

**Supersymmetric Dark Matter in the NMSSM and  
 $E_6$ SSM**

**Sophie Underwood**

**A thesis submitted for the degree of Doctor of Philosophy**

**The University of Adelaide**

**May 2016**



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# Declaration

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# Abstract

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Supersymmetric extensions of the standard model may resolve the outstanding dark matter problem by producing viable dark matter candidates, including a stable weakly interacting particle called a neutralino. The next-to-minimal supersymmetric standard model (NMSSM) is first explored with a scan of the parameter space for neutralino-hadron scattering using an updated value for the strange quark sigma commutator.

This is followed by an extensive exploration of the parameter space of the  $E_6$ -inspired supersymmetric standard model ( $E_6$ SSM). It is demonstrated that this model still provides neutralino dark matter candidates that may be detected in the near-future by upcoming experiments, despite tightening experimental constraints.





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