



# **Uniform-Electric-Field-Approximation Based Modelling of Longitudinal Piezoelectric Transducers**

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## Abstract

Longitudinal piezoelectric transducers (LPT), which collectively refer to piezoelectric actuators, vibrators, sensors and actuators designed for longitudinal deformations or vibrations, are the most widely used piezoelectric devices. LPT model, which can be used to predict the behavior or performance in time/frequency domain, plays a vital role in the design and optimization of these LPT-based applications. Existing models which can be used for dynamic behavior prediction, are based on the complex electro-mechanical coupled fundamentals of piezoelectricity, which involves a complex position-varying electric field. Therefore, solving these models for the design and optimization of LPT-based applications is very computationally inefficient.

After initial extensive investigations of possible effective simplifications in the complex fundamentals for modeling LPT, it is found that the electric field in LPT could be effectively approximated to be uniform (i.e. electric field is independent of its position) and this approximation could greatly simplify and facilitate the modeling of LPT-based applications. Therefore, the aim of this research is to study the uniform-electric-field approximation in simplifying the analysis, modelling and calculations of LPT for facilitating design and optimization of the LPT-based applications. LPT can, in principle, be divided into d31-mode LPT and d33-mode LPT. Both types are investigated in this thesis work.

The main contributions of this thesis work are presented in 6 chapters, with each based on an individual scientific paper.

*Paper 1* presents the rationale behind the uniform-electric-field-approximation for d33-mode LPT together with its scope and limitation. Then, based on the approximation, novel simplified fundamentals of both simple-layer-type and stack-type d33-mode LPT are formulated, which could provide a very simple analytical solution, especially for the stack-type.

To facilitate the modeling of free and loaded vibration of d33-mode LPT in a more straightforward way, a simple equivalent circuit is presented in *Paper 2*. The presented circuit is inspired by the network theory and formulated exactly based on the simplified fundamentals of d33-mode LPT presented in *Paper 1*.

In many LPT-based applications, LPT are joined with other layers, such as backing layers and propagating layers. For the calculations and analysis of a multilayer structure, a transfer matrix method is always used. Therefore, to further facilitate the calculation when LPT are joined with other layers, the simplified fundamentals of LPT in *Paper 1* is wrapped into a transfer matrix form as detailed in *Paper 3*.

When LPT are used in a complex structure, a finite element model is widely applied for computation and analysis. Based on the uniform-electric-field-approximation, two simple equivalent finite element models of LPT are presented in *Paper 4*, which can largely simplify the modeling process and reduce the computational efforts of direct finite element modeling of LPT.

Then, *Paper 5* presents the rationale behind the uniform-electric-field-approximation for d31-mode LPT, which is different in nature to those of d33-mode. Also, an equivalent mixing method is proposed to consider electrode and adhesive layers within d31-mode LPT. The related equivalent circuit and transfer matrix of d31-mode LPT are formulated.

Inspired by d33-mode, *Paper 6* presents simple equivalent finite element models of d31-mode LPT.

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## Declarations

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## List of Publications

This thesis is not written in the conventional narrative format but in the publication format as a portfolio of publications either published or submitted for publication by peer-reviewed journals according to the ‘Academic Program Rules’ of the University of Adelaide. The research outcomes of this thesis have results in the generation of 6 either published or submitted journal papers and 2 refereed conference papers which are listed below:

### Journal Papers

*Paper 1* Zhang, Y., Lu, T. F., & Al-Sarawi, S. (2015). Formulation of a simple distributed-parameter model of multilayer piezoelectric actuators. *Journal of Intelligent Material Systems and Structures*, 1045389X15595294.

*Paper 2* Zhang, Y., Lu, T. F., & Peng, Y. (2015). Three-port equivalent circuit of multi-layer piezoelectric stack. *Sensors and Actuators A: Physical*, 236, 92-97.

*Paper 3* Zhang, Y., Lu, T. F., Al-Sarawi, S., Tu, Z. (2016). A Simplified Transfer Matrix of Multi-layer Piezoelectric Stack. *Journal of Intelligent Material Systems and Structures*, 1045389X16651153.

*Paper 4* Zhang, Y., & Lu, T. F. (2016). Simple Equivalent Finite Element Models of D33-Mode Multi-layer Piezoelectric Actuator, submitted to *Smart Materials and Structures*

*Paper 5* Zhang, Y., Lu, T. F., & GAO, W. (2016). Model D31-mode Longitudinal Piezoelectric Transducers. Submitted to *Sensors and Actuators A: Physical*

*Paper 6* Zhang, Y., Lu, T. F., & Al-Sarawi, S. (2016). Simple Equivalent Finite Element Models of D31-Mode Multi-Layer Piezoelectric Actuator. Submitted to *Journal of Intelligent Material Systems and Structures*

### Conference Papers

Conference-1 Yangkun Zhang and Tien-Fu Lu 2014, ‘Investigation on the Uniform-Electrical-Field Assumption for Modeling multi-layer piezoelectric actuators’, *8<sup>th</sup> Asia International conference on Mathematical Modeling and computer simulation*, 23-25 September, Taipei, Taiwan, page 251-254

Conference-2 Yangkun Zhang and Tien-Fu Lu 2014, ‘A Simple Distributed Parameter Analytical Model of Multi-layer Piezoelectric Actuator’, *8<sup>th</sup> Asia International conference on Mathematical Modeling and computer simulation*, 23-25 September, Taipei, Taiwan, page 247-250

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