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Problems with extracting m_s from flavor breaking in hadronic τ decays

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A numerical error is pointed out in the existing expression for the $\mathcal{O}(\alpha_s^2)$ longitudinal component of the squared-mass ($D=2$) contribution to the hadronic τ decay rate. The corrected version is found to be such that, to $\mathcal{O}(\alpha_s^2)$, each term in the resulting series is larger than the previous one, hence ruling out the direct use of flavor breaking in hadronic τ decays as a means of extracting the strange quark mass. An alternate approach, in which one uses the model spectral functions previously developed for the strangeness-changing scalar channel (and employed in alternate sum rule analyses of m_s) as input to the τ decay analysis, is shown to provide mutual consistency checks for the two different methods. The results for m_s implied by τ decay data and the existing model spectral function are also presented. [S0556-2821(98)03921-6]

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I. INTRODUCTION

As is well known, the inclusive hadronic τ decay ratio

$$R_\tau \equiv \frac{\Gamma[\tau^- \rightarrow \nu_\tau \text{hadrons}(\gamma)]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]} \quad (1)$$

[where (γ) indicates additional photons or lepton pairs] may be rather reliably computed using techniques based on the operator product expansion (OPE) and perturbative QCD (PQCD) [1–10]. (A very clear recent review of the theoretical situation is given in Ref. [11].) One begins with the representation of this ratio in terms of hadronic spectral functions:

$$R_\tau = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \times \left[\left(1 + 2 \frac{s}{m_\tau^2}\right) \text{Im} \Pi^{(1)}(s) + \text{Im} \Pi^{(0)}(s) \right] \quad (2)$$

where $S_{EW} = 1.0194$ represents the leading electroweak corrections [12], $J=0,1$ labels the hadronic rest-frame angular momentum, and the $\Pi^{(J)}(q^2)$ are given in terms of the vector and axial vector current spectral functions $\Pi_{ij;V,A}^{(J)}(q^2)$ (with $ij=ud,us$) by

$$\begin{aligned} \Pi^{(J)}(s) \equiv & |V_{ud}|^2 [\Pi_{ud,V}^{(J)}(s) + \Pi_{ud,A}^{(J)}(s)] \\ & + |V_{us}|^2 [\Pi_{us,V}^{(J)}(s) + \Pi_{us,A}^{(J)}(s)], \end{aligned} \quad (3)$$

where

$$\begin{aligned} i \int d^4x e^{iqx} \langle 0 | T(J_{ij;V,A}^\mu(x) J_{ij;V,A}^\nu(0)^\dagger) | 0 \rangle \\ \equiv (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{ij;V,A}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij;V,A}^{(0)}(q^2), \end{aligned} \quad (4)$$

with $J_{ij;V,A}^\mu$ the standard vector and axial vector currents involving flavors ij , defines $\Pi_{ij;V,A}^{(J)}(q^2)$. Using the analyticity properties of the correlators, the hadronic representation, Eq. (2), can then be converted into a contour integral representation

$$R_\tau = 6\pi S_{EW} i \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \times \left[\left(1 + 2 \frac{s}{m_\tau^2}\right) \Pi^{(0+1)}(s) - 2 \frac{s}{m_\tau^2} \Pi^{(0)}(s) \right]. \quad (5)$$

Since $m_\tau^2 \sim 3 \text{ GeV}^2$, the latter expression is amenable to evaluation using the OPE. Keeping operators up to dimension $D=6$ in the OPE, and evaluating the mass-independent ($D=0$) and mass-dependent ($D=2$) perturbative terms to $\mathcal{O}(\alpha_s^3)$ and $\mathcal{O}(\alpha_s^2)$, respectively, it is known that an excellent fit to hadronic τ decay data is obtained using a value of $\alpha_s(m_\tau)$ compatible (after running) with that obtained directly at a scale M_Z (see Ref. [11], and earlier references cited therein). Of relevance to the present paper is the observation, made by numerous earlier authors, that, although the $D=2ud$ contributions to R_τ are tiny, the $D=2us$ terms can alter the prediction for the us contribution to R_τ by $\sim 20\%$, for conventional values of m_s . (See, for example, Refs. [4,6,13,14].) These contributions, of course, play little role in determining R_τ , owing to the suppression by the factor of $|V_{us}/V_{ud}|^2$ relative to the non-strange current contributions. As noted by Davier [13], however, sufficiently precise τ decay data allow one to look explicitly for this effect, and hence, in principle, determine m_s .

The most convenient way of implementing this extraction of m_s is to consider the contributions to inclusive hadronic τ decay mediated separately by the strangeness-changing (us) and strangeness-non-changing (ud) weak currents, and take the difference of these contributions after rescaling each by the inverse of the square of the corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix element. The $D=0$ contributions then cancel in the difference, leaving a leading $D=2$ contribution proportional (to leading order in m_u/m_s) to

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m_s^2 . Numerical estimates show that this term is not only formally leading, but also numerically leading at the scale m_τ fixed by the radius of the circular contour in Eq. (5) [4,6].

The extraction of m_s , as performed by the ALEPH Collaboration [13,14], using flavor breaking in hadronic τ decays, relies on the expressions for the $D=2$ mass-dependent terms worked out by Chetyrkin and Kwiatkowski [6]. Having determined the $D=2$ contributions to the correlators $\Pi_{us;V,A}^{(0+1),(0)}(q^2)$ and performed the contour integration indicated in Eq. (5), they obtain the result, to leading order in m_u/m_s , quoted in their Eq. (26). This result implies, summing ‘‘longitudinal’’ ($J=0$) and ‘‘transverse’’ ($J=0+J=1$) contributions, a us current contribution to R_τ given by

$$[R_\tau]_{us;V,A}^{D=2} = -12|V_{us}|^2 S_{EW} \frac{m_s^2(m_\tau)}{m_\tau^2} \times \left[1 + \frac{16}{3} a(m_\tau) + 11.03a(m_\tau)^2 \right]. \quad (6)$$

In Eq. (6), $m_s(m_\tau)$ is the running strange quark mass at a scale m_τ , and $a(\mu) \equiv \alpha_s(\mu)/\pi$, with $\alpha_s(\mu)$ the standard modified minimal subtraction scheme (MS) running coupling at scale μ . The polynomial $P(a) = 1 + 16a/3 + 11.03a^2$ appearing in Eq. (5) is evidently rather well converged to $\mathcal{O}(a^2)$, allowing, once experimental data are sufficiently precise, a reliable determination of $m_s(m_\tau)$. However, as we will see below, the coefficient 11.03 multiplying a^2 in $P(a)$ is incorrect. Not only is the corrected value (46.002) such that the convergence of $P(a)$ is not very compelling [$P(a) = 1 + .602 + .587 + \dots$ at the scale m_τ] but, worse, this large value is generated mostly by an enormous $\mathcal{O}(a^2)$ coefficient in the longitudinal contribution. The series for the longitudinal contribution then turns out to be such that each subsequent term is larger than the previous, making it impossible to rely on the OPE representation of this contribution, and hence impossible to use the OPE representation of the $D=2$ terms to extract m_s .

We will, however, see that it is still possible to make some use of the data on flavor breaking in τ decay, since the spectral function of the badly behaved longitudinal contribution is related to that occurring in existing sum rule analyses of m_s based on the strangeness-changing scalar channel. Although the spectral function has not been experimentally determined in that channel, model versions exist [15–17], and information on the low-energy portion of it is available using the value of the scalar $K\pi$ form factor measured in K_{l3} , together with the Omnès representation of the form factor, which takes experimentally measured $K\pi$ phase shifts as input [17].

The rest of this paper is organized as follows. In Sec. II we present the corrected results for the longitudinal and transverse $D=2$ us contributions to R_τ and demonstrate the non-convergence of the longitudinal series in $a(m_\tau)$. In Sec. III we will see how to reformulate the expression for the us contribution to R_τ in such a way that it involves only (1) a longitudinal contribution which can be calculated directly from any given (model-dependent or otherwise) version of

the spectral function for the strangeness-changing scalar channel, (2) reasonably well-converged $D=2$ transverse contributions and (3) higher dimension longitudinal and transverse contributions which are either quite accurately known or small. In Sec. IV we will then use existing models for the scalar spectral function, together with recent τ decay data, to perform a consistency check on the values of m_s extracted using the two different analyses.

II. PERTURBATIVE SERIES FOR THE $D=2$ CONTRIBUTION

The $D=2$ us contributions to R_τ follow immediately from Eq. (5) and the expressions for the longitudinal and transverse pieces of the relevant vector and axial vector current correlators to next-to-next-to-leading order, as given by Chetyrkin and Kwiatkowski [6]. In what follows we will drop terms of order m_u/m_s since, as we will see below, it is possible to employ τ decay data (once we are aware of the poor convergence of the longitudinal contributions) only if we neglect such contributions. Note also that the same extremely poor convergence of the longitudinal contributions also afflicts the $D=2$ ud contributions; however, in this case, the squared-mass factors are so tiny that this is of no numerical significance in existing analyses of the isovector hadronic weak current mediated decays.

The contributions to the correlators, in this approximation, are then given by [6]

$$\Pi_{us;V,A}^{(0)D=2} = \frac{3}{2\pi^2} \frac{m_s(Q^2)}{Q^2} \left[\frac{1}{a(Q^2)} - \frac{5}{2} - 1.713804a(Q^2) \right] + \dots \quad (7)$$

and

$$\Pi_{us;V,A}^{(0+1)D=2} = -\frac{3}{4\pi} \frac{m_s(Q^2)}{Q^2} \left[1 + \frac{7}{3} a(Q^2) + 19.5831a(Q^2)^2 \right] \quad (8)$$

where $a(Q^2) = \alpha_s(Q^2)/\pi$ and $m_s(Q^2)$ are the running coupling and running strange quark mass, both at scale $\mu^2 = Q^2 = -s$, in the MS scheme. In Eq. (7), $+\dots$ stands for the second set of terms of Eq. (16) of Ref. [6], which terms do not survive the contour integration involved in obtaining R_τ . Note that, in Eq. (8), the coefficient 19.5831 includes the contribution of the three-loop graph containing two quark loops, which contribution is also present in the corresponding ud contribution to R_τ and hence cancels in forming the flavor-breaking difference discussed below. (The term which survives corresponds to the replacement 19.5831 \rightarrow 19.9332.) Note that there is a typographical error in Eq. (25) of Ref. [6], where the correct result 19.5831 (~ 19.6) has inadvertently been recorded as 17.6.

It is now a straightforward matter to evaluate the $D=2$ us contributions to R_τ . One first expands $a(Q^2)$ in terms of $a(m_\tau^2) \equiv a$ and $m_s(Q^2)$ in terms of a and $m_s(m_\tau^2) \equiv \bar{m}_s$, the coefficients in these expansions being polynomials

in $\log(Q^2/m_\tau^2)$ [18]. The remaining integrals are then elementary ones, involving products of powers of Q^2 and powers of $\log(Q^2/m_\tau^2)$. Performing these integrals, one finds, for the contributions to R_τ corresponding to the terms written explicitly in Eqs. (7) and (8) above,

$$[R_\tau^{(0)}]_{us;V,A}^{D=2} = -3|V_{us}|^2 S_{EW} \frac{\bar{m}_s^2}{m_\tau^2} \left[1 + \frac{28}{3}a + 109.9889a^2 \right] \quad (9)$$

$$[R_\tau^{(0+1)}]_{us;V,A}^{D=2} = -9|V_{us}|^2 S_{EW} \frac{\bar{m}_s^2}{m_\tau^2} [1 + 4a + 24.6707a^2], \quad (10)$$

whose sum yields, as claimed above,

$$[R_\tau]_{us;V,A}^{D=2} = -12|V_{us}|^2 S_{EW} \frac{\bar{m}_s^2}{m_\tau^2} \left[1 + \frac{16}{3}a + 46.002a^2 \right]. \quad (11)$$

(In the transverse contribution, $24.6707 \rightarrow 25.0207$ when one takes into account the cancellation of the terms proportional to $\alpha_s^2 m_s^2$ common to the ud and us correlators, and associated with the graphs involving a quark bubble on the internal gluon line.) The revised numerical values quoted above have been confirmed by Chetyrkin [19].

If one uses the value of $\alpha_s(m_\tau)$ determined from the ALEPH analysis of non-strange hadronic τ decays [20] (this choice being the most sensible since, in contrast to the global analysis, the results are not affected numerically by the use of the erroneous form of the $D=2$ contributions), one finds, for the polynomial appearing in the Eq. (9),

$$1 + \frac{2}{3}8a + 109.9889a^2 = 1 + 1.052 + 1.397. \quad (12)$$

It is quite clear that the series is simply too poorly converged to allow one to consider the coefficient of m_s^2 appearing in the expression for R_τ reliably determined by the OPE analysis. As such, the conventional approach to obtaining m_s from flavor breaking in hadronic τ decay is ruled out.

III. MODIFIED ANALYSIS FOR m_s

Two observations make it possible to utilize flavor breaking in hadronic τ decay to constrain m_s , in spite of the non-convergence of the series for the longitudinal contribution noted above. The first is that, to the extent that one ignores corrections of order m_u/m_s , the vector and axial vector $D=2$ us contributions are identical. (Beyond leading order in m_s , this statement is, however, no longer valid.) One may then, as far as the $D=2$ contributions are concerned, concentrate on, say, the vector contributions. This is of relevance because of the second observation, namely that the vector current spectral function is related to the spectral function of the strangeness-changing scalar channel, about which some (though not complete) experimental information is available. Combining these two observations, one sees that the problematic $D=2$ contributions can be handled by replacing the

full ($D=2,4,6,\dots$ and vector plus axial vector) longitudinal contributions with twice the full vector contribution, providing one makes corrections reflecting the difference of the vector and axial vector contributions. These corrections then (to leading order in m_s) begin at $D=4$. The full vector contribution is obtained, as described below, from the model spectral functions employed in the strangeness-changing scalar channel.

Let us define $R_{ij;V,A}^{(J);D}$ to be the vector (V) or axial vector (A) flavor $ij=ud$ or us contributions to R_τ associated with those operators of dimension D in the OPE of the longitudinal ($J=0$) or transverse ($J=0+1$) terms in Eq. (5) *after scaling out the factors* $|V_{ij}|^2 S_{EW}$, and R_{ij} to be the corresponding sum, for given ij , over D , $V+A$ and $J=0,0+1$. As noted above, to be able to effectively employ the model spectral functions from the scalar channel as input it is necessary to drop corrections of $\mathcal{O}(m_u/m_s)$ to $R_{us;V,A}^{(0);D=2}$. For consistency one must then also drop other terms proportional to the light quark u,d masses. For $D=2$, this includes all of $R_{ud;V,A}^{(J);D=2}$ apart from the ‘‘quark bubble’’ transverse contribution proportional to $\alpha_s^2 m_s^2$ mentioned above, which simply cancels against the corresponding term in $R_{us;V,A}^{(0);D=2}$. For $D=4$, since the leading contributions are proportional to $\langle m_s \bar{l}l \rangle$, $l=u,d$, and m_s^4 , this means dropping also terms proportional to $\langle m_l \bar{l}l \rangle$ and terms fourth order in the quark masses, other than those proportional to m_s^4 . This removes all contributions to $R_{ud;V,A}^{(J);D=4}$ apart from those terms in the transverse contribution proportional to the gluon condensate, which terms cancel in forming the difference of ud and us contributions. Note also that, to the order so far computed, the $D=6$ contributions are pure transverse. With the set of approximations just discussed, one thus obtains

$$\begin{aligned} R_{ud} - R_{us} = & - \left[2 \sum_D R_{us;V}^{(0);D} \right] - [R_{us;A}^{(0);D=4} - R_{us;V}^{(0);D=4}] \\ & - [R_{us;V}^{(0+1);D=2} + R_{us;A}^{(0+1);D=2}]', \\ & + \sum_{D=4,6,\dots} [R_{ud;V}^{(0+1);D} + R_{ud;A}^{(0+1);D}] \\ & - R_{us;V}^{(0+1);D} - R_{us;A}^{(0+1);D}], \quad (13) \end{aligned}$$

where terms with $D>6$ have been dropped. The first term on the right-hand side (RHS) of Eq. (13) is to be obtained using the input model spectral function obtained from the scalar channel. The remaining terms will be obtained from the OPE representation. The prime on the third term in Eq. (13) reminds us that the ‘‘strange quark bubble’’ contribution cancelled by the corresponding contribution to the ud correlator has to be removed in evaluating this term. The ud contributions in the last term serve only to remove all $D=4$ contributions proportional to the gluon condensate. As we will see below, all of the remaining terms are either very well, or relatively well, known. Equation (13) thus isolates the problems of the $D=2$ part of the longitudinal term in the OPE representation in such a way that, to the extent one can obtain a good phenomenological representation of the scalar

spectral function, one can avoid the poorly converging OPE representation and instead evaluate this contribution phenomenologically. While, at present, it is not possible to do this in a completely satisfactory manner (see the discussion on the existing constraints on the scalar spectral function below), the situation is amenable to future improvement.

Let us turn then to the evaluation of the remaining terms in Eq. (13). We stress, again, that in writing down all the expressions below, corrections proportional to the light quark masses have, for consistency's sake, been systematically dropped, with the exception of one term for which there is a numerical enhancement, as discussed below. Since the expressions for the relevant contributions to the correlators are well known in the literature [4,6], we present only the OPE representations after the relevant contour integrations have been performed. We then find

$$\begin{aligned}
& -[R_{us;A}^{(0);D=4} - R_{us;V}^{(0);D=4}] \\
& = \frac{24\pi^2}{m_\tau^4} [-2\bar{m}_s \langle \bar{l}l \rangle] + 12 \left[\frac{12}{7a} + \frac{5}{7} \right] \left(\frac{\bar{m}_s}{m_\tau} \right)^4 \left(\frac{m_l}{m_s} \right), \\
& -[R_{us;V}^{(0+1);D=2} + R_{us;A}^{(0+1);D=2}], \\
& = -18 \left[\frac{\bar{m}_s}{m_\tau} \right]^2 [1 + 4a + 25.0207a^2 + f_3 a^3], \\
& [R_{ud;V}^{(0+1);D=4} + R_{ud;A}^{(0+1);D=4} - R_{us;V}^{(0+1);D=4} - R_{us;A}^{(0+1);D=4}] \\
& = \frac{81\pi^2 a^2}{m_\tau^4} [-2\bar{m}_s \langle \bar{s}s \rangle] - 54 \left(\frac{\bar{m}_s}{m_\tau} \right)^4, \\
& [R_{ud;V}^{(0+1);D=6} + R_{ud;A}^{(0+1);D=6} - R_{us;V}^{(0+1);D=6} - R_{us;A}^{(0+1);D=6}] \approx 0.
\end{aligned} \tag{14}$$

A few comments are in order concerning Eqs. (14). First, the leading term fourth order in the quark masses in the first expression, though formally an $\mathcal{O}(m_l/m_s)$ correction to the m_s^4 contribution appearing in the third expression, has a numerical enhancement due to the presence of a factor $1/a$ in the coefficient multiplying it, which enhancement largely undoes m_l/m_s suppression. This term has, therefore, been kept, though it turns out that, in fact, both terms are numerically tiny, and play a negligible role in the final analysis. The presence of a $1/a$ factor in the coefficient multiplying $m_s^3 m_l$ is due to the well-known fact that the quark condensate terms obtained from the standard Wick's theorem reduction must be modified to absorb additional long-distance mass logarithm terms. The modified condensates are then no longer scale invariant. Defining new scale invariant versions (see Ref. [4] for details) one finds that, rewriting the $D=4$ contributions in terms of these condensates, such an inverse power dependence on a shows up in the modified fourth order mass terms. The quark condensates appearing in Eq. (14) are thus those defined in the Appendix of Refs. [22, 4]. Second, in the $D=2$ transverse terms of the second line, the coefficient f_3 of a^3 has not, as yet, been computed. Because of the slow convergence of the $\mathcal{O}(a, a^2)$ terms in the series, we have, however, made an estimate of f_3 . This is done by

first estimating the corresponding coefficient in the series for the $D=2$ contribution to the correlator using the method of Ref. [23], and then integrating the result around the circular contour. The procedure of Ref. [23] is known to produce values for the $\mathcal{O}(a^3)$ coefficient accurate to ± 25 in the case of those mass-dependent observables for which the $\mathcal{O}(a^3)$ terms have been computed [23]. We find $f_3 = 217.8$ from the ‘‘fastest apparent convergence’’ version of the method and $f_3 = 219.6$ for the ‘‘principle of minimal sensitivity version.’’ As a conservative estimate of the effect of higher order terms we thus take $f_3 = 220 \pm 220$. At present the uncertainties in \bar{m}_s that result from that in f_3 are much smaller than those associated with the errors in the experimental input. Finally, the reason for setting the $D=6$ (purely transverse) contributions to zero needs an explanation. It is straightforward to read off the explicit forms of these contributions from Eq. (3.12) of Ref. [4]. As discussed in that reference, however, the full set of $D=6$ condensates is not known empirically, and it is customary to make an estimate based on a rescaled version of the vacuum saturation hypothesis [Eq. (3.13) of Ref. [4]]. This estimate has relatively large errors, but can be checked by employing the method of spectral moments [5,11], which allows one to make an empirical extraction of (at least the ud) $D=6$ contributions using the measured spectral data. This extraction has been performed by the ALEPH Collaboration [20,21], who obtain, for the sum of vector and axial vector ud $D=6$ contributions, a value seven times smaller than the central value obtained in Ref. [4], but compatible with it, within the estimated theoretical errors. With this empirical information as input, and the realization that the difference of the ud and us $D=6$ contributions should be further suppressed by the approximate $SU(3)_F$ flavor symmetry, the numerical value of the combined $D=6$ contribution, though not well determined, becomes completely negligible numerically, and hence is neglected in the analysis which follows.

IV. MODIFIED EXTRACTION OF m_s

In what follows we detail the input required to numerically evaluate the contributions listed in the previous section. We employ, from the various ALEPH fits for $a = \alpha_s(m_\tau)/\pi$, that obtained from the analysis of the non-strange modes alone, since the slow convergence of the longitudinal contributions in the strange case makes that extraction unreliable. In quoting values for m_s , we will also run the value extracted at scale m_τ down to a scale 1 GeV^2 , using the recently determined four-loop β [24] and γ [25] functions, in order to provide comparisons to other results for m_s [15–17,26,27], all of which are quoted at that lower scale. Other necessary inputs, apart from the longitudinal vector contribution, to be discussed in detail below, are the light quark mass ratio $m_s/\hat{m} = 24.4 \pm 1.5$ [28] [where $\hat{m} = (m_u + m_d)/2$] and

$$\begin{aligned}
-2m_s \langle \bar{s}s \rangle & = \left(\frac{m_s}{\hat{m}} \right) \left(\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \right) (-2\hat{m} \langle \bar{u}u \rangle) \\
& = \left(\frac{m_s}{\hat{m}} \right) \left(\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \right) f_\pi^2 m_\pi^2,
\end{aligned} \tag{15}$$

where, following Refs. [15,16], we take the ratio of strange to light quark condensates to lie between 0.7 and 1.

The crucial piece of input, however, which can, in principle at least, allow us to evade the problems with the longitudinal $D=2$ terms, is the full longitudinal vector contribution, $\Sigma_D[R_{us;V}^{(0);D}]$. This can be obtained straightforwardly from any model (or empirical determination) of the spectral function of the strangeness-changing scalar channel since the longitudinal us vector current spectral function is simply $1/s^2$ times that of the scalar channel. The latter spectral function begins at $K\pi$ threshold, the next open channel being $K\pi\pi\pi$ [experimentally, however, the $K\pi$ channel is purely elastic below $(1.3 \text{ GeV})^2$ [29]]. Given that the $K_0^*(1430)$ has a branching ratio of $93 \pm 4 \pm 9\%$ to $K\pi$ [30], it is safe to assume that the $K\pi$ mode dominates the scalar spectral function out to $s \sim 2 \text{ GeV}^2$. This observation is particularly relevant because the $K\pi$ contribution to the spectral function is determined by the timelike $K\pi$ scalar form factor, which satisfies an Omnes representation [15]. Using as input the experimental value of the form factor determined in K_{l3} , together with the measured $K\pi$ phase shifts, it is thus, in principle, possible to determine the $K\pi$ contribution to the physical scalar spectral function [15,17]. In practice, since the $K\pi$ phases have only been measured out to $s = (1.7 \text{ GeV})^2$, certain assumptions are required about the behavior of the phase beyond this point. In Ref. [17] it has been assumed that the asymptotic value, π , of the phase (associated with the known asymptotic $1/s$ -dependence of the scalar form factor) is achieved for all $s > (1.7 \text{ GeV})^2$. At present, neither theoretical nor experimental checks of this assumption exist, though the experimental phase does approach the asymptotic value as $s \rightarrow (1.7 \text{ GeV})^2$. One may also check that, if for example the phase were to make an excursion into the third quadrant as one passed through the $K_0^*(1950)$ region, and then return to π , the longitudinal spectral integral relevant to τ decay would be increased by less than $\sim 10\text{--}15\%$, producing a decrease of m_s of less than $\sim 2\%$. Were the onset of the asymptotic behavior to occur significantly beyond this point, however, the scalar form factor obtained in Ref. [17] could be significantly altered, even at low s . This difficulty is, of course, potentially surmountable in the future. Lacking any additional information at present, and in light of the discussion above, we will tentatively accept the assumption of Ref. [17], and hence consider the $K\pi$ portion of the spectral function, and hence the full scalar spectral function below $\sim 2 \text{ GeV}^2$, to be as determined there.

Even if this assumption is correct, however, this leaves the problem that one expects additional channels to become increasingly important as s is increased beyond the location of the $K_0^*(1430)$. Since the $K_0^*(1950)$ $K\pi$ branching fraction is only $52 \pm 14\%$ [30], it is likely that employing only the $K\pi$ contribution to the full spectral function will become a questionable approximation significantly before one has reached the end of the range of the spectral integral relevant to τ decay. This likelihood is reinforced by the observation that, attempting to perform a standard Borel transformed QCD sum rule analysis of the scalar channel using only the

$K\pi$ portion of the spectral function determined in Ref. [17], one finds no stability plateau for the ‘‘extracted’’ strange quark mass in the range $s \sim 3 \text{ GeV}^2$ [31]. Since

$$-2 \sum_D [R_{us;V}^{(0);D}] = 12\pi^2 S_{EW} \times \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - 2 \frac{s}{m_\tau^2}\right)^2 \left(4 \frac{\rho_{us}^s(s)}{m_\tau^2 s}\right) \quad (16)$$

where $\rho_{us}^s(s)$ is the spectral function of the scalar strange channel, such an underestimate of the spectral function corresponds to an *underestimate* of the full longitudinal contribution to τ decay, and hence [because of the overall sign (+) with which the contribution in Eq. (16) enters $R_{ud} - R_{us}$] to an *overestimate* of m_s . While the weight function entering the spectral integral in Eq. (16) has a double zero at $s = m_\tau^2$, this does not help the situation as much as one might hope: although the maximum of the weight function occurs for $s/m_\tau^2 = 1/3$, the weight has decreased by only a factor of 2 from its maximum value for $s/m_\tau^2 = 2/3$. At present there is, thus, no satisfactory extension of the full spectral function beyond $s \sim 2 \text{ GeV}^2$, and this limits our ability to extract m_s from the hadronic τ decay data. Note, however, that, while neglect of the non- $K\pi$ portion of the spectral function leads to an overestimate of m_s when one employs the τ decay analysis, it leads to an *underestimate* of m_s when one employs the direct QCD sum rule analysis of the scalar channel. The extent to which the two different extractions of m_s (both of which depend on the longitudinal spectral function) agree thus provides some *post facto* information of the degree of reliability of the model spectral function.

Using the $K\pi$ portion of the spectral function as determined in Ref. [17], we then find

$$-2 \sum_D [R_{us;V}^{(0);D}] = .055. \quad (17)$$

One can only guess at the error on this number at present. Varying the parameters of the fit to the experimental phases as described in Ref. [17] one finds a variation of $\pm .0012$, while, as explained above, the uncertainty associated with the unknown behavior of the $K\pi$ phase above $(1.7 \text{ GeV})^2$ could easily produce uncertainties of the order of $10\text{--}15\%$ or more. By way of contrast, the model spectral function employed in Refs. [15,16] produces a value .090. Note, however, that, as pointed out in Ref. [17], the behavior of this spectral function below $s \sim 2 \text{ GeV}^2$ is incompatible with the Omnes representation and known $K\pi$ phases, and hence is unphysical.

For the experimental input to the analysis we employ the following. First, for the strange and non-strange current contributions to R_τ in Eq. (1), we employ the latest ALEPH results [13,14,21,34]

$$R_{\tau,ud} = 3.493 \pm .026$$

$$R_{\tau,us} = 0.155 \pm .008. \quad (18)$$

These are compatible with those of CLEO [32,33]; the latter collaboration, however, has not yet released an official number for the strange branching fraction [33]. For the CKM matrix elements, we take the value of V_{us} from K_{e3} data, where the extraction is under best theoretical control, and V_{ud} from three-family unitarity (there exist, for example, effective isospin-breaking electromagnetic contact interactions, which involve no explicit photons in the effective hadronic theory, and which are, therefore, not taken into account in treatments of super-allowed β decays; it is not clear how to determine the probable error associated with this neglect). With this input, and the electroweak correction from Ref. [12], we obtain

$$[R_{ud} - R_{us}]_{\text{expt}} = 0.446 \pm 0.169. \quad (19)$$

The cancellation present in Eq. (19), of course, magnifies the errors. For example, if we employed the older ALEPH value $R_{\tau,us} = 0.156$, the central value would be shifted from 0.446 to 0.426. The overall error quoted is dominated by that in R_{us} .

From Eqs. (13), (14), (15), (17) and (19), one obtains the following upper bound for m_s :

$$m_s(1 \text{ GeV}^2) < 220_{-77}^{+56} \pm 33 \text{ MeV}, \quad (20)$$

where the first set of (asymmetric) errors is associated with the experimental uncertainties in Eq. (19), and the second with the estimate above of the uncertainties due to higher order contributions in the perturbative series for the $D=2$ transverse terms.

In quoting errors in Eq. (20) we have refrained from including those due to possible shortcomings in the model longitudinal spectral function [associated with (1) the unknown high energy behavior of the $K\pi$ phase and (2) neglected $K\pi\pi\pi, \dots$ contributions to the spectral function]. We remind the reader that these corrections, if made, would produce a value *lower* than the bound shown in Eq. (20), though it is not possible to make a sensible estimate of the size of this correction at present. In contrast, the same set of corrections would serve to *raise* the value

$$125 \text{ MeV} < m_s(1 \text{ GeV}^2) < 160 \text{ MeV}, \quad (21)$$

obtained from the analysis of the scalar sum rule in Ref. [17]. The true result for m_s will, in general, lie between the values

obtained from the τ decay analysis and the scalar sum rule, as long as one uses only the $K\pi$ portion of the longitudinal spectral function in both analyses. At present, the two results, owing to the large errors, especially in Eq. (20), are compatible, but one would like to see the errors in both reduced.

To conclude, we stress that flavor breaking in hadronic τ decays can still be employed as a means of extracting m_s , despite the poor convergence of the perturbative series for the $D=2$ longitudinal terms, provided empirical input can be obtained for the longitudinal spectral function. As pointed out in Refs. [35,36], it is possible to separate the longitudinal and transverse components experimentally by analyzing the dependence of the cross-section on the angle, β , between the direction of the K and the laboratory, as seen from the hadronic rest frame. One can thus, using τ decay data alone, obtain an estimate for m_s , in the approximation that one neglects the higher multiplicity ($K\pi\pi\pi, \dots$) contributions to the longitudinal spectral function. Since measuring the $K\pi$ component of the longitudinal spectral function would simultaneously determine the $K\pi$ component of the spectral function for the scalar channel, a comparison of the m_s values extracted using τ decay and the scalar channel sum rule would set upper and lower bounds for m_s , and provide an explicit check of the reliability of the approximation of neglecting higher multiplicity intermediate states. The experimental determination of the $K\pi$ component of the longitudinal spectral function is, of course, also of interest in that it would provide implicit information on the high energy behavior of the $K\pi$ phase.

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