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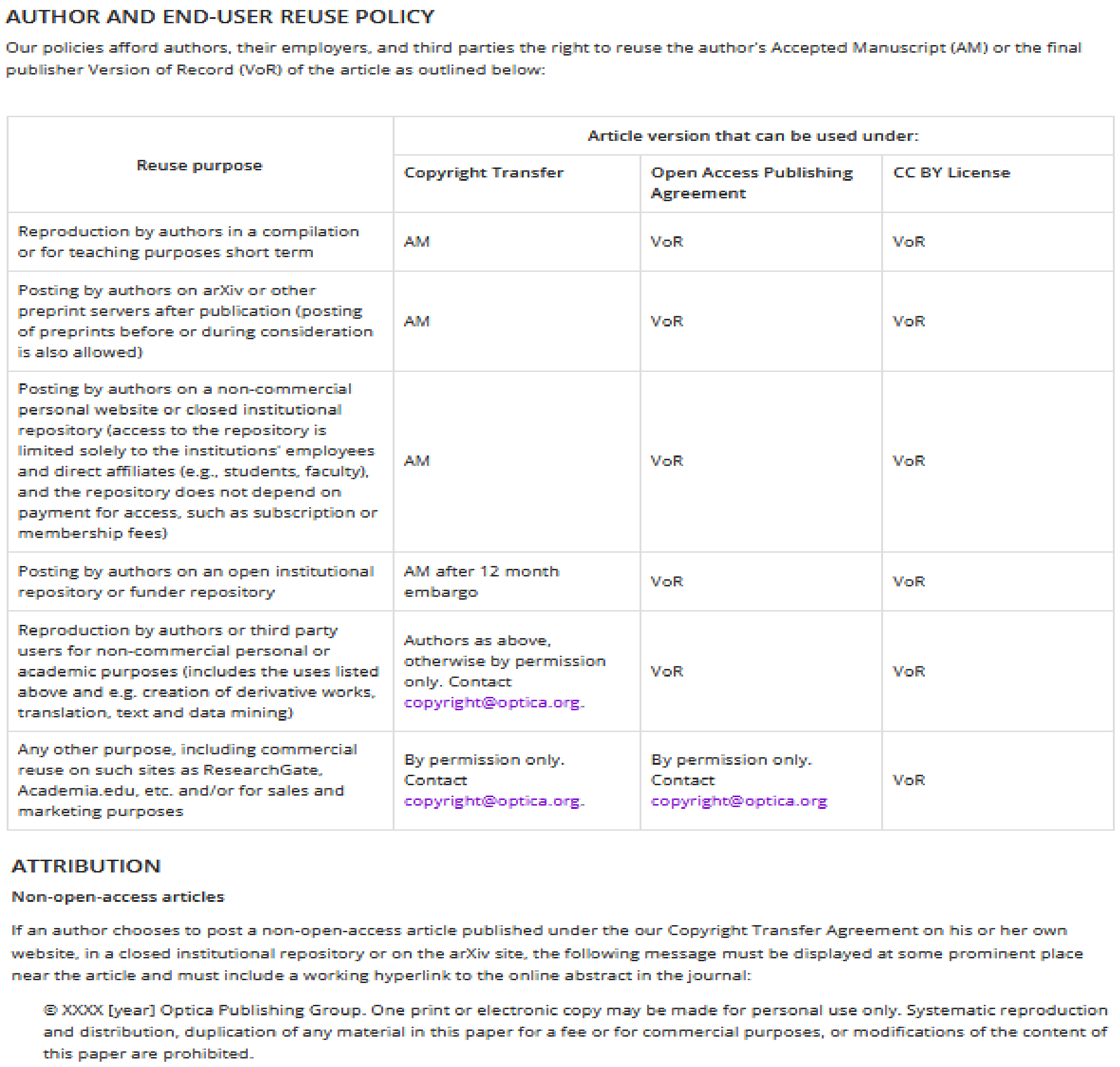

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Orthogonalizing the control of frequency combs for optical clockworks

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Frequency combs play a crucial supporting role for optical clocks by allowing coherent frequency division of their output signals into the electronic domain. This task requires the stabilization of the comb's offset frequency and of an optical comb mode to the clock laser. However, the two actuators used to control these quantities often influence both degrees of freedom simultaneously. This non-orthogonality leads to artificial limits to the control bandwidth and unwanted noise in the comb. Here, we orthogonalize the two feedback loops with a linear combination of the measured signals in an FPGA. We demonstrate this idea using a fiber frequency comb stabilized to a clock laser at 259 THz, half the frequency of the $^1S_0 \rightarrow {}^3P_0$ Yb transition. The decrease in coupling between the loops reduces the comb's optical phase noise by 20 dB. This approach could potentially improve the performance of any comb stabilized to any optical frequency standard.

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Optical clocks offer the possibility to measure and define time with a precision that surpasses that of microwave clocks by many orders of magnitude [1–3]. The operation of these clocks relies on three fundamental elements: a long-lived oscillatory phenomenon (an atomic transition), a source of excitation (a laser), and a way to count the cycles of oscillation. In traditional clocks, the oscillation is slow enough that cycles can be counted using an electronic circuit. However, optical clocks must first undergo some form of frequency division before any electronic device can be used to count the cycles. This division is achieved using a frequency comb, whose optical pulse train acts as the clockwork to bridge the gap between the optical and microwave domains in a single step [4–6]. This can be understood from the relation that defines the frequencies of each comb mode:

$$\nu_n = n f_r + f_0, \quad (1)$$

where ν_n is an optical frequency, n is an integer of order 10^6, f_r is the repetition rate of the pulse train, and f_0 is the offset frequency. Full stabilization of the comb can thus be achieved by controlling f_0 as well as by stabilizing any optical mode ν_n to the clock laser. Examination of Eq. 1 shows that f_r is now a highly divided version of the optical frequency: $(\nu_n - f_0)/n$.

Due to the reliability and maturity of telecommunications technology, we find that most existing frequency combs are based on erbium-doped fiber lasers emitting around 192 THz (1560 nm). Unfortunately, this spectral range is a poor match for many clock-worthy optical atomic transitions that lie in the visible or UV bands of the electromagnetic spectrum, e.g. Yb at 518 THz, Sr^+ at 445 THz, and Al^+ at 1121 THz [2]. Even with the addition of harmonic generation, the usual outcome is that the comb will be stabilized via a mode ν_n that is located far from the center of its emission spectrum. Although viable, this situation is problematic as it is difficult to find a pair of actuators that provides orthogonal control over f_0 and ν_n, unless ν_n falls near the comb's center. This introduces cross-coupling between the two required feedback loops and increases the amount of residual phase noise in the stabilized comb.

There have only been a few studies on the cross-coupling of feedback loops in frequency combs. It has previously been noted that a strong anti-correlation existed between f_0 and f_r for certain types of noise or actuation signals, leading to a set of optical modes near the comb's center with little sensitivity to these effects [7, 8]. The orthogonalization of two feedback loops has also been demonstrated for the cases of a diode laser [9] and of a solid-state laser [10], which were both stabilized via f_0 and f_r. Importantly, we note that these two demonstrations focused on the orthogonal control of the comb's microwave parameters and are thus not compatible with the case of optical-to-microwave frequency division, where one must control f_0 and ν_n to derive a stable signal at f_r. To circumvent the orthogonality issue, one can instead use the transfer method [11] to subtract f_0 from the signal of interest and work with one degree of freedom.

In this paper, we demonstrate, for the first time, orthogonal control of a frequency comb locked to an optical frequency reference, thus enabling its use as a frequency divider for an optical atomic clock. To achieve this, we digitally orthogonalize the feedback loops stabilizing the offset frequency f_0 and an optical mode $\nu_n \approx 259$ THz relative to the output of a clock laser, whose second harmonic at 518 THz can interrogate the $^1S_0 \rightarrow {}^3P_0$ transition in neutral ytterbium [3]. Here, the frequency of 259 THz is far from the emission center of this erbium-fiber comb and is reached via supercontinuum generation in a nonlinear fiber. The orthogonalization calculations and a 2-channel phase-locked-loop controller are all implemented on a field-programmable gate array (FPGA) that controls the pump diode's power and a piezoelectric fiber stretcher (PZT), which allows tuning the system in real-time without the need to modify an analog circuit. There are two important advantages to orthogonal control: First, the proportional-integral-derivative (PID) gains of the controller can be significantly increased before the system becomes unstable, which translates into less residual noise at the lock point near 259 THz. Second, as shown later, it prevents the generation of unnecessary noise on the comb modes around 192 THz, which is the useful band for optical communications and time transfer [12]. We show these benefits by measuring transfer functions and the phase noise for both the standard and orthogonal cases.

The frequency comb used in this demonstration is a home-built mode-locked erbium fiber laser with $f_r = 180$ MHz emitting light around 192 THz [13]. A nonlinear stage increases its spectral range to cover 145-310 THz and to detect f_0 with the $1f - 2f$ scheme. The actuators available for its control include the power of the pump diode laser, a fast, short-range PZT, and a slow, long-range PZT. Here, we only consider the first two actuators since the slow PZT is solely used to keep the fast one within range. This all-fiber comb design is commonly used for optical clockworks, which is why our work focuses on this particular configuration for the first time. Figure 1a) depicts a frequency comb in the spectral domain and illustrates the impact of each actuator on the frequency of the comb modes. Each actuator exhibits a fixed point in the spectrum where it has no effect and around which the comb expands or contracts [14, 15].

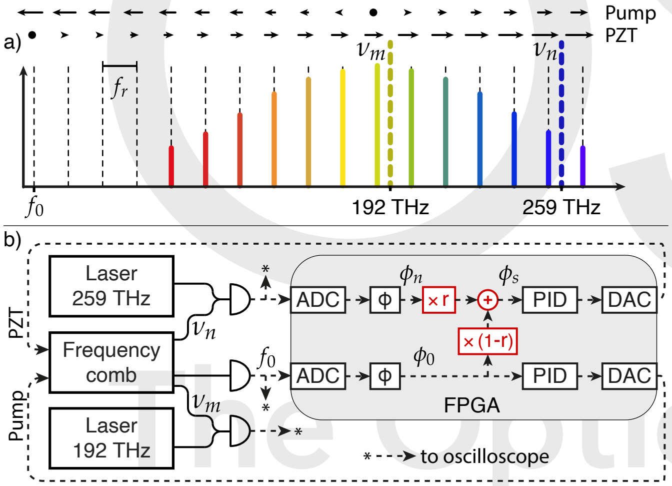

Fig. 1. a) Spectral representation of a frequency comb illustrating the effect of the pump and PZT actuators around their fixed points. b) Schematic of the control system. An FPGA is used to stabilize the offset frequency f_0 of a fiber frequency comb and its optical mode ν_n relative to a laser at 259 THz. The blocks in red are used to orthogonalize the system. The in-loop signals along with another optical mode ν_m measured relative to a laser at 192 THz are also monitored separately.

To obtain two orthogonal control channels, each actuator should control a mode that coincides with the fixed point of the other actuator. For example, in an erbium fiber laser, the PZT and the pump power have fixed points near DC and near the comb's center (the carrier frequency), respectively [14]. Using the pump power to control f_0 and the PZT to control the frequency of an optical mode $\nu_m \approx 192$ THz (1560 nm) would thus result in two near-orthogonal feedback loops. However, in most optical clockworks, it is not possible to use modes near the comb's emission center for its stabilization since this is determined by the frequency of the atomic clock. In the example presented here, we are required to control f_0 and a mode at $\nu_n \approx 259$ THz (1157 nm), which corresponds to half the frequency of the clock transition in neutral ytterbium. This transition can be interrogated using the second harmonic of a 259 THz clock laser (Toptica DL pro) that serves as a reference to stabilize the comb mode ν_n.

A schematic of the control system is given in Fig. 1b). The detected offset frequency f_0 and optical mode ν_n are sent to an FPGA board with fast analog inputs and outputs (Red Pitaya, STEMlab 125-14) that is augmented with a daughterboard with PZT drivers (Octosig Consulting). The FPGA is programmed to operate as a 2-channel phase-locked-loop controller as described in [16]. Each signal is digitized, its instantaneous phase is extracted via complex demodulation (denoted by the ϕ block in the figure), and sent to a PID filter, before being fed back to the comb. As explained earlier, these two feedback loops are coupled due to the pump power affecting both f_0 and ν_n, as shown in Fig. 1a). In order to counteract this coupling, we modify the FPGA firmware in [16] to include an orthogonalization step.

In detail, we form a linear combination of the instantaneous phase of the comb mode that we wish to stabilize, $\phi_n(t)$, and the instantaneous phase of the offset frequency, $\phi_0(t)$, to synthesize the phase $\phi_s(t)$ of a comb mode whose frequency ν_s has no dependence on the pump power (it is a fixed point for the pump actuator). From Eq. 1, this synthetic phase can be described as:

$$\phi_s(t) = r\phi_n(t) + (1-r)\phi_0(t), \tag{2}$$

where $r = s/n \approx \nu_s/\nu_n$. This operation is depicted by the blocks in red in Fig. 1b) and the result is fed to the PID. The challenge is thus to determine a value for r that results in maximal orthogonality. The same approach could be used to modify the feedback loop for f_0, although this is not necessary here as the fixed point of the PZT actuator is already near DC, in close coincidence with f_0. However, combs controlled via electro-optic modulators, which have inconveniently located fixed points [13], would benefit from this additional step. The method presented here uses the fixed-point formalism to determine the optimal combination of the measured signals, but we could equally use the well-known matrix diagonalization approach [9, 10] to describe the relation between the actuators and the measured signals. Our new approach is physically more intuitive since one only needs to determine the particular wavelength for which the control usually provides the best performance.

We quantify the effect of each actuator on f_0 and ν_s for different values of r by measuring all four possible transfer functions $h_{ij}(f)$ using the network analyzer feature built in the digital controller [16], where $i = 1,2$ and $j = 1,2$ are the inputs and outputs of the device under test. Here, the $i = 1,2$ inputs correspond to the pump and PZT control signals, while the $j = 1,2$ outputs correspond to the f_0 and ν_s measurements, respectively. We define the normalized transfer functions:

$$H_{ij}(f) = \frac{h_{ij}(f)}{\sqrt{|h_{ii}(0)||h_{jj}(0)|}}, \tag{3}$$

where $|h_{11}(0)| = 4.1$ GHz/V and $|h_{22}(0)| = 5.2$ MHz/V as measured from the analog outputs of the Red Pitaya board.

Figure 2 shows these normalized transfer functions prior to the orthogonalization process, i.e. with $r = 1$ (solid lines). On the left-hand side, the response of the offset frequency f_0 to the pump actuator (blue) shows a smooth transfer function with a cutoff at 5-10 kHz, typical of erbium fiber lasers [14]. In turn, the response to PZT actuation (red) is a factor of 20 smaller, as is expected from the fact that f_0 is near its fixed point. The residual effect of the PZT on f_0 is partly due to electrical crosstalk and to unwanted losses induced in the laser cavity when the PZT is excited at its resonant frequency (∼650 kHz) [7].

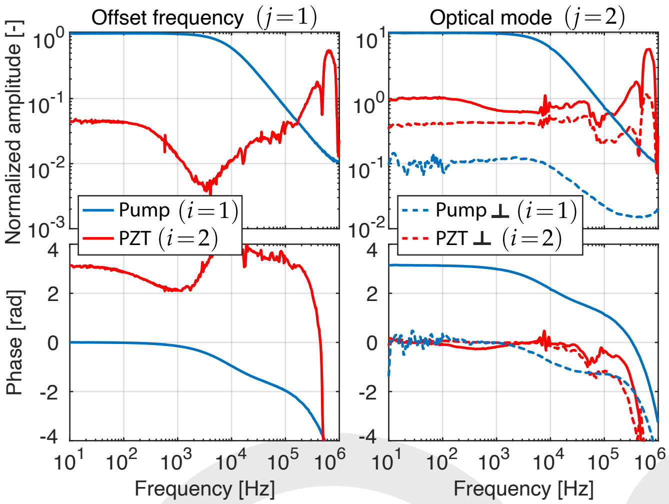

Fig. 2. Normalized transfer functions $H_{ij}(f)$ showing the influence of the pump and PZT actuators on the offset frequency f_0 (left) and the optical mode ν_s (right) with $r = 1$. The orthogonal (⊥) case is also shown for the optical mode with $r = 0.73$.

The right hand side of Fig. 2 shows the response of an optical mode ν_n near 259 THz (equal to ν_s with $r = 1$) to both the PZT actuator (solid red) and the pump actuator (solid blue). The PZT is found to have a transfer function that is relatively flat up to tens of kHz, but deviations in the frequency response and a strong mechanical resonance at ∼650 kHz limit the use of this actuator beyond ∼ 50 kHz. We also observe that the effect of the pump is 10 times larger than that of the PZT, while being out of phase by π rad. Direct control of this optical mode with feedback to the PZT would cause instability and add noise.

To remedy this problem, we orthogonalize this feedback loop by adjusting the parameter r so as to minimize the influence of the pump actuator on the newly synthesized optical mode. We find experimentally that $r = 0.73$ is ideal, which indicates that the pump fixed point is at $r\,\nu_n = 189$ THz, close to the center of the comb. This leads to new transfer functions for the optical mode with the orthogonalization in place ($r = 0.73$), as shown in Fig. 2 (dashed lines). The curve corresponding to the pump actuator (dashed blue) was reduced 100 fold and now sits below the PZT curve (dashed red). The latter also became flatter due to decreased competition between the actuators and its amplitude was slightly reduced by a factor $\sim r$, as expected from Eq. 2. The rejection of the pump's effect is currently limited by the resolution with which the parameter r is defined in the FPGA.

We now stabilize the frequency comb and compare the non-orthogonal and orthogonal operating conditions. We evaluate the performance by measuring the power spectral density (PSD) of in-loop phase fluctuations on the offset frequency f_0, and on the optical mode $\nu_n \approx 259$ THz by comparison to the the clock laser. Figure 3a) shows the standard non-orthogonal control case ($r = 1$). First, the free-running behaviour of f_0 (light blue) and ν_n (light red) is given by the curves that run off the top of the plot. Without any control, the phase fluctuations on f_0 are dominated by spontaneous emission in the laser cavity and potentially by pump intensity noise, which both have fixed points near the comb's center [14]. The large distance between f_0 and the comb's center means that f_0 is heavily impacted by these effects (see Fig. 1a)). The same argument applies to noise on the optical mode $\nu_n \approx 259$ THz, since its distance from the comb's center is around 35% of that of f_0. Indeed, their free-running phase noise PSDs have a similar shape and differ by this expected factor 0.35^2. Once stabilized, the noise on f_0 (blue) is found to be effectively suppressed by the feedback loop up to 100 kHz thanks to the derivative gain. The stabilized optical mode ν_n (red) displays noise at a level that is comparable to that on f_0, except that it also includes excursions that can be attributed to the non-ideality of the PZT transfer function. We note that stabilizing f_0 only still results in a 10 to 20 dB reduction of the free-running noise on ν_n between 10-100 kHz (not shown in the figure) compared to the completely unstabilized case [14].

We can use the measured data along with Eq. 2 to synthesize the phase fluctuations of the mode $\nu_s \approx 189$ THz, which is at the fixed point. The corresponding PSD is also given in Fig. 3a) (purple). We notice that its level is comparable to that of f_0 and ν_n, although one might have expected this 189 THz mode to be significantly quieter since this is where pump noise and spontaneous emission should have the smallest effect. This points to one of the challenges of controlling the comb via a mode that is far from its center: minimizing the noise at 259 THz results in the PZT adding unwanted noise throughout the rest of the spectrum. Even in the hypothetical situation where the noise at 259 THz was perfectly cancelled, the noise at 189 THz would be lifted to a level comparable to that on f_0 to enable total cancellation at 259 THz. In other words, setting $\phi_n(t) = 0$ in Eq. 2 would lead to $\phi_s(t) = (1 - r)\phi_0(t)$.

In order to verify our predicted phase noise for modes near the centre of the spectrum, we employ another laser (NKT, Koheras Boostik E15) at 192 THz. Its frequency is independently locked to the 259 THz laser through a two-color fiber interferometer, as described in [17]. The measured noise of a mode $\nu_m \approx 192$ THz from the stabilized comb relative to this auxiliary laser is also shown in Fig. 3a) (green) and is in near-perfect agreement with the synthesized noise at $\nu_s \approx 189$ THz for Fourier frequencies above 10^3 Hz. However, the two curves disagree below this point because the two-color interferometer technique that was used to lock one laser to the other has a noise floor at $\sim f^{-2}$ rad^2/Hz (dashed black line in the figure) [17].

Figure 3b) shows the phase noise PSDs obtained when the system is made orthogonal with $r = 0.73$. The two feedback loops now experience far less competition, which allows us to increase the PID gains for the PZT actuator by 20 dB before reaching instability. This results in a reduced amount of noise at $\nu_n \approx 259$ THz (red), while the noise on f_0 (blue) is similar to that in the non-orthogonal case. We also note that the noise at ν_n is now free of PZT resonances and displays the same behaviour as f_0 with a $\sim 0.35^2$ scaling factor, as was the case while the comb was free-running. This means that its performance is now entirely limited by the residual noise on f_0. We emphasize that the aim here is not to minimize the noise on a singular mode at 259 THz, but rather to minimize the overall noise across the comb to improve the performance of the optical frequency division. In this instance, the phase noise on f_r would also be limited by f_0 and a prediction for its PSD can be obtained by scaling the PSD for f_0 down by $n^2 \approx 10^{12}$. However, performing this measurement with sufficient sensitivity is challenging because

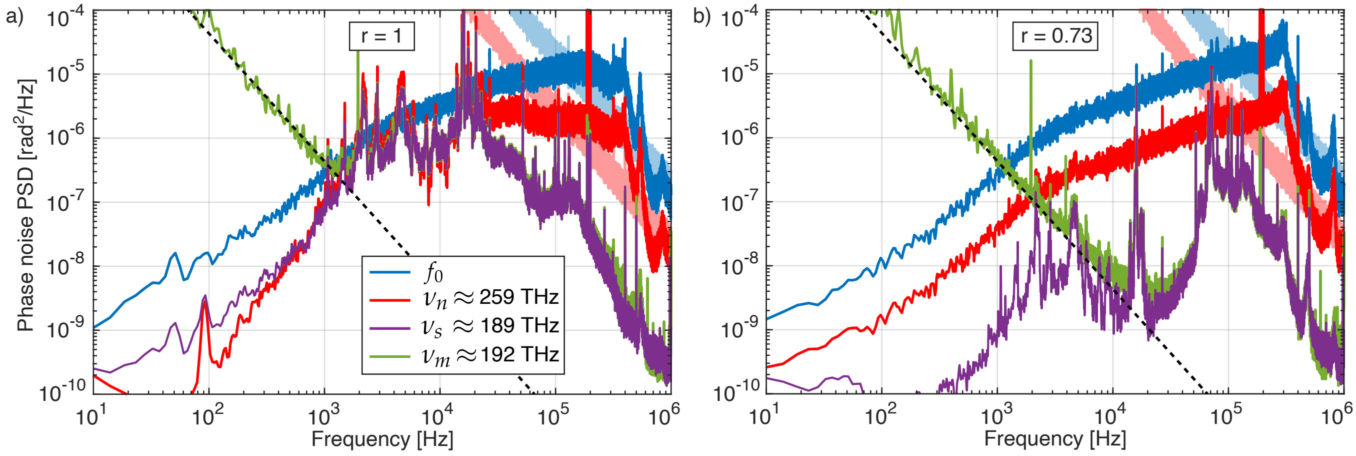

Fig. 3. In-loop phase noise PSDs a) without orthogonalization ($r = 1$) and b) with orthogonalization ($r = 0.73$) for the offset frequency f_0, the mode ν_n relative to a 259 THz laser, the synthetic mode ν_s at 189 THz, and the measured mode ν_m relative to a 192 THz laser. Free-running PSDs are also given for f_0 and ν_n in light colors.

of detection noise sources and using a high-harmonic of f_r facilitates this task. For 10 mW of detected power at 192 THz, we estimate the contribution of shot noise on f_r at $\sim 10^{-17}$ rad^2/Hz.

Another advantage of the orthogonal approach is that it prevents feeding unnecessary noise at the fixed point by allowing the PID to control the synthetic mode $\nu_s \approx 189$ THz instead, which minimizes noise in this spectral band. The noise displayed by ν_s in Fig. 3b) (purple) is reduced by 2 orders of magnitude within the $\sim$50 kHz control bandwidth when compared to the non-orthogonal case. It is also well below the noise displayed by ν_n in Fig. 3a) (red), even when scaled by $r = 0.73$ for direct comparison with ν_s, which further highlights the benefit of the orthogonalization. The data measured at $\nu_m \approx 192$ THz (green) with the auxiliary laser is again in good agreement with the synthetic data, although the noise floor (dashed line) arising from its stabilization [17] is more apparent here. Minimizing phase noise in the 192 THz band is highly important for optical time transfer, clock comparisons, and telecommunications applications that often make use of modes in this spectral range [12].

In conclusion, we demonstrated the orthogonal control of an erbium-doped fiber frequency comb locked to an optical frequency reference. We demonstrated the technique by stabilizing the comb against a laser at 259 THz chosen to interrogate the clock transition in ytterbium. This region of the spectrum is far from the fixed points of the comb's actuators, which normally results in strongly coupled feedback loops. To neutralize this unwanted effect, we digitally synthesized a comb mode coinciding with an actuator fixed point and fed this signal to the PID. We finally stabilized the comb without and with orthogonalization and were able to show that the residual phase noise for comb modes at both 189 THz and 259 THz was significantly lower in the latter case. Our method can also be used in a different context to actively suppress the f_0 contribution from any chosen mode within the comb by trading off the level of orthogonality. With the orthogonalized feedback loops implemented digitally, the only hardware requirement here is a cheap digital electronics board. We believe that this approach can be widely deployed to improve the performance of comb-based optical clockworks.

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Disclosures. The authors declare no conflicts of interest.

Data availability. Data underlying the results presented in this paper may be obtained from the authors upon reasonable request.

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