Underdetermined DOA Estimation of Deterministic Signals Using High Order Statistics and Noncircularity

by

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To my parents

Contents

Contents v

Abstract xi

Statement of Originality xiii

Acknowledgments xv

Conventions xvii

Publications xix

Abbreviations xxi

Notations xxv

List of Figures xxvii

List of Tables xxxi

Chapter 1. Introduction and Motivation 1

1.1 Research Area and Motivation 2

1.2 Original Contributions 4

1.3 Thesis Structure 6

Chapter 2. Background and A Review of Array Processing with Nonideal Models 9

2.1 Introduction 10

2.2 Development of Coherent Signals Processing 11

2.2.1 Dimension Reduction Processing 11

2.2.2 Full Dimension Processing 16
2.3 Development of Unknown Noise Processing 22
2.3.1 Parametric Model for Unknown Noise 23
2.3.2 Sparse Model for Unknown Noise 24
2.4 Development of Noncircular Signal Processing 26
2.4.1 Description 27
2.4.2 Research Status of DOA Estimation of Noncircular Signal 27
2.4.3 Direction of Noncircular Signal Research 31
2.5 Conclusion 33

Chapter 3. Linear Algebra Prerequisites **35**

3.1 Introduction 36
3.2 Rank of a Matrix 36
3.3 Matrix Inverses 38
3.4 Moore-Penrose Pseudo-inverse 38
3.5 Derivative Operations 40
3.6 Special Matrices 41
3.6.1 Vandermonde Matrix 42
3.6.2 Toeplitz Matrix 42
3.6.3 Hermitian Matrix 44
3.7 Subspaces 45
3.8 Projection Matrices 48
3.8.1 Orthogonal Projection 48
3.8.2 Oblique Projection 49
3.9 Conclusion 50

Chapter 4. Basis of DOA Estimation **51**

4.1 Introduction 52

4.2 Mathematical Model of Signals Received by a Sensor Array 52
4.3 Classical DOA Estimation Algorithms of Uncorrelated Signals 54
 4.3.1 Beamforming Techniques 55
 4.3.2 Subspace-based Methods 58
4.4 Classical DOA Estimation Algorithms of Coherent Signals 64
 4.4.1 Signal Model Under Multipath Propagation 64
 4.4.2 Conventional DOA Estimation Algorithms of Coherent Signals . 66
4.5 Source Enumeration 70
4.6 Conclusion 72

Chapter 5. Efficient DOA Estimation Using Cumulants in Multipath 73

5.1 Introduction 74
5.2 Cumulants Preliminaries—Definitions and Properties 76
5.3 Problem Formulation 77
5.4 Two-stage Processing for Mixed Signals Under Multipath Propagation . 80
 5.4.1 Stage 1: DOA Estimation of the Uncorrelated Signals 81
 5.4.2 Stage 2: DOA Estimation of the Coherent Signals 84
 5.4.3 Separable Signal Number 91
 5.4.4 Simulation Results and Discussion 92
5.5 DOA Estimation in the Presence of Multipath with Fewer Sensors 102
 5.5.1 Space-time Cumulant Matrix 104
 5.5.2 Computational Complexity 109
 5.5.3 Simulation Results and Discussion 110
5.6 Conclusion 120

Chapter 6. Noncircularity to Array: Novel Approaches with Enhanced DOFs 123

6.1 Introduction 124
6.2 DOA Estimation of Noncircular Signals with Fewer Sensors: A Cumulant-based Approach 125

6.2.1 Signal Model for Independent Noncircular Sources 125
6.2.2 Improved Noncircular Signals Direction Finding Using FOC . . . 126
6.2.3 Connection with Other Work 129
6.2.4 Simulation Results and Discussion 132
6.3 Direction Finding of Noncircular Signals in the Presence of Multipath Propagation 139
6.3.1 Array Model for Noncircular Signals 139
6.3.2 Separable Signal Number 149
6.3.3 Computational Complexity 151
6.3.4 Derivation of Deterministic CRLB for the Noncircular Signals in the Presence of Multipath 152
6.3.5 Simulation Results and Discussion 155
6.4 Conclusion 163

Chapter 7. Conclusions and Future Work 167
7.1 Review of Research Results 168
7.2 Recommendations on Future Work 170
7.3 Conclusion 172

Appendix A. Proof of Equations and Statements in Chapter 5 173
A.1 Proof of $\mathbf{C}_x^+$ in (5.24) 174
A.2 Proof of (5.33) 175
A.3 Proof of $\mathbf{J}\mathbf{B}_c^* = \mathbf{B}_c\mathbf{\Phi}^{1-m}$ 175
A.4 Proof of (5.37) 177
A.5 Analysis of the k-th Row of $\mathbf{D}_1 - \lambda\mathbf{I}$ 178

Appendix B. Proof of Equations and Statements in Chapter 6 181
B.1 Interpretation of **G** and Corresponding Examples 182
B.2 Interpretation of (6.51) 183
B.3 Interpretation of (6.52) 185
B.4 Detailed Derivation of Some FIM Blocks 186

Bibliography **189**

Abstract

Sensor arrays play important roles in signal transmission/reception, estimation, and tracking, and have been successfully applied to many engineering fields such as radar, sonar, wireless communications to name a few. Practically, sensor array systems usually suffer from nonideal factors such as signal coherency, spatially coloured noise, and a limited number of sensors. In this thesis, problems of direction of arrival (DOA) estimation in the presence of nonideal factors are addressed, and new algorithms to tackle these problems are developed that achieve improved performance with a limited number of sensors.

Under multipath propagation, independent and coherent signals coexist, resulting in rank deficiency of the cumulant matrix. To tackle this problem, two methods for DOA estimation of mixed independent and coherent signals using fourth-order cumulants (FOC) are proposed, and both algorithms can make efficient use of the array degrees of freedom (DOFs). The first algorithm implements the estimation via two-stage processing by separating the independent and coherent signals. In this method, new matrix reconstruction techniques for independent signal cumulants and rank restoration are developed, and the DOAs of both the independent and coherent signals can be estimated by polynomial rooting without performing a spectral grid search. Its superiority over existing methods is demonstrated by simulation results.

The second algorithm considers the case when a large number of coherent signals, greater than the number of sensors, exist due to the propagation channel. Here, we exploit temporal correlation in the signals to form an array output matrix with pseudo snapshots, spanning the same signal subspace as the one using real snapshots. By incorporating this property, new augmented cumulant matrices are constructed and the corresponding method for coherent group separation is derived. Compared with the existing method, the proposed one achieves better performance in terms of estimation accuracy and robustness of the spatial signature, especially for weak signals.

Apart from signal with circular statistics discussed above, we study the noncircularity embedded in modern wireless communication signals to further extend the effective aperture, enhance DOFs, and improve the estimation performance. A new FOC-based direction finding method which can extend the array aperture as well as maximise the DOFs is proposed. By combining noncircularity with high order cumulants and optimsing geometric arrangement of the virtual array arising accordingly, the resultant identifiability of DOA estimation can be up to twice larger compared with the using the same order cumulants for circular signals. Simulation results validate that the proposed method offers better performance in terms of identifiability as well as accuracy.

Last, we revisit the case when uncorrelated and coherent signals coexist and utilise the noncircularity of signals in this scenario. To the best of our knowledge, there are no publications addressing the class of DOA estimation problem, and a novel two-stage second order statistics (SOS) estimator is introduced accordingly to further increase the DOFs. In this method, a more robust approach is presented to identify the true DOA estimates from the pseudo ones, the estimates of noncircular phases are derived in closed-form, and a novel spatial smoothing technique based on the eigenvectors is developed to restore the rank deficiency. Additionally, new deterministic Cramér-Rao lower bounds (CRLBs) are derived for the considered mixture model of noncircular signals. The theoretical analysis justifies that the number of identifiable signals is larger than the current algorithms. Extensive simulation results show that the proposed method offers sufficient DOFs as well as improving the estimation accuracy of both the uncorrelated and coherent signals.

Statement of Originality

This work contains no material that has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Signed

Date

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Yuexian Wang (September 2015)

Conventions

Typesetting

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Referencing

Referencing and citation style in this thesis are based on the Institute of Electrical and Electronics Engineers (IEEE) Transaction style [1].

For electronic references, the last accessed date is shown at the end of a reference.

Units

The units used in this thesis are based on the International System of Units (SI units) [2].

Spelling

The Australian English spelling is adopted in this thesis.

Publications

Journal

[1] Y. Wang, M. Trinkle, and B. W.-H. Ng, "DOA estimation under unknown mutual coupling and multipath with improved effective array aperture," *Sensors*, vol. 15, no. 12, pp. 30 856–30 869, Dec. 2015.

[2] Y. Wang, M. Trinkle, and B. W.-H. Ng, "Two-stage DOA estimation of independent and coherent signals in spatially coloured noise," *Signal Process.*, vol. 128, pp. 350–359, Nov. 2016.

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Conference

[1] Y. Wang and M. Trinkle, "DOA estimation for coherent signals with symmetric virtual array," in *International Global Navigation Satellite Systems Symposium, IGNSS 2013*, Outrigger Gold Coast, Australia, Jul. 2013.

[2] Y. Wang and M. Trinkle, "Coherent signals DOA estimation in the presence of complex noise," in *International Global Navigation Satellite Systems Symposium, IGNSS 2013*, Outrigger Gold Coast, Australia, Jul. 2013.

Abbreviations

AIC	Akaike Information Criterion
AM	Amplitude Modulation
AP	Alternating Projection
AR	Autoregressive
ARMA	Autoregressive Moving Average
ASK	Amplitude Shift Keying
BPSK	Binary Phase Shift Keying
CB	Conventional Beamforming
CCI	Co-Channel Interferences
CRLB	Cramér-Rao Lower Bound
CW	Continuous Wave
DML	Deterministic Maximum Likelihood
DOA	Direction Of Arrival
DOFs	Degrees Of Freedom
EM	Expectation-Maximization
ESPRIT	Estimation of Signal Parameter by Rotation Invariance Techniques
FBSS	Forward/Backward Spatial Smoothing
FOC	Fourth Order Cumulants
GDE	Gerschgorin Disk Estimator

HOC	High Order Cumulants
HOS	High Order Statistics
LL	Log-Likelihood
MDL	Minimum Description Length
ML	Maximum Likelihood
MLE	Maximum Likelihood Estimator
MRAs	Minimum Redundancy Arrays
MSE	Mean Squared Error
MUSIC	MUltiple Signal Classification
MVDR	Minimum Variance Distortionless Response
NSF	Noise Subspace Fitting
OQPSK	Offset Quadrature Phase Shift Keying
PAM	Pulse Amplitude Modulation
PDF	Probability Density Function
RMSE	Root Mean Squared Error
SAGE	Space Alternating Generalized EM
SF	Subspace Fitting
SML	Stochastic Maximum Likelihood
SNR	Signal-to-Noise Ratio
SORTE	Second ORder sTatistic of Eigenvalues
SOS	Second Order Statistics
SS	Spatial Smoothing

SSF	Signal Subspace Fitting
SVD	Singular Value Decomposition
TAM	Toeplitz Approximation Method
ULA	Uniform Linear Array
VESPA	Virtual ESPRIT Algorithm
WSSF	Weighted Signal Subspace Fitting

Notations

λ	The wavelength of corresponding frequency
θ	Azimuth angle
c	The speed of light
d	The spacing between adjacent sensors
$\mathbf{x} \in \mathbb{R}^M$	A vector with dimension M in real domain
$\mathbf{x} \in \mathbb{C}^M$	A vector with dimension M in complex domain
$(\,\cdot\,)^T$	The transpose operation
$(\,\cdot\,)^*$	The conjugate operation
$(\,\cdot\,)^H$	The conjugate transpose operation
$(\,\cdot\,)^{-1}$	The inverse operation
$(\,\cdot\,)^+$	The pseudo-inverse operation
$\lceil\,\cdot\,\rceil$	The ceiling operation of a decimal number
$\lfloor\,\cdot\,\rfloor$	The flooring operation of a decimal number
$E_t[\,\cdot\,]$	Statistical expectation implemented by $\lim_{L\to\infty}\frac{1}{L}\sum_{t=1}^{L}[\,\cdot\,]$
$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}}$	The first derivative of the function $f(\mathbf{x})$ with respect to $\mathbf{x}$
$\frac{\partial^2 f(\mathbf{x})}{\partial \mathbf{x}^2}$	The second derivative of the function $f(\mathbf{x})$ with respect to $\mathbf{x}$
$\mathrm{Re}\{\,\cdot\,\}$	Real part of a complex number
$\mathrm{Im}\{\,\cdot\,\}$	Imaginary part of a complex number
$\mathrm{span}\{\,\cdot\,\}$	Linear span
$\dim\{\,\cdot\,\}$	Dimension of linear space
$\mathcal{R}(\,\cdot\,)$	Range (or signal space) of a matrix
$\mathcal{N}(\,\cdot\,)$	Null space of a matrix
$\mathcal{V}^{\perp}$	Subspace orthogonal to a linear space $\mathcal{V}$
$\mathbf{P}_{\mathbf{A}}^{\perp}$	Orthogonal projection to the range of $\mathbf{A}$
$\mathbf{E}_{\mathbf{A}\mid\mathbf{B}}$	Oblique projection with the range $\mathcal{R}(\mathbf{A})$ and null space $\mathcal{N}(\mathbf{B})$
$\max\{a, b\}$	The maximum value between a and b
$\min\limits_{x} f(x)$	Minimise the function f with respect to the variable x
$\lVert\cdot\rVert_2$	Euclidean (ℓ_2) norm

$\|\cdot\|_F$	Frobenius norm
$\det\{\cdot\}$	Matrix determinant
$\circ$	Khatri-Rao product
$\otimes$	Kronecker product
$\odot$	Schur-Hadamard product, i.e., element-wise product
$\mathrm{tr}\{\cdot\}$	Matrix trace
$\mathrm{rank}(\cdot)$	Matrix rank
$\mathrm{diag}\{z_1, z_2\}$	Diagonal matrix with diagonal entries z_1, z_2
$\mathrm{blkdiag}\{\mathbf{Z}_1, \mathbf{Z}_2\}$	Block diagonal matrix with diagonal entries $\mathbf{Z}_1, \mathbf{Z}_2$
$\mathbf{Z}(a:b, c:d)$	Submatrix by the entries from rows a to b and columns c to d of $\mathbf{Z}$
$\mathbf{Z}(a, b)$	Entry in the a-th row and b-th column of $\mathbf{Z}$
$\mathbf{I}$	Identity matrix
$\mathbf{J}$	Exchange matrix
$\mathbf{R}$	Covariance matrix
$\mathbf{U}_s$	Signal subspace
$\mathbf{U}_n$	Noise subspace
$\mathbf{a}(\theta)$	Steering vector
$\mathbf{A}(\theta)$	Array manifold

List of Figures

1.1 Coherent addition of signals 3

4.1 Uniform linear array 53

4.2 Digital beamforming network 56

4.3 Spatial spectrum of CB, MVDR, and MUSIC, for a 10-element ULA with $\frac{\lambda}{2}$ spacing between adjacent elements. SNR $=$ 5dB, snapshots number is 500 61

4.4 Polar of estimates using ESPRIT 63

4.5 Subarrays structure with forward spatial smoothing 66

4.6 Subarrays structure with backward spatial smoothing 67

4.7 Spatial spectrum comparison between (a) without rank restoration; (b) with FBSS. SNR $=$ 5dB, snapshots number is 500 68

4.8 MUSIC spatial spectrum with Toeplitz matrix reconstruction 70

5.1 Detection accuracy versus (a) SNR when the number of snapshots is 2000; (b) the number of snapshots when SNR $= -$6dB. 94

5.2 RMSE of the DOA estimates in the overdetermined case versus (a) SNR when the number of snapshots is 4000; (b) the number of snapshots when SNR $=$ 0dB. 95

5.3 Probability of resolution of the DOA estimates in the overdetermined case versus (a) SNR when the number of snapshots is 4000; (b) the number of snapshots when SNR $=$ 0dB. 97

5.4 RMSE of the DOA estimates in the underdetermined case versus (a) SNR when the number of snapshots is 4000; (b) the number of snapshots when SNR = 0dB. 99
5.5 Probability of resolution of the DOA estimates in the underdetermined case versus (a) SNR when the number of snapshots is 4000; (b) the number of snapshots when SNR = 0dB. 100
5.6 RMSE of the DOA estimates for T_m versus (a) SNR when the number of snapshots is 4000; (b) the number of snapshots when SNR = 0dB. 101
5.7 RMSE of the DOA estimates versus in the case of the small snapshot sizes (a) SNR when the number of snapshots is 700; (b) the number of snapshots when SNR = 0dB. 103
5.8 DOA estimation results of 50 runs in the overdetermined case. SNR = 0dB, $L_s = 5000$, and $L_p = 300$ 112
5.9 RMSE of DOA estimates in the overdetermined case. (a) The number of snapshots is 5000. (b) SNR = 0dB, and $L_p = \frac{3}{50}L_s$. 115
5.10 DOA estimation results of 50 runs in the underdetermined case. SNR = 0dB, $L_s = 5000$, and $L_p = 300$ 116
5.11 RMSE of DOA estimates in the underdetermined case. (a) The number of snapshots is 5000. (b) SNR = 0dB,and $L_p = \frac{3}{50}L_s$ 119

6.1 The schematics of virtual aperture extension for circular signals 130
6.2 The schematics of virtual aperture extension for noncircular signals . . . 132
6.3 DOA estimation results of 50 runs under the maximum processing capacity. SNR = 20dB, the number of snapshot is 20000 133
6.4 RMSE of the DOA estimates in the overdetermined case versus (a) SNR when the number of snapshots is 5000; (b) the number of snapshots when SNR = 5dB 135
6.5 Probability of resolution in the overdetermined case versus (a) SNR when the number of snapshots is 10000; (b) the number of snapshots when SNR = 10dB 136

6.6 RMSE of the DOA estimates in the underdetermined case versus (a) SNR when the number of snapshots is 5000; (b) the number of snapshots when SNR $= 5$dB 137

6.7 Probability of resolution in the underdetermined case versus (a) SNR when the number of snapshots is 10000; (b) the number of snapshots when SNR $= 10$dB 138

6.8 RMSE of the uncorrelated signal estimates in the overdetermined case versus (a) SNR when the number of snapshots is 800; (b) the number of snapshots when SNR $= -5$dB 157

6.9 RMSE of the noncircularity phase estimates for 3 uncorrelated signals. (a) The number of snapshots is 800. (b) The SNR $= -5$dB 159

6.10 RMSE of the coherent signal estimates in the overdetermined case versus (a) SNR when the number of snapshots is 800; (b) the number of snapshots when SNR $= -5$dB 160

6.11 RMSE of the uncorrelated signal estimates in the underdetermined case versus (a) SNR when the number of snapshots is 800; (b) the number of snapshots when SNR $= -5$dB 161

6.12 RMSE of the noncircularity phase estimates for 5 uncorrelated signals. (a) The number of snapshots is 800. (b) The SNR $= -5$dB 162

6.13 RMSE of the coherent signal estimates in the underdetermined case versus (a) SNR when the number of snapshots is 800; (b) the number of snapshots when SNR $= -5$dB 164

List of Tables

5.1 Minimum number of array elements required 92

5.2 Average runtime (in seconds) 111

5.3 Estimated amplitude of ξ_1 versus SNR in the overdetermined case (True value $\rho_1 = 0.7522$) 113

5.4 Estimated amplitude of ξ_2 versus SNR in the overdetermined case (True value $\rho_2 = 0.9821$) 113

5.5 Estimated phase of ξ_1 versus SNR in the overdetermined case (True value $\psi_1 = 2.1533$rad) 113

5.6 Estimated phase of ξ_2 versus SNR in the overdetermined case (True value $\psi_2 = 1.9644$rad) 114

5.7 Estimated amplitude of ξ_1 versus SNR in the underdetermined case (True value $\rho_1 = 0.7386$) 117

5.8 Estimated amplitude of ξ_2 versus SNR in the underdetermined case (True value $\rho_2 = 1.5385$) 117

5.9 Estimated amplitude of ξ_3 versus SNR in the underdetermined case (True value $\rho_3 = 1.8732$) 117

5.10 Estimated phase of ξ_1 versus SNR in the underdetermined case (True value $\psi_1 = 1.6613$rad) 118

5.11 Estimated phase of ξ_2 versus SNR in the underdetermined case (True value $\psi_2 = -0.7055$rad) 118

5.12 Estimated phase of ξ_3 versus SNR in the underdetermined case (True value $\psi_3 = -2.4933$rad) 118

6.1 Minimum number of array elements required 150

6.2 Average runtime (in milli-seconds) 156