

PUBLISHED VERSION

C. D. Froggatt, R. Nevzorov, H. B. Nielsen, A. W. Thomas

On the smallness of the cosmological constant

Nuclear and Particle Physics Proceedings, 2016; 273-275:1465-1470

© 2015 Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

Originally published at:

<http://doi.org/10.1016/j.nuclphysbps.2015.09.237>

PERMISSIONS

<http://creativecommons.org/licenses/by/4.0/>



Attribution 4.0 International (CC BY 4.0)

This is a human-readable summary of (and not a substitute for) the [license](#).

[Disclaimer](#)



You are free to:

Share — copy and redistribute the material in any medium or format

Adapt — remix, transform, and build upon the material

for any purpose, even commercially.

The licensor cannot revoke these freedoms as long as you follow the license terms.

Under the following terms:



Attribution — You must give **appropriate credit**, provide a link to the license, and **indicate if changes were made**. You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use.

No additional restrictions — You may not apply legal terms or **technological measures** that legally restrict others from doing anything the license permits.

<http://hdl.handle.net/2440/103298>



On the smallness of the cosmological constant

C. D. Froggatt^a, R. Nevzorov^{b,c}, H. B. Nielsen^d, A. W. Thomas^b

^a*School of Physics and Astronomy, University of Glasgow, Glasgow, UK*

^b*ARC Centre of Excellence for Particle Physics at the Tera-scale,*

School of Chemistry and Physics, University of Adelaide, Adelaide SA 5005, Australia

^c*Institute for Theoretical and Experimental Physics, Moscow, 117218, Russia*

^d*The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark*

Abstract

In $N = 1$ supergravity the scalar potential of the hidden sector may have degenerate supersymmetric (SUSY) and non-supersymmetric Minkowski vacua. In this case local SUSY in the second supersymmetric Minkowski phase can be broken dynamically. Assuming that such a second phase and the phase associated with the physical vacuum are exactly degenerate, we estimate the value of the cosmological constant. We argue that the observed value of the dark energy density can be reproduced if in the second vacuum local SUSY breaking is induced by gaugino condensation at a scale which is just slightly lower than Λ_{QCD} in the physical vacuum. The presence of a third degenerate vacuum, in which local SUSY and electroweak (EW) symmetry are broken near the Planck scale, may lead to small values of the quartic Higgs self-coupling and the corresponding beta function at the Planck scale in the phase in which we live.

Keywords: Supergravity, Cosmological constant, Higgs boson

1. Introduction

It is commonly expected that the exploration of TeV scale physics at the LHC may lead to the discovery of new physics phenomena beyond the Standard Model (SM) that can shed light on the stabilisation of the EW scale. Indeed, if the SM is embedded in a more fundamental theory characterized by a much larger energy scale (e.g. the Planck scale $M_{Pl} \approx 10^{19}$ GeV) than the EW scale, then due to the quadratically divergent radiative corrections, the Higgs boson tends to acquire a mass of the order of the larger energy scale; excessive fine-tuning is then required to keep the Higgs mass around the observed value ~ 125 GeV.

Despite the compelling arguments for physics beyond the SM, no signal or indication of its presence has been detected at the LHC so far. Besides there are some reasons to believe that the SM is extremely fine-tuned. Indeed, astrophysical and cosmological observations indicate that there is a dark energy spread all over the

Universe which constitutes 70% – 73% of its energy density. A fit to the recent data shows that its value is $\rho_\Lambda \sim 10^{-123} M_{Pl}^4 \sim 10^{-55} M_Z^4$ [1, 2]. At the same time much larger contributions should come from electroweak symmetry breaking ($\sim 10^{-67} M_{Pl}^4$) and QCD condensates ($\sim 10^{-79} M_{Pl}^4$). The contribution of zero-modes is expected to push the vacuum energy density even higher up to $\sim M_{Pl}^4$, i.e.

$$\rho_\Lambda \approx \sum_{bosons} \frac{\omega_b}{2} - \sum_{fermions} \frac{\omega_f}{2} \quad (1)$$

$$= \int_0^\Omega \left[\sum_b \sqrt{|\vec{k}|^2 + m_b^2} - \sum_f \sqrt{|\vec{k}|^2 + m_f^2} \right] \frac{d^3\vec{k}}{2(2\pi)^3} \sim -\Omega^4,$$

where the m_b and m_f are the masses of bosons and fermions while $\Omega \sim M_{Pl}$. Because of the cancellation needed between the contributions of different condensates to ρ_Λ , the smallness of the cosmological constant should be regarded as a fine-tuning problem.

Here, instead of trying to alleviate fine-tuning, we postulate the exact degeneracy of different vacua. The presence of such degenerate vacua was predicted by the so-called Multiple Point Principle (MPP) [3, 4], according to which Nature chooses values of coupling constants such that many phases of the underlying theory should coexist. This scenario corresponds to a special (multiple) point on the phase diagram of the theory where these phases meet. The vacuum energy densities of these different phases are degenerate at the multiple point.

The MPP applied to the SM implies that the Higgs effective potential which is given by

$$V_{eff}(H) = m^2(\phi)H^\dagger H + \lambda(\phi)(H^\dagger H)^2, \quad (2)$$

where H is a Higgs doublet and ϕ is a norm of the Higgs field, i.e. $\phi^2 = H^\dagger H$, has two rings of minima in the Mexican hat with the same vacuum energy density [5]. The radius of the little ring equals the EW vacuum expectation value (VEV) of the Higgs field, whereas the second vacuum is taken to be at the Planck scale. The degeneracy of these vacua can be achieved only if

$$\lambda(M_{Pl}) \simeq 0, \quad \beta_\lambda(M_{Pl}) \simeq 0, \quad (3)$$

where $\beta_\lambda = \frac{d\lambda(\phi)}{d \log \phi}$ is the beta-function of $\lambda(\phi)$, which depends on $\lambda(\phi)$ itself, gauge $g_i(\phi)$ and top quark Yukawa $g_t(\phi)$ couplings. Using MPP conditions (3) the values of the top quark and Higgs masses were computed [5]

$$M_t = 173 \pm 5 \text{ GeV}, \quad M_H = 135 \pm 9 \text{ GeV}. \quad (4)$$

The value of the Higgs mass specified above basically coincides with the lower bound on the Higgs mass in the SM that comes from the vacuum stability constraint. In previous papers the application of the MPP to the two Higgs doublet extension of the SM was considered [6, 7, 8]. In particular, it was argued that the MPP can be used as a mechanism for the suppression of the flavour changing neutral current and CP-violation effects [8].

Of critical importance here is the observation that the mass of the Higgs boson discovered at the LHC is very close to the theoretical lower bound on M_H in the SM mentioned above. Thus the parameters of the SM can be extrapolated all the way up to M_{Pl} without any inconsistency. Recently, using the extrapolation of the SM parameters up to the Planck scale with full 3-loop RGE precision, it has been shown that (see [9])

$$\lambda(M_{Pl}) = -0.0143 - 0.0066 \left(\frac{M_t}{\text{GeV}} - 173.34 \right) + 0.0018 \left(\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \right) + 0.0029 \left(\frac{M_H}{\text{GeV}} - 125.15 \right). \quad (5)$$

The computed value of $\beta_\lambda(M_{Pl})$ also tends to be very small, so that the MPP conditions (3) are basically satisfied.

The success of the MPP in predicting the Higgs mass [5] suggests that we might also use it for explaining the extremely low value of the dark energy density. In principle the smallness of this energy density could be related to an almost exact symmetry. However at this moment none of the available generalizations of the SM provides a satisfactory explanation for the smallness of the cosmological constant. An exact global supersymmetry (SUSY) ensures zero value for the energy density at the minimum of the potential of the scalar fields. Since superpartners of quarks and leptons have not been observed yet, supersymmetry must be broken. In general the breakdown of SUSY induces a huge and positive contribution to the total vacuum energy density of order M_S^4 , where M_S is the SUSY breaking scale. The non-observation of superpartners of quarks and leptons implies that $M_S \gtrsim 1 \text{ TeV}$.

Here the MPP assumption is adapted to models based on ($N = 1$) local supersymmetry – supergravity (SUGRA), in order to provide an explanation for the small deviation of the cosmological constant from zero.

2. Dark energy density in SUGRA models

The full ($N = 1$) SUGRA Lagrangian is specified in terms of an analytic gauge kinetic function $f_a(\phi_M)$ and a real gauge-invariant Kahler function $G(\phi_M, \phi_M^*)$, which depend on the chiral superfields ϕ_M . The function $f_a(\phi_M)$ determines the kinetic terms for the fields in the vector supermultiplets and the gauge coupling constants $Re f_a(\phi_M) = 1/g_a^2$, where the index a designates different gauge groups. The Kahler function is a combination of two functions

$$G(\phi_M, \phi_M^*) = K(\phi_M, \phi_M^*) + \ln |W(\phi_M)|^2, \quad (6)$$

where $K(\phi_M, \phi_M^*)$ is the Kahler potential whose second derivatives define the kinetic terms for the fields in the chiral supermultiplets. $W(\phi_M)$ is the complete analytic superpotential of the SUGRA model. Here we shall use standard supergravity mass units: $\frac{M_{Pl}}{\sqrt{8\pi}} = 1$.

The SUGRA scalar potential can be presented as a sum of F - and D -terms

$$V_{SUGRA}(\phi_M, \phi_M^*) = V_F(\phi_M, \phi_M^*) + V_D(\phi_M, \phi_M^*), \quad (7)$$

where the F - and D -parts are given by

$$\begin{aligned} V_F(\phi_M, \phi_M^*) &= \sum_{M, \bar{N}} e^G \left(G_M G^{\bar{M}\bar{N}} G_{\bar{N}} - 3 \right), \\ V_D(\phi_M, \phi_M^*) &= \frac{1}{2} \sum_a (D^a)^2, \\ D^a &= g_a \sum_{i,j} \left(G_i T_{ij}^a \phi_j \right), \end{aligned} \quad (8)$$

$$G_M \equiv \partial_M G \equiv \partial G / \partial \phi_M, \quad G_{\bar{M}} \equiv \partial_{\bar{M}} G \equiv \partial G / \partial \phi_M^*.$$

In Eq. (8) g_a is the gauge coupling constant associated with the generator T^a of the gauge transformations. The matrix $G^{M\bar{N}}$ is the inverse of the Kahler metric $K_{\bar{N}M}$, i.e.

$$G_{\bar{N}M} \equiv \partial_{\bar{N}} \partial_M G = \partial_{\bar{N}} \partial_M K \equiv K_{\bar{N}M}.$$

In order to break supersymmetry in ($N = 1$) SUGRA models, a hidden sector is introduced. It contains superfields z_i , which are singlets under the SM $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge group. It is assumed that the superfields of the hidden sector interact with the observable ones only by means of gravity. If, at the minimum of the scalar potential, hidden sector fields acquire VEVs so that at least one of their auxiliary fields

$$F^M = e^{G/2} G^{M\bar{P}} G_{\bar{P}} \quad (9)$$

is non-vanishing, then local SUSY is spontaneously broken. At the same time a massless fermion with spin $1/2$ – the goldstino, which is a combination of the fermionic partners of the hidden sector fields giving rise to the breaking of SUGRA, is swallowed up by the gravitino which thereby becomes massive $m_{3/2} = \langle e^{G/2} \rangle$. This phenomenon is called the super-Higgs effect.

Usually the vacuum energy density at the minimum of the SUGRA scalar potential (7)–(8) tends to be negative. To show this, let us suppose that, the function $G(\phi_M, \phi_M^*)$ has a stationary point, where all derivatives $G_M = 0$. Then it is easy to check that this point is also an extremum of the SUGRA scalar potential. In the vicinity of this point local supersymmetry remains intact while the energy density is $-3 \langle e^G \rangle$. It implies that the vacuum energy density must be less than or equal to this value. Therefore, in general, an enormous fine-tuning must be imposed, in order to keep the total vacuum energy density in SUGRA models around the observed value of the cosmological constant.

3. MPP inspired SUGRA models

The successful implementation of the MPP in ($N = 1$) supergravity requires us to assume the existence of a vacuum in which the low-energy limit of the considered theory is described by a pure supersymmetric model in flat Minkowski space [10, 11, 12, 13, 14, 15]. According to the MPP this vacuum and the physical one must be degenerate. Since the vacuum energy density of supersymmetric states in flat Minkowski space is just zero, the cosmological constant problem is thereby solved to first approximation. Such a second vacuum is realised only if the SUGRA scalar potential has a minimum where $m_{3/2} = 0$. The corresponding minimum

is achieved when the superpotential W for the hidden sector and its derivatives vanish [10], i.e.

$$W(z_m^{(2)}) = 0, \quad (10)$$

$$\left. \frac{\partial W(z_i)}{\partial z_m} \right|_{z_m = z_m^{(2)}} = 0 \quad (11)$$

where $z_m^{(2)}$ denote VEVs of the hidden sector fields in the second vacuum.

The simplest Kahler potential and superpotential that satisfy conditions (10) and (11) can be written as [10]

$$K(z, z^*) = |z|^2, \quad W(z) = m_0(z + \beta)^2. \quad (12)$$

The hidden sector of this SUGRA model contains only one singlet superfield z . If the parameter $\beta = \beta_0 = -\sqrt{3} + 2\sqrt{2}$, the corresponding SUGRA scalar potential possesses two degenerate minima with zero energy density at the classical level. One of them is a supersymmetric Minkowski minimum that corresponds to $z^{(2)} = -\beta$. In the other minimum of the SUGRA scalar potential ($z^{(1)} = \sqrt{3} - \sqrt{2}$) local supersymmetry is broken; so it can be associated with the physical vacuum. Varying the parameter β around β_0 one can obtain a positive or a negative contribution from the hidden sector to the total energy density of the physical vacuum. Thus β can be fine-tuned so that the physical and second vacua are degenerate.

In general Eq. (10) represents the extra fine-tuning associated with the presence of the supersymmetric Minkowski vacuum. This fine-tuning can be to some extent alleviated in the no-scale inspired SUGRA models with broken dilatation invariance [11, 12, 13]. Let us consider a model with two hidden sector supermultiplets T and z . These superfields transform differently under the imaginary translations ($T \rightarrow T + i\beta$, $z \rightarrow z$) and dilatations ($T \rightarrow \alpha^2 T$, $z \rightarrow \alpha z$). If the superpotential and Kahler potential of the hidden sector of the SUGRA model under consideration are given by

$$\begin{aligned} K(T, z) &= -3 \ln \left[T + \bar{T} - |z|^2 \right], \\ W(z) &= \kappa \left(z^3 + \mu_0 z^2 \right), \end{aligned} \quad (13)$$

then the corresponding tree level scalar potential of the hidden sector is positive definite

$$V(T, z) = \frac{1}{3(T + \bar{T} - |z|^2)^2} \left| \frac{\partial W(z)}{\partial z} \right|^2, \quad (14)$$

so that the vacuum energy density vanishes near its global minima. The scalar potential (14) possesses two

minima at $z = 0$ and $z = -\frac{2\mu_0}{3}$ that correspond to the stationary points of the hidden sector superpotential. In the first vacuum, where $z = -\frac{2\mu_0}{3}$, local supersymmetry is broken so that the gravitino becomes massive

$$m_{3/2} = \frac{4\kappa\mu_0^3}{27\left\langle\left(T + \bar{T} - \frac{4\mu_0^2}{9}\right)^{3/2}\right\rangle}. \quad (15)$$

and all scalar particles get non-zero masses. Since one can expect that $\mu_0 \lesssim M_{Pl}$ and $\kappa \lesssim 1$, SUSY is broken in this vacuum near the Planck scale. In the second minimum, with $z = 0$, the superpotential of the hidden sector vanishes and local SUSY remains intact, so that the low-energy limit of this theory is described by a pure SUSY model in flat Minkowski space.

Of course, the inclusion of perturbative and non-perturbative corrections to the Lagrangian of the no-scale inspired SUGRA model, which should depend on the structure of the underlying theory, are expected to spoil the degeneracy of vacua inducing a huge energy density in the vacuum where SUSY is broken. Moreover in this SUGRA model the mechanism for the stabilization of the VEV of the hidden sector field T remains unclear. The model discussed above should therefore be considered as a toy example only. This SUGRA model demonstrates that, in ($N = 1$) supergravity, there might be a mechanism which ensures the vanishing of vacuum energy density in the physical vacuum. This mechanism may also lead to a set of degenerate vacua with broken and unbroken supersymmetry, resulting in the realization of the multiple point principle.

4. Estimation of the dark energy density

Let us now assume that a phenomenologically viable SUGRA model with degenerate vacua of the type discussed in the previous section is realised in Nature. This implies that there are at least two vacua which are exactly degenerate. In the first (physical) vacuum the spontaneous breakdown of SUSY takes place near the Planck scale. We shall assume that in the second vacuum SUSY remains intact and only vector supermultiplets, which correspond to the unbroken gauge symmetry in the hidden sector, remain massless. These vector supermultiplets can give rise to the breakdown of SUSY in the second vacuum which is caused by the formation of a gaugino condensate induced in the hidden sector at the scale $\Lambda_{SQCD} \ll M_{Pl}$.

In this context it is worth to recall that the gaugino condensate does not actually break global SUSY. Nonetheless, we can have a non-trivial dependence of

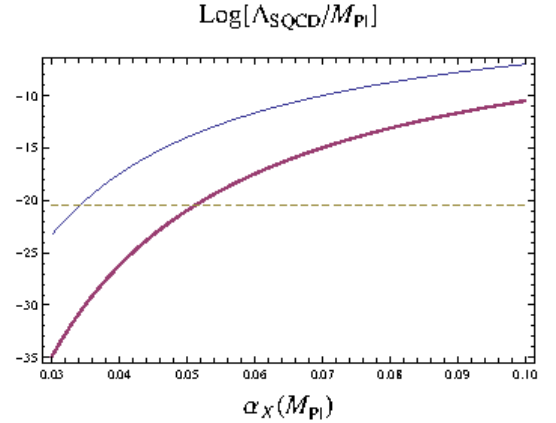


Figure 1: The value of $\log[\Lambda_{SQCD}/M_{Pl}]$ versus $\alpha_X(M_{Pl})$. The thin and thick solid lines correspond to the $SU(3)$ and $SU(2)$ gauge symmetries, respectively. The horizontal line is associated with the value of Λ_{SQCD} that leads to the observed value of the cosmological constant.

the gauge kinetic function $f_X(z_m)$ on the hidden sector superfields z_m . Such a dependence leads to auxiliary fields corresponding to the hidden fields z_m

$$F^{z_m} \propto \frac{\partial f_X(z_k)}{\partial z_m} \bar{\lambda}_a \lambda_a + \dots \quad (16)$$

acquiring non-zero VEVs, which are set by $\langle \bar{\lambda}_a \lambda_a \rangle \simeq \Lambda_{SQCD}^3$. Thus it is only via the effect of a non-renormalisable term that this gaugino condensate causes the SUSY breaking. As a consequence the SUSY breaking scale $M_S^2 \sim \frac{\Lambda_{SQCD}^3}{M_{Pl}}$ is many orders of magnitude lower than Λ_{SQCD} [16]. Therefore the formation of a gaugino condensate gives rise to a vacuum energy density

$$\rho_\Lambda^{(2)} \sim M_S^4 \sim \frac{\Lambda_{SQCD}^6}{M_{Pl}^2}. \quad (17)$$

The postulated exact degeneracy of vacua then requires that the physical vacuum, in which SUSY is broken near the Planck scale, has the same energy density as the phase where local supersymmetry breakdown is induced by the gaugino condensate. From Eq. (17) it follows that the observed value of the cosmological constant can be reproduced if Λ_{SQCD} is relatively close to Λ_{QCD} in the physical vacuum [15], i.e.

$$\Lambda_{SQCD} \sim \Lambda_{QCD}/10. \quad (18)$$

Although there is no compelling theoretical reason to expect a priori that the two scales Λ_{SQCD} and Λ_{QCD} should be relatively close or related, Λ_{SQCD} and M_{Pl} can

be considered as the two most natural choices for the scale of dimensional transmutation in the hidden sector.

For each given value of the gauge coupling $\alpha_X(M_{Pl})$ of the hidden sector gauge interactions one can estimate the energy scale, Λ_{SQCD} , where the supersymmetric QCD-like interactions become strong in the second vacuum. The analytical solution of the one-loop renormalization group equation for $\alpha_X(Q)$ is given by

$$\frac{1}{\alpha_X(Q)} = \frac{1}{\alpha_X(M_{Pl})} + \frac{b_X}{4\pi} \ln \frac{M_{Pl}^2}{Q^2}, \quad (19)$$

where b_X is the one-loop beta function of the hidden sector gauge interactions. In particular, $b_X = -9$ and -6 for the $SU(3)$ and $SU(2)$ gauge groups respectively. Setting $\frac{1}{\alpha_X(\Lambda_{SQCD})} \rightarrow 0$ one finds

$$\Lambda_{SQCD} = M_{Pl} \exp \left[\frac{2\pi}{b_X \alpha_X(M_{Pl})} \right]. \quad (20)$$

The dependence of Λ_{SQCD} on $\alpha_X(M_{Pl})$ is shown in Fig. 1. From Fig. 1 it follows that the value of Λ_{SQCD} diminishes with decreasing $\alpha_X(M_{Pl})$. The measured value of the dark energy density is reproduced when $\alpha_X(M_{Pl}) \simeq 0.051$ in the case of the model based on the $SU(2)$ gauge group and $\alpha_X(M_{Pl}) \simeq 0.034$ in the case of the $SU(3)$ SUSY gluodynamics. These values of $\alpha_X(M_{Pl})$ correspond to $g_X(M_{Pl}) \simeq 0.801$ and $g_X(M_{Pl}) \simeq 0.654$ respectively. Thus in the case of the $SU(3)$ model the gauge coupling $g_X(M_{Pl})$ is just slightly larger than the value of the QCD gauge coupling at the Planck scale, i.e. $g_3(M_{Pl}) = 0.487$ (see Ref. [9]), in the physical vacuum where we live.

5. Implications for Higgs phenomenology

Now we shall discuss the possible implications of SUGRA models with degenerate vacua for Higgs phenomenology. The presence of two degenerate vacua does not rule out the possibility that there can exist another vacuum with the same energy density where EW symmetry is broken near the Planck scale. Since in this third vacuum the Higgs VEV is somewhat close to M_{Pl} one must consider the interaction of the Higgs and hidden sector fields. Thus the full scalar potential can be written:

$$V = V_{hid}(z_m) + V_0(H) + V_{int}(H, z_m) + \dots, \quad (21)$$

where $V_{hid}(z_m)$ is the part of the scalar potential associated with the hidden sector, $V_0(H)$ is the part of the full scalar potential that depends on the Higgs field only and $V_{int}(H, z_m)$ corresponds to the interaction of the SM

Higgs field with the hidden sector fields. Here we assume that in the observable sector only one Higgs doublet acquires a non-zero VEV and all other observable fields can be ignored in the first approximation.

Let us first consider the limit when $V_{int}(H, z_m) \rightarrow 0$. In this case, the Planck scale VEV of the Higgs field would not lead to substantial variations of the VEVs of hidden sector fields. Because of this the gauge couplings and $\lambda(M_{Pl})$ in the third and physical vacua are expected to be approximately equal. Then the requirement of the degeneracy of all three vacua should lead to the conditions (3).

It is worth noting that in general in the vacuum with the Planck scale VEV of the Higgs doublet the VEVs of the hidden sector fields should be very different from those in the physical vacuum when $V_{int}(H, z_m)$ is not vanishingly small. Therefore in the third vacuum the gauge couplings at the Planck scale, $\lambda(M_{Pl})$ and $m^2(M_{Pl})$, that in general depend on the VEVs of the hidden sector fields, are not the same as in the physical vacuum.

Nonetheless the interactions between the SM Higgs doublet and the hidden sector fields can be rather weak near the third vacuum, i.e. $V_{int}(H, z_m) \ll M_{Pl}^4$, if the VEV of the Higgs field is considerably smaller than M_{Pl} (say $\langle H \rangle \sim M_{Pl}/10$) and the couplings of the SM Higgs doublet to the hidden sector fields are suppressed. Then the VEVs of the hidden sector fields in the third and physical vacua can be basically identical. As a result the gauge couplings and $\lambda(M_{Pl})$ in the third vacuum remain almost the same as in the physical vacuum. On the other hand, the absolute value of m^2 in the Higgs effective potential should - although fixed to be small at the weak scale according to experiment - be much larger in the third vacuum. Indeed, in the physical vacuum $|m^2|$ can be small because of the cancellation of different contributions. However, in this case even small variations of the VEVs of the hidden sector fields should spoil such cancellations. Although in the third vacuum $|m^2|$ is expected to be many orders of magnitude larger than the EW scale, it can still be substantially smaller than M_{Pl}^2 and $\langle H^\dagger H \rangle$ if the interactions between the SM Higgs doublet and hidden sector fields are weak. In this limit the value of $V_{hid}(z_m)$ in the third vacuum remains almost the same as in the physical vacuum where $V_{hid}(z_m^{(1)}) \ll M_{Pl}^4$. This means that the requirement of the degeneracy of vacua implies that in the third vacuum $\lambda(M_{Pl})$ and $\beta_\lambda(M_{Pl})$ are approximately zero. Since in this case the couplings in the third and physical vacua are basically identical, the presence of such a third vacuum results in the predictions (3) for $\lambda(M_{Pl})$ and $\beta_\lambda(M_{Pl})$ in the physical vacuum.

6. Conclusions

In $N = 1$ supergravity (SUGRA) supersymmetric (SUSY) and non-supersymmetric Minkowski vacua originating in the hidden sector can be degenerate. This allows for consistent implementation of the multiple point principle (MPP) which implies that such vacua have the same vacuum energy densities. We presented SUGRA models where the MPP assumption is realised. In the supersymmetric phase in flat Minkowski space SUSY may be broken dynamically inducing a tiny vacuum energy density which can be assigned, by virtue of MPP, to all other phases including the one in which we live. We have argued that SUGRA models with degenerate vacua can lead to the measured value of the dark energy density, as well as small values of $\lambda(M_{Pl})$ and $\beta_\lambda(M_{Pl})$. This is realised in a scenario where the existence of at least three exactly degenerate vacua is postulated. In the first (physical) vacuum SUSY is broken near the Planck scale and the small value of the cosmological constant appears as a result of the fine-tuned precise cancellation of different contributions. In the second vacuum the breakdown of local supersymmetry is induced by gaugino condensation, which is formed at a scale which is slightly lower than Λ_{QCD} in the physical vacuum. In the third vacuum local SUSY and EW symmetry are broken near the Planck scale.

It is worth noting that our estimate of the tiny value of the cosmological constant makes sense only if the vacua mentioned above are degenerate to very high accuracy. In principle, a set of approximately degenerate vacua can arise if the underlying theory allows only vacua which have a similar order of magnitude of space-time 4-volumes at the final stage of the evolution of the Universe (This may imply the possibility of violation of the principle that the future can have no influence on the past [3]). Since the sizes of these volumes are determined by the expansion rates of the corresponding vacua associated with them, only vacua with similar order of magnitude of dark energy densities are allowed. Thus all vacua are degenerate to the accuracy of the value of the cosmological constant in the physical vacuum.

7. Acknowledgements

This work was supported by the University of Adelaide and the Australian Research Council through the ARC Center of Excellence in Particle Physics at the Terascale and through grant LFO 99 2247 (AWT). HBN thanks the Niels Bohr Institute for his emeritus status.

CDF thanks Glasgow University and the Niels Bohr Institute for hospitality and support.

References

- [1] WMAP Collaboration, First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Preliminary maps and basic results, *Astrophys. J. Suppl.* 148 (2003) 1, [astro-ph/0302207].
- [2] WMAP Collaboration, First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Determination of cosmological parameters, *Astrophys. J. Suppl.* 148 (2003) 175, [astro-ph/0302209].
- [3] D. L. Bennett, C. D. Froggatt, H. B. Nielsen, Nonlocality as an explanation for fine tuning and field replication in nature, [hep-ph/9504294].
- [4] D. L. Bennett, H. B. Nielsen, Predictions for nonAbelian fine structure constants from multicriticality, *Int. J. Mod. Phys. A* 9 (1994) 5155, [hep-ph/9311321].
- [5] C. D. Froggatt, H. B. Nielsen, Standard model criticality prediction: Top mass 173 ± 5 GeV and Higgs mass 135 ± 9 GeV, *Phys. Lett. B* 368 (1996) 96, [hep-ph/9511371].
- [6] C. D. Froggatt, L. V. Laperashvili, R. B. Nevzorov, H. B. Nielsen, M. Sher, Implementation of the multiple point principle in the two-Higgs doublet model of type II, *Phys. Rev. D* 73 (2006) 095005, [hep-ph/0602054].
- [7] C. D. Froggatt, R. Nevzorov, H. B. Nielsen, D. Thompson, Fixed point scenario in the Two Higgs Doublet Model inspired by degenerate vacua, *Phys. Lett. B* 657 (2007) 95, [arXiv:0708.2903 [hep-ph]].
- [8] C. D. Froggatt, R. Nevzorov, H. B. Nielsen, D. Thompson, On the origin of approximate custodial symmetry in the Two-Higgs Doublet Model, *Int. J. Mod. Phys. A* 24 (2009) 5587, [arXiv:0806.3190 [hep-ph]].
- [9] D. Buttazzo, G. Degrandi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, A. Strumia, Investigating the near-criticality of the Higgs boson, *JHEP* 1312 (2013) 089, [arXiv:1307.3536 [hep-ph]].
- [10] C. D. Froggatt, L. V. Laperashvili, R. Nevzorov, H. B. Nielsen, Cosmological constant in SUGRA models and the multiple point principle, *Phys. Atom. Nucl.* 67 (2004) 582, [hep-ph/0310127].
- [11] C. D. Froggatt, L. V. Laperashvili, R. Nevzorov, H. B. Nielsen, No-scale supergravity and the multiple point principle, [hep-ph/0411273].
- [12] C. D. Froggatt, R. Nevzorov, H. B. Nielsen, On the smallness of the cosmological constant in SUGRA models, *Nucl. Phys. B* 743 (2006) 133, [hep-ph/0511259].
- [13] C. D. Froggatt, R. Nevzorov, H. B. Nielsen, Smallness of the cosmological constant and the multiple point principle, *J. Phys. Conf. Ser.* 110 (2008) 072012, [arXiv:0708.2907 [hep-ph]].
- [14] C. D. Froggatt, R. Nevzorov, H. B. Nielsen, Dark Energy density in models with Split Supersymmetry and degenerate vacua, *Int. J. Mod. Phys. A* 27 (2012) 1250063, [arXiv:1103.2146 [hep-ph]].
- [15] C. D. Froggatt, R. Nevzorov, H. B. Nielsen, A. W. Thomas, Cosmological constant in SUGRA models with Planck scale SUSY breaking and degenerate vacua, *Phys. Lett. B* 737 (2014) 167, [arXiv:1403.1001 [hep-ph]].
- [16] H. P. Nilles, Gaugino Condensation and Supersymmetry Breakdown, *Int. J. Mod. Phys. A* 5 (1990) 4199.