PERTURBATION THEORY IN QUANTUM ELECTRODYNAMICS

by

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CONTENTS

SUMMARY

STATEMENT

ACKNOWLEDGEMENTS

		Page
CHAPTER 1	The Free Electromagnetic Field	1
CHAPTER 2	The Interacting Electromagnetic Field	20
CHAPTER 3	Application of the Fermi Method to the Interacting Electromagnetic Field	33
CHAPTER 4	Evaluation of S-matrix Elements in the Fermi Method	43
CHAPTER 5	The Self-Energy of the Electron	51
CHAPTER 6	Conclusions	62
REFERENCES		64

SUMMARY

It is possible to give an algebraic description of the quantized free electromagnetic field in which all quantization schemes are regarded as methods of obtaining representations of a quotient algebra of a certain subalgebra of the field algebra of the free electromagnetic field. In particular, the Fermi method of quantization, which was used for many early calculations, can be reformulated as such a representation, called the Fermi representation. In its original form, the Fermi method involved normalization difficulties which led to ambiguities in the calculations. The re-interpretation eliminates these ambiguities.

The Fermi representation is obtained by writing the Fock representation of the field algebra as a direct integral over the spectrum of the supplementary condition operators, and choosing a certain component representation from this decomposition. The component representations are representations of the commutator algebra of the supplementary condition operators, in which the supplementary condition operators act as scalars. They therefore determine a representation of a quotient algebra of the commutator algebra with respect to an ideal generated by the supplementary condition operators. This quotient algebra describes the physical degrees of freedom of the free electromagnetic field.

So far, our description has applied to the free field. The aim of this thesis is to show that the Fermi method can be used to derive methods for perturbation calculations in quantum electrodynamics, and to justify the Feynman rules for S-matrix calculations. Of course there remain the usual problems of interpretation of perturbation methods, since they involve representations of the free field in the infinite past and future, and inequivalent representations at other times. These difficulties are present not only in the Fermi method but in any perturbative study of quantum electrodynamics. Therefore the Fermi method will prove to be a

suitable alternative to more standard methods of quantization, such as the Gupta-Bleuler method.

Certain early papers which used the Fermi method for perturbation calculations, and to derive the S-matrix rules, point out the normalization difficulty which led to the rejection of this method in its original form. In one sense all we need to do in order to justify perturbation calculations on the basis of the Fermi method is to demonstrate that the re-interpretation eliminates this difficulty, and renders valid the treatments given in these early papers.

STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma, and to the best of my knowledge and belief, contains no material previously published or written by another person except where due reference is made in the text.

Jill D. Wright

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