Vector meson dominance and the ρ meson

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We discuss the properties of vector mesons, in particular the ρ^0, in the context of the hidden local symmetry (HLS) model. This provides a unified framework to study several aspects of the low energy QCD sector. First, we show that in the HLS model the physical photon is massless, without requiring off field diagonalization. We then demonstrate the equivalence of HLS and the two existing representations of vector meson dominance, VMD1 and VMD2, at both the tree level and one loop order. Finally the S matrix pole position is shown to provide a model and process independent means of specifying the ρ mass and width, in contrast with the real axis prescription currently used in the Particle Data Group tables. [S0556-2821(99)02807-6]

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I. INTRODUCTION

There is no reliable analytic means for calculating low and medium energy strongly interacting processes with the underlying theory, QCD. Despite progress in numerical studies of QCD both via the lattice [1] and QCD-motivated models [2], pre-QCD effective Lagrangians involving the observed hadronic spectrum continue to play an important role in studies of this sector. We shall be concerned in this work with the interactions of the pseudoscalar and vector mesons as described by the hidden local symmetry (HLS) model [3].

The focus of our paper will be the ρ resonance. As the lightest and broadest of the vector octets it plays an important role in phenomenology and is presently the subject of interest as a possible indicator of chiral symmetry restoration in heavy ion collisions [4]. It also serves as a guide for physics in other sectors. As we shall see the interaction of vector mesons and photons in the HLS construction is closely analogous to the SU(2)⊗U(1) symmetry breaking of the electroweak interaction. The traditional determination of the ρ mass and width has been plagued by model dependence, which as we show can be avoided by use of the S-matrix pole, closely following developments in the study of the Z^0.

Our paper is structured as follows: we begin with a brief outline of the HLS model and discuss the generation of vector boson masses by the spontaneous breaking of the global chiral symmetry through the Higgs-Kibble (HK) mechanism [5]. In Sec. III we consider the relationship between these HK masses and the *physical* vector masses, using a two-channel propagator matrix for the photon-ρ system. This allows one to consider the effect of mixing in the dressing of the bare propagators and when this is properly considered the dressed photon is seen to be massless as required, without the need for a change of basis.

Section IV is devoted to a comparison of the two commonly used representations of vector meson dominance (VMD) (referred to hereafter as VMD1 and VMD2 following the convention of Ref. [6]), both of which can be obtained as special cases from the HLS Lagrangian of Ref. [3]. Note that VMD2 is the most commonly used version and is often in the literature, simply referred to as VMD or ''the vector dominance model'' [7]. Using the pion and kaon form factors we explicitly demonstrate their equivalence at the tree level (which is trivial) and at one-loop order, where care is required. This treatment is performed in the general case $a \neq 2$, where a is the HLS parameter [3], for which we introduce $\mathrm{VMD1}_a$ and $\mathrm{VMD2}_a$ in an obvious manner.

Finally, as the vector mesons are resonant particles, we study the effect of the large width on the determination of model independent ρ parameters. The S-matrix pole position is shown to provide a truly model-independent and, furthermore, process-independent parametrization of the ρ meson.

II. HIDDEN LOCAL SYMMETRY AND VMD

The HLS model allows us to produce a theory with vector mesons as the gauge bosons of a hidden local symmetry. These then become massive because of the spontaneous breaking of a chiral $U(3)_L \otimes U(3)_R$ global symmetry. Let us consider the chiral Lagrangian [8]

$$\mathcal{L}_{\mathrm{chiral}} = \frac{1}{4}\mathrm{Tr}[\partial_\mu F \partial^\mu F^\dagger], \tag{1}$$

where $F(x) = f_\pi U(x)$ in the usual notation. This exhibits the chiral $U(3)_L \otimes U(3)_R$ symmetry under $U \to g_L U g_R^\dagger$. We can write this in exponential form and expand

$$F(x) = f_\pi e^{2iP(x)/f_\pi} = f_\pi + 2iP(x) - 2P^2(x)/f_\pi + \cdots; \tag{2}$$

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therefore, substituting into Eq. (1) we see that the vacuum corresponds to $P=0$, $U=1$. That is, F has a non-zero vacuum expectation value which spontaneously breaks the $U(3)_L \otimes U(3)_R$ symmetry as this symmetry of the Lagrangian is not a symmetry of the vacuum. The massless Goldstone bosons contained in P, associated with the spontaneous symmetry breaking, then correspond to the perturbations about the QCD vacuum and we can think of expansions in this field as given by the Hermitian matrix $P=P^aT^a$ where the Gell-Mann matrices are normalized such that $\mathrm{Tr}[T^aT^b]=\delta^{ab}/2$. So for the pseudoscalars one has

$$P=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\pi_8+\frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\pi_8+\frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K^0} & -\sqrt{\frac{2}{3}}\pi_8+\frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}. \tag{3}$$

However, in addition to the global chiral symmetry, G, the HLS scheme includes a local symmetry, H, in Eq. (1) in the following way [3]. Let

$$U(x)\equiv\xi_L^\dagger(x)\xi_R(x) \tag{4}$$

where

$$\xi_{R,L}(x)=\exp[iS(x)/f_S]\exp[\pm iP(x)/f_\pi] \tag{5}$$

$$\xi_{R,L}(x)\to h(x)\xi_{R,L}(x)g_{R,L}^\dagger. \tag{6}$$

Note that this introduces the scalar field, $S(x)$, analogous to the pseudoscalar, $P(x)$, of Eq. (3), though $S(x)$ does not appear in the chiral field $U(x)$ of Eq. (4). The general forms of the transformations are given by $g_{L,R}=\exp(i\epsilon_{L,R}^aT^a)$ and $h(x)=\exp[i\epsilon_H^a(x)T^a]$.

One now seeks to incorporate HLS into the low energy Lagrangian in a non-trivial way, thereby introducing the lightest vector meson states [3,9]. The procedure is to first re-write $\mathcal{L}_{\mathrm{chiral}}$ explicitly in terms of the ξ components:

$$\mathcal{L}_{\mathrm{chiral}}=-\frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu\xi_L\xi_L^\dagger-\partial_\mu\xi_R\xi_R^\dagger]^2. \tag{7}$$

The Lagrangian can be gauged for both electromagnetism and the hidden local symmetry by changing to covariant derivatives

$$D_\mu\xi_{L,R}=\partial_\mu\xi_{L,R}-igV_\mu\xi_{L,R}+ie\xi_{L,R}A_\mu Q \tag{8}$$

where A_μ is the photon field, $Q=\mathrm{diag}(2/3,-1/3,-1/3)$ and $V_\mu=V_\mu^aT^a$ where V_μ^a are the vector meson fields transforming as $V_\mu\to h(x)V_\mu h^\dagger(x)+ih(x)\partial_\mu h^\dagger(x)/g$. Suppressing for brevity the space-time index μ, we can write the vector meson field matrix V_μ as

$$V=\frac{1}{\sqrt{2}}\begin{pmatrix} (\rho^0+\omega)/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & (-\rho^0+\omega)/\sqrt{2} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & \phi \end{pmatrix}. \tag{9}$$

Here we have used ideal mixing in defining the bare ω and ϕ mesons. Note that the vector meson fields $V_\mu^a=K^*,\rho,\omega,\phi$ are introduced in the role of gauge fields for $H\equiv$flavor SU(3). However, $\mathcal{L}_A\equiv\mathcal{L}_{\mathrm{chiral}}$ is independent of V, and in the HLS model a second piece of the Lagrangian, $\mathcal{L}_V$, is introduced:

$$\mathcal{L}_A=-\frac{f_\pi^2}{4}\mathrm{Tr}[(D_\mu\xi_L\xi_L^\dagger-D_\mu\xi_R\xi_R^\dagger)]^2$$

$$\mathcal{L}_V=-\frac{f_\pi^2}{4}\mathrm{Tr}[(D_\mu\xi_L\xi_L^\dagger+D_\mu\xi_R\xi_R^\dagger)]^2. \tag{10}$$

The full HLS Lagrangian is then finally defined by [3]

$$\mathcal{L}_{\mathrm{HLS}}=\mathcal{L}_A+a\mathcal{L}_V, \tag{11}$$

where we see that the HLS parameter a has now been introduced.

In the absence of the gauge fields, V_μ and A_μ, we see that Eq. (10) reduces to

$$\mathcal{L}_A=\frac{1}{2}\mathrm{Tr}[\partial_\mu P\partial^\mu P],\quad \mathcal{L}_V=\frac{f_\pi^2}{2f_S^2}\mathrm{Tr}[\partial_\mu S\partial^\mu S] \tag{12}$$

to quadratic order in bosons. In this case P and S, would both be Goldstone bosons, where $P(x)$ is associated with the spontaneous breaking of the usual, global chiral symmetry, G, of $\mathcal{L}_A$ arising from the vacuum expectation value of the $U(x)$ fields and analogously for $S(x)$. However, gauge invariance allows for their elimination. It is usual to take a special gauge, the unitary gauge, for H, for which the scalar field no longer appears, $S(x)=0$ [9] (for a discussion of the unitary gauge and spontaneously broken symmetries, see for example Sec. 12-5-3 of Ref. [10]). This is phenomenologically reasonable as no chiral partner for the pion has been observed. With this choice we have

$$\xi_L^\dagger(x)=\xi_R(x)\equiv\xi(x)=\exp[iP(x)/f_\pi]. \tag{13}$$

By demanding $S(x)=0$ the local symmetry, H, is lost, but the g transformations of the global chiral symmetry group G will regenerate the scalar field through

$$\xi(x)\to\xi'(x)=\xi(x)g^{\dagger}$$
$$=\exp[iS'(P(x),g)/f_S]\exp[iP'(x)/f_\pi]. \quad (14)$$

However, the system can still maintain the unitarity gauge condition $S(x)=0$ and global chiral symmetry, G, under the combined transformation [9]

$$\xi(x)\to\xi'(x)=h(P(x),g)\xi(x)g^{\dagger},$$
$$h(P(x),g)=\exp[-iS'(P(x),g)/f_S] \quad (15)$$

where the particular choice of local transformation, $h(P(x),g)$, ''kills'' the scalar field created by the global transformation g. The physical meaning of this is that vector fields acquire longitudinal components by ''eating'' the scalar S field through a transformation of the form (to lowest order in Goldstone fields)

$$V_\mu\to V_\mu-\frac{1}{gf_S}\partial_\mu S'. \quad (16)$$

Once the scalar field is effectively removed by the unitarity gauge choice the traditional VMD Lagrangian, that of VMD2, is obtained from an expansion of $\mathcal{L}_{\rm HLS}$ in the pion field, as per Eq. (2). The HLS model actually generalizes VMD2, through the additional parameter, a of Eq. (11); so we shall refer to the resulting Lagrangian as VMD2$_a$. It has the form

$$\mathcal{L}_{\rm VMD2a}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}V_{\mu\nu}V^{\mu\nu}-aef_\pi^2g\left[\rho+\frac{\omega}{3}-\frac{\sqrt{2}}{3}\phi\right]\cdot A$$
$$+\frac{2}{3}ae^2f_\pi^2A^2+\frac{af_\pi^2g^2}{2}(\rho^2+\omega^2+\phi^2)+af_\pi^2g^2(\rho^+\cdot\rho^-+K^{*+}\cdot K^{*-}+\overline{K}^{*0}\cdot K^{*0})$$
$$+\partial\pi^+\cdot\partial\pi^-+\partial K^+\cdot\partial K^-+\partial K^0\cdot\partial\overline{K}^0+\frac{1}{2}[\partial\pi^0\cdot\partial\pi^0+\partial\pi^8\cdot\partial\pi^8+\partial\eta^0\cdot\partial\eta^0]$$
$$+\frac{i}{2}[ag\rho+e(2-a)A]\cdot[\partial\pi^+\pi^--\partial\pi^-\pi^+]$$
$$+\frac{i}{4}[ag(\rho+\omega-\sqrt{2}\phi)+2e(2-a)A]\cdot(\partial K^+K^--\partial K^-K^+)$$
$$+\frac{iag}{4}[\rho-\omega+\sqrt{2}\phi]\cdot[\partial\overline{K}^0K^0-\partial K^0\overline{K}^0]+\frac{iag}{2\sqrt{2}}\rho^+\cdot[\partial K^0K^--\partial K^-K^0+\sqrt{2}(\partial\pi^-\pi^0-\partial\pi^0\pi^-)]$$
$$+\frac{iag}{2\sqrt{2}}\rho^-\cdot[\partial K^+\overline{K}^0-\partial\overline{K}^0K^++\sqrt{2}(\partial\pi^0\pi^+-\partial\pi^+\pi^0)]$$
$$+\frac{iag}{4}K^{*0}\cdot[\partial\pi^0\overline{K}^0-\partial\overline{K}^0\pi^0+\sqrt{2}(\partial K^-\pi^+-\partial\pi^+K^-)+\sqrt{3}(\partial\overline{K}^0\pi^8-\partial\pi^8\overline{K}^0)]$$
$$+\frac{iag}{4}\overline{K}^{*0}\cdot[\partial K^0\pi^0-\partial\pi^0K^0+\sqrt{2}(\partial\pi^-K^+-\partial K^+\pi^-)+\sqrt{3}(\partial\pi^8K^0-\partial K^0\pi^8)]$$
$$+\frac{iag}{4}K^{*-}\cdot[\partial\pi^0K^+-\partial K^+\pi^0+\sqrt{2}(\partial\pi^+K^0-\partial K^0\pi^+)+\sqrt{3}(\partial\pi^8K^+-\partial K^+\pi^8)]$$
$$+\frac{iag}{4}K^{*+}\cdot[\partial K^-\pi^0-\partial\pi^0K^-+\sqrt{2}(\partial\overline{K}^0\pi^--\partial\pi^-\overline{K}^0)+\sqrt{3}(\partial K^-\pi^8-\partial\pi^8K^-)], \quad (17)$$

where for brevity we used the notation $\rho \equiv \rho^0$. Here $F_{\mu\nu}$ and $V_{\mu\nu}$ are the standard Abelian and non-Abelian field strength tensors for the photon and vector meson octet respectively. The photon and vector mesons therefore acquire Lagrangian masses through the HK mechanism [5] with the vector mesons also obtaining longitudinal components through Eq. [16]. The HK mass generated for the vector mesons is given by

$$m_{HK}^2 \equiv M^2 = a g^2 f_\pi^2. \tag{18}$$

The choice $a=2$ [3] reduces VMD2$_a$ [Eq. (17)] to the traditional VMD2, eliminating the photon-pion contact term and reproducing the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [11] through Eq. (18).

For simplicity we shall not consider SU(3) breaking due to strangeness in the vector meson sector [12,13], which results in changes in the vector meson coupling constants and HK masses for the K^* and ϕ. One should also note that the VMD2$_a$ Lagrangian does not generate couplings of the vectors to the *singlet* component of the pseudoscalar mesons. To be perfectly clear, VMD2$_a$ is nothing but the standard HLS Lagrangian used to deduce the HLS pion form factor in Ref. [14]. It is a non-trivial extension of the traditional vector meson dominance model VMD2, since $a \neq 2$ generates not only a violation of the KSRF relation ($m_\rho^2 = 2 g^2 f_\pi^2$), but also introduces a direct coupling of the photon to the pseudoscalars.

III. DRESSED VECTOR PROPAGATORS AND THE PHOTON POLE

The physical photon is massless, but the VMD2$_a$ Lagrangian in Eq. (17) possesses a photon mass term. To find the physical results one obtains from VMD2$_a$, we need to dress the vector propagators by means of a matrix equation which accounts for the possible mixings between vector particles [15]. The matrix Dyson-Schwinger equation is given by

$$iD_{\mu\nu} = iD^0_{\mu\nu} + iD^0_{\mu\alpha} iG^{\alpha\beta} iD_{\beta\nu} \tag{19}$$

where $D_{\mu\nu}$ is the dressed propagator matrix and $D^0_{\mu\nu}$ is the bare propagator obtained from the Lagrangian,

$$D^0_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2} \right) \frac{1}{q^2 - M^2}. \tag{20}$$

Inverting Eq. (19) we obtain

$$D^{(-1)}_{\mu\nu} = D^{0(-1)}_{\mu\nu} + G_{\mu\nu}. \tag{21}$$

The self-energy matrix $G_{\mu\nu}$ is composed of two entities. The first is the polarization function $\Pi_{\mu\nu}$ generated from loop effects. As can be shown from Eq. (17), the vector mesons couple to loops only through conserved currents [13], and hence the polarization tensor is transverse

$$\Pi_{\mu\nu} \equiv (g_{\mu\nu} - q_\mu q_\nu / q^2) \Pi(q^2) \tag{22}$$

and from the node theorem [15]

$$\Pi(0) = 0. \tag{23}$$

The function $\Pi(s)$ has a branch cut along the real axis beginning at the production threshold (for the ρ this is $s = 4m_\pi^2$) and extending to infinity. The second dressing term comes from the Lagrangian mixing terms between the vector mesons and photon. For simplicity let us consider the case with only the photon and the $\rho \equiv \rho^0$ [3]; we then have

$$G_{\mu\nu} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \begin{pmatrix} \Pi_{\gamma\gamma} & \Pi_{\gamma\rho} \\ \Pi_{\rho\gamma} & \Pi_{\rho\rho} \end{pmatrix} + g_{\mu\nu} \frac{eM^2}{g} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{24}$$

where M is the HK mass of the ρ given in Eq. (18).

As we shall here only consider amplitudes such as $e^+e^- \to \pi^+\pi^-$ where the vectors couple to external conserved currents (lepton or pion) the $q_\mu q_\nu$ pieces of Eq. (21) can be ignored. So defining the scalar part of the propagator through $D_{\mu\nu}(q) \equiv g_{\mu\nu} D(q^2)$ the surviving part of the dressed propagator is given by

$$\begin{pmatrix} D_{\gamma\gamma} & D_{\gamma\rho} \\ D_{\rho\gamma} & D_{\rho\rho} \end{pmatrix} = \begin{pmatrix} e^2M^2/g^2 + \Pi_{\gamma\gamma} - s & eM^2/g + \Pi_{\gamma\rho} \\ eM^2/g + \Pi_{\rho\gamma} & M^2 + \Pi_{\rho\rho} - s \end{pmatrix}^{-1}, \tag{25}$$

where $s \equiv q^2$. A similar formalism has been discussed by Hung and Sakurai for the $\gamma - Z^0$ system in the electroweak interaction [16]. The poles, p_i, $i=1,2$, of the two channel propagator are obtained from

$$\mathrm{Det}[D^{-1}(p_i)] = 0. \tag{26}$$

The VMD2$_a$ Lagrangian given in Eq. (17) is $\mathcal{O}(\epsilon^2)$, where $\epsilon \equiv e/g$; so we work to this order in the calculation of the pole positions in the complex s plane. Equation (26) gives

$$2p = M^2(1+\epsilon^2) + \Pi_{\rho\rho} + \Pi_{\gamma\gamma} \pm \left\{ M^2(1+\epsilon^2) + \Pi_{\rho\rho} - \Pi_{\gamma\gamma} + \frac{2(\epsilon^2(\Pi_{\gamma\gamma} - \Pi_{\rho\rho})M^2 + 2\epsilon M^2 \Pi_{\gamma\rho} - \Pi_{\gamma\rho}^2)}{M^2(1+\epsilon^2) + \Pi_{\rho\rho} - \Pi_{\gamma\gamma}} + \mathcal{O}(\epsilon^3) \right\}. \tag{27}$$

In power counting with respect to $\Pi_{\rho\rho}$, we see that $\Pi_{\gamma\rho}$ and $\Pi_{\gamma\gamma}$ are intrinsically of $\mathcal{O}(\epsilon)$ and $\mathcal{O}(\epsilon^2)$ respectively: so we can further simplify:

$$2p = M^2(1+\epsilon^2) + \Pi_{\rho\rho} + \Pi_{\gamma\gamma} \pm \left\{ M^2(1+\epsilon^2) + \Pi_{\rho\rho} - \Pi_{\gamma\gamma} - \frac{2(\epsilon^2 M^2 \Pi_{\rho\rho} - 2\epsilon M^2 \Pi_{\gamma\rho} + \Pi_{\gamma\rho}^2)}{M^2 + \Pi_{\rho\rho}} + \mathcal{O}(\epsilon^3) \right\}. \tag{28}$$

Thus the poles are, to $\mathcal{O}(\epsilon^2)$,

$$p_\gamma = \Pi_{\gamma\gamma}(p_\gamma) + \frac{\epsilon^2 M^2 \Pi_{\rho\rho}(p_\gamma) - 2\epsilon M^2 \Pi_{\gamma\rho}(p_\gamma) + \Pi_{\gamma\rho}^2(p_\gamma)}{M^2 + \Pi_{\rho\rho}(p_\gamma)} \tag{29}$$

$$p_\rho = M^2(1+\epsilon^2) + \Pi_{\rho\rho}(p_\rho) - \frac{\epsilon^2 M^2 \Pi_{\rho\rho}(p_\rho) - 2\epsilon M^2 \Pi_{\gamma\rho}(p_\rho) + \Pi^2_{\gamma\rho}(p_\rho)}{M^2 + \Pi_{\rho\rho}(p_\rho)}. \tag{30}$$

Since they couple only through conserved currents, the polarization functions, $\Pi(0)=0$ [15], and we see that the photon pole resides at $q^2=0$ as required. The explicit photon mass term in Eq. (17) is canceled by the photon self-energy. Similarly the ρ pole is shifted only by corrections of $\mathcal{O}(\epsilon^2)$ by mixing with the photon. This result is as we might expect, since the vacuum state $F_0=f_\pi$ in invariant under the U(1) EM transformation [17]

$$F \to F + i\epsilon(x)[Q,F], \tag{31}$$

and massive vectors correspond to spontaneously broken symmetries. However, it is slightly more subtle than the usual case where invariance under a transformation of the form $F_0 \to F_0 + i\epsilon^a(x) T^a F_0$ requires that there be no Lagrangian mass term (as $T^a F_0 = 0$). The generation of vector meson masses in the presence of a massless photon is treated in the general case by Gottlieb [18].

HLS can therefore be used to demonstrate ''from first principles'' that the VMD2 Lagrangian is perfectly consistent with physical expectations concerning the photon and vector meson masses. However, this is only realized after the above cancellation of the photon mass. For this reason, an alternative representation of vector meson dominance, VMD1, was introduced. In the next section we examine the relationship between VMD1 and VMD2 at both tree level and one-loop order.

IV. VMD1 vs VMD2

The VMD2$_a$ Lagrangian, Eq. (17), contains a mass term for the photon. As the physical photon is massless, one might consider an alternative version with no Lagrangian mass term for the photon [19], which has been referred to as VMD1 [6,20]. We shall introduce the term VMD1$_a$ for the general version of VMD1 derived from HLS. As in the previous representation of VMD, VMD1$_a$ reduces to standard VMD1 for the special case $a=2$.

In VMD1$_a$ the photon can couple directly to the pseudoscalars and the photon–vector-meson mixing term is linear in q^2 and hence vanishes at $q^2=0$. As will be shown the two representations of VMD, VMD1$_a$ and VMD2$_a$, are related by a field transformation, and as such should be physically equivalent. At the tree level, this equivalence is complete and easy to prove; however, at the one loop level care needs to be taken to ensure all terms are included [21]. We begin with a discussion of one-loop effects in VMD, starting from the HLS Lagrangian.

A. VMD2$_a$ form factors

So far we have discussed the generation of masses and longitudinal components for the vector mesons through the HK mechanism. These masses are necessarily real valued, but are not exactly what is seen in experiment. The vector meson propagator is modified by the vacuum polarization function $\Pi(q^2)$ away from $q^2=0$, i.e.,

$$D_V(s) = \frac{1}{s - m^2_{HK}} \to \frac{1}{s - m^2_{HK} - \Pi_{VV}(s)}, \quad s \equiv q^2. \tag{32}$$

As the physical pseudoscalar fields appear as vacuum fluctuations we may assume a weak field expansion and work to first order in pseudoscalar loops [9]. The polarization functions, $\Pi_{VV}(s)$, are composed of loops, $l(PP')$, from the VPP' couplings

$$\Pi_{\rho\rho} = g^2a^2 l(\pi^+\pi^-)/4 + g^2a^2 l(K^+K^-)/16 + g^2a^2 l(K^0\bar{K}^0)/16 \tag{33}$$

$$\Pi_{\omega\omega} = g^2a^2 l(K^+K^-)/16 + g^2a^2 l(K^0\bar{K}^0)/16 \tag{34}$$

$$\Pi_{K^{*0}K^{*0}} = g^2a^2 l(K^-\pi^+)/2 + g^2a^2 l(K^0\pi^0)/4 \tag{35}$$

$$\Pi_{K^{*+}K^{*+}} = g^2a^2 l(\bar{K}^0\pi^+)/2 + g^2a^2 l(K^+\pi^0)/4 \tag{36}$$

$$\Pi_{\phi\phi} = g^2a^2 l(K^+K^-)/8 + g^2a^2 l(\bar{K}^0K^0)/8 \tag{37}$$

where we have factored out the couplings leaving only the generic loop integrals involving the appropriate pseudoscalars. Note that we have not included the anomalous $VPPP$ and VVP couplings [22] in the polarization functions, since these effects are expected to be rather small. Numerical studies show that the $\rho \to \pi\pi \to \rho$ contribution to the real part of $\Pi_{\rho\rho}$ is a few percent of $m_{HK} \equiv M$, but the contribution to the ω physical mass from $\omega \to 3\pi \to \omega$ is negligible [23]. Similar behavior is observed in the imaginary part, as the ρ width (generated by $\mathrm{Im}\,\Pi_{\rho\rho}$) is *much* larger than that of the ω or ϕ. We may conclude that the two pion loop is the dominant influence[1] on the mass shift and that only the ρ physical mass is significantly different from its HK value. This is supported by the observation that the Bando-Kugo-Yamawaki relation between the HK masses of the ω, K^* and ϕ in models of the SU(3) breaking [12],

$$m_\omega m_\phi = m^2_{K^*} \tag{38}$$

holds to $0.1\% - 0.4\%$ (for charged or neutral K^*), whereas the observed ρ and ω masses differ by a few percent despite having identical HK masses.

In a similar manner, we can obtain the hadronic contributions loop corrections to vector meson mixing, and can derive relations between them and the polarization functions. The pure pseudoscalar loop mixings are given by

[1] See, however, Ref. [21] for a discussion of the ϕ meson.

$$\Pi_{\rho\omega}=\frac{g^2a^2}{16}[l(K^+K^-)-l(K^0\overline{K^0})] \tag{39}$$

$$\Pi_{\rho\phi}=-\frac{\sqrt{2}g^2a^2}{16}[l(K^+K^-)-l(K^0\overline{K^0})] \tag{40}$$

$$\Pi_{\phi\omega}=-\frac{\sqrt{2}g^2a^2}{16}[l(K^+K^-)+l(K^0\overline{K^0})]. \tag{41}$$

Note that if isospin invariance is assumed, as it has been so far, only ω-ϕ mixing survives and we notice that

$$\Pi_{\phi\phi}=2\Pi_{\omega\omega}=-2\sqrt{2}\Pi_{\phi\omega}. \tag{42}$$

Hence ρ-ω mixing is only allowed in the present analysis if isospin violation is admitted through allowing $l(K^+K^-)\neq l(K^0\overline{K^0})$. However, once isospin violation is allowed, we should in principle consider the possibility of additional isospin violating effects arising from u-d splitting in our HLS Lagrangian. For example, to first order in isospin violation the possibility of an $\omega\pi\pi$ coupling must be considered in ρ-ω mixing [24,25]. Such a term arises naturally via loop effects when higher order pseudoscalar terms, for example $VPPPP$, are considered [26]. Alternatively it could be generated through SU(2) breaking in $\mathcal{L}_V$ itself, analogous to the existing studies of SU(3) breaking [12,13]. So far, in studies of the HLS model such explicit isospin breaking effects have been considered only for the anomalous sector [27,28]. For an investigation of isospin violation in a chiral meson theory see Ref. [29].

The possibility of a direct coupling of the photon to the pseudoscalar field allows for a loop-induced photon–vector-meson mixing, which in the isospin limit (where $m_{K^0}=m_{K^+}$) gives

$$\Pi_{\rho\gamma}=\frac{e(2-a)}{ag}\Pi_{\rho\rho},\quad \Pi_{\omega\gamma}=\frac{e(2-a)}{ag}\Pi_{\omega\omega},$$

$$\Pi_{\phi\gamma}=-\frac{e(2-a)}{\sqrt{2}ag}\Pi_{\phi\phi}. \tag{43}$$

Note that these mixings vanish if $a=2$ or $q^2=0$. Since these vanish at $q^2=0$, we see that the previous proof of the masslessness of the photon remains unaffected.

From the Lagrangian we can now write expressions for the pion and kaon form factors, defined from the Feynman amplitude via $\mathcal{M}_{\gamma PP}=-eF_P$. In the following we shall write the HK mass simply as M [see Eq. (18)]. We can go from the tree level result to the one-loop result by replacing the ordinary Lagrangian interaction pieces by an effective Lagrangian for the photon pseudoscalar couplings. At the tree level, where the propagators are simply $D_V^0=1/(s-M^2)$, the form factors are given by

$$F_\pi^{\rm tree}(s)=1-a/2-aM^2D_\rho^0/2=1-asD_\rho^0/2 \tag{44}$$

$$F_{K^+}^{\rm tree}(s)=1-a/2-aM^2[D_\rho^0+D_\omega^0/3+2D_\phi^0/3]/4$$
$$=1-(as/4)[D_\rho^0+D_\omega^0/3+2D_\phi^0/3] \tag{45}$$

$$F_{K^0}^{\rm tree}(s)=-(aM^2/4)[D_\rho^0-D_\omega^0/3-2D_\phi^0/3]. \tag{46}$$

We note here that in this SU(3) flavor symmetric tree result $F_{K^0}^{\rm tree}(s)=0$.

We now consider the effects of loops. As the resonant structure of the vector mesons, generated by loops, is an important part of the phenomenology we might expect loops to play an important role. The contact term between the photon and the pseudoscalars when $a\neq 2$ induces a one loop contribution to the photon–vector-meson vertex, which we can introduce through the effective interaction Lagrangian

$$\mathcal{L}_{\gamma\rho}^{\rm eff}=-\frac{eM^2}{g}+\Pi_{\gamma\rho}=-\frac{eM^2}{g}+\frac{e(2-a)}{ag}\Pi_{\rho\rho}$$

$$\mathcal{L}_{\gamma\omega}^{\rm eff}=-\frac{eM^2}{3g}+\Pi_{\gamma\omega}=-\frac{eM^2}{3g}+\frac{e(2-a)}{ag}\Pi_{\omega\omega}$$

$$\mathcal{L}_{\gamma\phi}^{\rm eff}=\frac{\sqrt{2}eM^2}{3g}+\Pi_{\gamma\phi}=\frac{\sqrt{2}eM^2}{3g}-\frac{e(2-a)}{\sqrt{2}ag}\Pi_{\phi\phi}. \tag{47}$$

In a similar way the tree-level propagators are replaced by their one-loop forms to give $D_V^1=1/[s-M_V^2-\Pi_{VV}(s)]$. Similarly the vector mesons can now mix through pseudoscalar loops. For example, the effect of ρ-ω mixing, which is important for the pion form factor, is easily incorporated by replacing the ρ propagators by

$$D_\rho^1\rightarrow D_\rho^1[1+(f_{\omega\gamma}/f_{\rho\gamma})A_{\omega\pi\pi}D_\omega^1]. \tag{48}$$

As it is not realistic, in the sense of data fitting [24], to draw a distinction between isospin violation occurring through ρ-ω mixing and intrinsic isospin violation in the $\omega\pi\pi$ vertex (which could could either be present in the original Lagrangian or be generated by loop effects [26]), Eq. (48) uses $A_{\omega\pi\pi}$, the ''effective mixing function'' that absorbs both effects to couple the ω to the 2 pion final state, through the ρ [24,25],

$$A_{\omega\pi\pi}=-3500\pm 300\ {\rm MeV}^2. \tag{49}$$

Using Eqs. (42) and (47) the leading resonant terms in the one-loop expressions for the VMD2$_a$ model form factors are given by

$$F_\pi^{1\ {\rm loop}}(s)=[(1-a/2)s-M^2]D_\rho^1[1+(f_{\omega\gamma}/f_{\rho\gamma})A_{\omega\pi\pi}D_\omega^1] \tag{50}$$

$$F_{K^+}^{1\ {\rm loop}}(s)=(1/2)[(1-a/2)s-M^2][D_\rho^1+(1/3)D_\omega^1$$
$$+(2/3)D_\phi^1-(2\sqrt{2}/3)D_\omega^1\Pi_{\omega\phi}D_\phi^1] \tag{51}$$

$$F_{K^0}^{1\,\mathrm{loop}}(s)=(1/2)[(1-a/2)s-M^2][D_\rho^1-(1/3)D_\omega^1 -(2/3)D_\phi^1+(2\sqrt{2}/3)D_\omega^1\Pi_{\omega\phi}D_\phi^1]. \quad (52)$$

These expressions are one loop in the sense that each of the components in the tree-level amplitudes has been replaced by its one-loop generalization. Without the vector meson mixing loops, these expressions agree identically with the tree level results for $a=2$ (as the photon decouples from the pseudoscalars and pseudoscalar loops cannot be generated). We see that the tree level results are protected from major corrections at the one-loop level for $a\simeq 2$, since vector meson mixing effects are small [30].

B. VMD1$_a$ form factors

Although the photon mass term appearing in Eq. (17) does not result in a massive photon, it is tempting to choose a field redefinition in which it does not appear [3]. The transformation most resembling that used to eliminate the photon mass term in the electro-weak theory actually removes the pointlike coupling of the photon to the ρ and thus obscures VMD [3,31]. We shall consider an alternative transformation [19] that generates ''current mixing'' [32] between the photon and vector mesons,

$$A\to A,\quad \rho\to\rho+\epsilon A,\quad \omega\to\omega+(\epsilon/3)A,$$

$$\phi\to\phi-(\epsilon\sqrt{2}/3)A, \quad (53)$$

to derive VMD1$_a$ from VMD2$_a$ [Eq. (17)]. Making the substitutions of Eq. (53) in Eq. (17) we find that the resulting Lagrangian for VMD1$_a$ is given by

$$\begin{aligned}
\mathcal{L}_{\mathrm{VMD1}a}=&-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}V_{\mu\nu}V^{\mu\nu}-\epsilon s\left[\rho+\frac{\omega}{3}-\frac{\sqrt{2}}{3}\phi\right]\cdot A\\
&+\frac{1}{2}af_\pi^2g^2(\rho^2+\omega^2+\phi^2)+af_\pi^2g^2(\rho^+\cdot\rho^-+K^{*+}\cdot K^{*-}+\bar{K}^{*0}\cdot K^{*0})+\partial\pi^+\cdot\partial\pi^-\\
&+\partial K^+\cdot\partial K^-+\partial K^0\cdot\partial\bar{K}^0+\frac{1}{2}[\partial\pi^0\cdot\partial\pi^0+\partial\pi^8\cdot\partial\pi^8+\partial\eta^0\cdot\partial\eta^0]\\
&+\frac{i}{2}[ag\rho+2eA]\cdot[\partial\pi^+\pi^--\partial\pi^-\pi^+]\\
&+\frac{i}{4}[ag(\rho+\omega-\sqrt{2}\phi)+4eA]\cdot(\partial K^+K^--\partial K^-K^+)\\
&+\frac{iag}{4}[\rho-\omega+\sqrt{2}\phi]\cdot[\partial\bar{K}^0K^0-\partial K^0\bar{K}^0]+\frac{iag}{2\sqrt{2}}\rho^+\cdot[\partial K^0K^--\partial K^-K^0+\sqrt{2}(\partial\pi^-\pi^0-\partial\pi^0\pi^-)]\\
&+\frac{iag}{2\sqrt{2}}\rho^-\cdot[\partial K^+\bar{K}^0-\partial\bar{K}^0K^++\sqrt{2}(\partial\pi^0\pi^+-\partial\pi^+\pi^0)]\\
&+\frac{iag}{4}K^{*0}\cdot[\partial\pi^0\bar{K}^0-\partial\bar{K}^0\pi^0+\sqrt{2}(\partial K^-\pi^+-\partial\pi^+K^-)+\sqrt{3}(\partial\bar{K}^0\pi^8-\partial\pi^8\bar{K}^0)]\\
&+\frac{iag}{4}\bar{K}^{*0}\cdot[\partial K^0\pi^0-\partial\pi^0K^0+\sqrt{2}(\partial\pi^-K^+-\partial K^+\pi^-)+\sqrt{3}(\partial\pi^8K^0-\partial K^0\pi^8)]\\
&+\frac{iag}{4}K^{*-}\cdot[\partial\pi^0K^+-\partial K^+\pi^0+\sqrt{2}(\partial\pi^+K^0-\partial K^0\pi^+)+\sqrt{3}(\partial\pi^8K^+-\partial K^+\pi^8)]\\
&+\frac{iag}{4}K^{*+}\cdot[\partial K^-\pi^0-\partial\pi^0K^-+\sqrt{2}(\partial\bar{K}^0\pi^--\partial\pi^-\bar{K}^0)+\sqrt{3}(\partial K^-\pi^8-\partial\pi^8K^-)]. \quad (54)
\end{aligned}$$

This removes the photon mass term as well as the constant $V\cdot A$ terms from Eq. (17) and introduces a new coupling of the photon to the pseudoscalar fields

$$\mathcal{L}_{\gamma P}^{\mathrm{VMD1}}=ieA^\mu[(\partial_\mu\pi^+\pi^--\partial_\mu\pi^-\pi^+)+(\partial_\mu K^+K^--\partial_\mu K^-K^+)]. \quad (55)$$

The field redefinition in Eq. (53) also introduces the $q^2=0$ vanishing mixing of the photon and vector mesons through the vector meson kinetic terms (this is discussed in detail in Ref. [6]). Together with Eq. (55) we then have the effective interaction Lagrangian

$$\mathcal{L}_{\gamma V}^{\mathrm{VMD1}_a}=\frac{e}{g}\left[\left(-s+\frac{2}{a}\Pi_{\rho\rho}\right)\rho_\mu+\left(-\frac{s}{3}+\frac{2}{a}\Pi_{\omega\omega}\right)\omega_\mu+\left(\frac{\sqrt{2}s}{3}-\frac{\sqrt{2}}{a}\Pi_{\phi\phi}\right)\phi_\mu\right]A^\mu. \tag{56}$$

We are now in a position to write the VMD1$_a$ form factors, as derived from the HLS model. The tree level result is

$$F_\pi^{\mathrm{tree}}(s)=1-asD_\rho^0/2 \tag{57}$$

$$F_{K^+}^{\mathrm{tree}}(s)=1-(as/4)[D_\rho^0+D_\omega^0/3+2D_\phi^0/3] \tag{58}$$

$$F_{K^0}^{\mathrm{tree}}(s)=-(as/4)[D_\rho^0-D_\omega^0/3-2D_\phi^0/3]. \tag{59}$$

We see that Eqs. (44)–(46) and Eqs. (57)–(59) are identical [recalling that Eqs. (52) and similarly (59) are zero] and that the two representations of VMD are thus equivalent, while the leading resonant terms for the one-loop result are given by

$$F_\pi^{1\ \mathrm{loop}}(s)=[(1-a/2)s-M^2]D_\rho^1[1+(f_{\omega\gamma}/f_{\rho\gamma})A_{\omega\pi\pi}D_\omega^1] \tag{60}$$

$$F_{K^+}^{1\ \mathrm{loop}}(s)=(1/2)[(1-a/2)s-M^2][D_\rho^1+(1/3)D_\omega^1+(2/3)D_\phi^1-(2\sqrt{2}/3)D_\omega^1\Pi_{\omega\phi}D_\rho^1] \tag{61}$$

$$F_{K^0}^{1\ \mathrm{loop}}(s)=(1/2)[(1-a/2)s-M^2][D_\rho^1-(1/3)D_\omega^1-(2/3)D_\phi^1+(2\sqrt{2}/3)D_\omega^1\Pi_{\omega\phi}D_\rho^1], \tag{62}$$

which are identical to Eqs. (50)–(52).

It is worthwhile now to briefly explain how the one loop results have been obtained. The simplest case is the pion form factor of VMD1$_a$. From the effective Lagrangian one has (ignoring, for simplicity, the isospin violating piece)

$$F_\pi^{1\ \mathrm{loop}}(s)=1-[as/2-\Pi_{\rho\rho}(s)]D_\rho^1. \tag{63}$$

The procedure we have followed is to eliminate the $\Pi_{\rho\rho}(s)$ term appearing in the numerator of Eq. (63). Doing this also cancels the leading one associated with the pointlike coupling of the photon to the pion current,

$$F_\pi^{1\ \mathrm{loop}}(s)=\{[s-M^2-\Pi(s)]-[as/2-\Pi(s)]\}D_\rho^1=[s(1-a/2)-M^2]D_\rho^1. \tag{64}$$

The other expressions are obtained similarly. Recalling $\Pi(0)=0$ [15] all form-factor normalization conditions are clearly maintained ($A_{\omega\pi\pi}$ in our model is generated only by loops). We can now clearly see the agreement between the one loop results for VMD1$_a$ and VMD2$_a$. This highlights the importance of including loop effects in VMD studies.

So we see the equivalence of VMD1$_a$ and VMD2$_a$, which we might suspect on principle as the physics must be independent of the choice of fields. However, this is only true if one works consistently to one loop order in the form factors, including loop-induced photon-vector-meson mixings. In addition to this, care must be taken to distinguish between the HK and the physical mass of the ρ. Such effects were not included in Ref. [14], leading to somewhat different results for data fits using VMD1$_a$ and VMD2$_a$ (note that in this paper the ''HLS'' fits refer to what we call VMD2$_a$ here). For the kaon form factors it would be interesting to more systematically include and study the effects of SU(3) symmetry breaking on this analysis [12]. One should note, however, that including SU(3) symmetry breaking effects will not spoil the effect of the transformation, given in Eq. (53), on the VMD Lagrangian.

It should also be noted that the initial investigation into generalizations of VMD1 and VMD2 performed in Ref. [14] can be interpreted as evidence for preferring VMD1$_a$/VMD2$_a$ to VMD1/VMD2. Indeed, in order to get an acceptable fit to the form factor data, it was necessary to introduce an additional term $(1+\epsilon)=1.167\pm0.008$ affecting the resonance contribution in the standard VMD1 form factor and interpret this as an *ad hoc* universality violation.[2] We might, in light of the above discussion, make a connection between $(1+\epsilon)$ and the HLS $a/2$. Similarly, the ''HLS'' fit of Ref. [14], i.e., a VMD2$_a$ type fit, finds significant excursion from the traditional $a=2$, namely, $a/2=1.182\pm0.008$. One should then remark that the results of Ref. [14] give additional evidence for the underlying equivalence of VMD1$_a$ and VMD2$_a$. It would be very interesting to refit the data with the full 1-loop expressions for the form factors given here.

[2]Here ϵ is merely a fitting parameter, not e/g as defined earlier.

V. MASSES, WIDTHS AND *S* MATRIX POLES

Because $m_{\rm HK}$ is above threshold, the vector meson pole, p_V, ''slips down'' onto the second Riemann sheet to a location on the complex plane satisfying

$$p_V - m_{HK}^2 - \Pi_{VV}(p_V) = 0. \tag{65}$$

In some sense the discussion of masses and widths for unstable particles is an artifact of convention, because only the pole position, p_V, defined in Eq. (65) is process independent and hence physically meaningful [33]. Model dependence is, of course, unavoidable when comparing various resonance models and their different associated vector meson masses and widths.

However, model-independent definitions of the masses and widths can be conveniently defined from the pole position through the identification

$$p_V \equiv \overline{m}^2 - i\overline{m}\overline{\Gamma} \tag{66}$$

(or through the slightly old-fashioned form $\sqrt{p_V} \equiv \tilde{m} - i\tilde{\Gamma}/2$). For narrow resonances such as the ω and ϕ, the pole is directly accessed by the choice of a momentum independent Breit-Wigner form in the fit to data and the masses and widths generally quoted correspond very closely to those in Eq. (66). However, as a result of the large width of the ρ, one generally sees an attempt to model the possible s dependence of Π_{VV} modifying the naive Breit Wigner (see, e.g., Refs. [34,35]). For the pion form factor, where ρ parameters are extracted, this amounts to the isospin pure contribution (that not including the ω [25]) being given by a function

$$F_\pi(s) = \frac{f(0)}{f(s)} h(s), \quad h(0) = 1 \tag{67}$$

where $f(s)$ has the appropriate cut beginning at $s = 4m_\pi^2$ and the function $h(s)$ models any small deviations from elasticity. Though this in itself is perfectly acceptable, the quoted values of mass and width are defined through their values on the real axis [35,36]

$$\mathrm{Re}\, f(m_{\rm Re}^2) = 0, \quad \mathrm{Im}\, f(m_{\rm Re}^2) = -i m_{\rm Re}\Gamma_{\rm Re}, \quad m_{\rm Re}, \Gamma_{\rm Re} \in \Re \tag{68}$$

to emulate the Breit-Wigner form factor along the real axis [37],

$$\lim_{s \to m_{\rm Re}^2} f(s) = s - m_{\rm Re}^2 + i m_{\rm Re}\Gamma_{\rm Re}. \tag{69}$$

These are not tied to the pole position and are hence very model dependent. Indeed, phenomenology suggests specific forms for the function $f(s)$, which in turn influence the parameter values for the function $f(s)$. This has led to considerable difficulties in comparing ρ parameters obtained by various authors as has been noted by the Particle Data Group (PDG) as early as 1971 (see p. S62 of Ref. [38]) and is also discussed in the most recent Particle Data listing (see Eidelman's review on p. 364 of Ref. [39]). Indeed the differences between many quoted values for the ρ mass and width arise merely from model dependence, rather than discrepancies in physical data, as discussed by Gardner and O'Connell [25] for the $e^+e^- \to \pi^+\pi^-$ data of Barkov *et al.* [36].

This is a well-known problem for the parametrization of broad resonances. The model dependence of the determination of the mass and width of the $\Delta(1236)$ was discussed in the 1971 Particle Data listing [38]. The solution, through the model independent pole prescription, was then provided in the 1972 listing [40]. Höhler's recent review of this (see p. 624 of Ref. [39]) concludes that in contrast to the conventional real axis parameters, the pole positions have a well-defined relation to *S* matrix theory and are generally more useful. Our aim here is to present the pole results for various recent fits to ρ data in one place to fully highlight both the model independence and process independence.

In Ref. [25] generalized form factors were used to obtain very good fits to the pion form-factor data [36] for four choices of model (A, B, C and D). In this manner the function $f(s)$ is fixed, in terms of fitting parameters, along the real s axis. The resulting real axis determinations of the mass and width showed a significant model dependence. However, in Ref. [41] the model independence of the pole prescription can clearly be seen in the quoted values of $\overline{m}_\rho$ and $\overline{\Gamma}_\rho$ as defined in Eq. (66) by solving $f(p_V) = 0$ for each of the four models. To give an example this procedure, model A of Ref. [25] uses $f(s) = as^2 + bs + c + f_1(s)$, where $f_1(s)$ is a specified *complex* function. The parameters a, b, c, are fixed by the fit to data, allowing one to find the zero of $f(s)$ through a step iteration on the complex plane. The correlations associated with the parameters a, b and c allow one to determine the statistical errors associated with $\overline{m}_\rho$ and $\overline{\Gamma}_\rho$ (for a discussion of error analysis see, for example, Ref. [42]). The results are given in Table I along with the masses and widths obtained from the traditional real-axis prescription (which show a large model dependence).

We have also repeated this procedure for other determinations of the ρ mass and width; although without knowing correlations of the fitting parameters, we have simply solved $f(p_V) = 0$ using MATHEMATICA [43] and do not quote statistical errors for $\overline{m}_\rho$ and $\overline{\Gamma}_\rho$. This is done for the fits of Ref. [14] as well as the original fit of Barkov *et al.* [36] and the recent fit for the *charged* ρ seen in hadronic τ decay [44]. The latter results should allow one to look for any statistically significant isospin violation in the pole position. As $m_{\rho(770)^0} - m_{\rho(770)^\pm}$ is quoted by the PDG [39], a more detailed study would be of considerable interest for a determination of this mass splitting based on model-independent quantities.

Not only are the results of the pole determination virtually model independent, they agree completely with previous pole determinations from both $e^+e^- \to \pi^+\pi^-$ [45] *and* π-π [46,47] scattering data (further demonstrating process independence). We see in Table I that the central values for the ρ mass and width, defined by the standard real-axis prescription, cover ranges of 763–780 and 141–157 respectively, while those from the pole prescription lie in the ranges 756–759 and 140–145 respectively (all figures quoted in MeV). This illustrates the model independence of the ρ pole location. Indeed, if one takes into account the errors on the pole

TABLE I. Extraction of ρ masses and widths for both the real part of the complex pole [$\overline{m}_\rho$ and $\overline{\Gamma}_\rho$ defined in Eq. (66)] and the standard real axis [m_ρ and Γ_ρ as given in Eq. (68)] prescriptions. Fits I (Ref. [36]), II (Refs. [25,41]), III (Ref. [45]) and IV (the results of the unitarized fits of Ref. [14]; see Table 2 therein) use the $e^+e^- \to \pi^+\pi^-$ data obtained in I. Fit V (Ref. [47], adjusted to $\overline{m}$ from $\tilde{m}$) uses $\pi\pi$ scattering data and fit VI (Ref. [44] using two different fitting functions; see Table 3 therein) gives results for the charged ρ obtained from τ decay.

Fit	$\chi^2/N_{\rm DF}$	m_ρ (MeV)	Γ_ρ (MeV)	$\overline{m}_\rho$ (MeV)	$\overline{\Gamma}_\rho$ (MeV)
I	129/133	775.9 ± 1.1	150.5 ± 3.0	758.3	145.0
IIA	68/75	763.1 ± 3.9	153.8 ± 1.2	756.3 ± 1.2	141.9 ± 3.1
IIB	66/76	771.3 ± 1.3	156.2 ± 0.4	757.0 ± 1.0	141.7 ± 3.0
IIC	67/76	773.9 ± 1.2	157.0 ± 0.4	757.0 ± 1.0	141.7 ± 3.0
IID	68/76	773.9 ± 1.2	146.9 ± 3.4	757.0 ± 1.0	141.7 ± 3.0
III				757.5 ± 1.5	142.5 ± 3.5
IV[VMD1$_a$]	65/77	774.7 ± 0.7	147.1 ± 1.6	758.5	142.8
IV[VMD2]	81/77	780.4 ± 0.7	155.4 ± 2.0	760.0	146.8
IV[HLS(VMD2$_a$)]	65/77	775.2 ± 0.7	147.7 ± 1.5	759.0	143.4
IV[WCCWZ]	61/76	$770.9^{+1.8}_{-1.5}$	$140.6^{+3.2}_{-2.9}$	759.2	139.6
V		771	147	756	142
VI[KS]	81/65	774.9 ± 0.9	144.2 ± 1.5	759.1	139.9
VI[GS]	54/65	776.4 ± 0.9	150.5 ± 1.6	758.8	145.0

parameters, when they are available, the difference between the various determinations of the pole position is never greater than 2σ, i.e., not statistically significant.

The power of this method is useful not only to finding a model independent means of parametrizing a broad resonance such as the ρ mesons, but also in the standard model for the Z^0 where the data has very high precision and $\Pi_{VV}(s)$ can actually be theoretically calculated [48,49] (see also the Z^0 review in Ref. [39]). The use of S-matrix poles is also potentially important for the Higgs boson [50] and CP violation in the kaon system [51].

VI. CONCLUSION

We have applied a field theoretic treatment to the vector meson sector and, in particular, the ρ. Starting with the HLS model allows one to produce a Lagrangian for the low energy hadronic sector, which, when carefully treated, is seen to be consistent with physical expectations. Working with this model, we have demonstrated the automatic preservation of the massless photon and the equivalence of VMD1$_a$ and VMD2$_a$ (the HLS generalizations of VMD1 and VMD2). Previous fits of the pion form factor, preferred VMD1$_a$ relative to the standard VMD1, which may have further phenomenological consequences. Finally, as predicted from S-matrix theory, we have shown that the determination of the complex pole position of the ρ meson from a large number of existing fits to data is model and process independent. This will be of particular use for the comparison of ρ parameter determinations from different experiments and for exploring the usefulness and possible limitations of different realizations of vector meson dominance.

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