=12) of the teachers took the survey in the United States and 29% (n=5) took it in Australia. The average age of the tertiary teachers who responded to the survey instrument was 45.3 years. Their average teaching time was 17.8 years. Their education level was as follows: 17.6% (n=3) had a bachelor's degree, 47.1% (n=8) had a master's degree and 35.3% (n=6) had a doctoral degree. (See Tables 4.8 through 4.10).

Table 4.8 Tertiary Teacher Age in Years at Year End

Age	Frequency	Percent	Cumulative Percent
30	3	17.6	17.6
40	3	17.6	35.3
50	9	52.9	88.2
55	2	11.8	100.0
Total	17	100.0	

Average Age = 45.29

The majority of tertiary teachers were at least 50 years old.

Table 4.9 Tertiary Teacher Years of Teaching Experience

Years Exp.	Frequency	Percent	Cumulative Percent
3	1	5.9	5.9
5	1	5.9	11.8
8	2	11.8	23.5
9	1	5.9	29.4
20	6	35.3	64.7
25	6	35.3	100.0
Total	17	100.0	

Average/Median Teaching Experience = 17.82/20.0 years

Over 70% of the tertiary teacher survey respondents had at least 20 years of teaching experience.

Table 4.10 Tertiary Teacher Education Level

Education Level*	Frequency	Percent	Cumulative Percent
1	3	17.6	17.6
2	8	47.1	64.7
3	6	35.3	100.0
Total	17	100.0	

*1= Bachelor, 2=Master, 3=Doctorate
Average educational level = 2.8 (Master and above)

The large percentage of tertiary teachers with at least a master's degree reflects the expectation that college/university professors will have higher degrees than secondary school teachers.

The remainder of the chapter describes how the survey population was validated as being representative of the universal population.

4.3. Sample Validation Methodology

The methodology applied to build a population representation so that variance, without a "true" population, could be calculated merged multiple validation schemes to generate a generalizability index score, β, that reflected the extent to which the sample population that responded to the survey instrument represented the population at large. The methodology included the following steps (Johnson & Bell, 1985):

- Minimizing the confounding variables in the survey via sample stratification.
- Validating equivalency by comparison with existing research data.
- Validating representative sample with survey responses.
- Calculating the generalizability index score, β, such that $0 \leq \beta \leq 1$.

4.3.1. Minimizing Confounding Variables via Sample Stratification.

The sample population was stratified into two population sets: secondary students and tertiary students.

4.3.1.1. Secondary Students

The sampling consisted of male and female students who were studying elective mathematical courses in year 11 and year 12; that is, the survey targets were students whose superior mathematical aptitude was already established. In simple random sampling, the probability of selection of any of the $C(N, n)$ possible combinations of n out of N sampling units is a function solely of the probabilities of selection assigned to each of the *n* units. With stratified random sampling, on the other hand, the selection of units is partially controlled in that the probability of selection of the combination depends on the strata with which the *n* units are associated (Goodman et al., 1950, as cited in Pruhs & Manber, 1991). Stratification introduces restrictions or controls in the process of selection (Wirkala & Kuhn, 2011).

In this study, sampling stratification was controlled so that there was a positive probability that the student had interest in further mathematical study and a zero probability that the student had no interest in further mathematical study. Due to the smaller class sizes in the secondary schools (common in the elective mathematics courses) the sampling stratification was further controlled by minimizing the effect of confounding classroom variables. For example, as demonstrated in the validation below, the smaller class size permits, in any classroom meeting, a much higher probability for (1) students' questions to be answered, (2) individual tutoring to be provided, and (3) more detailed topic presentation to be given. Table 4.11 illustrates the influence of smaller class size on student-teacher interaction based on 560 respondents. Note that for tables in this chapter, 1=strongly agree, 3.5=neutral, 6=strongly disagree. An average > 3.5 indicates a mild to strong general disagreement. Mode is the indicator of tendency in the average.

Table 4.11 Influence of Smaller Class Size on Secondary Student/Teacher Interaction

SPSS / Question Number	Question Verbiage	Average	Mode	Standard Deviation	Variance	State
V22/18	I prefer the notes from the teacher than the textbook	2.24	1.0	1.28	1.64	Agree
V23/19	Without the teacher, the textbook/written materials would be useless	2.71	2	1.39	1.95	Agree
V24/20	With textbook only (no teacher) I could understand the topics clearly	3.89	4	1.22	1.49	Disagree

The surveyed students, all of whom were in smaller class sizes, were able to interact with the teacher, get their questions answered, and receive detailed explanations of topics. Although early studies suggested that class size reduction is principally beneficial only for academically lower-level students (Boozer & Rouse, 2001; Nye et al., 2001), more recent studies indicate that, with regard to science and mathematics (usually academically higher students), the smaller class size in the secondary school also influenced the performance of students who go on to take science and mathematics courses in college/university (Wyss et al., 2007). The availability of interaction between the teacher and students in this setting provides validation for the secondary teacher's confidence in the student's capabilities relative to the material being presented in the secondary mathematics textbook, since the opportunity for personal interaction as well as assessment results, are much more available in the smaller class settings (Cann, 2009; Krueger, 1999).

4.3.1.2. Tertiary Students

The sampling consisted of male and female students who were studying elementary calculus in year 13; that is, the survey targeted students whose superior mathematical aptitude was already established and whose chosen study emphasis included higher level mathematics. These students were sampled near the beginning of the midterm so that they could better self-assess their preparedness for the course based on perception of readiness. Additionally, both universities selected were and are world-renowned science and engineering schools so that the tertiary student sample was controlled to minimize the spurious correlations that can occur with confounding variables issues in larger and subject decentralized cohorts (Tilaki, 2012). By virtue of the controls on the sample data (i.e., stratification of year-12 public and private school students and advanced mathematics and engineering school year-13 calculus students), the variance commonly caused by confounding variables has been minimized (Gallaher, 1973).

4.3.2. Validating Equivalency Using Existing Research Data.

As noted in Chapter 1, year-13 students are generally unprepared for university mathematics. Declining test scores, increasing attrition rates, and the proliferation of remedial mathematics courses, all reflect this diminished preparedness of year-13 students for higher level mathematics. The literature suggests:

- Secondary students experience problems transitioning to college calculus due to inadequate maturity in elementary concepts (Clark & Lovric, 2009; CUPM Panel, 1987; Hourigan & O'Donoghue, 2007; Nortvedt & Siqveland, 2019).
- Universities are redesigning precalculus courses because incoming students who were supposedly ready for calculus were not even ready for precalculus (Jones & Lanaghan, 2021).
- There is a 50% failure and/or dropout rate from freshman calculus courses at the University of Manitoba despite students believing they were ready for the course; these same students were quite surprised when they could not pass (Froese, 2019), and 90% of the identified (yet supposedly prepared) high-risk students at the University of Victoria, British Columbia were unsuccessful (Dame, 2012).
- Scholars see a crisis looming for calculus readiness as electronic tools and computer programs are emphasized over concept acquisition (Cipra, 1988; Hieb et al., 2015).
- Over 60% of U.S. community college students taking mathematics placement tests are placed in remedial or developmental courses (Ngo & Kwon, 2015).

- Countries with high student mathematics confidence had low student mathematics scores and vice versa in the TIMSS 2007 (Yoshino, 2012). See section 4.3.3.2 for details.
- Further evidence of declining mathematical skills has been the proliferation of remedial mathematical coursework at both two- and four-year institutions in the United States. In 1975 there were 245,000 students enrolled in remedial courses at two-year colleges and 141,000 students enrolled in remedial courses at four-year institutions (Mathematical Association of America, 1981). In 2019, a workshop of the National Academies of Sciences, Engineering, and Medicine addressing the success and failure of students in developmental mathematics reported that 70% of incoming students at two-year colleges and 40% of incoming students at four-year institutions had to take remedial mathematics classes (Zachry Rutschow, 2019) and that incoming engineering students have greater mathematics deficiencies than students in the past (Hall et al., 2015).

Based in part on the evidence above, the survey questions were designed to inquire as to secondary student and teacher perception for university calculus preparedness. Additionally, the survey questions attempted to determine the influence of the textbook on these perceptions. The tertiary student survey questions were designed to confirm perceptions of incoming students. The tertiary professor survey questions inquired as to the actual student preparedness for the university calculus course.

4.3.3. Validating the Representative Sample with Survey Responses

Two separate analyses were conducted—one using survey results from this study and the other using the 2007 TIMSS survey results. Based on the consensus of research data cited above, it was expected that a representative sample of students and instructors would be the set in which:

- Secondary student math confidence is a positive score.
- Tertiary teacher math capability appraisal of student is a negative score.

4.3.3.1. Analysis using Survey Results

Table 4.12 illustrates secondary students' positive mathematical confidence. Table 4.13 reflects secondary teachers' neutral to positive assessment of secondary student confidence and an acknowledgement of possible lack of skill due to frequency of help needed. Table 4.14 illustrates tertiary students' neutral to positive math confidence while Table 4.15 reflects university professors' assessment that tertiary students had a lack of mathematical maturity and skill.

Table 4.12 Secondary Students' Mathematical Confidence

SPSS/ Question Number	Question Verbiage	Average	Mode	Standard Deviation	Variance	State
V5/1	I was ready for Year 12 math	2.254	2	1.012	1.024	Agree
V6/2	I am going to do well in the final exams	2.515	3	0.996	0.992	Agree

Table 4.13 Secondary Teachers' Assessment of Student Maturity and Skill

SPSS/ Question Number	Question Verbiage	Average	Mode	Standard Deviation	Variance	State
V4/4	Students generally lack confidence in mathematics	3.25	4	1.05	1.12	Disagree
V6/6	Students ask for help with homework often	2.62	2	0.96	0.92	Agree
V8/8	Students are more mature this year than last year	3.09	3	1.24	1.54	Agree
V9/9	Students are more academically ready for math this year than last year	3	2	1.28	1.65	Agree

Table 4.14 Tertiary Students' Mathematical Confidence

SPSS / Question Number	Question Verbiage	Average	Mode	Standard Deviation	Variance	State
V11/11	My progress so far is better than I expected	3.15	3	1.21	1.46	Agree
V15/15	I am going to do very well in terminal exams	2.98	3	1.83	1.40	Agree

Table 4.15 University Professors' Assessment of Tertiary Student Maturity and Skill

SPSS / Question Number	Question Verbiage	Average	Mode	Standard Deviation	Variance	State
V1/1	Students believe the textbook/written material is valuable to them	3.625	5.0	1.54	2.257	Disagree
V3/3	Students are disciplined and make mature use of the text/written material	4.533	5.0	0.618	1.195	Disagree
V9/9	Students are more academically ready for math this year than last year	3.687	4.0	0.704	0.492	Disagree

4.3.3.2. Analysis using TIMSS 2007 Data Findings

The Trends in International Mathematics and Science Study (TIMSS) for 2007 assessed Japanese and North American middle school mathematics student confidence and mathematics maturity (Mullis et al., 2007). While not necessarily given in the same cultural context, the questions on the TIMSS study were similar to those used in Tables 4.12 and 4.14 to appraise U.S. and Australian secondary and tertiary student mathematical maturity and confidence and, thus, the TIMSS survey is useful for validating the methodology and conclusions of this study. The list below is a sampling of the types of questions used in multiple TIMSS to ascertain a mathematical "self-concept" score based on two indices: "Students' Positive Affect Toward Mathematics" and "Students' Self-Confidence in Learning Mathematics":

- I enjoy learning mathematics.
- Mathematics is boring.
- I like mathematics.
- I usually do well in mathematics.
- I would like to take more mathematics in school.
- Mathematics is more difficult for me than for many of my classmates.
- Mathematics is not one of my strengths.
- I learn things quickly in mathematics (Mullis et al., 2016).

The TIMSS survey found that Japanese students, though lacking confidence in mathematics, were far more mature in their mathematical skills than their North American counterparts (Yoshino, 2012). The 8th grade students who took the 2007 TIMSS survey would have been year-13 students in 2012; thus, they are closely representative of the year-12 and year-13 students who took the survey for this study in 2013-2016. In an earlier study of mathematics across Australian, American, and Japanese 7th and 8th grade mathematics students, it was noted that the Japanese students had lower confidence scores but higher spatial relationship skills than their Australian and American counterparts (Iben, 1991). Thus, both the survey instrument and TIMSS affirm that sample data are logically representative of the universal population in that at both U.S. and Australian schools:

- Student math confidence is a positive score
- Student math skill is a negative score.

4.3.4. Calculating the Generalizability Index Score, β

This section details the algorithm used to compare known population data with data gathered from the survey instrument used for this study. The resulting generalizability index score measures the extent to which the sample set for this study represents the "true" population of all students in Australian and U.S. universities who studied calculus in their first year of university following advanced mathematics coursework in their secondary schools.

4.3.4.1. Definition

Rentz's observations on generalizability form the basis for algorithm construction: "Generalizability, then, refers to the extent to which one can generalize from the observations in hand to a universe of generalization." (1987, p. 20). For the purposes of this study, what has been done to facilitate a sample that is representative of the universal population is to target, or focus, the sample and the survey questions so that there would be a positive probability of truthful maturity inferences and a zero probability of interest lack in mathematics. Because outliers and unwanted variation can confound results, Rentz notes that "the measurement instrument should minimize variance arising from these sources" (1987, p. 20). Steps were taken to do this.

In the algorithm applied below:

- The scores calculated as 0 will indicate NOT representative of the population.
- The scores calculated as 1 will indicate FULLY representative of the population.
- The score (generalizability index score), β, is such that $0 \leq \beta \leq 1$ (Tipton, 2014).
- β will be evaluated as ${s^2}/{\sigma^2}$ or the (sample variance/population variance) (Tipton, 2014).
- Therefore, s^2 and σ^2 will need to be calculated and derived respectively.

4.3.4.2. Specifications and Considerations

The need to derive a representative population variance σ^2 is due to the shortage of available data on secondary to tertiary transition for mathematics students. In fact, such studies are quite rare (Clark & Lovric, 2008, p. 34). Thus, in developing a generalizable index score, this study uses summarized, available data for the population calculations and the collected survey data as the sample population.

To enhance the accuracy and validity of the derived population variance, student performance data from the United Kingdom (UK) was also used since the UK is a Western nation that exhibits positive student math confidence and negative math skill like her Australian and U.S. counterparts. That data included the following observations:

- USA. According to the MAA study of college calculus, 25-40% of students enrolled in Calculus I failed to achieve a grade that allowed entrance to Calculus II (Bressoud, 2015).

- USA. Community colleges reported that 60% of students were enrolled in remedial Math and English courses (Redden, 2010).

- AUS. According to the *International Journal of Innovation in Science and Mathematics Education*, 30% of students enrolled in a tertiary elementary calculus class failed to acquire passing grades (Nicholas et al., 2015).

- UK. According to a report, "Tackling the Mathematics Problem," prepared by the London Mathematical Society, the Institute of Mathematics and its Applications, and the Royal Statistical Society:

> There is unprecedented concern…about the mathematical preparedness of new undergraduates…The serious problems perceived by those in higher education are: (i) a serious lack of essential technical facility—the ability to undertake numerical and algebraic calculation with fluency and accuracy; (ii) a marked decline in analytical powers when faced with simple problems requiring more than one step; (iii) a changed

perception of what mathematics is—in particular of the essential place within it of precision and proof (1995, p. 3).

- According to Darlington and Bowyer, the mathematical sciences had a 24% dropout rate in year 13 (Darlington & Bowyer, 2017).

4.3.4.3. Determining the Sample Calculus Failure Statistic (SF_c)

When the failure rate is averaged between the U.S. population and the Australian population, the Population Calculus Failure rate (PF_c) is $27.5\% \leq PF_c \leq 35\%$ (assumed population statistic). Note that 27.5% is the average of the U.S. lowest failure rate and the Australian failure rate. Likewise, 35% is the average of the U.S. highest failure rate and the Australian failure rate. Also note that if the U.S. community college data were taken into consideration, the interval for the PF_c would move 29.1% such that the "umbrella" interval would become $36\% \leq PF_c \leq 46\%$. Nevertheless, the expected Sample Calculus Failure statistic (SF_c) should be in the PF_c interval of $27.5\% \leq SF_c \leq 35\%$ since surveys did not include any U.S. community colleges.

SF_c was calculated using tertiary teacher responses to survey questions V3 and V9. SF_c will be the aggregation of university professors that believe students are *unprepared* in skill and necessary maturity in both Australia and the United States as seen in Table 4.16. In this table, question V3 assesses student maturity while question V9 assesses evidence of mathematical skills.

Table 4.16 Calculating SFc

Tertiary Teacher Survey Questions	Mean Survey Answer (MSA)*	MSA Ratio (MSAR) (MSA*0.171)	Agreement (1-MSAR)
V3. Students are self-driven, disciplined and make mature use of the textbook	4.44	0.75	0.25

Table 4.16 (continued)

Tertiary Teacher Survey Questions	Mean Survey Answer (MSA)*	MSA Ratio (MSAR) (MSA*0.171)	Agreement (1-MSAR)
V.9 Students are more academically ready for math this year than last year	3.64	0.61	0.39
Calculated SF_c (mean)			0.32

*1=Strongly Agree, 2 = Agree, 3 = Neutral Agree, 4 =Neutral Disagree, 5 =, Disagree, 6=Strongly Disagree
See tables C-1 and C-2 in Appendix C for MSAR supporting data and computations.

$SF_c = 0.32$ (or 32%) is in the interval $27.5\% \leq PF_c \leq 35\%$; thus, the SF_c (student maturity and student mathematical readiness) is in the acceptable range of the *calculated population calculus failure rate*.

Alternatively, a broad agreement on the percentage of professors responding to V3 with 1, 2, or 3 and to V9 with 1, 2, or 3 yielded 17.6% and 47.1% respectively. The average of these two statistics is 32.3% which is in agreement with the MSA and MSAR calculations for SF_c.

In Table 4.16, MSA is the average of the university professors' answers for the questions while MSAR is the ratio of the MSA and the denominator constant that is calculated as an authentication score based on the professor's sex, credentials, age, and experience using the four criteria below:

1. Sex Differentiated Considerations:

i. Female dispersal for V3 is the product of the proportion of female respondents and the average of the female responses.

ii. Male dispersal for V3 is the product of the proportion of male respondents and the average of the male responses.

iii. Due to multiple studies that debunk real cognitive differences between male and female (Hyde, 2005; Hyde & Linn, 2006), the gender coefficients are merged by calculating the absolute value of the difference between the male and female dispersal score. That is, rather than calculating and assigning a statistical weight to professor gender, equivalence is assumed.

2. Credential Differentiated Considerations:

i. Male and female responses though accumulated distinctly were processed as i and ii above and for the same reasons.

3. Experience Differentiated Considerations:

i. Male and female responses though accumulated distinctly were processed as i and ii above and for the same reasons.
ii. The Experience Factor is calculated by the product of the gender proportion and the ratio of the sum of the gender segregated ages and the gender segregated experience. Research indicates that this factor has a non-linear relationship with the Credential factor hence the calculation distinction (Avolio et al., 1990).

4. Age Differentiated Considerations:

i. The V3 and V9 coefficients are the mean of the gender, credential, and experience data. V3, a more character-based question included Gender Dispersal and Experience/Age factor and omitted Credential factor whereas V9, skill assessment, used Gender Dispersal and Credential Factor and omitted Experience/Age factor as it was determined that age and experience tend to make better character judgment calls and Credential and knowledge a better empirical (actual skill measurement) call (Hill et al., 2005).

Final Calculation: The mean of the V3 and V9 coefficients were averaged for the Averaged Coefficient of 0.171 as noted in Table 4.16 and Table C.2 in Appendix C.

4.3.4.4. Generalizability Index Score

Generalizability of the tertiary data has been established by the calculated sample and population calculus failure rate mean and interval, respectively. Nevertheless, to further qualify SF_c, we will compute β, as mentioned above, the generalizability index score, such that $0 \leq \beta \leq 1$ where a value of zero (0) indicates the sample is NOT representative and a value of one (1) indicates equivalence to the population (Tipton, 2014).

β = (Sample Variance) ÷ (Population Variance).

To calculate the generalizability index score, we calculate the ratio of the sample variance to the population variance across the calculated intervals in Table 4.17.

Table 4.17 Ratio of Sample Variance to Population Variance

	Population	Sample	Ratio= β
Low Overall	25	26.0	
High Overall	40	39.0	
Variance	56.25 (σ^2)	42.25 (s^2)	0.751 (s^2/σ^2)

- Data collected for the interval construction was used to arrive at the variance for the Population, PF_c.
- Using PF_c, the following data points are both provided and derived: 25, 27.5, 35, 36, 40 and 46 (see Table C-3 in Appendix C).
- There were 17 contributing surveyed university math professors, so 11 filler data points were inserted in linear order and labelled, “filler.” (See Table C-3 in Appendix C).
- V3 and V9 were row-wise (from the same professor) added together and the missing data were left blank.

- Mean and Variance were calculated for the Population and the Sample with the assumption that all data points have equal weight.

Initial Result (summarized from Table C-3 in Appendix C):

Population variance of 32.67 and sample variance of 2.86 resulted in the generalizability score (Sample Variance/ Population variance or β) as 0.0871 which is not considered a favorable score (Tipton, 2014) so MSAR mitigations were implemented in an effort to determine sample validity for continued analysis.

- As noted from the MSAR, professor education showed the Degree factor of 2.13 for female and 2.33 for male with an overall weighting of 2.24 (See Tables C-2 and C-5 in Appendix C).
- Sample data was weighted for the professors' educational level, as seen in Table C-4 of Appendix C.

Final Result:

Population variance of 32.67 and sample variance of 14.23 resulted in the weighted generalizability score (β_w) of 0.439. Further weightings are not required as the MSAR coupled with β_w=0.439 is adequate (Tipton, 2014).

4.4. Conclusions and Concerns

The methodology implemented multiple strategies for validating that the sample population used in this study was representative of the "true" population of all students in Australian and U.S. universities who studied calculus in their first year of university following advanced mathematics coursework in their secondary school. Each of the measurements presented provide a valid basis to accept the sample set with population affinity. This was accomplished in that:

- the data sample set was stratified.
- the data sample set was shown to be representative of existing research with MSAR=0.32.
- the generalizability index score was $\in [0,1]$.

With a favorable ratio of the variances, reported student mathematics skills being consistent between the population and the sample, student confidence, and a stratification scheme to minimize confounding variables, it is established that the sample set used in this study adequately represents the population.

Chapter 5. Survey Data Analysis – Exploratory Factor Analysis

5.1. Introduction

This chapter will demonstrate the methodology used to screen, cleanse, and analyze the data collected through the survey instrument described in Chapter 3 and will present the results of an Exploratory Factor Analysis (EFA) run on secondary and tertiary student survey responses in preparation for doing a Confirmatory Factor Analysis (CFA) and Structured Equation Model (SEM) described in Chapter 6. EFA is the tool of choice for identifying latent factors in preparation for CFA and SEM even though the *a priori* model already identified theoretical latent variables for secondary student data (Anderson et al., 1988, as cited in Hu & Li, 2015). EFA was used to further refine the latent constructs (i.e., to provide a covariance matrix for later comparison) to enable a more refined initial CFA.

The survey instrument was designed to collect data that could be used to assemble a model or models that would investigate mathematics textbook rigor as an essential component in the readiness of year-12 students to successfully complete their year-13 calculus course. The survey was completed by 560 secondary students answering 27 survey questions and 2,154 tertiary students answering 32 survey questions over the course of three years.

One of the difficulties inherent in surveys is that they cannot directly quantify attitudes or indirect actions in the lives and livelihood of participants. Because the underlying perceptions and beliefs that cause recipients to answer certain questions in certain ways are generally latent personality traits in the targeted survey recipients, they must be mined from within the data using various tools available to social scientists. That is, an investigation of the common implied links between the survey answers must be designed and implemented in order to discover such links or any meaningful commonality in the set of variables (DeCoster, 1998).

Factor analysis is a method that may be used to estimate possible latent connections in survey data and, for this reason, was the method selected to analyze data commonality links, to test the linking, to test the existence of any latent commonality between survey responses, to investigate the theoretical model presented in Chapter 3.

5.2. Organization

Section 5.3 is organized so that general data quality and suitability for factor analysis is detailed using the following steps:

1. Sample size considerations. Determining whether the stability of the latent variables derived by the EFA and CFA will be affected by sample size.
2. Data screening. Removing non-reflective variables (column removal).
3. Data cleaning. Removing spurious or unengaged responses (row removal).
4. Missing data replacement. Determining which algorithm to use and when to use it based on data type and randomness of the missing data (MCAR).
5. Validating data normality. Addressing skewness, kurtosis, and outliers.
6. Statistical Tests After Missing Data Replacement. Determining whether missing data replacement has statistically significantly altered the data sets.

Section 5.4 describes the factor analysis process (EFA), to include selection and naming of factors and the summary analysis of the application of reverse coding where the need to do so was revealed in the factor analysis procedures.

5.3. Assessing General Data Quality and Suitability for Factor Analysis

5.3.1. Sample Size Considerations

There is no exact parametric model for sample sizing, but the general consensus is 5-10 observations per estimated parameters (Bentler & Chou, 1987; Schreiber et al., 2006). For this study there are 55 parameters (6 proposed factors, 20 Likert variable error terms, and 15 possible covariances between the factors and 14 factor loadings). With the secondary student dataset at 560 observations and the tertiary student dataset at 2,154 observations, this provides a quotient of 10.18 for secondary and 39.16 for tertiary, both of which are greater than 10.

The scores for the Kaiser-Meyer-Olkin (KMO) Measure of Sampling Adequacy and the Bartlett's test for homogeneity of variances in the collected survey data indicate that the data samples are adequate for factor analysis. (See Table 5.1). The secondary student data KMO score of 0.757 and the tertiary student data score of 0.755 both exceed the minimum acceptable adequacy score of > 0.60 (Hair et al., 2017). The Bartlett score of 0.00 for secondary and tertiary student data indicates there was no reason to assume unequal variances.

Table 5.1 KMO and Bartlett's Adequacy Scores for Secondary and Tertiary Student Data

		Secondary Student Data	Tertiary Student Data
Kaiser-Meyer-Olkin Measure of Sampling Adequacy		.757	.755
Bartlett's Test of Sphericity	Approx. Chi-Square	2367.986	9929.056
	df	190	190
	Sig.	0.000	0.000

5.3.2. Data Screening

Because factor analysis is focused on deriving latent variables from Likert scale variables, demographic data was omitted from processing because it is not used for factor analysis. Refer to chapter 4, Section 4.2, for demographic details.

5.3.3. Data Cleaning

Row removal may be done where clearly spurious or unengaged responses are present. Spurious responses are those in which students purposely answer questions in a pattern of their own choosing or when students inadvertently mark their answers in the wrong order. Unengaged responses are those in which a student answers all the questions the same or agrees to conflicting statements. The secondary and tertiary surveys were written so that some questions were asked in the negative and the positive. For example, the secondary student survey question 5 states “The year 12 math textbook is too complicated” and question 16 states, “There is enough detail in the textbook to master the topics.” The tertiary student survey question 2 states “12th grade mathematics textbook needed more depth” and question 16 states “There is enough detail in the textbook to master the topics.” Students who answered “Agree,” to both conflicting statements were likely unengaged, so their records were removed (Slater et al., 2017).

To determine which rows to delete, the standard deviation was calculated for each row—that is, the measure of individual consistency within a single respondent’s survey answers. Standard deviation and variance may detect patterned and/or unengaged responses (Curran, 2016). Unengaged responses typically yield a variance of $0.00 \leq V \leq 0.50$ so that no variance demonstrated, given a small survey, indicate the same response for each question (Huang et al., 2015) and when the variance was ≤ 0.50 it was observed that students had answered questions in a pattern such as 1, 1, 1, 6, 6, 6, 2, 2, 2, 5, 5, 5, etc. (Marjanovic et al., 2015). Data cleaning resulted in removal of five secondary student and twenty-two tertiary student responses (i.e., rows of data) for spurious and unengaged responses.

5.3.4. Missing Data Replacement

Prior to analysis, the data was analyzed to determine a methodology for handling missing data (Rubin, 1996). Even though missing data is directly proportional to a good quality statistical inference, scholars have not reached a consensus on an acceptable percentage of missing data

(Dong & Peng, 2013; Schafer, 1999). Dong states that Schafer "asserted 5% or less was inconsequential," while others, like Bennett holding to 10% (Bennett, 2001) have varied criteria (2013, p. 2). Prior to choosing a missing data post-processing procedure, missing data elements were counted and Little's Missing Completely At Random (MCAR) test (Little, 1988) was run to determine whether the missing data was randomly distributed. These steps are detailed below.

5.3.4.1. Counting Missing Data

Missing data quantities/proportions must be under certain rule-of-thumb thresholds to facilitate appropriate analysis (Clavel et al., 2014). Although gender and age were not used for the EFA and CFA preparation for the SEM, they were retained for the missing data analysis and for potential validation of student maturity findings (a hypothesized latent variable), and for possible corroboration with teacher and professor survey responses.

5.3.4.1.1. Secondary Student Missing Data

There were 22 total questions on the secondary student survey taken by 560 students. Thus, there were 12,320 responses possible if all questions had been answered by all students. Data analysis revealed that the secondary students provided 12,249 (99%) responses, (total number of questions multiplied by total students) leaving approximately 1% of the data elements missing. Table 5.2 shows the missing data of the secondary students indicating percentage and variables where data is missing. There is missing data in all the variables except V17 and v23, but in all the data the percentage of missing data was well within acceptable limits according to the literature (Abir et al., 2021; Bennett, 2001; Dong & Peng, 2013). The question about gender had the highest level of missing data on the secondary student survey. This contrasted with the tertiary student survey which had no missing gender data, leading to interest in why so many secondary students had omitted it. In the final analysis, however, since gender carried no weight in the calculation of student mathematical maturity (Steinberg et al., 1989; Zenkl, 2021), the anomaly was not pursued. A similar anomaly was also seen in the secondary and tertiary response to age with no tertiary students omitting age and 1.8% of secondary students omitting it. Since age was of no statistical value in secondary school data because it is controlled (by grade level), this anomaly was also ignored for further data processing. Question V21, "My parents like the math textbook," which had 2% missing responses was probably due to students' parents having inadequate knowledge to personally assist with the mathematics homework—a speculation reinforced by responses to Question V13, "My parents are/have been able to help me with my year 12 math homework," showing that the majority of students disagreed with the statement (see discussion in Section 5.3.5.1).

Table 5.2 Secondary Student Missing Data Statistics

Variables	Survey Questions	N	Missing	
			Count	Percent
Gender		544	16	2.9
Age		550	10	1.8
V5	I was ready for year 12 math	559	1	0.2
V6	I am going to do well in the final exams	559	1	0.2
V7	My mid-year progress was better than I expected	559	1	0.2
V8	Year 11 math prepared me very well for year 12 math	556	4	0.7
V9	The year 12 textbook is too complicated	558	2	0.4
V10	Handouts were better, sometimes, than the textbook	554	6	1.1
V11	My year 12 math teacher uses the textbook and refers to it in class	558	2	0.4
V12	My year 12 math textbook examples helped me understand the topic	559	1	0.2
V13	My parents are/have been able to help me with my year 12 math homework	558	2	0.4
V14	The chapters in the textbook follow each other pretty well	557	3	0.5

Table 5.2 (continued)

Variables	Survey Questions	N	Missing	
			Count	Percent
V15	My teacher likes the textbook	559	1	0.2
V16	I have regular help with my math homework	559	1	0.2
V17	I often bring the mathematics textbook/written materials home or to my study location	560	0	0
V18	If there were more problems in the textbook, I would practice more	559	1	0.2
V19	I do not need to ask the teacher for homework help	559	1	0.2
V20	There is enough detail in the textbook to master the topics	556	4	0.7
V21	My parents like the math textbook	549	11	2
V22	I prefer the notes from the teacher than from the textbook	558	2	0.4
V23	Without the teacher, the textbook would be useless	560	0	0
V24	With textbook only (no teacher) I could understand the topics clearly	559	1	0.2

5.3.4.1.2. Tertiary Student Missing Data

Statistical analysis was done on 22 questions answered by 2,154 tertiary students. There were 47,388 responses possible if all questions had been answered by all students. Data analysis revealed that the tertiary students provided 46,307 (97.7%) responses, leaving 2.3% of the data elements missing. Table 5.3 shows the missing data of the tertiary students indicating percentage and variables where data is missing. Only gender and age questions had no missing data.

Table 5.3 Tertiary Student Missing Data Statistics

Variables	Survey Questions	N	Missing	
			Count	Percent
Gender		2154	0	0
Age		2154	0	0
V1	Year 12 mathematics was very good preparation for this course	2109	45	2.1
V2	12th grade mathematics textbook needed more depth	1624	530	24.6
V3	Without the teacher, my 12th grade textbook would have been useless	2101	53	2.5

Table 5.3 (continued)

Variables	Survey Questions	N	Missing	
			Count	Percent
V4	There was too much homework in my 12th grade mathematics class	2098	56	2.6
V5	There were enough exercises in 12th grade textbook for me to be well practiced	2103	51	2.4
V6	12th grade Handouts were better, sometimes, than the textbook	2100	54	2.5
V7	My 12th grade math teacher used the textbook and referred to it in class	2102	52	2.4
V8	My 12th grade math book examples helped me understand the topic	2099	55	2.6
V9	The textbook/written material examples help me understand the topic	2138	16	0.7
V10	The chapters in the textbook/written materials follow each other pretty well	2137	17	0.8
V11	My progress so far is better than I expected	2134	20	0.9
V12	I have regular help with my mathematics	2145	9	0.4
V13	I often bring the mathematics textbook/written materials home or to my study location	2142	12	0.6
V14	The teacher uses the textbook/written materials and refers to it in class	2140	14	0.6
V15	I am going to do very well in the terminal exams	2143	11	0.5
V16	There is enough detail in the textbook/written materials to master the topics	2137	17	0.8
V17	I was ready for mathematics this year	2140	14	0.6
V18	Extra handouts are sometimes better than the textbook/written materials	2128	26	1.2
V19	Without the teacher, the textbook/written materials would be useless	2138	16	0.7
V20	With textbook/written materials only (no lectures) I could understand the topics clearly	2141	13	0.6

The missing data percentages for secondary and tertiary surveys are well within the limits prescribed for appropriate application of a missing data algorithm and data replacement (Dong & Peng, 2013) with the possible exception of tertiary student question V2 (“12th grade mathematics textbook needed more depth”) which has 530 missing responses—well above the 5% threshold. The large number of missing responses to V2 was of interest as a possible error on coding responses but was determined to be most likely related to the fact that university students (unlike secondary students) could have taken the survey many years after graduating. Table 4.3 indicates that 139 tertiary students had graduated from high school in the 1960s through 2000s (from 4 years to 44 years before they took the survey). Additionally, numerous tertiary student paper and pencil survey sheets contained comments relating to the fact that they had graduated many years earlier and could not remember their year-12 textbook.

5.3.4.2. Determining Missing Data Randomness with MCAR

When data is missing completely at random, there are numerous procedures for replacing it. To determine whether missing data was completely at random, Little’s MCAR test was run against both the secondary and tertiary student data. Under Little’s MCAR rubric, if Chi Square significance is less than .05, then we reject the NULL hypothesis that the missing data is random (Little, 1988). When Little’s MCAR was run against all the secondary student variables (Table 5.4), it was discovered that the Estimated Marginal (EM) Means indicated the Chi Square significance was 0.000; thus, it was determined that the missing data in the secondary student responses is not random.

Table 5.4 Little’s MCAR Results for Secondary Student Data

V5	V6	V7	V8	V9	V10	V11	V12	V13	V14
2.3	2.5	3.0	2.7	3.7	2.8	1.8	2.6	4.7	3.2

V15	V16	V17	V18	V19	V20	V21	V22	V23	V24
2.9	3.9	2.0	3.7	3.8	3.5	3.7	2.2	2.7	3.9

Little's MCAR test: Chi-Square = 651.971, DF = 509, Sig. = .000

When Little’s MCAR was run against all the tertiary student variables (Table 5.5), it was discovered that the EM Means indicated the Chi Square significance was 0.000; thus, it was determined that the missing data in the tertiary student responses was not random.

Table 5.5 Little's MCAR Results for Tertiary Student Data

V1	V2	V3	V4	V5	V6	V7	V8	V9	V10
2.6	3.2	2.9	3.8	2.6	2.6	2.4	2.7	2.8	2.7

V11	V12	V13	V14	V15	V16	V17	V18	V19	V20
3.1	3.5	3.0	3.3	2.9	3.1	2.7	2.5	2.7	3.9

Little's MCAR test: Chi-Square = 1635.204, DF = 891, Sig. = .000

5.3.4.3. Selecting the Missing Data Algorithm

Because the secondary and tertiary student data was not missing completely at random, it was determined that a multiple imputation (MI) procedure was likely optimal for replacing the missing data (Jakobsen et al., 2017). However, it was first necessary to ensure that the missing data did not conform to adjacent groupings (i.e., were non-monotonic) (Zhang, 2003). By using the data imputation analysis tools, it was possible to determine that the missing data values as seen in Figures 5.1 and 5.2 below were non-monotonic for both secondary and tertiary data.

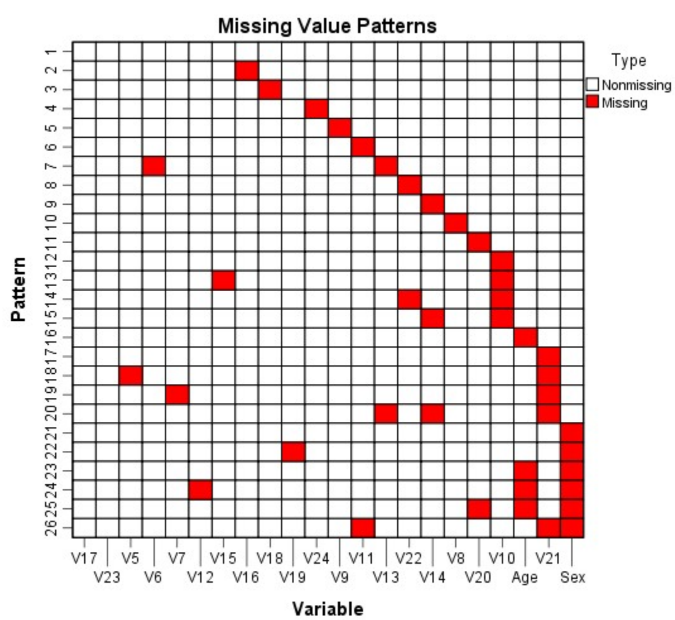


Figure 5.1 Non-Monotonic Secondary Student Missing Data Patterns

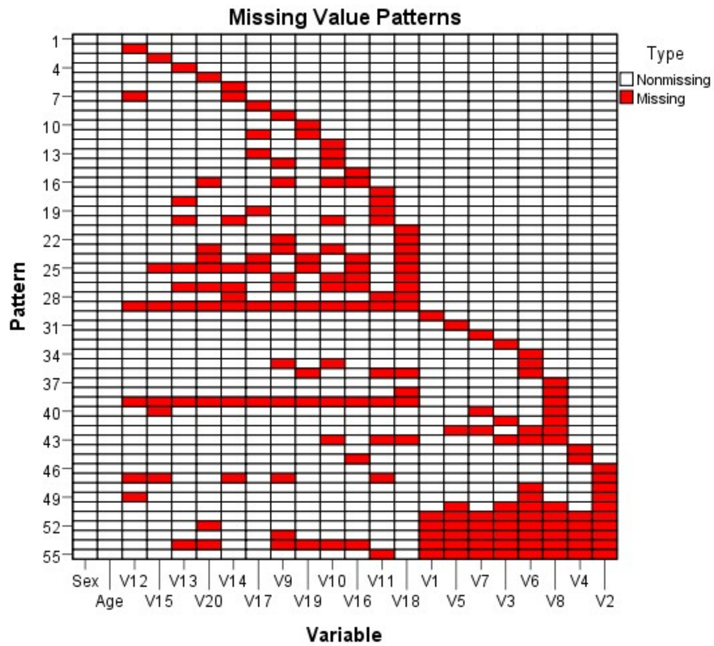


Figure 5.2 Non-Monotonic Tertiary Student Missing Data Patterns

Since the data is non-monotonic, the utilization of a Markov Chain Monte Carlo (MCMC) method of multiple imputation is warranted (Gilks et al, (Eds.) 1996, as cited in Zhang, 2003). SPSS v20.0 was selected as the tool for implementing multiple imputation on these data sets.

5.3.4.4. Missing Data Replacement

Table 5.6 illustrates the secondary student data after data cleaning and prior to missing data replacement.

Table 5.6 Secondary Student Cleaned Data Before Missing Data Replacement

	Gender	**Age**	**V5**	**V6**	**V7**	**V8**	**V9**	**V10**	**V11**	**V12**	**V13**
Valid	539	545	552	552	552	549	551	547	551	553	551
Missing	14	8	1	1	1	4	2	6	2	0	2

	V14	**V15**	**V16**	**V17**	**V18**	**V19**	**V20**	**V21**	**V22**	**V23**	**V24**	**Δ**
Valid	550	552	552	553	552	552	549	542	551	553	552	12100
Missing	3	1	1	0	1	1	4	11	2	0	1	66

Table 5.7 illustrates the tertiary student data after data cleaning and prior to missing data replacement.

Table 5.7 Tertiary Student Cleaned Data Before Missing Data Replacement

	Gender	Age	V1	V2	V3	V4	V5	V6	V7	V8	V9
Valid	2132	213	208	160	207	207	208	207	208	207	211
Missing	0	0	45	526	53	56	51	54	52	55	16

	V10	V11	V12	V13	V14	V15	V16	V17	V18	V19	V20	Δ
Valid	2115	211	212	212	211	212	211	211	210	211	211	4582
Missing	17	20	9	12	14	11	17	14	26	16	13	1077

5.3.4.5. Missing Data Replaced

Five imputation data sets were created during MCMC Multiple Imputation procedure (MCMC MI) and Cronbach's Alpha statistic computed for each as shown in 5.4.1 and 5.4.2, the result being one imputation dataset with the highest Cronbach's Alpha selected from the individual imputed datasets of secondary and tertiary student data.

5.3.5. Statistical Data Analysis

After data replacement and imputation choices were completed for the secondary and tertiary students, the data was analyzed for normality using skewness and kurtosis. Per Hair and Byrne, skewness in a range of -2 to +2 and kurtosis in a range of -7 to +7 indicates data normality (Byrne, 2010; Hair et al., 2010). Data was also analyzed for centrality and spread. Per Triola (2010), for standard normal distribution, 68% of the scores need to be to be within ±1 standard deviation of the mean (1SD) and 95% of the scores need to be within ± 2 standard deviations of the mean (2SD).

5.3.5.1. Secondary Student Data

Table 5.8 shows that skewness and kurtosis for all variables on the secondary student survey were within acceptable ranges. With the exception of gender (a binary response) and age, the rest of the analyzed student responses were collected via a Likert, 6-gradation scale, where 1="Strongly Agree" and 6="Strongly Disagree." This permitted an interpretation of the average (centrality) of these scores to exhibit a tendency for agreement or disagreement where less than 3.5 indicates agreement and greater than 3.5 indicates disagreement. For variables (questions) V5 – V24 on Table 5.8, the Mean column indicated a mixture of agreement and disagreement with some sharp distinction. For example, question V13, "My parents are/have been able to help me with my year 12 math homework" showed strong disagreement at 4.67 and question V11, "My year 12 math teacher uses the textbook and refers to it in class" showed sharp agreement at 1.83. The spread of these scores was small, with a normalized score of 57% within 1SD and 94.7% within 2SD, and this is consistent with data that is distributed normally.

Table 5.8 Pre-MI Secondary Student Descriptive Statistics

Variables	N	Minimum	Maximum	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Gender	544	0	1	.56	.496	-.260	.105	-1.939	.209
Age	550	17	18	17.51	.500	-.058	.104	-2.004	.208
V5	559	1	6	2.25	1.012	.733	.103	.609	.206
V6	559	1	6	2.52	.996	.258	.103	-.270	.206
V7	559	1	6	2.99	1.083	.189	.103	-.116	.206
V8	556	1	6	2.68	1.184	.486	.104	-.240	.207
V9	558	1	6	3.72	1.260	-.430	.103	-.468	.206
V10	554	1	6	2.77	1.346	.391	.104	-.723	.207
V11	558	1	6	1.83	.996	1.340	.103	1.761	.206
V12	559	1	6	2.59	1.228	.736	.103	.110	.206
V13	558	1	6	4.67	1.304	-1.125	.103	.759	.206
V14	557	1	6	3.17	1.121	.222	.104	-.568	.207
V15	559	1	6	2.89	1.374	.624	.103	-.280	.206
V16	559	1	6	3.13	1.325	.207	.103	-.589	.206
V17	560	1	6	1.97	1.283	1.499	.103	1.715	.206
V18	559	1	6	3.65	1.448	-.160	.103	-.888	.206
V19	559	1	6	3.81	1.269	-.277	.103	-.464	.206
V20	556	1	6	3.46	1.256	.016	.104	-.555	.207
V21	549	1	6	3.71	1.171	.304	.104	-.072	.208
V22	558	1	6	2.24	1.279	.929	.103	.228	.206
V23	560	1	6	2.71	1.397	.501	.103	-.693	.206
V24	559	1	6	3.89	1.222	-.281	.103	-.469	.206
Valid N (listwise)	510								

5.3.5.2. Tertiary Student Data

Table 5.9 shows that skewness and kurtosis for all variables on the tertiary student survey were within acceptable ranges. With the exception of gender (a binary response) and age, the rest of the analyzed student responses were collected via a Likert, 6-gradation scale, where 1="Strongly Agree" and 6="Strongly Disagree." This permitted an interpretation of the average (centrality) of these scores to exhibit a tendency for agreement or disagreement where less than 3.5 indicates agreement and greater than 3.5 indicates disagreement. As in the secondary student data, the Mean column in Table 5.9 indicates a mixture of agreement and disagreement, however the differences are less distinct, ranging from 2.48 (agreement) to 3.99 (disagreement). Throughout the responses the spread of these scores was smaller than the secondary data with a normalized score of 70% within 1SD and 90% within 2SD; this is also consistent with data that is distributed normally.

Table 5.9 Skewness and Kurtosis for Tertiary Student Data

Variables	N	Minimum	Maximum	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Gender	2154	0	8	.77	.599	5.215	.053	66.683	.105
Age	2154	8	18	17.85	1.121	-8.516	.053	71.866	.105
V1	2109	1	6	2.66	1.373	.636	.053	-.337	.107
V2	1624	1	6	3.24	1.212	.150	.061	-.370	.121
V3	2101	1	6	2.97	1.407	.305	.053	-.756	.107
V4	2098	1	6	3.88	1.179	-.250	.053	-.131	.107
V5	2103	1	6	2.63	1.253	.651	.053	-.196	.107
V6	2100	1	6	2.66	1.267	.545	.053	-.306	.107
V7	2102	1	6	2.48	1.378	.819	.053	-.167	.107
V8	2099	1	6	2.76	1.340	.526	.053	-.431	.107
V9	2138	1	6	2.89	1.197	.499	.053	-.195	.106
V10	2137	1	6	2.72	1.079	.619	.053	.220	.106
V11	2134	1	6	3.15	1.208	.309	.053	-.388	.106
V12	2145	1	6	3.52	1.393	-.100	.053	-.905	.106

Table 5.9 (continued)

Variables	N	Minimum	Maximum	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
V13	2142	1	6	3.02	1.648	.416	.053	-1.113	.106
V14	2140	1	6	3.30	1.616	.173	.053	-1.183	.106
V15	2143	1	6	2.98	1.183	.370	.053	-.162	.106
V16	2137	1	6	3.18	1.228	.292	.053	-.477	.106
V17	2140	1	6	2.79	1.349	.553	.053	-.420	.106
V18	2128	1	6	2.54	1.150	.530	.053	-.121	.106
V19	2138	1	6	2.72	1.384	.522	.053	-.542	.106
V20	2141	1	6	3.99	1.331	-.329	.053	-.603	.106
Valid N (listwise)	1514								

5.3.6. Assessing Outliers

Part of the process of validating data normality is to apply a standard Z-Score to the data and then look for Z-scores that fall outside a prescribed interval with the outlier Z-Score at 95% two-tailed, meaning 0.4750 or z=1.96. Thus, any scores less than -1.96 or greater than 1.96 indicated an outlier (Liu, 2011). In the secondary data, only one outlier record was identified and removed; in that case, 7 of 20 questions fell outside the 1.96 limit. In the tertiary data, no outliers were found.

5.3.7. Statistical Tests after Missing Data Replacement

Once missing data has been replaced, it is necessary to ensure that additions have not statistically significantly altered the original data sets. Table 5.10 shows Independent Samples t-test results comparing means and variances before and after missing data replacement for the secondary student survey. Each variable is shown with “Equal Variance Assumed” (EVA) and “Equal Variance Not Assumed” (EVNA).

Table 5.10 Secondary Student Survey Independent Samples T-Test Results

Variables		Levene's Test F	Levene's Test Sig.	t-test for Equality of Means t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of Diff. Lower	Upper
V5	EVA	.000	.993	.026	1117	.980	.002	.061	-.117	.120
	EVNA			.026	1116.995	.980	.002	.061	-.117	.120
V6	EVA	.001	.978	.040	1117	.968	.002	.060	-.115	.119
	EVNA			.040	1116.999	.968	.002	.060	-.115	.119
V7	EVA	.005	.946	.050	1117	.960	.003	.065	-.124	.130
	EVNA			.050	1117.000	.960	.003	.065	-.124	.130
V8	EVA	.000	.994	.070	1114	.945	.005	.071	-.134	.144
	EVNA			.070	1113.944	.945	.005	.071	-.134	.144
V9	EVA	.000	.993	-.006	1116	.996	.000	.075	-.148	.147
	EVNA			-.006	1115.984	.996	.000	.075	-.148	.147
V10	EVA	.005	.941	-.017	1112	.986	-.001	.081	-.160	.157
	EVNA			-.017	1111.818	.986	-.001	.081	-.160	.157
V11	EVA	.020	.888	-.034	1116	.973	-.002	.060	-.119	.115
	EVNA			-.034	1115.977	.973	-.002	.060	-.119	.115
V12	EVA	.000	.998	.025	1117	.980	.002	.073	-.142	.146
	EVNA			.025	1116.995	.980	.002	.073	-.142	.146
V13	EVA	.000	.983	.025	1116	.980	.002	.078	-.151	.155
	EVNA			.025	1115.972	.980	.002	.078	-.151	.155
V14	EVA	.000	.998	.003	1115	.998	.000	.067	-.131	.132
	EVNA			.003	1114.967	.998	.000	.067	-.131	.132
V15	EVA	.001	.973	-.002	1117	.998	.000	.082	-.161	.161
	EVNA			-.002	1116.992	.998	.000	.082	-.161	.161
V16	EVA	.000	.985	.017	1117	.986	.001	.079	-.154	.157
	EVNA			.017	1116.994	.986	.001	.079	-.154	.157
V17	EVA	.000	1.000	0.000	1118	1.000	0.000	.077	-.150	.150
	EVNA			0.000	1118.000	1.000	0.000	.077	-.150	.150
V18	EVA	.001	.978	-.012	1117	.991	-.001	.087	-.171	.169
	EVNA			-.012	1116.993	.991	-.001	.087	-.171	.169
V19	EVA	.009	.925	.065	1117	.948	.005	.076	-.144	.154
	EVNA			.065	1116.997	.948	.005	.076	-.144	.154
V20	EVA	.006	.936	-.109	1114	.913	-.008	.075	-.156	.139
	EVNA			-.109	1113.971	.913	-.008	.075	-.156	.139
V21	EVA	.024	.876	-.003	1107	.998	.000	.070	-.138	.137
	EVNA			-.003	1106.266	.998	.000	.070	-.138	.137
V22	EVA	.003	.958	.021	1116	.983	.002	.076	-.148	.152
	EVNA			.021	1115.974	.983	.002	.076	-.148	.152

Table 5.10 (continued)

Variables		Levene's Test F	Levene's Test Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of Diff. Lower	95% Confidence Interval of Diff. Upper
V23	EVA	.000	1.000	0.000	1118	1.000	0.000	.083	-.164	.164
	EVNA			0.000	1118.000	1.000	0.000	.083	-.164	.164
V24	EVA	.001	.981	.030	1117	.976	.002	.073	-.141	.146
	EVNA			.030	1116.996	.976	.002	.073	-.141	.146

Note that the dataset indicates that the Levene's significance for V5-V24 is all > 0.05 and the Significance values for V5-V24 for the means are all greater than 0.05.

Table 5.11 shows Independent Samples t-test results comparing means and variances before and after missing data replacement for the tertiary student survey. Each variable is shown with "Equal Variance Assumed" (EVA) and "Equal Variance Not Assumed" (EVNA).

Table 5.11 Tertiary Student Survey Independent Samples T-Test Results

Variables		Levene's Test F	Levene's Test Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of Diff. Lower	95% Confidence Interval of Diff. Upper
V1	EVA	.029	.864	.142	4261	.887	.006	.042	-.076	.088
	EVNA			.142	4258.794	.887	.006	.042	-.076	.088
V2	EVA	.147	.702	-.959	3776	.338	-.038	.040	-.116	.040
	EVNA			-.958	3480.285	.338	-.038	.040	-.116	.040
V3	EVA	.017	.897	-.264	4253	.791	-.011	.043	-.096	.073
	EVNA			-.264	4250.208	.791	-.011	.043	-.096	.073
V4	EVA	.000	.993	.015	4250	.988	.001	.036	-.070	.071
	EVNA			.015	4246.952	.988	.001	.036	-.070	.071
V5	EVA	.027	.871	-.082	4255	.934	-.003	.038	-.078	.072
	EVNA			-.082	4251.887	.934	-.003	.038	-.078	.072
V6	EVA	.006	.936	.086	4252	.932	.003	.039	-.073	.080
	EVNA			.086	4249.388	.932	.003	.039	-.073	.080
V7	EVA	.018	.893	-.131	4254	.896	-.006	.042	-.088	.077
	EVNA			-.131	4251.010	.896	-.006	.042	-.088	.077
V8	EVA	.005	.942	-.208	4251	.835	-.009	.041	-.089	.072
	EVNA			-.208	4248.240	.835	-.009	.041	-.089	.072

Table 5.11 (continued)

Variables		Levene's Test		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of Diff.	
									Lower	Upper
V9	EVA	.002	.964	.022	4290	.982	.001	.037	-.071	.072
	EVNA			.022	4289.716	.982	.001	.037	-.071	.072
V10	EVA	.001	.978	.034	4289	.973	.001	.033	-.063	.066
	EVNA			.034	4288.681	.973	.001	.033	-.063	.066
V11	EVA	.005	.945	.008	4286	.994	.000	.037	-.072	.073
	EVNA			.008	4285.728	.994	.000	.037	-.072	.073
V12	EVA	.017	.896	-.022	4297	.983	-.001	.042	-.084	.082
	EVNA			-.022	4296.856	.983	-.001	.042	-.084	.082
V13	EVA	.000	.990	.085	4294	.933	.004	.050	-.094	.103
	EVNA			.085	4293.906	.933	.004	.050	-.094	.103
V14	EVA	.002	.964	.105	4292	.916	.005	.049	-.092	.102
	EVNA			.105	4291.899	.916	.005	.049	-.092	.102
V15	EVA	.002	.967	.027	4295	.979	.001	.036	-.070	.072
	EVNA			.027	4294.903	.979	.001	.036	-.070	.072
V16	EVA	.004	.948	.053	4289	.957	.002	.037	-.071	.075
	EVNA			.053	4288.682	.957	.002	.037	-.071	.075
V17	EVA	.001	.975	-.088	4292	.930	-.004	.041	-.084	.077
	EVNA			-.088	4291.890	.930	-.004	.041	-.084	.077
V18	EVA	.002	.962	-.039	4280	.969	-.001	.035	-.070	.068
	EVNA			-.039	4279.529	.969	-.001	.035	-.070	.068
V19	EVA	.005	.946	-.103	4290	.918	-.004	.042	-.087	.079
	EVNA			-.103	4289.841	.918	-.004	.042	-.087	.079
V20	EVA	.000	.991	.046	4293	.963	.002	.041	-.078	.081
	EVNA			.046	4292.831	.963	.002	.041	-.078	.081

Note that the dataset indicates that the Levene's significance for V1-V20 is all > 0.05 and the significance values for V5-V24 for the means are all greater than 0.05. It was concluded that the data replacement via MI had not significantly changed the secondary or tertiary student data and that, therefore, the MI dataset could be used for EFA.

5.4. Exploratory Factor Analysis (EFA)

EFA is intended to mine through data to locate a set of latent factors that identify connections implied (but not expressed) in the observed data. EFA is, in a sense, data analysis exploration. The EFA procedure reveals the need for variable elimination and/or reverse coding that are done in preparation for Confirmatory Factor Analysis (CFA) which confirms commonality of latent factors and necessary preparation for Structured Equation Modelling (SEM) that enables a theoretical model to be tested.

Chapter 3 detailed the development of the survey instrument constructed to validate the theoretical model developed to explain why year-12 students, despite being confident that they were ready for advanced mathematical studies, were unprepared to succeed in year-13 university calculus courses. The remainder of this chapter details the EFA conducted to explore whether the six factors proposed in the theoretical model shown in Figure 3.6 can explain at least 50% of the common variation in the observed factors. It should be noted that the theoretical model was based on secondary student data only; however, because the tertiary student survey responses will be helpful in analyzing the factor connections between the latent secondary and latent tertiary factors, the chapter also includes details on the EFA conducted on tertiary student data.

Secondary Student EFA

SPSS Reliability was run against the cleansed secondary student data set described in 5.3 above to determine the best Alpha valued imputation. This data set uses the multiple imputation iterations (0,5).

Cronbach's Alpha for Imputations 1-5

Imputation 1: 0.752

Imputation 2: 0.754

Imputation 3: 0.757

Imputation 4: 0.754

Imputation 5: 0.756

Imputation 0 does not have MI data replacement: Alpha = 0.750. Imputation 3 has the highest Alpha Value and is best suited to the next steps in the factor definitions. Moreover, as seen from Table 5.1, reliability has improved through data cleansing and missing data replacement. Therefore, Imputation 3 is chosen for EFA. Appendix D details the iteration strategy through ProMax and Varimax rotation schemes to systematically hone the EFA such that variable cross-loading was minimized, and high probability factors would present themselves. By increasing the suppression of coefficients from <0.33 to < 0.41, all cross-loadings were hidden. Table 5.12 displays those loadings.

Table 5.12 Imputation 3 Pattern Matrix by ProMax Rotation Coefficient Suppression < 0.41

Variables	Component					
	1	2	3	4	5	6
V5			.836			
V6			.691			
V7			.599			
V8			.662			
V9	.552					
V10	.756					
V11		.706				
V12		.470				
V13					.876	
V14		.423				
V15		.779				
V16R				.663		
V17						.628
V18						.848
V19				.891		
V20		.412				
V21					.721	
V22	.861					
V23	.698					
V24				.517		

After removing from view the cross-loaded variables, the following six factors were more clearly presented:

- Factor 1: V9, V10, V22, V23
- Factor 2: V11, V12, V14, V15, V20
- Factor 3: V5, V6, V7, V8
- Factor 4: V16R, V19, V24
- Factor 5: V13, V21
- Factor 6: V17, V18

5.4.1.1. Secondary Student EFA-Directed Latent Variable Operationalization

The secondary student factors with their associated variables derived from the EFA are:

Factor 1: Student Textbook Dislike (STD)

V9/Q5. The year 12 textbook is too complicated.

V10/Q6. Handouts were better, sometimes, than the textbook.

V22/Q18. I prefer the notes from the teacher than from the textbook.

V23/Q19. Without the teacher, the textbook would be useless.

As the experiment progressed, this variable was reversed to *Student Textbook Like* and combined with *Student Textbook Trust* to become *Student Textbook Comfort* (Section 6.3.1).

Factor 2: Student Textbook Trust (STT)

V11/Q7. My year 12 math teacher uses the textbook and refers to it in class.

V12/Q8. My year 12 math textbook examples helped me understand the topic.

V14/Q10. The chapters in the textbook follow each other pretty well.

V15/Q11. My teacher likes the textbook.

V20/Q16. There is enough detail in the textbook to master the topics.

The concept implied by this grouping of questions is that students trust their textbook. In cases where students are using rigorous textbooks, this would be a positive factor; where textbooks are not rigorous, students are given a false sense of readiness for year-13 calculus. As the experiment progressed, this variable was combined with *Student Textbook Like* to become *Student Textbook Comfort* (Section 6.3.1).

Factor 3: Student Perceived Readiness (SPR)

V5/Q1. I was ready for year 12 math.

V6/Q2. I am going to do well in the final exams.

V7/Q3. My mid-year progress was better than I expected.

V8/Q4. Year 11 math prepared me very well for year 12 math.

This latent variable emerges from questions relating to year-12 students' perceptions of their readiness for year-12 mathematics. In cases where students' year-11 preparation was rigorous, this would be a positive factor; where it was not rigorous, students would have a false sense of readiness for year-12 precalculus.

Factor 4: Student Mathematical Maturity (SMM)

V16R/Q12R. I (*do not*) have regular help with my math homework.

V19/Q15. I do not need to ask the teacher for homework help.

V24/Q20. With textbook only (no teacher) I could understand the topics clearly.

This latent variable emerges from questions relating to year-12 students' mathematical maturity as measured by the extent to which they needed external help beyond their textbook to complete homework and master topics.

Factor 5: Student Parental Involvement (SPI)

V13/Q9. My parents are/have been able to help me with my year 12 math homework.

V21/Q17. My parents like the math textbook.

The underlying assumption behind these questions was that even a parent without a strong mathematical background would be able to help their child if the textbook was rigorous enough.

Factor 6: Student Textbook Use (STU)

V17/Q13. I often bring the mathematics textbook/written materials home or to my study location.

V18/Q14. If there were more problems in the textbook, I would practice more.

This variable also indirectly reflects students' valuing of, trust in, and comfort with their textbook.

The secondary student EFA reveals potential latent variables that are derived from the observed data. This derivation, or construct, serves as the shared (EFA vs. *a priori* 6 factor model) starting point for the secondary student CFA detailed in Chapter 6.

5.4.2. Tertiary Student EFA

Though secondary and tertiary student survey questions were similar, the expectation of a model distinction between secondary and tertiary students based on literature highlighting the disparity between perceived readiness and actual readiness of first-year mathematical students, led to running a tertiary EFA to assist affirmation of secondary textbook centrality.

SPSS Reliability was run against the cleansed tertiary student data set to determine the best Alpha valued imputation. This data set uses the multiple imputation iterations (0,5).

Cronbach's Alpha for Imputations 1-5

Imputation 1: 0.754

Imputation 2: 0.754

Imputation 3: 0.752

Imputation 4: 0.753

Imputation 5: 0.755

Imputation 0 does not have MI data replacement: Alpha = 0.740. Imputation 5 has the highest Alpha Value and is likely best suited to the next steps in the factor definitions. Moreover, when compared to Table 5.1, reliability has improved through data cleansing and missing data replacement. The EFA on the Imputation 5 dataset indicates a possible 7-factor model as seen in Table 5.13.

Table 5.13 Tertiary Student Data Imputation 5 Rotated Component Matrix

Variables	Component							MAX	SD
	1	2	3	4	5	6	7		
V1	.588	-.015	.521	.023	-.011	.114	.043	0.59	0.24
V2	-.333	.174	-.330	.097	-.037	.068	.557	0.56	0.29
V3	.131	-.065	-.005	.492	.003	.187	.507	0.51	0.22
V4	.036	-.080	.162	-.103	.071	-.006	.789	0.79	0.28
V5	.378	-.066	.657	-.010	.047	.173	.058	0.66	0.24
V6	.042	-.066	-.026	.042	.015	.810	.199	0.81	0.28
V7	-.061	.133	.758	.086	-.056	-.032	.039	0.76	0.27
V8	.021	.169	.825	-.052	.029	-.082	-.050	0.83	0.30
V9	.105	.799	.144	-.128	.100	-.016	-.015	0.80	0.28
V10	.121	.776	.122	-.049	.138	.048	-.015	0.78	0.26
V11	.513	.527	.019	.219	-.090	-.122	.039	0.53	0.25
V12	-.295	.341	.032	.314	.315	.100	.077	0.34	0.21
V13	-.086	.158	.060	.020	.822	.000	.019	0.82	0.29
V14	.127	.099	-.067	-.014	.830	.008	.026	0.83	0.29
V15	.743	.350	-.056	-.009	.030	.019	-.005	0.74	0.27
V16	.357	.598	-.037	-.250	.216	.036	-.054	0.60	0.27
V17	.764	.159	.158	-.069	.022	.108	-.056	0.76	0.26
V18	.064	.095	.042	.123	.007	.814	-.096	0.81	0.28
V19	.116	-.014	.044	.789	.089	.081	.063	0.79	0.26
V20	.232	.216	.025	-.699	.103	-.029	.146	0.23	0.30

V3 ("Without the teacher, my 12th grade textbook would have been useless") and V12 ("I have regular help with my mathematics") had the lowest standard deviation (loading evenly across all factors). Models were re-run with V3 and V12 removed. Removing V12 did not enhance the model whereas elimination of V3 yielded an improved Alpha = 0.759 and a 6-factor model. Table 5.14 shows the Cronbach's Alpha value with V3 removed and Table 5.15 shows the resulting 6-factor model with V3 removed and Variance Explained.

Table 5.14 KMO and Bartlett's Test without V3

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.759
Bartlett's Test of Sphericity	Approximate Chi-Square	9399.968
	df	171
	Sig.	0.000

Table 5.15 Tertiary Student Data Imputation 5, Total Variance Explained Without V3

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.786	19.929	19.929	3.786	19.929	19.929	3.004	15.812	15.812
2	2.116	11.136	31.064	2.116	11.136	31.064	2.350	12.366	28.178
3	1.768	9.304	40.368	1.768	9.304	40.368	1.732	9.118	37.297
4	1.525	8.029	48.397	1.525	8.029	48.397	1.613	8.488	45.785
5	1.211	6.374	54.772	1.211	6.374	54.772	1.490	7.841	53.625
6	1.086	5.716	60.488	1.086	5.716	60.488	1.304	6.862	60.488
7	.997	5.249	65.737						
8	.807	4.248	69.985						
9	.772	4.064	74.048						
10	.689	3.627	77.675						
11	.615	3.237	80.912						
12	.560	2.948	83.860						
13	.532	2.801	86.661						
14	.505	2.656	89.316						
15	.466	2.453	91.770						
16	.441	2.319	94.089						
17	.403	2.123	96.212						
18	.395	2.079	98.291						
19	.325	1.709	100.000						

With a Mean of 2.9, a Median of 3, a Mode of 3, a Maximum of 6, and a Minimum of 1, it was determined that the removal of V3 is justified as a non-decisive contributor. Table 5.16 shows the Varimax Rotated Component Matrix for the 6-factor model.

Table 5.16 Tertiary Student Data Varimax Rotated Component Matrix

Variables	Component					
	1	2	3	4	5	6
V1		.601				
V2						.668
V4						.665
V5		.706				
V6				.758		
V7		.737				
V8		.810				
V9	.689					
V10	.675					
V11	.743					
V12			.398			
V13			.821			
V14			.769			
V15	.720					
V16	.677					
V17	.556					
V18				.687		
V19					.748	
V20					-.734	

Appendix D details the iteration strategy through Varimax rotation schemes to systematically hone the EFA such that cross-loading was minimized and high probability factors would present themselves. Additionally, different variables were removed and/or reversed during the rotations which continued to improve the factor loadings as seen in Table 5.17.

Table 5.17 Imputation 5 Pattern Matrix by Varimax Rotation with Coefficient Suppression < 0.395

Variables	Component					
	1	2	3	4	5	6
V1		.601				
V2						.668
V4						.665

Table 5.17 (continued)

Variables	Component					
	1	2	3	4	5	6
V5		.706				
V6				.758		
V7		.737				
V8		.810				
V9	.689					
V10	.675					
V11	.743					
V12			.398			
V13			.821			
V14			.769			
V15	.720					
V16	.677					
V17	.556					
V18				.687		
V19					.748	
V20R					.734	

The following six factors were derived from the tertiary student data EFA with cross-loading coefficient suppression < 0.395:

- Factor 1: V9, V10, V11, V15, V16, V17
- Factor 2: V1, V5, V7, V8
- Factor 3: V12, V13, V14
- Factor 4: V6, V18
- Factor 5: V19, V20R
- Factor 6: V2, V4

5.4.2.1. Contrary Initial Tertiary Student EFA Analytics

The initial EFA analysis revealed that Factor 4 posed a problem in the initial tertiary student EFA because its variables referred to both year-12 and year-13 handouts. V6 (“12th grade handouts were better, sometimes, than the textbook”) is a question asked in the secondary experience section of the tertiary student survey while V18 (“13th grade extra handouts are sometimes better than the textbook/written materials") refers to the tertiary experience. Because the experiences are not logically related in this model, either V6 or V18 needed to be omitted from the EFA. The details of the iterations of rotations, variable removals, and reversals are located in Appendix D where the final model is shown in Table 5.18.

Table 5.18 Rotated Component Matrix with Additional Reverse Coding and Loading Adjustments

Variables	Component					
	1	2	3	4	5	6
V1	.607					
V2R						.611
V4R						.855
V5			.650			
V7			.766			
V8			.822			
V9		.820				
V10		.799				
V11	.533					
V13				.820		
V14				.841		
V15	.753					
V16		.595				
V17	.791					
V19R					.822	
V20					.759	

5.4.2.2. Tertiary Student EFA-Directed Latent Variable Operationalization

Here are the tertiary student factors with their associated variables derived from the EFA: (Note that, in the tertiary student data, variable numbers and question numbers are the same. V1=Q1, etc.).

Factor 1: Perceived Readiness (PR)

V1. 12th grade math was very good preparation for college mathematics.

V11. My progress so far is better than I expected.

V15. I am going to do very well in the terminal exams.

V17. I was ready for mathematics this year.

In cases where students' year-12 preparation was rigorous, this would be a positive factor; where it was not rigorous, students would have a false sense of readiness for year-13 calculus.

Factor 2: Tertiary Student Like of Year 13 Textbook (L13T)

V9. The textbook/written material examples help me understand the topic.

V10. The chapters in the textbook/written materials follow each other pretty well.

V16. There is enough detail in the textbook/written materials to master the topics.

This latent variable emerges from questions relating to the extent to which year-13 students value their textbook.

Factor 3: Tertiary Student Like Y12 Textbook (L12T)

V5. There were enough exercises in the Y12 textbook for me to be well practiced.

V7. My 12th grade math teacher used the textbook & referred to it in class.

V8. My 12th grade math book examples helped me understand the topic.

The logic behind these questions was to explore the relationship between year-13 students' perceptions of their year-12 and year-13 textbooks.

Factor 4: Tertiary Student Y13 Textbook Use (U13T)

V13. I often bring the mathematics textbook home or to my study location.

V14. The teacher uses the textbook and refers to it in class.

This variable also indirectly reflects students' valuing of, trust in, and comfort with their textbook.

Factor 5: Tertiary Student Mathematical Maturity (M13M)

V19R. Without the teacher, the textbook would NOT be useless.

V20. With textbook only (no lectures) I could understand the topics clearly.

(Note: V19 and V20R were reversed so that the factor was positively named.)

This latent variable emerges from questions relating to year-13 students' mathematical maturity as measured by the extent to which they needed external help beyond their textbook to complete homework and master topics.

Factor 6: Tertiary Student Confidence in the Year-12 Textbook (C12T)

V2R. 12-grade mathematics textbook DID NOT need more depth.

V4R. There was NOT too much homework in my 12th-grade mathematics class.

(Note: V2 and V4 were reversed so that the factor is positively named.)

This variable indirectly measures the perceived readiness of year-13 students based on their experience with their year-12 textbook.

The tertiary student EFA has revealed potential latent variables that are derived from the observed data. This derivation, or construct, will serve as the starting point for the tertiary student CFA in Chapter 6.

5.5. Conclusion

This chapter identified six factors for both the secondary (the theoretical modeled) and tertiary (not previously modeled) student data through EFA. The following chapter will present the CFA and SEM for this data. It will also compare and contrast data from secondary students/teachers with tertiary students/professors regarding their perception of students' mathematical readiness.

Chapter 6. Survey Data Analysis – Confirmatory Factor Analysis and Structured Equation Modeling

6.1. Introduction

The chapter will explain how CFA and SEM were employed to refine the theoretical model described in Chapter 3. SEM is a multivariate statistical analysis technique that is used to test and evaluate multivariate causal relationships (Brandmaier et al., 2013; Tarka, 2017). This technique is the combination of factor analysis and multiple regression analysis. A SEM has two components. It is initialized with the CFA which yields the measurement model depicting the relationship between the latent variables and their measures. SEM is finalized with the structural equation model (path analysis) which shows how the survey data and constructs are related. In addition to explaining how CFA and SEM were used to model the survey data, this chapter also includes a section on descriptive statistics used to amplify connections between student perceived readiness and year-12 mathematical textbooks.

6.2. Chapter Organization

This chapter is divided into six sections. The first section describes the process through which CFA was applied to the secondary student data to build a measurement model and to refine the proposed model pictured in Figure 6.1. The second section details how the CFA-generated (measurement model) factor loadings and correlation coefficients were applied to the secondary student SEM. The third and fourth sections describe the same process applied to the tertiary student data. The fifth section uses descriptive statistics to highlight students' opinion of their year-12 textbook and to link secondary teacher and tertiary professor survey responses to the secondary and tertiary models. The final section details how the above processes drove alterations to the theoretical model originally proposed in Chapter 3.

6.3. Secondary Student Confirmatory Factor Analysis

CFA is a technique that is designed to confirm the validity of a theoretical model. In the CFAs to follow, the theoretical model developed for secondary student data was used to estimate a population covariance matrix and to compare that to the observed covariance matrix with the objective of minimizing the difference between the estimated and the observed covariance matrices (Schreiber et al., 2006). Figure 6.1 shows the proposed CFA model for secondary student data that will be used to confirm the theoretical model. The latent variables (i.e., unobserved variables) are designated with ellipses and the observed variables (i.e., survey answers) are designated with rectangles.

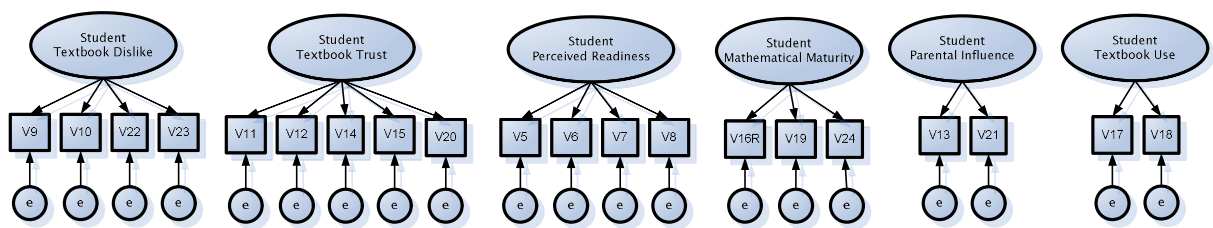


Figure 6.1 Proposed Secondary Student Data Confirmatory Factor Analysis Model

Multiple iterations of the model were experimented with in order build a confirmatory measurement model suitable for SEM analytics.

6.3.1. Development of the Measurement Model Through CFA

In preparation for SEM, multiple CFAs were run in Mplus v7.1 for measurement modeling. In the tables below, the model fit indices (Table 6.1), factor loadings (Table 6.2) and latent constructs correlations and significances (Table 6.3) are identified. Good model fit is when the Root Mean Squared Error of Approximation (RMSEA) is less than 0.08, when the Comparative Fit Index (CFI) and Tucker Lewis Index (TLI) are greater than .90, and when Standardized Root Mean Square Residual (SRMSR) is less than 0.08 (Browne & Cudeck, 1992; Hu & Bentler, 1999; Kline, 2015).

Table 6.1 Secondary Student Initial Measurement Model Fit Indices

RMSEA	Estimate	0.06	
	90% C.I.	0.05	0.07
	Prob. RMSEA <=	0.01	
CFI/TLI	CFI	0.87	
	TLI	0.83	
SRMR	Value	0.06	

Note: STD (Student Textbook Dislike) negatively correlated with STT (Student Textbook Trust) in the initial CFA and, as a result, the proposed model was modified to reverse code STD as STL (Student Textbook Like) so that each scale would be positive.

Table 6.2 Secondary Student Initial Measurement Model Factor Loadings

Latent Variable	Indicator Variable	Est. Cor.	S.E.	2-Tailed P-Value
STL	V9R	0.65	0.04	0.00
	V10R	0.54	0.04	0.00
	V22R	0.79	0.04	0.00
	V23R	0.55	0.04	0.00
STT	V11	0.32	0.05	0.00
	V12	0.73	0.03	0.00
	V14	0.51	0.04	0.00
	V15	0.40	0.04	0.00
	V20	0.63	0.03	0.00
SPR	V5	0.68	0.06	0.00
	V6	0.73	0.06	0.00
	V7	0.25	0.05	0.00
	V8	0.76	0.07	0.00
SPI	V13	0.34	0.06	0.00
	V21	0.98	0.12	0.00
STU	V17	0.64	0.08	0.00
	V18	0.50	0.07	0.00
SMM	V16R	0.28	0.06	0.00
	V19	0.47	0.05	0.00
	V24	0.74	0.06	0.00

Indicator (manifest) variables with factor loadings > 0.45 are considered fair and over 0.71 as excellent (Comrey & Lee, 1992, as cited in Davies et al., 2010). This meant that V11 and V15 loading onto STT at 0.32 and 0.40 and V13 loading onto SPI at 0.34 needed justification for retention in the model. Nevertheless, the P-value < 0.05 for all factor loadings in Table 6.2 indicate that the factor loadings, including those < 0.45, were statistically significant.

Table 6.3 Secondary Student Initial Measurement Model Latent Variable Correlations and Significances

Regressed On	Regressed	Est. Cor.	S.E.	2-Tailed P-Value
STT	STL	0.63	0.04	0.00
SPR	STL	0.14	0.05	0.01
	STT	0.33	0.05	0.00
SPI	STL	0.03	0.05	0.52
	STT	0.45	0.07	0.00
	SPR	0.25	0.05	0.00
STU	STL	0.01	0.07	0.94
	STT	0.36	0.07	0.00
	SPR	0.15	0.07	0.02
	SPI	0.24	0.07	0.00
SMM	STL	0.53	0.05	0.00
	STT	0.51	0.06	0.00
	SPR	0.23	0.07	0.00
	SPI	0.25	0.06	0.00
	STU	0.06	0.08	0.49

Note in Table 6.3 that STL is highly correlated with STT. In other words, it is apparent that STL and STT are strongly connected (r=.63), but it is unclear if Like loads onto Trust or Trust loads onto Like. It is also observed that STL and STT are loading on Student Mathematical Maturity (SMM) as comparable descriptors. Hair et al. (2017) recommends r >=.80 to drive combining variables and, while this was a consideration, the lack of clarity as to loading as stated above led to the development of a second level latent construct reflected by STL and STT. The added benefit, in spite of the lower r value helped to avoid possible multicollinearity problems when both STL and STT are included in the SEM. The second level latent construct was labelled Student Textbook Comfort (STC). In this case, "comfort" connotes the student's level of liking and trusting the textbook content to the degree that the student is able to answer the questions, understand the examples, and follow how the teacher uses the textbook. Student

Parental Influence (SPI) and Student Perceived Readiness (SPR) are also correlated moderately and similarly with SMM. Figure 6.2 shows the secondary student modified measurement model to be used for SEM with the addition of the second level latent construct, STC.

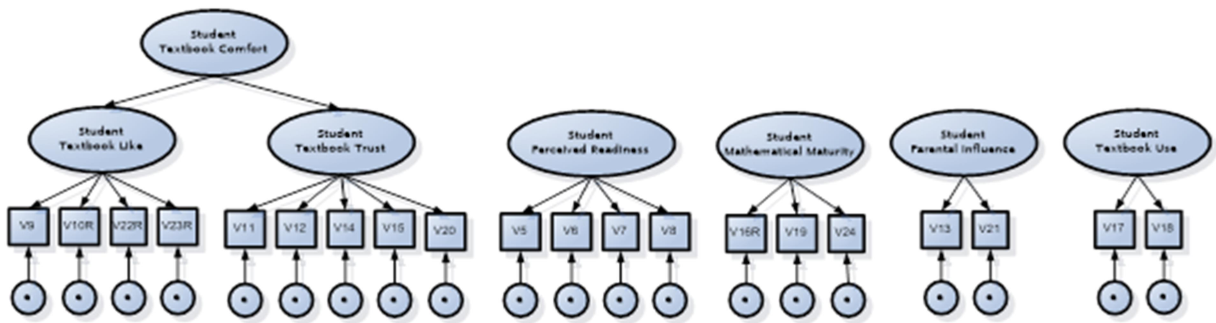


Figure 6.2 Secondary Student Modified Measurement Model

The next step in SEM preparation was to examine the constructs validity. Reliability needed to be investigated with regard to the constructs in the measurement model and this was accomplished by calculating the average variance extracted to determine Convergent Validity (CV) and then taking the square root of these averages to determine Discriminant Validity (DV). CV refers to the degree to which two measures of constructs that theoretically should be related are, in fact, related. DV tests whether concepts or measurements that are supposed to be unrelated are, in fact, unrelated (Campbell & Fiske, 1959). CV and DV were used to validate that the factors defined by the observed variables are distinct (i.e., discriminating) and that the observed variables fairly converge on the defined unobserved (or latent factors). Applying the Campbell algorithm and analytics to all the variables in the measurement model revealed that CV was fair. The second level latent variable, STC, derived from STL and STT was omitted from the CV analysis in favor of the variables with observed variables. See Table 6.4.

Table 6.4 Secondary Student Convergent and Discriminant Validity

Indicator Variables	Latent Variables	Standardized Factor Loadings (Correlations)	Square of Standardized Loadings	Sum of the Squared Standardized Loadings	Number of Indicators	Average variance Extracted = Convergent Validity (CV)	Square Root Average = Discriminant Validity (DV)	Largest Correlation Between Latent Variables (BC)	DV-BC (>0)
V9R	STL	0.65	0.42						
V10R		0.54	0.29						
V22R		0.79	0.62						
V23R		0.55	0.31	1.63	4	0.41	0.64	0.63	0.01
V11	STT	0.32	0.10						
V12		0.73	0.53						
V14		0.51	0.26						
V15		0.40	0.16						
V20		0.63	0.40	1.44	5	0.29	0.54	0.63	- [illegible]
V5	SPR	0.68	0.46						
V6		0.73	0.53						
V7		0.25	0.06						
V8		0.76	0.57	1.63	4	0.41	0.64	0.25	0.39
V16R	SMM	0.28	0.08						
V19		0.47	0.22						
V24		0.74	0.54	0.84	3	0.28	0.53	0.25	0.28
V13	SPI	0.34	0.11						
V21		0.98	0.95	1.07	2	0.53	0.73	0.24	0.49
V17	STU	0.64	0.40						
V18		0.50	0.25	0.65	2	0.33	0.57	0.36	0.21

6.3.2. Further Modifications to the Secondary Student Measurement Model

Note in Table 6.4 that V7 and V16R loaded at only 0.25 and 0.28 respectively. They were removed from the model. Table 6.5 records the CV and DV after this step.

Table 6.5 Secondary Student CV and DV with V7 and V16R Removed

Indicator Variables	Latent Variables	Standardized Factor Loadings (Correlations)	Square of Standardized Loadings	Sum of the Squared Standardized Loadings	Number of Indicators	Average variance Extracted = Convergent Validity (CV)	Square Root Average = Discriminant Validity (DV)	Largest Correlation Between Latent Variables (BC)	DV-BC (>0)
V9R	STL	0.65	0.43						
V10R		0.53	0.28						
V22R		0.80	0.64						
V23R		0.55	0.30	1.65	4	0.41	0.64	0.63	0.0

Table 6.5 (continued)

Indicator Variables	Latent Variables	Standardized Factor Loadings (Correlations)	Square of Standardized Loadings	Sum of the Squared Standardized Loadings	Number of Indicators	Average variance Extracted = Convergent Validity (CV)	Square Root Average = Discriminant Validity (DV)	Largest Correlation Between Latent Variables (BC)	DV-BC (>0)
V11		0.32	0.10						
V12		0.73	0.53						
V14	STT	0.51	0.26						
V15		0.41	0.16						
V20		0.62	0.39	1.44	5	0.29	0.54	0.63	-
V5		0.54	0.29						
V6	SPR	0.87	0.75						
V8		0.98	0.96	2.00	3	0.67	0.82	0.25	0.5
V19	SMM	0.38	0.14						
V24		0.91	0.82	0.96	2	0.48	0.69	0.25	0.4
V13	SPI	0.36	0.13						
V21		0.93	0.86	0.98	2	0.49	0.70	0.24	0.4
V17	STU	0.62	0.39						
V18		0.51	0.26	0.65	2	0.32	0.57	0.36	0.2

Though CV is not as high as the preferred ideal model, the DV is well within acceptable range. This acceptability is based on the expectation that lower factor loadings mathematically affect the CV and, following Hand et al. (2018), factor loadings ≥0.32 are expected and considered to be acceptable. Thus, the modified measurement model does have fair CV (Fornell & Larcker, 1981).

Tables 6.6 and 6.7 show the model fit indices and factor loadings for the modified measurement model with V7 and V16R removed. The modifications have brought the CFI and the TLI into the fully acceptable range for model fit.

Table 6.6 Secondary Student Modified Measurement Model Fit Indices

RMSEA	Estimate	0.05	
	90% C.I.	0.04	0.06
	Prob. RMSEA <=	0.65	
CFI/TLI	CFI	0.93	
	TLI	0.90	
SRMR	Value	0.05	

Table 6.7 Secondary Student Modified Measurement Model Factor Loadings

Latent Variable	Indicator Variable	Factor Loadings	S.E.	2-Tailed P-Value
STL by	V9R	0.65	0.04	0.00
	V10R	0.53	0.04	0.00
	V22R	0.80	0.04	0.00
	V23R	0.55	0.04	0.00
STT by	V11	0.32	0.05	0.00
	V12	0.73	0.03	0.00
	V14	0.51	0.04	0.00
	V15	0.41	0.04	0.00
	V20	0.62	0.03	0.00
SPR by	V5	0.54	0.08	0.00
	V6	0.87	0.12	0.00
	V8	0.98	0.14	0.00
SPI by	V13	0.36	0.05	0.00
	V21	0.93	0.10	0.00
STU by	V17	0.62	0.08	0.00
	V18	0.51	0.07	0.00
SMM by	V19	0.38	0.05	0.00
	V24	0.91	0.09	0.00

Factor loadings for V11 and V15 loading onto STT at 0.32 and 0.41, and V13 loading onto SPI at 0.36, and V19 loading onto SMM at 0.38 needed justification for retention in the model. The P-value < 0.05 for all factor loadings in Table 6.7 indicate that the factor loadings, including those < 0.45, were statistically significant. The justification for the retention fell into the realm of the Latent Variable and the loaded variables and the literature. V24 (With textbook only (no teacher) I could understand the topics clearly) loaded strongly (0.91) onto SMM. V19 (I do not need to ask the teacher for homework help) used different wording to ask a similar thing and, thus, the average of these factor loadings (0.645) could be used and is considered by Comrey and Lee to be very good (Comrey & Lee, 1992, as cited in Davies et al., 2010). Per (Hand et al., 2018), even factor loadings >= 0.32 can be considered significant. Using either criterion (averaging factor loadings or factor loadings ≥ 0.32), the modified measurement model is valid for subsequent use and analysis.

6.4. Secondary Student Structured Equation Model

Table 6.8 records the initial structured equation model path coefficients and significances showing that several of the regressions were not significant.

Table 6.8 Secondary Student Initial SEM Path Coefficients and Significances

Regressed On	Regressed	Path Coefficients	S.E.	2-Tailed P-Value
STT	STL	0.62	0.04	0.00
SPR	STL	0.09	0.05	0.05
	STT	0.29	0.06	0.00
SPI	STL	0.03	0.05	0.52
	STT	0.47	0.06	0.00
	SPR	0.22	0.06	0.00
STU	STL	0.01	0.07	0.85
	STT	0.37	0.07	0.00
	SPR	0.15	0.06	0.01
	SPI	0.26	0.07	0.00
SMM	STL	0.44	0.06	0.00
	STT	0.47	0.07	0.00
	SPR	0.08	0.06	0.20
	SPI	0.26	0.06	0.00
	STU	0.12	0.07	0.07

The path coefficients that were not significant (p>0.05) were removed. The SEM fit results are shown in Table 6.9.

Table 6.9 Final Secondary Student Measurement Model Fit

RMSEA	Estimate	0.05	
	90% C.I.	0.46	0.6
	Prob. RMSEA <=	0.24	
CFI/TLI	CFI	0.91	
	TLI	0.89	
SRMR	Value	0.05	

Table 6.10 lists the remaining regressions that are significant.

Table 6.10 Final Secondary Student Structural Model Results

Regressed On	Regressed	Est. Cor.	S.E.	2-Tailed P-Value
STU	SPI	0.18	0.06	0.00
	SPR	0.15	0.07	0.03
STC	SPR	0.24	0.06	0.00
	SPI	0.33	0.05	0.00
	STU	0.15	0.07	0.03
SMM	STC	0.57	0.07	0.00

As seen in Table 6.10, Student Parental Influence (SPI) and Student Perceived Readiness (SPR) are influencing Student Textbook Use (STU). Together these three latent variables are influencing Student Textbook Comfort (STC) which is influencing Student Mathematical Maturity (SMM). The paths all had significance < 0.05 and path coefficient > 0.10. This indicates that the paths are significant for asserting influential relationships between the latent (unobserved) variables. With the regressions in Tables 6.7 and 6.10 the SEM takes on the form shown in Figure 6.3.

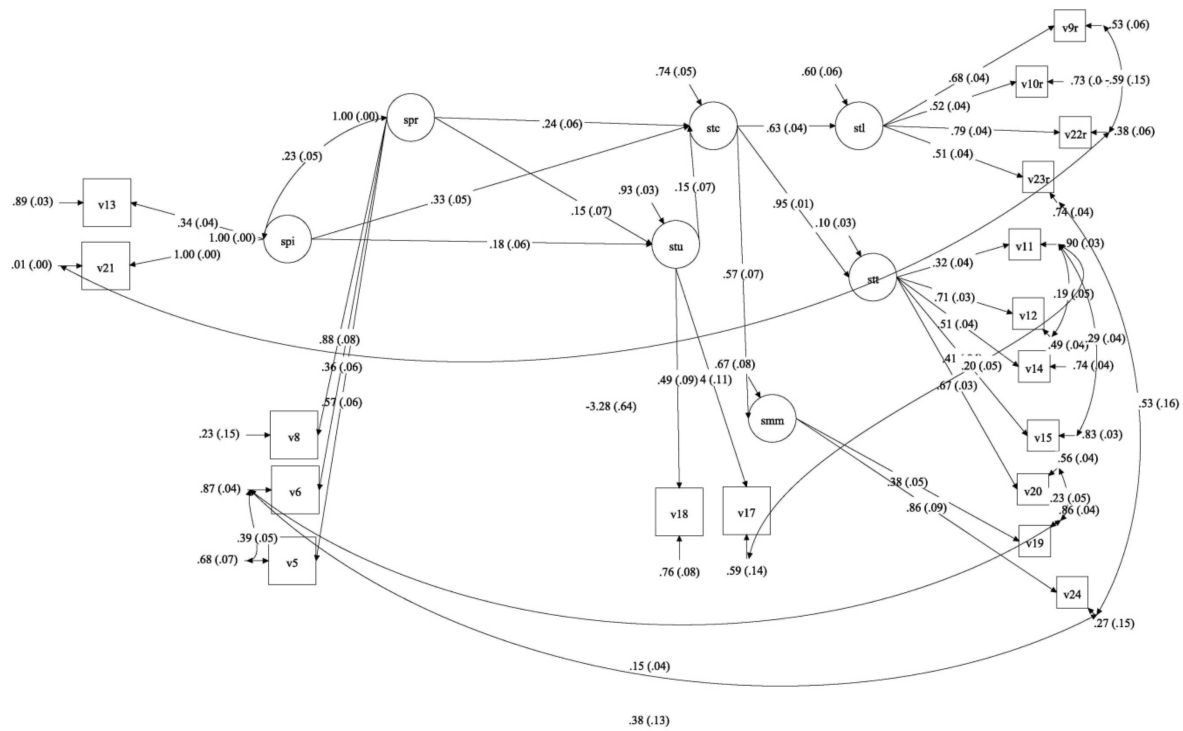


Figure 6.3 Final Secondary Student SEM diagram

Tables 6.11 and 6.12 provide the STDYX details for Figure 6.3 in tabular format.

Table 6.11 Residual Variances

Residual Variances	Est.	S.E.	2-Tailed P-Value
V5	0.68	0.07	0.00
V6	0.87	0.04	0.00
V8	0.23	0.15	0.12
V9R	0.53	0.06	0.00
V10R	0.73	0.04	0.00
V11	0.90	0.03	0.00
V12	0.49	0.04	0.00
V13	0.89	0.03	0.00
V14	0.74	0.04	0.00
V15	0.83	0.03	0.00
V17	0.59	0.14	0.00
V18	0.76	0.08	0.00
V19	0.86	0.04	0.00
V20	0.56	0.04	0.00
V21	0.01	0.00	0.00
V22R	0.38	0.06	0.00

Table 6.11 (continued)

Residual Variances	Est.	S.E.	2-Tailed P-Value
V23R	0.74	0.04	0.00
V24	0.27	0.15	0.08
STL	0.60	0.06	0.00
STT	0.10	0.03	0.00
STU	0.93	0.03	0.00
SMM	0.67	0.08	0.00
STC	0.74	0.05	0.00

Table 6.12 CFA Measurement Model Factor Loadings

Latent Variable	Observed Variables	Factor Loadings	S.E.	2-Tailed P-Value
STL by	V9R	0.68	0.04	0.00
	V10R	0.52	0.04	0.00
	V22R	0.79	0.04	0.00
	V23R	0.51	0.04	0.00
STT by	V11	0.32	0.04	0.00
	V12	0.71	0.03	0.00
	V14	0.51	0.04	0.00
	V15	0.41	0.04	0.00
	V20	0.67	0.03	0.00
SPR by	V5	0.57	0.06	0.00
	V6	0.36	0.06	0.00
	V8	0.88	0.08	0.00
SPI by	V13	0.34	0.04	0.00
	V21	1.00	0.00	0.00
STU by	V17	0.64	0.11	0.00
	V18	0.49	0.09	0.00
SMM by	V19	0.38	0.05	0.00
	V24	0.86	0.09	0.00
STC by	STL	0.63	0.05	0.00
	STT	0.95	0.01	0.00

As seen in Table 6.12, Student Textbook Like (STL) is strongly supported by the reverse coded variables seen in the table above as are all the other latent variables. The factor loadings all had significance < 0.05 and correlations > 0.30 and this is the statistical assurance that the correlations are reliable and that the factor loadings are suitable for use as seen in Table 6.13 (Cohen, 1992).

Table 6.13 Pearson Correlation Coefficient r

Effect Size	*r*
Small	0.10
Medium	0.30
Large	0.50

This led to the final first and second level latent variables for the structural model in Table 6.14 demonstrating the R-Square of the latent variables with the observed variables.

Table 6.14 Secondary Student SEM R-Square Results

Latent Variable	R-Square	Std. Err. *SE*	Sig. *p*
STL	0.396	0.056	0.000
STT	0.899	0.026	0.000
STU	0.067	0.031	0.032
SMM	0.330	0.079	0.000
STC	0.264	0.047	0.000

Notice that all R-Square values have $p < 0.05$ indicating that R-Square values have statistical significance with regard to the factor loadings seen in Table 6.12.

6.4.1. Secondary Student SEM Summary

The analysis of secondary student data revealed that all the indices (textbook use, textbook trust, parental influence, and students' perception of readiness) flowed into textbook comfort (the student use and trust of the textbook) which flowed into student mathematical maturity. Thus, it is concluded that the textbook is a positive contributor to a general mathematical maturity (i.e., the better the textbook, the better the students' mathematical maturity). Figure 6.4 shows the final secondary student path diagram with associated path coefficients.

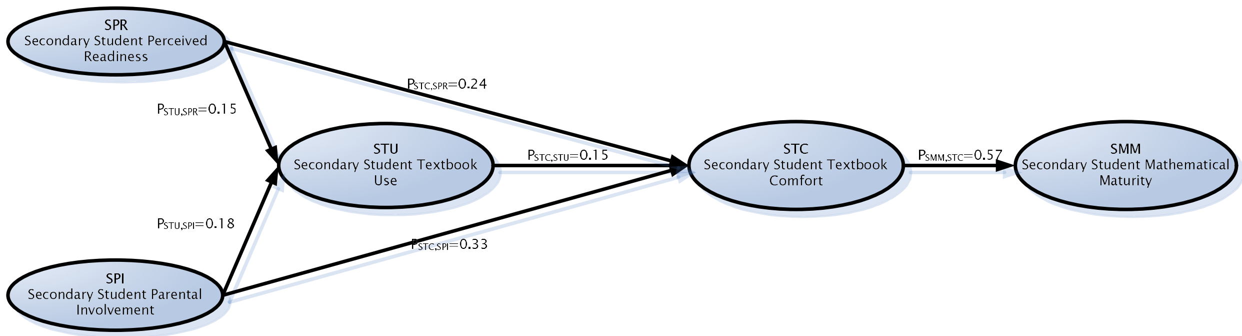


Figure 6.4 Final Secondary Student Path Diagram with Path Coefficients

The secondary path diagram (Figure 6.4) displays how latent variables are related (i.e., how they influence, or are influenced by, each other) based on the data in Table 6.10. Moving from left to right, Student Perceived Readiness (SPR) influences both Student Textbook Use (STU) and Student Textbook Comfort (STC). The path coefficient between SPR and STU is 0.15 and between SPR and STC is 0.24, indicating the comfort or trust in the textbook is influenced by the students' perceived readiness and the perceived readiness influenced the use of the textbook. STU also influenced STC with a path coefficient of 0.15 while being influenced by the student's parental involvement (SPI) at 0.18. SPI influenced STC with a path coefficient of 0.33, likely indicating that parental influence and perceived readiness influence on textbook use are catalysts for trust in the textbook as shown by the textbook comfort. This suggests textbook centrality in the mathematical experience of the student at home and in the classroom. As presented, the education of mathematics students includes the necessary mathematical topics, content, structure, and theory, and an implementation of those characteristics must include recursive building and maturing of those characteristics as the data and diagram suggest. With student mathematical maturity (SMM) influenced strongly by STC (0.57), student mathematical maturity becomes a key goal and, coupled with textbook centrality, the textbook has the responsibility to mature the student. Moreover, secondary student textbook centrality in the secondary students' curriculum is statistically significant, whether rigorous or not, is clear from the data and its graphical representation in Figure 6.4. Thus, it is concluded that the textbook is a statistically significant influencer to a general mathematical maturity (i.e., the better the textbook, the better the students' mathematical maturity and vice-versa). When textbook rigor and content presentation are out of alignment with year-13 university calculus prerequisites, the secondary student will have acquired (false) confidence (from using the non-rigorous textbook) and maturity (consistent with the textbook presentation and assessments). Parental involvement will affirm and agree with the teacher that the student is mastering the textbook material and is being rigorously prepared and made ready when, in actuality, as Chapter 7 demonstrates, many secondary textbooks give a false sense of adequate preparation because the mastery of non-rigorous textbook content does not equate to subject mastery.

Having finalized the SEM for secondary students, the next step was to analyze tertiary student data to determine whether year-12 student mathematical maturity was a contributor to tertiary student readiness for success in year-13 elementary calculus.

6.5. Tertiary Student Confirmatory Factor Analysis

6.5.1. Preparation for CFA

There was no *a priori* model for tertiary student data. The conclusion of the EFA yielded the skeleton factor configuration for the Mplus CFA. The tertiary student EFA was completed with SPSS v20.0. The final EFA model had communalities seen in Table 6.15.

Table 6.15 Tertiary Student Variable Communalities Using Principal Components Analysis Extraction

OV	Initial	Extraction
V1	1.00	0.63
V2R	1.00	0.60
V4R	1.00	0.80
V5	1.00	0.61
V7	1.00	0.62
V8	1.00	0.71
V9	1.00	0.73
V10	1.00	0.69
V11	1.00	0.59
V13	1.00	0.71
V14	1.00	0.74
V15	1.00	0.69
V16	1.00	0.60
V17	1.00	0.67
V19R	1.00	0.72
V20	1.00	0.68

The final tertiary student EFA shows the six factor loadings with 67.44% of the variance accounted for in those loadings as shown in Table 6.16.

Table 6.16 Tertiary Student Total Variance Explained Using Principal Components Analysis Extraction

Comp.	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% Variance	Cumulative %	Total	% Variance	Cumulative %	Total	% Variance	Cumulative %
1	3.76	23.49	23.49	3.76	23.49	23.49	2.34	14.63	14.63
2	2.05	12.83	36.32	2.05	12.83	36.32	2.23	13.96	28.59
3	1.48	9.24	45.55	1.48	9.24	45.55	2.15	13.44	42.03
4	1.33	8.29	53.84	1.33	8.29	53.84	1.52	9.49	51.52
5	1.10	6.88	60.73	1.10	6.88	60.73	1.41	8.80	60.32
6	1.07	6.71	67.44	1.07	6.71	67.44	1.14	7.12	67.44
7	0.78	4.89	72.32						
8	0.65	4.08	76.41						
9	0.61	3.79	80.20						
10	0.56	3.47	83.67						
11	0.54	3.35	87.02						
12	0.49	3.05	90.08						

Table 6.16 (continued)

Comp.	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% Variance	Cumulative %	Total	% Variance	Cumulative %	Total	% Variance	Cumulative %
13	0.45	2.81	92.89						
14	0.41	2.55	95.44						
15	0.40	2.51	97.94						
16	0.33	2.06	100.00						

Table 6.17 shows the EFA factor loadings with small coefficient suppression set to 0.525. Varimax with Kaiser Normalization was used. The rotation converged in eight iterations.

Table 6.17 Tertiary Student Rotated Component Matrix Using Principal Components Analysis Extraction

Observed Variables	Component					
	1	2	3	4	5	6
V1	0.61					
V2R						0.61
V4R						0.85
V5			0.65			
V7			0.77			
V8			0.82			
V9		0.82				
V10		0.80				
V11	0.53					
V13				0.82		
V14				0.84		
V15	0.75					
V16		0.60				
V17	0.79					
V19R					0.82	
V20					0.76	

6.5.2. Tertiary Student EFA Factor (Latent Variable) Naming

The six factors (latent variables) yielded by the tertiary student EFA are shown in Table 6.18 along with their associated observed variables and loadings. (Note: V2, V4, V19 and V20R were reversed so that the factor was positively named.)

Table 6.18 Tertiary Student EFA Mplus Factor Loadings

Factor Code	Factor Name	Observed Variables	Survey Question	Loadings
PR	Perceived Readiness (PR)	V1	12th grade math was very good preparation for college mathematics	0.58
		V11	My progress so far is better than I expected	0.51
		V15	I am going to do very well in the terminal exams	0.72
		V17	I was ready for mathematics this year	0.77
L13T	Tertiary Student Like of Year 13 Textbook (L13T)	V9	The textbook/written material examples help me understand the topic	0.64
		V10	The chapters in the textbook/written materials follow each other pretty well	0.58
		V16	There is enough detail in the textbook/written materials to master the topics	0.79
L12T	Tertiary Student Like Y12 Textbook (L12T)	V5	There were enough exercises in the Y12 textbook for me to be well practiced	0.72
		V7	My 12th grade math teacher used the textbook & referred to it in class	0.36
		V8	My 12th grade math book examples helped me understand the topic	0.54
U13T	Tertiary Student Y13 Textbook Use (U13T)	V13	I often bring the mathematics textbook home or to my study location	0.99
		V14	The teacher uses the textbook and refers to it in class	0.47
M13M	Tertiary Student Mathematical Maturity (M13M)	V19R	Without the teacher, the textbook would (NOT) be useless	0.34
		V20	With textbook only (no lectures) I could understand the topics clearly	0.99
C12T	Tertiary Student Confidence in the Year-12 Textbook (C12T)	V2R	12-grade mathematics textbook (DID NOT) need more depth	0.99
		V4R	There was (NOT) too much homework in my 12th-grade mathematics class	0.14

6.5.3. Development of the Initial Tertiary Student Measurement Model

When the SPSS v20.0 EFA was imported into Mplus v7.1, the initial fit indices were as shown in Table 6.19.

Table 6.19 Tertiary Student Initial Fit Indices

RMSEA	Estimate	0.07	
	90% C.I.	0.06	0.07
	Prob. RMSEA <=	0	
CFI/TLI	CFI	0.90	
	TLI	0.86	
SRMR	Value	0.06	

Initial Fit indices indicate a fair to good model to data fit since a good model fit is when the Root Mean Squared Error of Approximation (RMSEA) is less than 0.08, when the Comparative Fit Index (CFI) and Tucker Lewis Index (TLI) are greater than .90, and when Standardized Root Mean Square Residual (SRMSR) is less than 0.08 (Browne & Cudeck, 1992; Hu & Bentler, 1999; Kline, 2015). As seen in Tables 6.22 and 6.28, subsequent improvements made iteratively brought RMSEA, CFI, TLI, and SRMR values to the point where the model and the data show an adequate fit (Tóth-Király et al., 2019) with RMSEA ≤ 0.6, SRMR ≤ 0.05, CFI ≥ 0.90, and TLI ≥ 0.90.

The standardized model results from the initial tertiary student EFA model imported to Mplus are shown in Table 6.20 while the initial measurement model diagram is shown in Figure 6.5.

Table 6.20 Tertiary Student Initial Measurement Model Factor Loadings

Latent Variable	Observed Variables	Factor Loadings	S.E.	2-Tailed P-Value
PR by	V1	0.55	0.02	0.00
	V11	0.58	0.02	0.00
	V15	0.72	0.02	0.00
	V17	0.74	0.02	0.00
L13T by	V9	0.79	0.02	0.00
	V10	0.74	0.02	0.00
	V16	0.66	0.02	0.00
L12T by	V5	0.57	0.03	0.00
	V7	0.61	0.27	0.00
	V8	0.79	0.26	0.00

Table 6.20 (continued)

Latent Variable	Observed Variables	Factor Loadings	S.E.	2-Tailed P-Value
U13T by	V13	1.00	0.00	0.00
	V14	0.46	0.02	0.00
M13M by	V20	1.00	0.00	0.00
	V19R	0.34	0.02	0.00
C12T by	V2R	1.00	0.00	0.00
	V4R	0.14	0.02	0.00

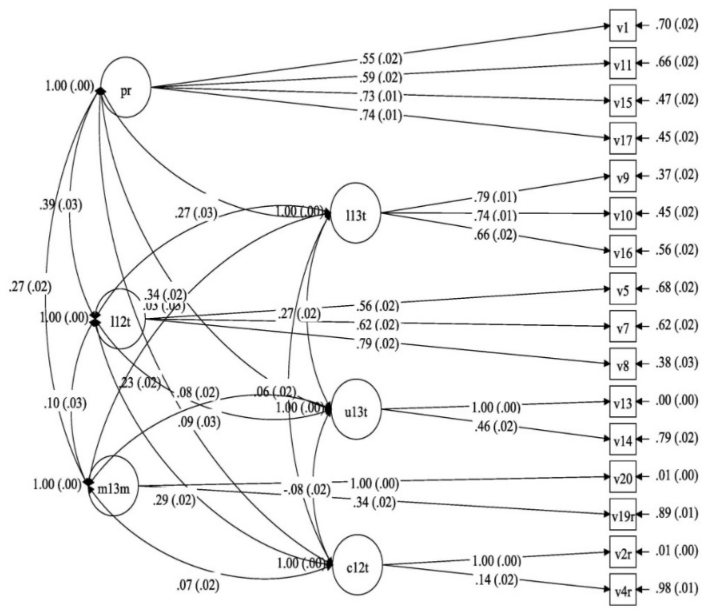


Figure 6.5 Tertiary Student Initial Measurement Model Factor Loadings Diagram

Multiple iterations of this model were experimented with in order build a confirmatory measurement model suitable for SEM analytics. For the initial measurement model, most factor loadings are ≥ 0.45 and average factor loadings onto a latent variable, as noted above in 6.3.2., may be considered significant. When coupled with the P values <0.05, the initial measurement model results demonstrate that the tertiary students' answers to questions V1, V11, V15 and V17 show a solid perception of readiness for the year-13 university calculus course while responses to V2R and V4R supported the tertiary student confidence in the year-

12 textbook for course preparation such that the completion, use and liking of the year-12 mathematics textbook was sufficient for the tertiary student to be confident of mathematical skill and mathematical maturity commensurate with year-13 calculus requirements. Noted was the use and liking for the year-13 textbook (although some of the university professors used only handouts or a combination of handouts and textbook) was partially predicted by the year-12 textbook use and liking. Based on the informal interviews and focus group comments, the university professors saw need for instructional resources other than the unchanging, standard year-13 calculus textbook but, since the year-13 textbook was identified in the curriculum, the students answered survey questions on the textbook/handouts content that may be the reason students used and liked the year-13 text/handouts as they were more closely aligned to the student readiness. The data supports this initial model with a good fit.

6.5.4. Analysis for Tertiary Student Measurement Model

Analysis of correlation coefficients and latent variable interactions did not reveal any need for modifying the SPSS-computed EFA factor loadings.

6.5.5. Tertiary Student Measurement Model Construct(s) Validity

The next step in SEM preparation for the tertiary student data was to examine constructs validity. Reliability needed to be investigated with regard to the constructs in the initial measurement model and this was accomplished by calculating the average variance extracted to determine Convergent Validity (CV) and then taking the square root of these averages to determine Discriminant Validity (DV). respectively.

Applying the Campbell algorithm and analytics to all the variables in the initial measurement model revealed that CV was fair to poor. Table 6.21 records the CV and DV.

Table 6.21 Tertiary Student Convergent and Discriminant Validity

Indicator Variables	Latent Variables	Standardized Factor Loadings (Correlations)	Square of Standardized Loadings	Sum of Squared Standardized Loadings	Number of Indicators	Average variance Extracted = Convergent Validity (CV)	Square Root Average= Discriminant Validity (DV)	Largest Correlation Between Latent Variables (BC)	DV-BC (>0)
V1	PR	0.55	0.30						
V11		0.58	0.34						
V15		0.72	0.52						
V17		0.74	0.55	1.70	4	0.43	0.65	0.51	0.14
V9	L13T	0.79	0.62						
V10		0.74	0.55						
V16		0.66	0.44	1.61	3	0.54	0.73	0.51	0.22
V5	L12T	0.57	0.32						
V7		0.61	0.37						
V8		0.79	0.62	1.32	3	0.44	0.66	0.50	0.16
V13	U13T	1.00	1.00						
V14		0.46	0.21	1.21	2	0.61	0.78	0.38	0.40
V20	M13M	1.00	1.00						
V19R		0.34	0.12	1.12	2	0.56	0.75	0.26	0.49
V2R	C12T	1.00	1.00						
V4R		0.14	0.02	1.02	2	0.51	0.71	-0.09	0.80

The preference for CV will ideally be 0.5 or greater and the preference for DV is that each measure is greater than the largest correlation between the latent variables (Fornell & Larcker, 1981). CV is only low in PR and L12T and the DV is well within acceptable range. Thus, the initial measurement model does have fair CV.

6.5.6. Tertiary Student Data Model Fit Indices

When the RMSEA, CFI, TLI, and SRMR were analyzed for the tertiary student measurement model by co-varying the necessary modification indices, it was determined that the model had a good fit on RMSEA and SRMR and a fair fit on CFI and TLI. Table 6.22 displays results of the analysis.

Table 6.22 Initial Tertiary Student CFA Model Fit

RMSEA	Estimate	0.06	
	90% C.I.	0.06	0.07
	Prob. RMSEA <=	0	
CFI/TLI	CFI	0.92	
	TLI	0.88	
SRMR	Value	0.05	

6.6. Tertiary Student Structured Equation Model

Table 6.23 depicts the initial SEM configuration from the measurement model output. It contains the syntax used to regress the variables as per the measurement model in Mplus and Figure 6.6.

Table 6.23 Tertiary Student Initial Mplus Regressed On Syntax

Regressed On	Regressed
PR	L12T
C12T	L12T, PR
M13M	L12T, PR, C12T
L13T	L12T, PR, C12T, M13M
U13T	L12T, PR, C12T, M13M, L13T

Figure 6.6 depicts the structural model with path coefficients derived from the Tertiary Student Initial SEM data shown in Table 6.24.

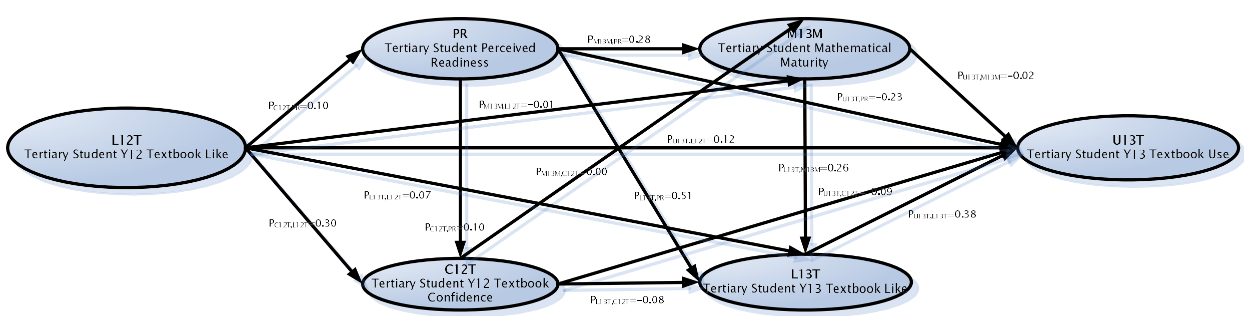


Figure 6.6 Tertiary Student Initial Latent Variable Path Diagram

Table 6.24 Tertiary Student Initial Structural Model Results

Regressed On	Regressed	Path Coefficients	S.E.	2-Tailed P-Value
PR	L12T	0.50	0.03	0.00
C12T	L12T	0.30	0.03	0.00
	PR	0.10	0.03	0.00
M13M	L12T	-0.01	0.03	0.72
	PR	0.28	0.03	0.00
	C12T	0.00	0.02	0.97
L13T	L12T	0.07	0.04	0.06
	PR	0.51	0.03	0.00
	C12T	-0.08	0.03	0.00
	M13M	0.26	0.02	0.00
U13T	L12T	0.12	0.03	0.00
	PR	-0.23	0.04	0.00
	C12T	-0.09	0.02	0.00
	M13M	-0.02	0.02	0.36
	L13T	0.38	0.04	0.00

Figure 6.7 displays the Modification Indices-altered tertiary student SEM diagram.

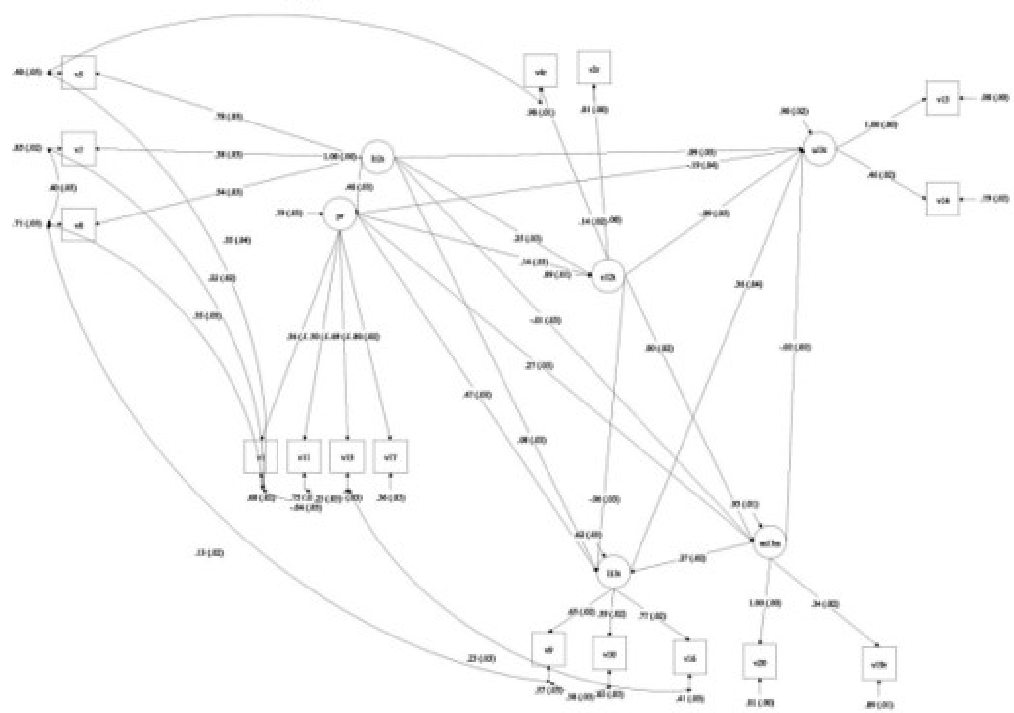


Figure 6.7 Tertiary Student Modified SEM Showing STDYX Standardized Results

Tables 6.25, 6.26, and 6.27 are the indices of the diagram in Figure 6.7. Any p-value over 0.05 indicates a non-significant loading. These were targets of removal for the final model. The highlighted (*) P-Values in the table indicate a non-significant correlation.

Table 6.25 Tertiary Student Measurement Model Factor Loadings

Latent Variable	Observed Variable Influenced By	Factor Loadings	S.E.	Sig.
PR	V1	0.56	0.02	0.00
	V11	0.50	0.02	0.00
	V15	0.69	0.02	0.00
	V17	0.80	0.02	0.00
L13T	V9	0.65	0.02	0.00
	V10	0.59	0.02	0.00
	V16	0.77	0.02	0.00
L12T	V5	0.78	0.03	0.00
	V7	0.38	0.03	0.00
	V8	0.54	0.03	0.00
U13T	V13	0.99	0.00	0.00
	V14	0.46	0.02	0.00
M13M	V20	0.99	0.00	0.00
	V19R	0.34	0.02	0.00
C12T	V2R	0.99	0.00	0.00
	V4R	0.14	0.02	0.00

Table 6.26 Tertiary Student Modified Structural Model Results

Latent Variable	Latent Variable Regressed On	Path Coefficients	S.E.	Sig.
PR	L12T	0.46	0.03	0.00
C12T	PR	0.14	0.03	0.00
	L12T	0.25	0.03	0.00
M13M	L12T	-0.01	0.02	0.70*
	PR	0.27	0.03	0.00
	C12T	0.00	0.02	0.98*
L13T	L12T	0.08	0.03	0.01
	PR	0.47	0.03	0.00
	C12T	-0.06	0.03	0.01
	M13M	0.27	0.02	0.00
U13T	L12T	0.09	0.03	0.00
	PR	-0.19	0.04	0.00
	C12T	-0.09	0.02	0.00
	M13M	-0.02	0.02	0.49*
	L13T	0.36	0.04	0.00

Table 6.27 Tertiary Student Correlated Error Terms

Observed Variable	Observed Variable Co-Varied With	Estimate	S.E.	Sig.
V5	V4R	-0.19	0.03	0.00
	V1	0.55	0.04	0.00
V7	V1	0.22	0.02	0.00
V8	V1	0.35	0.03	0.00
	V7	0.40	0.02	0.00
V9	V8	0.13	0.02	0.00
	V10	0.38	0.03	0.00
V15	V1	-0.04	0.03	0.10
	V11	0.23	0.03	0.00
V16	V15	0.23	0.03	0.00

The initial and modified latent variable correlated error terms indicated a strategy for removing non-significant regressions. This was done in an iterative process to retain paths of statistical significance. The final SEM fit results are show in Table 6.28.

Table 6.28 Tertiary Student Final SEM Fit

RMSEA	Estimate	0.06	
	90% C.I.	0.06	0.06
	Prob. RMSEA <=	0	
CFI/TLI	CFI	0.92	
	TLI	0.89	
SRMR	Value	0.05	

Figure 6.8 is the final Mplus diagrammatical illustration of the independent and dependent relationship between the latent variables.

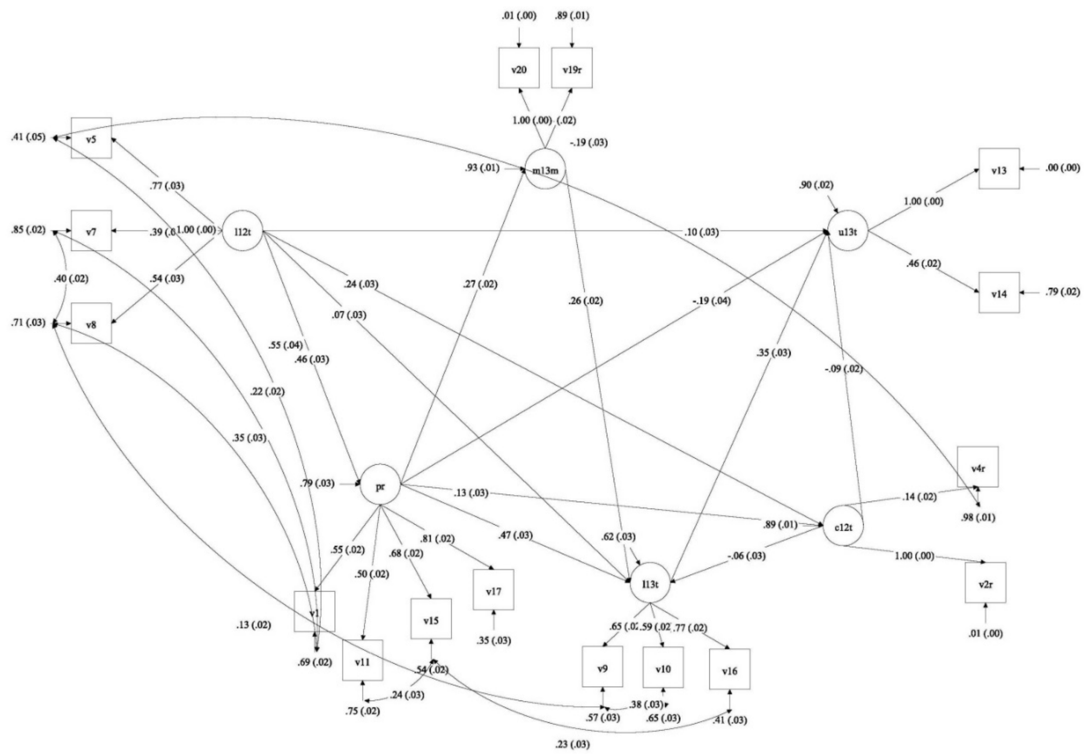


Figure 6.8 Tertiary Student Final SEM Diagram

Tables 6.29, 6.30, and 6.31 are the indices of the diagram in Figure 6.8. Table 6.29 shows the loading of each of the observed variables onto the latent constructs. Perceived Readiness (PR) is strongly supported by the variables as are Tertiary Student Like of Year 13 Textbook (L13T) and Tertiary Student Like Y12 Textbook (L12T). Tertiary Student Y13 Textbook Use (U13T) and Tertiary Student Mathematical Maturity (M13M) were supported solidly and though Tertiary Student Confidence in the Year-12 Textbook (C12T) had weaker support in one of the observed variables, all factor loadings had significance < 0.05 and all but C12T with reversed V4R loading, had correlations > 0.30, giving the statistical assurance that the correlations are reliable and that the factor loadings are suitable for use (Cohen, 1992).

Table 6.29 Tertiary Student CFA Measurement Model Factor Loadings

Latent Variable	Observed Variable	Factor Loadings	S.E.	Sig.
PR	V1	0.55	0.02	0.00
	V11	0.50	0.02	0.00
	V15	0.68	0.02	0.00
	V17	0.80	0.02	0.00
L13T	V9	0.65	0.02	0.00
	V10	0.59	0.02	0.00
	V16	0.77	0.02	0.00
L12T	V5	0.77	0.03	0.00
	V7	0.39	0.03	0.00
	V8	0.54	0.03	0.00
U13T	V13	0.99	0.00	0.00
	V14	0.46	0.02	0.00
M13M	V20	0.99	0.00	0.00
	V19R	0.34	0.02	0.00
C12T	V2R	0.99	0.00	0.00
	V4R	0.14	0.02	0.00

Table 6.30 demonstrates that L12T is influencing the student's PR and this was consistent with the literature that the year-13 university student was confident in their preparation based on the successful completion of the year-12 precalculus textbook. Similarly, student PR and L12T influenced C12T. PR influencing M13M may be understood as the tertiary student who was influenced to the perception of readiness was similarly influenced to believe that readiness was a measure of maturity in the level to which they had been taught and the rigor of the year-12 precalculus textbook (i.e., maturity in subject matter that was simple and likely inadequate). PR and L12T and M13M were influencing L13T, indicating a likelihood that the confidence gained as a result of the successful completion of the year-12 precalculus textbook coursework was influential for the student to be disposed to like their year-13 calculus textbook. Lastly, L12T and L13T are positively influencing U13T which seems intuitive whereas PR and C12T are negatively influencing U13T, indicating an increase in complexity in material presentation or that textbooks were being replaced by handouts as discussed earlier

which may further the lack of readiness of the student after all. All paths had significance <0.05 and almost all the path coefficients are either greater than 0.10 or less than -0.10, indicating that the paths are significant for asserting influential relationships between the latent (unobserved) variables.

Table 6.30 Tertiary Student Final Structural Model Results

Latent Variable	Latent Variable Regressed On	Path Coefficient	S.E.	Sig.
PR	L12T	0.46	0.03	0.00
C12T	PR	0.13	0.03	0.00
	L12T	0.25	0.03	0.00
M13M	PR	0.27	0.02	0.00
L13T	PR	0.47	0.03	0.00
	L12T	0.08	0.03	0.52
	C12T	-0.06	0.03	0.02
	M13M	0.26	0.02	0.00
U13T	PR	-0.19	0.04	0.00
	L12T	0.10	0.03	0.00
	C12T	-0.09	0.02	0.00
	L13T	0.35	0.03	0.00

Table 6.31 shows the correlated error terms, (or disturbances) remaining after the index modifications (Mplus modification of indices) were performed to optimize the model fit shown in Table 6.28. The correlations, if strong enough, may suggest information yet to be exploited in the model.

Table 6.31 Tertiary Student Final SEM Results Correlated Error Terms

Observed Variable	Observed Variable Co-Varied With	Estimate	S.E.	Sig.
V5	V4R	-0.19	0.03	0.00
	V1	0.55	0.04	0.00
V7	V1	0.22	0.02	0.00
V8	V1	0.35	0.03	0.00
	V7	0.40	0.02	0.00

Table 6.31 (continued)

Observed Variable	Observed Variable Co-Varied With	Estimate	S.E.	Sig.
V9	V8	0.13	0.02	0.00
	V10	0.38	0.03	0.00
V15	V11	0.24	0.03	0.00
V16	V15	0.23	0.03	0.00

As shown in Table 6.32, all latent variable R-Square values have $p < 0.05$ indicating statistical significance with regard to the observed variables seen in Table 6.29.

Table 6.32 Tertiary Student SEM R-Square Results

Latent Variable	R-Squared	Std. Err. *S.E.*	Sig. *p*
PR	0.243	0.028	0.000
L13T	0.421	0.027	0.000
U13T	0.110	0.017	0.000
M13M	0.074	0.013	0.000
C12T	0.127	0.017	0.000

6.6.1. Tertiary Student SEM Summary

Figure 6.9 shows the final path diagram for the tertiary student data.

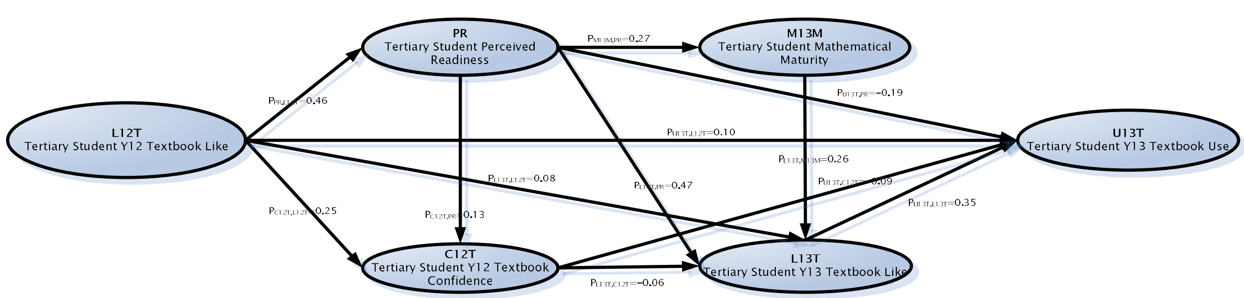


Figure 6.9 Final Tertiary Student Path Diagram with Path Coefficients

The tertiary student path diagram graphically displays how latent variables are related (i.e., how they influence, or are influenced by, each other). Moving from left to right, L12T influences both student PR and C12T. The path coefficient between L12T and PR is 0.46 and the path coefficient between L12T and C12T is 0.25 indicating that the tertiary students' appreciation of the year-12 textbook was influential in the tertiary students' perceived readiness for year-13 mathematics and the students' confidence in the year-12 textbook as adequate preparation for year 13. Table 6.30 shows that those influences are statistically significant.

C12T is also influenced by PR with a path coefficient of 0.13 which is also statistically significant. That is, student perceived readiness influences the student confidence in the value of the textbook. Yet, as chapter 7 indicates, the textbooks in use by the secondary students (as reflected by the tertiary student data) were textbooks with insufficient rigor standards. The lack of rigor made the textbooks easier to use which likely explains tertiary students' confidence in their year-12 textbooks and perceived readiness for year-13 mathematics.

In Sections 6.8 and 6.9 below, the secondary and tertiary teacher data affirm the consensus of perceived readiness in the secondary setting but not in the tertiary setting. Both year-12 students and secondary teachers believe students are ready for year-13 calculus, but tertiary professors report a lack of readiness in the incoming year-13 students even though year-13 students believe they are ready for year-13 mathematics.

The model goes on to show a path coefficient between L12T and L13T of 0.08 and a non-significant p value of 0.052, probably affirming that the year-13 student memory of liking their year-12 textbook is not likely to influence their liking of their year-13 textbook (or handouts), but their perceived readiness for year-13 calculus *is* significantly influencing their liking of the year-13 textbook (or handouts) with a $p < 0.05$. It is interesting to note that the tertiary students' confidence in the year-12 textbook (C12T) has a negative influence on the tertiary students' liking of the year-13 textbook with a path coefficient value of -0.06. In other words, the confidence that the year-13 student had in the year-12 textbook failed to positively influence the liking of the year-13 textbook. This also may indicate that the tertiary students' experience with the year-12 textbook inadequately prepared them for using and liking the year-13 calculus textbook. As the general rigor of the year-13 calculus textbooks has remained somewhat static (see chapter 7), the general dislike of the year-13 textbooks by tertiary students may very well be associated with the ease of use and trivial (non-rigorous) approach to mastery learning and concept acquisition in many modern year-12 precalculus textbooks.

The diagram also indicates a path coefficient between tertiary student mathematical maturity (M13M) and their liking of their year-13 textbook (L13T) of 0.26. This positive correlation likely reflects the fact that a mathematically mature student would appreciate a rigorous textbook while a less mathematically mature student would not appreciate a rigorous textbook. There is also a path coefficient between PR and M13M of 0.27 because students who perceive themselves to be ready for higher level mathematics will demonstrate mathematical maturity by confidence in their textbooks and their ability to learn independently of their teacher. Note, however, that a mistaken perceived readiness will result in a *lack* of mathematical maturity.

The diagram further shows that the tertiary students' use of the year-13 textbook (U13T) is positively influenced by L12T (0.10) and L13T (0.35), but negatively influenced by C12T (-0.09) and PR (-.19). The positive correlation is logical since students who like their textbooks are likely to use them. The negative correlation can be explained by the fact that the year-12 textbook confidence and their perceived readiness for year-13 mathematics did not in fact prepare the students to use the more rigorous year-13 calculus textbooks. This was affirmed by the fact that many calculus professors did not use a textbook in the first-semester university calculus but rather their own notes and handouts to present mathematical material. As shown in Chapter 7, rigor was not absent from calculus textbooks; thus, the professors' decision to use notes and handouts was not due to any insufficiency of the year-13 calculus textbook.

6.7. Summary of Results for Secondary and Tertiary Students

Year-12 students believe they are prepared for year-13 calculus and the year-12 teachers believe the same. This is logically connected since the teacher and the student use, like, and trust the year-12 precalculus textbook. Moreover, the year-13 students, confident of skill and maturity for their year-13 calculus course, not surprisingly at the early part of the term recalled liking and using the year-12 textbook (the previous year for most) and were confident in their use and like of the year-13 textbook/handouts. Yet, the overwhelming consensus of the university professors was that their students lacked both skill and mathematical maturity. The next section examines and formally integrates descriptive statistics into the conclusions of this chapter.

6.8. Linking the Descriptive Results to SEM Results

Secondary student CFA and SEM provide a basis for the assertion that the content of the year-12 precalculus textbook, coupled with use, trust, parental influence and students' perception of readiness are contributors to a general mathematical maturity and that this maturity will increase (improve) according to the listed influences. That is, the more rigorous the year-12 textbook, the greater will be the mathematical maturity in preparation for year-13 mathematics; additionally, the maturity of the incoming year-13 mathematics students will present itself by the student embracing the year-13 textbook and using it in a mature way. It is recognized, however, that there are some limitations to conclusions that can be drawn from data modeling because of certain limitations in survey design and limitations on data collection. In the case of this study, privacy issues prevented linking specific year-12 students with specific outcomes in year-13 mathematics courses. Additionally, because data was collected in early- to mid-semester, end-of-year results were not available to include in models. This section employs descriptive statistics relating to tertiary student assessment of their year-13 textbook and of secondary teacher and tertiary professor responses to survey questions relating to textbooks, student confidence, and student readiness.

The descriptive statistics below indicate that the year-13 mathematical textbook is neither highly used nor preferred by students or teachers. This confirms the argument that there is a deficiency in year-12 mathematical textbooks since, had year-12 mathematical textbooks been of equivalent maturity and rigor as the year-13 mathematical textbooks, a smooth transition from year-12 mathematical textbook use to year-13 mathematical textbook use likely would have occurred.

6.8.1. Descriptive Statistics Related to Tertiary Student Opinion of Y13 Textbook

This section draws conclusions using descriptive statistics applied to V18, V19R and V20 of the tertiary student survey:

V18. Extra handouts are sometimes better than the textbook/written materials. *

V19R. Without the teacher, the textbook/written materials would (NOT) be useless.

V20. With textbook/written materials only (no lectures) I could understand the topics clearly.

**Dropped from the EFA since factor loadings were inadequate for modelling use but included for analyzing descriptive statistics.*

These questions queried student comfort with their year-13 textbook. Figure 6.10 is an abbreviated scatter plot of survey responses to these questions. Table 6.33 shows the statistics for these responses.

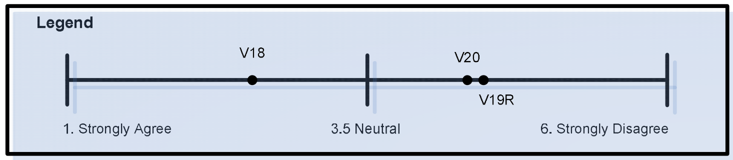


Figure 6.10 Abbreviated Scatter Plot of Tertiary Student Responses to V18, V19R and V20

Table 6.33 Statistics Relating to Tertiary Student Responses to V18, V19R and V20

	N	Mode	Mean	Std. Deviation
V18	2132	2	2.54	1.151
V19R	2132	2	2.73	1.392
V20	2132	4	4.00	1.337
Valid N (listwise)	2132			

Tables 6.34, 6.35, and 6.36 indicate that year-13 textbooks were not well liked over the teacher explanations and the teacher handouts. Table 6.34 addresses how students responded to V18. Clearly, the preponderance of responses falls into the agreement side of neutral on the question of whether extra handouts are sometimes better than the textbook/written materials.

Table 6.34 Tertiary Student Responses to V18

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	419	19.7	19.7	19.7
	2	702	32.9	32.9	52.6
	3	586	27.5	27.5	80.1
	4	312	14.6	14.6	94.7
	5	87	4.1	4.1	98.8
	6	26	1.2	1.2	100.0
	Total	2132	100.0	100.0	

Table 6.35 addresses how students responded to V19R. In this case, the preponderance of responses falls into the disagreement side of neutral on the question of the effectiveness of textbooks/written materials without the teacher. That is, most students believed that the textbook/written materials alone are inadequate. ("Written materials" was included in the survey because a number of professors indicated that they had abandoned use of textbooks.)

Table 6.35 Tertiary Student Responses to V19R

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	80	3.8	3.8	3.8
	2	190	8.9	8.9	12.7
	3	323	15.2	15.2	27.8
	4	495	23.2	23.2	51.0
	5	564	26.5	26.5	77.5
	6	480	22.5	22.5	100.0
	Total	2132	100.0	100.0	

Table 6.36 addresses how students responded to V20, which is simply a summated scale check of responses to V19R. The preponderance of responses falls into the disagree side of neutral on the question of whether the teacher was unnecessary.

Table 6.36 Tertiary Student Responses to V20

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	88	4.1	4.1	4.1
	2	224	10.5	10.5	14.6
	3	421	19.7	19.7	34.4
	4	561	26.3	26.3	60.7
	5	553	25.9	25.9	86.6
	6	285	13.4	13.4	100.0
	Total	2132	100.0	100.0	

6.8.2. Secondary and Tertiary Teacher/Professor Descriptive Statistics

Although the study cannot supply a statistical connection from the tertiary student SEM to the secondary student SEM, connections may be extracted descriptively from scholarly literature and anecdotally from the survey data collected. In this section, descriptive statistics are employed to analyze secondary and tertiary teacher survey responses in order to demonstrate a causal flow as depicted in Figure 6.11.

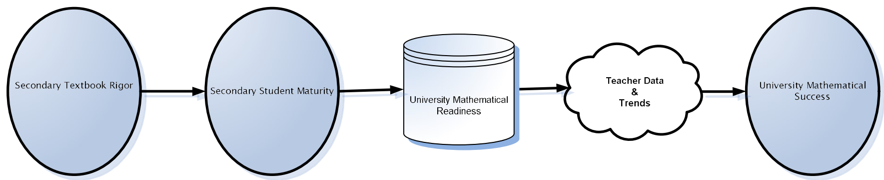


Figure 6.11 Causal Flow Chart

6.8.2.1. Secondary Teacher Survey Response Analysis

Based on secondary teachers' average age (41.17) when they responded to the survey and their assumed age at secondary school graduation (18-19 years), it was determined that most would have used mathematics textbooks published between 1991 and 1994 when they were in school. Their experience with these textbooks would have influenced their responses to survey questions relating to their and their students' perception of year-12 textbooks.

Table 6.37 shows how secondary teachers responded to questions related to year-12 textbooks and student confidence, maturity, and perception of those textbooks. The table key is:

- 1 <= Agree (A)<=3.25
- 3.25 < Neutral (N) < 3.75
- <= 3.75 Disagree (D) <= 6

Table 6.37 Secondary Teacher Survey Instrument Responses

Question Number	Question	Mean Answer Response	Agreement (A/N/D)
1	Students Believe the textbook is valuable	2.44	A
2	A good textbook is valuable to the teacher	1.59	A
3	Students are self-driven, disciplined and make mature use of the textbook	2.76	A
4	Students generally lack confidence	3.28	N
5	Students generally complete assignments	2.32	A
6	Students ask for help often	2.75	A

Table 6.37 (continued)

Question Number	Question	Mean Answer Response	Agreement (A/N/D)
7	Very little textbook material is remedial in content	2.24	A
8	Students are more mature this year than last year	3.10	A
9	Students are more academically ready for math this year than last year	3.00	A
10	Section and chapter exercises reinforce the current topic	2.02	A
11	I am very satisfied with the current textbook	2.76	A
12	Textbook is taken home and used by the student on a regular basis	1.95	A
13	Students that are academically mature use the textbook as a focus for study	1.90	A
14	Sample questions in the textbook are very helpful in explaining the concept	2.37	A
15	Textbook has concept explanations that are very good	2.63	A
16	The year 11 math textbook flows coherently into the year 12 math textbook	2.76	A
17	Textbook is thorough and mastery oriented	2.93	A
18	Section and chapter exercises range from simple to difficult	2.63	A
19	I have used a better textbook than the current textbook	4.00	D
20	Section and chapter exercises utilize previously learned topics	2.63	A
21	Textbook is used as a focus in the classroom for topic presentation	2.90	A

Secondary teacher appraisal of student confidence, mathematical readiness, textbook value and textbook use indicate an overall understanding by the teachers that what they are teaching the student from the textbook is fundamentally sound, concept-building, and sufficient preparation for the next mathematical course(s). Moreover, for the average time of teaching (18.2 years), the teachers agree that the current textbook (published between 1998 and 2016) is the best they have used, indicating that prior years of textbooks had been less satisfactory. It is quite likely that prior year textbooks, being more rigorous, were considered more difficult to use by teachers and students.

6.8.2.2. Tertiary Teacher Survey Response Analysis

Based on tertiary teachers' average age (45.29) when they responded to the survey and their assumed age at secondary school graduation (18-19 years), it was determined that most would have used mathematics textbooks published between 1984 and 1989 when they were in school. Their experience with these textbooks would have influenced their responses to survey questions relating to their and their students' perception of year-13 textbooks.

Table 6.38 shows how tertiary teachers responded to questions related to year-13 textbooks and student confidence, maturity, and perception of those textbooks. The table key is:

- 1 <= Agree (A)<=3.25
- 3.25 < Neutral (N) < 3.75
- <= 3.75 Disagree (D) <= 6

Table 6.38 Tertiary Teacher Survey Instrument Responses

Question Number	Question	Mean Answer Response	Agreement (A/N/D)
1	Students believe the textbook/written material is valuable to them	3.59	N
2	The textbook is valuable to the teacher	2.00	A
3	Students are self-driven disciplined and make mature use of textbook	4.51	D
4	Students generally lack confidence in mathematics	3.00	A
5	Students generally complete their assignments	3.24	A
6	Students ask for help with homework often	3.59	N
7	Very little textbook/written material is remedial in content	2.47	A
8	Students are more mature this year than last year	3.44	N
9	Students are more academically ready for math this year than last year	3.65	N
10	Students are less prepared for mathematics this year than ever before	3.50	N

6.8.2.3. Analysis of Secondary and Tertiary Teacher Responses

The common questions and answers were matched up to determine if the experiences and opinions of the secondary and tertiary teachers differed. In table 6.39 the trend in the answers to the common questions between the tertiary and secondary teacher responses are shown in the trend column as to whether the experiences of the teachers are static or degrading. (-1 = degraded, 0 = unchanged, 1 = improved).

Table 6.39 Comparison of Secondary and Tertiary Teacher Survey Responses to Common Questions

Common Questions	Secondary Response	Tertiary Response	Trend
Students believe the textbook is valuable	A	N	-1
A good textbook is valuable to the teacher	A	A	0
Students are self-driven, disciplined and make mature use of the textbook	A	D	-1
Students generally lack confidence	N	A	1
Students generally complete assignments	A	A	0
Students ask for help often	A	N	-1
Very little textbook material is remedial in content	A	A	0
Students are more mature this year than last year	A	N	-1
Students are more academically ready for math this year than last year	A	N	-1

In general, secondary teacher survey answers indicate that textbook confidence, student maturity, confidence, willingness to ask for help, academic readiness and study-related self-discipline are acceptable in year 12 students, while tertiary teachers noted decline or degradation in these categories in year-13 students.

The following conclusions were drawn from the secondary and tertiary teacher responses: First, teacher evaluation of student textbook value (like, trust, confidence) degrades from the secondary to the tertiary level which may indicate that students are either unwilling or unable to use the tertiary textbook. (Refer back to Tables 6.34 through 6.37 which indicate student inability to use the textbook to understand the topic.) Second, if the difference in rigor between year-12 and year-13 textbooks is great, this may explain student inability to use the year-13 textbook by virtue of being trained to use less rigorous textbooks in year 12 and prior. Third, the differing opinion of secondary and tertiary teachers with regard to student maturity and mature use of the textbook indicate a student inability or reluctance to use the more rigorous year-13 textbook. (Chapter 7 describes the static nature of year-13 calculus textbooks over time and chapter 8 demonstrates the systematic degradation of secondary precalculus textbook rigor.). Fourth, teacher evaluation of mathematical confidence shows the secondary teacher reporting that students were mathematically confident whereas year-13 teachers did not report mathematical confidence. Finally, teacher evaluation of students' mathematical maturity and readiness indicate the secondary teacher believes strongly that students are ready and equipped for year-13 mathematics, whereas the tertiary teacher is neutral to negative in this matter.

6.8.3. Confirmation of Tertiary Teacher Observations Regarding Student Readiness

Statistical data regarding placement of tertiary student survey respondents in first-year mathematics courses supports tertiary teacher observations regarding mathematical maturity and readiness of these students. The tertiary survey instrument included data indicating if students were in their first university semester, whether they had been placed into a mathematics class and, if so, into which class they had been placed. In the tertiary survey given in the United States and Australia, 88.5% (n=1,906) of the 2,154 year-13 students who provided their year of graduation had graduated from high school between 2012 and 2016. See Table 4.3. During those years, 72.0% (n=1,581) of the students had completed high school the previous year and were enrolled in their first semester at university. Regarding placement tests, 44.7% (n=982) of tertiary survey respondents indicated that they were placed into their year-13 mathematics course via placement test and, of those, 34.4% (n=358) were placed into Calculus, 11.8% (n=115) into Algebra, and 51.8% (n=509) into Trigonometry (Table 4.4).

The descriptive statistics above, coupled with scholarly studies reporting a high failure and drop-out rate for year-13 calculus students (See chapter 2), reinforce the argument of this study that year-12 students are ill-prepared for their year-13 calculus course.

6.9. Professor and Teacher Discussion Groups

As mentioned in Section 3.6, informal discussion groups were conducted with secondary teachers and university professors. While the primary purpose of the groups was to develop and refine survey questions, the discussions about teacher and professor classroom experiences provided affirmation of the findings articulated above that there is disagreement between secondary teachers and university professors on the readiness of students for their year-13 calculus course. Additionally, the discussion groups reinforced the finding that textbooks are considered central to the learning of mathematics by secondary teachers so much so that they considered completion of textbook-prescribed coursework an indication that students were prepared for year-13 calculus. Lastly, those informal discussions also revealed that many university professors had created remedial material (in place of the textbook) for their year-13 calculus students, affirming the survey findings that students were either not ready for their year-13 calculus course or unable to use the year-13 textbook due to its higher rigor level and demanding pace, or both.

6.10. Conclusion

Secondary teachers and students and tertiary students have generally concurred with one another as to the adequacy of preparation of students for year-13 mathematics based on the completion of the year-12 curriculum and textbook choice for that year. Tertiary teachers have not generally agreed with this conclusion. If year-12 precalculus textbooks used by survey respondents had been adequately rigorous to have covered the necessary year-13 calculus prerequisites, degradation in mathematical skill or degradation in mathematical maturity could logically be attributed to other factors such as the teacher, classroom size, and physical amenities. But, as chapters 7 and 8 will demonstrate, though the prerequisites for elementary calculus in year-13 have not changed (as evidenced by the static nature of year-13 textbooks), year-12 precalculus textbooks have degraded over time to the extent that they can be identified as the primary cause of lack of general competence of secondary students for their year-13 elementary calculus course. Year-12 textbooks no longer maintain the alignment of topics and presentation format necessary to ensure that tertiary calculus requirements are addressed in a systematic and effective way.

Chapter 7. Rigor Algorithm Development

7.1. Introduction

Considering the results expressed in the secondary and tertiary SEMs showing the centrality of the secondary mathematics textbooks as a key element in the mathematical maturity of year-12 students and their mathematical readiness for year-13 calculus, the next consideration was to determine whether a method could be devised for secondary teachers and mathematics departments to use to assess the existence of misalignment between year-12 exit and year-13 entrance standards in their secondary precalculus textbooks. This chapter describes the development of a unique tool, based on MRP principles of Bills of Material and Critical Path, for assessing the rigor of mathematical textbooks designed to prepare year-12 students for their year-13 calculus course. Identifying the university mathematics department as "the customer," the MRP approach acquired, from multiple calculus textbooks published over multiple years, an assessment of "build requirements" that were fashioned into a list of minimum topics and rigor standards needed to complete the built product (mathematically mature year-12 students ready for their year-13 calculus course). This chapter details how the textbook rigor algorithm was developed and explains its component parts.

7.2. Developing Mathematical Maturity in Students

The typical mathematics student is both taught and exposed to practical mathematical skills. The order in which the student is introduced to mathematical concepts and computational techniques is a critical indicator of how quickly the student will be able to master relevant concepts and how prepared the student will be to apply that mathematical maturity to master more complex concepts and computational techniques (Raubenheimer et al., 2010). For example, suppose a student is told to fetch a certain number of certain objects from a certain location. If understood, the student may comply with this request by advancing to the location of the objects, sorting for the specified objects, and then counting the objects to verify compliance. To successfully complete the task, the student must exercise elementary reasoning, to include object identification and arithmetic order. Suppose that the student is then asked to acquire a certain number of *different types* of objects from certain locations. The diligent student will ponder the request, develop a strategic plan, and implement the plan. That is, the student will take that which was experientially acquired in previous endeavors and apply relevant knowledge to the more complicated request. Nevertheless, at some point, the requests will become sufficiently complex that previous experience alone will not enable successful compliance; the student will then have to research alternatives or abandon the activity (Miller & Maellaro, 2016).

The same iterative process holds true for mathematics students striving to prepare for the complexities of calculus. As students tackle mathematical exercises, they apply previous experience and mathematical maturity to successfully master more complex exercises until such time as the computational strategies they are employing become too unwieldy or simply inadequate for the level of complexity presented. While concept mastery promotes further sophistication of computational techniques, computational/procedural sophistication does not necessarily promote concept mastery (Siegler & Lortie-Forgues, 2015) and may even hinder it (Havard et al., 2018; Mao et al., 2017; OECD, 2015; Saxon, 1986; Siegler & Lortie-Forgues, 2015). Thus, concepts, not computational techniques, should introduce topics because, in the absence of the proper order of presentation (i.e., teaching students to rely on computational techniques without grasping underlying concepts), computational techniques inhibit the necessary conceptual appropriation and, as a result, contribute to the interruption of the cohesive continuum and dilution of rigor needed in mathematics education (Bergsten et al., 2015; Clement et al., 1981; Klymchuk et al., 2010; Pollatsek et al., 1981; Siegler & Oppenzato, 2021). An optimally rigorous mathematical textbook is one that balances concept mastery and computational techniques, introducing them recursively and in the proper order. Pollatsek et al. (1981) wrote: "Learning a computational formula is a poor substitute for gaining an understanding of the basic underlying concept" (p. 191). As an example, they note that for many students, "dealing with the mean is a computational rather than a conceptual act. Knowledge of the mean seems to begin and end with an impoverished computational formula" (p. 191). They concluded:

> The source of the difficulty appears to be that students' knowledge often seems limited to computational formulas, and many simple problems (such as weighted mean problems) require more general, relational, knowledge of concepts. One pedagogical point seems clear. In many introductory courses, students are taught to use formulas in a rote manner with the justification that thorough understanding of the material can wait until the second course (or later). While it is undeniably true that students can solve some problems with this approach, our data suggest that the range of problems that can be solved with only instrumental knowledge is vanishingly small (Pollatsek et al., p. 202).

As detailed in Section 2.4.2, mathematical maturity is promoted when concepts are presented recursively in a logical sequence of interconnected and relevant concepts (Choppin, 2011; Fonger et al., 2018; Henningsen & Stein, 1997). It is imperative that a continuum be maintained as new concepts mature and the cement of the foundation of previously mastered concepts properly sets. In fact, the properly ordered presentation of precalculus material and the increasingly challenging exercises presented in a rigorous course are what are needed for maturing mathematics students and preparing them to be successful in tertiary elementary calculus (Lorch, 1977).

The remainder of this chapter will describe the rigor tool development for measuring the rigor of mathematical textbooks designed to prepare year-12 students for the year-13 calculus course.

7.3. Method—Designing the Rigor Algorithm

This section of the chapter describes the process through which a unique tool—the rigor algorithm—was designed to determine a rigor "score" for precalculus textbooks.

7.3.1. Data Collection and Textbook Acquisition

Surveys were designed and distributed to collect data about students' experience with their textbook and about professors' experience with the students using those textbooks. Two of the data elements collected were the name of the high school where the year-12 students took their calculus preparation class and the year that students graduated. Subsequently, high schools were contacted, and mathematics departments were queried as to the textbook in use in each graduation year that students identified on the survey instrument. A statistical sampling of precalculus textbooks from several eras (1965-1985, 1986-2003, and 2004-2012) was acquired in preparation for calculating a rigor score for each.

7.3.2. Identifying Elementary Calculus Prerequisite Topics

The first step in developing the rigor measurement tool was to identify the list of attributes to be measured. The MRP approach (a.k.a. backward scheduling) places the focus for defining requirements on the deliverable—the end product—in this case, the "calculus-ready" student. MRP defined the requirements needed to produce the desired deliverable and was the driver for the analysis of year-13 calculus textbooks to determine what common topics they included.

The following eight college/university calculus textbooks published between 1965 and 2003 were selected as a representative sampling of year-13 textbooks, then analyzed to identify the similar and dissimilar, and expressed and implied, recommended requirements that formed the prerequisite minimum for successful course completion. Whether the textbooks were ranked as exemplar samples was not the focus of the selection, rather, the criterion was whether they were in common use across the years listed so that their contents could form the basis of the MRP-inspired "build requirements" used to develop the algorithm tool for assessing how well year-12 precalculus textbooks were preparing secondary students for their year-13 calculus courses. Though "goodness" was not the criteria for selection of exemplar books, their contents were in alignment with Michael Spivak's *Calculus* (1994), heralded by one reviewer as "in the running for the best calculus book ever written (Terrell, 2003, p. 69). Spivak included eighty-nine pages of prerequisite topics in the "Prologue" and "Foundations" sections of his book that included number properties, analytic geometry, graphing, functions, inequalities, proofs, factoring and trigonometry—the same foundational material assumed by the exemplar books used to create the rigor algorithm.

Books are listed in chronological order of publication.

i. John F. Randolph, *Calculus and Analytic Geometry*, 2d ed., (Belmont, CA: Wadsworth Publ. Co., 1965), LOCCCN: 65-11579.

ii. Richard E. Johnson and Fred L. Kiokemeister, *Calculus with Analytic Geometry*, 4th ed., (Boston: Allyn and Bacon, Inc., 1969), LOCCCN: 75-78925.

iii. Louis Leithold, *The Calculus with Analytic Geometry*, 2d ed., (New York: Harper & Row, Publ., 1972), LOCCCN: 74-168364.

iv. Lipman Bers and Frank Karal, *Calculus*, 2d ed., (New York: Holt, Rinehart and Winston, 1976), ISBN: 0-03-089268-6.

v. Howard Anton, *Calculus with Analytic Geometry*, 1st ed., (New York: John Wiley and Sons, 1980), ISBN: 0-471-03248-4.

vi. C.H. Edwards, Jr. and David E. Penney, *Calculus and Analytic Geometry*, 3rd ed., (Englewood Cliffs, NJ: Prentice Hall, 1990), ISBN: 0-13-111253-8.

vii. Roland E. Larson, Robert P. Hostetler, and Bruce Edwards, *Calculus with Analytic Geometry*, 4th ed., (Lexington: D.C. Heath & Co., 1990), ISBN: 0-669-16406-2.

viii. James Stewart, *Calculus*, 5th ed., Belmont, CA: Brooks/Cole-Thomson Learning, 2003), ISBN: 0-534-39339-X.

Given that limits are universally understood to be the central topic that defines the calculus (Adams, 2013; Boyer, 1959; Strang et al., 2016), the analysis of the selected textbooks focused on topics (algebraic and trigonometric) that were introduced prior to limits (or in an appendix). These topics, by definition, comprise the prerequisites for the study of calculus and formed part of the framework used to devise the rigor algorithm described later in the chapter.

Tables 7.1 and 7.2 contain the raw data from the textbook analysis showing which algebraic and trigonometric topics were covered in each of the eight calculus textbooks. The analysis identified 14 topics.

7.3.2.1. Algebraic Topics Included in the Textbooks

Table 7.1 uses a binary key (1=true, 0=false) to indicate whether a textbook covered a topic either before limits or in an appendix. The columns are as follows:

Book Numbers i-viii, (as listed above)

Topic 1 Real Number Systems, to include axioms, number line, operations, theorems

Topic 2 Cartesian Systems, to include coordinate systems, linear and non-linear graphs on the coordinate systems, linear and non-linear inequalities on coordinate systems

Topic 3 Equations Methods, to include solving linear and non-linear equations of degree 2 and 3 graphically and algebraically

Topic 4 Factoring (including polynomial long division) and Paper & Pencil (P&P) Graphing Solutions, to include zeros (real factors factor and imaginary/complex number factoring), polar representations of real and imaginary zeros ($x=r\cos\theta$, $y=r\sin\theta$ and $e^{ix}=\cos x+i\sin x$ *or* $\text{cis}\,x$), and orientations and concavities (tangential slope changes over the domain)

Topic 5 Function Analytics & Applications, to include domain, range, composition, inverse (exponential and logarithmic)

Topic 6 Analytic Geometry, to include factoring and general equations for the conic circle, parabola, ellipse, hyperbola (asymptotes)

Topic 7 Algebraic Proofs, to include proofs and derivations for elementary theorems (rational root, quadratic, Pythagorean, and binomial)

Table 7.1 Algebraic Topics

Book Number	1. Real Number Systems	2. Cartesian Systems	3. Equations of Degree 0-2	4. Factoring & Graphing	5. Function Analytics & Applications	6. Analytic Geometry	7. Algebraic Proofs
i.	1	1	1	1	1	1	1
ii.	1	1	1	1	1	1	1
iii.	1	1	1	1	1	1	1
iv.	1	1	1	1	1	1	1
v.	1	1	1	1	1	1	1
vi.	1	1	1	0	0	0	1
vii.	1	1	1	0	0	0	0
viii.	1	0	1	0	1	0	0
Total	**8**	**7**	**8**	**5**	**6**	**5**	**6**

7.3.2.2. Trigonometric Topics Included in the Textbooks

The eight textbooks vary in how they introduce prerequisite trigonometric topics. Some cover them in a 'chapter 0' review section, some include them in appendices, and others lace remedial trigonometry into various chapters on an as-needed basis before, during, and after introducing derivatives of the trigonometric functions. Table 7.2 uses a binary key (1=true, 0=false) to indicate whether a textbook covered a topic either in a review section, a chapter, or an appendix. The columns are as follows:

Book Numbers	i-viii, (as listed above)
Topic 8	Angles, Arcs & Unit Circle, to include definitions and theorems (degrees and radians), identification of rectangular quadrants (rotation), significance of the unit circle (trigonometric function evaluation and trigonometric function behavior)
Topic 9	Trigonometric Functions, to include sine, cosine, tangent and their inverses, domain, range, and asymptotic behavior
Topic 10	Exact Value Triangles, to include degree and radian measure for the sine, cosine, tangent and inverses of 30º, 60º, 90º triangles and 45º, 45º, 90º triangles
Topic 11	Algebraic Factoring and Paper & Pencil (P&P) Graphing, to include orientations and characteristics of sine, cosine, and tangent and their inverses (on the Cartesians system and in the unit circle), amplitude, and frequency
Topic 12	Identities, Equations and Laws, to include sine, cosine, tangent and inverses identities (simplify trigonometric equations and solve trigonometric

equations), and implement identities to simplify and solve trigonometric equations

Topic 13 Paper & Pencil Graphing, to include representation of periodicity of sine, cosine, and tangent (calculation of asymptotes for tangents and calculation of sine^{-1}, cosine^{-1}, and tangent^{-1}) and representation of periodicity of sine^{-1}, cosine^{-1}, and tangent^{-1}

Topic 14 Trigonometric Proofs, to include proofs of theorems and laws (Pythagorean theorem, Law of Sines, and Law of Cosines)

Table 7.2 Trigonometric Topics

Book Number	8. Angles, Arcs & Unit Circle	9. Functions & Application	10. Exact Value Triangle	11. Factoring & Graphing (Algebraic)	12. Identities & Equations	13. Graphing (Trig.)	14. Trig. Proofs
i.	0	0	1	0	1	0	1
ii.	0	1	0	0	0	0	0
iii.	1	0	1	0	1	0	0
iv.	0	1	1	0	0	0	1
v.	1	1	1	1	1	1	1
vi.	1	0	1	0	0	0	0
vii.	0	1	1	1	0	1	0
viii.	0	1	1	0	1	1	0
Total	**3**	**5**	**7**	**2**	**4**	**3**	**3**

7.3.3. Refining Prerequisite Topic List through Statistical Analysis

The next step in determining which calculus prerequisite deliverables to include in the rigor algorithm was to statistically derive whether any of the topics not included in all eight books should be discarded. Sample average and sample standard deviation were collected in order to calculate a test statistic based on a population mean (μ=1) that reflects each topic being in each book. The Student t-test was implemented since *n*, the number of books used in the test, was less than 30 and because inclusion of topics 1 through 14 were distributed normally (Triola, 2010). No test statistic was needed for topics 1 and 3 since they appeared in all eight textbooks.

7.3.3.1. Student *t* – test Analysis of Algebraic Topics

The test statistic for the Student t-test is given by Triola (p. 356) as:

$$\left(t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}\right) \text{ where,}$$

t is the test statistic,
$\bar{x}$ is the sample mean,
μ is the claimed population mean,
s is the sample standard deviation and
n is the number of books

The test involves two tails since $H_0 : \mu = 1$ and $H_1: \mu \neq 1$, where the null hypothesis of $H_0: \mu = 1$ is that each topic in each book (using 1=true and 0=false) as logged in table 7.3 below, the Degrees of Freedom is equal to 7 (t(df) $n - 1 = 7$), with Significance of $\alpha = 0.05$ (two-tailed) gives a Critical t = ± 2.365.

Table 7.3 Algebraic Topic Analysis

Book Number	1. Real Number Systems	2. Cartesian Systems	3. Equations of Degree 0-2	4. Factoring & Graphing	5. Function Analytics & Applications	6. Analytic Geometry	7. Algebraic Proofs
i.	1	1	1	1	1	1	1
ii.	1	1	1	1	1	1	1
iii.	1	1	1	1	1	1	1
iv.	1	1	1	1	1	1	1
v.	1	1	1	1	1	1	1
vi.	1	1	1	0	0	0	1
vii.	1	1	1	0	0	0	0
viii.	1	0	1	0	1	0	0
Total	8	7	8	5	6	5	6
-($\bar{x}$)	1.000	0.875	1.000	0.625	0.750	0.625	0.750
Sample σ	0.000	0.354	0.000	0.518	0.463	0.518	0.463
n	8.000	8.000	8.000	8.000	8.000	8.000	8.000
Student t	n/a	-1.000	n/a	-2.049	-1.528	-2.049	-1.528
Critical t	n/a	±2.365	n/a	±2.365	±2.365	±2.365	±2.365

Table 7.3 shows that the Student *t* score is not within the Critical *t* region. This is also illustrated in Figure 7.1.

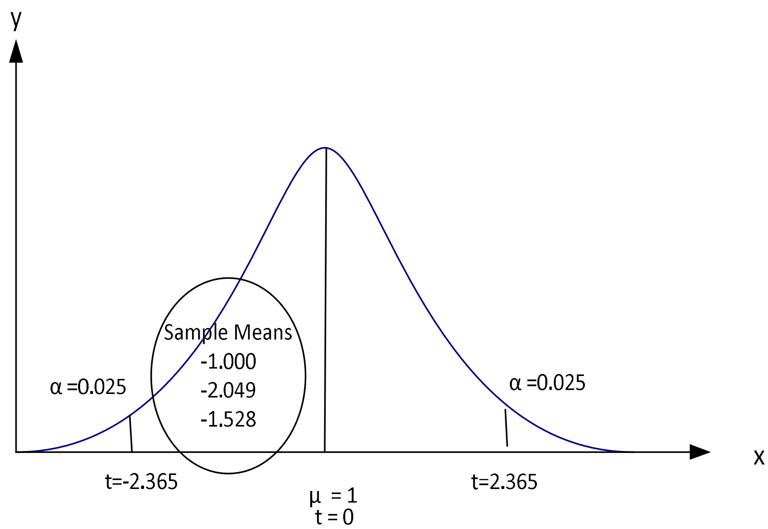


Figure 7.1 Algebraic Topic Analysis

The sample means (-1.000, -2.049, -1.528) for the five algebraic topics analyzed are all larger than ±2.365, so the forecast population mean ($\mu = 1$) is supported by the data. In other words, "fail to reject the null hypothesis" is concluded since the test statistics are not in the critical region. Thus, there is insufficient evidence to reject the claim that the five prerequisite topic selections be excluded simply because they do not appear in all eight textbooks. All seven topics listed in Table 7.1 were retained for the purposes of designing the rigor algorithm.

7.3.3.2. Student *t* – test Analysis of Trigonometric Topics

The same type of analysis was done for the seven trigonometric topics identified in the calculus textbooks—none of which appeared in all eight textbooks. As noted in Table 7.4 and Figure 7.2, all but one of the sample means for the seven trigonometric topics were larger than -2.365 so the null Hypothesis is not rejected for those six topics. Only algebraic factoring and graphing (Topic 11) fell into the critical region. Because this topic is already included in the algebraic prerequisites, it was omitted as a required trigonometric topic in the rigor algorithm.

Table 7.4 Trigonometric Topic Analysis

Book Number	8. Angles, Arcs & Unit Circle	9. Functions & Application	10. Exact Value Triangle	11. Factoring & Graphing (Algebraic)	12. Identities & Equations	13. Graphing (Trig.)	14. Trig. Proofs
i.	0	0	1	0	1	0	1
ii.	1	1	0	0	0	1	0
iii.	1	0	1	0	1	1	1
iv.	0	1	1	0	0	0	1
v.	1	1	1	1	1	1	1
vi.	1	0	1	0	1	0	0
vii.	0	1	1	1	1	1	0
viii.	1	1	1	0	1	1	1
Total	3	5	7	2	4	3	3
Sample ($\bar{x}$)	0.375	0.625	0.875	0.250	0.500	0.375	0.375
Sample σ	0.518	0.518	0.354	0.463	0.535	0.518	0.518
n	8.000	8.000	8.000	8.000	8.000	8.000	8.000
Student t	-2.049	-2.049	-1.000	-4.583	-1.528	-2.049	-2.049
Critical t	±2.365	±2.365	±2.365	±2.365	±2.365	±2.365	±2.365

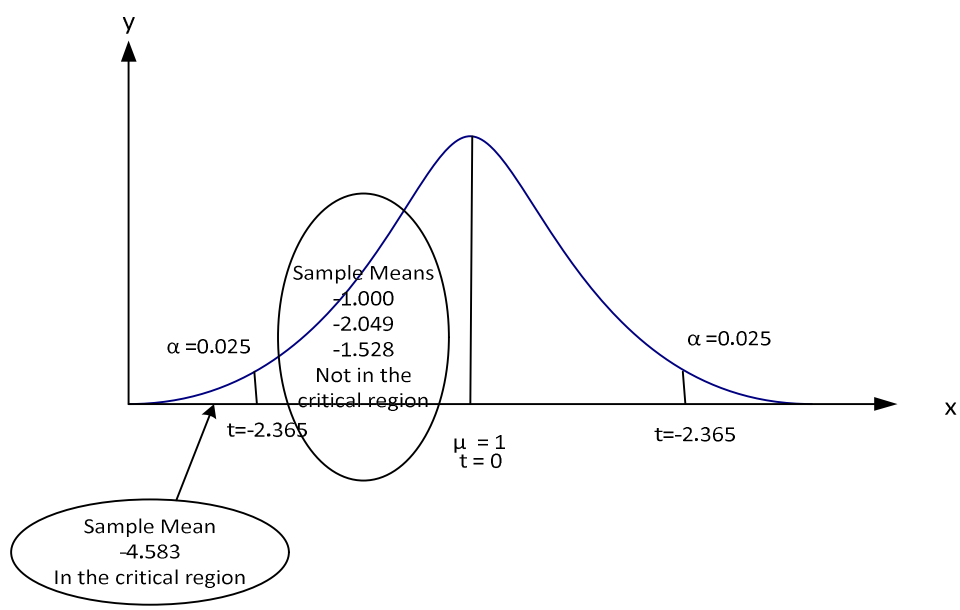


Figure 7.2 Trigonometric Topic Analysis

7.3.4. Validation of Selected Prerequisites

The conclusions of the statistical analysis are further validated by subjective observations regarding the inclusion of factoring and analytic geometry. For example, the statistical analysis indicates that both factoring (to include polynomial long division, completing the square, factor and remainder theorems, and deriving the quadratic formula) and analytic geometry (conic section identification and use) should be retained as prerequisites even though some modern textbooks are omitting them. In fact, Judson and Nishimori (2005) note that mastery of the skill called “reduction of operational complexity” is key for the elementary calculus student’s success. They also note that conic section identification, simplification, and manual graphing require factoring maturity that is gained only through first principle applications such as the derivation of the quadratic formula and ‘completing the square.’ Thus, it is concluded that advanced factoring skill is a critical requirement because much of elementary calculus is laden with complex algebraic gymnastics. Accordingly, function simplification or the reducing of operational complexity, such as through factoring, must be stressed as a mastered prerequisite for the calculus. Additionally, since the trigonometric functions are circular functions, the basis of the trigonometric approach requires, at minimum, unit circle mastery as a central concept. Thus, unit circle emphasis in the precalculus will prove vital in the calculus. The rigor algorithm emphasizes mastery of trigonometric identities and trigonometric equations as well as paper and pencil graphing literacy.

7.3.5. Other Considerations in the Rigor Algorithm

A textbook that incorporates all prerequisite topics may still lack rigor if it includes material that is pedagogically unnecessary or if it presents topics in a less than optimal sequence.

7.3.5.1. Technology—Pedagogically Necessary?

There is considerable debate among educators about the integration of technology into mathematics textbooks and classrooms. In developing a rigor measuring tool, the issue of technology in the classroom (e.g., graphing calculators and computer exercises) should be weighted as either a positive factor, a negative factor, or a neutral factor. Some argue that technology contributes much to students’ understanding of the connection between algebraic and geometric systems and they support the increasing emphasis in textbooks on teaching students to use graphing calculators and to complete computerized exercises (Koop, 2016; Thomson, 2008). Other educators counter that there are no mathematical concepts that require technology to either teach or assess. They contend that an abundance of technologically-related exercises and applications actually interferes with the mathematical maturing of students because technology tends to force the introduction of applications prematurely (Foster & Ollerton, 2020), thus truncating the concept acquisition phase of the pedagogical process and leaving students device rich and concept poor (Mao et al., 2017; White, 1998; Wilson & Naiman, 2004). Professor James Stewart of McMaster University (2003), author of the most recently published calculus textbook in this analysis, took a middle ground on technology, writing his book so that it could be used with or without technology. He noted, however, that the "availability of technology makes it not less important but more important to clearly understand the concepts that underlie the images on the screen" (p. xvii).

Eight years after Stewart made his observations about the need to keep concept mastery preeminent in mathematical teaching, the MAA Calculus I instructor survey for 2011 revealed that nearly 47% of college Calculus I professors prohibited the use of graphing calculators in examinations (Bressoud, 2011). Like Stewart, many university professors recognized a diminishing return for their students from the use of technology and set examination requirements that would motivate them to acquire manual mastery in the assessed topics (Mao et al., 2017).

Because technology (with its emphasis on computation) interrupts the "precept upon precept" process (thus hindering students from a recursively-maturing mastery of mathematical concepts), the rigor algorithm negatively scores textbooks that emphasize procedural or technological skills over mastery of concepts (Bergsten et al., 2015; Klein, 2000).

7.3.6. Defining the Components of the Rigorous Textbook

The learning of precalculus mathematics, like the learning of any other set of concepts, requires that each new topic or concept presented is solidly and recursively built on the previous topics and concepts. More than most subjects, mathematics mastery demands mastery of prerequisite material and, recursively, prerequisite material requires mastery of its prerequisite material and so on. Recursively presented topics and concepts is vital to the concept acquisition continuum (Carlson et al., 2010; Klein, 2000; "Our work of iteration," 1877). The rigor algorithm is designed to assess how well textbooks facilitate this approach to mathematical instruction. It considers a textbook to have an acceptable level of rigor when:

- Continuity of topics/concepts is upheld (Lin et al., 2020; Newton et al., 2020; Roehrig et al., 2021).
- Topic perspicuity is not stagnated by irrelevant discussions (Jackson et al., 2021; Roehrig et al., 2021; Siegler & Oppenzato, 2021).
- A maturing precept upon precept structure is maintained (Newton et al., 2020; Poast et al., 2021; Roehrig et al., 2021; Rohrer et al., 2020).
- The prerequisite topics (as described in Section 7.3.2) are presented recursively (Bowen et al., 2019; Jackson et al., 2021; Zerger, 2010).

7.4. Calculating Final Rigor Scores for Precalculus Textbooks

This section describes the three measurements (derived from the four components listed in section 7.3.6) incorporated into the rigor algorithm to calculate a Final Rigor Score (FRS) for textbooks that can be used to predict how effectively each textbook has incorporated the prerequisites for year-13 calculus to facilitate year-12 student mastery of calculus prerequisites. The three measurements are: the Cohesive Continuum Score, the Presentation Score, and the Maturity Score.

7.4.1. Measuring Topic Order (A)—Calculating the Cohesive Continuum Score (CCS)

The concept of cohesive continuum looks at the order in which topics are presented in a textbook. That is, it assesses the extent to which a textbook maintains a continuum of prerequisite topics to prerequisite topics while maturing the previous topics in the current topic. TIMSS testing results consistently rank students from countries that maintain curriculum coherence higher than those from countries that do not (Hong & Mi Choi, 2014; Schmidt, 2004; Schmidt et al., 2001). Thus, the rigor algorithm negatively scores textbooks that fail to maintain cohesive continuum as defined below.

Cohesive: Effectively blending strategies and content for subject presentation and content consistency (Lin et al., 2020) while ordering mathematical concepts in such a way as to promote increasing abstraction capability in seemingly uncorrelated information that the student needs in complex problem solving (Yin et al., 2020).

Continuum: "A coherent whole characterized as a collection, sequence, or progression of values or elements varying by minute degrees"(Webster, 1828). Learning requires "a rigorous, *continuous* sequence of concept building through listening, practicing, memorizing, and repeating" (Wagner, 2006, p. 88). A continuum is required in the support of the appropriately ordered prerequisite learning, proficiency, and mathematical maturity (Faulkner et al., 2020; Wheeler, 1986).

Cohesive Continuum: A union of "cohesive" and "continuum" provides a definition for "cohesive continuum": the effective blending of strategies and content for subject presentation in support of the appropriately ordered prerequisite learning, proficiency, and mathematical maturity (Lin et al., 2020). It is “a bonded progression of elements or concepts, increasing in complexity while remaining succinct with adjacent elements”(Burton, 2013). It ensures scholarly communication in a knowledge continuum such that the dissemination of new knowledge may be utilized (Vassallo, 1999).

A cohesive continuum in mathematics instruction will reflect a strict adherence to a progressive (step by step) mastery of basic concepts, maintenance of succinctness of progressing adjacent concepts, and a commitment to presenting material, examples, and exercises for the current topic that include previous concepts and that connect them to upcoming ideas (Anselone & Lee, 2005; Burton, 2013; Hirsch, 1996; Most & Wellmon, 2015).

Topic order—the methodical arrangement of subject materials—is the foundation of cohesive continuum in a textbook (Retnawati et al., 2018). The optimal mathematics textbook is one that incorporates “cyclic reinforcement” of ordered topics as it strategically implements the preservation of concept order and arrangement as the topic matures (Carlson et al., 2010).

The order of topics (chapters and sections) is integral in the maintenance of the concept continuum. That is, without a cohesive topic order, the concept continuum will fragment, and the cyclic reinforcement will fail to maintain the continuum within the topic order (Carlson et al., 2010; Yin et al., 2020). Figure 7.3 visually depicts the essential components of a rigorous presentation that will be measured in the rigor algorithm. Note how the simplicity of this continuum acts as a funnel toward the calculus.

- <u>(A) Precalculus Topic Order</u> (concept-upon-concept), calculated as a “Cohesive Continuum Score” (CCS) in the algorithm. The algorithm penalizes textbooks that do not follow the optimal topic order.

- (B) Concept Continuum (concept flow to sustain continuity and perspicuity), calculated as a "Presentation Score" (PS) in the algorithm (see section 7.4.2). The algorithm penalizes textbook attributes that interfere with concept flow.
- (C) Cyclic Reinforcement (concept practice to promote concept maturity), calculated as a "Maturity Score" (MS) in the algorithm (see section 7.4.3). The algorithm rewards textbooks that promote concept maturity.

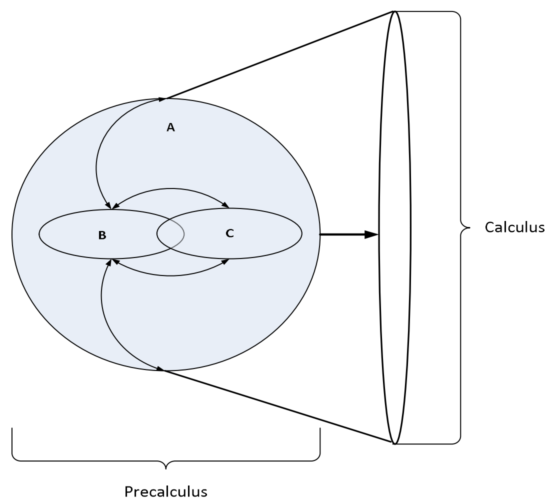


Figure 7.3 Essential Components of a Rigorous Presentation

7.4.1.1. Determining Optimal Topic Order

In order to avoid the fragmentation of topics and topic order, preceding and subsequent topics may be ordered into sequential reasoning and computational abilities (Yin et al., 2020). Each subsequent level of requirements should be introduced within the reasoning abilities category. For example, the elementary calculus student must understand the concept of a function to especially include the composition and inverse concepts. However, since the very definition of a function requires a precise understanding of what the domain and range are, the optimally rigorous textbook must first introduce students to domain and range (covariational reasoning) *before* formally addressing functions. Only then should the textbook present the algebraic concept and associated computations for function composition and function inverse (Carlson et al., 2010). According to Carlson's assertion, the elementary calculus student needs to be able to:

Set up, solve, and understand:

- Algebraic equations/functions
- Logarithmic equations & functions
- Exponential equations & functions
- Trigonometric equations/functions

Graph:

- Algebraic equations/functions
- Logarithmic equations & functions
- Exponential equations & functions
- Trigonometric equations/functions
- Interpret said graphs

Prove:

- Algebraic identities and theorems
- Trigonometric identities and theorems
- Understand and implement said proofs

Solve:

- Linear systems
- Nonlinear systems

7.4.1.2. Carlson-derived Topic Order

Given the broad agreement among scholars with the principles underlying Carlson's suggested topic order for the algebraic and trigonometric topics listed in Tables 7.1 and 7.2 (Klein, 2000; Wakefield et al., 2018; Weir, 2020), it was decided to use the following "Carlson-derived" topic sequence to design the rigor algorithm:

- General Equations and Inequalities Review to include:
    - algebraic structures, concepts (domain and range, symmetry)
    - linear equations, graphs, and applications
    - quadratic equations and applications
    - complex numbers
    - other types of equations (circles)
    - inequalities
- Polynomials Functions and their Graphs
    - quadratic functions
    - higher order polynomial functions
    - division of polynomials
    - zeros of polynomials
    - applications
- Functions and Graphs
    - functions
    - graphs of functions (manually)
    - parent functions (function family)
    - transformation of functions

    - composite functions
    - inverse functions
- Exponential and Logarithmic Functions and Graphs
    - properties of exponents and logarithms
    - exponential functions and graphs
    - logarithmic functions and graphs
    - exponential and logarithmic equations and applications
- Trigonometry (Functional, Graphical, and Analytic)
    - degree and radian measures
    - unit circle
    - definitions of the trigonometric functions as circular
    - standard trigonometric identities (recognition, use, and proof)
    - graphs of trigonometric functions
    - inverse trigonometric functions
    - law of sines and law of cosines
    - equations and solutions
    - applications
- Polar Coordinates and Complex Numbers and Graphs
    - polar coordinates and graphs
    - geometric representation of complex numbers
    - powers and roots of complex numbers
    - De Moivre's Theorem
- Vectors and Determinants

- vector Properties and operations
- applications of vectors
- parametric equations
- 3D vectors (optional)

- Analytic Geometry (Conic Sections) and Graphs
  - ellipse, hyperbola, parabola and circle
  - rotations and transformations
- Sequences and Series
  - notation and factorials
  - induction
  - summations
  - finite and infinite considerations

Table 7.5 lists the prerequisite topic order derived from Carlson and Figure 7.4 illustrates the dependency construct depicted in Figure 7.3. (Dashed lines indicate optional coursework dependencies.)

Table 7.5 Carlson-Derived Precalculus Topic Order

Topic Order	Topic
1	General and Prerequisite Review
2	Polynomials and Rational Functions
3	Equations and Inequalities
4	Functions: Theory, Operations and Graphs
5	Exponential and Logarithmic Functions
6	Trigonometry (Functional, Graphical, and Analytic)
7	Polar concepts and Complex Number integration
8	Vectors and Determinants
9	Analytic Geometry
10	Sequences and Series with Combinatorics
11	*Linear Systems and Matrices- Optional*

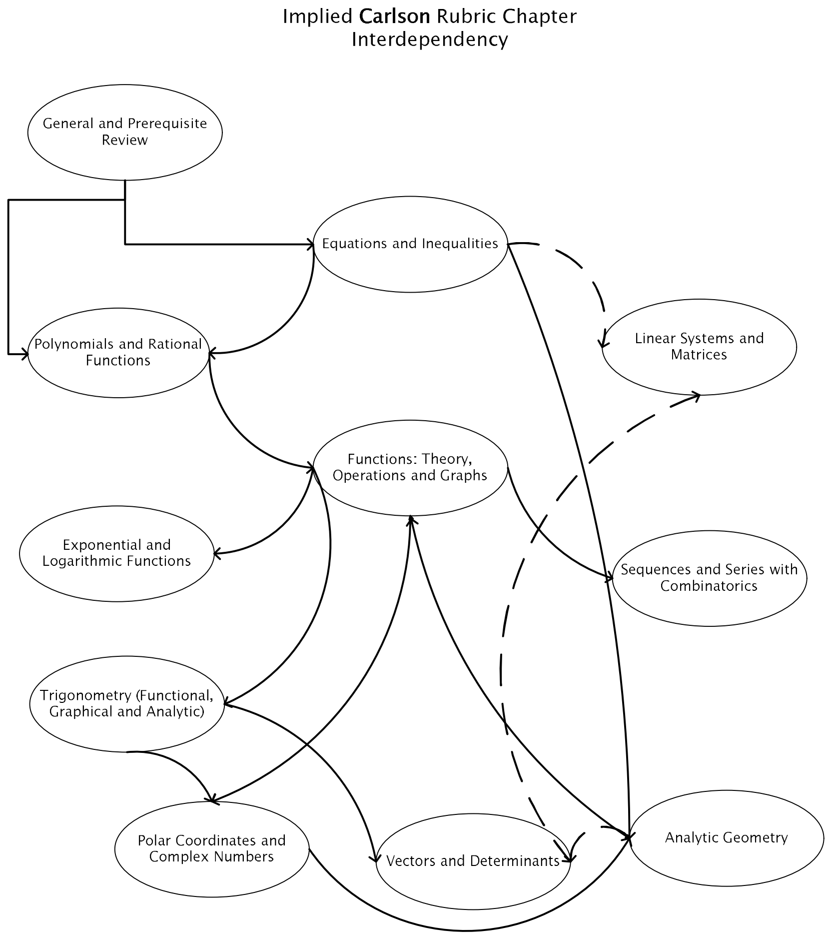


Figure 7.4 Dependency Construct of Carlson-Derived Topics

7.4.1.3. Final Topic Order for Rigor Algorithm

The rigor algorithm considers the following order of precalculus topics to be the optimal ordering for a maximum rigor score:

Chapter 0/1, or prerequisite review chapter (often chapter 1) will reinforce the minimum required mastery for the essential concepts that will be presented in the precalculus textbook. These topics commonly include:

- polynomial, rational, exponential, logarithmic, and trigonometric functions
- functions of complex numbers
- linear systems and tools

- sequences and series (optional based on infinite series topic in either 1st or 2nd term/semester calculus requirements)

Accordingly, the review topics should include:

- real numbers
- exponents and radicals
- polynomials
- rational expressions
- coordinate systems
- finite and transfinite arithmetic

Chapter 2 will flow from some of the review chapter's concepts.

- factoring polynomial equations
- graphing polynomial functions
- roots of polynomial equations
- proofs and derivations

Chapter 3 will flow from polynomials into applications of polynomials.

- linear and polynomial inequalities
- applications of the inequalities and in particular the quadratic
- using the discriminant
- proofs and derivations

Chapter 4 will expand polynomials into functions and function attributes.

- function definition and graphs
    - domain and range
- functional relations and specifications

    - function composition
    - applications
    - inverse functions

Chapter 5 will flow from functions and inverses into the acquisition and use of an inverse function.

- exponents
    - integral
    - rational
    - exponential functions
    - proofs and derivations
- logarithms
    - logarithmic functions
    - properties of logarithms
    - proofs and derivations
    - exponential equations
- base changing
- exponential growth
    - natural logarithms and the number e

Since functions are now formally defined, the optimally-rigorous textbook will next address the application of functions to trigonometry.

Chapter 6 will introduce and define trigonometric functions.

- angles, arcs, and sectors
    - angle measure

    - application to the circle
    - proofs and derivations
    - unit circle
- circular trigonometric functions
    - sine
    - cosine
    - tangent
    - co-functions
    - graphing the trigonometric functions
- identities and equations
    - functional relationships
    - solving trigonometric equations
    - proofs and derivations

Chapter 7 will apply the unit circle and angle measure to triangle trigonometry.

- solving triangles
    - right triangle trigonometry
    - triangle area
    - law of sines and cosines
    - proofs and derivations
- trigonometric inverses
    - inverse trigonometric relations
    - domains and "Arc" or "$^{-1}$" (Arcsin, $\sin^{-1}$)
- trigonometric graphing

    - stretching
    - reflecting
    - asymptotes
    - applications
- trigonometric equations
    - solving trigonometric equations

Chapter 8 will advance the identities to the trigonometric addition formulae.

- addition and subtraction of sine, cosine and tangent
- double and half angle
- derivations
- applications
- solving trigonometric equations

Chapter 9 will apply sine and cosine equations to polar coordinates and complex numbers.

- polar coordinates and graphs
    - move between polar coordinates and rectangular coordinates
- geometric representations of complex numbers
- powers and De Moivre's theorem
- roots

Chapter 10 will apply the polar representations and associated trigonometry to vectors and determinants.

At this point in the optimally rigorous textbook, all prerequisite algebraic and trigonometric functions have been covered. The textbook should now introduce the following calculus topics because both appear in all sampled calculus textbooks and, thus, have been incorporated into the rigor algorithm as prerequisites.

- Conic Sections (Analytic Geometry)
- Sequences and Series

The rigor algorithm penalizes any precalculus textbook that does not include both topics and it penalizes any textbook that includes one or both topics before completing instruction in all algebraic and trigonometric topics outlined above. Below is an example of how these two topics might appear as chapters 11 and 12 in an optimally rigorous precalculus textbook.

Chapter 11 will apply function analytics, parabolas, circles, and polynomial manipulations to the analytic geometry of the cone (i.e., conic sections).

- circles
- ellipses
- hyperbolas
- parabolas
- applications and common properties

Chapter 12 will apply logic and proof experience, as well as algebraic maturity, to sequences and series.

- finite sequences and series
    - arithmetic and geometric sequences
    - arithmetic and geometric series and their sums
- infinite sequences and series
    - limits of infinite sequences
    - sums of infinite series

- sigma notation
- power series
- mathematical induction

Precalculus textbooks may also include additional topics such as:

- Probability & Statistics
- Advanced Linear Systems with Matrices

The rigor algorithm does not reward textbooks that include these (and other) topics; it will not penalize textbooks that include these (and other) topics *unless* they are introduced prior to the presentation of all the prerequisite topics outlined above in chapters 0/1 through chapter 10.

7.4.2. Measuring Concept Continuum (B)—Calculating the Presentation Score

In addition to presenting topics in the proper order, an optimally rigorous textbook will ensure that all material presented is *relevant* to the concepts being mastered. The antithesis of concept continuum is the interruption of topic instruction with irrelevant discussions, technology-centric verbiage, or concept-irrelevant imagery (photographs, illustrations, cartoons) (Henningsen & Stein, 1997). The rigor algorithm Presentation Score (PS) decrements textbooks for extraneous material that disrupts the concept continuum. The PS is calculated by collecting demographics from each textbook and then applying a mathematical formula that measures concept continuum.

7.4.2.1. Textbook Demographics Collected:

A. Teaching Page Count
    i. All teaching pages (excluding solutions and indexes)
    ii. All remedial review pages (e.g., Chapter 0)

B. Chapter Count

i. Total number of non-calculus chapters

C. Section Count

i. Total number of sections in the counted chapters

D. Irrelevant imagery pages count (if page has one or more images)

E. Exercises/Examples Count

i. All Exercises/Examples

ii. Technology, Discovery, Focus or Group exercises/examples

iii. Concept exercises/examples (Ei.-Eii.)

7.4.2.2. Components of the PS Mathematical Formula:

Since textbooks generally contain both relevant/cogent material and irrelevant material, it was determined that content relevance could be described as the *density* of relevant/cogent material (Ipek, 2011). Thus, the following PS mathematical formula was created using the following variables:

#1. Proportion of Review Pages to Teaching Pages = A.ii/A.i. When the rigor algorithm was being designed, it was unknown whether the number of remedial or review pages would be a positive or a negative variable in maintaining the concept continuum. The number of remedial or review pages was analyzed by inclusion using a variation of Rational Choice or Multi Attribute Utility Theory (Jonassen, 2012). The Presentation Score highs and lows were ordered equally so the remedial variable was determined to be unneeded and was disqualified using the argumentation decision process described by Jonassen.

#2. Imagery per Teaching Page = D/A.i. Because excessive or irrelevant images degrade student mastery of concepts, they are considered a negative variable in the PS calculation.

#3. Technology Exercises/Examples per Teaching Page = E.ii./A.i. Because excessive technology infusion contributes to the student's inability to acquire a mastery-level understanding of the concept presented and group or "discovery" exercises (Chun-Hung Lin et al, 2013), having the same effect, it is regarded as a negative variable.

#4. Concept-Building Exercises/Examples per Teaching Page = (E.iii)/A.i. Given the positive nature of concept mastery practice (Schmidt et al., 2001), the number of concept exercises per teaching page is calculated as a positive variable.

The PS formula was created using accounting principles. It creates a balance sheet of assets and liabilities within textbooks in which conceptual assets in a textbook (relevance, topic flow/ordering, application, practice, and integration) are considered to be positives in the assets column while concept liabilities (interference or interruption of any or all of the conceptual assets) are considered to be positives in the liability column (Barth, 2014; Miller-Nobles et al., 2018). A balance sheet is the sum of assets minus the sum of liabilities (Gangwar & Gangwar, 2008) and, thus, the final PS is determined by calculating the difference between the concept-building exercises/examples (concept assets) per teaching page and the sum of the material that degrades the concept-building exercises/examples (concept liabilities) in the textbook using this formula: **PS = #4 - (#3 + #2 + #1)**. The positive components of the PS (PS_p) = #4 and the negative components of the PS (PS_n) = (#3 + #2 + #1), thus, $PS_{final} = (PS_p - PS_n)$. The final PS can be positive, negative, or even zero.

7.4.3. Measuring Cyclic Reinforcement (C)—Calculating the Maturity Score

In addition to presenting topics in the proper order and sustaining the concept continuum by minimizing irrelevant or distracting material, the optimally rigorous textbook will continually reinforce topics in a cyclic manner in which previously-mastered topics are reintroduced in increasingly complex scenarios. The rigor algorithm calculates a Maturity Score (MS) to measure how well a textbook achieves cyclic reinforcement of concepts.

There are two components used to calculate the Maturity Score. The first (called Maturity Analytics) assesses whether a textbook includes four key categories of instruction that promote cyclic reinforcement of concepts. The categories are:

- Mathematical Tables (see section 7.4.3.1)
- Reduction of Functional or Operational Complexity (see section 7.4.3.2)
- Functional Analysis (see section 7.4.3.3)
- Trigonometric Function and Identities (see section 7.4.3.4)

The second component of the MS, (called Maturity Numerics) counts how many pages in the textbook are devoted to each of these four instructional categories. Using a similar approach to the PS calculation, it measures the proportion of each textbook given to the key categories (i.e., it determines the conceptual asset density of these categories in each textbook) (Petersson et al., 2020; Schubring & Fan, 2018).

7.4.3.1. Mathematical Tables.

Logarithms are significantly important in calculus (Rachael & Signe, 2013). In the past tables of logarithms were commonly used in conjunction with the teaching of logarithms. Today, tables of logarithms are absent from most textbooks; they have been replaced by electronic calculators. Though logarithms can be taught in the absence of logarithmic tables, depriving students of an understanding of how the tables were constructed in the first place leaves conceptual gaps in the presentation of the topics (Panagiotou, 2010). Mulqueeny (2012) found that presenting students with the historical account of the development of the logarithms and introducing the logarithmic tables as the initiation of the study of logarithms provided understanding that was needed for acquiring the concept. Tables of logarithms are an implied part of the collection of unused yet valuable necessities for modern mathematical curricula cited by Boas (1993), because knowing and using them contributes to thought processes that are needed in subsequent learning and thinking experiences. Cook (2013) asserts that logarithmic tables fill the "conceptual gaps" mentioned above by making logs "more tangible" and that using the tables teaches students how to interpolate. The skill of interpolation is important for precalculus students to learn in preparation for their calculus course since interpolation is needed in approximation theory as it relates to where functions cross the x axis (the solution) and approximation of areas under curves using numeric methods. (Yang & Gordon, 2014). Because understanding and using mathematical tables promotes conceptual mastery of logarithms and skill in interpolation, the rigor algorithm includes an assessment of whether (and to what extent) a textbook includes mathematical tables.

7.4.3.1.1. Maturity Numerics for Mathematical Tables.

The MS is based on how many pages of mathematical tables addressing the following items are included in the textbook:

- Logarithmic (Base 10)
- Logarithmic (Base *e*)
- Trigonometric tables for sine, cosine, and tangent

7.4.3.2. Reduction of Functional or Operational Complexity

Elementary factoring, advanced factoring, or factoring in general, facilitates concept maturity by enabling the student to see or visualize a problem in its entirety (DiBello & Stout, 2007; Ponce, 2007). Advanced factoring mastery is fundamental because the elementary calculus student is immediately and constantly faced with algebraic complexity that must be simplified; thus, maturity in the application of advanced factoring is enhanced by student participation and hands-on exposure to the techniques (Dobbs & Peterson, 1991; Fitzherbert, 2016). Therefore, the rigor algorithm includes an assessment of whether (and to what extent) a textbook includes multiple techniques for reduction of functional or operational complexity.

7.4.3.2.1. Maturity Numerics for Reduction of Functional or Operational Complexity

The MS is based on how many teaching pages (not appendices) of the textbook are devoted to the following topics and techniques:

- Completing the Square
- Quadratic Formula
- Polynomial Long Division (and Synthetic Division)
- Remainder and Factor Theorem
- Rational Roots Theorems
- Fundamental Theorem of Algebra
- Min-Max Applications

7.4.3.3. Functional Analysis

The application of domain and range to the inverse or composed function and the trigonometric functions work to coerce thinking that is organizational in nature (Hajizah et al., 2020). To be able to see and understand what the problem is asking is a maturing event for the student as this promotes connectedness of other concepts (Ponce, 2007). Therefore, the rigor algorithm includes an assessment of whether (and to what extent) a textbook comprehensively covers elementary function and elementary function theory.

7.4.3.3.1. Maturity Numerics for Functional Analysis.

The MS is based on how many pages of the textbook are devoted to the following topics:

- Definitions of Functions
- Domain and Range
- Function Inverse
- Function Composition

7.4.3.4. Trigonometric Functions and Identities

Trigonometric functions and identities and their proofs/derivations constitute a significant contribution to the calculus student's problem-solving strategy of a derivative or an anti-derivative problem because operation and functional complexity can be dramatically reduced by re-writing the trigonometric function as a simpler quantity (Kindle & Gentimis, 2018). For example: Given: $f(x) = sin2x$, calculate $f'(x)$. To solve this example, the calculus student must implement the chain rule, which is an algorithm for calculating the derivative of the composition of two functions. $The\ two\ functions\ forming\ f(x)$: $h(x) = 2x\ and\ g(x) = sinx. Thus, f(x) = g(h(x))$. However, complexity may be reduced to needing only the product rule algorithm by altering $f(x) = sin2x$ to become $f(x) = 2sinxcosx$. This is accomplished with trigonometric identities. Therefore, the rigor algorithm includes an assessment of whether (and to what extent) a textbook addresses trigonometric functions and identities.

7.4.3.4.1. Maturity Numerics for Trigonometric Functions and Identities

The MS is based on how many pages of the textbook are devoted to the following topics:

- Trigonometric Functions (to include Unit Circle)
- Trigonometric Identities

7.4.3.5. Calculating the Maturity Score

The calculation for the Maturity Score is as follows:

$$\boldsymbol{MS} = \frac{\boldsymbol{A+B+C+D}}{\boldsymbol{Number\ of\ Teaching\ Pages}}$$

where **A**, **B**, **C**, and **D** are the number of pages of relevant instruction in the four instructional categories. Though word count per page can vary from textbook to textbook, because A through D and the number of teaching pages are all derived from the same textbook, the density/proportionality retains integrity across all examined books.

7.4.4. Combining PS and MS into a Presentation Maturity (PM) Score

The PS and MS intersect in that cyclic reinforcement, as measured by the MS, affects the PS because concept building exercises contain the cyclic reinforcement that is measured in the MS. Since this was an intersection of densities, a new score (the PM, for Presentation Maturity) is derived as the product of the PS and the MS. As stated in 7.4.2.2, the final PS is determined by subtracting the concept interference density from the concept building density: $PS_{final} = (PS_p – PS_n)$. The PM score is calculated as $MS(PS_p) – MS(PS_n)$ with the MS applied to both positive and negative PS factors because even concept building on irrelevant concepts (such as technology-centric exercises) is calculated as a negative impact in the rigor algorithm.

7.4.5. Calculating the Final Rigor Score

The Final Rigor Score is derived by adding the product of the Presentation Score and Maturity Score (the PM score) to the Cohesive Continuum Score for each textbook.

$\boldsymbol{FRS = CCS + (PS \text{ x } MS)}$, and becomes in the final computation

$$\boldsymbol{FRS = CCS + (PM)}$$

The FRS formula was developed based on the following:

1. Since a final asset measure is affected by addition of assets and the subtraction of liabilities, the FRS is derived by adding assets and subtracting liabilities.
2. The CCS (the assessment of topic order adherence) is the high priority asset in the textbook; a teacher may augment an orderly set of topics in any way that suits the class but will run into problems when the topic order fails to promote the "preceding-current-subsequent" model (Carlson et al., 2010; Yin et al., 2020). The CCS is an asset, but it is assigned a negative value in the algorithm and thus, for the purposes of determining the FRS, the PM score (deficiency mitigation) is added.

7.5. Conclusion

The concept of MRP and deliverable end-product necessitates that tertiary math departments set requirements for year-12 mathematics curriculum. The rigor algorithm described in this chapter used the tertiary elementary calculus textbook as the driver for understanding the "deliverable end-product"—i.e., what was needed for secondary students to be ready for their elementary calculus course. The eight sampled calculus textbooks shared much assumed knowledge for the incoming student to be both well-versed and well-practiced. The triumvirate scoring model was derived so that it could be applied to the precalculus textbooks in an orderly and repeatable way by secondary mathematics departments to enable them to assess their precalculus textbooks to suggest modifications and upgrades to better prepare their mathematics students for tertiary coursework.

In Chapter 8, the rigor algorithm will be demonstrated. A CCS, PM, and FRS will be calculated for a selected sampling of textbooks identified by the year-13 student survey.

Chapter 8. Mathematical Textbook Rigor Tool Applied

8.1. Introduction

This chapter demonstrates how to use the rigor tool presented in Chapter 7 using a selection of eight precalculus textbooks identified through the survey as well as some textbooks published prior to the widespread use of handheld electronic calculators. The details of chapter and topic order for these textbooks will be included in an appendix so the process used to create a final rigor score for each book can be followed step-by-step. The chapter will then apply the algorithm in an abbreviated fashion to an additional eleven precalculus textbooks published in Australia and the United States for the purposes of providing a pool of textbook data for analysis. Four of the nineteen books assessed with the rigor tool were published between 1965 and 1985, six were published between 1986 and 2003, and nine were published between 2004 and 2012. Section 8.2 describes the demonstration of the use of the rigor algorithm, section 8.3 provides results of the application of the algorithm to the remaining 11 textbooks, and section 8.4 tabulates the results for all 19 textbooks and provides analysis.

8.2. Verifying the Rigor Algorithm

This section demonstrates how the reliability of the rigor algorithm was verified by applying it to a sampling of eight precalculus textbooks. Since the purpose of the rigor algorithm is to measure the rigor of textbooks, it was decided to use a sampling of textbooks distributed across the graduation years of survey respondents plus a sampling of older textbooks commonly used in the 1970s and the 1980s[7] to verify the algorithm and to test the hypothesis that rigor in precalculus textbooks has degraded over time. One of the selected books, number ii, *Advanced Mathematics: A Precalculus Course*, (1984), had a published review available

[7] Based on the author's personal experience.

that was used as an independent validation of the algorithm's assessment of textbook rigor. (See comments following Table 8.14).

The following eight secondary school precalculus textbooks published between 1972 and 2012 were selected as representative of textbooks designed to prepare students for success in year-13 calculus:

i. Earl Swokowski, *Fundamentals of Algebra and Trigonometry*, 2d ed., (Boston: Prindle, Weber & Schmidt, Inc., 1972).

ii. Richard G. Brown and David P. Robbins, *Advanced Mathematics: A Precalculus Course*, 1st ed., (Boston: Houghton Mifflin Co., 1984).

iii. Richard G. Brown, *Advanced Mathematics: Precalculus with Discrete Math and Data Analysis*, 1st ed., (Boston: Houghton Mifflin Co., 1992).

iv. Roland E. Larson, Robert P. Hostetler, *Precalculus*, 3rd ed., (Lexington, MA: D.C. Heath & Co., 1993).

v. Franklin Demana, Bert K. Waits, Gregory D. Foley & Daniel Kennedy, *Precalculus: Graphical, Numerical, Algebraic*, 5th ed., (Reading, MA: Addison Wesley Longman, Inc., 2001).

vi. James Stewart, Lothar Redlin & Saleem Watson, *Precalculus: Mathematics for Calculus*, 4th ed., (Pacific Grove, CA: Wadsworth Group, 2002).

vii. Roland E. Larson, Robert P. Hostetler, and Bruce H. Edwards, *Precalculus with Limits: A Graphing Approach*, 4th ed., (Boston: Houghton Mifflin Co., 2005).

viii. Paul A. Foerster, *Precalculus with Trigonometry: Concept and Applications*, 1st ed., (Emeryville, CA: Key Curriculum Press, 2012).

Section 8.2.1 describes how the algorithm was used to analyze Topic Order (A) and generate a Cohesive Continuum Score. Section 8.2.2 describes how the algorithm was applied to calculate a Presentation Score that reflects Concept Continuum (B). Section 8.2.3 describes how the algorithm was used to calculate a Presentation Maturity Score (PM) to assess Cyclic Reinforcement (C). Section 8.2.4 gives the Final Rigor Score calculated for each of the eight sampled textbooks.

8.2.1. Analyzing Topic Order (A)—Calculating the Cohesive Continuum Score (CCS). Topic order was compared to the Carlson-derived topic order to calculate the CCS for each textbook. Table 8.1 depicts which topics each of the precalculus textbooks included and in what order. (Note: Calculus/Limits topics were omitted because none of the calculus textbooks surveyed to create the MRP-based prerequisites for precalculus assumed a knowledge of limits. Additionally, because precalculus books should focus on preparation for calculus and not calculus itself, calculus topics were omitted from the rigor algorithm.) The table uses a generic topic nomenclature. The numbers in each column, reading left to right, indicate in what order each textbook introduced the topic. The numbers do not directly correspond to chapter numbers; rather, they designate the relative order in which a book introduced a topic. (Appendix E provides a detailed chapter outline for each textbook to include the exact nomenclature used for topics and the chapter numbers for each topic.) Multiple trigonometry chapters were grouped together into a single chapter. A zero indicates that the book did not cover that topic. The table also includes some descriptive statistics (mode, mean, median, standard deviation) related to topic order. Topic numbers correspond to the following topics:

1. General and Prerequisite Review
2. Polynomials and Rational Functions

3. Equations and Inequalities

4. Functions: Theory, Operations and Graphs

5. Exponential and Logarithmic Functions

6. Trigonometry (Functional, Graphical, Analytic)

7. Polar and Complex

8. Vectors and Determinants

9. Analytic Geometry

10. Sequences and Series with Combinatorics

An eleventh topic, *Linear Systems and Matrices*, was included in many precalculus textbooks. However, because it is not mandatory in the Carlson-derived rubric, it was not included in the rigor algorithm scoring.

Table 8.1 Analyzing Topic Order (A)

Book	Topic Order									
i.	1	**8**	2	3	4	5	7	6	0	9
ii.	1	2	3	4	5	6	7	9	8	10
iii.	1	2	3	4	5	7	8	9	6	10
iv.	1	3	1	2	4	5	6	6	10	9
v.	1	3	2	2	4	5	6	6	8	9
vi.	1	3	1	2	4	5	6	6	8	9
vii.	1	3	1	2	4	5	6	6	10	9
viii.	1	3	0	2	**2**	4	6	**4**	5	8
Mode	1	3	2	2	4	5	6	6	8	9
Mean	1.00	3.38	1.71	2.63	4.00	5.25	6.50	6.50	6.88	9.13
Median	1.00	3.00	2.00	2.00	4.00	5.00	6.00	6.00	8.00	9.00
SD	0.00	1.92	1.11	0.92	0.93	0.89	0.76	1.69	3.27	0.64

Bold font designates statistical outliers.

8.2.1.1. Outlier Considerations

An outlier is an observation that lies outside the overall pattern of a distribution (Moore & McCabe, 1999). Usually, the presence of an outlier indicates some sort of problem. This can be a case which does not fit the model under study or an error in measurement. Table 8.2 calculates the Inter Quartile Range (IQR) to determine if there are outliers of topic order in the selected textbooks; that is, it calculates whether there are significant differences in where a topic is placed in relative order. The numbers in the rows are the chapters in which a particular topic was introduced. Columns are sorted lowest to highest on chapter number to acquire first quartile (Q1) and third quartile (Q3). As seen in the table, an outlier is a point which falls more than 1.5 times the IQR above the third quartile or below the first quartile [Q3+I and Q1-I]. It will be shown that including the outliers is an important consideration for "relative chapter order" because textbook authors place their chapters in the order they believe is consistent with preceding and subsequent material.

Table 8.2 Outlier Table: (IQR=Inter Quartile Range)

	Carlson Rubric Topic Order									
Carlson Rubric Topic Order	1	2	3	4	5	6	7	8	9	10
	1	2	0	2	**2**	4	6	**4**	0	8
	1	2	1	2	4	5	6	6	5	9
	1	3	1	2	4	5	6	6	6	9
	1	3	2	2	4	5	6	6	8	9
	1	3	2	2	4	5	6	6	8	9
	1	3	3	3	4	5	7	6	8	9
	1	3	3	4	5	6	7	9	10	10
	1	**8**	3	4	5	7	8	9	10	10
Q1=MEDIAN Low	1	2.5	1	2	4	5	6	6	5.5	9
Q1 – I	1	1.75	-1.25	-0.25	3.25	4.25	4.5	3.75	0.25	8.25
Q3=MEDIAN High	1	3	2.5	3.5	4.5	5.5	7	7.5	9	9.5
Q3+ I	1	3.75	4.75	5.75	5.25	6.25	8.5	9.75	14.25	10.25
IQR	0	0.5	1.5	1.5	0.5	0.5	1	1.5	3.5	0.5
I= IQR * 1.5	0	0.75	2.25	2.25	0.75	0.75	1.5	2.25	5.25	0.75

Bold font designates statistical outliers.

8.2.1.2. Calculating Means for Topic Order

In Table 8.3 the Means were calculated for the topic order. The numbered table columns record the following calculations:

1. Mean order including the outliers of topic order

2. Absolute value of the difference between the Outlier Mean and the Topic
3. Mean without outliers
4. Absolute value of the difference between the "no Outlier Mean" and the Topic
5. Outlier and no Outlier Means
6. Means of Difference (1) and Difference (2)
7. Mean of 1, 3 and 5

Table 8.3 Calculating Means for Carlson-Derived Topic Order

Topic	Carlson's Calculus Order	1. Mean Order with Outliers	2. Difference (1)	3. Mean Order without Outliers	4. Difference (2)	5. Outlier-No Outlier Means	6. Difference Means	7. Grand Mean
1	General and Prerequisite Review	1.00	0.00	1.00	0.00	1.00	0.00	1
2	Polynomials and Rational Functions	3.38	1.38	2.78	0.71	3.04	1.04	3.07
3	Equations and Inequalities	1.71	1.29	1.71	1.29	1.71	1.29	1.71
4	Functions: Theory, Operations and Graphs	2.63	1.38	2.63	1.37	2.63	1.37	2.63
5	Exponential and Logarithmic Functions	4.00	1.00	4.29	0.71	4.15	0.86	4.15
6	Trigonometry (Functional, Graphical and Analytic)	5.25	0.75	5.25	0.75	5.25	0.75	5.25
7	Polar and Complex	6.50	0.50	6.50	0.50	6.50	0.50	6.5
8	Vectors and Determinants	6.50	1.50	6.86	1.14	6.68	1.32	6.68
9	Analytic Geometry	6.88	2.13	6.88	2.12	6.88	2.12	6.88
10	Sequences and Series with Combinatorics	9.13	0.88	9.13	0.87	9.13	0.87	9.13

When the Grand Mean (Average of the Outlier Means and the Difference Means) and Carlson-derived chapter numbers are aligned and sorted by the Grand Mean, as seen in Table 8.4, there is a conformational tendency to the Carlson-derived order with disagreement in the placement of topic 2 being placed ahead of the ordered topics 3 and 4. Thus the disagreement between the Carlson-derived topic order and the calculated topic order is that *Equations and Inequalities* have been placed prior to the correctly-ordered topics 3 and 4 (*Equations and Inequalities* and *Functions: Operations and Graphs*). See analysis in Tables 8.4 and 8.5.

Table 8.4 Mean Chapter Placement

Grand Mean	Chapter
1	1
3.07	3
1.71	4
2.63	2
4.15	5
5.25	6
6.5	7
6.68	8
6.88	9
9.13	10

Table 8.5 compares the calculated topic order with the Carlson-derived topic order. An "X" indicates agreement between the two lists.

Table 8.5 Calculated Textbook Topic Order and Carlson-derived Topic Order

Calculated Textbook Topic Order		Carlson-Derived Topic Order
General and Prerequisite Review	X	General and Prerequisite Review
		Polynomials and Rational Functions
Equations and Inequalities	X	Equations and Inequalities
Functions: Theory, Operations and Graphs	X	Functions: Theory, Operations and Graphs
Polynomials and Rational Functions		
Exponential and Logarithmic Functions	X	Exponential and Logarithmic Functions
Trigonometry (Functional, Graphical and Analytic)	X	Trigonometry (Functional, Graphical and Analytic)
Polar and Complex	X	Polar and Complex
Vectors and Determinants	X	Vectors and Determinants
Analytic Geometry	X	Analytic Geometry

8.2.1.3. Analysis of Topics 2 and 3 Ordering

All ten prerequisite topics are included in the two rubrics in Table 8.5. Chapters 1, 3, 4 and 6-10, match the derived order except the placement of chapter 2, Polynomials and Rational Functions. The next step was to determine whether the deviation from the order specified by the Carlson-derived rubric should be penalized in the Cohesive Continuum Score calculation or whether this order is acceptable. The working hypothesis was that topic order *does* matter and that the topic, Polynomials and Rational Functions, is best done before Equations and Inequalities. The following justification is offered in support of this hypothesis:

First, polynomials are simple functions having domain and range $(-\infty,\infty)$ and other elementary attributes of functions and though they may serve as an introduction to functions, the notion of function extends far beyond polynomials. Thus, formal functional domain and range discussion (incorporating function composition and function inverse) should come after the polynomial. Second, though insignificantly different, the formal treatment of functions should immediately precede the applications of function inverse and function composition as exercised in logarithms and exponents. Formal domain and range, compostion and inverse discussion should immediately precede the implementation of those topics in the exponential and logarithmic chapter/topic which is consistent with the Carlson-derived topic order. Third, polynomials can be expressions, equations, or inequalities. As expressions, polynomials may be succintly defined; as equations they may be solved using simple or complex techniques. All these techniques will be applicable to the inequality problems but at a more complicated level (McLaurin, 1985). Complexity-reducing techniques, such as polynomial long division, factor and remainder theorems, and even the Fundamental Theorem of Algebra, are all presented in their simplest form in the presentation of polynomials and polynomial equations, and are assumed as polynomial inequalities are presented (Dobbs & Peterson, 1991). Thus, it is concluded that the Carlson-derived topic order is optimal for the Cohesive Continuum measurement; therefore, textbooks that place Equations and Inequalities before Polynomials and Rational Functions are penalized in the rigor algorithm.

Tables 8.6 further confirms the correctness of the Carlson-derived chapter order asserted above. It depicts the Z-score (Hayek & Buzas, 2010, pp. 27-28) that was calculated for the chapter order of each chapter in each book.

Table 8.6 Z-Score Calculations for Topic Order

Book	Topic Order Z-Score for Each Topic										Mean	SD
i.	1 -1.22	8 1.22	2 -0.87	3 -0.52	4 -0.17	5 0.17	7 0.87	6 0.52	0 -1.57	9 1.57	4.50	2.87
ii.	1 -1.57	2 -1.22	3 -0.87	4 -0.52	5 -0.17	6 0.17	7 0.52	9 1.22	8 0.87	10 1.57	5.50	2.87
iii.	1 -1.57	2 -1.22	3 -0.87	4 -0.52	5 -0.17	7 0.52	8 0.87	9 1.22	6 0.17	10 1.57	5.50	2.87
iv.	1 -1.25	3 -0.57	1 -1.25	2 -0.91	4 -0.24	5 0.10	6 0.44	6 0.44	10 1.79	9 1.45	4.70	2.97
v.	1 -1.42	3 -0.63	2 -1.02	2 -1.02	4 -0.24	5 0.16	6 0.55	6 0.55	8 1.34	9 1.73	4.60	2.54
vi.	1 -1.32	3 -0.56	1 -1.32	2 -0.94	4 -0.19	5 0.19	6 0.56	6 0.56	8 1.32	9 1.69	4.50	2.66
vii.	1 -1.25	3 -0.57	1 -1.25	2 -0.91	4 -0.24	5 0.10	6 0.44	6 0.44	10 1.79	9 1.45	4.70	2.97
viii.	1 -1.09	3 -0.22	0 -1.53	2 -0.65	2 -0.65	4 0.22	6 1.09	4 0.22	5 0.65	8 1.96	3.50	2.29

Table 8.7 shows the same data as Table 8.6 except that each chapter Z-score is sorted from smallest to largest numeric order (as read left to right).

Table 8.7 Z-Scores Sorted Smallest to Largest

Book	Z-Order of Topics Chapter Where Topic is Found Z-Score for Each Topic Ranked Smallest to Largest									
i.	9 0 -1.57	1 1 -1.22	3 2 -0.87	4 3 -0.52	5 4 -0.17	6 5 0.17	8 6 0.52	7 7 0.87	2 8 1.22	10 9 1.57
ii.	1 1 -1.57	2 2 -1.22	3 3 -0.87	4 4 -0.52	5 5 -0.17	6 6 0.17	7 7 0.52	9 8 0.87	8 9 1.22	10 10 1.57

Table 8.7 (continued)

Book	Z-Order of Topics / Chapter Where Topic is Found / Z-Score for Each Topic Ranked Smallest to Largest									
	1	2	3	4	5	9	6	7	8	10
iii.	1	2	3	4	5	6	7	8	9	10
	-1.57	-1.22	-0.87	-0.52	-0.17	0.17	0.52	0.87	1.22	1.57
	1	3	4	2	5	6	7	8	10	9
iv.	1	1	2	3	4	5	6	6	9	10
	-1.25	-1.25	-0.91	-0.57	-0.24	0.10	0.44	0.44	1.45	1.79
	1	3	4	2	5	6	7	8	9	10
v.	1	2	2	3	4	5	6	6	8	9
	-1.42	-1.03	-1.03	-0.63	-0.24	0.16	0.55	0.55	1.34	1.73
	1	3	4	2	5	6	7	8	9	10
vi.	1	1	2	3	4	5	6	6	8	9
	-1.32	-1.32	-0.94	-0.57	-0.19	0.19	0.57	0.57	1.32	1.70
	1	3	4	2	5	6	7	8	10	9
vii.	1	1	2	3	4	5	6	6	9	10
	-1.25	-1.25	-0.91	-0.57	-0.24	0.10	0.44	0.44	1.45	1.79
	3	1	4	5	2	6	8	9	7	10
viii.	0	1	2	2	3	4	4	5	6	8
	-1.53	-1.09	-0.66	-0.66	-0.22	0.22	0.22	0.66	1.09	1.96

8.2.1.4. Calculating the final Cohesive Continuum Score (CCS)

To retain the outliers (since they are not erroneous measures), the following strategy was used to construct Tables 8.8 which depicts the final CCS based on the Relative Chapter Order (RCO). In order to give credit for chapters that were in order *relatively*, as opposed to a one-to-one match against the Carlson-derived order, the scores for each *individual* textbook (B1-B8) were sorted low to high by Z-Score. Once this sort was finished, the Z-order chapter numbers were calculated to determine if chapters were in a Carlson-derived relative order. If the chapter order standard was met, the score was zero (0). If the chapter order standard was not met, the score was negative one (-1). Scores for each book were then totaled and the largest negative number (closest to zero) earned the highest CC score.

Table 8.8 Final Cohesive Continuum Scores for Books 1-8

Book	Continuity Across Chapters									Cohesive Continuum Score
	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10	
i	-1	-1	0	0	0	-1	-1	-1	-1	-6
ii	0	0	0	0	0	0	-1	-1	-1	-3
iii	0	0	0	0	-1	-1	0	0	-1	-3
iv	-1	0	-1	-1	0	0	0	-1	-1	-5
v	-1	0	-1	-1	0	0	0	0	0	-3
vi	-1	0	-1	-1	0	0	0	0	0	-3
vii	-1	0	-1	-1	0	0	0	-1	-1	-5
viii	-1	-1	0	-1	-1	-1	0	-1	-1	-7

8.2.2. Analyzing Concept Continuum (B)—Calculating Presentation Scores (PS)

Table 8.9 displays the demographic information collected for each of the eight textbooks, *omitting any specific calculus topics*. Note that when the score in any section is a 1, it is a non-zero placeholder for texts that have no pages in that category.

Table 8.9 Presentation Score Raw Data

Book	Teaching Pages	Review Pages	Chapters	Sections	Imagery Pages	Technology Exercises	Concept Exercises	Imagery Infusion*	Tech Infusion**
i	412	46	10	75	1	1	1758	0.24%	0.06%
ii	565	46	15	96	75	75	3326	13.27	2.25%
iii	749	52	18	107	180	510	3401	24.03%	15.00%
iv	810	86	11	74	308	1110	3223	38.02%	34.44%
v	751	59	10	64	290	2090	3420	38.62%	61.11%
vi	905	141	11	74	264	1105	3367	29.17%	32.82%
vii	704	71	10	65	300	2593	5377	42.61%	48.22%
viii	786	62	15	90	225	1665	2280	28.63%	73.03%

** Calculated by dividing the number of pages containing imagery by the number of teaching pages.*

*** Calculated by dividing the number of technology exercises/examples by the number of concept exercises/examples.*

8.2.2.1. Analyzing the Importance of the Remedial/Review Pages as a Variable

Table 8.10 shows the results of applying the mathematical formula described in chapter 7 (Proportion of Review Pages to Teaching Pages = A.ii/A.i) to the demographics. The analysis included applying two different formulas—one in which the ratio of review pages to teaching pages was considered as a variable, and one in which it was not considered as a variable. The result of the analysis was that this ratio was optional in the calculation of the Presentation Score (PS) because the rank order of books did not change with either option.

Table 8.10 Presentation Score Analytics

Book	Review Pages per Teaching Pages	Imagery per Teaching Pages	Calculated * Technology Exercises per Teaching Pages	Concept Exercises per Teaching Pages	PS Including Review Pages**	PS Without Review Pages***
i	0.11	0.00	0.01	4.27	4.15	4.26
ii	0.08	0.13	0.40	5.89	5.27	5.36
iii	0.07	0.24	2.04	4.54	2.19	2.26
iv	0.11	0.38	4.11	3.98	-0.62	-0.51
v	0.08	0.39	8.35	4.55	-4.26	-4.18
vi	0.16	0.29	3.66	3.72	-0.39	-0.23
vii	0.10	0.43	11.05	7.64	-3.94	-3.84
viii	0.08	0.29	6.35	2.90	-3.82	-3.74

**Calculated by dividing the number of pages containing technology exercises/examples by the number of teaching pages and then multiplying by a factor of 3 to reflect that, while images can be a distraction, technology exercises/examples likely interfere with concept reinforcement.*

***PS=Concept-building exercises/examples per teaching page minus the sum of technology exercises/examples per teaching pages, imagery per teaching pages and review pages per teaching pages. (See chapter 7, section 7.4.2.2 for details.)*

**** PS=Concept-building exercises/examples per teaching page minus the sum of technology exercises per teaching pages and imagery per teaching pages.*

The final rank order of books based on Presentation Score is shown in Table 8.11 indicating that with or without review pages the order does not change, therefore review pages visibility will remain. Note that the rank order of books did not change.

Table 8.11 Final Presentation Score Ranking

Book	Presentation Score With Review Pages	Presentation Score Without Review Pages
ii	5.27	5.36
i	4.15	4.26
iii	2.19	2.26
vi	-0.39	-0.23
iv	-0.62	-0.51

Table 8.11 (continued)

Book	Presentation Score With Review Pages	Presentation Score Without Review Pages
viii	-3.82	-3.74
vii	-3.94	-3.84
v	-4.26	-4.18

8.2.3. Measuring Cyclic Reinforcement—Calculating Maturity Scores (MS)

Table 8.12 shows the number of pages devoted to each of the four categories of mathematical instruction described in Section 7.3.3 of Chapter 7 and gives the final MS. (The MS is the total of columns A, B, C, and D divided by the total number of teaching pages.)

Table 8.12 Final Maturity Score

Book Number	A. Mathematical Tables	B. Reduction of Functional or Operational Complexity	C. Functional Analysis	D. Trigonometric Functions and Identities	# of Teaching Pages	Final MS
i	16	17	16	90	412	0.34
ii	19	17	18	126	565	0.32
iii	20	21	17	110	749	0.22
iv	8	29	11	130	810	0.22
v	0	13	7	93	751	0.15
vi	0	15	8	133	905	0.17
vii	0	10	8	130	804	0.18
viii	0	8	2	92	786	0.13
Average	7.88	16.25	10.88	113.00	722.75	0.22

8.2.4. Calculating the Final Rigor Score (FRS)

Determining the Final Rigor Score (FRS) for each textbook was a two-step process. The first step was to calculate a Presentation Maturity (PM) score for each book by multiplying the Presentation Score (PS) and the Maturity Score (MS). Multiplication was used because the Final Rigor Score target is 0 (a zero indicates no rigor adjustment is needed). Multiplying Maturity Scores ranging from 0 to <1 with Presentation Scores centralizes the results (i.e., groups them closer to zero. The results are shown in Table 8.13.

Table 8.13 Calculating Presentation Maturity Scores

Book Number	Presentation Score	Maturity Score	PM Score
i	4.15	0.34	1.41
ii	5.27	0.32	1.69
iii	2.19	0.22	0.48
iv	-0.62	0.22	-0.14
v	-4.26	0.15	-0.64
vi	-0.39	0.17	-0.07
vii	-3.94	0.18	-0.71
viii	-3.82	0.13	-0.50

The second step was to add the PM score to the Cohesive Continuum Score (see Table 8.8). The FRS closest to zero is the most rigorous textbook. Table 8.14 presents the data sorted by the FRS.

Table 8.14 Final Rigor Scores Sorted by Rigor Score

Rigor Ranking	Book Number	CC Final	PM Score	Final Rigor Score
1	ii	-3	1.69	-1.31
2	iii	-3	0.48	-2.52
3	vi	-3	-0.07	-3.07
4	v	-3	-0.64	-3.64
5	i	-6	1.41	-4.59
6	iv	-5	-0.14	-5.14
7	vii	-5	-0.71	-5.71
8	viii	-7	-0.50	-7.50

One of the textbooks, number ii, *Advanced Mathematics: A Precalculus Course*, (1984), had a published review available. After running the algorithm, this review was consulted for comparison. The review identified the textbook as being a very good resource that covered the "usual" calculus prerequisites (Kenneth, 1984). The algorithm placed it as the most rigorous of the selected textbooks. The algorithm was then applied to 11 additional precalculus textbooks in order to provide a larger sample to facilitate analysis of whether precalculus textbooks have become less rigorous over time.

8.3. Applying the Rigor Algorithm to Additional Textbooks

8.3.1. Identifying Additional Textbooks

The following 11 secondary school precalculus textbooks published between 1965 and 2012 were selected as representative of textbooks designed to prepare mathematically savvy students for success in year-13 calculus:

ix. Mary P. Dolciani, Simon L. Berman, William Wooton, *Modern Algebra and Trigonometry: Book 2, Structure and Method,* 2d ed., (Boston: Houghton Mifflin Company, 1965).

x. J.D. Harmer, *Senior Mathematics Book 1 & 2,* Revised ed., (E. Herman, 1978) OCLC#: 220214667 (2 Volumes Book 1 and Book 2). Only Book 1 was reviewed as Book 2 was Abstract Algebra and Elementary Calculus only.

xi. Roland E. Larson, Robert. P. Hostetler, *Precalculus*, (Lexington MA: D.C. Heath & Co.,1989).

xii. Robert Haese, Sandra Haese, Michael Haese, Roger Dixon, Jon Roberts, Michel Teubner, Anthony Thompson, *Specialist Mathematics; Mathematics for Year 12*, 1st Ed., (Adelaide: Raksar Nominees Ry Ltd., 2002) and Robert Haese, Sandra Haese, Tom Van Dulken, Kim Harris, Anthony Thompson, Mark Bruce, Michael Haese, *Mathematical Studies; Mathematics for Year 12*, 2nd Ed., (Adelaide: Raksar Nominees Ry Ltd., 2006).

xiii. Mal Coad, Glen Wiffen, John Owen, Robert Haese, Sandra Haese, Mark Bruce, *Mathematics for the International Student*, 1st Ed., (Adelaide: Raksar Nominees Ry Ltd., 2006).

xiv. Ron Larson, Robert Hostetler, *Precalculus with Limits*, 1st Ed., (Boston: Haughton Mifflin Co., 2007).

xv. Paul Urban, David Martin, Robert Haese, Sandra Haese, Michael Haese, Mark Humphries, *Mathematics for the international student; Mathematics HL (Core)*, 2nd Ed.,(Adelaide SA: Raksar Nominees Pty Ltd.,2008).

xvi. Robert Blitzer, *Precalculus*, 4eth Ed., (Upper Saddle River, NJ: Prentice-Hall, Inc., 2009).

xvii. Michael Sullivan, Michael Sullivan III, *Precalculus: Concepts Through Functions – A Right Triangle Approach To Trigonometry*, 2nd Ed., (Boston: Prentice Hall Pearson Education, 2011).

xviii. Eric Connally, Deborah Hughes-Hallett, Andrew M. Gleason, et al, *Functions Modeling Change: A Preparation for Calculus*, 4th Ed., (Hoboken NJ: John Wiley & Sons, Inc., 2011).

xix. Mal Coad, Glen Wiffen, John Owen, Sandra Haese, Michael Haese, Mark Humphries, *Mathematics for the International Student; Mathematical Studies SL*, 3rd Ed., (Adelaide SA: Haese and Harris Publications, 2012).

8.3.2. Calculating Cohesive Continuum Scores

The same process used to validate the rigor algorithm in Section 8.2.1 was used to determine final CCS for the 11 additional precalculus textbooks. Results are shown in Table 8.15.

Table 8.15 Cohesive Continuum Scores

Book	Continuity Across Chapters									Cohesive Continuum Score
	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10	
ix	-1	0	-1	-1	0	0	-1	-1	-1	-6
x	0	-1	-1	-1	-1	-1	-1	-1	-1	-8
xi	-1	-1	-1	0	0	0	-1	-1	-1	-6
xii	-1	-1	0	0	-1	-1	0	-1	-1	-6
xiii	0	-1	-1	0	-1	-1	-1	-1	-1	-7
xiv	-1	-1	-1	0	0	-1	0	-1	0	-5
xv	-1	-1	0	-1	-1	-1	-1	-1	-1	-8
xvi	-1	-1	-1	0	0	-1	-1	0	0	-5
xvii	-1	-1	-1	-1	0	0	0	0	0	-4
xviii	-1	-1	0	0	-1	-1	-1	-1	-1	-7
xix	-1	0	0	-1	0	-1	-1	-1	0	-5

8.3.3. Calculating Presentation Scores (PS)

The same process used to validate the rigor algorithm in Section 8.2.1 was used to determine final PS for the 11 additional precalculus textbooks. Results are shown in Table 8.16. Note that when the score in any section is a 1, it is a non-zero placeholder for texts that have no pages in that category.

Table 8.16 Presentation Score Raw Data

Book	Teaching Pages	Review Pages	Chapters	Sections	Imagery Pages	Technology Exercises	Concept Exercises	Imagery Infusion*	Tech Infusion**
ix	572	76	15	139	15	45	4858	2.62%	0.93%
x	124	6	4	31	1	1	280	0.81%	0.36%
xi	730	86	11	73	220	473	5544	30.14%	8.53%
xii	346	18	10	68	110	150	683	31.79%	21.96%
xiii	553	147	17	119	374	84	1208	67.63%	6.95%
xiv	810	39	10	67	360	1020	7880	44.44%	12.94%
xv	832	48	29	200	218	667	3335	26.20%	20.00%
xvi	1037	134	10	76	340	1193	7607	32.79%	15.68%
xvii	996	121	12	84	309	944	7728	31.02%	12.22%
xviii	608	102	14	67	14	1	4288	2.30%	0.02%
xix	540	126	19	63	342	266	1102	63.33%	24.14%

* *Calculated by dividing the number of pages containing imagery by the number of teaching pages.*

** *Calculated by dividing the number of technology exercises/examples by the number of concept exercises/examples.*

Table 8.17 shows the presentation score analytics.

Table 8.17 Presentation Score Analytics

Book	Review Pages per Teaching Pages	Imagery per Teaching Pages	Calculated* Technology Exercises per Teaching Pages	Concept Exercises per Teaching Pages	PS Including Review Pages**
ix	0.13	0.03	0.24	8.49	8.10
x	0.05	0.01	0.02	2.26	2.18
xi	0.12	0.30	1.94	7.59	5.23
xii	0.05	0.32	1.30	1.97	0.30
xiii	0.27	0.68	0.46	2.18	0.79
xiv	0.05	0.44	3.78	9.73	5.46
xv	0.06	0.26	2.41	4.01	1.28
xvi	0.13	0.33	3.45	7.34	3.43
xvii	0.12	0.31	2.84	7.76	4.48
xviii	0.17	0.02	0.00	7.05	6.86
xix	0.23	0.63	1.48	2.04	-0.30

**Calculated by dividing the number of pages containing technology exercises/examples by the number of teaching pages and then multiplying by a factor of 3 to reflect that, while images can be a distraction, technology exercises/examples likely interfere with concept reinforcement.*

***PS=Concept-building exercises/examples per teaching page minus the sum of technology exercises/examples per teaching pages, imagery per teaching pages and review pages per teaching pages. (See chapter 7, section 7.4.2.2 for details.)*

The final rank order of books based on Presentation Score is shown in Tables 8.18.

Table 8.18 Final Ranking of Books Based on Presentation Score

Book	PS Including Review Pages
ix	8.10
xviii	6.86
xiv	5.46
xi	5.23
xvii	4.48
xvi	3.43
x	2.18
xv	1.28
xiii	0.79
xii	0.30
xix	-0.30

8.3.4. Calculating Maturity Scores (MS)

Table 8.19 shows the number of pages devoted to each of the four categories of mathematical instruction described in Section 7.3.3 of Chapter 7 and gives the final MS. (The MS is the total of columns A, B, C, and D divided by the total number of teaching pages.)

Table 8.19 Final Maturity Score

Book Number	A. Mathematical Tables	B. Reduction of Functional or Operational Complexity	C. Functional Analysis	D. Trigonometric Functions and Identities	# of Teaching Pages	Final MS
ix	12	13	2	108	572	0.24
x	0	7	0	0	124	0.06
xi	7	15	15	157	730	0.27
xii	0	14	1	26	346	0.12
xiii	0	1	6	10	553	0.03
xiv	0	8	11	130	810	0.18
xv	0	11	13	8	832	0.04
xvi	0	21	15	177	1037	0.21
xvii	0	11	18	98	996	0.13
xviii	0	5	11	82	608	0.16
xix	0	26	2	32	540	0.11
Average	1.73	10.50	8.55	75.27	597.50	0.14

8.3.5. Calculating Final Rigor Score

The first step in calculating a final rigor score for the additional 11 textbooks was to calculate a Presentation Maturity (PM) score for each book by multiplying the Presentation Score (PS) and the Maturity Score (MS). The results are shown in Table 8.20.

Table 8.20 Calculating Presentation Maturity Scores

Book Number	Presentation Score	Maturity Score	PM Score
ix	8.10	0.24	1.94
x	2.18	0.06	0.13
xi	5.23	0.27	1.41
xii	0.30	0.12	0.04
xiii	0.79	0.03	0.02
xiv	5.46	0.18	0.98
xv	1.28	0.04	0.05
xvi	3.43	0.21	0.72
xvii	4.48	0.13	0.58

Table 8.20 (continued)

Book Number	Presentation Score	Maturity Score	PM Score
xviii	6.86	0.16	1.10
xix	-0.30	0.11	-0.03

The second step was to add the PM score to the Cohesive Continuum Score (see Table 8.15). The FRS closest to zero is the most rigorous textbook. Table 8.21 presents the data sorted by the FRS.

Table 8.21 Final Rigor Scores Sorted by Rigor Score

Rigor Ranking	Book Number	CC Final	PM Score	Final Rigor Score
1	xvii	-4	0.58	-3.42
2	xiv	-5	0.98	-4.02
3	ix	-6	1.94	-4.06
4	xvi	-5	0.72	-4.28
5	xi	-6	1.41	-4.59
6	xix	-5	-0.03	-5.03
7	xviii	-7	1.10	-5.90
8	xii	-6	0.04	-5.96
9	xiii	-7	0.02	-6.98
10	x	-8	0.13	-7.87
11	xv	-8	0.05	-7.95

8.4. Pooling Results and Final Analysis

8.4.1. Pooled Results

Table 8.22 shows the final rigor score calculations for all 19 textbooks sorted by FRS and includes the year of publication for each book.

Table 8.22 Results Sorted by Final Rigor Score Including Publication Year

Rigor Rank	Book #	Rigor Score	Year	Teaching Pages	CCS	PM Score	Imagery Infusion	Tech Infusion
1	ii	-1.31	1984	565	-3	1.72	13.27%	2.25%
2	iii	-2.52	1992	749	-3	0.48	24.03%	15.00%
3	vi	-3.07	2002	905	-3	-0.07	29.17%	32.82%
4	xvii	-3.42	2011	996	-4	0.58	31.02%	12.22%

Table 8.22 (continued)

Rigor Rank	Book #	Rigor Score	Year	Teaching Pages	CCS	PM Score	Imagery Infusion	Tech Infusion
5	v	-3.64	2001	751	-3	-0.64	38.62%	61.11%
6	xiv	-4.02	2007	810	-5	0.98	44.44%	12.94%
7	ix	-4.06	1965	572	-6	1.94	2.62%	0.93%
8	xvi	-4.28	2010	1037	-5	0.72	32.79%	15.68%
9	i	-4.59	1972	412	-6	1.41	0.24%	0.06%
10	xi	-4.59	1989	730	-6	1.41	30.14%	8.53%
11	xix	-5.03	2012	540	-5	-0.03	63.33%	24.14%
12	iv	-5.14	1993	810	-5	-0.14	38.02%	34.44%
13	vii	-5.71	2005	704	-5	-0.71	42.61%	48.22%
14	xviii	-5.90	2011	608	-7	1.10	2.30%	0.02%
15	xii	-5.96	2003	346	-6	0.04	31.79%	21.96%
16	xiii	-6.98	2004	553	-7	0.02	67.63%	6.95%
17	viii	-7.50	2012	786	-7	-0.50	28.63%	73.03%
18	x	-7.87	1978	124	-8	0.13	0.81%	0.36%
19	xv	-7.95	2008	832	-8	0.05	26.20%	20.00%

8.4.2. Analysis of Results

Final rigor scores ranged from as high as -1.31 to as low as -7.95 as shown in Table 8.22. There is a fairly strong correlation between CCS and FRS (0.875) evident in the final sorted order. Chronological correlation to lower rigor scores is less pronounced than expected, yet there are some notable correlations that can be exploited to draw conclusions here.

SPSS v20.0 was run against CCS, PM score, FRS, year of publication, number of teaching pages, Imagery Infusion and Technology Infusion for the purposes of revealing correlations both positive and negative. Table 8.23 displays the results.

Table 8.23 Rigor Score Correlations

		CCS	PM Score	FRS	Year	Teaching Pages	Imagery Infusion	Tech Infusion
Cohesive Continuum Score	Pearson Correlation	1	-.028	.875**	-.115	.265	.044	.123
	Sig. (2-tailed)		.913	.000	.649	.289	.862	.626

Table 8.23 (continued)

		CCS	PM Score	FRS	Year	Teaching Pages	Imagery Infusion	Tech Infusion
PM Score	Pearson Correlation	-.028	1	.460	-.624**	-.200	-.619**	-.816**
	Sig. (2-tailed)	.913		.055	.006	.426	.006	.000
Final Rigor Score	Pearson Correlation	.875**	.460	1	-.405	.138	-.261	-.286
	Sig. (2-tailed)	.000	.055		.096	.585	.296	.250
Year	Pearson Correlation	-.115	-.624**	-.405	1	.423	.566*	.414
	Sig. (2-tailed)	.649	.006	.096		.080	.014	.088
Pages	Pearson Correlation	.265	-.200	.138	.423	1	.144	.280
	Sig. (2-tailed)	.289	.426	.585	.080		.569	.261
Imagery Infusion	Pearson Correlation	.044	-.619**	-.261	.566*	.144	1	.348
	Sig. (2-tailed)	.862	.006	.296	.014	.569		.158
Tech Infusion	Pearson Correlation	.123	-.816**	-.286	.414	.280	.348	1
	Sig. (2-tailed)	.626	.000	.250	.088	.261	.158	

**. Correlation is significant at the 0.01 level (2-tailed).
*. Correlation is significant at the 0.05 level (2-tailed).

Tech Infusion was negatively correlated significantly with the PM score, indicating that the more infusion of technology, the more rigor degradation; likewise, with Imagery Infusion. The year of the book is also negatively correlated to the PM score, indicating that newer books had a lower PM score.

8.4.2.1. Comments on Imagery and Technology

The expectation described in previous chapters was that newer textbooks were larger, primarily because of imagery and technology infusion. More study needs to be done in this area as there were notable exceptions to this expectation. Book xviii, for example, is relatively new, having been published in 2011; yet it scored 2.3% and 0.02% for imagery and technology infusion. Noteworthy was the fewer number of pages in this textbook compared to most others published around the same time—a reflection of minimal imagery and technology interferences. Its FRS reflected the fact that it failed to conform to the Carlson-derived topic order rather than that concept maturity was interrupted by imagery and technology infusion. In general, however, as textbooks became newer and imagery and technology infusion increased, the FRS was affected negatively.

8.4.2.2. Presentation Maturity Score Factors

The bivariate correlations in Table 8.23 show significant correlations negatively linking PM scores with: the age of the textbook, photos and other unnecessary imagery, and technology instruction and exercises. That is, the newer textbooks generally had lower PM scores. Imagery Infusion was strongly correlated with the newness of the textbook so that the newer the textbook, the more imagery present. When coupled with the negative correlation with presentation and maturity scores, this affirms the hypothesis that proliferation of imagery degrades the rigor of material presented. Technology Infusion is positively correlated but SPSS has evaluated it as only *almost* significant at 0.414 with 0.088 significance in a 2-tailed test. The same can be said for the size of the textbook and the year of publication, also positively correlated and close to significant at .080. It is likely that, processing more textbooks through the rigor algorithm would strengthen the already significant results and move the "close to significant" results into the significant category.

Further, though not manipulated to do so, the PM scores are not correlated to the CC scores, indicating that they are not measuring the same thing. Perhaps revisiting elements of the order of topics in the Carlson-derived topic order, such as placement of Polynomials and Rational Functions chapters in relation to chapters on Equations and Inequalities (as addressed in Chapter 7), would yield different results. Yet, the PM scores' almost significant score of 0.460 (2-tailed, 0.055) coupled with the very strong positive correlation between the FRS and the

CC scores seems to indicate that the Carlson-derived topic order is valuable for evaluating the rigor of a textbook.

8.4.3. Linking the Rigor Algorithm to SEM Results

The SEM described in Chapter 6 demonstrated the centrality and significance of the year-12 precalculus textbook and its use to students in year-12 precalculus classes; therefore, its use and centrality cannot be underemphasized. Accordingly, the year-12 precalculus textbook's ability to do this profoundly important task had to be critiqued as to its adequacy as measured against the rigorously presented minimum calculus prerequisites. This led to development of a measuring tool (called the rigor algorithm in this research) to provide a resource for teachers and mathematics departments to measure and, where needed, to augment topic items so that alignment between year-12 precalculus textbook content and year-13 calculus prerequisites is satisfied.

8.5. Conclusion

This chapter has demonstrated the implementation of an MRP-based rigor algorithm that provides mathematics department with a tool for evaluating year-12 precalculus textbooks as to whether the topics presented are adequately covering year-13 calculus prerequisites. Using the tool may reveal inadequacies in textbooks that can be addressed with extracurricular resources of the department's choice. The adequacy of those additional resources can also be assessed using the algorithm (or a portion of the algorithm). Moreover, the principles behind the algorithm can be applied to textbooks used at any stage in the mathematics curriculum to prepare students for their subsequent courses.

Chapter 9. Conclusions, Implications and Recommendations

9.1. Introduction

This project was fueled initially from the researcher's classroom experience in secondary advanced mathematics and university calculus courses in Australia and the United States where it became apparent that there was a misalignment of secondary advanced mathematics exit standards and university calculus entrance requirements. This fact was also readily apparent from the poor scores of university students on calculus tests, the number of students failing to complete their calculus courses, and from professor concerns about the inadequate skill set of the incoming mathematics students—particularly those needed for the first course in elementary calculus introducing limits, derivatives, and anti-derivatives, and applications of those three topics. Despite this strong evidence about the lack of preparedness of secondary students for advanced mathematics, there was also evidence that secondary school students and their teachers had full confidence in their mathematical readiness for university calculus. This project was designed to explore this misalignment.

The researcher, already sensitive to the fact that important complexity-reducing techniques had been omitted from many secondary curricula and textbooks, embarked on this project to determine whether lack of rigor in the secondary advanced mathematics textbook was a significant reason that many secondary students believed themselves ready for their university elementary calculus course even though they lacked the skill set needed to succeed in that course. This project sought to investigate the distinct possibility that some secondary precalculus textbooks were designed more as a casual resource for secondary student enjoyment of mathematics as opposed to solid preparation and practice for the next course in mathematics. When the study revealed a lack of rigor in the presentation of necessary topics, both in order and content in some secondary mathematics textbooks, it inspired development of a rigor algorithm tool based on Material Requirements Planning (MRP) that mathematics teachers could use to assess rigor shortcomings in their textbooks so that they could adapt their curricula to better align secondary exit requirements with tertiary entrance requirements.

9.2. Research Design

9.2.1. Incorporating MRP

The MRP strategy of backwards scheduling was incorporated into the project as an optimal tool for guiding curriculum design and planning. The underlying assumption was that MRP could be used to ensure that the minimum exit requirements of one course (as codified in curricula and syllabi) would satisfy the minimum entrance requirements of the subsequent course. Though the project focused specifically on the alignment of exit and entrance requirements for year-12 precalculus and year-13 calculus courses, it was understood that the same MRP strategy could be applied iteratively to all previous and subsequent mathematics courses and even to courses outside of the discipline of mathematics. Accordingly, teacher and professor survey questions were designed to reveal misalignment between secondary advanced mathematics curricula and university calculus entrance requirements.

9.2.2. Survey Instrument Design

Rigor has many facets that were outside the scope of this study, so it was necessary to design an experiment that could query students and teachers/professors in both secondary and university settings regarding various areas of student skill and textbook use to narrowly investigate how the textbook and its use contributed to the general successful completion of the advanced secondary mathematics curriculum but unsuccessful preparation for subsequent university coursework. As described in Chapter 3, the design of surveys for secondary students, secondary teachers, secondary principals, university students, and university professors went through a pilot and final phase. The intention in the design was to determine whether secondary curriculum completion (via the textbook) was perceived as finished and adequate preparation for university calculus.

Questions to secondary students targeted textbook use, textbook trust/comfort, textbook like, whether the teacher used the textbook, and sought to determine whether completion of the textbook requirements (i.e., homework, exercises, and assessments) gave students confidence that they were ready for the next level of mathematics. Questions to secondary teachers targeted their appraisal of student mathematical skills, confidence, use and like of the textbook plus whether the teachers used the textbook and whether they augmented it with other materials. Tertiary students were asked similar questions about their secondary and tertiary textbooks with the expectation that student responses to questions about their secondary textbooks would be influenced by how well or how poorly they were doing in their current course. Questions to tertiary professors asked about student use and trust of the textbook as well as solicited a general appraisal of current and past student readiness.

9.2.3. Rigor Algorithm Design

The rigor algorithm was developed to expose weaknesses in the textbook coverage of required material so that when the advanced secondary curriculum, delivered via the textbook, was analyzed by the tool, the discovered weaknesses and strengths could be assembled into a score revealing the extent to which a textbook was sufficient to the task of preparing students for the next course in mathematics. The tool was also designed so that, when the textbook fell short in the rigor score, the specific weakness could be implicitly exposed so that teachers could incorporate steps to mitigate those shortcomings.

9.3. Project Implementation

9.3.1. Data Collection

All student data was collected via in-person (on campus, in the classroom) and online surveys over a span of three years. In-person surveys were administered by the researcher and his assistants, and by teachers and professors. The online surveys were administered via Google Forms and Google Sheets. Teacher/professor data was collected in the same way.

The secondary student survey was given to 566 year-12 students in the United States and Australia. The secondary students in the United States were attending secular public and religious private schools; those in Australia were attending secular private and religious private schools. All schools in the U.S. and Australia were co-educational. The tertiary student survey was given to 2,195 students attending two engineering universities—one in Australia and one in the United States.

The secondary school teacher survey was given to 41 teachers, 68% of whom were in Australia and 32% of whom were in the United States. The tertiary professor survey was given to 17 professors, 29% of whom were in Australia and 71% of whom were in the United States.

9.3.2. Data Processing and Statistical Analysis

Excel spreadsheets were used as the initial data containers for the data collected from the paper surveys and Google Forms/Sheets. Initial data cleansing was performed via statistical capability of Excel. Data was then imported into SPSS v20.0 for descriptive statistics and continuation of data cleansing as detailed in Chapter 4.

SPSS was then used to implement Exploratory Factor Analysis (EFA) and initial Confirmatory Factor Analysis (CFA) on the cleansed student data as detailed in Chapters 5 and 6. The output of the CFA was then exported to Mplus v7.1 for finalizing the CFA in preparation for Structured Equation Modelling (SEM) in order to compile collected data and determine whether there were underlying relationships in the data that could be grouped as factors contributing to (or possibly, causing) a particular outcome.

9.4. Summary of Findings

With respect to the first research question, (Is the textbook central to instruction—used, trusted, and liked?), the findings reflected in the secondary and tertiary SEMs demonstrate the centrality of the year-12 mathematics textbook with its use, trust, and parental influence leading to the student's comfort with the textbook. Moreover, the tertiary SEM demonstrated that student comfort with the textbook continued into the beginning of the year-13 university calculus term and the literature affirmed the perception of the year-13 university calculus student in agreement with the model results. Literature solidly argues that the mathematical textbook is the key curriculum delivery system for the requirements contained in the curriculum that is aimed at preparing the year-12 precalculus student for year-13 university calculus.

With regard to the second research question, (Does the textbook contribute to the student and teacher perception of student readiness?), the research and literature agree that U.S. and Australian year-12 students are inadequately prepared for their year-13 calculus course. The research and literature also agree that U.S. and Australian year-12 students and their teachers are confident that the students are prepared for year-13 calculus and that year-13 students are confident in their readiness for calculus. In alignment with the literature that the textbook is the central component of curriculum delivery for year-12 precalculus mathematics students, the secondary SEM and the secondary teacher data presented in this research project affirm that the textbook is central as a source of content and confidence. Student and teacher perception of year-12 student readiness was based on the successful completion of curriculum requirements as delivered by the year-12 precalculus textbook, thus confirming the importance of the precalculus textbook containing content that is consistent with year-13 calculus prerequisites.

The tertiary SEM affirmed the year-13 students' confidence in their readiness by affirming their use and like of the year-12 precalculus textbook. That is, the year-13 calculus students' positive assessment of their previous year-12 precalculus textbook positively influenced their perception of readiness. An additional finding of the model was that year-13 calculus students were not comfortable using their year-13 textbook, but the model did not explain why this was the case. It is possible that this is because students were so accustomed to the non-rigorous presentation of topics in their year-12 precalculus textbooks that they found the year-13 calculus textbooks dry and complicated. Alternatively, it could be because, in some cases, calculus professors spent the first part of the course augmenting the textbooks with remedial handouts. Because the surveys were given early in a student's university experience, if professors were focused on remedial instruction through handouts, it would influence the students' responses to questions about year-13 textbook use.

The tertiary SEM modelled the students' perceived readiness as a result of the confidence acquired in the use and comfort with the year-12 precalculus textbook and, in this way, as supported by the literature, demonstrated the connection between the rigor of the secondary advanced mathematics textbook and university student perception of readiness in such a way that a less rigorous secondary textbook (i.e., easy coursework) promoted a higher level of perceived readiness. Coupling this finding with the university professors' appraisal of actual student mathematical maturity and readiness, the centrality of the year-12 precalculus textbook to both maturity and readiness (and its need for rigor) was affirmed.

Given the findings relating to the first two research questions, the following conundrum pointed to the third question: “Can MRP-derived year-13 calculus prerequisites measure rigor adequacy of year-12 precalculus textbooks?” Tertiary professors at the universities surveyed in the informal discussion groups and in the survey question answers agreed that the student, though confident in mathematical preparedness is actually not prepared for year-13 calculus. This led to research into why, if the year-12 precalculus mathematics textbooks are central in the presentation of precalculus mathematics and if the year-12 student and their teachers are confident that the successful completion of the textbook’s prescribed mathematics is adequate preparation for the year-13 calculus, are the students not prepared for their university course? With personal experience and literature indicating a misalignment of year-12 core mathematical outcomes with year-13 calculus prerequisites, a rigor tool was developed to enable inspection and measurement of the alignment of the content of year-12 precalculus textbooks with calculus prerequisites. The tool indicated the level of conformity via a rigor score. Misalignment was quantitatively measured by the rigor tool to show actual omissions from year-12 precalculus textbooks of necessary calculus requirements. It also measured the extent to which the necessary content continuum was interfered with by irrelevant material. The tool demonstrated that, as textbooks became newer and imagery and technology infusion increased, rigor scores were negatively affected.

The data collected from secondary students and teachers and from tertiary students and professors affirmed that rigor changes in year-12 precalculus textbooks were negatively impacting mathematical maturity and readiness for year-13 calculus courses. The factors developed in the secondary SEM (textbook use, textbook trust, parental influence, and students' perception of readiness) which contributed to textbook comfort led to the logical linear consequence that the textbook is an influencer of mathematical maturity (i.e., the better the textbook, the better the students' mathematical maturity and vice-versa). The secondary SEM affirmed the centrality of, and student comfort with, the textbook, whereby successful completion of the material presented in the textbook promoted confidence in the students, teachers, and parents regarding students' preparation for year-13 calculus. Accordingly, were the secondary mathematical textbooks sufficiently rigorous and appropriately aligned with year-13 tertiary mathematical prerequisites, it would be logical to assume that the mathematical inadequacies in year-12 preparation for year-13 calculus cited in the literature were caused by reasons other than the year-12 precalculus textbook. Nevertheless, the SEM has shown year-12 students and their teachers to be confident in readiness based on the standard of the textbook. In cases where the secondary precalculus textbook lacks rigor and is out of alignment with year-13 calculus prerequisites, this confidence does not reflect reality. Numerous studies have been completed, or are underway, to discover possible remedies, but the research presented here has positively identified the year-12 precalculus textbook as an addressable cause.

9.5. Limitations

9.5.1. Sample Size Constraints

Because it was infeasible to sample the universal population of secondary and tertiary mathematics students, it was necessary to create a representative sample. It is understood that when sample sizes are limited, inferential processes (as opposed to the whole population) mean that the representative sample is limited in its modeling of the actual population. These limitations were intentionally offset with stratified sampling, statistically based data cleansing strategies, and by comparing findings with anecdotal data collected from multiple resources (as presented in Chapter 4).

9.5.2. Student Identity Constraints

Due to privacy limitations, direct tracking of students from secondary to tertiary was not possible. Thus, this project could not draw a direct connection between specific students and their performance in secondary and tertiary mathematics courses or between specific students and their secondary and tertiary textbooks. However, these constraints were mitigated by collecting data in a narrow window of time (many tertiary students who took the survey in their first semester of college had graduated within the previous three months). The content of university calculus textbooks has changed little over the years while secondary precalculus textbooks are regularly revised. By conducting data collection within a three-year time span, it was possible to draw conclusions about textbook influence on readiness for tertiary courses for the representative sample of students surveyed. Thus, the findings of this study form a sturdy foundation for future studies designed and implemented to target declining rigor of secondary mathematics textbooks as a definite contributor to declining university mathematical readiness.

9.6. Recommendations

9.6.1. Affirm Findings through a Targeted, Longer Study

Additional data collection that directly tracked specific students from their secondary to tertiary experience across multiple revisions of precalculus textbooks will likely affirm the findings of this study and mitigate the limitation imposed by privacy requirements. Though this study (three years of data collection) coincided with the typical cycle of textbook revisions, a longer study spanning more revisions may more starkly show the secondary advanced mathematical exit standards diverging from the tertiary calculus prerequisites.

9.6.2. Reestablish Universities as the Drivers of Secondary Exit Requirements

For secondary mathematical textbooks to be aligned with university entrance requirements, tertiary institutions need to drive the requirements to the secondary systems so that mathematical curriculum can be reformed to promote the student (K-12) toward mathematical mastery for the student's exit sector. MRP strategies that utilize the clear understanding of minimal product requirements (secondary student mathematical readiness) for the customer (tertiary mathematics department elementary calculus prerequisites, for this study) convincingly surface as a rational option in secondary mathematics departments' adoption of curriculum and textbooks. Such a strategy would have a cascading effect, in that, if secondary advanced mathematical curricula and textbooks were driven by university prerequisites, then the requirements of the secondary advanced mathematics curricula would then drive the prerequisites for previous mathematics courses across a student's K-12 experience.

9.6.3. Using the Rigor Tool to Enhance Current Curricula and Textbooks

The altering of the K-12 curricula cannot happen in any one school year, but mathematics departments can use the rigor algorithm described in this study to survey their current precalculus textbooks to determine whether optimal topic order, reduction of operational complexity, and maturing of concepts through a smooth continuum are present. If they are not present, departments can augment the textbook with the missing material, skip over unnecessary material, and order/alter the presentation to align with the rubric presented in Chapters 7 and 8. Additionally, departments could adapt the rigor tool/algorithm to perform the same function for textbooks used in previous mathematics courses.

9.7. Conclusion

The data indicate that the rigor needed for student mathematical understanding and success in university elementary calculus is affected by the presentation of prerequisite material. This precept-upon-precept (generative) endeavor is sufficiently important that even the "best" of textbooks may not help prepare students for success in year-13 calculus if prerequisite topics are not included (in proper order, free of distractions) in year-12 precalculus courses. Additionally, the success of students in year-12 precalculus courses will be dependent on generative prerequisites being covered in prior mathematics courses.

Further, this study has found that mathematical textbook rigor and student mathematical preparation are connected in a profound way. Thus, the textbook, as the curriculum delivery tool, must align itself with the minimum deliverable requirements since it is central in the effective conveyance of necessary mathematical content, the ordering of that content, the maintenance of a cohesive continuum, and the exercising of that content for both teachers and students.

While other factors (teacher education and ability, environmental factors, student demographics, etc.) may also influence student mathematical equipping and maturity, this study has demonstrated the centrality of the textbook in the mathematical environment and places the mathematics textbook, by virtue of the findings, as foundational for conveying what is needed to meet and exceed student exit requirements.

Appendices

Appendix A. Data Driven Survey Instrument Construction

Secondary Student Final Survey Instrument

Likert Questions
Pre-year 12 textbook experience (1, 4, 5, 6)
Pre-year 12 environmental experiences (1, 2, 4, 7, 9, 11, 15, 17, 19)
Year 12 preparedness appraisal (1, 2, 4, 5)
Year 12 math textbook use (5, 6, 7, 8, 9, 13, 16)
Year 12 textbook value (3, 5, 6, 10, 11, 13, 14, 16, 17, 18, 19, 20)
Year 12 teacher experience (6, 7, 11, 15, 18, 19, 20)
Year 12 examination success estimation (2, 5, 14, 16)
Year 12 student maturity (1, 2, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20)

Demographic Questions
Gender (0/1)
Age (Years)

Secondary Teacher Final Survey Instrument

Likert Questions
Textbook evaluation (2, 7, 10, 11, 14, 15, 17, 18, 19, 20)
Textbook use (1, 3, 12, 13, 14, 15, 21)
Student evaluation (1, 12, 13, 16, 21)
Student maturity – compare and contrast (1, 3, 4, 5, 6, 8, 9, 12, 13, 21)

Demographic Questions
Gender (0/1)
Age (years)
Education Level
Years teaching mathematics
Number of different textbooks used

Tertiary Student Final Survey Instrument

Likert Questions
Pre-Year 13 preparation (1, 5, 6, 17)
Pre-Year 13 textbook value (1, 2, 3, 4, 5, 7, 8, 10)
Pre-Year 13 teacher experience (3, 4, 7, 16, 19, 20)
Year 13 textbook experience (6, 9, 10, 12, 13, 14, 16, 19)
Year 13 student maturity (11, 12, 13, 14, 15, 17)
Year 13 examination success estimation (11, 15, 17)
Year 13 math textbook value (6, 9, 10, 13,14, 16, 18)
Year 13 student maturity (1, 2, 8, 9, 12, 13, 15, 18, 20)

Demographic Questions
Gender (0/1)
Age (years)
Experience
Placement Test
Current mathematics class
High School Name
High School State
High School City
High School Country
High School Graduation Year

Tertiary Teacher Final Survey Instrument

Likert Questions
Textbook evaluation (1, 7)
Textbook use (1, 2, 3)
Student evaluation (3, 4, 5, 6, 8)
Student maturity—compare and contrast (1, 8, 9, 10)

Demographic Questions
Gender (0/1)
Age (years)
Number years teaching math
Education
Number years teaching 1st year engineering math

Appendix B. Survey Instruments

Secondary Student Pilot Survey

THE UNIVERSITY of ADELAIDE

The University of Adelaide

Faculty of the Professions
Level 8, 10 Pulteney Street, Adelaide SA 5005; Tel: (+618) 8313 5628
Attachment XI

Secondary Student Questionnaire Pilot

Project Title:	**Mathematical Rigor in the Year 12 Curriculum: A Focus on the Secondary Mathematics Textbook as a Tool for University Mathematical Success**
Approval Number:	**HP-2013-099**

Section 1: General Information

- *Please answer all the questions – leave blank if you cannot answer*

1. **Your Gender** ☐ Female ☐ Male
2. **Your Age at the end of the school year ________years**
3. **Your High School Name** __________________________________
4. **What is the name of your textbook(s)**
   a. __________________________________
   b. __________________________________
5. **Did / is the textbook helping you in math this year**
   ☐ Yes ☐ No

Please turn over to Section 2

For Researcher Use Only

School Code: ____________________

Date: _________________________

This research project will be conducted according to the NHMRC National Statement on Ethical Conduct in Human Research (see http://www.nhmrc.gov.au/publications/synopses/e72syn.htm)

Faculty of the Professions
Level 8, 10 Pulteney Street, Adelaide SA 5005; Tel: (+618) 8313 5628
Attachment XI

Secondary Student Questionnaire Pilot

Project Title:	Mathematical Rigor in the Year 12 Curriculum: A Focus on the Secondary Math Textbook as a Tool for University Mathematical Success
Approval Number:	HP-2012-099

Section 2: Preparedness for math this year

Please answer all the questions according to your level of agreement with the statement
1 – 2 = Agreement, 3 – 4 = Neutral, 5 – 6 = Disagreement where:
1 = agree strongly and 6 = disagree strongly

		Strongly Agree		Neutral		Strongly Disagree	
	Scale	1	2	3	4	5	6
1	I was ready for year 12 math						
2	I am going to do well in the final exams						
3	My mid-year progress was better than I expected						
4	Year 11 math prepared me very well for year 12 math						
5	The year 12 textbook is too complicated						
6	Handouts were better, sometimes, than the textbook						
7	My year 12 math teacher uses the textbook and refers to it in class						
8	My year 12 math textbook examples helped me understand the topic						
9	My parents are/have been able to help me with year 12 math						
10	The chapters in the textbook follow each other pretty well						
11	My teacher likes the textbook						
12	I have regular help with my math						
13	I often bring the math textbook home or to my study location						
14	If there were more problems in the textbook I would practice more						
15	I do not need to ask the teacher for homework help						
16	There is enough detail in the textbook to master the topics						
17	My parents like the textbook						
18	I prefer the notes from the teacher than from the textbook						
19	Without the teacher, the textbook would be useless						
20	With textbook only (no teaching) I could understand the topics clearly						

Please Comment on Next Page

Secondary Student Questionnaire Pilot
Comments

Please comment below on whether the questions were hard to understand or if you believe that a particular question may not be helpful or, if you think a question should be added.
Please write the details below.
The sections are for your convenience/use as you see fit for questionnaire:

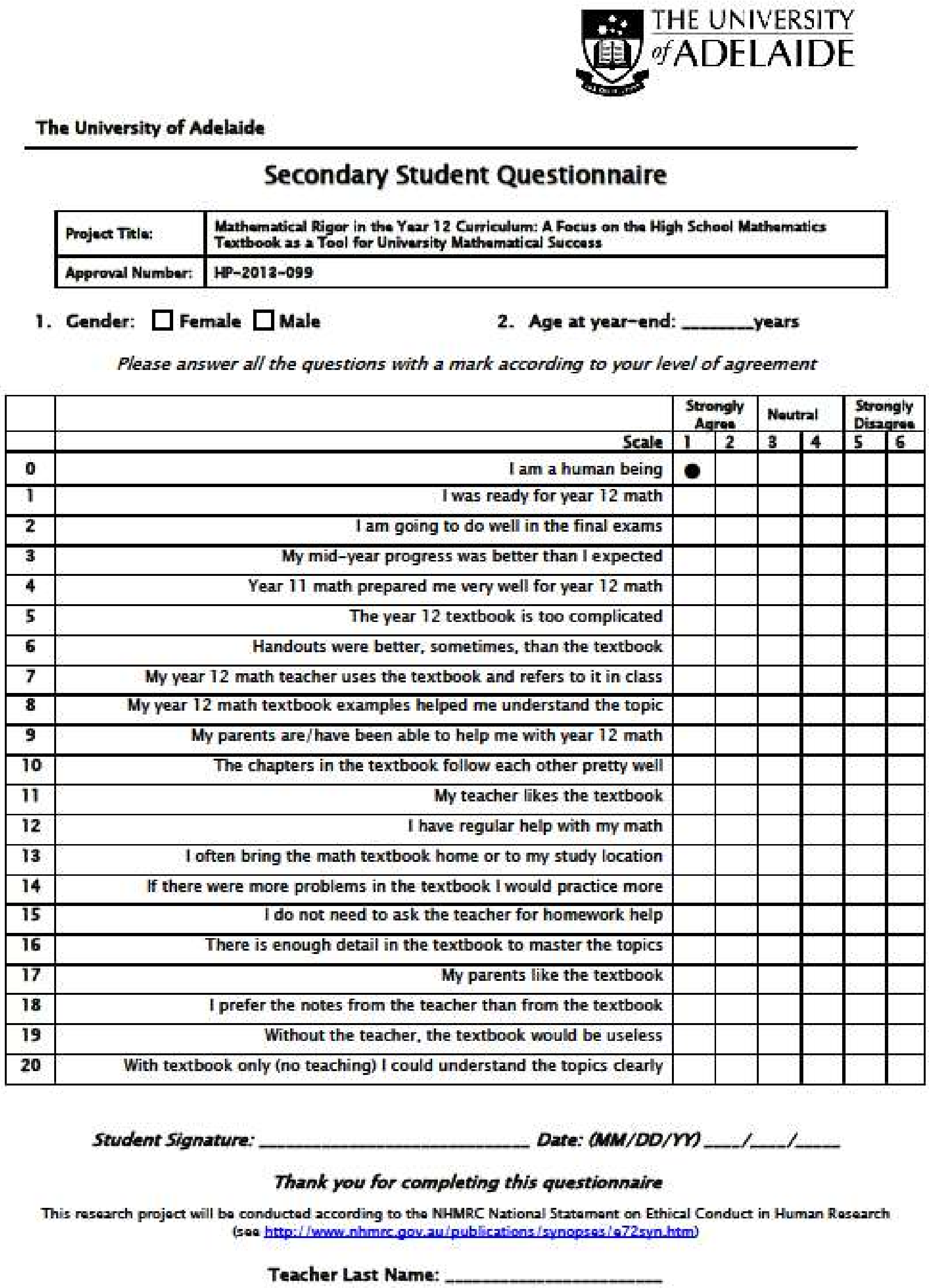
- **Understanding**
- **Removals**
- **Additions**

This research project will be conducted according to the NHMRC National Statement on Ethical Conduct in Human Research (see http://www.nhmrc.gov.au/publications/synopses/e72syn.htm) 3

Figure B-1 Secondary Student Pilot Survey Instrument

Secondary Student Final Survey Instrument

In the Secondary Student Survey, Column "C" refers to the categories and Column "Q" refers to question numbers.



THE UNIVERSITY of ADELAIDE

The University of Adelaide

Secondary Student Questionnaire

Project Title:	Mathematical Rigor in the Year 12 Curriculum: A Focus on the High School Mathematics Textbook as a Tool for University Mathematical Success
Approval Number:	HP-2012-099

1. Gender: ☐ Female ☐ Male   2. Age at year-end: _______years

Please answer all the questions with a mark according to your level of agreement

		Strongly Agree		Neutral		Strongly Disagree	
	Scale	1	2	3	4	5	6
0	I am a human being	●					
1	I was ready for year 12 math						
2	I am going to do well in the final exams						
3	My mid-year progress was better than I expected						
4	Year 11 math prepared me very well for year 12 math						
5	The year 12 textbook is too complicated						
6	Handouts were better, sometimes, than the textbook						
7	My year 12 math teacher uses the textbook and refers to it in class						
8	My year 12 math textbook examples helped me understand the topic						
9	My parents are/have been able to help me with year 12 math						
10	The chapters in the textbook follow each other pretty well						
11	My teacher likes the textbook						
12	I have regular help with my math						
13	I often bring the math textbook home or to my study location						
14	If there were more problems in the textbook I would practice more						
15	I do not need to ask the teacher for homework help						
16	There is enough detail in the textbook to master the topics						
17	My parents like the textbook						
18	I prefer the notes from the teacher than from the textbook						
19	Without the teacher, the textbook would be useless						
20	With textbook only (no teaching) I could understand the topics clearly						

Student Signature: ____________________________ *Date: (MM/DD/YY)* ____/____/_____

Thank you for completing this questionnaire

This research project will be conducted according to the NHMRC National Statement on Ethical Conduct in Human Research (see http://www.nhmrc.gov.au/publications/synopses/e72syn.htm)

Teacher Last Name: ______________________

Figure B-2 Secondary Student Final Survey Instrument

Secondary Teacher Pilot Survey Instrument

THE UNIVERSITY of ADELAIDE

The University of Adelaide

Faculty of the Professions
Level 8, 10 Pulteney Street, Adelaide SA 5005; Tel: (+618) 8313 5628

Attachment VII

Secondary Teacher Questionnaire Pilot

Project Title:	**Mathematical Rigor in the Year 12 Curriculum: A Focus on the Secondary Mathematics Textbook as a Tool for University Mathematical Success**
Approval Number:	**HP-2012-099**

Section 1: **General Information** (*Please answer all the questions*)

1. **Your Gender** ☐ Female ☐ Male
2. **Your Age at the end of the school year** _______ years
3. **Years teaching math** _______ years
4. **Your highest level of formal education**

   ☐ Diploma ☐ Bachelor ☐ Master ☐ Doctorate

   ☐ Other (Please specify):___________________________________

5. **Number of different textbooks you have taught from in year 12** _________
6. **Does the curriculum require more that 1 textbook:** ☐ Yes

   If no above, what is the name and author of the textbook:

   Name: ....

   Author: ....

   If yes above, what are the names/authors of the year 12 two primary textbook(s).

   Name:..

   Author: ....

   Name:..

   Author: ....

Please turn over to Section 2

This research project will be conducted according to the NHMRC National Statement on Ethical Conduct in Human Research (see http://www.nhmrc.gov.au/publications/synopses/e72syn.htm)

Secondary Teacher Questionnaire Pilot

<u>Section 2</u>: **Assessment of the year 12 math textbook use**
Please answer all the questions according to your level of agreement with the statement
<u>1 - 2 = Range of agreement / 3 - 4 = Neutral (Slight agreement-Slight disagreement) / 5 - 6 = Range of disagreement where: 1 = Strongly agree and 6 = Strongly disagree</u>

		Strongly Agree		Neutral		Strongly Disagree	
	Scale	**1**	**2**	**3**	**4**	**5**	**6**
1	Students believe the textbook is valuable to them						
2	A good textbook is valuable to the teacher (me)						
3	Students are self-driven, disciplined and make mature use of the textbook						
4	Students generally lack confidence in mathematics						
5	Students generally complete their assignments						
6	Students ask for help with homework often						
7	Very little textbook material is remedial in content						
8	Students are more mature this year than last year						
9	Students are more academically ready for math this year than last year						
10	Section and chapter exercises reinforce the current topic						
11	I am very satisfied with the current textbook						
12	Textbook is taken home and used by the student on a regular basis						
13	Students that are academically mature use the textbook as a focus for study						
14	Sample questions in the textbook are very helpful in explaining the concept						
15	Textbook has concept explanations that are very good						
16	The year 11 math textbook flows coherently into the year 12 math textbook						
17	Textbook is thorough and mastery oriented						
18	Section and chapter exercises range from simple to difficult						
19	I have used a better textbook than the current textbook						
20	Section and chapter exercises utilize previously learned topics						
21	Textbook is used as a focus in the classroom for topic presentation						

Please Comment on Next Page

....

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School Code: ___________________
Date: ___________________

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Secondary Teacher Questionnaire Pilot Comments

Please comment below on whether the questions were hard to understand or if you believe that a particular question may not be helpful or, if you think a question should be added.
Please write the details below.
The sections are for your convenience/use as you see fit for questionnaire:

- **Understanding**
- **Removals**
- **Additions**

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Figure B-3 Secondary Teacher Pilot Survey Instrument

Secondary Teacher Final Survey Instrument

THE UNIVERSITY of ADELAIDE

The University of Adelaide

Faculty of the Professions
Level 8, 10 Pulteney Street, Adelaide SA 5005; Tel: (+618) 8313 5628
Attachment VII

Secondary Teacher Questionnaire

Project Title:	**Mathematical Rigor in the Year 12 Curriculum: A Focus on the Secondary Mathematics Textbook as a Tool for University Mathematical Success**
Approval Number:	HP-2013-099

Please answer all the questions with a mark (●) according to your level of agreement

	Gender: ☐ Female ☐ Male **Age:**______(years at term end)	Strongly Agree		Neutral		Strongly Disagree	
	Scale	1	2	3	4	5	6
1	Students believe the textbook is valuable to them						
2	A good textbook is valuable to the teacher (me)						
3	Students are self-driven, disciplined and make mature use of the textbook						
4	Students generally lack confidence in mathematics						
5	Very little textbook material is remedial in content						
6	Students are more mature this year than last year						
7	Students are more academically ready for math this year than last year						
8	Section and chapter exercises reinforce the current topic						
9	I am very satisfied with the current textbook						
10	Textbook is taken home and used by the student on a regular basis						
11	Students that are academically mature use the textbook as a focus for study						
12	Sample questions in the textbook are very helpful in explaining the concept						
13	Textbook has concept explanations that are very good						
14	The year 11 math textbook flows coherently into the year 12 math textbook						
15	Textbook is thorough and mastery oriented						
16	Section and chapter exercises range from simple to difficult						
17	I have used a better textbook than the current textbook						
18	Section and chapter exercises utilize previously learned topics						
19	Textbook is used as a focus in the classroom for topic presentation						

1. **Your highest level of formal education:** ☐ Diploma ☐ Bachelor ☐ Master ☐ Doctor
2. **Years Teaching Mathematics:** __________ years
3. **Number of different textbooks used while teaching math:** ☐ 1 ☐ 2-5 ☐ 6+
4. **Current Textbook Name(s):** ______________________________ **Edition(s):** _________
5. **Current Textbook Author(s):** ____________________________________

Signature: ________________________ ***Date: (DD/MM/YY)*** ____/____/_15_

Thank you for completing this questionnaire

This research project will be conducted according to the NHMRC National Statement on Ethical Conduct in Human Research (see http://www.nhmrc.gov.au/publications/synopses/e72syn.htm)

Teacher Surname: __________________________

Figure B-4 Secondary Teacher Final Survey Instrument

Tertiary Student Pilot Survey Instrument

The University of Adelaide

THE UNIVERSITY of ADELAIDE

Faculty of the Professions
Level 8, 10 Pulteney Street, Adelaide SA 5005 Australia; Tel: (+618) 8313 5628
Attachment XII

University Student Questionnaire Pilot

Project Title:	**Mathematical Rigor in the 12th Grade Curriculum: A Focus on the High School Mathematics Textbook as a Tool for University Mathematical Success**
Approval Number:	**HP-2013-099**

Section 1: Preparedness for math this year (1 =agree strongly and 6=disagree strongly)
Please answer all the questions according to your level of agreement with the statement
1 - 2 = Agreement / 3 - 4 = Neutral / 5 - 6 = Disagreement

		Strongly Agree		Neutral		Strongly Disagree	
	Scale	1	2	3	4	5	6
	High School Related						
1	12th grade math was very good preparation for college mathematics						
2	12th grade mathematics textbook needed more depth						
3	Without the teacher, my 12th grade textbook would have been useless						
4	There was too much homework in my 12th grade mathematics class						
5	There were enough exercises for me to be well practiced						
6	Handouts were better, sometimes, than the textbook						
7	My 12th grade math teacher used the textbook & referred to it in class						
8	My 12th grade math book examples helped me understand the topic						
	College Related						
9	The textbook examples help me understand the topic						
10	The chapters in the textbook follow each other pretty well						
11	My mid-year progress was better than I expected						
12	I have regular help with my mathematics						
13	I often bring the mathematics textbook home or to my study location						
14	The teacher uses the textbook and refers to it in class						
15	I am going to do very well in the terminal exams						
16	There is enough detail in the textbook to master the topics						
17	I was ready for mathematics this year						
18	Handouts are sometimes better than the textbook						
19	Without the teacher, the textbook would be useless						
20	With textbook only (no lectures) I could understand the topics clearly						

Please turn over to Section 2 and 3

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School Code: ______________
Date: ____________________

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University Student Questionnaire Pilot

Section 2 of 3: **General Information**

1. **Your country of location:** ☐ AUS ☐ USA
2. **Your School Type:** ☐ Public ☐ Private
3. **Your Gender:** ☐ Female ☐ Male
4. **Your Age at the end of this year:** ________ **Years**
5. **Is this your first semester of mathematics at any university facility?** ☐ Yes ☐ No
6. **Did the COMPASS math placement test score assign this math class?** ☐ Yes ☐ No
7. **What is your current mathematics class?** ________________________

Begin High School Questions

8. **Your High School Name:** ________________________________
9. **Your High School City:** ________________________________
10. **Your High School Country:** ________________________________
11. **Your High School Graduation Year (CCYY):** ____________________

End High School Questions

Section 3 of 3: Comments on Questions in Section 1

Please comment below on whether the questions were hard to understand or if you believe that a particular question may not be helpful or, if you think a question should be added.
Please write the details below.
The sections are for your convenience/use as you see fit for questionnaire comments:

For example:

- Understanding
- Removal
- Addition

This research project will be conducted according to the NHMRC National Statement on Ethical Conduct in Human Research (see http://www.nhmrc.gov.au/publications/synopses/e72syn.htm)

Figure B-5 Tertiary Student Pilot Survey Instrument

Tertiary Student Final Survey Instrument

THE UNIVERSITY of ADELAIDE

Project Title:	Mathematical Rigor in the 12th Grade Curriculum: A Focus on the High School Mathematics Textbook as a Tool for University Mathematical Success
Approval Number:	HP-2013-099

University Student Questionnaire

1. Your Gender: ☐ Female ☐ Male
2. Your Age at the end of this year: ________ years
3. Is this your first semester of mathematics at any university facility? ☐ Yes ☐ No
4. Did the COMPASS math placement test score assign this math class? ☐ Yes ☐ No
5. What is your current mathematics class? ________________________
6. What was your High School Name: ________________________
7. What was your High School State: ________________________
8. What was your High School City: ________________________
9. What was your High School Country: ______________
10. What was your High School Graduation Year: _______

#	University Textbook can mean written material if no textbook for this course	Strongly Agree		Neutral		Strongly Disagree	
	Scale	1	2	3	4	5	6
	High School Related						
1	12th grade math was very good preparation for college mathematics						
2	12th grade mathematics textbook needed more depth						
3	Without the teacher, my 12th grade textbook would have been useless						
4	There was too much homework in my 12th grade mathematics class						
5	There were enough exercises for me to be well practiced						
6	Handouts were better, sometimes, than the textbook						
7	My 12th grade math teacher used the textbook & referred to it in class						
8	My 12th grade math book examples helped me understand the topic						
	College Related						
9	The textbook examples help me understand the topic						
10	The chapters in the textbook follow each other pretty well						
11	My mid-year progress was better than I expected						
12	I have regular help with my mathematics						
13	I often bring the mathematics textbook home or to my study location						
14	The teacher uses the textbook and refers to it in class						
15	I am going to do very well in the terminal exams						
16	There is enough detail in the textbook to master the topics						
17	I was ready for mathematics this year						
18	Handouts are sometimes better than the textbook						
19	Without the teacher, the textbook would be useless						
20	With textbook only (no lectures) I could understand the topics clearly						

________ Please initial to authenticate your responses

This research project will be conducted according to the NHMRC National Statement on Ethical Conduct in Human Research (see http://www.nhmrc.gov.au/publications/synopses/e72syn.htm)

Figure B-6 Tertiary Student Final Survey Instrument

Tertiary Professor Pilot Survey Instrument

The University of Adelaide

THE UNIVERSITY of ADELAIDE

Faculty of the Professions
Level 8, 10 Pulteney Street, Adelaide SA 5005 Australia; Tel: (+618) 8313 5628
Attachment XIII

University Teacher Questionnaire Pilot

Project Title:	**Mathematical Rigor in the Year 12 Curriculum: A Focus on the High School Mathematics Textbook as a Tool for University Mathematical Success**
Approval Number:	**HP-2012-099**

Section 1: Assessment of the 1st year engineering math students
1 - 2 = Agreement / 3 - 4 = Neutral / 5 - 6 = Disagreement
1 = agree strongly and 6 = disagree strongly

		Strongly Agree		Neutral		Strongly Disagree	
	Scale	**1**	**2**	**3**	**4**	**5**	**6**
1	Students believe the textbook is valuable to them						
2	The textbook is valuable to the teacher						
3	Students are self-driven, disciplined and make mature use of the textbook						
4	Students generally lack confidence in mathematics						
5	Students generally complete their assignments						
6	Students ask for help with homework often						
7	Very little textbook material is remedial in content						
8	Students are more mature this year than last year						
9	Students are more academically ready for math this year than last year						
10	Students are less prepared for mathematics this year than **ever** before						

Please turn to Section 2 and 3

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School Code: ___________________

Date: _________________________

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Section 2: General Information

1. **Your country of location** ☐ AUS ☐ USA
2. **Institution Type** ☐ Public ☐ Private
3. **Institution Name** ____________________________
4. **Your Gender** ☐ Female ☐ Male
5. **Your Age at the end of the school year:** ☐ 20 – 30 ☐ 31 – 40 ☐ 41 – 50 ☐ Over 50
6. **Years teaching math:** ___________ years
7. **Your highest level of formal education:** ☐ Diploma ☐ Bachelor ☐ Master ☐ Doctorate

   ☐ Other (Please specify):________________________________
8. **Years teaching 1st year engineering mathematics:** ________ years
9. **Name / Author of the textbook used in this class:** __________________________ / ______________________

Section 3 of 3: Comments on Questions in Section 1

Please comment below on whether the questions were hard to understand or if you believe that a particular question may not be helpful or, if you think a question should be added.
Please write the details below.
The sections are for your convenience/use as you see fit for questionnaire comments:

For example:

- Understanding
- Removal
- Addition

This research project will be conducted according to the NHMRC National Statement on Ethical Conduct in Human Research (see http://www.nhmrc.gov.au/publications/synopses/e72syn.htm)

Figure B-7 Tertiary Professor Pilot Survey Instrument

Tertiary Professor Final Survey Instrument

THE UNIVERSITY *of* ADELAIDE

University Teacher Questionnaire

Project Title:	Mathematical Rigor in the Year 12 Curriculum: A Focus on the High School Mathematics Textbook as a Tool for University Mathematical Success
Approval Number:	HP-2013-099

1. **Gender:** ☐ Female ☐ Male
2. **Age at the end of the school year:** ☐ 20 – 30 ☐ 31 – 40 ☐ 41 – 50 ☐ Over 50
3. **Number of years teaching math:** ________
4. **Level of formal education:** ☐ Bachelor ☐ Master ☐ Doctorate ☐ Other (Please specify):

   ----
5. **Years teaching 1st year engineering mathematics:** ___________

		Strongly Agree		Neutral		Strongly Disagree	
	Scale	**1**	**2**	**3**	**4**	**5**	**6**
1	Students believe the text/written material is valuable to them						
2	The text/written material is valuable to the me						
3	Students are disciplined and make mature use of the text/written material						
4	Students generally lack confidence in mathematics						
5	Students generally complete their assignments						
6	Students ask for help with homework often						
7	Very little text/written material is remedial in content						
8	Students are more mature this year than last year						
9	Students are more academically ready for math this year than last year						
10	Students are less prepared for mathematics this year than **ever** before						

Thank you for completing this questionnaire

This research project will be conducted according to the NHMRC National Statement on Ethical Conduct in Human Research (see http://www.nhmrc.gov.au/publications/synopses/e72syn.htm)

Figure B-8 Tertiary Professor Final Survey Instrument

Appendix C. Supporting Tables

Table C-1 Raw Survey Data For Final MSA Computation in Table C-2

School	Date	Country	Sex	Age	Exp	Edu	V1. Students believe the textbook/written material is valuable to	V2. The textbook is valuable to the teacher	V3. Students are self-driven, disciplined and make mature use of the textbook	V4. Students generally lack confidence in mathematics	V5. Students generally complete their assignments	V6. Students ask for help with homework often	V7. Very little textbook/written material is remedial in content	V8. Students are more mature this year than last year	V.9 Students are more academically ready for math this year than	V.10 Students are less prepared for mathematics this year than **ever** before
SDSM&T	08/09/2014	USA	0	30	5	MA	6	2	5	2	3	4	2	4	4	3
SDSM&T	08/09/2014	USA	1	50	25	MA	1	1	2	3	2	2	2	3	3	4
SDSM&T	08/09/2014	USA	0	50	20	MA	5	1	6	5	4	5	2	3	4	4
SDSM&T	08/09/2014	USA	1	50	20	PHD	5	4	5	3	4	4	2	4	4	3
SDSM&T	08/09/2014	USA	0	40	3	MA	2	1	4	3	4	3	2	3	3	5
SDSM&T	08/09/2014	USA	0	50	9	PHD	4	1	4	2	3	5	2	3	3	3
SDSM&T	08/09/2014	USA	0	50	20	BA	5	2	4.4375	4	4	5	3	3	5	4
SDSM&T	08/09/2014	USA	1	50	25	MA	3	1	5	3	5	5	2	3	3	5
SDSM&T	08/09/2014	USA	1	30		BA	1	1	6	3	4	4	3	4	4	5
SDSM&T	08/09/2014	USA	1	30	8	BA	5	2	5	2	5	4	2	5	5	3
U of A	03/02/2014	AUS	1	50	25	PHD	5	3	5	2	3	2	4	3.5	4	3.5
U of A	03/02/2014	AUS	0	40	25	PHD	3	2	3	3	2	2	2	3	3	3
SDSM&T	11/28/2015	USA	0	55	20	MA	4	1	5	4	3	5	3	4	4	2
U of A	03/21/2015	AUS	1	50	25	PHD	2	5	4	3	2	1	3	3	3	4
SDSM&T	11/26/2016	USA	0	55	20	MA	4	1	5	4	3	5	3	4	4	2
U of A	03/10/2016	AUS	1	50	25	PHD	3	4	4	2	1	2	3	3	3	3
U of A	03/16/2016	AUS	1	40	20	PHD	3	2	3	3	3	3	2	3	3	3
Ave.							3.5882353	2	4.4375	3.00	3.235294118	3.58823529	2.470588	3.441176	3.6470588	3.50
MODE							5	1	5	3	3	5	2	3	3	3
S.D.							1.502449	1.274754878	1.058817147	0.866025404	1.091410313	1.37198868	0.624264	0.609363	0.7018882	0.935414347
Var.							2.2573529	1.625	1.12109375	0.75	1.191176471	1.88235294	0.389706	0.371324	0.4926471	0.875

Table C-2 shows the merging of university professors' gender, age, education, and experience pertaining to survey answers for V3 and V9 (general maturity and mathematical skill). Credential factor pertained to skill analysis and experience factor to maturity analysis. Combined Coefficient is the mean of the coefficients calculated for V3 and V9.

Table C-2 Professor Demographic Data Relating to Survey Answers for V3 & V9

Gender					**Gender**					
	Female	**Male**	**Total**	**V3 Coefficient**		**Female**	**Male**	**Total**	**V9 Coefficient**	**Averaged Coefficient**
V3	8	9	17		**V9**	8	9	17		
Average	**4.550**	**4.330**			**Average**	**3.750**	**3.560**	99999999		
Dispersal	2.141	2.292		0.151	**Dispersal**	1.765	1.885		0.120	
Credential					**Credential**					
	Female	**Male**				**Female**	**Male**			
	8	9	17			8	9	17		
Degree	**2.13**	**2.33**			**Degree**	**2.13**	**2.33**			
Factor	1.002	1.234		0.231	**Factor**	1.002	1.234		0.231	
Experience					**Experience**					
	Female	**Male**				**Female**	**Male**			
	8	9	17			8	9	17		
Sum Ages	**370**	**360**			**Sum Ages**	**370**	**360**			
Sum Experience	**122**	**153**			**Sum Experience**	**122**	**153**			
Factor	1.427	1.246		0.182	**Factor**	1.427	1.246		0.182	
Average				0.166					0.176	0.171

Table C-3 Data Points Provided and Derived for University Professors

University Teachers	Distribution of Data from 3.a & 3. b	Population Data With Filler	Survey Items V3+V9
1	Low US	25	9
2	filler	27	5
3	Low US/AUS	28	10
4	filler	29	9
5	filler	31	7
6	filler		
7	filler	32	5
8	filler	33	8
9	filler	34	10
10	High US/AUS	35	10
11	Low w/CC	36	9
12	filler	37	6
13	filler	38	9
14	filler	39	7
15	High US	40	9
16	filler	43	7
17	High w/CC	46	7
	Mean	34.53125	7.9375
	Variance	32.67089844	2.8625
	Beta	**0.087616201**	

Table C-4 Application of the Weighting Calculation as shown in Table C-5

Education Credential	**University Teachers**	**Distribution of Data from 3.a & 3.b**	**Population Data With Filler**	**Survey Items V3+V9**	**Weighting 2.24**
MA	1	Low US	25	9	20.16
MA	2	filler	27	5	11.2
MA	3	Low US/AUS	28	10	22.4
PHD	4	filler	29	9	20.16
MA	5	filler	31	7	15.68
PHD	6	filler			
BA	7	filler	32	5	11.2
MA	8	filler	33	8	17.92
BA	9	filler	34	10	22.4
BA	10	High US/AUS	35	10	22.4
PHD	11	Low w/CC	36	9	20.16
PHD	12	filler	37	6	13.44
MA	13	filler	38	9	20.16
PHD	14	filler	39	7	15.68
MA	15	High US	40	9	20.16
PHD	16	filler	43	7	15.68
PHD	17	High w/CC	46	7	15.68
		Mean	34.53	7.94	17.78
		Variance	32.67	2.86	14.36
		Beta	0.439623049		

To calculate MSAR and the weighting for the sample data values, age, experience, gender and educational level were analyzed using the data in this table.

Table C-5 Calculating MSAR and Sample Data Weighting

Sex	Age	Exp	Edu	Edu Score
0	30	5	MA	2
1	50	25	MA	2
0	50	20	MA	2
1	50	20	PHD	3
0	40	3	MA	2
0	50	9	PHD	3
0	50	20	BA	1
1	50	25	MA	2
1	30		BA	1
1	30	8	BA	1
1	50	25	PHD	3
0	40	25	PHD	3
0	55	20	MA	2
1	50	25	PHD	3
0	55	20	MA	2
1	50	25	PHD	3
1	40	20	PHD	3
	45.29	18.44		**2.24**
1	50.00	25.00		2.00
	8.56	7.70		0.75
	73.35	59.33		0.57

Appendix D. Reverse Coding Process

Table D-1 shows the rotated matrix for Imputation 3 with |suppression of coefficients| <0.33 using Principal Component Analysis and Varimax rotation.

Table D-1 Secondary Student Data Imputation 3 Rotated Component Matrix

Variables	Component (Factor)					
	1	2	3	4	5	6
V5			.819			
V6			.697			
V7			.552			
V8		.360	.670			
V9	.555	-.347				
V10	.679					
V11		.671				
V12	-.467	.533				
V13					.794	
V14		.473				
V15		.727				
V16				-.604		
V17		.381				.633
V18						.838
V19				.798		
V20		.465		.421		
V21					.703	
V22	.796					
V23	.698					
V24	-.443			.581		

Note that four of the variables (V9, V12, V16, and V24) in Table 5.12 are negatively loaded but that V16 does not have any cross-loading. To determine if reverse coding would allow V16 (survey question #12, “I have regular help with my math”) to be loaded onto component (factor) 4 without affecting other variables, the rotated component matrix was re-run with the reverse coding (i.e., V16R as “I do not have regular help with my math.”) to make the direction of the item (question) consistent with V19, V20, and V24. Table D-2 shows the statistics for this variable (question) and Table D-3 depicts the rotated component matrix with V16R (reverse coded V16) with |suppression of coefficients ≤ 0.33 illustrating that V16R did load onto component 4 without affecting other variables.

Table D-2 Secondary Student Data V16/Q12 Statistics

Statistic	**V16/Q12**	**V16R/Q12R**
Median	3	4
Mean	3.1	3.9
Mode	3	4
Max	6	6
Min	1	1

1=Strongly Agree; 6=Strongly Disagree

Table D-3 Secondary Student Data Imputation 3 Rotated Component Matrix V16 Reverse Coded

Variables	Component					
	1	2	3	4	5	6
V5			.819			
V6			.696			
V7			.552			
V8		.360	.670			
V9	.555	-.347				
V10	.680					
V11		.670				
V12	-.467	.533				
V13					.794	
V14		.474				
V15		.727				
V16R				.602		
V17		.380				.633
V18						.838
V19				.798		
V20		.465		.421		
V21					.703	
V22	.796					
V23	.697					
V24	-.442			.582		

To address negative and positive cross-loaded variables (V8 loaded on factors 2 and 3, V9 and V12 loaded on factors 1 and 2, V17 loaded on factors 2 and 6, V20 loaded on factors 2 and 4, and V24 loaded on factors 1 and 4), the ProMax Rotation algorithm was used. Table D-4 shows the results.

Table D-4 Secondary Student Data Imputation 3 Pattern Matrix by ProMax Rotation

Variables	Component 1	2	3	4	5	6
V5			.836			
V6			.691			
V7			.599			
V8			.662			
V9	.552					
V10	.756					
V11		.706				
V12	-.408	.470				
V13					.876	
V14		.423				
V15		.779				
V16R				.663		
V17		.404				.628
V18						.848
V19				.891		
V20		.412		.384		
V21					.721	
V22	.861					
V23	.698					
V24				.517		

Note that only variables V12, V17, and V20 remain cross-loaded.

In the tertiary student data rotated component matrix, V20 (“With textbook only (no lectures) I could understand the topics clearly”) presents itself as a variable that requires reverse coding. Table D-5 shows the statistics for this variable.

Table D-5 Tertiary Student Data V20/Q20 Statistics

Statistic	V20/Q20	V20R/Q20R
Median	4	3
Mean	3.9	3.1
Mode	4	3
Max	6	6
Min	1	1

1=Strongly Agree
6=Strongly Disagree

Table D-6 shows the Varimax rotation with V3 removed for comparison and with |suppression of coefficients| < 0.395. The loading densities are equivalent save for the V12 loading. V12 is the question, “I have regular help with my mathematics.” The double loading in factor 3 and factor 5 when analyzed seem to align V12 with Factor 3.

Table D-6 Imputation 5 Pattern Matrix by Varimax Rotation with Coefficient Suppression < 0.395

Variables	Component					
	1	2	3	4	5	6
V1		.601				
V2						.668
V4						.665
V5		.706				
V6				.758		
V7		.737				
V8		.810				
V9	.689					
V10	.675					
V11	.743					
V12			.398			
V13			.821			
V14			.769			
V15	.720					
V16	.677					
V17	.556					
V18				.687		
V19					.748	
V20R					.734	

Table D-7 shows the improved KMO after removal of V6. Additionally, it was recognized that, with seven factors explaining only 64% of the variance, the model needed upgrades before being ready for Mplus (CFA and SEM).

Table D-7 KMO and Bartlett’s Test After Removal of V6 From Tertiary EFA

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.764
Bartlett's Test of Sphericity	Approx. Chi-Square	9358.668
	df	171
	Sig.	0.000

Table D-8 shows the new variance explained with V6 removed, indicating a 6-factor model.

Table D-8 Total Variance Explained with V6 Removed

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.781	19.902	19.902	3.781	19.902	19.902	2.526	13.292	13.292
2	2.122	11.166	31.069	2.122	11.166	31.069	2.229	11.731	25.023
3	1.847	9.723	40.792	1.847	9.723	40.792	2.118	11.145	36.168
4	1.491	7.849	48.641	1.491	7.849	48.641	1.724	9.072	45.240
5	1.143	6.013	54.654	1.143	6.013	54.654	1.628	8.569	53.809
6	1.093	5.754	60.408	1.093	5.754	60.408	1.254	6.599	60.408
7	.918	4.832	65.240						
8	.846	4.451	69.691						
9	.781	4.109	73.800						
10	.706	3.716	77.516						
11	.626	3.297	80.813						
12	.566	2.982	83.795						
13	.546	2.873	86.668						
14	.508	2.673	89.341						
15	.480	2.524	91.865						
16	.420	2.213	94.078						
17	.406	2.134	96.213						
18	.394	2.072	98.285						
19	.326	1.715	100.000						

After removing V6, V20 loaded negatively onto Factor 4 (Table D-9).

Table D-9 Rotated Component Matrix After Removal of V6

Variables	Component					
	1	2	3	4	5	6
V1	.066	.612	.490	.076	-.040	.037
V2	.118	-.346	-.317	.112	-.016	.572
V3	-.030	.099	.001	.535	-.037	.484
V4	-.089	.072	.156	-.084	.071	.796
V5	-.031	.447	.631	.061	.044	.063
V7	.105	-.054	.767	.065	-.046	.032
V8	.148	.037	.828	-.076	.039	-.052
V9	.795	.012	.159	-.122	.137	-.012
V10	.773	.034	.137	-.021	.175	-.010
V11	.633	.340	.036	.172	-.139	.010

Table D.9 (continued)

Variables	Component					
	1	2	3	4	5	6
V12	.264	-.314	.051	.327	.351	.092
V13	.125	-.081	.065	.027	.818	.020
V14	.110	.119	-.071	.002	.806	.014
V15	.468	.660	-.076	.022	-.003	-.009
V16	.633	.302	-.044	-.208	.238	-.044
V17	.262	.758	.118	-.005	.003	-.045
V18	-.011	.304	-.026	.408	.151	.007
V19	.006	.056	.046	.784	.050	.046
V20	.221	.289	-.002	-.649	.145	.185

Table D-9 also reveals that V3 and V20 (secondary and tertiary mix) are loading onto Factor 4. Since the communalities are too low for repeatable modeling in V18 and V12, variables V3, V12 and V18 were removed and V20 was reversed to V20R. Table D-10 indicates improved KMO, and Table D-11 shows a six-factor model with 67% of variance explained.

Table D-10 Improved KMO and Bartlett's Test After Additional Adjustments to Tertiary EFA

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.772
Bartlett's Test of Sphericity	Approximate Chi-Square	8364.427
	df	120
	Sig.	0.000

Table D-11 Total Variance Explained with V3, V12, and V18 Removed and V20 Reverse Coded

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.758	23.489	23.489	3.758	23.489	23.489	2.342	14.639	14.639
2	2.053	12.829	36.318	2.053	12.829	36.318	2.234	13.963	28.602

Table D.11 (continued)

Component	Initial Eigenvalues	Extraction Sums of Squared	Rotation Sums of Squared

					Loadings			Loadings	
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
3	1.478	9.237	45.555	1.478	9.237	45.555	2.149	13.433	42.035
4	1.326	8.289	53.844	1.326	8.289	53.844	1.518	9.487	51.522
5	1.102	6.889	60.732	1.102	6.889	60.732	1.408	8.800	60.322
6	1.073	6.709	67.441	1.073	6.709	67.441	1.139	7.119	67.441
7	.782	4.885	72.326						
8	.654	4.086	76.412						
9	.607	3.791	80.203						
10	.555	3.468	83.671						
11	.536	3.351	87.022						
12	.488	3.052	90.074						
13	.450	2.812	92.886						
14	.408	2.551	95.438						
15	.401	2.507	97.944						
16	.329	2.056	100.000						

Table D-12 is a rotated component matrix revealing cross-loading ambiguity in V1 and V11.

Table D-12 Rotated Component Matrix Showing Cross-Loading Ambiguity in V1 and V11

Variables	Component					
	1	2	3	4	5	6
V1	.607		.511			
V2						.611
V4						.855
V5			.650			
V7			.766			
V8			.822			
V9		.820				
V10		.798				
V11	.533	.513				
V13				.820		
V14				.841		
V15	.753					
V16		.595				
V17	.791					
V19					.822	
V20R					.759	

The following observations followed from an analysis of Table D-12. V11 ("My progress so far is better than I expected") is not logically connected to the other variables in Factor 2 which are: V9 ("textbook/written material examples help me understand the topic"), V10 ("The chapters in the textbook/written materials follow each other pretty well"), and V16

("There is enough detail in the textbook/written materials to master the topics"). Instead, V11 is logically connected to the variables in Factor 1 which are: V1 ("Year 12 mathematics was very good preparation for this course"), V15 ("I am going to do very well in the terminal exams"), and V17 ("I was ready for mathematics this year"). It was also noted that V1 cross loaded onto Factor 3 with observed variables that were not logically connected. Factor 3 variables are: V5 ("There were enough exercises in 12th grade textbook for me to be well practiced"), V7 ("My 12th grade math teacher used the textbook and referred to it in class") and V8 ("My 12th grade math book examples helped me understand the topic"). It was decided to suppress lower cross loadings on V-1 and V-11 by using SPSS to suppress coefficients ≤ 0.52. Results are seen in Table D-13. The extraction method used was Principal Component Analysis. The rotation method was Varimax with Kaiser Normalization.

Table D-13 Rotated Component Matrix Filtered with SPSS Suppression of Coefficients, ≤ 0.52

Variables	Component					
	1	2	3	4	5	6
V1	.607					
V2						.611
V4						.855
V5			.650			
V7			.766			
V8			.822			
V9		.820				
V10		.798				
V11	.533					
V13				.820		
V14				.841		
V15	.753					
V16		.595				
V17	.791					
V19					-.822	
V20R					.759	

Appendix E. Textbook Chapter Details

Book i. Earl Swokowski, *Fundamentals of Algebra and Trigonometry*, 2d ed., (Boston: Prindle, Weber & Schmidt, Inc., 1972), LOCCCN: 68-13118.

Generalized Content and Topic Order Summary:

1: General and Prerequisite Review

2: Equations and Inequalities

3: Functions: Theory, Operations and Graphs

4: Exponential and Logarithmic Functions

5: Trigonometry (Functional, Graphical and Analytic)

6: Linear Systems and Matrices

7: Polar Coordinates and Complex Numbers

8: Polynomials and Rational Functions

9: Sequences and Series with Combinatorics

Detailed Chapter Contents:

Chapter 1: Fundamental Concepts of Algebra
- Sets
- Real Numbers
- Coordinate Lines; Absolute Value
- Integral Exponents
- Rational Exponents
- Algebraic Expressions
- Factoring

Chapter 2: Equations and Inequalities
- Elementary Equations
- Applications
- Equations of Degree Greater than 1
- Miscellaneous Equations
- Elementary Inequalities
- More Inequalities

Chapter 3: Functions and Graphs
- Coordinate Systems Two Dimensions
- Relations and their Graphs
- Functions
- Graphs of Functions
- Linear Functions
- Composite and Inverse Functions

- Variation

Chapter 4: Exponential and Logarithmic Functions
- Exponential Functions
- Logarithms
- Logarithmic Functions
- Common Logarithms
- Linear Interpolation
- Computations with Logarithms
- Exponential and Logarithmic Equations and Inequalities

Chapter 5: The Trigonometric Functions
- Arc Length and the Unit Circle
- Trigonometric Functions
- Values of the Trigonometric Functions
- Angles and their Measurement
- Trigonometric Functions of Angles
- Right Triangle Trigonometry

Chapter 6: Analytic Trigonometry
- Trigonometric Identities
- Conditional Equations
- Addition Formulas
- Multiple Angle Formulas
- Sum and Product Formulas
- Summary of Formulas
- Trigonometric Graphs
- Graphs and their Applications
- Inverse Trigonometric Functions
- Law of Sines
- Law of Cosines

Chapter 7: Systems of Equations and Inequalities
- Solving for Function Intersections
- Systems of Two Variable Linear Equations
- Systems of More than Two Variable Linear Equations
- Matrix Methods
- Determinants
- Properties of Determinants
- Cramer's Rule
- Systems of Inequalities
- The Algebra of Matrices
- Inverses of Matrices

Chapter 8: Complex Numbers

- Definitions
- Conjugates and Inverses
- Complex Roots of Equations
- Trigonometric Form of Complex Numbers
- De Moivre's Theorem nth Roots of Complex Numbers

Chapter 9: Polynomials

- Algebra of Polynomials
- Polynomial Division (Long and Synthetic Division)
- Factorization Theory (FTA, RT, FT)
- Zeros of Polynomials with Real Coefficients

Chapter 10: Sequences and Series

- Mathematical Induction
- Infinite Sequences
- Summation Notation
- Arithmetic Sequences
- Geometric Sequences
- The Binomial Theorem
- Permutations and Combinations

Book ii. Richard G. Brown and David P. Robbins, *Advanced Mathematics: A Precalculus Course*, 1st ed., (Boston: Houghton Mifflin Co., 1984), ISBN: 0-395-32073-9.

Generalized Content and Topic Order Summary:

1: General and Prerequisite Review

2: Polynomials and Rational Functions

3: Equations and Inequalities

4: Functions: Theory, Operations and Graphs

5: Exponential and Logarithmic Functions

6: Trigonometry (Functional, Graphical, and Analytic)

7: Polar Coordinates and Complex Numbers

8: Analytic Geometry

9: Vectors and Determinants

10: Sequences and Series

11: Statistics*

12: Probability*

13: Introduction to Calculus*

* These topics do not appear in the minimum requirements for calculus but were needed for place holders in the scoring of the continuum

Detailed Chapter Contents:

Chapter 1: Coordinate Geometry
- Points and Lines
- Parallel and Perpendicular Lines
- Finding Equations of Lines
- Complex Numbers
- Quadratic Equations
- Circles and their Equations
- Intersections of Lines and Circles
- Coordinate Geometry Proofs

Chapter 2: Polynomials
- Factoring Polynomials
- Graphing Quadratics and Polynomials
- Polynomial Analytics (Division, FTA, FT, RT)
- Polynomial Applications

Chapter 3: Inequalities
- Linear Inequalities
- Absolute Value
- Polynomial Inequalities
- Applications of Inequalities
- Using the Discriminant

Chapter 4: Functions
- Definitions
- Graphs
- Composition
- Applications
- Inverses

Chapter 5: Exponents and Logarithms
- Integral Exponents
- Rational Exponents
- Exponential Functions
- Logarithmic Functions
- Properties of Logarithms
- Exponential Equations (Changing Bases)
- Exponential Growth
- Natural Logarithm and e

Chapter 6: Trigonometric Functions

- Arcs and Angles
- Sectors
- Unit Circle
- Sine and Cosine Functions
- Evaluating Sines and Cosines
- Other Trigonometric Functions
- Function Relationships
- Trigonometric Equations

Chapter 7: Triangle Trigonometry

- Right Triangle Trigonometry
- Area of a Triangle
- Law of Sines
- Law of Cosines
- Inverse Trigonometric Functions

Chapter 8: Trigonometric Graphs

- Periodic Function Graphing and Stretching
- Periodic Graphic Reflections and Symmetry
- Graph Translation
- Asymptote
- Applications

Chapter 9: Trigonometric Addition Formulas

- cosine($\alpha \pm \beta$) and sine($\alpha \pm \beta$)
- tan($\alpha \pm \beta$)
- Double and Half angle Formulas
- Solving Equations / Applications

Chapter 10: Polar Coordinates and Complex Numbers

- Polar Coordinates and Graphs
- Geometric form of Complex Numbers
- Powers of Complex Numbers
- De Moivre's Theorem
- Roots of Complex Numbers

Chapter 11: Analytic Geometry

- Ellipse
- Hyperbola
- Parabola
- Applications

Chapter 12: Vectors and Determinants

- Geometric and Algebraic Representation of Vectors
- Parametric Equations\
- Parallel and Perpendicular Vectors
- Vectors in three Dimensions and in the Plane
- Determinants
- Applications
- Determinants and Vectors in Three Dimensions

Chapter 13: Sequences and Series

- Arithmetic and Geometric Sequences
- Arithmetic and Geometric Sequences and their Sums
- Limits of Infinite Sequences
- Sums of Infinite Series
- Sigma Notation
- Mathematical Induction

Chapter 14: Statistics
Chapter 15: Probability
Chapter 16: Calculus Introduction

Book iii. Roland E. Larson, Robert P. Hostetler, *Precalculus*, 3rd ed., (Lexington, MA: D.C. Heath & Co., 1993), ISBN: 0-669-33236-4.

Generalized Content and Topic Order Summary:

1: General and Prerequisite Review

2: Functions: Theory, Operations and Graphs

3: Polynomials and Rational Functions

4: Exponential and Logarithmic Functions

5: Trigonometry (Functional, Graphical and Analytic)

6: Vectors without Determinants

7: Complex Numbers

8: Equations and Inequalities

9: Linear Systems and Matrices

10: Sequences and Series

11: Analytic Geometry

12: Polar Coordinates and Complex Numbers

Detailed Chapter Contents:

Chapter 1: Review of Basic Algebra
- Real Number System
- Exponents and Radicals
- Polynomials
- Fractional Expressions
- Solving Equations
- Solving Inequalities
- Remedial Activities

Chapter 2: Functions and Graphs
- The Cartesian Plane
- Graphs of Equations
- Lines in the Plane
- Function
- Graphs of Functions
- Combinations of Functions
- Inverse Functions
- Variation

Chapter 3: Polynomial and Rational Functions
- Quadratic Equations
- Polynomial Functions of Higher Degree
- Polynomial Division (Long and Synthetic)
- Real Zeros of Polynomial Functions
- Complex Numbers
- FTA
- Rational Functions
- Partial Fractions

Chapter 4: Exponential and Logarithmic Functions
- Exponential Functions
- Logarithmic Functions
- Properties of Logarithms
- Solving Exponential and Logarithmic Equations
- Applications

Chapter 5: Trigonometry
- Radian and Degree Measure
- The Trigonometric Functions and the Unit Circle
- The Trigonometric Functions and Right Angles
- Trigonometric Functions of Any Angle
- Graphs of Sine and Cosine Functions
- Graphs of Other Trigonometric Functions

- Other Graphing
- Inverse Trigonometric Functions
- Applications

Chapter 6: Analytic Trigonometry
- Applications of Fundamental Identities
- Verifying Trigonometric Identities
- Solving Trigonometric Equations
- Sum and Difference Formulas
- Multiple-Angle and Product-Sum Formulas

Chapter 7: Additional Topics in Trigonometry
- Law of Sines
- Law of Cosines
- Vectors in the Plane
- Trigonometric Form of a Complex Number
- De Moivre's Theorem

Chapter 8: Systems of Equations and Inequalities
- Systems of Equations
- Systems of Linear Equations in Two Variables
- Linear Systems in More than Two Variables
- Systems of Inequalities
- Linear Programming

Chapter 9: Matrices and Determinants
- Matrices and Systems of Linear Equations
- Operations with Matrices
- The Inverse of a Square Matrix
- The Determinant of a Square Matrix
- Properties of Determinants
- Applications of Determinants and Matrices

Chapter 10: Sequences, Counting Principles and Probability
- Sequences and Summation Notation
- Arithmetic Sequences
- Geometric Sequences
- Mathematical Induction
- The Binomial Theorem
- Counting Principles, Permutations and Combinations
- Probability

Chapter 11: Topics in Analytic Geometry
- Lines
- Parabolas

- Ellipses
- Hyperbolas
- Rotation and Second Degree Equations
- Graphs of Polar Equations
- Polar Equations of Conics
- Plane Curves and Parametric Equations

Book iv. Richard G. Brown, *Advanced Mathematics: Precalculus with Discrete Math and Data Analysis*, 1st ed., (Boston: Houghton Mifflin Co., 1994), ISBN: 0-395-42169-1.

Generalized Content and Topic Order Summary:

1: General and Prerequisite Review

2: Polynomials and Rational Functions

3: Equations and Inequalities

4: Functions: Theory, Operations and Graphs

5: Exponential and Logarithms Functions

6: Analytic Geometry

7: Trigonometry (Functional, Graphical and Analytic)

8: Polar Coordinates and Complex Numbers

9: Vectors and Determinants

10: Sequences and Series with Combinatorics

11: Linear Systems and Matrices

12: Sequences and Series With Combinatorics

13: Probability*

14: Statistics*

15: Curve Fitting and Model*

16: Limits, Series and Iterated Functions*

17: An Introduction to Calculus*

* These topics do not appear in the minimum requirements for calculus but were needed for place holders in the scoring of the continuum

Detailed Chapter Content:

Chapter 1: Linear and Quadratic Functions
- Points and Lines
- Slopes of Lines
- Finding Equations of Lines
- Linear Functions and Models

- The Complex Numbers
- Solving Quadratic Equations
- Quadratic Functions and their Graphs
- Quadratic Models

Chapter 2: Polynomial Functions
- Polynomials
- Synthetic Division; The Remainder Theorem; Factor Theorems
- Graphing Polynomial Functions
- Finding Maximums and Minimums of Polynomial Functions
- Using Technology to Approximate Roots of Polynomial Equations
- Solving Polynomial Equations by Factoring
- General Results for Polynomial Equations

Chapter 3: Inequalities
- Linear Inequalities; Absolute Value
- Polynomial Inequalities in One Variable
- Polynomial Inequalities in Two Variables
- Linear Programming

Chapter 4: Functions
- Functions
- Operations on Functions
- Reflecting Graphs; Symmetry
- Periodic Functions; Stretching and Translating Graphs
- Inverse Functions
- Functions of Two Variables
- Forming Functions from Verbal Descriptions

Chapter 5: Exponents and Logarithms
- Growth and Decay: Integral Exponents
- Growth and Decay: Rational Exponents
- Exponential Functions
- The Number e and the Function ex
- Logarithmic Functions
- Laws of Logarithms
- Exponential Equations; Changing Bases

Chapter 6: Analytic Geometry
- Coordinate Proofs
- Equations of Circles
- Ellipses
- Hyperbolas
- Parabolas
- Systems of Second-Degree Equations

- A New Look at Conic Sections

Chapter 7: Trigonometric Functions
- Measurement of Angles
- Sectors of Circles
- The Sine and Cosine Functions
- Evaluating and Graphing Sine and Cosine
- The Other Trigonometric Functions
- The Inverse Trigonometric Functions

Chapter 8: Trigonometric Equations and Applications
- Simple Trigonometric Equations
- Sine and Cosine Curves
- Modeling Periodic Behavior
- Relationships Among the Functions
- Solving More Difficult Trigonometric Equations

Chapter 9: Triangle Trigonometry
- Solving Right Triangles
- The Area of a Triangle
- The Law of Sines
- The Law of Cosines
- Applications of Trigonometry to Navigation and Surveying

Chapter 10: Trigonometric Addition Formulas
- Formulas for cos $(\alpha \pm \beta)$ and sin $(\alpha \pm \beta)$
- Formulas for tan $(\alpha \pm \beta)$
- Double-Angle and Half-Angle Formulas
- Solving Trigonometric Equations

Chapter 11: Polar Coordinates and Complex Numbers
- Polar Coordinates and Graphs
- Geometric Representation of Complex Numbers
- Powers of Complex Numbers
- Roots of Complex Numbers

Chapter 12: Vectors and Determinants
- Geometric Representation of Vectors
- Algebraic Representation of Vectors
- Vector and Parametric Equations: Motion in a Plane
- Parallel and Perpendicular Vectors; Dot Product
- Vectors in Three Dimensions
- Vectors and Planes
- Determinants
- Applications of Determinants

- Determinants and Vectors in Three Dimensions

Chapter 13: Sequences and Series
- Arithmetic and Geometric Sequences
- Recursive Definitions
- Arithmetic and Geometric Series and Their Sums
- Limits of Infinite Sequences
- Sums of Infinite Series
- Sigma Notation
- Mathematical Induction

Chapter 14: Matrices
- Matrix Addition and Scalar Multiplication
- Matrix Multiplication
- Applying Matrices to Linear Systems
- Communication Matrices
- Transition Matrices
- Transformation Matrices

Chapter 15: Combinatorics
- Venn Diagrams
- The Multiplication, Addition, and Complement Principles
- Permutations and Combinations
- Permutations with Repetition; Circular Permutations
- The Binomial Theorem; Pascal's Triangle

Chapter 16: Probability
- Introduction to Probability
- Probability of Events Occurring Together
- The Binomial Probability Theorem
- Probability Problems Solved with Combinations
- Working with Conditional Probability
- Expected Value

Chapter 17: Statistics
- Tables, Graphs, and Averages
- Box-and-Whisker Plots
- Variability
- The Normal Distribution
- Sampling
- Confidence Intervals for Surveys and Polls

Chapter 18: Curve Fitting and Models
- Introduction to Curve Fitting; The Least-Squares Line
- Fitting Exponential Curves

- Fitting Power Curves
- Choosing the Best Model

Chapter 19: Limits, Series and Iterated Functions
- Limits of Functions
- Graphs of Rational Functions
- Using Technology to Approximate the Area under a Curve
- Power Series
- Analyzing Orbits
- Applications of Iterated Functions

Chapter 20: Introduction to Calculus
- The Slope of a Curve
- Using Derivatives in Curve Sketching
- Extreme Value Problems
- Velocity and Acceleration

Book v. Franklin Demana, Bert K. Waits, Gregory D. Foley & Daniel Kennedy, *Precalculus: Graphical, Numerical, Algebraic*, 5th ed., (Reading, MA: Addison Wesley Longman, Inc., 2001), ISBN: 0-201-69974-5.

Generalized Content and Topic Order Summary:

1: General and Prerequisite Review

2: Functions: Theory, Operations and Graphs

3: Polynomials and Rational Functions

4: Exponential and Logarithms Functions

5: Trigonometry (Functional, Graphical and Analytic)

6: Polar Coordinates and Complex Numbers

7: Linear Systems and Matrices

8: Analytic Geometry

9: Sequences and Series with Combinatorics

10: An Introduction to Calculus: Limits, Derivatives and Integrals*

*These topics do not appear in the minimum requirements for calculus but were needed for place holders in the scoring of the continuum.

Detailed Chapter Content

Chapter P: Prerequisites
- Real Numbers
- Cartesian Coordinate System
- Linear Equations and Inequalities

- Lines in the Plane
- Solving Equations Graphically, Numerically and Algebraically
- Solving Inequalities Algebraically and Graphically

Chapter 1: Functions and Graphs
- Modeling and Equation Solving
- Functions and Their Properties
- Ten Basic Functions
- Building Functions from Functions
- Graphical Transformations
- Modeling with Functions

Chapter 2: Polynomial, Power and Rational Functions
- Linear and Quadratic Functions with Modeling
- Power Functions with Modeling
- Polynomial Functions of Higher Degree with Modeling
- Real Zeros of Polynomial Functions
- Complex Numbers
- Complex Zeros and the Fundamental Theorem of Algebra
- Rational Functions and Equations
- Solving Inequalities in One Variable

Chapter 3: Exponential, Logistic, and Logarithmic Function
- Exponential and Logistic Functions
- Exponential and Logistic Modeling
- Logarithmic Functions and Their Graphs
- Properties of Logarithmic Functions
- Equation Solving and Modeling
- Mathematics of Finance

Chapter 4: Trigonometric Functions
- Angles and their Measures
- Trigonometric Functions of Acute Angles
- Trigonometry Extended: The Circular Functions
- Graphs of Sine and Cosine: Sinusoids
- Graphs of Tangent, Cotangent, Secant, and Cosecant
- Graphs of Composite Trigonometric Functions
- Inverse Trigonometric Functions
- Solving Problems with Trigonometry

Chapter 5: Analytic Trigonometry
- Fundamental Identities
- Proving Trigonometric Identities
- Sum and Difference Identities
- Multiple-Angle Identities

- Law of Sines
- Law of Cosines

Chapter 6: Vectors, Parametric Equations, and Polar Equations
- Vectors in the Plane
- Dot Product of Vectors
- Parametric Equations and Motion
- Polar Coordinates
- Graphs of Polar Equations
- De Moivre's Theorem and nth Roots

Chapter 7: Systems and Matrices
- Solving Systems of Two Equations
- Matrix Algebra
- Multivariate Linear Systems and Row Operations
- Partial Fractions
- Systems of Inequalities in Two Variables

Chapter 8: Analytic Geometry in Two and Three Dimensions
- Conic Sections and Parabolas
- Ellipses
- Hyperbolas
- Translations and Rotations of Axes
- Polar Equations of Conics
- Three Dimensional Cartesian Coordinate System

Chapter 9: Discrete Mathematics
- Basic Combinatorics
- The Binomial Theorem
- Probability
- Sequences and Series
- Mathematical Induction
- Statistics and Data (Graphical)
- Statistics and Data (Algebraic)

Chapter 10: An Introduction to Calculus: Limits, Derivatives, and Integrals
- Limits and Motion: The Tangent Problem
- Limits and Motion: The Area Problem
- More on Limits
- Numerical Derivatives and Integrals

Book vi. James Stewart, Lothar Redlin & Saleem Watson, *Precalculus: Mathematics for Calculus*, 4th ed., (Pacific Grove, CA: Wadsworth Group, 2002), ISBN: 0-534-38541-9.

Generalized Content and Topic Order Summary

1: General and Prerequisite Review

2: Functions: Theory, Operations and Graphs

3: Polynomials and Rational Functions

4: Exponential and Logarithmic Functions

5: Trigonometry (Functional, Graphical and Analytic)

6: Polar Coordinates and Complex Numbers

7: Equations and Inequalities

8: Vectors and Determinants

9: Analytic Geometry

10: Sequences and Series

11: Sequences and Series

12: Probability*

13: Introduction to Calculus*

* These topics do not appear in the minimum requirements for calculus but were needed for place holders in the scoring of the continuum

Detailed Chapter Content

Chapter 1: Fundamentals

- Real Numbers
- Exponents and Radicals
- Algebraic Expressions
- Fractional Expressions
- Equations
- Modeling with Equations
- Inequalities
- Coordinate Geometry
- Solving Equations and Inequalities Graphically
- Lines

Chapter 2: Functions

- What Is a Function?
- Graphs of Functions
- Applied Functions: Variation
- Average Rate of Change: Increasing and Decreasing Functions
- Transformations of Functions
- Extreme Values of Functions
- Modeling with Functions

- Combining Functions
- One-to-One Functions and Their Inverses

Chapter 3: Polynomial and Rational Functions
- Polynomial Functions and Their Graphs
- Dividing Polynomials
- Real Zeros of Polynomials
- Complex Numbers
- Complex Zeros and the Fundamental Theorem of Algebra
- Rational Functions

Chapter 4: Exponential and Logarithmic Functions
- Exponential Functions
- Logarithmic Functions
- Laws of Logarithms
- Exponential and Logarithmic Equations
- Modeling with Exponential and Logarithmic Functions

Chapter 5: Trigonometric Functions of Real Numbers
- The Unit Circle
- Trigonometric Functions of Real Numbers
- Trigonometric Graphs
- More Trigonometric Graphs

Chapter 6: Trigonometric Functions of Angles
- Angle Measure
- Trigonometry of Right Triangles
- Trigonometric Functions of Angles
- The Law of Sines
- The Law of Cosines

Chapter 7: Analytic Trigonometry
- Trigonometric Identities
- Addition and Subtraction Formulas
- Double-Angle, Half-Angle, and Product-Sum Formulas
- Inverse Trigonometric Functions
- Trigonometric Equations
- Trigonometric Form of Complex Numbers: DeMoivre's Theorem
- Vectors
- The Dot Product

Chapter 8: Systems of Equations and Inequalities
- Systems of Equations
- Systems of Linear Equations in Two Variables
- Systems of Linear Equations in Several Variables

- Systems of Linear Equations: Matrices
- The Algebra of Matrices
- Inverses of Matrices and Matrix Equations
- Determinants and Cramer's Rule
- Systems of Inequalities
- Partial Fractions

Chapter 9: Topics in Analytic Geometry
- Parabolas
- Ellipses
- Hyperbolas
- Shifted Conics
- Rotation of Axes
- Polar Coordinates
- Polar Equations of Conics
- Parametric Equations

Chapter 10: Sequence and Series
- Sequences and Summation Notation
- Arithmetic Sequences
- Geometric Sequences
- Annuities and Installment Buying
- Mathematical Induction
- The Binomial Theorem

Chapter 11: Counting and Probability
- Counting Principles
- Permutations and Combinations
- Probability
- Expected Value

Chapter 12: Limits: A Preview of Calculus
- Finding Limits Numerically and Graphically
- Finding Limits Algebraically
- Tangent Lines and Derivatives
- Limits at Infinity; Limits of Sequences
- Areas

Book vii. Roland E. Larson, Robert P. Hostetler, and Bruce H. Edwards, *Precalculus with Limits: A Graphing Approach*, 4th ed., (Boston: Houghton Mifflin Co., 2005), ISBN: 0-618-39480-X.

Generalized Content and Topic Order Summary

1: General and Prerequisite Review

2: Functions: Theory, Operations and Graphs

3: Polynomials and Rational Functions

4: Exponential and Logarithms Functions

5: Trigonometry (Functional, Graphical and Analytic)

6: Linear Systems and Matrices

7: Sequences and Series with Combinatorics

8: Probability*

9: Analytic Geometry

10: Vectors and Determinants

11: Limits and in Introduction to Calculus*

* These topics do not appear in the minimum requirements for calculus but were needed for place holders in the scoring of the continuum

Detailed Chapter Content

Chapter 1: Functions and their Graphs

- Lines in the Plane
- Functions
- Graphs of Functions
- Shifting, Reflecting, and Stretching Graphs
- Combinations of Functions
- Inverse Functions
- Exploring Data: Linear Models and Scatter Plots

Chapter 2: Polynomial and Rational Functions

- Quadratic Functions
- Polynomial Functions of Higher Degree
- Real Zeros of Polynomial Functions
- Complex Numbers
- The Fundamental Theorem of Algebra
- Rational Functions and Asymptotes
- Graphs of Rational Functions
- Exploring Data: Quadratic Models

Chapter 3: Exponential and Logarithmic Functions

- Exponential Functions and their Graphs
- Logarithmic Functions and Their Graphs
- Properties of Logarithms
- Solving Exponential and Logarithmic Equations
- Exponential and Logarithmic Models
- Exploring Data: Nonlinear Models

Chapter 4: Trigonometric Functions

- Radian and Degree Measure
- Trigonometric Functions: The Unit Circle
- Right Triangle Trigonometry
- Trigonometric Functions of Any Angle
- Graphs of Sine and Cosine Functions
- Graphs of Other Trigonometric Functions
- Inverse Trigonometric Functions
- Applications and Models

Chapter 5: Analytic Trigonometry

- Using Fundamental Identities
- Verifying Trigonometric Identities
- Solving Trigonometric Equations
- Sum and Difference Formulas
- Multiple-Angle and Product-to-Sum Formulas

Chapter 6: Additional Topics in Trigonometry

- Law of Sines
- Law of Cosines
- Vectors in the Plane
- Vectors and Dot Products
- Trigonometric Form of a Complex Number

Chapter 7: Linear Systems and Matrices

- Solving Systems of Equations
- Systems of Linear Equations in Two Variables
- Multivariable Linear Systems
- Matrices and Systems of Equations
- Operations with Matrices
- The Inverse of a Square Matrix
- The Determinant of a Square Matrix
- Applications of Matrices and Determinants

Chapter 8: Sequences, Series, and Probability

- Sequences and Series
- Arithmetic Sequences and Partial Sums
- Geometric Sequences and Series
- Mathematical Induction
- The Binomial Theorem
- Counting Principles
- Probability

Chapter 9: Topics in Analytic Geometry

- Introduction to Conics: Parabolas

- Ellipses
- Hyperbolas
- Rotation and Systems of Quadratic Equations
- Parametric Equations
- Polar Coordinates
- Graphs of Polar Equations
- Polar Equations of Conics

Chapter 10: Analytic Geometry in Three Dimensions

- The Three-Dimensional Coordinate System
- Vectors in Space
- The Cross Product of Two Vectors
- Lines and Planes in Space

Chapter 11: Limits and an Introduction to Calculus

- Introduction to Limits
- Techniques for Evaluating Limits
- The Tangent Line Problem
- Limits at Infinity and Limits of Sequences
- The Area Problem

Book viii. i. Paul A. Foerster, *Precalculus with Trigonometry: Concept and Applications*, 1st ed., (Emeryville, CA: Key Curriculum Press, 2012), ISBN: 978-1-60440-044-1.

Generalized Content and Topic Order Summary

1: General and Prerequisite Review

2: Functions: Theory, Operations and Graphs

3: Functions: Theory, Operations and Graphs

4: Exponential and Logarithms Functions

5: Statistics*

6: Polynomials and Rational Functions

7: Trigonometry (Functional, Graphical and Analytic)

8: Analytic Geometry /6

9: Polar Coordinates and Complex Numbers /7

10: Vectors and Determinants /8

11: Linear Systems and Matrices /9

12: Probability* /10

13: Sequences and Series /11

14: Introduction to Limits, Derivatives and Integrals* /12

* These topics do not appear in the minimum requirements for calculus but were needed for place holders in the scoring of the continuum

Detailed Chapter Contents

Chapter 1: Functions and Mathematical Models

- Functions: Graphically, Algebraically, Numerically, and Verbally
- Types of Functions
- Dilation and Translation of Function Graphs
- Composition of Functions
- Inverse Functions and Parametric Equations
- Reflections, Absolute Values, and Other Transformations
- Precalculus Journal

Chapter 2: Properties of Elementary Functions

- Shapes of Function Graphs
- Identifying Functions from Graphical Patterns
- Identifying Functions from Numerical Patterns
- Properties of Logarithms
- Logarithms: Equations and Other Bases
- Logarithmic Functions
- Logistic Functions for Restrained Growth

Chapter 3: Fitting Functions to Data

- Introduction to Regression for Linear Data
- Deviations, Residuals, and the Correlation Coefficient
- Regression for Nonlinear Data
- Linearizing Data and Logarithmic Graph Paper
- Residual Plots and Mathematical Models

Chapter 4: Polynomial and Rational Functions

- Introduction to Polynomial and Rational Functions
- Quadratic Functions, Factoring, and Complex Numbers
- Graphs and Zeros of Higher-Degree Functions
- Fitting Polynomial Functions to Data
- Rational Functions: Asymptotes and Discontinuities
- Partial Fractions and Operations with Rational Expressions
- Fractional Equations and Extraneous Solutions

Chapter 5: Periodic Functions and Right Triangle Problems

- Introduction to Periodic Functions
- Measurement of Rotation
- Sine and Cosine Functions
- Values of the Six Trigonometric Functions
- Inverse Trigonometric Functions and Triangle Problems

Chapter 6: Applications of Trigonometric and Circular Functions

- Sinusoids: Amplitude, Period, and Cycles
- General Sinusoidal Graphs
- Graphs of Tangent, Cotangent, Secant, and Cosecant Functions
- Radian Measure of Angles
- Circular Functions
- Inverse Circular Relations: Give y, Find x
- Sinusoidal Functions as Mathematical Models
- Rotary Motion

Chapter 7: Trigonometric Function Properties and Identities, and Parametric Equations

- Introduction to the Pythagorean Property
- Pythagorean, Reciprocal, and Quotient Properties
- Identities and Algebraic Transformation of Expressions
- Arcsine, Arctangent, Arccosine, and Trigonometric Equations
- Parametric Functions
- Inverse Trigonometric Relation Graphs

Chapter 8: Properties of Combined Sinusoids

- Introduction to Combinations of Sinusoids
- Composite Argument and Linear Combination Properties
- Other Composite Argument Properties
- Composition of Ordinates and Harmonic Analysis
- The Sum and Product Properties
- Double and Half Argument Properties

Chapter 9: Triangle Trigonometry

- Introduction to Oblique Triangles
- Oblique Triangles: The Law of Cosines
- Area of a Triangle
- Oblique Triangles: The Law of Sines
- The Ambiguous Case
- Vector Addition
- Real-World Triangle Problems

Chapter 10: Conic Sections and Quadric Surfaces

- Quadratic Relations and Conic Sections
- Cartesian Equations for Conic Sections
- Parametric Equations for Conic Sections
- Quadric Surfaces and Inscribed Figures
- Analytic Geometry of the Conic Sections
- Parametric and Cartesian Equations for Rotated Conics
- Applications of Conic Sections

Chapter 11: Polar Coordinates, Complex Numbers, and Moving Objects

- Introduction to Polar Coordinates
- Polar Equations for Conics and Other Curves
- Intersections of Polar Curves
- Complex Numbers in Polar Form
- Parametric Equations for Moving Objects

Chapter 12: Three-Dimensional Vectors
- Review of Two-Dimensional Vectors
- Two-Dimensional Vector Practice
- Vectors in Space
- Scalar Products and Projections of Vectors
- Planes in Space
- Vector Product of Two Vectors
- Direction Angles and Direction Cosines
- Vector Equations for Lines in Space

Chapter 13: Matrix Transformations and Fractal Figures
- Introduction to Iterated Transformations
- Matrix Operations and Solutions of Linear Systems
- Rotation and Dilation Matrices
- Translation with Rotation and Dilation Matrices
- Strange Attractors for Several Iterated Transformations
- Fractal Dimensions

Chapter 14: Probability, and Functions of a Random Variable
- Introduction to Probability
- Words Associated with Probability
- Two Counting Principles
- Probabilities of Various Permutations
- Probabilities of Various Combinations
- Properties of Probability
- Functions of a Random Variable
- Mathematical Expectation

Chapter 15: Sequences and Series
- Introduction to Sequences and Series
- Arithmetic, Geometric, and Other Sequences
- Series and Partial Sums

Chapter 16: Introduction to Limits, Derivatives and Integrals
- Exploring Limits, Derivatives, and Integrals
- Limits
- Rate of Change of a Function: The Derivative
- Accumulated Rates: The Definite Integral

Appendix F. Ethics Approval

THE UNIVERSITY of ADELAIDE

RESEARCH BRANCH
OFFICE OF RESEARCH ETHICS, COMPLIANCE AND INTEGRITY

BEVERLEY DOBBS
EXECUTIVE OFFICER
LOW RISK HUMAN RESEARCH ETHICS REVIEW GROUPS
THE UNIVERSITY OF ADELAIDE
SA 5005 AUSTRALIA

TELEPHONE +61 8 8313 4725
FACSIMILE +61 8 8313 7325
email: beverley.dobbs@adelaide.edu.au
CRICOS Provider Number 00123M

7/11/2013

Dr I Darmawan
School: Education

Dear Dr Darmawan

ETHICS APPROVAL No: **HP-2013-099**
PROJECT TITLE: **Mathematical rigor in the Year 12 curriculum: A focus on the textbook as a tool for mathematical success**

The ethics application for the above project has been reviewed by the Low Risk Human Research Ethics Review Group (Faculty of Humanities and Social Sciences and the Faculty of the Professions) and is deemed to meet the requirements of the *National Statement on Ethical Conduct in Human Research (2007)* involving no more than low risk for research participants. You are authorised to commence your research on **7 November 2013**.

Ethics approval is granted for three years and is subject to satisfactory annual reporting. The form titled *Project Status Report* is to be used when reporting annual progress and project completion and can be downloaded at http://www.adelaide.edu.au/ethics/human/guidelines/reporting. Prior to expiry, ethics approval may be extended for a further period.

Participants in the study are to be given a copy of the Information Sheet and the signed Consent Form to retain. It is also a condition of approval that you **immediately report** anything which might warrant review of ethical approval including:

- serious or unexpected adverse effects on participants,
- previously unforeseen events which might affect continued ethical acceptability of the project,
- proposed changes to the protocol; and
- the project is discontinued before the expected date of completion.

Please refer to the following ethics approval document for any additional conditions that may apply to this project.

Yours sincerely

ASSOCIATE PROFESSOR RACHEL A. ANKENY
Co-Convenor
Low Risk Human Research Ethics Review Group (Faculty of Humanities and Social Sciences and Faculty of the Professions)

ASSOCIATE PROFESSOR PAUL BABIE
Co-Convenor
Low Risk Human Research Ethics Review Group (Faculty of Humanities and Social Sciences and Faculty of the Professions)

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