Pressure and flow within the umbilical vessels

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Abstract

This thesis considers a fluid dynamic study of blood flow within the umbilical vessels of the human maternal-fetal circulatory system. In particular it evaluates the efficacy of the umbilical coiling index (UCI) in predicting the blood vessel pressure drop, and develops clinically viable indices for the quantification of umbilical cord flow resistance. A numerical approach is developed employing the finite element method via the open-source `c++` code `oomph-lib`, available from the University of Manchester. This permits the description of an umbilical vessel geometry as a fully three-dimensional rigid tube in order to consider a wide range of cords subject to steady and unsteady boundary conditions.

The thesis is composed of three main studies. The first concerns steady incompressible Newtonian flow through model geometries representative of the umbilical vessels. It is found that the UCI is unable to distinguish between cords of significantly varying pressure and flow characteristics, which are typically determined by the vessel curvature, torsion and length. Larger scale geometric non-uniformities superposed over the inherent coiling, including cords exhibiting width and/or local UCI variations as well as loose true knots, typically produce a small effect on the total pressure drop. Crucially, this implies that a helical geometry of mean coiling may be used to determine the steady vessel pressure drop through a more complex cord. The presence of vessel constriction, however, drastically increases the steady pressure drop and alters the flow profile.

The second study provides an analysis of pulsatile incompressible Newtonian flow through arterial geometries. The steady pressure drop is found to approximate the time-averaged value with high accuracy over a wide range of arteries. Furthermore, the relative peak systolic pressure measured over the period is found to remain virtually constant and approximately 25% below the equivalent straight pipe value for a large range of non-straight vessels. Interestingly, this suggests that the coiled structure dampens extreme pressures within the arterial cycle and may provide another possible evolutionary benefit to the coiled structure of the cord.

Having shown the UCI ineffective at predicting the vessel pressure drop, the third and final study presents two alternative indices for the diagnosis of cord pathology based on steady calculations. The umbilical pressure index, PX, and flow index, QX, quantify the deviation of a cord geometry from typical conditions by considering the steady pressure and flow-rate, respectively. These indices are calculated based on a combination of empirical and interpolated numerical data and require only one additional geometric measurement to the calculation of the UCI; namely the cord width. Together the indices provide a non-invasive measure of the flow-resistance inherent to a particular cord geometry, and allow comparison with typical values in pregnancy. Further testing of the indices is required to determine their efficacy in a clinical setting, however, their simple and robust nature ensures that they are promising candidates.

Signed Statement

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Dedication

For my mother, Rosemary.