



Maximum Consensus with Mixed Integer Linear Programming

by

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Abstract

Maximum consensus is fundamentally important in computer vision as a criterion for robust estimation, where the goal is to estimate the parameters of a model of best fit. It is computationally demanding to solve such problems exactly. Instead, conventional methods employ randomised sample-and-test techniques to approximate a solution, which fail to guarantee the optimality of the result. This thesis makes several contributions towards solving the maximum consensus problem exactly in the context of Mixed Integer Linear Programming. In particular, two preprocessing techniques aimed at improving the speed and efficiency of exact methods are proposed.

Declaration

I, Yang Heng KEE, certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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Date

Preface

During my studies at the University of Adelaide from 2014 to 2016, I have contributed to the production of one conference paper. Chapter 4 of this thesis is based on the content presented in the aforementioned paper, with details of the paper listed below:

Conference Publication

- T.-J. Chin, Y.H. Kee, A. Eriksson, and F. Neumann. Guaranteed outlier removal with mixed integer linear programs. In *2016 IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*.

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